

Light scattering from spin waves in quantum dots and wires

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Plan of the lectures:

Part 1: Basics of spin dynamics

Magnetic interactions
Dynamic of spins
Spin waves
Brillouin light scattering

Part 2: Spin wave confinement in dots&wires

Spin wave modes in dots& wires
Lateral quantization
Spin wave wells

Part 3: Static and dynamic interaction between dots&wires

Collective static properties
Anisotropic interaction
Dynamic mode coupling

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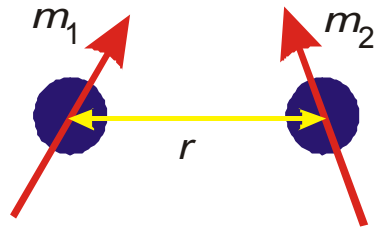
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Light scattering from spin waves in quantum dots and wires

Part I

Basics of spin dynamics

Magnetic interactions



Exchange interaction :

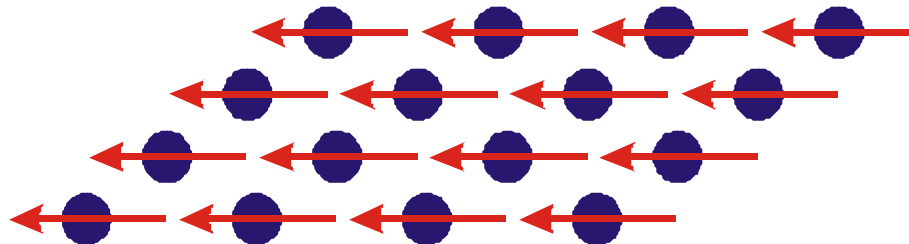
$$E = -Jm_1 \cdot m_2$$

Magnetic dipolar interaction:

$$E = \frac{m_1 \cdot m_2}{r^3} - \frac{3(m_1 \cdot r)(m_2 \cdot r)}{r^5}$$



$J > 0$, ferromagnet



Shape anisotropy:

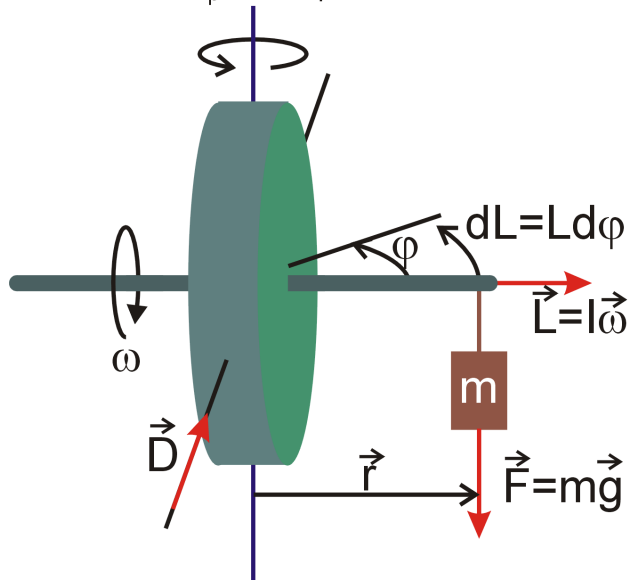
due to dipole interaction the spins in a film are „in-plane“

Zeeman energy : $E = -B \cdot m$

Precession of a spin

precession frequency

$$\omega_p = d\phi/dt$$



angular momentum $\vec{L} = I \vec{\omega}$

acting torque \vec{D} :

$$\vec{D} = \vec{r} \times \vec{F} = \vec{r} \times m\vec{g} \stackrel{!}{=} \frac{d\vec{L}}{dt} \Rightarrow d\vec{L} \parallel \vec{D}$$

since $\vec{D} \perp \vec{L}$: $|L| = \text{const}$ and $|dL| = L d\phi$

$$\Rightarrow |D| = \frac{d|L|}{dt} = L \frac{d\phi}{dt} = L\omega_p$$

$$\Rightarrow \boxed{\omega_p = \frac{D}{L} = \frac{mrg}{I\omega}} \quad \text{frequency of precession}$$

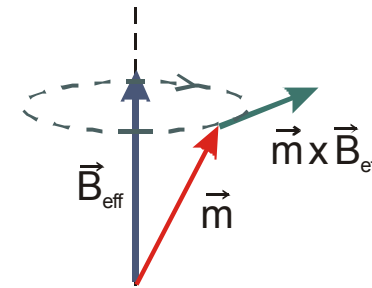
Precession of a magnetic moment in a field

Relation between magnetic moment \vec{m} and angular momentum \vec{l} :

$$\vec{m} = -\frac{g \mu_B}{\hbar} \vec{l} = \gamma \cdot \vec{l} \quad \text{with} \quad \gamma = \gamma_e \cdot g / 2$$

$\gamma_e = 175,8 \text{ GHz/T}$: gyromagnetic factor of a single electron

g : g-factor



Equation of motion (Landau-Lifshitz equation):

$$\vec{D} = \frac{d}{dt} \vec{l} = \frac{1}{\gamma} \frac{d\vec{m}}{dt} = \vec{m} \times \vec{B}_{\text{eff}}$$

or, using magnetization M (density of magnetic moments):

$$\frac{1}{\gamma} \frac{d\vec{M}}{dt} = \vec{M} \times \vec{B}_{\text{eff}}$$

$$B_{\text{eff}} = B_e \Rightarrow \omega = \gamma B_e$$

Precession of a magnetic moment in a field (2)

\vec{B}_{eff} : magnetic field acting on magnetic moment:

$$\vec{B}_{\text{eff}} = \vec{B}_0 + \vec{B}(t) + \vec{B}_{\text{exch}} + \vec{B}_{\text{ani}}$$

\vec{B}_0 : external magnetic field

$\vec{B}(t)$: time dependent field
(caused by precession; plus
microwave field, if applicable)

\vec{B}_{exch} : exchange field between neighbored
moments

$$\vec{B}_{\text{exch}} = \frac{2A}{M_s^2} \nabla^2 \vec{M} = \frac{D}{M_s} \nabla^2 \vec{M}$$

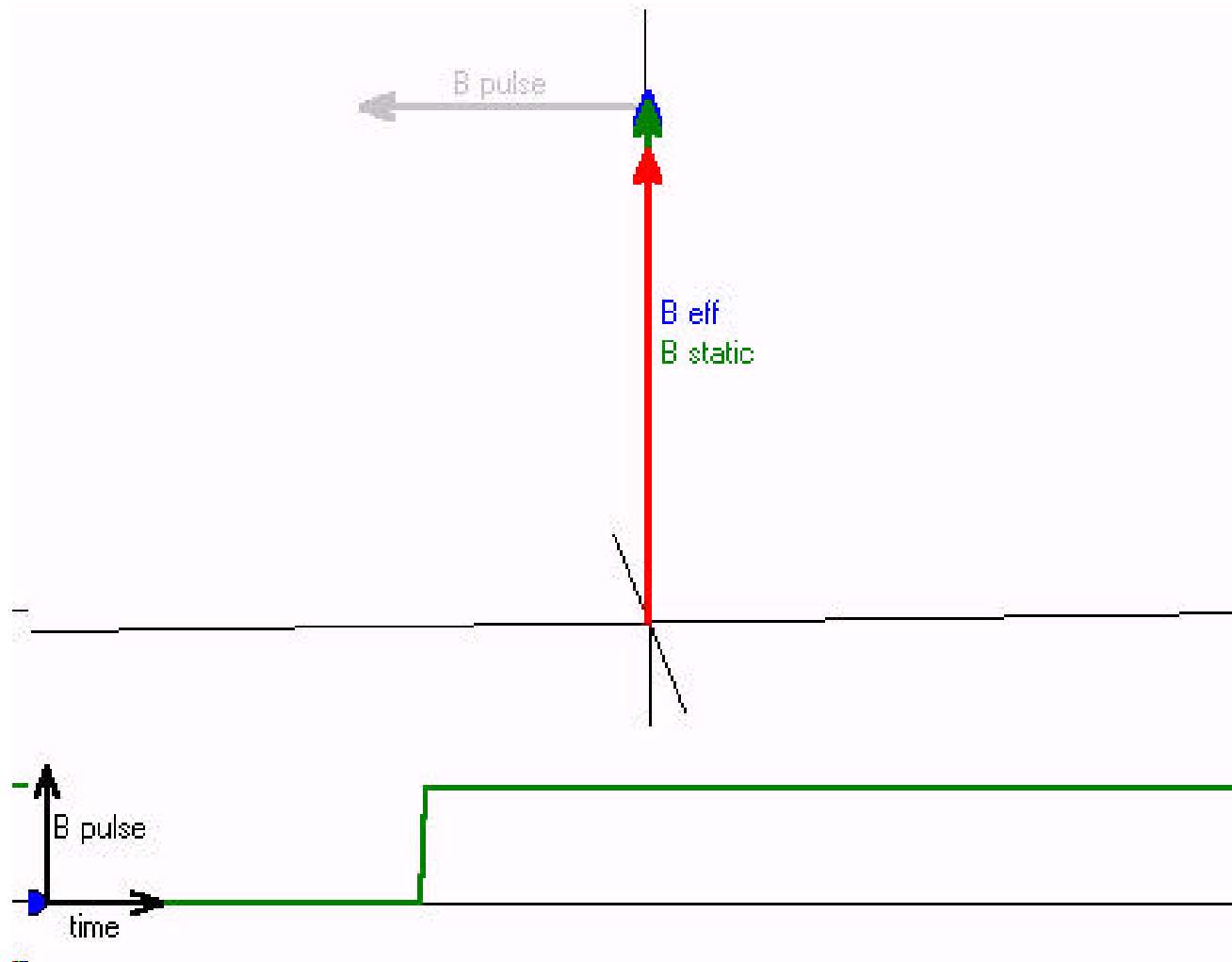
\vec{B}_{ani} : anisotropy field

$$\vec{B}_{\text{ani}} = -\frac{1}{M} \nabla_{\vec{\alpha}} E_{\text{ani}} \quad \text{with} \quad \vec{\alpha} = \vec{M} / |M|$$

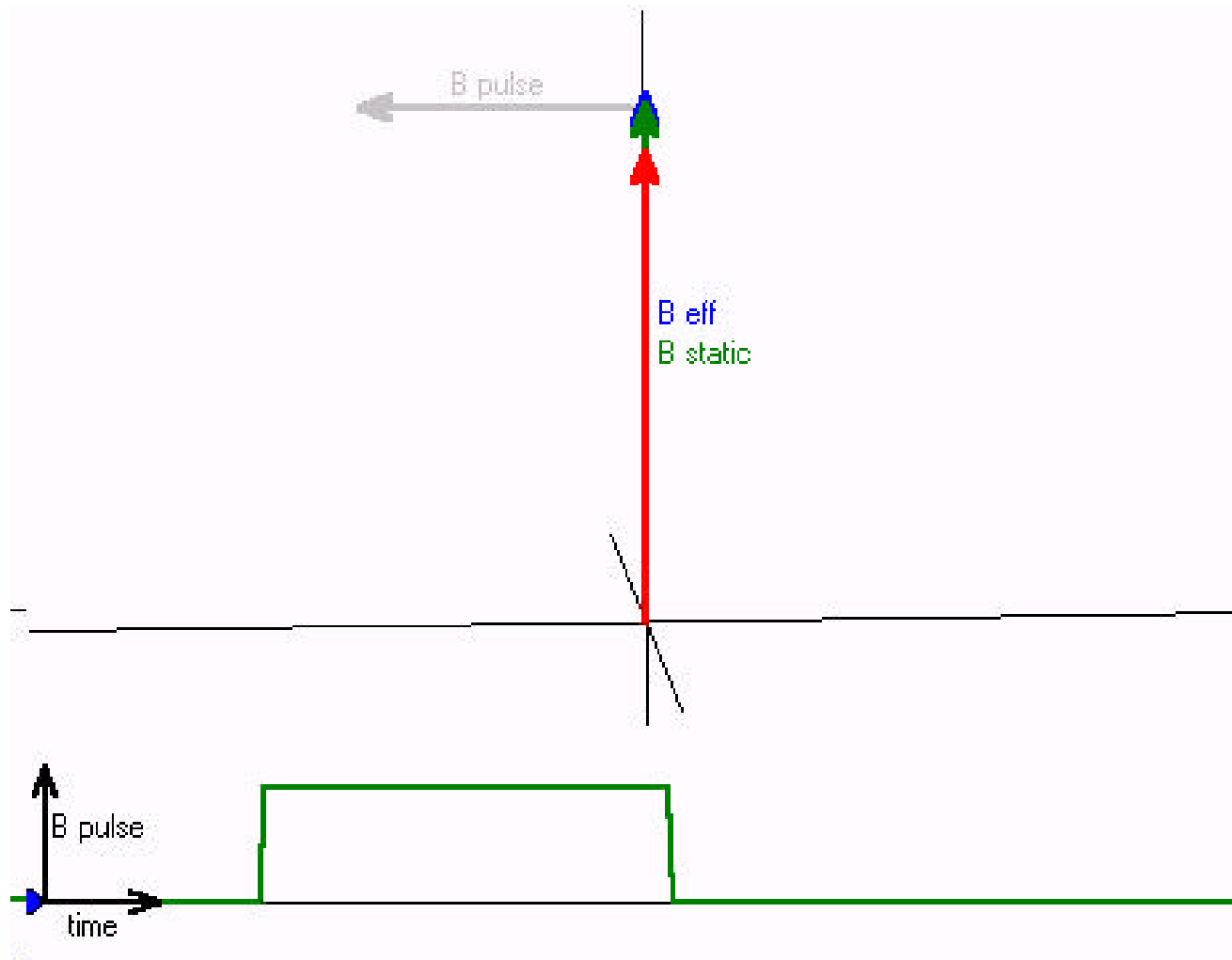
E_{ani} : magnetic free anisotropy density
cubic anisotropy:

$$E_{\text{ani}} = K_1 (\alpha_x^2 \alpha_y^2 + \alpha_y^2 \alpha_z^2 + \alpha_z^2 \alpha_x^2)$$

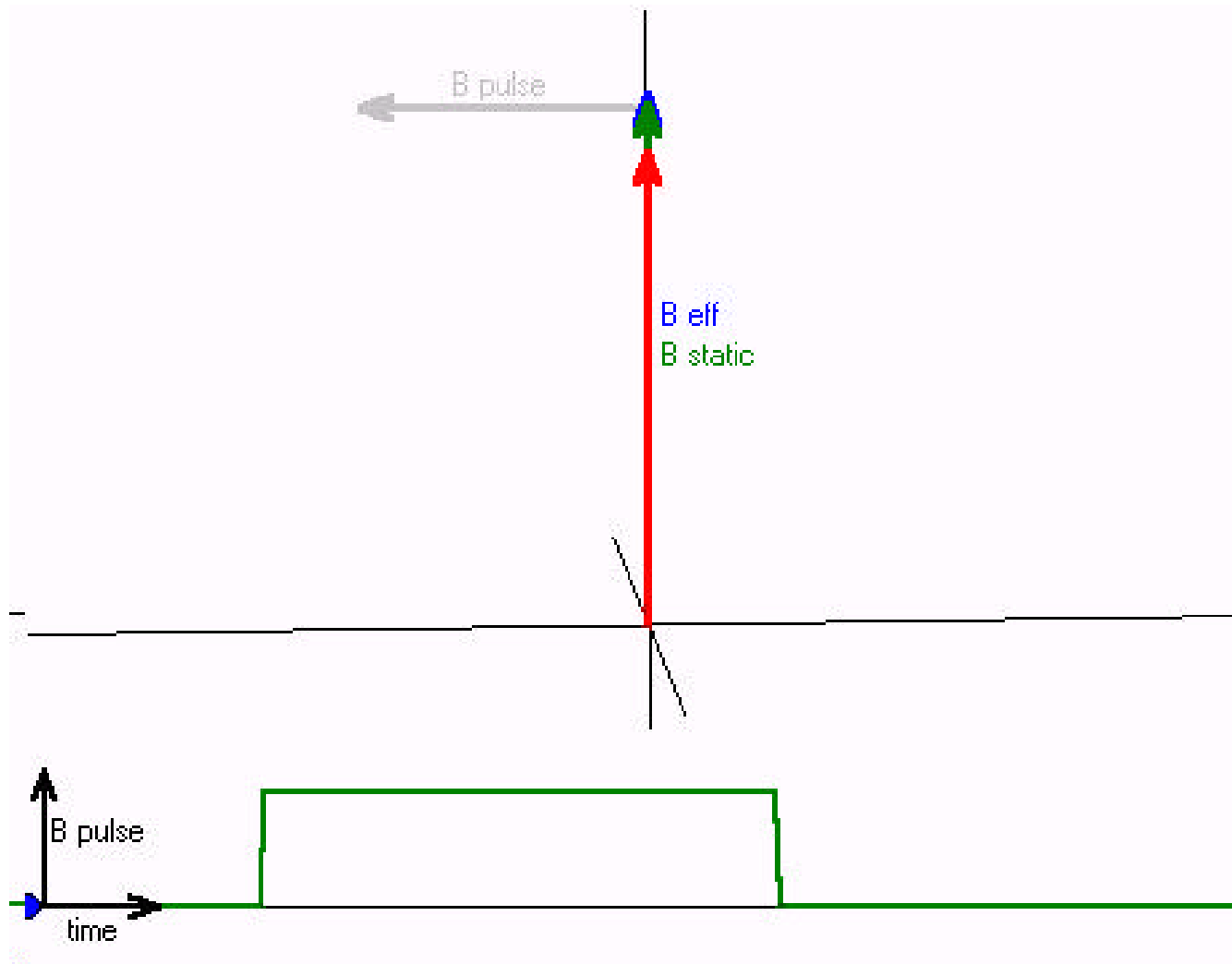
Precession of magnetization



Precession of magnetization



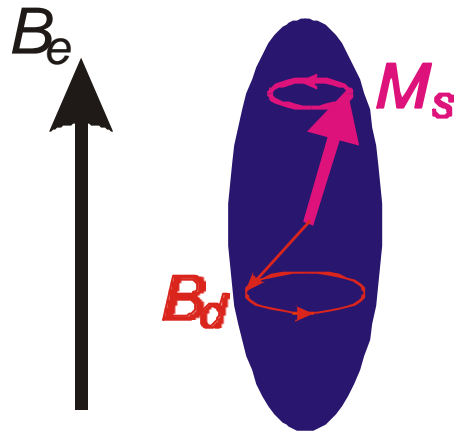
Precession of magnetization



Dynamic of confined systems

Example: ellipsoid

Dipolar interaction can be characterized by a demagnetizing tensor N_x , N_y and N_z :



Uniform precession with frequency

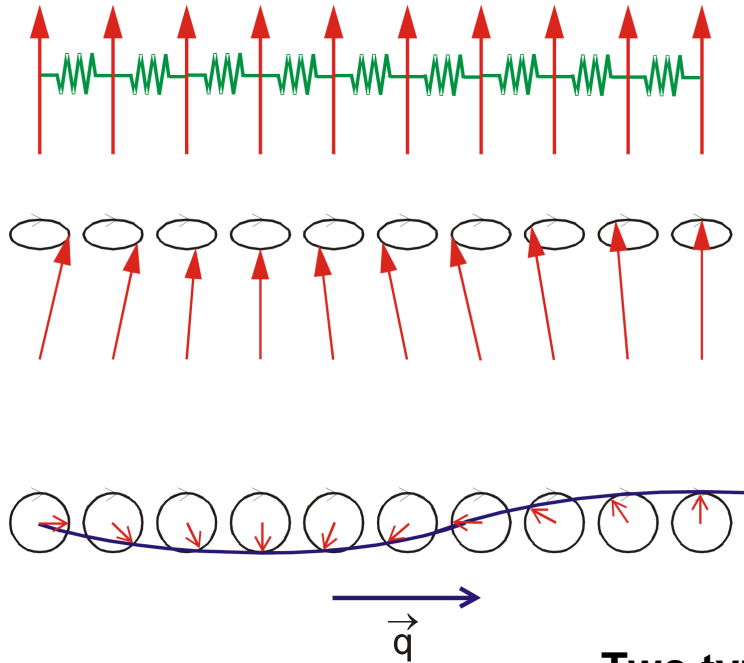
$$\left(\frac{\omega}{\gamma}\right)^2 = (B_e + (N_x - N_z)J_s) \cdot (B_e + (N_y - N_z)J_s)$$

with :

$J_s = \mu_0 M_s$ (magnetic saturation polarization) in z-direction

Uniform eigen-mode of the system (FMR)

Spin waves



Non-uniform eigen-modes of the magnetic system

Two types of energy contributions

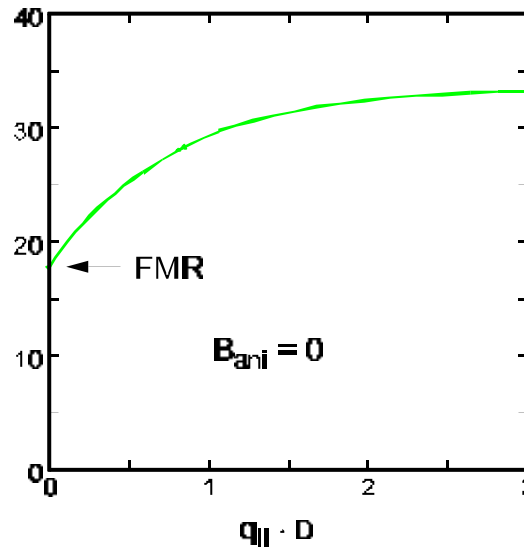
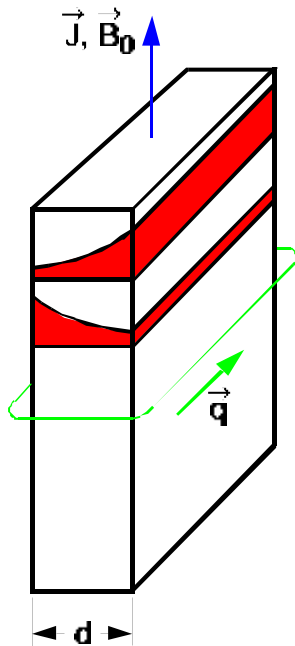
- exchange energy:
generated by twist of neighbored spins
- dipolar energy:
generated by magnetic poles in long-wavelength spin waves

Spin waves in films

Solution to equation of motion (without anisotropies):

$$\left(\frac{\omega}{\gamma}\right)^2 = B_0(B_0 + J_s) + \left(\frac{J_s}{2}\right)^2 (1 - e^{-2q_{\parallel}d})$$

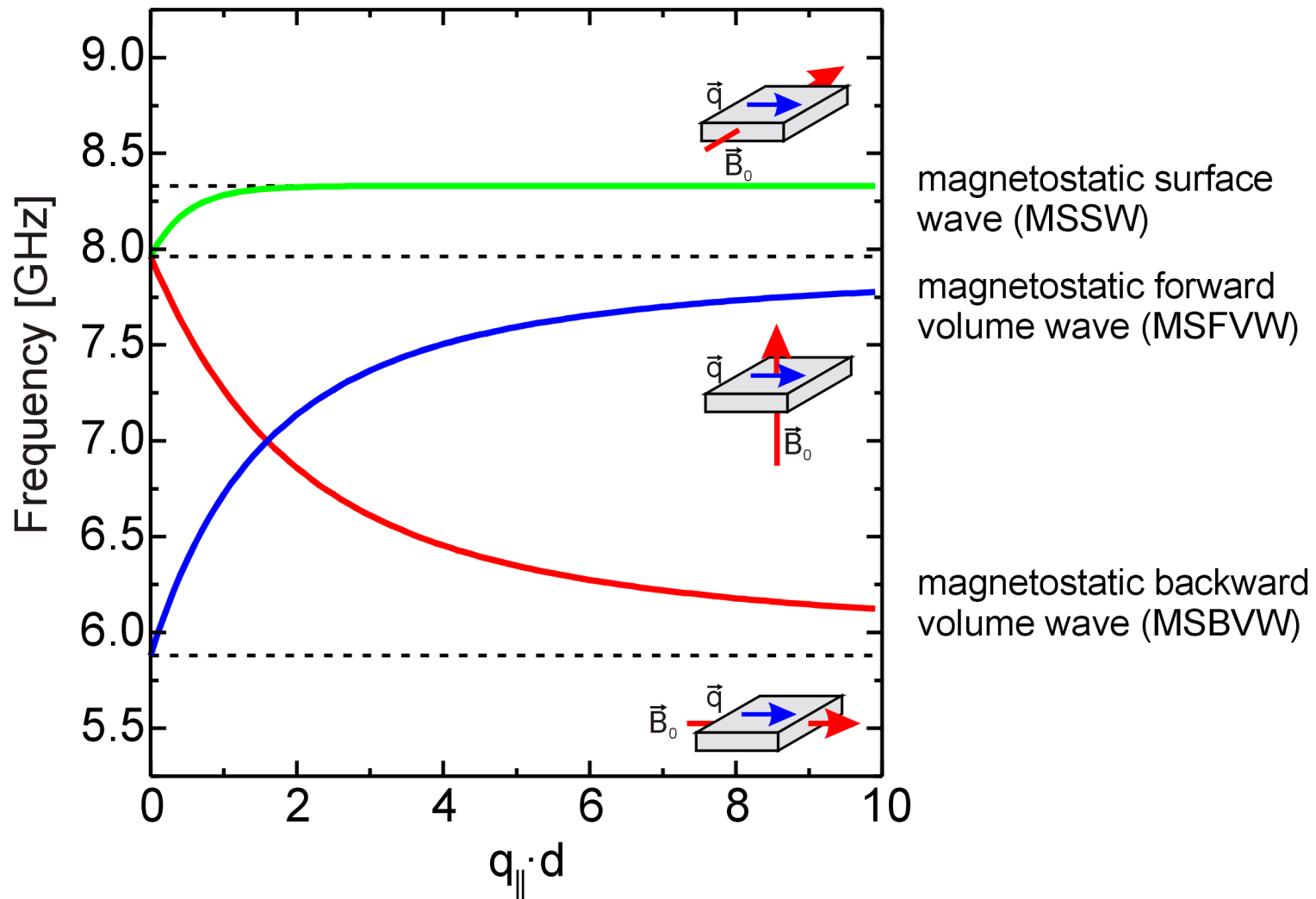
with: $J_s = \mu_0 M_s$ (magnetic saturation polarization)



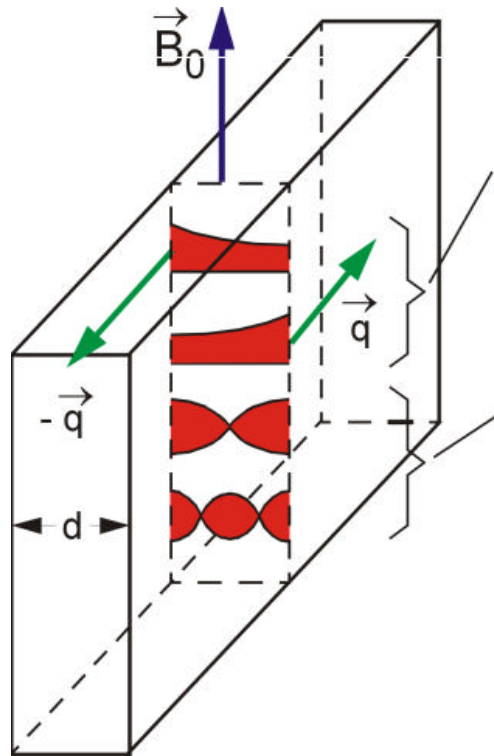
Mode is localized on the film surfaces

Surface (so-called Damon-Eshbach) modes

Spin waves in films: field geometry



Dipolar and exchange spin waves



Dipolar Damon-Eshbach modes

$$\omega^2/\gamma^2 = [B_0(B_0 + J_s) + (J_s/2)^2 (1 - e^{-2qd})]$$

Standing spin waves

$$\frac{\omega}{\gamma} = \frac{2A}{M_s} \cdot q^2 = \frac{2A}{M_s} \left(\frac{n\pi}{d}\right)^2 \quad n = 1, 2, \dots$$

A: exchange constant

M_s : magnetization

Solving the Landau-Lifshitz equation

1st step:

solve LL equation of motion

$$\frac{1}{\gamma} \frac{d\vec{M}}{dt} = \vec{M} \times \vec{B}_{\text{eff}}$$

P 6 partial wave solutions

- non-linearity intrinsically built in due to $M_i B_{\text{eff},j}$ products

2nd step:

formulate boundary conditions

P coupled partial waves

a) boundary conditions from Maxwell equations

b) Rado-Weertman boundary condition
(from LL-equation)

$$\vec{M} \times \left(A \frac{\partial}{\partial \vec{n}} \vec{M} - k_s \vec{n} (\vec{n} \cdot \vec{M}) \right) = 0$$

with:

\vec{n} : unit vector normal to surface

$\partial \vec{M} / \partial \vec{n}$: derivative of \vec{M} in direction of \vec{n}

k_s : out-of-plane interface
anisotropy constant

Exchange modes

small wavelength: exchange interaction

$$\vec{B}_{\text{exch}} = \frac{2A}{M_s} \nabla^2 \vec{M} = \frac{D}{M_s} \nabla^2 \vec{M}$$

must be considered

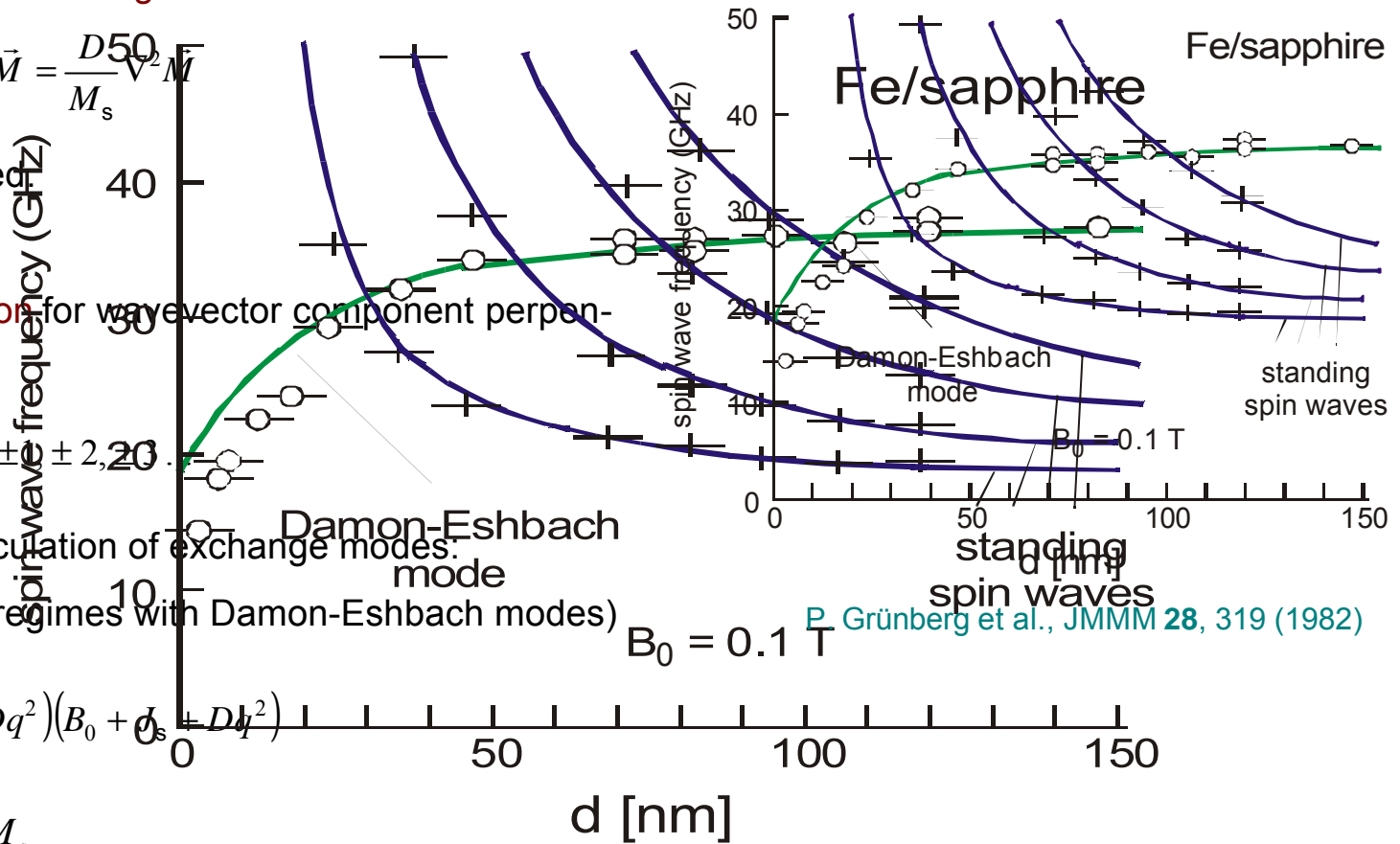
resonance condition for wavevector component perpendicular to film:

$$q_{\perp} = n \frac{\pi}{d}; \quad n = \pm 1, \pm 2, \dots$$

approximative calculation of exchange modes:
(outside crossing regimes with Damon-Eshbach modes)

$$\left(\frac{\omega}{\gamma}\right)^2 = (B_0 + Dq^2)(B_0 + Dq^2)$$

with: $D = 2A/M_s$



Experiments on spin dynamics

Experiments on the time scale:

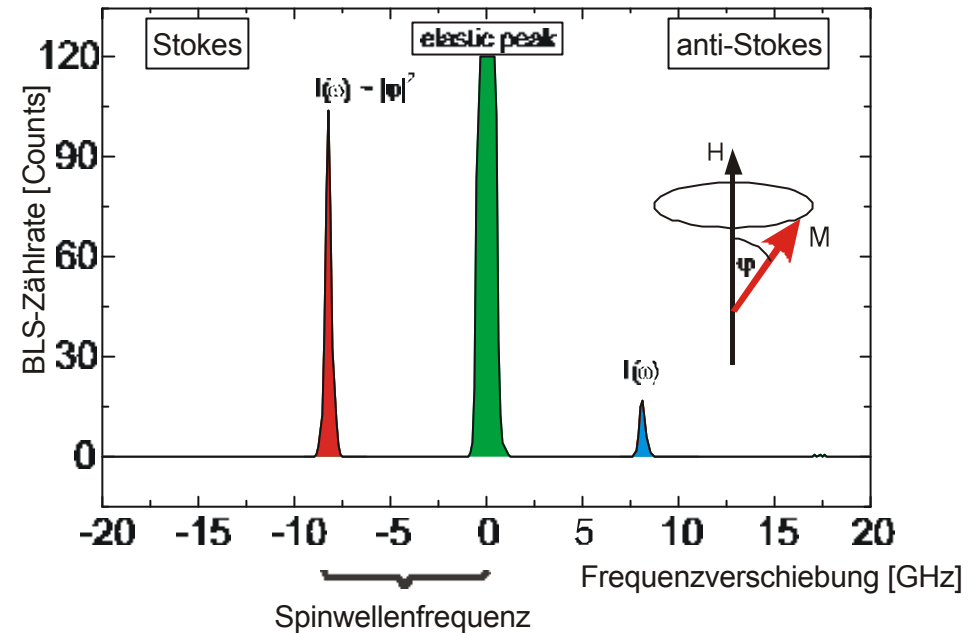
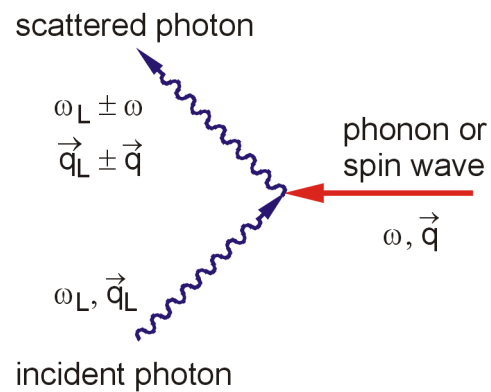
- **suppression of precession**
- **magnetic switching**

Experiments on the frequency scale:

- **dynamic eigen-excitations of islands**
- **excitations in inhomogeneous internal fields**

Brillouin light scattering (BLS) process

= inelastic scattering of photons from spin waves

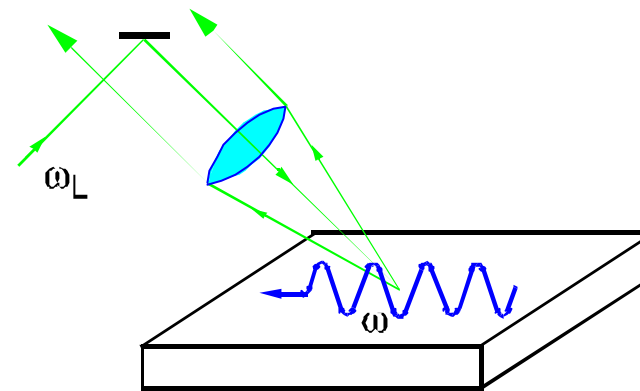
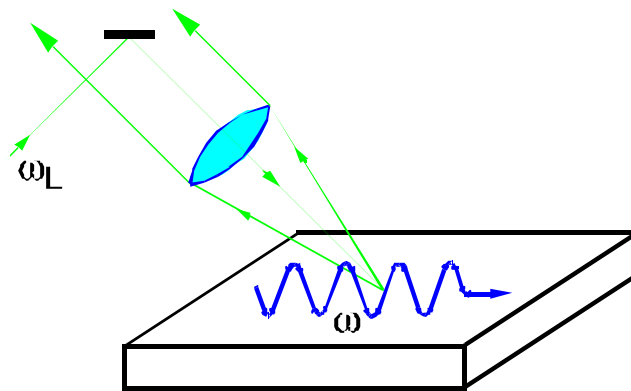
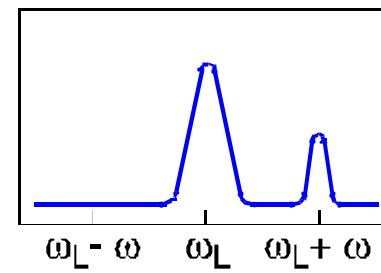
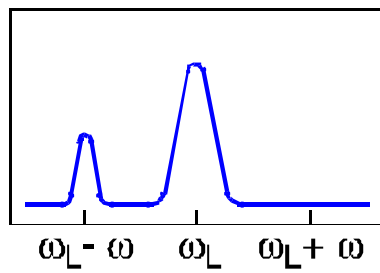


Conservation laws:

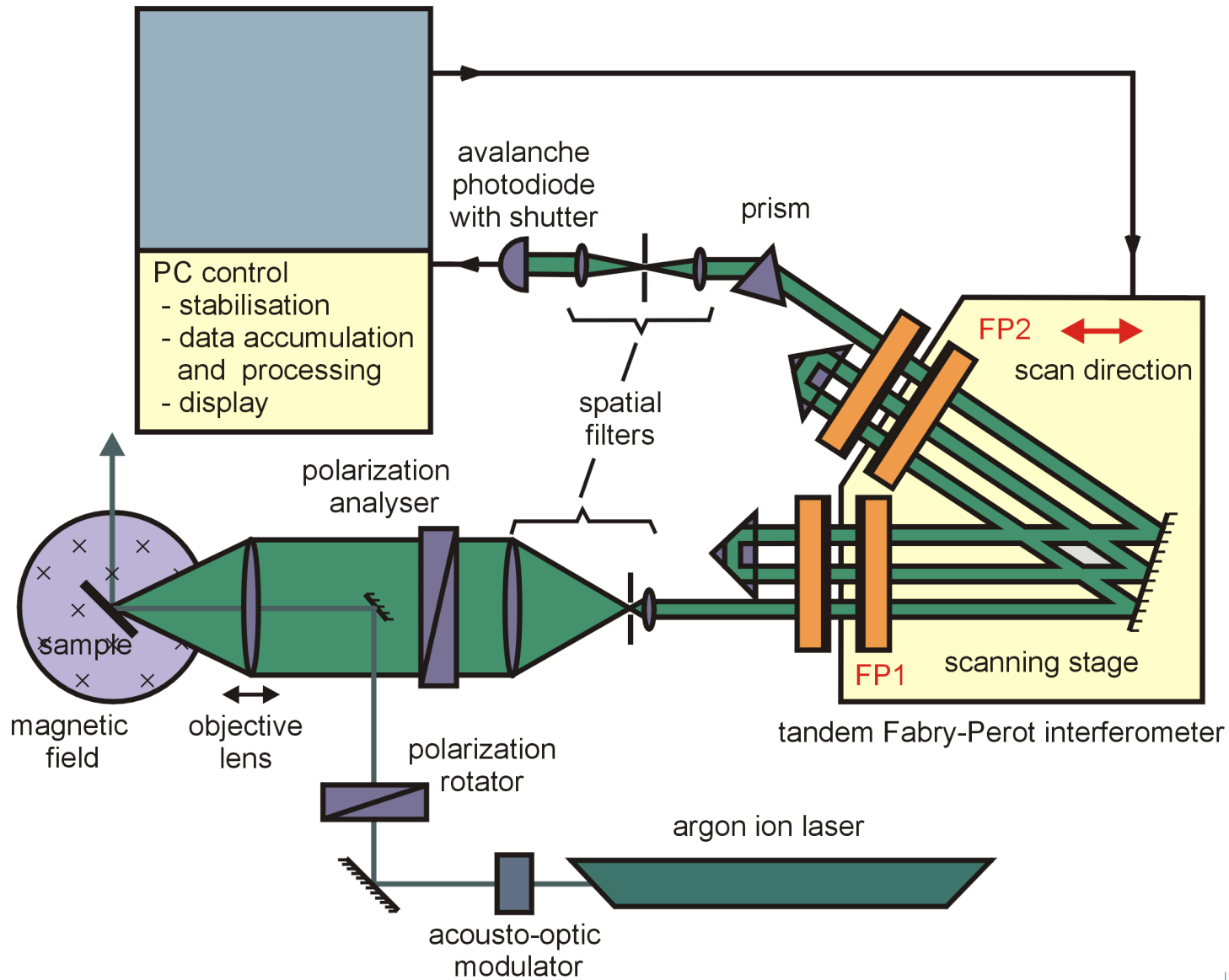
- Time invariance
- In-plane translational invariance

$$\begin{aligned} \mathbf{P} \quad \omega_{sc} &= \omega_L \pm \omega \\ \mathbf{P} \quad \vec{q}_{sc} &= \vec{q}_L \pm \vec{q} \end{aligned}$$

Light scattering cross section



BLS spectrometer



BLS spectrometer (2)

- frequency range: 1–500 GHz
- resolution: 0.1 GHz (or 1% of free spectral range)
- contrast: $> 1:10^{10}$ (multipass arrangement)
- laser spot diameter: 40 μm
- magnetic sensitivity: $7 \cdot 10^{10}$ Co atoms ($2 \cdot 10^{-9}$ emu) at 300 K
- wavevector range: $0 - 2.4 \cdot 10^5 \text{ cm}^{-1}$
- determined quantities: dispersion of
- surface acoustic waves
 - spin waves
 - nonlinear spin wave phenomena

BLS spectroscopy

Observation of:

- dispersion of acoustic phonons in the volume, on surfaces and in films
⇒ elastic properties
- dispersion of spin waves in magnetic materials
⇒ anisotropies, magnetic moments, magnetic exchange constant
- nonlinear spin waves at large precession angles

Light scattering cross section

- spin orbit coupling generates phase grating in refractive index propagating with speed of spin wave

electric polarization P in medium:

$$\vec{P} = (\epsilon_{11} - 1)\epsilon_0\vec{E} + K / M_s (\vec{E} \times \vec{M}_s)$$

with K : magneto-optic coefficient

- light is refracted at phase grating
- frequency is Doppler shifted

Conclusions (I)

exchange and dipole coupling are the most important interactions in spin systems

spin dynamics is determined by the Landau-Lifshitz-equation

spin waves are non-uniform eigenmodes of the system

properties of spin waves reflect the dimensionality of the system

Brillouin light scattering(BLS) is a powerful tool for studies of spin waves

different spin wave modes can be investigated separately

BLS sensitivity is high enough to allow experiments on ultra-thin films