The Lightest Strange Scalar Meson

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INTRODUCTION

- $f_0(400-1200)$, $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_J(1700)$
- $a_0(980)$, $a_0(1450)$
- $qq\bar{q}\bar{q}$, $K\bar{K}$ molecules, glueballs
- 2 nonets? (1 GeV 4q, 1.4 GeV $q\bar{q}$)
- Only 1 strange scalar, $K_0^*(1430)$
- $\kappa(900)$?
- A resonance $\neq$ BW but is a pole in S-matrix at complex energy.

Mapping

$$y(s) = \left(\frac{s - s_c}{s + s_c}\right)^2$$

$$z(s) = \frac{i\beta\sqrt{y(s)} - \sqrt{y(s) - y(s_{+W})}}{i\beta\sqrt{y(s)} + \sqrt{y(s) - y(s_{+W})}}$$

β real, chosen to minimise distance

- continuation must cover
- Physical data could never cover
- the whole circle.
- Mapping is highly 'non-linear'!

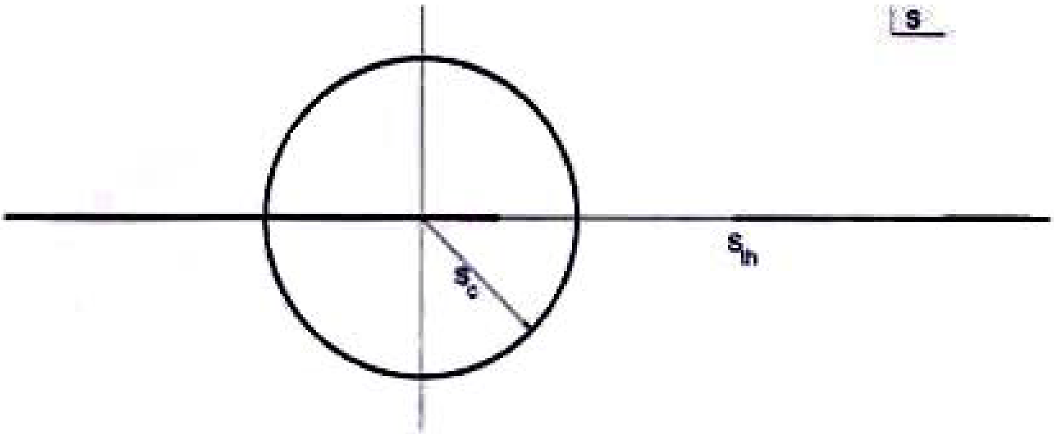


Figure 1: The cut structure of the πK partial wave amplitude, where $s_{th} = (m_K + m_\pi)^2$ and the radius of the circular cut is $s_c = m_K^2 - m_\pi^2$.

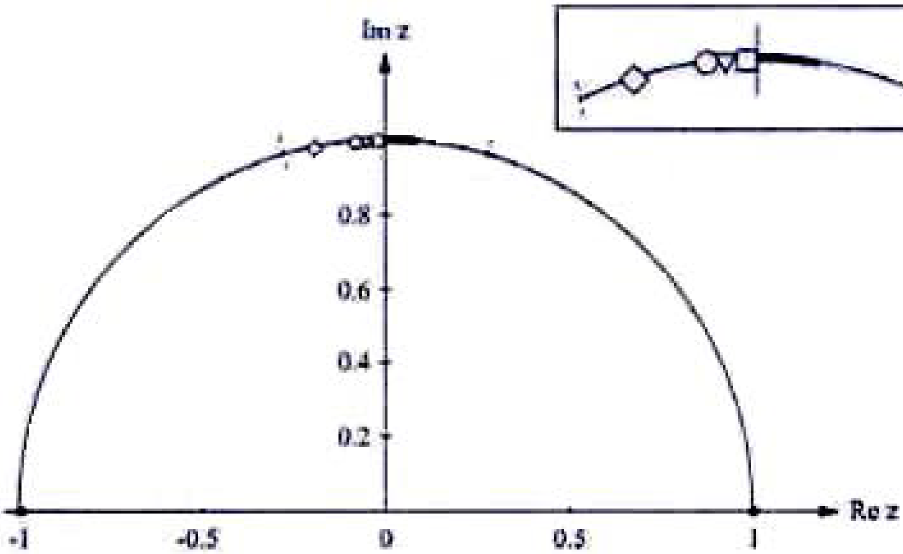


Figure 2: The z-plane showing how points in s-plane map. The mapping parameter β is chosen so that $s = (1.4 + 0.15i)^2$ GeV^2 is mapped to the imaginary axis. The thicker line shows the arc covered by the LASS data. The inset shows an enlargement of the key region close to $z = i$, from where we analytically continue. The symbols mark particular values of s as follows: $\blacklozenge\ s = s_{th}$, $\Box\ s = \infty$, $\nabla\ s = -s_c$, $\circ\ s = is_c$, $\Diamond\ s = \frac{1}{\sqrt{2}}(s_c + is_c)$, $\bullet\ s = s_c$.

CONTINUATION

$Y(z)$ = continuos data function.

$\mathcal{E}(z)$ = " error ".

$F(z)$ = test function.

$$\chi^2 = \frac{1}{2\pi}\oint_c \left|\frac{F(z)-Y(z)}{\mathcal{E}(z)}\right|^2 |dz|$$

· Define · $g(z) = 0$

$g(z^*) = g^*(z)$

$|g(z)| = \mathcal{E}(z)$ on the circle

Write

$$y(z) = \frac{Y(z)}{g(z)} = \sum_{k=-\infty}^{\infty} y_k z^k$$

$$f(z) = \frac{F(z)}{g(z)} = \sum_{k=-\infty}^{\infty} a_k z^k$$

$$\chi^2 = \sum_{k=1}^{\infty} (a_{-k} - y_{-k})^2 + \sum_{k=0}^{\infty} (a_k - y_k)^2$$

To test if amplitude has zero resonances, set $a_{-k} \equiv 0 \quad \forall k > 0$.

Then,

$$\chi_0^2 = \sum_{k=1}^{N} y_{-k}^2 \quad \text{has } \chi^2\text{-distribution}$$

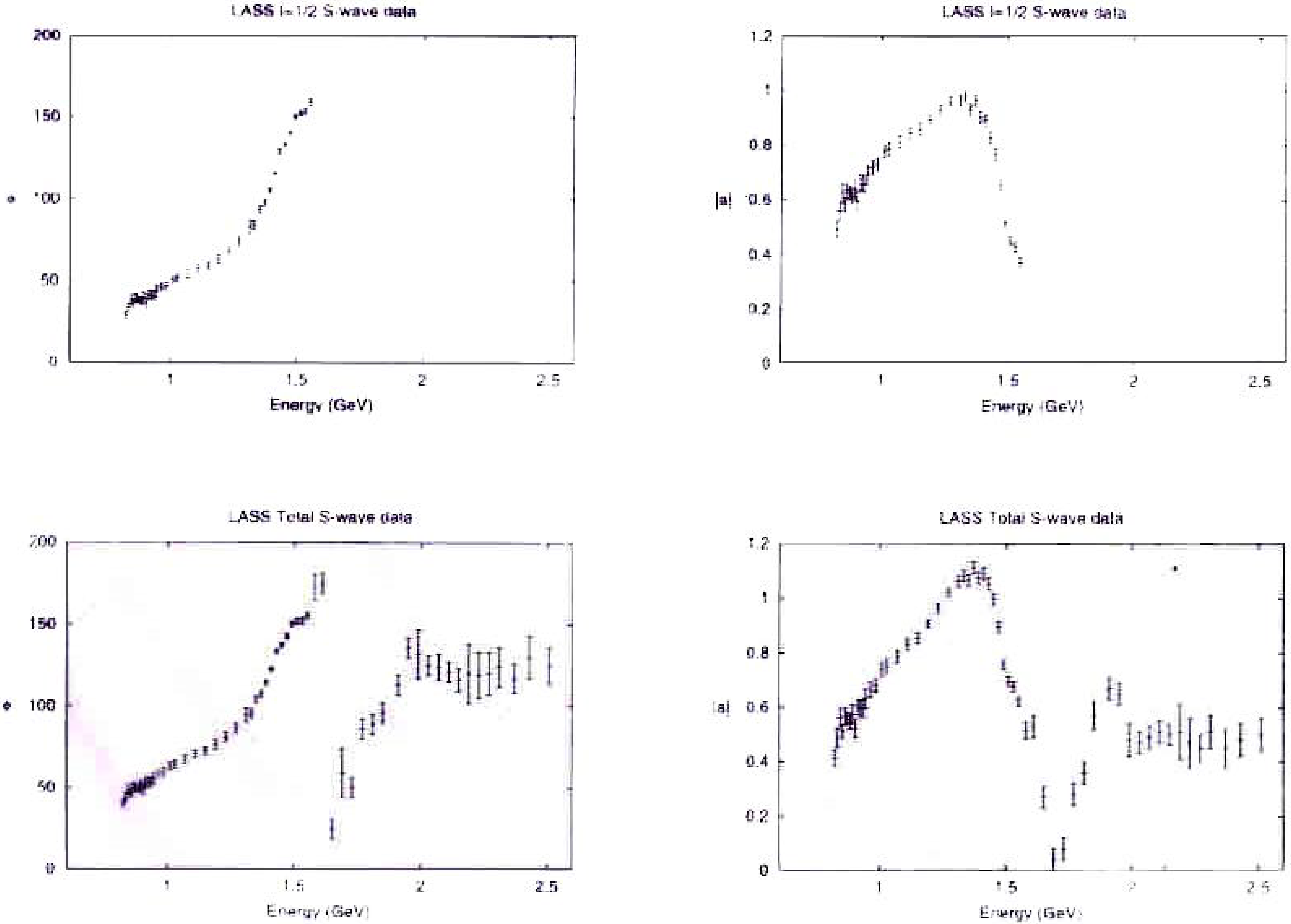
To test if amplitude has one resonance, define

$$\tilde{y}(z) = \frac{B_\lambda(z)\, Y(z)}{g(z)} = \sum_{k=-\infty} \tilde{y}_k z^k$$

with $B_\lambda(z) = \frac{(z-\lambda)(z-\lambda^*)}{(1-z\lambda)(1-z\lambda^*)}$.

Then,

$$\chi_1^2 = \sum_{k=1}^{N} \tilde{y}_{-k}^2 \quad \text{has } \chi^2\text{-distribution.}$$



LASS I=1/2 S-wave data
φ
Energy (GeV)
LASS I=1/2 S-wave data
|a|
Energy (GeV)
LASS Total S-wave data
φ
Energy (GeV)
LASS Total S-wave data
|a|
Energy (GeV)

INPUTS

$$f^{I}(s) = \frac{a(s)\, e^{i\phi(s)}}{\rho(s)} \quad , \quad \rho(s) = \frac{2q(s)}{\sqrt{s}}$$

$$f^{II}(s) = f^{I}(s)_{\phi \to -\phi} \longmapsto f_i(z).$$

$$\Delta_i(s) \longmapsto \Delta_i(z)$$

$$\epsilon_i(z) = \Delta_i \sqrt{\frac{|\delta z|}{2\pi}}$$

- Below data use Lass fit to create a few guide points.

- In unphysical region, we guess a few guide point and assign large errors.

Define

$$g(z) = e^{\sum_{n=1}^{M} c_n z^n}$$

$$f_i(z) \longrightarrow Y(z)$$

$$\epsilon_i(z) \longrightarrow \epsilon(z)$$

$$y_{-k} = \frac{1}{2\pi}\oint_c \frac{Y(z)}{g(z)} z^k \, |dz|.$$

$$\tilde{y}_{-k} = \frac{1}{2\pi}\oint_c \frac{Y(z) B_\lambda(z)}{g(z)} z^k \, |dz|.$$

- We look for value of λ which minimises χ_1^2 and compare final value of χ_1^2 with χ_0^2.
- We carry out a similar procedure to test for 2 resonances and compare χ_2^2 with χ_1^2 and χ_0^2.

Option	No. of resonances	$\sqrt{s_{pole}}$ (MeV)	χ^2
1	0	na	4081
	1	$1201 \pm 131i$	281
	2	$1398 \pm 142i$ $903 \pm 234i$	0.5
2	0	na	3759
	1	$1173 \pm 111i$	417
	2	$1392 \pm 140i$ $900 \pm 233i$	1.0

Table 1: Pole positions and χ^2's for model data. (na = not applicable.)
Option 1 has the unphysical errors set to 5. Option 2 has the unphysical errors set to 2.5.

Max. Energy	χ_0^2	χ_1^2	χ_2^2	λ MeV	λ_1 MeV	λ_2
22.51	2320	43.32	2.015	$1395 \pm 53i$	$1938 \pm 234i$	$1453 \pm 137i$
12.51	2361	37.02	1.982	$1394 \pm 56i$	$1951 \pm 253i$	$1456 \pm 133i$
8.51	2389	35.01	1.958	$1394 \pm 58i$	$1959 \pm 264i$	$1457 \pm 130i$
6.51	2380	18.88	1.843	$1395 \pm 69i$	$2024 \pm 331i$	$1458 \pm 114i$
3.51	2297	11.40	1.767	$1396 \pm 78i$	$2099 \pm 441i$	$1454 \pm 102i$
2.91	1959	7.330	1.699	$1398 \pm 84i$	$2178 \pm 788i$	$1445 \pm 91i$
2.51	1529	4.583	1.569	$1399 \pm 92i$	$899 \pm 1457i$	$1422 \pm 82i$

Table 2: χ^2's and pole positions for trial data described in the text. Maximum energies are in GeV. λ is the pole position found when searching for one resonance. λ_1 and λ_2 are the pole positions found when searching for two resonances. χ_0^2, χ_1^2, and χ_2^2 are the χ^2's assuming no, one and two resonances respectively. The actual pole positions are $1421 \pm 119i$ MeV for the κ_2 and $1957 \pm 106i$ MeV for the κ_3.

Option	No. of resonances	$\sqrt{s_{pole}}$ (MeV)	χ^2
1	0	na	1373
	1	$1433 \pm 149i$	4.7
	2	$1432 \pm 148i$ $805 \pm 13i$	1.8
2	0	na	1629
	1	$1423 \pm 157i$	9.0
	2	$1423 \pm 154i$ $969 \pm 6i$	8.8
3	0	na	1133
	1	$1446 \pm 144i$	2.6
	2	$1432 \pm 126i$ $1060 \pm 130i$	1.2

Table 3: Pole positions and χ^2's for LASS $S-$wave data from 0.825 GeV to 2.51 GeV. (na = not applicable)
Option 1: Amplitude in unphysical region equals amplitude at threshold, error is 5.
Option 2: Amplitude in unphysical region equals amplitude at threshold, error is 2.5.
Option 3: Amplitude in unphysical region varies with θ and data points not evenly spaced.

Option	No. of resonances	$\sqrt{s_{pole}}$ (MeV)	χ^2
1	0	na	210
	1	$1467 \pm 224i$	2.1
	2	$1464 \pm 221i$ $808 \pm 7i$	1.1
2	0	na	367
	1	$1455 \pm 245i$	5.3
	2	$1439 \pm 237i$ $503 \pm 238i$	2.4

Table 4: Pole positions and χ^2's for LASS $I = \frac{1}{2}$ data.
Option 1 has unphysical errors set to 5. Option 2 has unphysical errors set to 2.5.

CONCLUSION

- Method we use is most sensitive to lighter resonances.
- We can identify two overlapping states.
- πK scattering data is found to contain only one resonance < 1.8 GeV.
- This state is identifiable as the $K_0^*(1430)$.
- There is no evidence for a $\kappa(900)$.