Study of hadronic form factors relating DAΦNE and BABAR

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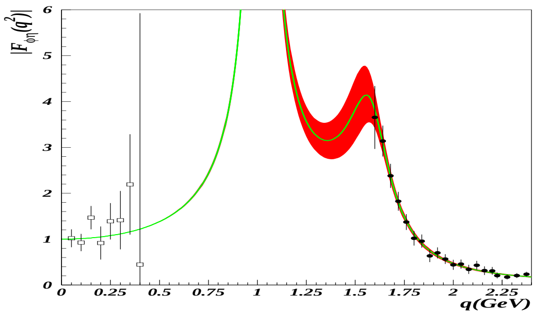
General outline

- Analytic continuation of the $\phi\eta$ transition form factor (tff)
  - We find that the steep enhancement of the cross-section data near by the threshold is the tail of a resonance, with mass $\sim 1600\ MeV$ and width $\sim 200\ MeV$.



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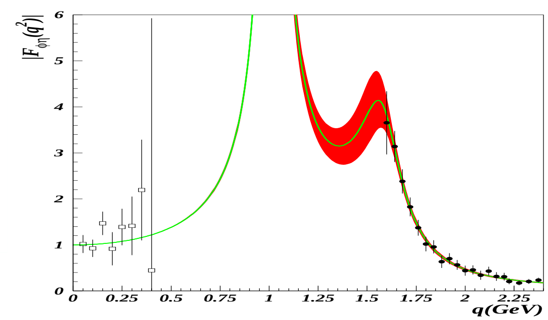

- Analytic considerations on the $\phi\pi^0$ tff
  - A theoretical analysis of the $\phi\pi^0$ tff reveals that the only expected resonance ρ^0 is not enough to justify the value of the tff at $q^2 = 0$.

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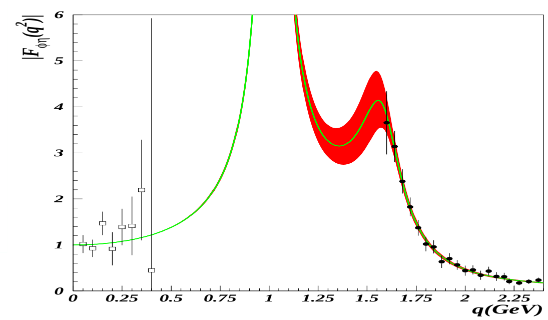

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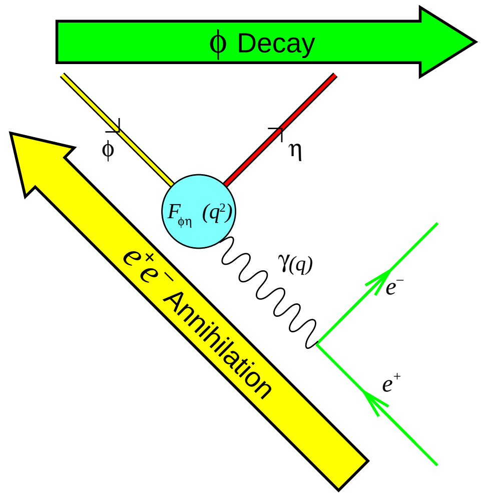
  - A theoretical analysis of the $\phi\pi^0$ tff reveals that the only expected resonance ρ^0 is not enough to justify the value of the tff at $q^2 = 0$.
- Analytic estimate of the decay rate $\Gamma(\phi \to \gamma f_0)$
  - The analysis of the ϕf_0, and $\phi\eta$ BaBar data, performed by using a form factor continuation method, gives an estimate of $BR(\phi \to \gamma f_0)$ which is more than 300 times lower than the PDG value. In the hypothesis of 4-quarks state the discrepancy is reduced.

Extracting the tff from the data



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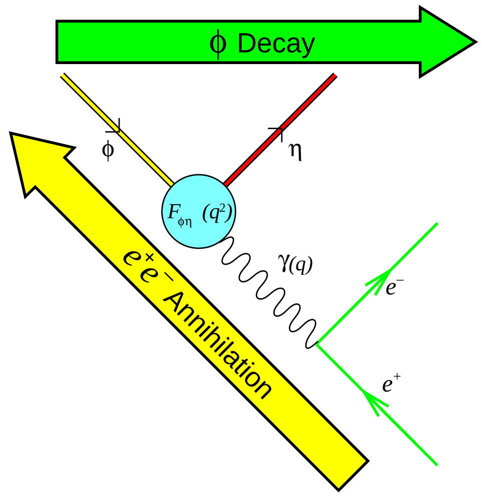
$$e^+e^- \to \gamma^* \to \eta\phi$$



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$$\sigma = \sigma_{\text{QED}} \cdot |f_{\phi\eta}(q^2)|^2$$

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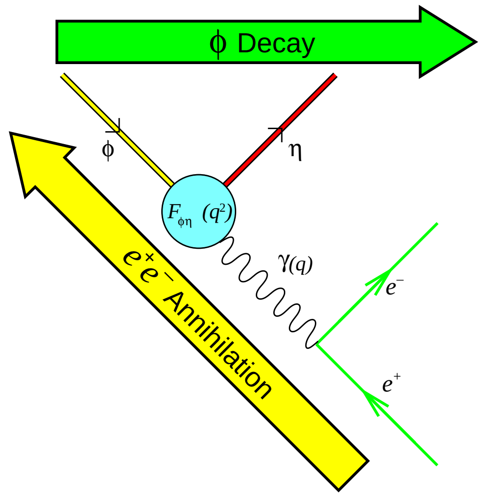

$$\phi \to \gamma^*\eta \to e^+e^-\eta$$

The diff. decay rate is:

$$\frac{d\Gamma}{dq^2} = \left[\frac{d\Gamma}{dq^2}\right]_{\text{QED}} \cdot |f_{\phi\eta}(q^2)|^2$$

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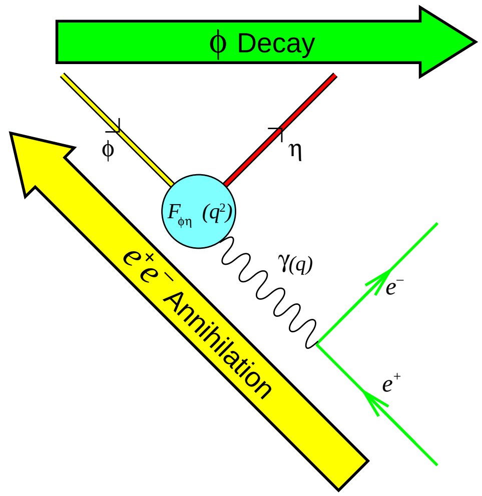

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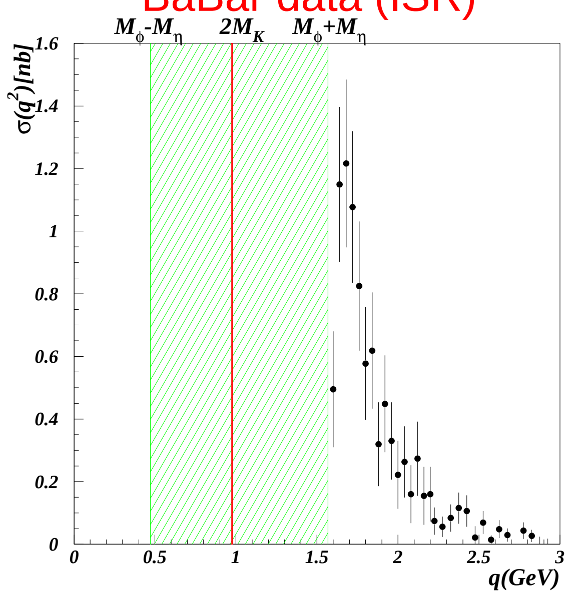

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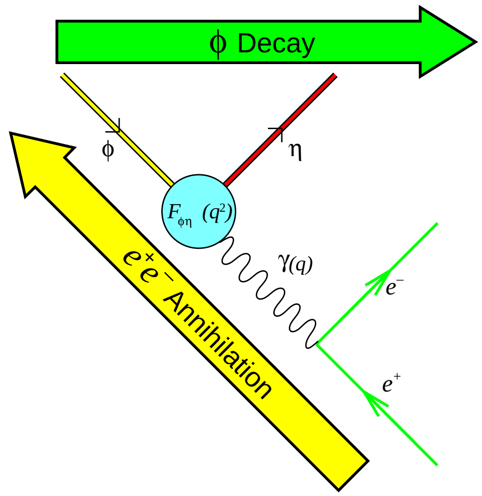

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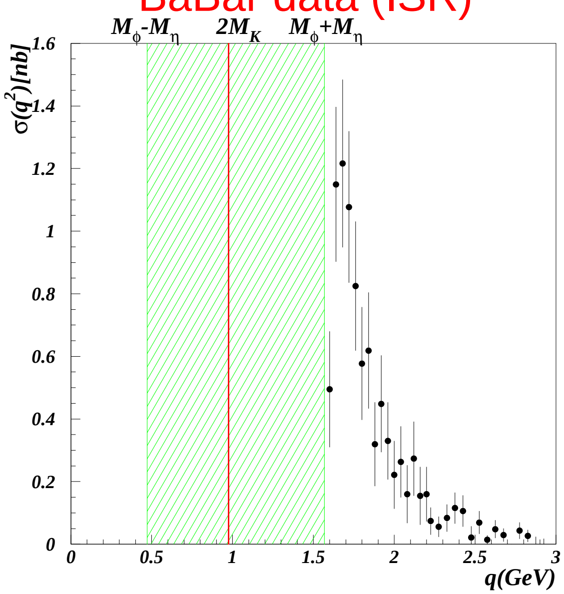

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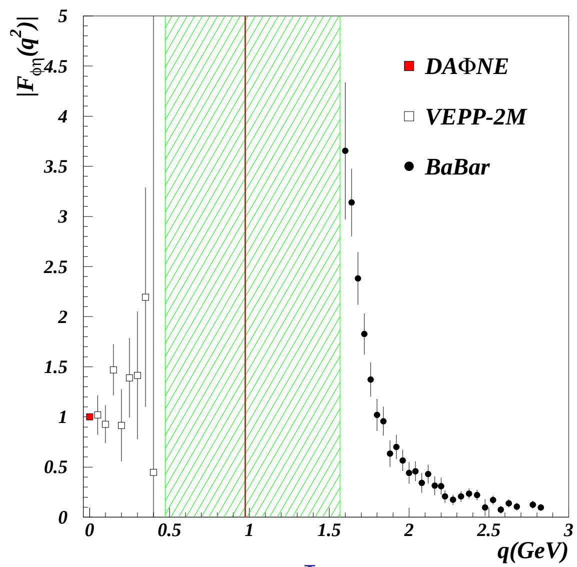

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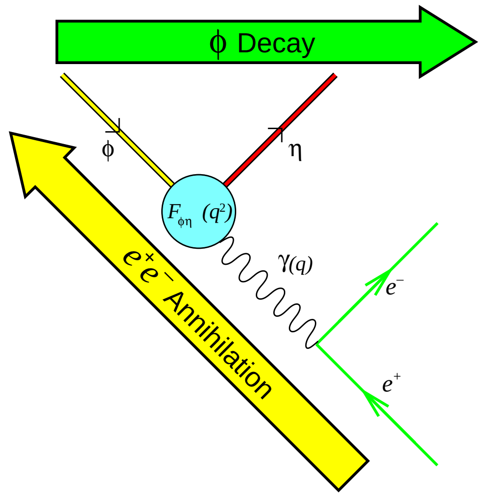

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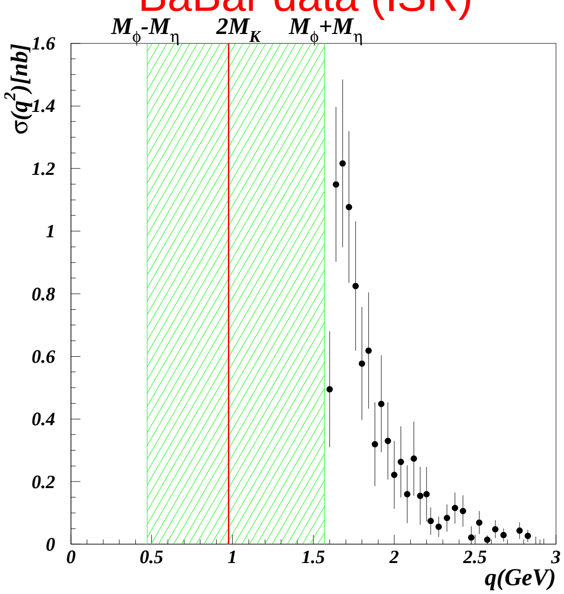

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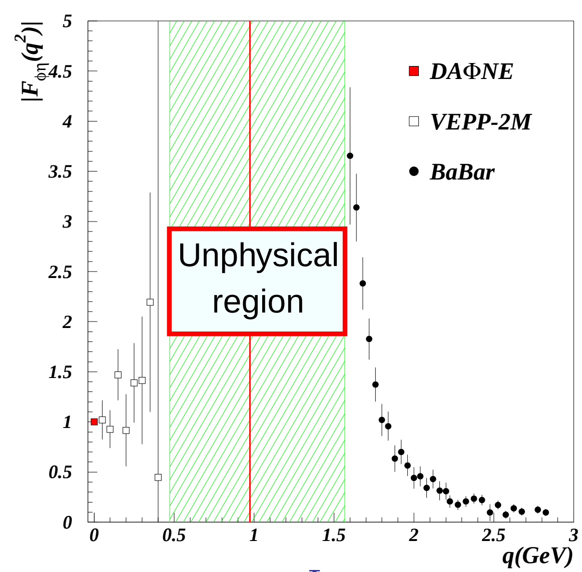

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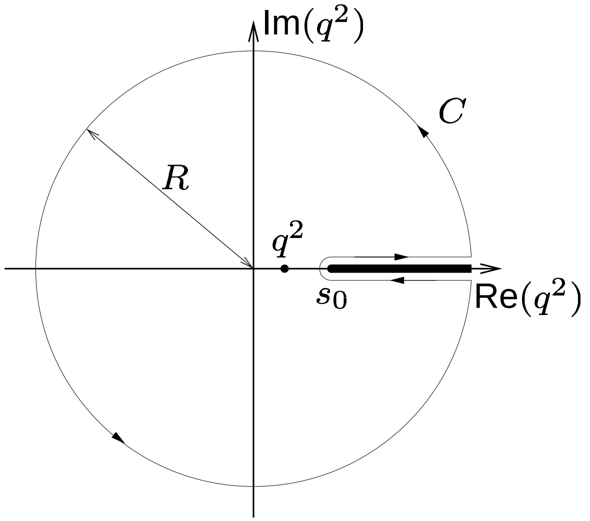

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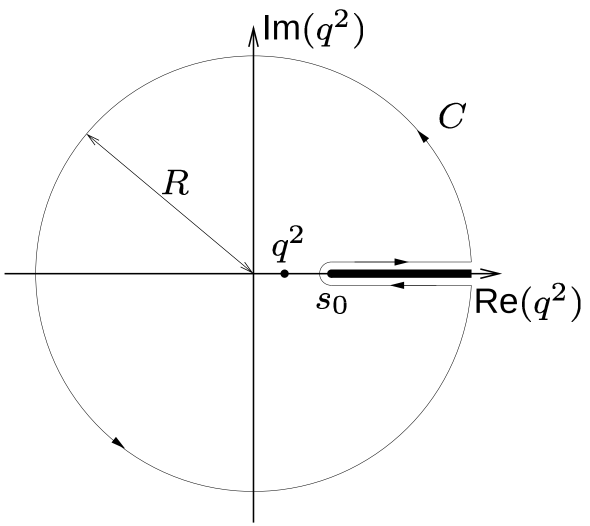

- The modulus $|f(s)|$ only is measured by BABAR $\Rightarrow$ dispersion relations on $\ln(f)$, if no important zero in the physical sheet (check in the following):

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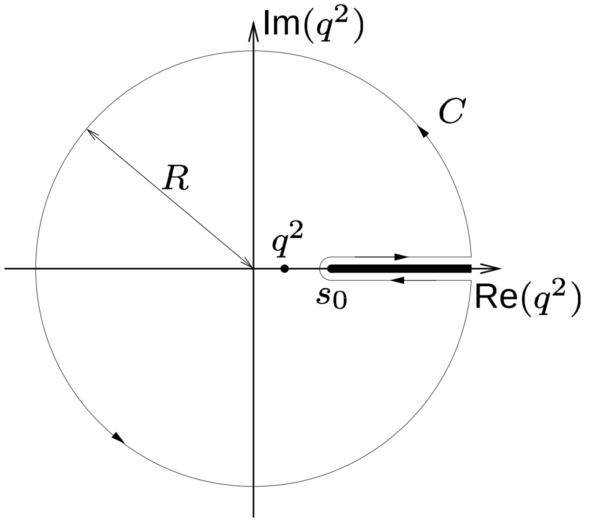

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- Furthermore:

$$\delta(s) = -\frac{\sqrt{s - s_0}}{\pi}\,\text{Pr}\int_{s_0}^{\infty} \frac{\ln|f(s')|ds'}{(s' - s)\sqrt{s' - s_0}}, \qquad s > s_0$$

The phase gives unique information about any bump of $f(s)$

First Model for $F_{\phi\eta}(q^2)$

What we know

- $\phi(1020)\eta$ final state
  Isospin $= 0$
  2 $s\bar{s}$ valence quark pairs

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First Model
tff dominated by
the $\phi(1020)$

$$F_{\phi\eta}(s) = \begin{cases} BW_\phi(s) = a_\phi \frac{-M_\phi\Gamma_\phi}{M_\phi^2 - s - i\Gamma_\phi M_\phi} & s \leq s_{\text{asy}} \\ BW_\phi(s_{\text{asy}}) \left(\frac{s_{\text{asy}}}{s}\right)^2 & s > s_{\text{asy}} \end{cases}$$

χ^2 definition and first result

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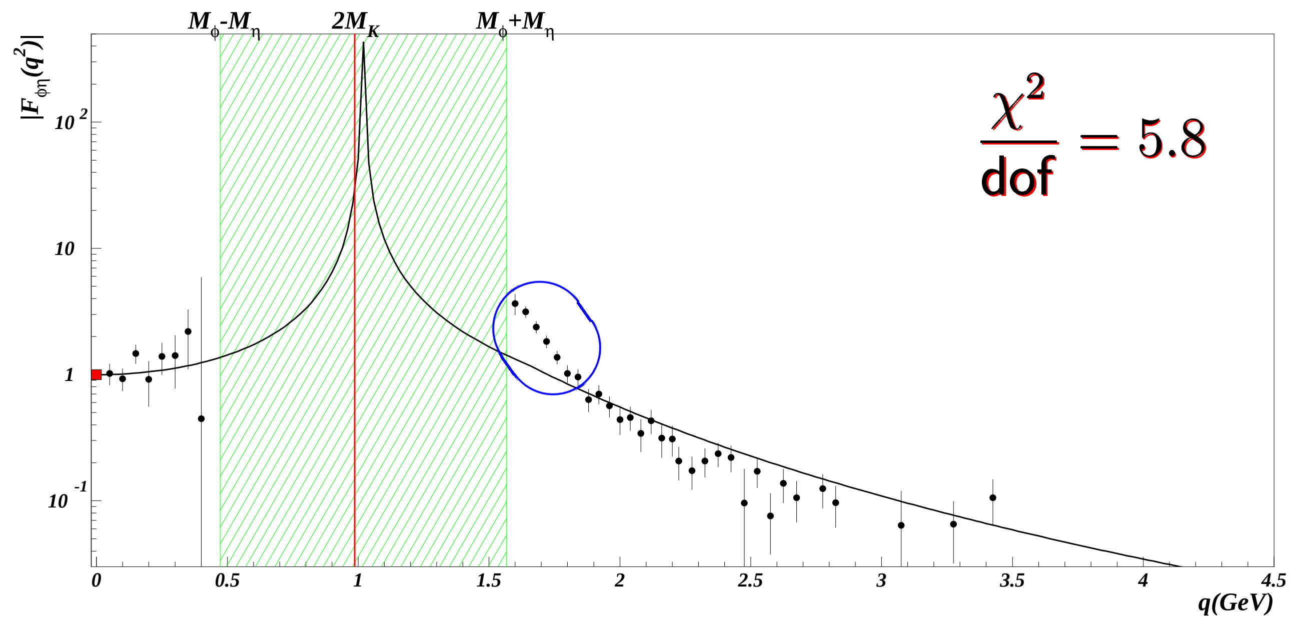
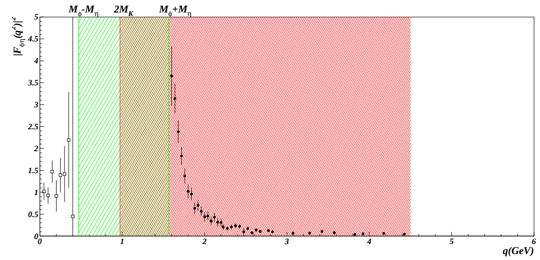
Only $\phi(1020)$



A more complete approach

- In the unphysical region above the threshold and in BABAR data region $[(2M_K)^2, (4.5\ GeV)^2]$:

$$F(s) = BW_\phi(s) + \sum_{j=1}^{N} C_j T_j(x)$$



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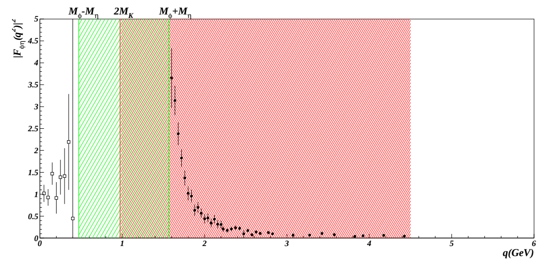

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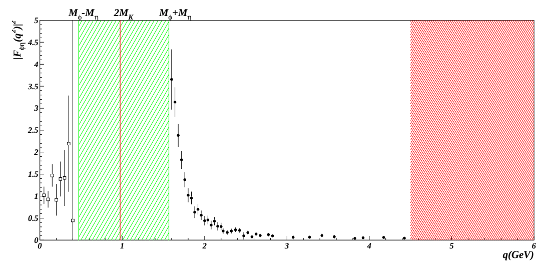
- In the asymptotic region $s > (4.5\ GeV)^2$, we use:

$$F(s) \propto \left(\frac{1}{s}\right)^2$$



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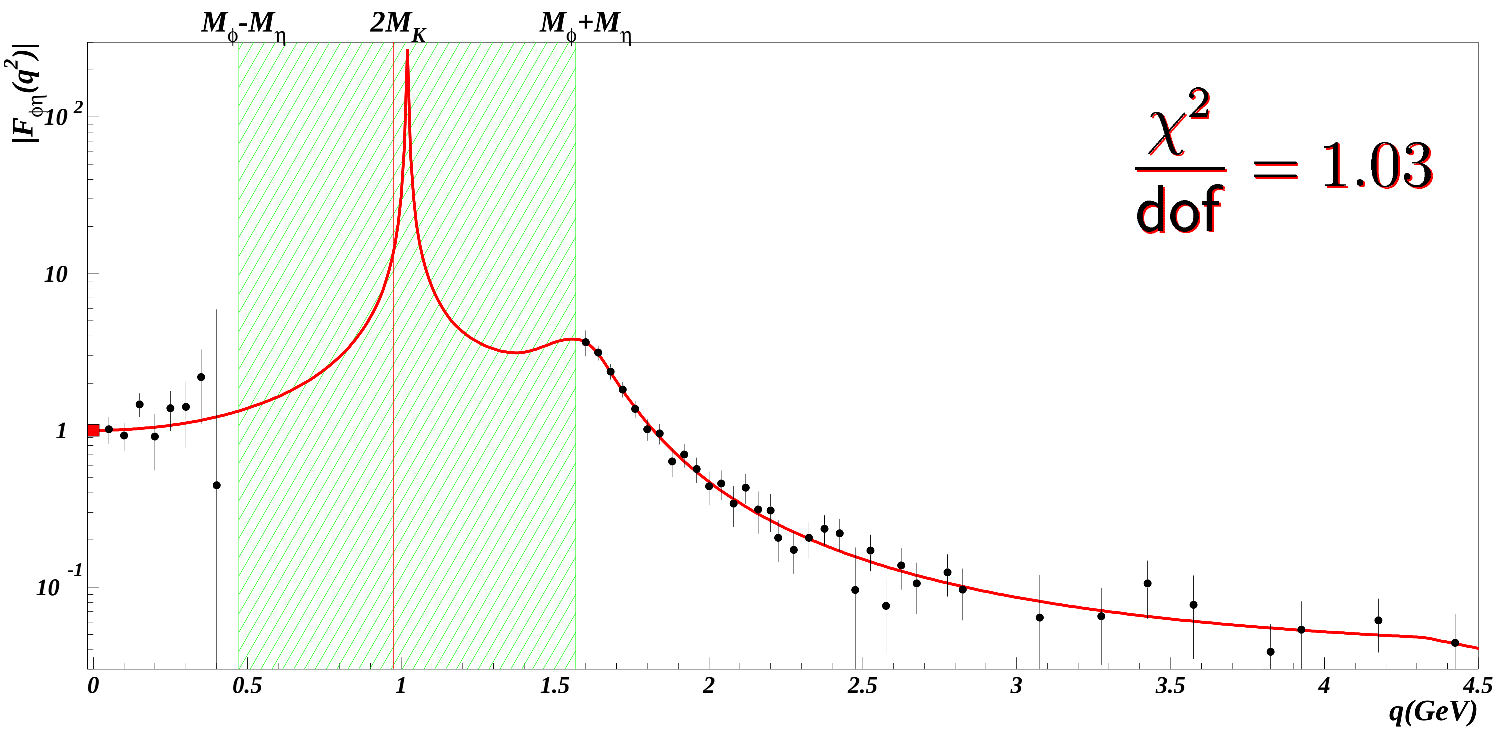

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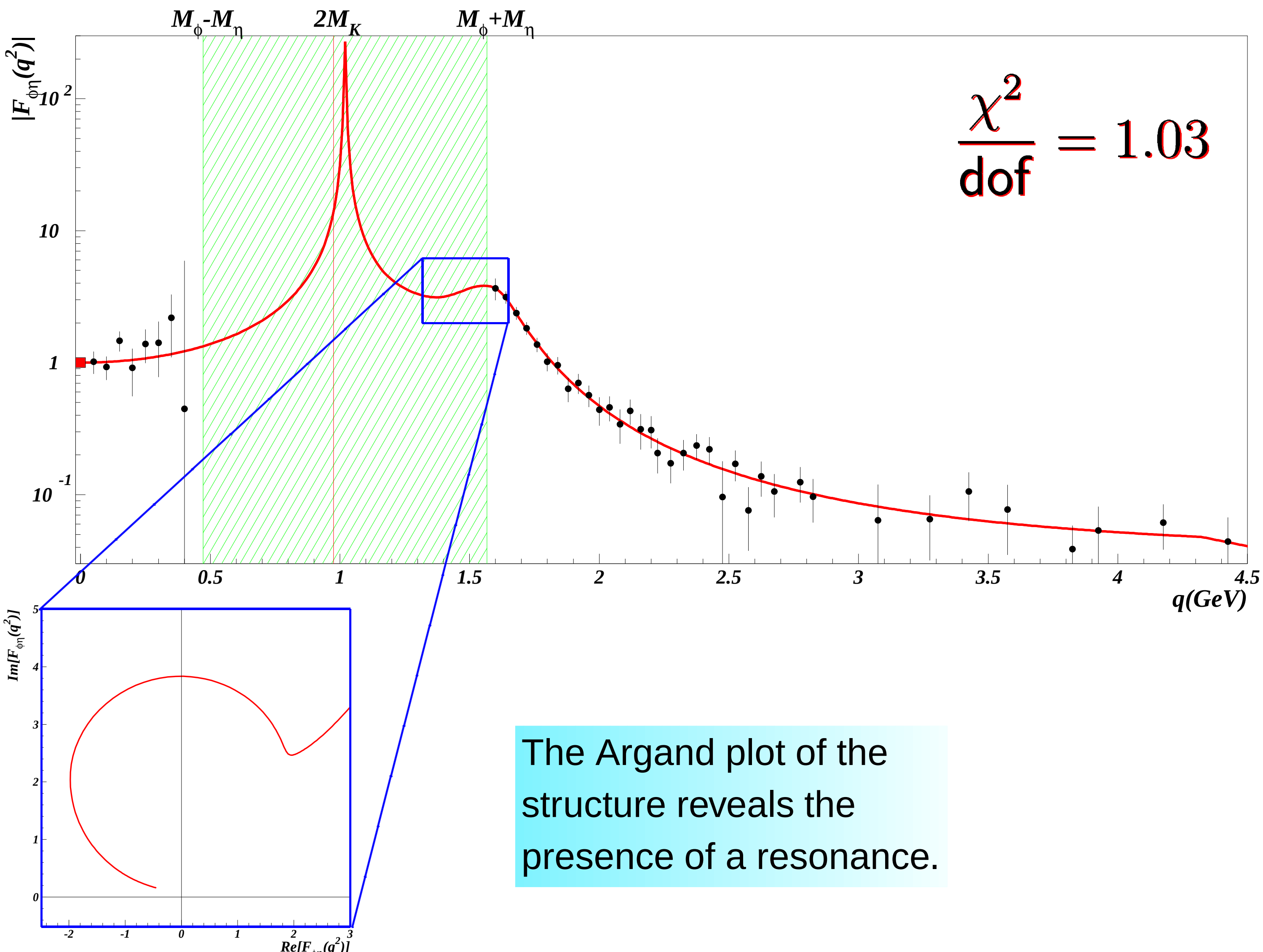

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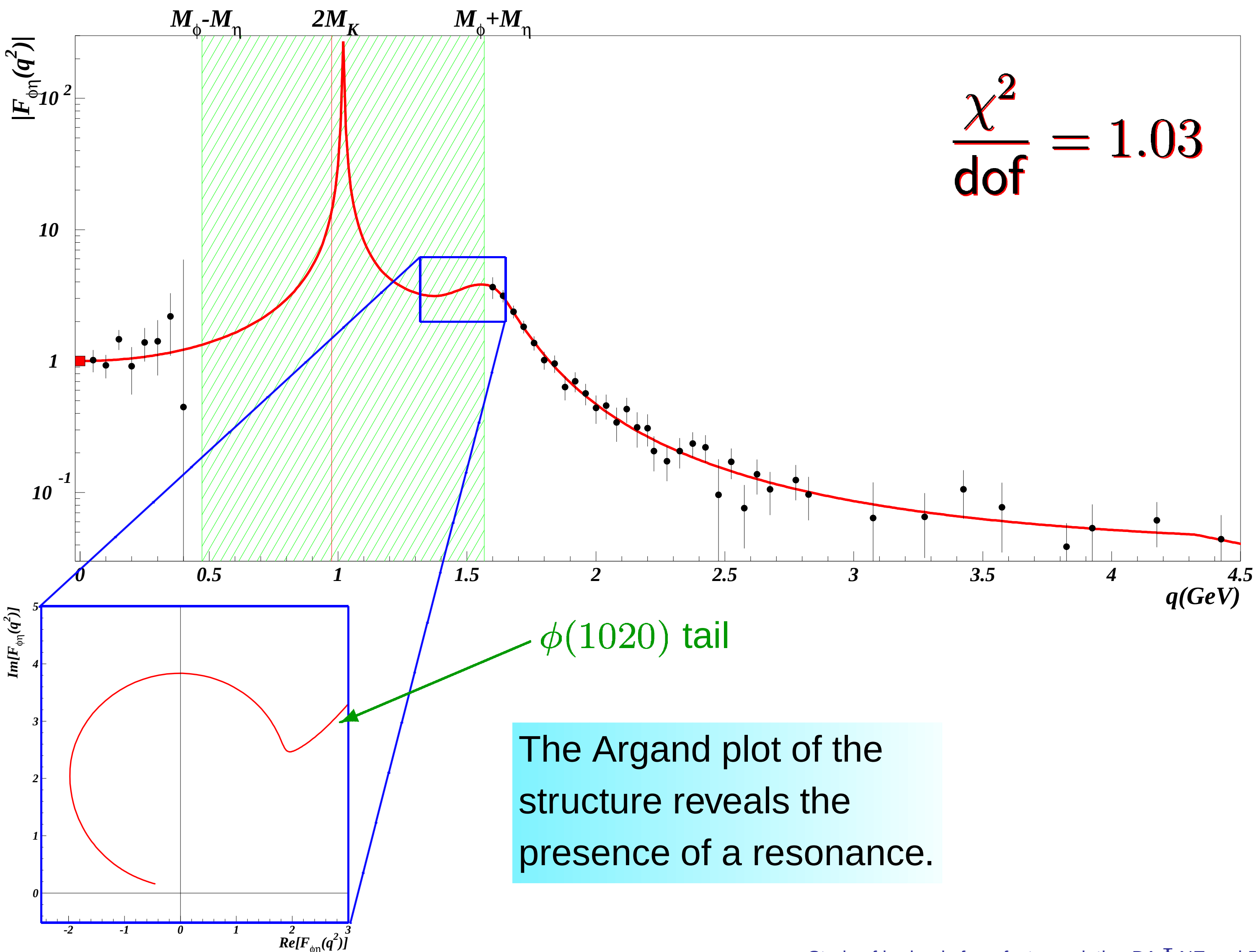

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Analysis of the $\phi(1600)$ candidate

To analyze the properties of the pole we use this function:

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the coefficients r_j are complex.

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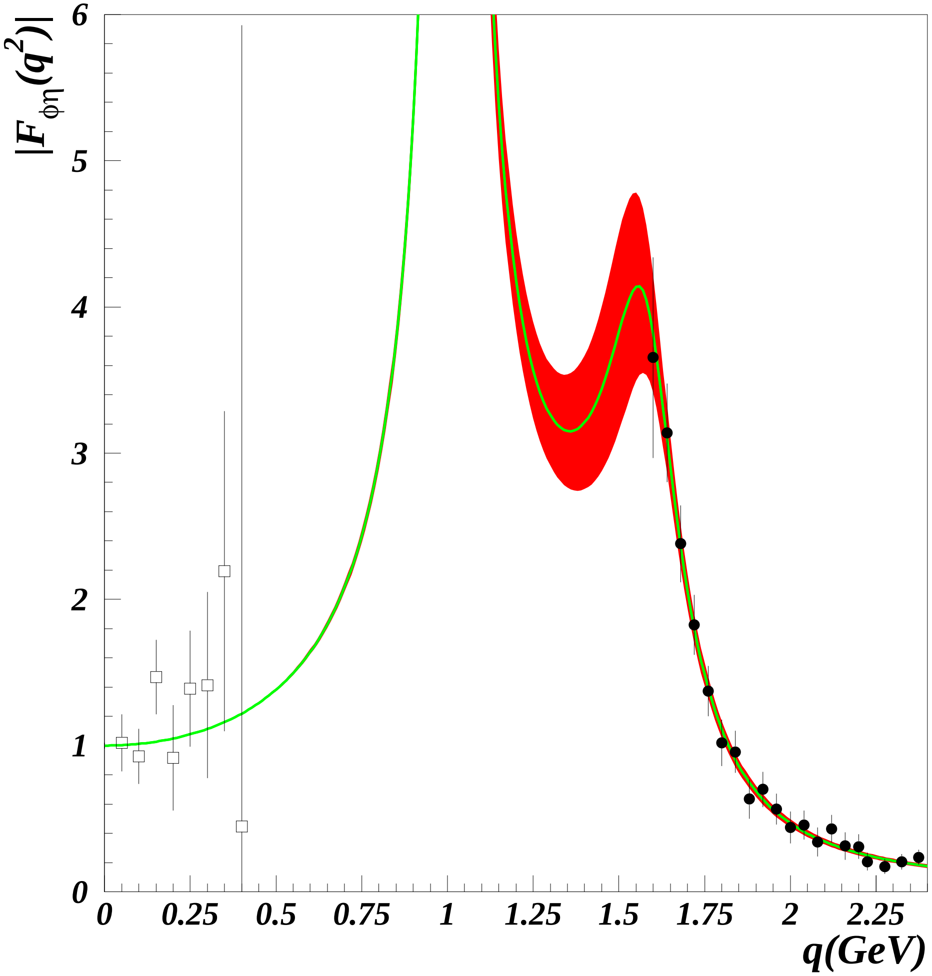

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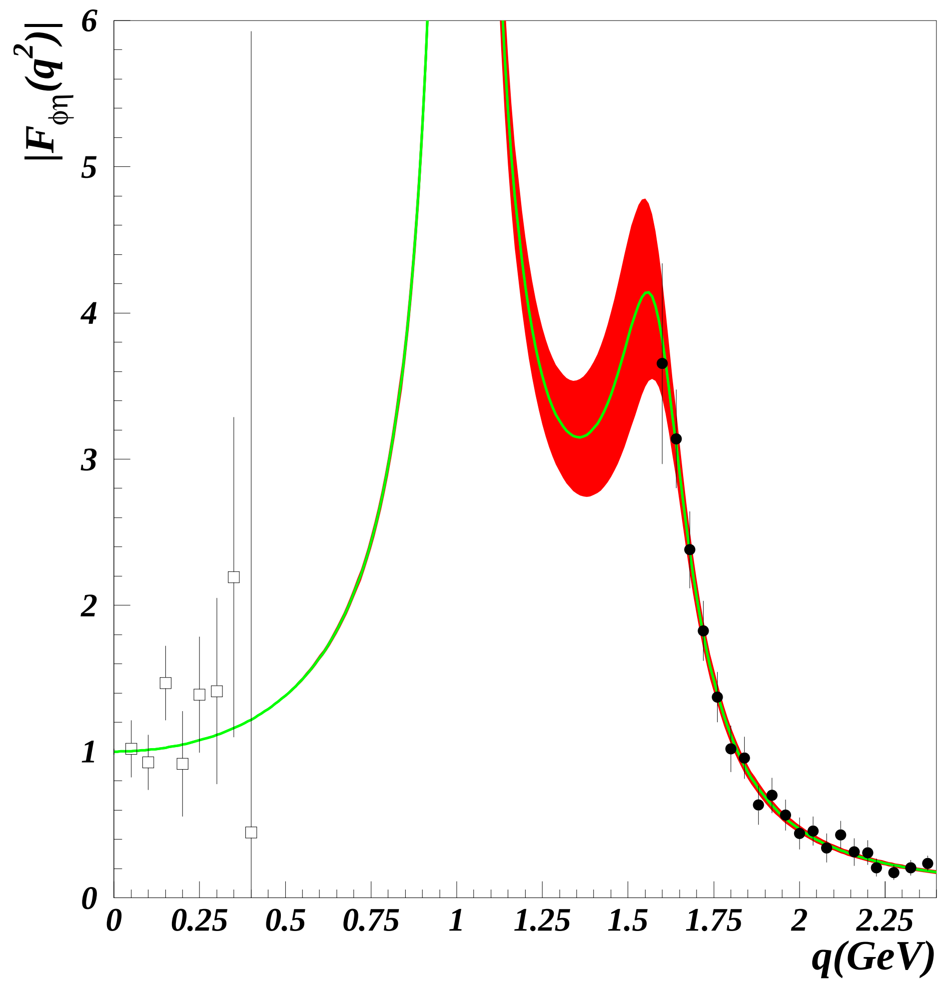
$\phi(1600)$	
M	$(1582 \pm 25)\ MeV$
Γ	$(198 \pm 30)\ MeV$
Phase	$(0.09 \pm 0.28)\ rad$



Phase and consistency check

By using the DR:

$$\delta(s) = -\frac{\sqrt{s-s_0}}{\pi}\,\mathrm{Pr}\int_{s_0}^{\infty}\frac{\ln|F_{\phi\eta}(s')|}{(s'-s)\sqrt{s'-s_0}}ds',$$

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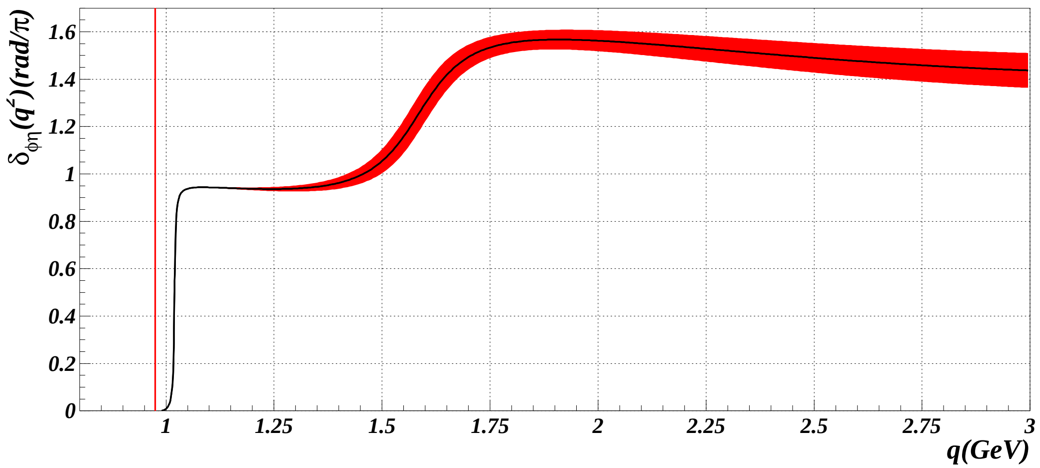

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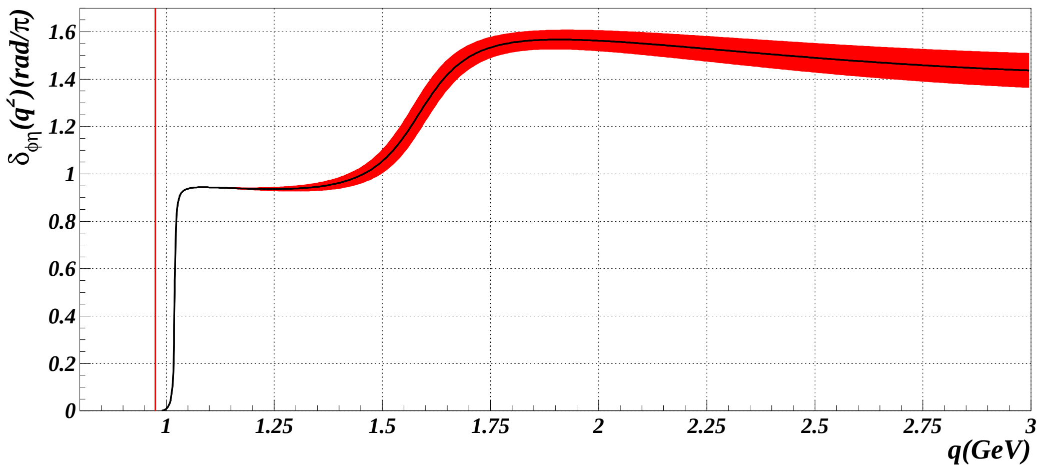

By using the dispersion relation for the imaginary part, the value at $s=0$ can be tested:

$$F_{\phi\eta}(0) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im}F_{\phi\eta}(s)}{s}ds = 1.0008 \pm 0.0025$$

the expectation is 1.

This result is a check of:

- the self-consistency of the approach;
- the hypothesis of no important contributing zero.

The $\phi\pi^0$ transition form factor

What we know

- $\phi(1020)\pi^0$ final state
  Isospin $= 1$
  One $s\bar{s}$ valence quark pair

$\Longrightarrow$

ρ-recurrences only could contribute.
Double-suppressed:

- OZI rule
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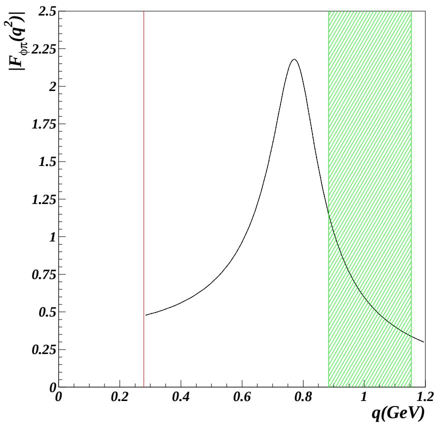
- The main known contribution to the tff should come from: $\phi \to \pi^0\rho^0$, $\rho^0 \to e^+e^-$
- By means of the analyticity and dispersion relations we can establish if another contribution, in addition to the known ρ^0, is needed...

Contribution at $s = 0$ of the ρ^0 form factor and data

By extrapolating the value at $s = 0$ of the ρ^0 contribution to the $\phi\pi^0$ tff one obtains:

$$\exp\left[\frac{\sqrt{s_0}}{\pi}\int_{s_0}^{\infty}\frac{\ln|F_{\rho^0}(s)|}{s\sqrt{s-s_0}}ds\right] \simeq 0.4.$$

The expected value in the case of ρ^0 domination is 1.



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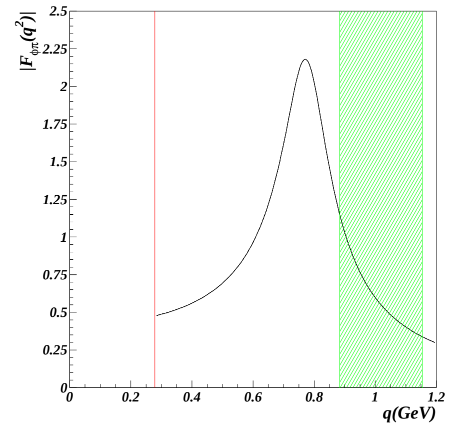
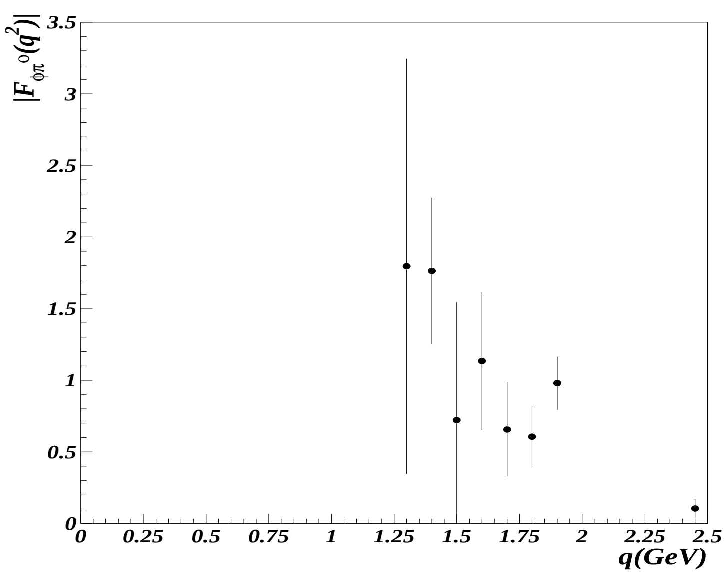
$$\exp\left[\frac{\sqrt{s_0}}{\pi}\int_{s_0}^{\infty}\frac{\ln|F_{\rho^0}(s)|}{s\sqrt{s-s_0}}ds\right] \simeq 0.4.$$



The expected value in the case of ρ^0 domination is 1.



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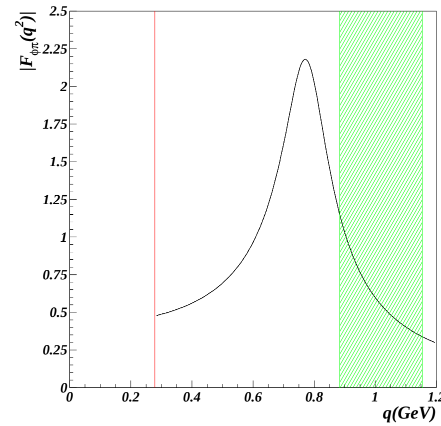

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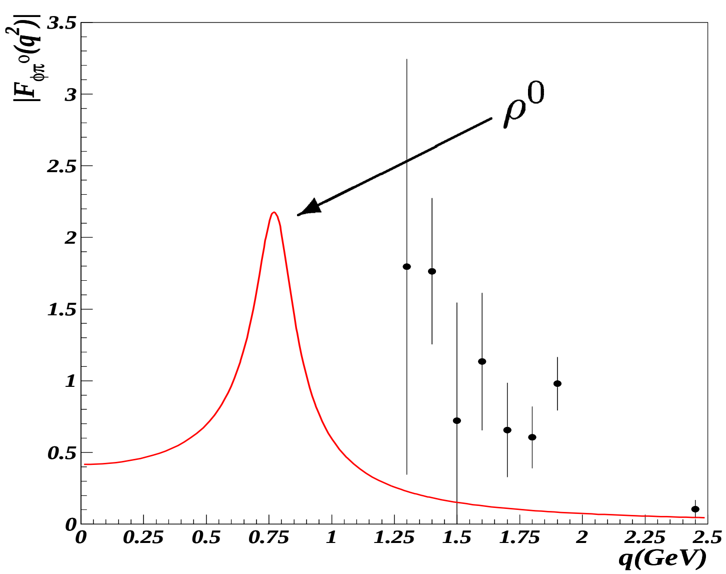

Preliminary data of the $\phi\pi^0$ tff appear in agreement with the hypothesis of the presence of other contributions in addition to the ρ^0 one.

Conclusions

- A general procedure has been applied to link the amplitudes of: $\phi \to M\gamma$, $\phi \to M e^+ e^-$ and $e^+ e^- \to \phi M$, where M is a generic light meson.
- In the case $M \equiv \eta$, by using the DAΦNE, VEPP-2M and BABAR data as inputs in a dispersive approach, the $\phi\eta$ tff has been reconstructed in the whole time-like region, in a model independent way.
- The analysis of this tff reveals a resonance, $\phi(1600)$, near by the threshold.

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- In the case $M \equiv \pi^0$, a theoretical analysis of the $\phi\pi^0$ tff shows that the contribution of the ρ^0 meson seems unable to explain the value of the tff at $s = 0$.
- Preliminary data of the cross-section for the process $e^+e^- \to \phi\pi^0$ appear in agreement with the hypothesis concerning the presence of other contributions in addition to the ρ^0 one.
- The analysis of the data are still in progress and the present statistics does not permit to verify any hypothesis concerning the nature of this further contribution.