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Quantum Loops in the Resonance Chiral Theory: The Vector Form Factor

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May 2004

1. Introduction

MOTIVATION

- Confinement: We do NOT know how to work with the $\mathcal{L}_{\rm QCD}$ at low energies $\longrightarrow$ effective field theories
- At intermediate energies ($M_\rho \lesssim E \lesssim 2\,{\rm GeV}$) there is no natural expansion parameter $\longrightarrow$ expansion in $1/N_C$
- The Effective Theory for QCD at intermediate energies which uses $1/N_C$ as a expansion parameter $\longrightarrow$ Resonance Chiral Theory (RχT)

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PROBLEM!	SOLUTION
The renormalization is not clear: the theory has been used only at tree level.	A study of any function at NLO is needed → VFF of the pion.

EFFECTIVE FIELD THEORIES (EFT)

The only remnant of the high-energy dynamics are in the low-energy couplings and in the symmetries of the EFT.

- Theoretical interpretation: integration of the heavy particles

$$e^{i\,\Gamma_{eff}[\Phi_l]} = \int [\mathrm{d}\Phi_h]\; e^{iS[\Phi_l,\Phi_h]}$$

- Example: the Fermi Theory of Weak Interactions

- The choice of the suitable degrees of freedom is fundamental.
- Interchanges with heavy particles ⟶ local operators with light particles.
- They can be worked as renormalizables if one stops at certain precision.

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where $g_{L,R} \in SU(n_f)_{L,R}$. As the symmetry is global, one expects an influence into the hadronic spectroscopy:

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- With dimensional regularization the counterterms are in the Lagrangian.

THE $1/N_C$ EXPANSION

Which expansions can be used for QCD EFT's?

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- $M_\rho \lesssim E \lesssim 2\,\mathrm{GeV}$: **???**

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- Main features in the limit $N_C \to \infty$

  1. Mesons and glue states are free and stable.
  2. The number of meson states is infinity.
  3. The hadronic physics can be studied trough an effective Lagrangian with mesons $\longrightarrow$ EFT's.

THE RESONANCE CHIRAL THEORY (RχT)

- Basic features

1. EFT for $E \sim M_R$ ($M_\rho \lesssim E \lesssim 2\,\mathrm{GeV}$).
2. Expansion in powers of momentum does not work ⟶ $1/N_C$ expansion.
3. Goldstone bosons and resonances as degrees of freedom:

$$\mathcal{L}_{R\chi T}(u,V,A,S,P) \; = \; \mathcal{L}_u(u) \; + \; \mathcal{L}_R(u,V,A,S,P)\,.$$

4. Construction of the Lagrangian *à la* CCWZ, using the chiral symmetry.
5. The antisymmetric formalism is used for the massive spin-1 fields.
6. The matching with QCD bans the introduction of $\mathcal{L}_{\chi PT}$ local terms of $\mathcal{O}(p^4)$ or higher at LO in $\mathcal{L}_{R\chi T}$ and fixes the values of the couplings.

2. VFF in RχT at NLO

[I. ROSELL, J.J. SANZ-CILLERO, A. PICH, in preparation]

- The VFF of the pion for the neutral current, $\mathcal{F}(q^2)$, is defined through the matrix element:

$$\langle\, \pi^+(p_1)\, \pi^-(p_2)|\, \left(\frac{1}{2}\overline{u}\gamma_\mu u \;-\; \frac{1}{2}\overline{d}\gamma_\mu d\right)\, |0\,\rangle \;=\; \mathcal{F}(q^2)\,(p_1 - p_2)^\mu\,.$$

- The VFF at tree level has the contributions

,

which give the result

$$\mathcal{F}_{LO}(q^2) \;=\; 1 \;+\; \frac{F_V G_V}{F^2}\,\frac{q^2}{M_V^2 - q^2}\,.$$

- In our work this observable is studied at NLO.

GREEN FUNCTIONS

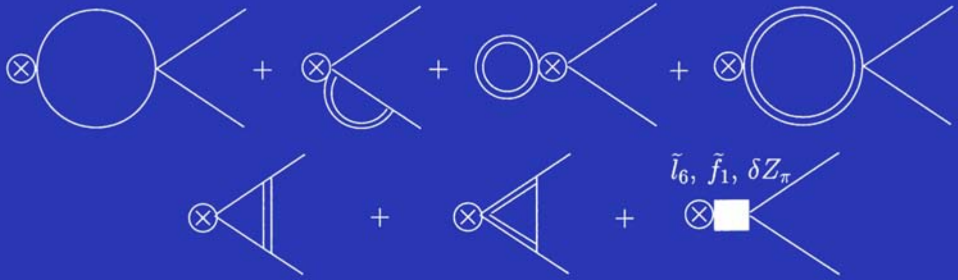
1. Pion self-energy

2. ρ self-energy

3. $\overline{q}\gamma^\mu q \longrightarrow V_{\rho\sigma}$ vertex

4. $V_{\mu\nu} \longrightarrow \pi\pi$ vertex

5. $\overline{q}\gamma^\mu q \longrightarrow \pi\pi$ vertex



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- Future work: functional integration $\longrightarrow$ a fundamental step to work with RχT at NLO.