Chiral Perturbation Theory and the $1/N_c$ Expansion

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- The large N_c limit of QCD
- The effective Lagrangian and the η'
- The δ expansion
  (as opposed to the p expansion)
- Kaplan-Manohar transformation
- Results
- Summary and outlook

Work done in collaboration with H. Leutwyler

The large N_c limit of QCD

- $N_c \to \infty$, $g^2 N_c$ fixed — G.'t Hooft, 74

- As a consequence the $U(1)_A$ anomaly disappears in this limit

- Spontaneous symmetry breakdown
  $U(3)_L \times U(3)_R \to U(3)_V$

- Nonet of Goldstone bosons: π, K, η, η'
  E.Witten, 79

- Effective prescription : need to include the degree of freedom of the η'

- Perturbative analysis yields counting rules for correlation functions
  The η' mass is a special case

Effective Lagrangian

- Leading order ($\sim$ current algebra):
  - take Lagrangian of SU(3): $U(x) \in \mathrm{SU}(3) \rightarrow U(x) \in \mathrm{U}(3)$
  - $\det U = e^{i\psi}$, ψ describes the η'
  - add a mass term for the ψ field
  - chiral symmetry: $\psi^2 \rightarrow (\psi + \theta)^2$

$$\mathcal{L} = \frac{F^2}{4} \langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \rangle - \frac{\tau_{GD}}{2} (\psi + \theta)^2$$

Di Vecchia & al, Rosenzweig & al, Witten, 80

- $\theta(x)$: source field for the winding number density $\omega(x)$

$$\omega = \frac{1}{16\pi^2} \mathrm{tr}_c\, G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- η' mass ($m_q = 0$) : $M_{\eta'}^2 = 6\, \tau_{GD}/F^2$

- $F^2 = O(N_c)$, $\tau_{GD} = O(1)$: $M_{\eta'}^2 = O(1/N_c)$

Effective Lagrangian, cont.

- Leading order

$$\mathcal{L} = \frac{F^2}{4}\langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U\rangle - \frac{\tau_{GD}}{2}(\psi + \theta)^2$$

- Low energy constants are independent of the light quark masses m_u, m_d, m_s

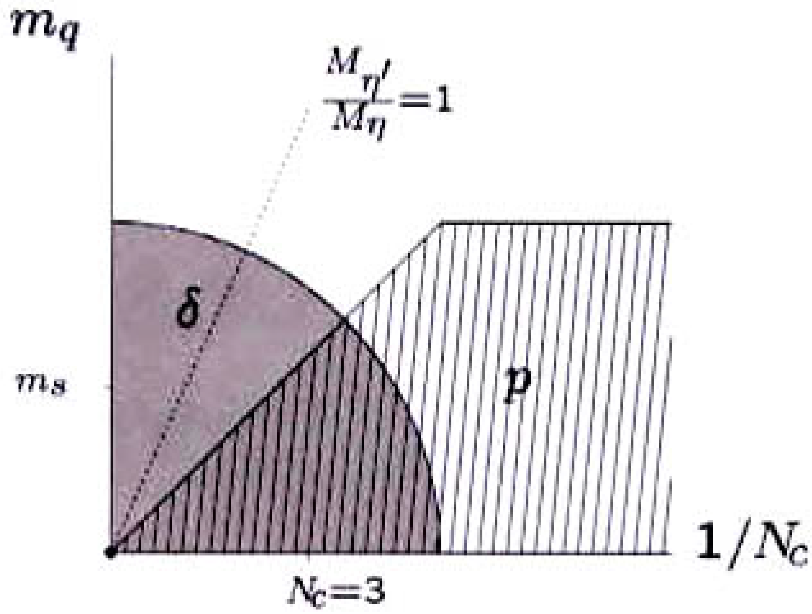
- Low energy constants do know about N_c: $F^2 = O(N_c)$, $B = O(1)$, $\tau_{GD} = O(1)$, ...

- Introduce counting parameter δ: $p = O(\sqrt{\delta})$, $m = O(\delta)$, $1/N_c = O(\delta)$

- Leading order Lagrangian $= O(1)$

- $\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$

Domain of validity



- Equation of Motion for the η'

$$(\Box + M_{\eta'}^2)\psi(x) = f_{\text{ext}}(x) + \dots$$

- $N_c = \infty$: the η' is massless

- $p^2 \ll M_{\eta'}^2$: $\psi(x) = M_{\eta'}^{-2} f_{\text{ext}}(x) + \dots$

- p^2, $M_{\eta'}^2$ both small : δ expansion

δ expansion

- Chiral symmetry

- Large N_c counting rules

  $$\langle 0|T j_1 \ldots j_n \omega_1 \ldots \omega_m|0\rangle = O(N_c^{2-l-m})$$

  $j_i = \bar{q}\,\Gamma_i\, q$, l = # of quark loops

  In particular: correlation functions involving ω are suppressed

- Translates into counting rules for the effective coupling constants...

  There are, however, ambiguities $\leftrightarrow$ freedom in choice of the effective variables

- Nevertheless, it can be shown that the input is sufficient for the construction of an effective Lagrangian which

  (a) describes QCD correctly and

  (b) has the desired properties, namely...

R.K. & H.Leutwyler, to appear

δ expansion, cont.

- To any given order in δ only a finite number of terms is needed

- The coupling constants in the Lagrangian do obey the 'canonical counting rules' of large N_c J.Gasser & H.Leutwyler, 1985

- In particular, the coupling constants c_n are bounded, $c_n \leq O(N_c)$, $\forall\, n$

- The Lagrangian of order δ^n is a polynomial of degree $n+2$ in θ

- Note that it is easy to find a Lagrangian which does not have these properties: a suitable redefinition of the effective fields, $\psi \to \psi + f(\psi + \theta)$, $f = O(N_c^2)$, e.g., generates one

Kaplan-Manohar transformations

- SU(3) effective Lagrangian up to and including $O(p^4)$ is invariant under
  $m_u \to m_u + \lambda\, m_d\, m_s$, cyclic
  $L_8 \to L_8 + \lambda F^2/(16B)$, L_6,L_7 similar

  D.Kaplan & A.Manohar, 86

- In order to preserve the U(1)-transformation properties of m we must include θ:

$$m \to m + \lambda\, e^{-i\theta} m^{\dagger -1} \det m^\dagger$$

- This implies the change

$$\langle U^\dagger \chi \rangle \;\to\; \langle U^\dagger \chi \rangle + \frac{\lambda}{4B} e^{-i(\psi+\theta)} \left(\langle \chi^\dagger U \rangle^2 - \langle \chi^\dagger U \chi^\dagger U \rangle \right)$$

- Introduces a non-polynomial dependence on θ
  Conclusion: λ must vanish to all orders in $1/N_c$

Summary and outlook

- The occurrence of a new low energy scale ($M_{\eta'}$) leads to a more complicated low energy structure

- The formulation of the effective theory at large N_c involves a simultaneous expansion in powers of p, m_q and $1/N_c$: δ expansion

- The Kaplan-Manohar transformation is forbidden in this framework

- Promising results at order δ - extension to order δ^2 (loops)