

Low Fermi energy and superconductivity in C_{60} compounds



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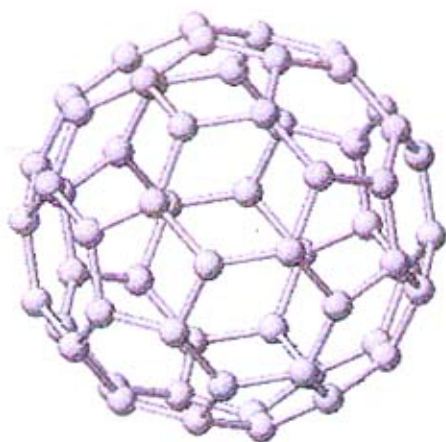
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Outline

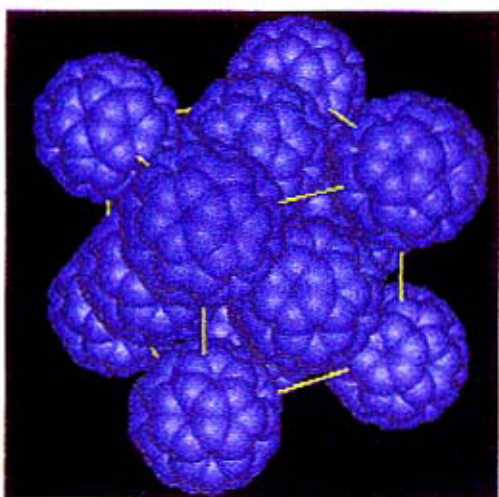
- High- T_c superconductivity in C_{60}
- Fullerenes: low carrier density; small Fermi energy
- Electron-phonon: breakdown of adiabatic hypothesis
- Nonadiabatic superconductivity
- Nonadiabatic Fermi liquid

C_{60} Fullerene

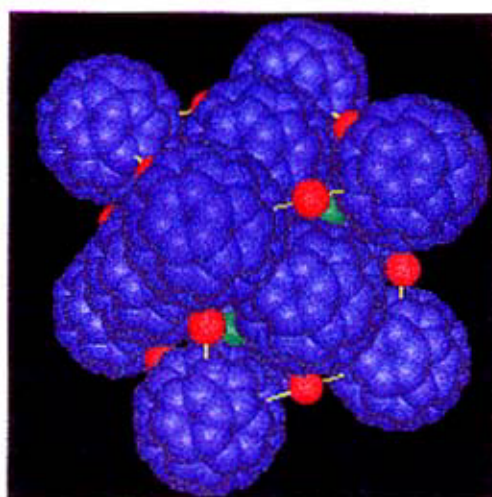
Molecular structure



Crystal structure

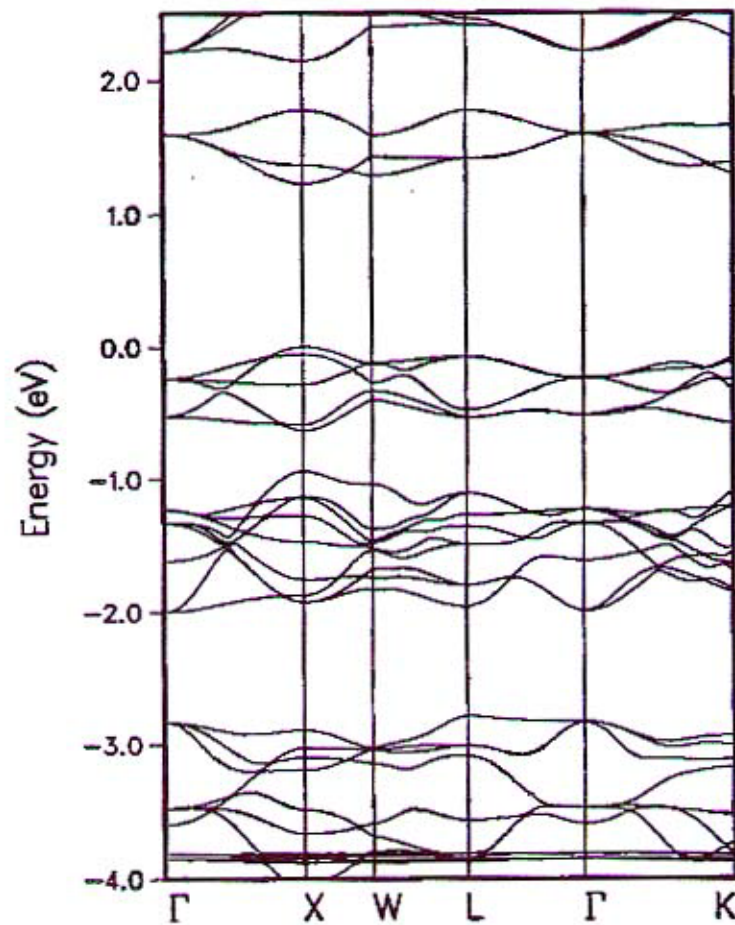


C_{60}



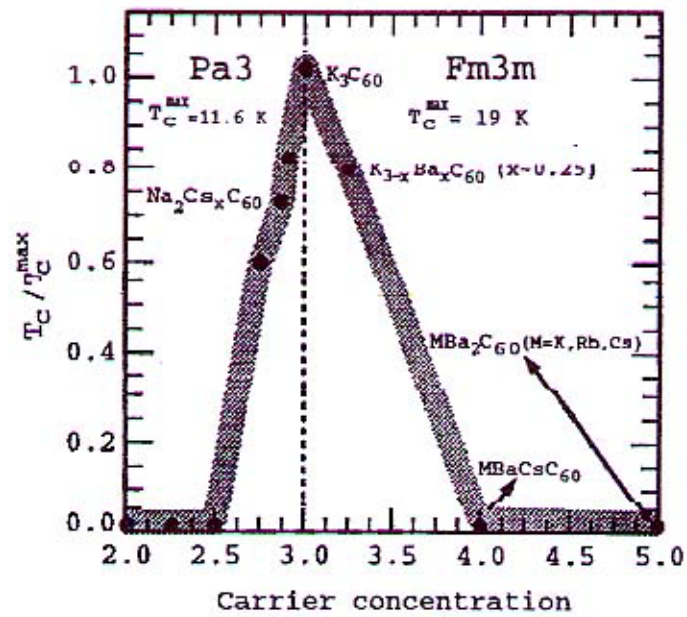
A_3C_{60}

Electronic band structure

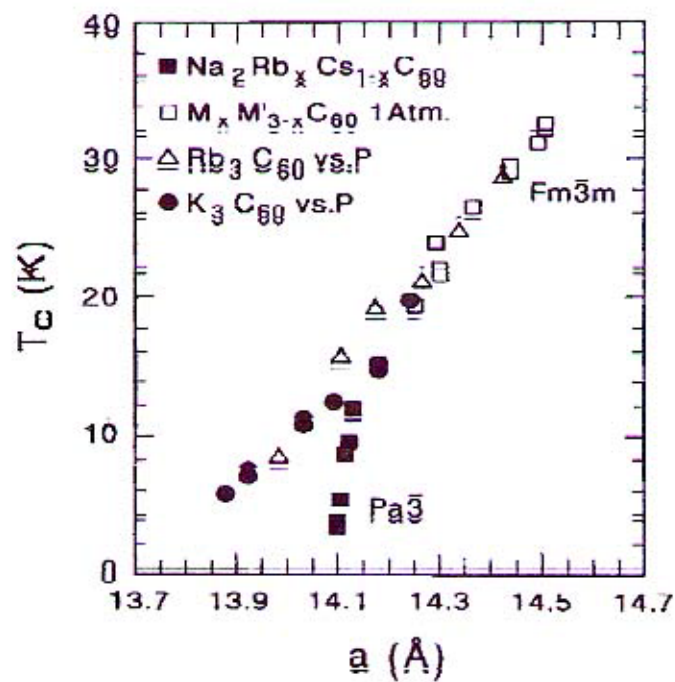


[S.C. Erwin, in *Buckminsterfullerenes*, eds. W.E. Billups and M.A. Ciufolini (1993)]

Phase diagram

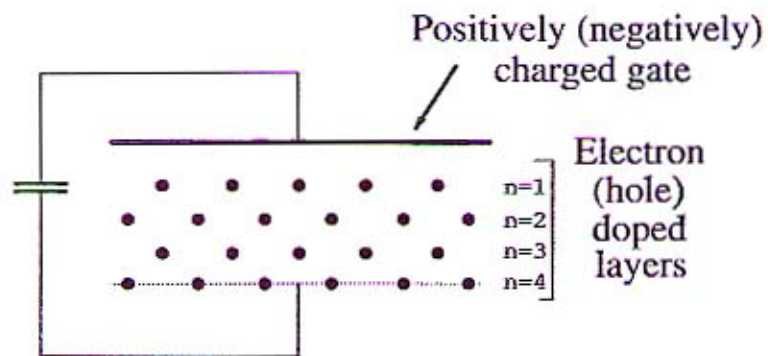
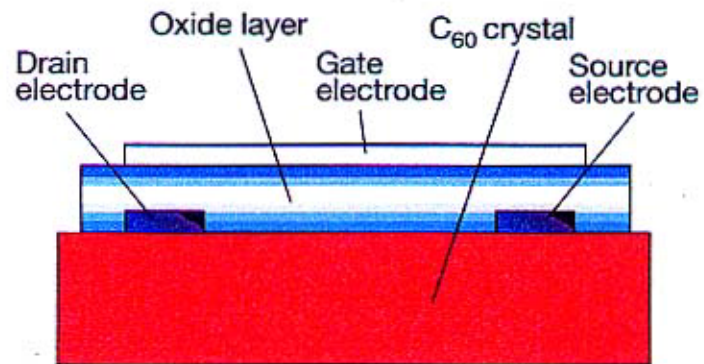


[Yildirim et al. Phys. Rev. Lett. 1996]

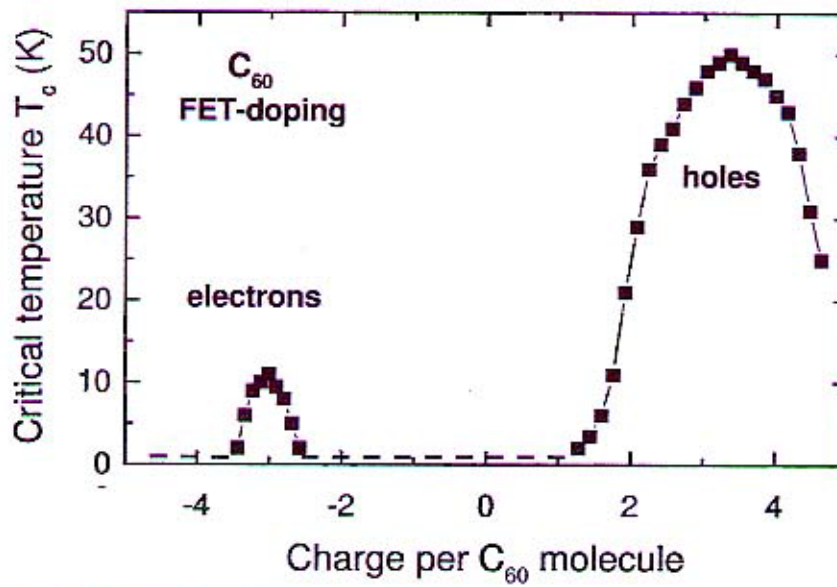


[Yildirim et al. Solid State Commun. 1995]

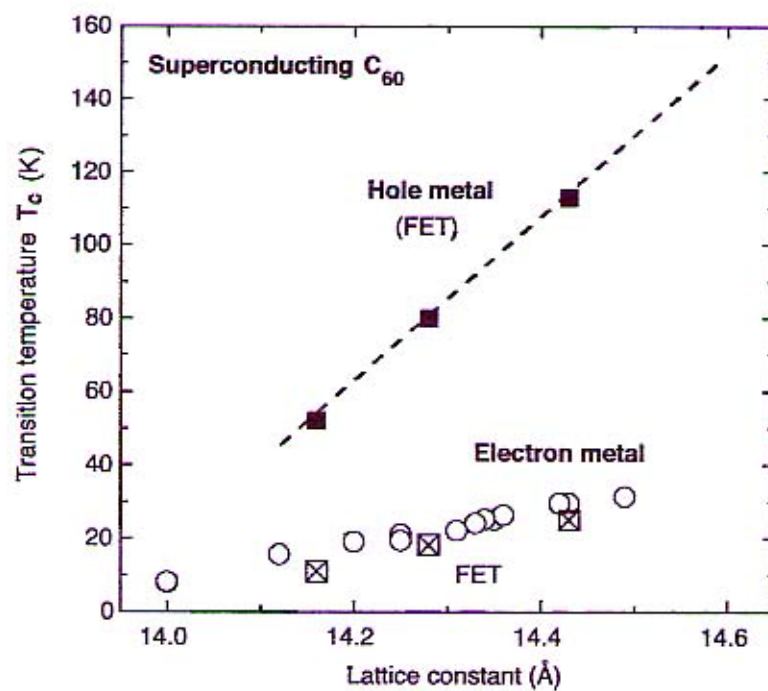
Field Effect Transistor doping



FET results



[Schön et al. Nature 2000]



[Schön et al. Science 2001]

Superconductivity in fullerenes

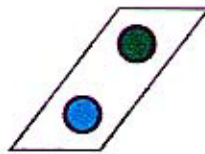
- $T_c^{\text{max}} = 117 \text{ K!}$ (so far ...)
[BCS empirical limit $T_c \lesssim 20 - 25 \text{ K}$]
- Strong electron-electron repulsion
[Cooper pairing disfavoured]
- Small density of carriers
[poor metal]

⇒ Superconductivity in fullerenes
in spite of
these negative features ????

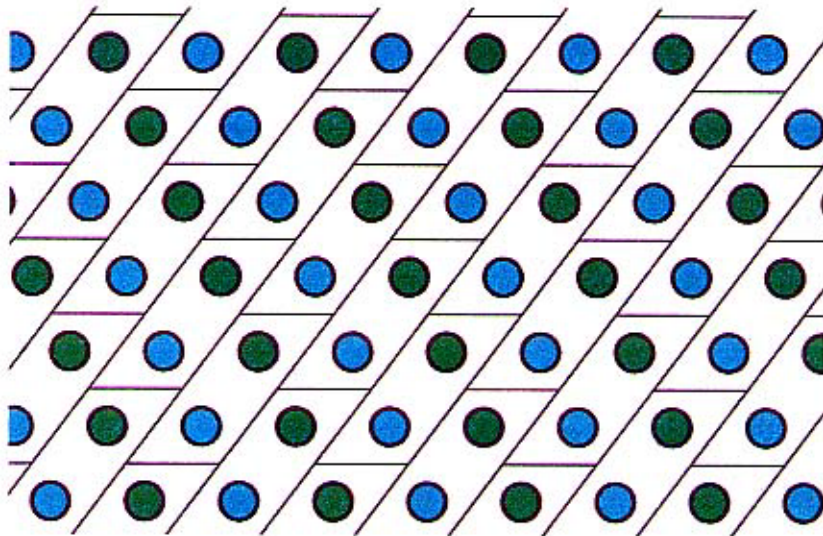
⇒ Unconventional superconductivity:
new context in which
these negative features favour pairing

Common metals

(a) Few atoms in unit cell



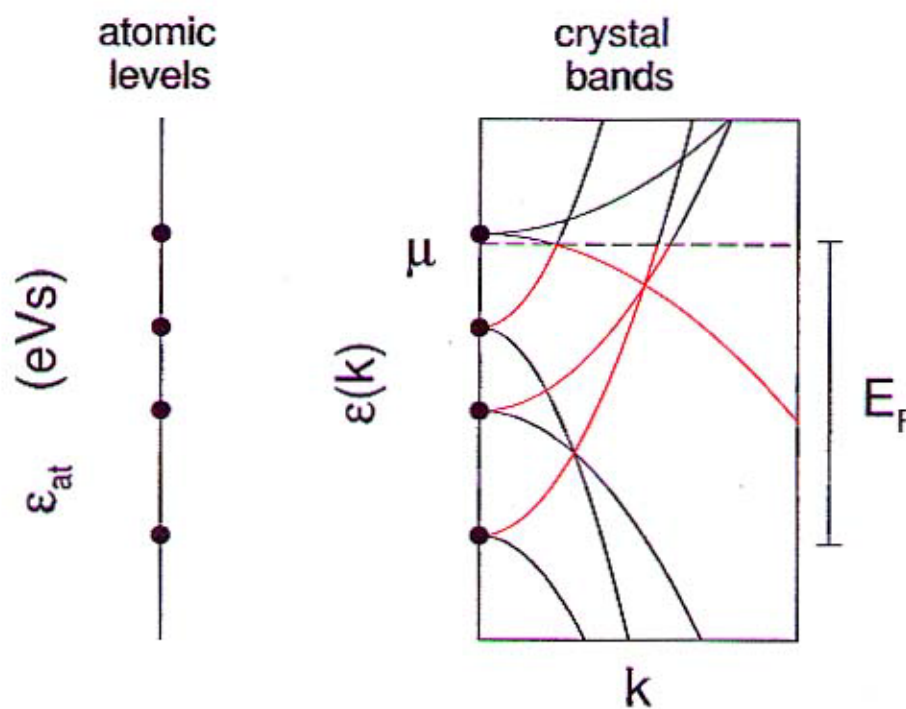
(b) Close packed unit cells



Electronic states

(a) High energy spaced atomic levels

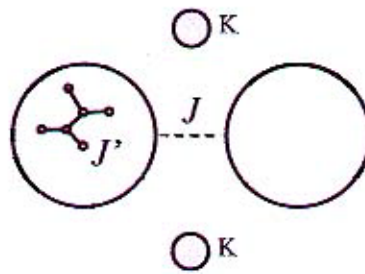
(b) Strong hybridization $\Rightarrow$ large bands



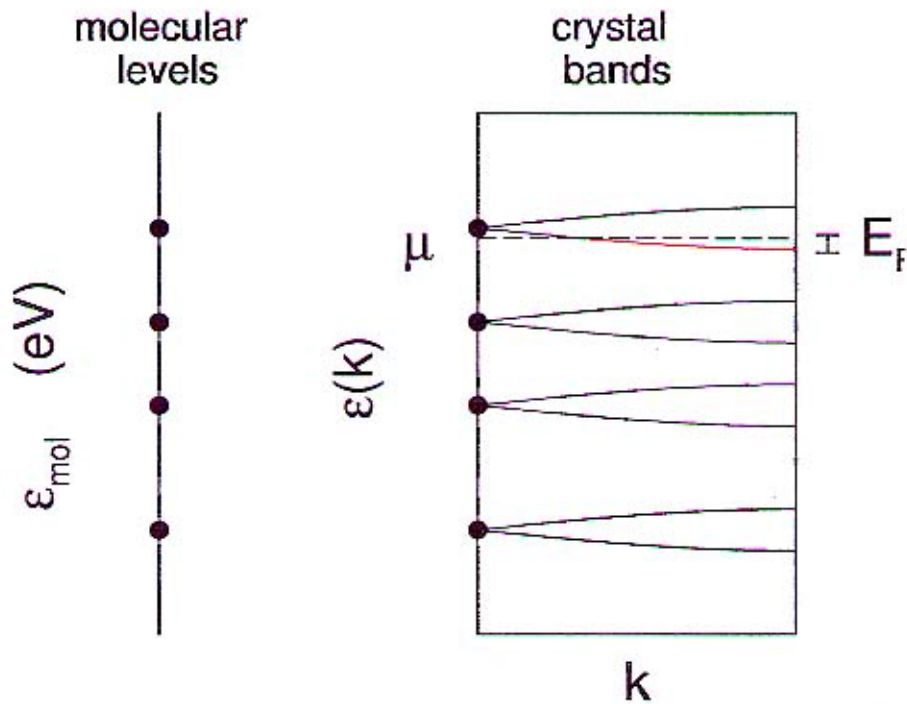
$\Rightarrow$ Large carrier density

$\Rightarrow$ Large Fermi energy E_F

Fullerenes

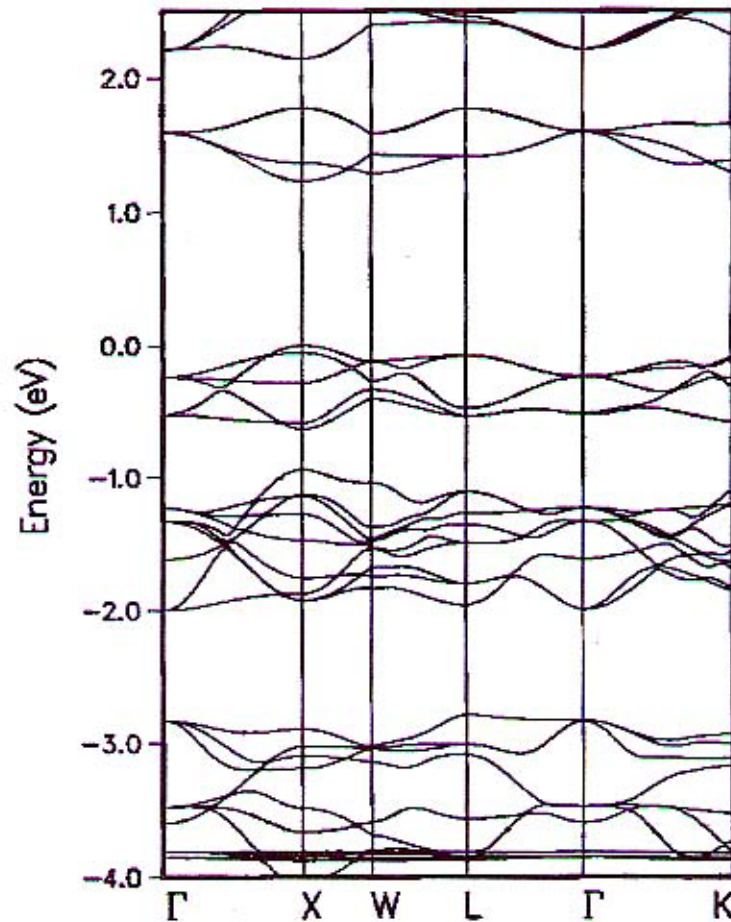


- Large cluster unit block
- Weak hybridization



- ⇒ Small carrier density
- ⇒ Narrow bands
- ⇒ Small Fermi energy E_F

Band structure C_{60}



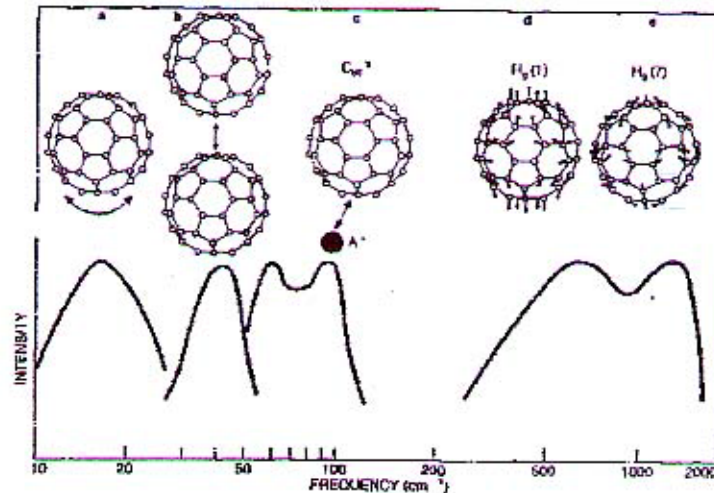
[S.C. Erwin, in *Buckminsterfullerenes*, eds. W.E. Billups and M.A. Ciufolini (1993)]

Narrow bandwidths $W \sim 0.5 - 0.6$ eV

Small Fermi energies $E_F \sim 0.2 - 0.3$ eV

Nonadiabaticity

- Phonon frequencies $\omega_{\text{ph}} \sim 80 - 200 \text{ meV}$



A.F. Hebard et al., Phys. Today (1992)

- Fermi energies $E_F \sim 0.25 - 0.35 \text{ eV}$

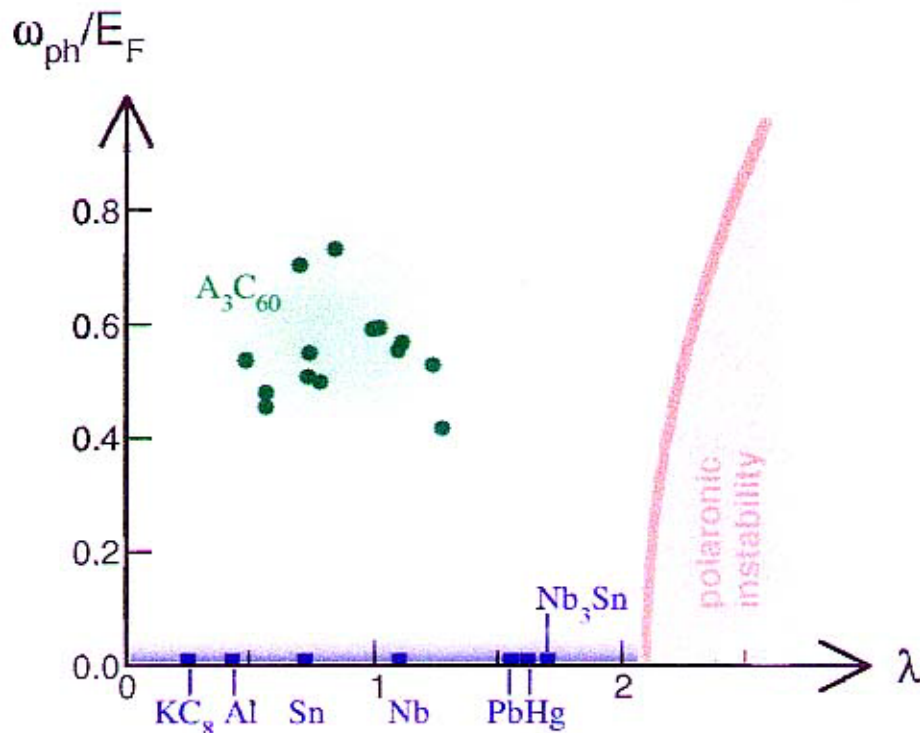


$$\omega_{\text{ph}}/E_F \simeq 0.4 - 0.8$$

The adiabatic principle
(Born-Oppenheimer)
breaks down

Nonadiabatic metal

Adiabatic phase diagram



$$N(E_F) \simeq 8 \text{ states/eV } C_{60}$$

$$E_F \simeq 0.25 \text{ eV}$$

Data collected by LDA calculations:

Schluter et al., Journ. Phys. Chem. Sol., 1992

Schluter et al., Phys. Rev. Lett., 1992

Varma et al., Science, 1991

Faulhaber et al., Phys. Rev. B, 1993

Antropov et al., Phys. Rev. B, 1993

Breda et al., Chem. Phys. Lett., 1998

Onida et al., Europhys. Lett., 1992

Adiabatic hypothesis

- Vertex function

$$\Lambda = 1 + P + \dots$$

Migdal's theorem

$$P \sim \lambda \frac{\omega_{ph}}{E_F}$$

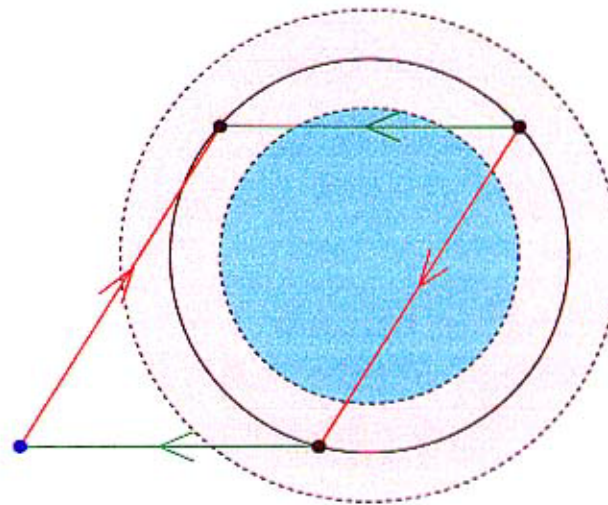
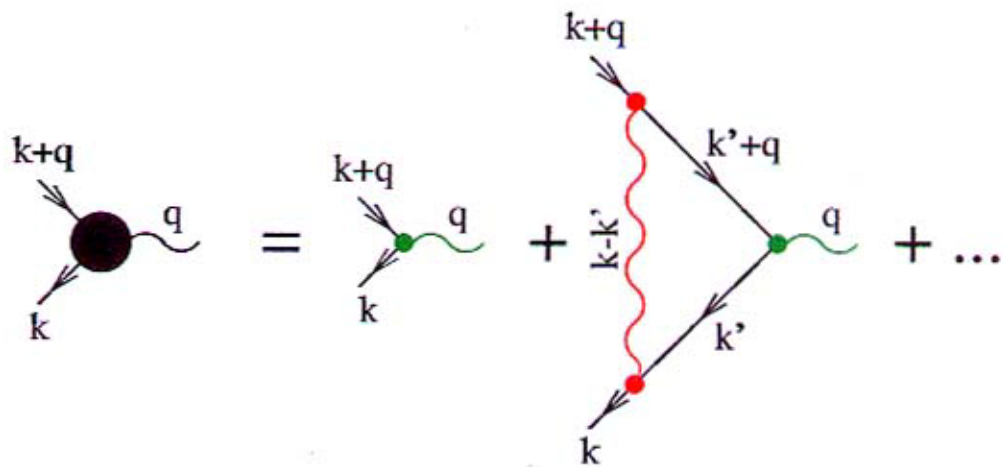
⇒ Conventional ME superconductors ($\omega_{ph} \ll E_F$):

$$P \ll 1 \quad \Rightarrow \quad \Lambda = 1$$

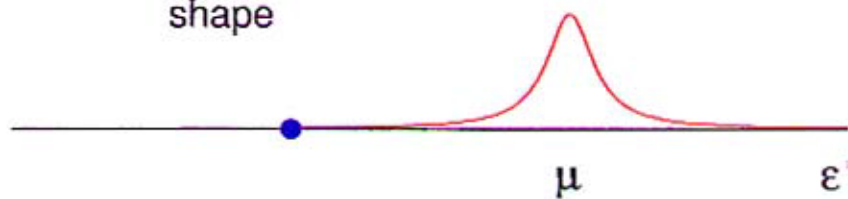
⇒ Fullerenes ($\omega_{ph} \sim E_F$)

$$P \sim 1 \quad \Rightarrow \quad \Lambda = 1 + P$$

Vertex interferences



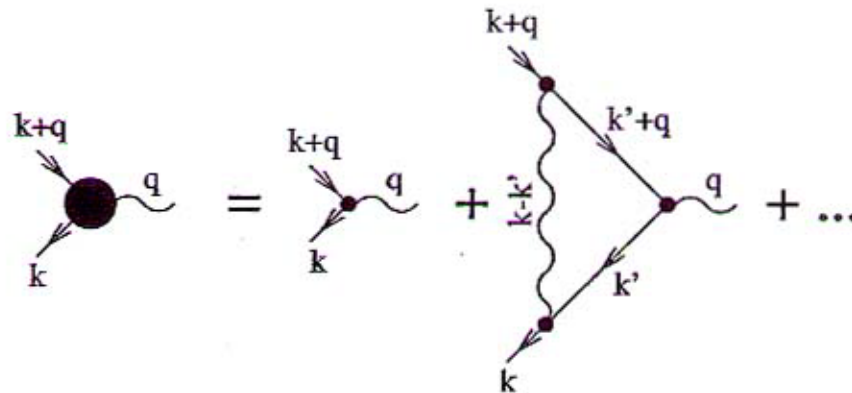
phonon shape



$$D(\omega) = \frac{\omega_{\text{ph}}}{\omega_{\text{ph}}^2 + \omega^2} \Rightarrow D(E_F)E_F \sim \frac{\omega_{\text{ph}}}{E_F^2}E_F \sim \frac{\omega_{\text{ph}}}{E_F}$$

Beyond Migdal's theorem

- Vertex function



Expansion parameter: $P = \lambda\omega_{\text{ph}}/E_F$

- Migdal-Eliashberg: nonperturbative in λ
- Nonadiabatic theory: nonperturbative in λ ,
perturbative in $\lambda\omega_{\text{ph}}/E_F$
 $\Rightarrow$ counting of vertex diagrams

First order in vertex



Nonadiabatic channels

- Self-energy

$$\Sigma = \text{diagram 1} + \text{diagram 2}$$

- Superconducting Cooper pairing

$$\Delta = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4}$$

Nonadiabatic metal parameters

- **Not** a parameter renormalization
Es.: Electronic mass $m^*(\lambda) \rightarrow m^*(\lambda')$

$$m^*(\lambda) \rightarrow m^*(\lambda, \omega_{\text{ph}}/E_F)$$

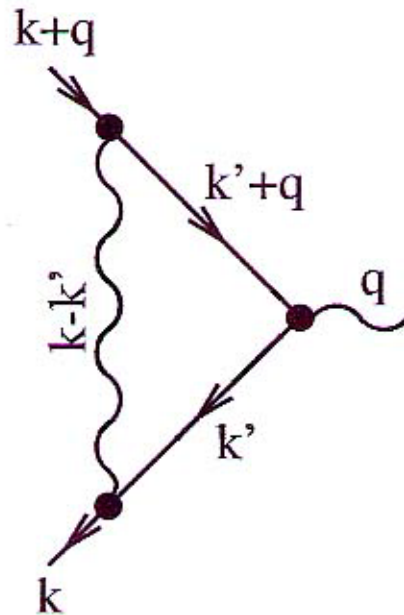
$$t_c(\lambda) \rightarrow t_c(\lambda, \omega_{\text{ph}}/E_F)$$

$$\chi_P \rightarrow \chi_P(\lambda, \omega_{\text{ph}}/E_F)$$

$$\left[t_c = \frac{T_c}{\omega_{\text{ph}}} \right]$$

- **Different** nonadiabatic effects
on **different** physical quantities

Key element: vertex function

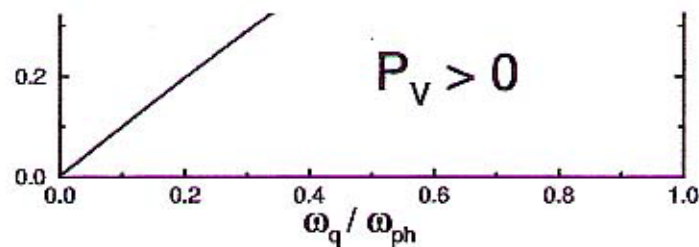


$$P(\mathbf{k}, \mathbf{q}; m, n) = -T \sum_l \sum_{\mathbf{q}} |g(\mathbf{k} - \mathbf{k}')|^2 D(m - l) \\ \times G(\mathbf{k}' + \mathbf{q}, l + n) G(\mathbf{k}', l).$$

Smooth dependence on the external electronic momenta-frequencies $\mathbf{k}-\omega_m$

Complex structure in the exchanged phonon momenta-frequencies $\mathbf{q}-\omega_n$

Sign of vertex corrections



- Static limit $q \rightarrow 0, \omega = 0$

$$\lim_{q \rightarrow 0, \omega = 0} P < 0$$

- Dynamic limit $\omega \rightarrow 0, q = 0$

$$\lim_{\omega \rightarrow 0, q = 0} P > 0$$

Grabowski and Sham 1984: $\lim_{\omega=0} P < 0$

Electronic correlation

- Metals: long range el-el interaction screened; short-range (on-site)?

Simplest (Hubbard) model

$$H = \sum_{\mathbf{k}, \sigma} \epsilon(\mathbf{k}) c_{\mathbf{k}, \sigma}^{\dagger} c_{\mathbf{k}, \sigma} + U \sum_i n_{i, \uparrow} n_{i, \downarrow}$$

- Competition between propagating [$\epsilon(\mathbf{k})$] and local [U] terms
- Half-filling (one electron per site) for $U \gg E_F$: no propagation (strong correlation)
- At low density $n_i \ll 1$: weak correlation no matter U/E_F

Strongly correlated systems

- Textbook example

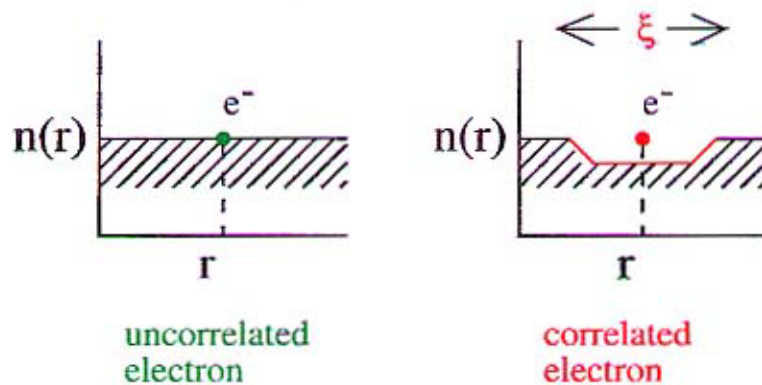
	low density	high density
$U \ll E_F$		
$U \gg E_F$		✓

- More complex structures

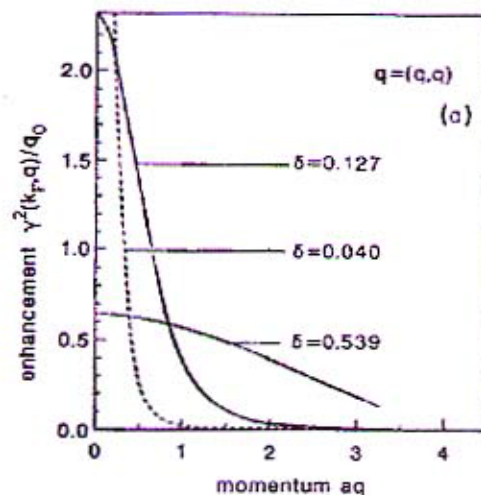
	$k_F \ll \frac{\pi}{2a}$	$k_F \simeq \frac{\pi}{2a}$
large E_F		
small E_F		✓

Electron-phonon interaction in strongly correlated metals

- Narrow bands $\Rightarrow E_F \sim U$
 $\Rightarrow$ strong correlation



Large correlation hole induces **small q -momenta selection**: $q_c \propto 1/\xi$



[M. Kulić and R. Zeyher, PRB1994]

High T_c in nonadiabatic metals

Low charge carrier density



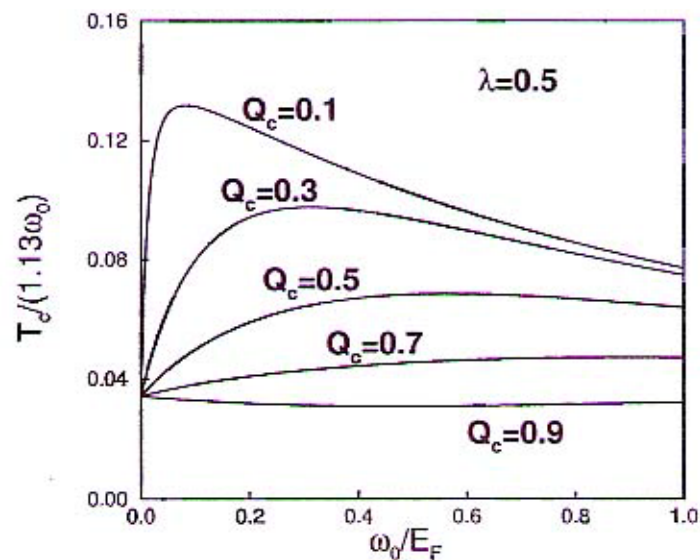
small E_F



nonadiabatic regime
(+ strong correlation)



enhancement of superconducting pairing



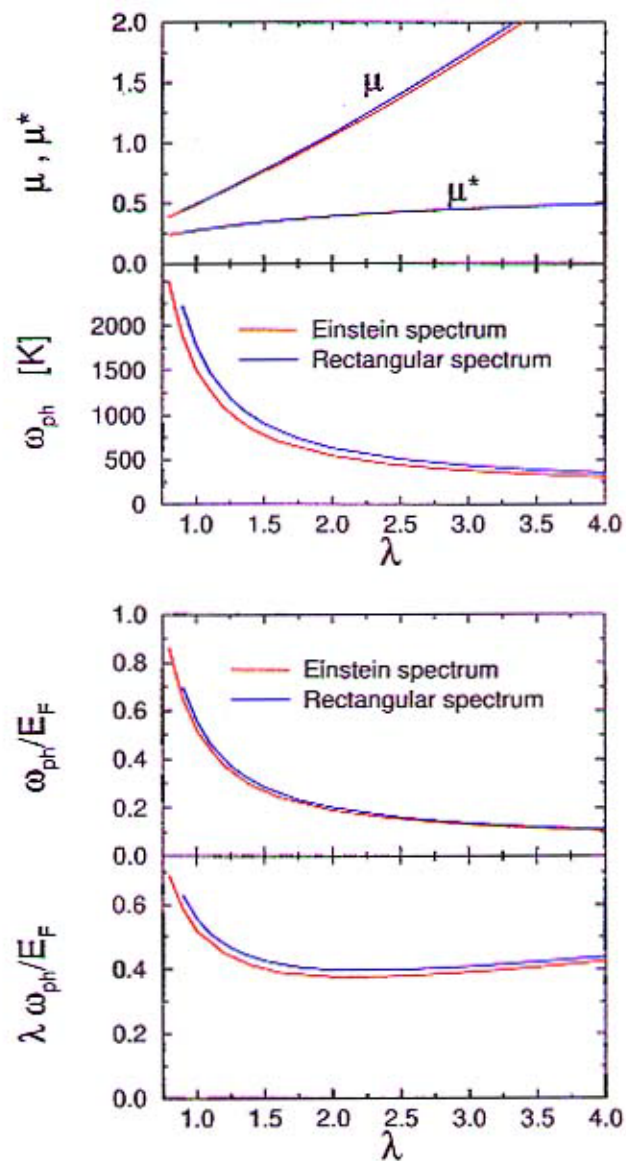
High- T_c : **conventional** parameters in a **new** theory

Superconductivity Rb_3C_{60}

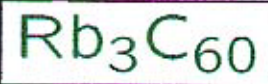
Inconsistency of ME theory

- $T_c = 30 \text{ K};$

- $\alpha_{T_c} = 0.21$

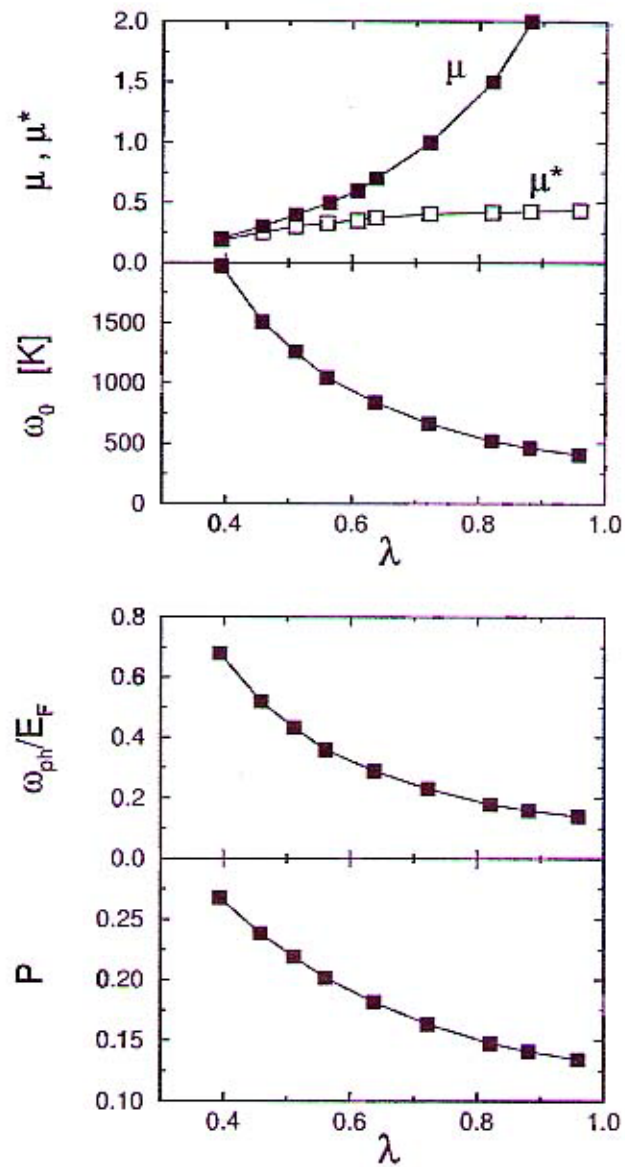


Nonadiabatic analysis



- $T_c = 30 \text{ K};$

- $\alpha_{T_c} = 0.21$



Nonadiabatic characteristic features

Experimental tests of nonadiabatic effects:

anomalous features **not expected** in conventional theory

In general, **isotope** effects provide important informations about nonadiabatic interaction

- Isotope effect on quantities expected **not to have it**

Effective mass m^*

$$\Sigma = \text{[Feynman diagram: a horizontal line with an arrow pointing right, connected at both ends to a wavy loop above it, representing a self-energy correction.]}$$

$$\frac{m^*}{m} = 1 - \left. \frac{\Sigma(i\omega_n)}{i\omega_n} \right|_{n=0}$$

In Migdal-Eliashberg: $m^*/m = 1 + \lambda$

$\Rightarrow$ no isotope effect

No more true in the nonadiabatic theory

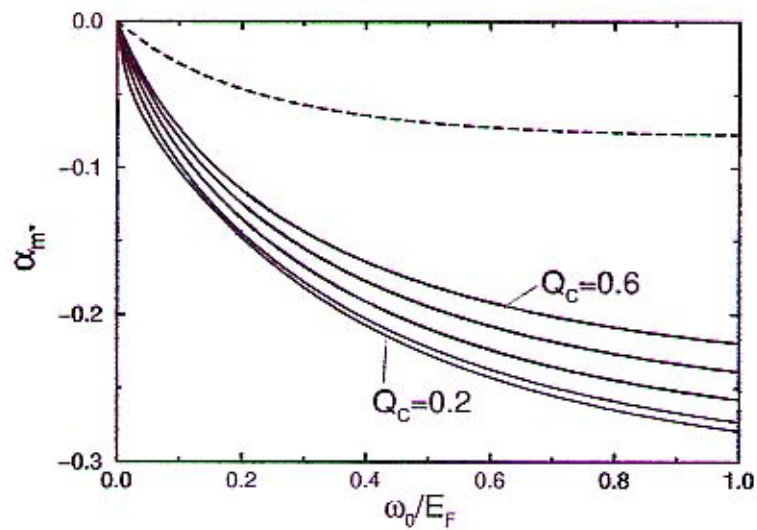
Nonadiabatic effective mass m^*

$$\Sigma = \text{[Feynman diagram 1]} + \text{[Feynman diagram 2]}$$

The first diagram shows a horizontal fermion line with an arrow pointing right, starting and ending at dots. A wavy line (phonon) is attached to the top of this line. The second diagram shows a horizontal fermion line with an arrow pointing right, starting and ending at dots. Two wavy lines are attached to this line: one to the top and one to the bottom, forming a loop.

- $m^*(\lambda) \longrightarrow m^*(\lambda, \omega_{\text{ph}}/E_F)$

Finite isotope effect



Pauli spin susceptibility χ

$$\chi = -\lim_{q \rightarrow 0} \mu_B^2 T \sum_{k,n} G(k, \omega_n) G(k+q, \omega_n) \times \Gamma_s(k+q, k; \omega_n)$$

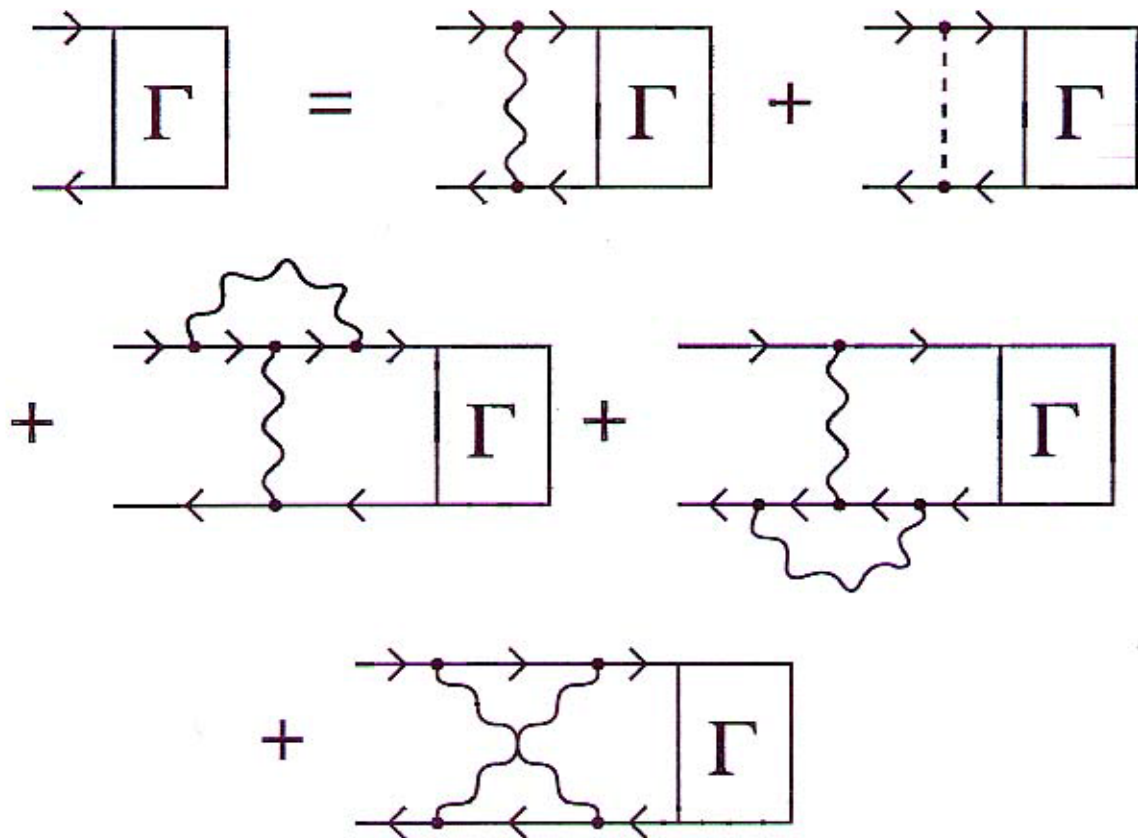
- Migdal-Eliashberg

$$\Gamma = \Gamma + \text{phonon correction} + \text{electron correction}$$

$$\Rightarrow \chi = \mu_B^2 N(E_F) \quad [\mu_B^2 N(E_F)/(1+I)]$$

- no el-ph renormalization of χ
- no isotope effect on spin susceptibility

Nonadiabatic Pauli susceptibility



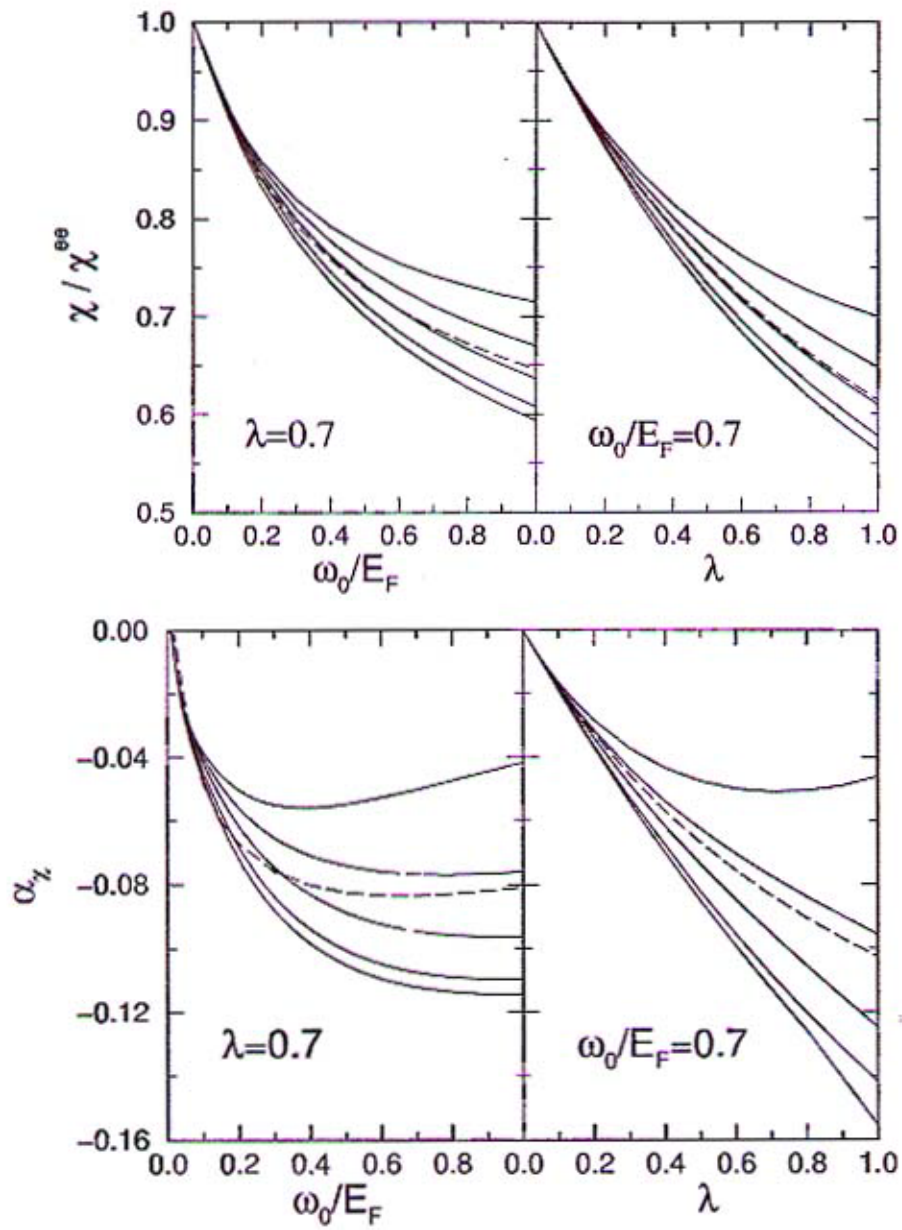
again:

- $\chi \longrightarrow \chi(\lambda, \omega_{\text{ph}}/E_F)$

$\Rightarrow$ Electron-phonon renormalization

$\Rightarrow$ Finite isotope effect

Dependence of χ on λ and ω_{ph}

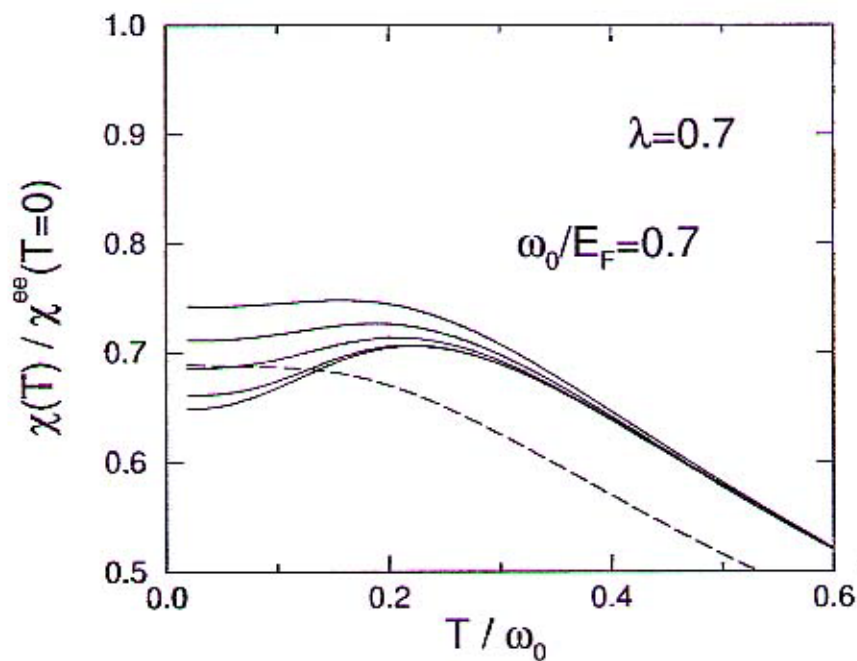


Temperature dependence of χ

In Migdal-Eliashberg: $\chi = \mu_B^2 N(E_F)$ constant in temperature $T \ll E_F$

Finite bandwidth effects: decrease of χ with temperature

Non monotonic temperature dependence in **nonadiabatic** theory



Conclusions

- Fullerenes: molecular crystal
- Small Fermi energies
- New physics (nonadiabaticity)
- Anomalous metal properties
- Anomalous superconductivity (high- T_c)
- New perspectives (material engineering)