Effective theories for hadronic atoms:

The universality conjecture

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LNF Spring School, Frascati, 18 May 2000

- Determination of the πN scattering lengths from pionic hydrogen and pionic deuterium measurements

ETH Zürich - Neuchâtel - PSI collaboration

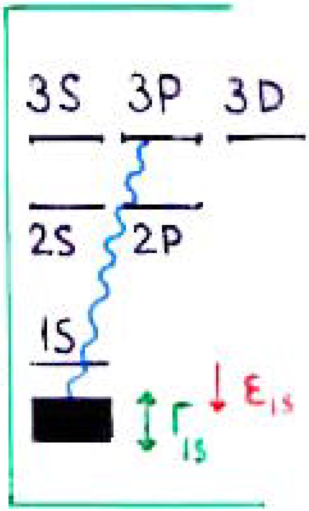
→ Measured: Energy and line shape of the 3p-1s x-ray transitions in pionic hydrogen and deuterium

⇓

$$\frac{\varepsilon_{1s}}{E_{1s}} = \frac{\Delta E_{1s}^{tot} - \Delta E_{1s}^{em}}{E_{1s}} = -4\,\frac{a_{\pi^- p\to\pi^- p}}{r_B}\,(1+\delta_\varepsilon)$$

$$\frac{\Gamma_{1s}}{E_{1s}} = 8\,\frac{q}{r_B}\left(1+\frac{1}{P}\right)\left\{a_{\pi^- p\to\pi^0 n}\,(1+\delta_\Gamma)\right\}^2$$

↖ Panofsky ratio



- Output: Strong scattering lengths b_0, b_1

$$a_{\pi^- p\to\pi^- p} = b_0 - b_1 = \frac{1}{3}(2a_1 + a_3)$$

$$a_{\pi^- p\to\pi^0 n} = \sqrt{2}\,b_1 = \frac{\sqrt{2}}{3}(a_3 - a_1)$$

→ b_1: input to GMO sum rule to determine πNN coupling const.

→ b_0: input to determine the value of πN Σ-term

- Other experiments on hadronic atoms:

DIRAC at CERN:	$\pi^+\pi^-$
DEAR at DAΦNE:	pK^-, dK^-
KEK:	pK^-

- **Agenda** Evaluate the isospin-breaking corrections $\hat{\delta}_\varepsilon, \hat{\delta}_\Gamma$ to the bound-state characteristics in the accuracy that matches the experimental precision

- **Potential model** D. Sigg et al, NPA 609, 310 (1996), ...

Simplified version

$$\begin{pmatrix} y''_{cc}(r) \\ y''_{nc}(r) \end{pmatrix} = \left(\frac{\ell(\ell+1)}{r^2} + \hat{M}^2 - (\hat{\mathcal{E}} - \hat{V})^2 + 2\hat{M}\hat{U} \right) \begin{pmatrix} y_{cc}(r) \\ y_{nc}(r) \end{pmatrix}$$

$$\hat{M} = \begin{pmatrix} \mu_c & 0 \\ 0 & \mu_n \end{pmatrix}; \quad \hat{\mathcal{E}} = \begin{pmatrix} \mu_c + E & 0 \\ 0 & \mu_n + \Delta M + E \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} -\alpha/r & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{U} = \begin{pmatrix} \frac{1}{3}(2U_1(r) + U_3(r)) & \frac{\sqrt{2}}{3}(U_3(r) - U_1(r)) \\ \frac{\sqrt{2}}{3}(U_3(r) - U_1(r)) & \frac{1}{3}(U_1(r) + 2U_3(r)) \end{pmatrix}$$

→ Assume no isospin breaking, no Coulomb interactions ⟹

⟹ Calculate strong scattering lengths

→ Include isospin breaking and Coulomb ⟹

⟹ Calculate the energy-level shift for the same potential

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Isospin-breaking corrections to the bound-state characteristics

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Assumes: No isospin-breaking in strong potential!

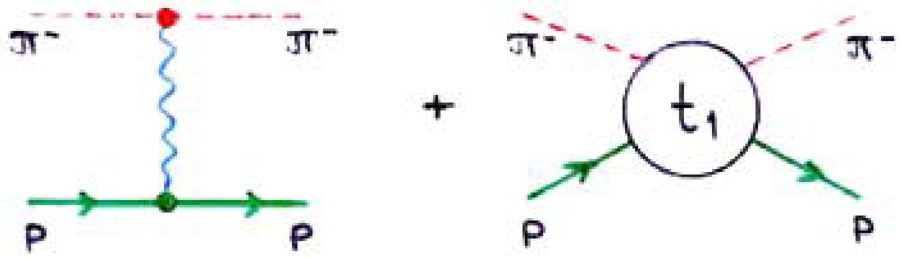
* Nonrelativistic effective Lagrangian approach + ChPT

$$\mathcal{E}_{1s} = -2\alpha^3\mu_c^2 A_{\pi N}\left(1 - 2\alpha\mu_c(\ln\alpha - 1)A_{\pi N}\right)$$

$O(\alpha)$, $O(m_d - m_u)$

$\mu_c = \dfrac{m_p M_{\pi^+}}{m_p + M_{\pi^+}}$

$t_{\pi^- p \to \pi^- p} =$



$$\mathrm{Re}\left[e^{-2i\theta_c} t_1\right]\Big|_{|\vec p| \to 0} \longrightarrow \frac{\bullet}{|\vec p|} + \bullet \ln\frac{|\vec p|}{\mu_c} - \frac{2\pi}{\mu_c} A_{\pi N}$$

Coulomb phase

* Isospin-breaking corrections, $O(p^2)$, ChPT

$$A_{\pi N}^{(2)} = \underbrace{b_0 - b_1}_{\text{Isospin symm.}} + \underbrace{\epsilon^{(2)}}_{\text{Isospin breaking}}$$

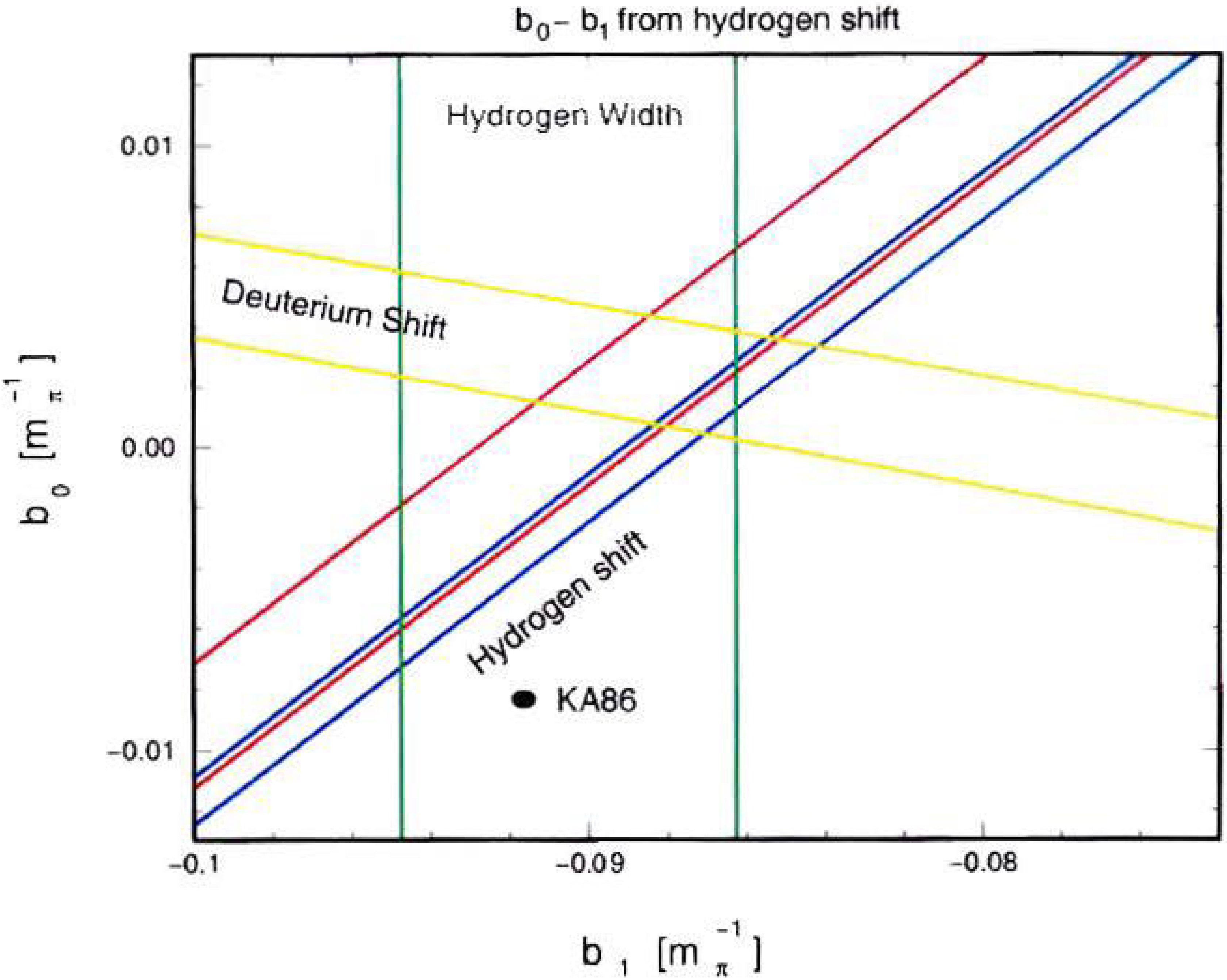
$$\epsilon^{(2)} = \frac{m_p\left\{8c_1(M_{\pi^+}^2 - M_{\pi^0}^2) - 4e^2 f_1 - e^2 f_2\right\}}{8\pi(m_p + M_{\pi^+})F^2}$$

c_1, f_1, f_2 – low-energy constants in ChPT

→ Disagreement with the potential model:

↳ ChPT, $O(p^2)$: $\delta_\epsilon = (-4.7 \pm 2.0) \times 10^{-2}$

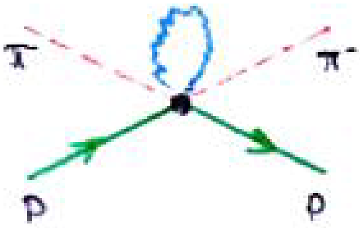
↳ Potential model: $\delta_\epsilon = (-2.1 \pm 0.5) \times 10^{-2}$



b0- b1 from hydrogen shift
Hydrogen Width
Deuterium Shift
Hydrogen shift
KA86
0.01
0.00
-0.01
-0.1
-0.09
-0.08
b 0 [m π -1]
b 1 [m π -1]

- What is missing in the potential model?

1). Direct quark-photon coupling (encoded in f_1, f_2)



$$\longrightarrow e^2 \sum_i f_i \mathcal{L}_i^{(p^2)}$$

2) Dependence on quark mass ($M_\pi^2 = M_{\pi^+}^2$ – convention)

$$A_{\pi N} = \cdots + \bullet\, 2\hat{m}B + \cdots = \cdots + \bullet \{ M_{\pi^+}^2 + (M_{\pi^0}^2 - M_{\pi^+}^2) \} + \cdots$$

Isospin-symmetric   Isospin breaking

- Agenda: resolving the potential model puzzle

  Give a recipe for constructing the short-range potential, that includes the full content of isospin breaking in ChPT

$\Uparrow$

* Tool: the universality conjecture

- Potential model: one-channel case

$$H_I = \underbrace{-\frac{\alpha}{r}}_{\text{Coulomb}} + \underbrace{\hat{u}}_{\text{Short-range}}$$

$$\hat{u} = V_s + \underbrace{\alpha V' + (m_d - m_u) V'' + \cdots}_{\text{isospin breaking}}$$

$$\Downarrow$$

$$T(z) = H_I + H_I \frac{1}{z - H_0} T(z)$$

- Pole position in the Scattering amplitude $T(z)$

$$z - E_0 - \langle \psi_0 | \tau(z) | \psi_0 \rangle = 0$$

$$\tau(z) = \hat{u} + \hat{u} \frac{1 - |\psi_0\rangle\langle\psi_0|}{z - H_0 + \alpha/r} \tau(z)$$

- Universality conjecture:

The relation between the bound-state pole position and the scattering amplitude does not depend on the details of the potential and is the same in the field theory and in the potential model

(first nonleading order in isospin breaking)

$$\varepsilon_{1S} = -2\alpha^3\mu^2 A \,(1 - 2\alpha\mu(\ln\alpha - 1) A)$$

i) Introduce scattering amplitude with short-range potential only:

$$t(z) = \hat{u} + \hat{u} \frac{1}{z - H_0} t(z)$$

$$t(z;\vec{p},\vec{k}) = \langle \vec{p} | t(z) | \vec{k} \rangle = 4\pi \sum_{\ell m} Y_{\ell m}(\hat{p})\, t_\ell(z;p,k)\, Y^*_{\ell m}(\hat{k}) \longrightarrow$$

$\hookrightarrow$ Retain only S-waves: $t(z;\vec{p},\vec{k}) = t(z;p,k)$; $\quad t_0 \equiv t(0;0,0)$

ii) Energy-level shift from the bound-state equation:

$$\mathcal{E}_{1S} = z - E_0 = \frac{\alpha^3\mu^3}{\pi} \left\{ t_0 - \frac{\alpha\mu^2}{\pi} t_0^2 - \frac{\alpha\mu^2}{\pi} t_0^2 \ln\frac{b}{2\alpha\mu} + \frac{8\alpha\mu}{\pi} Q[t] + \frac{\alpha\mu^2}{\pi^3} R[t;b] \right\}$$

$$Q[t] = \int_0^\infty \frac{dp}{p^2} \left[t(0;p,0) - t_0 \right]$$

$$R[t;b] = \int_0^\infty \frac{dp}{p} \int_0^\infty \frac{dk}{k} \ln\frac{(p-k)^2 + b^2}{(p+k)^2 + b^2}\, t(0;0,p)\, t(0;k,0) +$$

$$+ \int_0^\infty \frac{dp}{p} \int_0^\infty \frac{dk}{k} \left\{ \ln\frac{(p-k)^2}{(p+k)^2} - \ln\frac{(p-k)^2 + b^2}{(p+k)^2 + b^2} \right\} \left[t(0;0,p)\, t(0;k,0) - t_0^2 \right]$$

- Scattering amplitude near threshold, order α

$$T = -\frac{\alpha}{r} + t + \left(-\frac{\alpha}{r}\right)\frac{1}{z-H_0}t + t\frac{1}{z-H_0}\left(-\frac{\alpha}{r}\right) + t\frac{1}{z-H_0}\left(-\frac{\alpha}{r}\right)\frac{1}{z-H_0}t$$

$$\Downarrow$$

$$\mathrm{Re}\left(e^{-2i\theta_c}\left(T+\frac{\alpha}{r}\right)\right)\Big|_{|\bar{p}|\to 0} \longrightarrow \frac{\bullet}{|\bar{p}|} + \bullet \ln\frac{|\bar{p}|}{\mu} - \frac{2\pi}{\mu}A$$

$$\Downarrow$$

$$-\frac{2\pi}{\mu}A = t_0 - \frac{\alpha\mu^2}{\pi}t_0^2 \ln\frac{b}{2\mu} + \frac{8\alpha\mu}{\pi}Q[t] + \frac{\alpha\mu^2}{\pi^3}R[t;b]$$

$$\Downarrow$$

$$\mathcal{E}_{1s} = -2\alpha^3\mu^2 A\,(1 - 2\alpha\mu(\ln\alpha - 1)A)$$ Universality!

- Construction of the isospin-breaking part of the potential

ChPT: $A = A_{strong} + A_1$

$$\hat{U} = V_s + \alpha V' + (m_d - m_u)V'' \doteq (1+\lambda)V_s$$

$$t_s(z) = V_s + V_s\frac{1}{z-H_0}t_s(z)$$

$$\Downarrow$$

- Matching conditions at threshold ($\bar{t}_s \equiv t_s(0;0,0)$)

i) $-\frac{2\pi}{\mu}A_{strong} = \bar{t}_s$

ii) $-\frac{2\pi}{\mu}A_1 = \lambda\left(\bar{t}_s + G[t_s]\right) + \frac{8\alpha\mu}{\pi}Q[t_s] + \frac{\alpha\mu^2}{\pi^3}R[t_s;b] - \frac{\alpha\mu^2}{\pi}\bar{t}_s^2\ln\frac{b}{2\mu}$

$\rightarrow$ $G[t_s] = -\frac{\mu}{\pi^2}\int_0^\infty dp\; t_s(0;0,p)\,t_s(0;p,0)$

- Discussion & outlook

→ Universality conjecture:

For a short-range potentials, the relation between the bound-state energy and the scattering amplitude at threshold does not depend on the details of the potential and is the same in the field theory, and in the potential model

→ Based on the universality, a constructive algorithm is provided for the derivation of the isospin-breaking part of the potential from field theory

* We hope that the potential obtained can be used to carefully reexamine the πN scattering data near threshold on the basis of ChPT, with respect to the isospin-breaking effects in the scattering amplitudes