Lecture n. 4

(Non−leptonic weak interactions)

Non−leptonic weak interactions

Lowest−order non−leptonic weak lagrangian

$O(p^2)$ **predictions with** $L_W^{(2)}$

Non−leptonic weak interactions at NLO

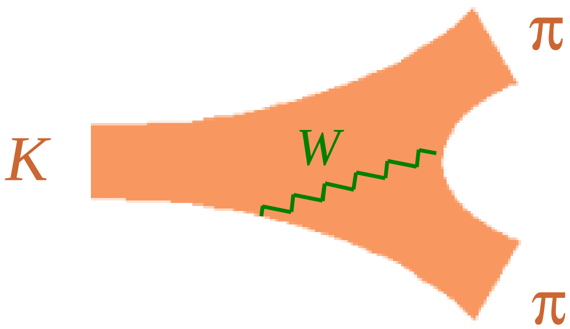
A particular case: $K_S \to \gamma\gamma$

Classification of the $O(p^4)$ **operators**

Non−trivial chiral tests at NLO

Beyond $O(p^4)$

Non−leptonic weak interactions



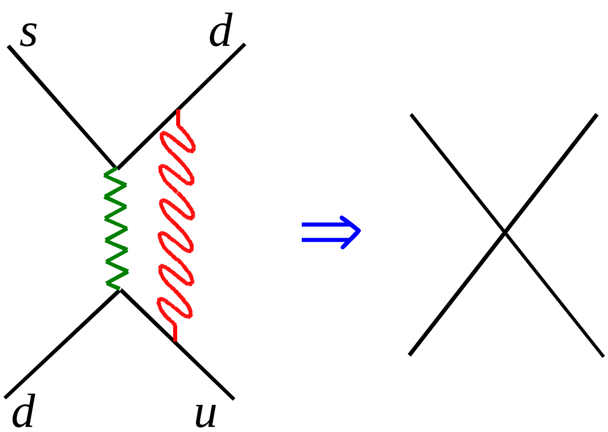
the W cannot be considered as an external field

two very different scales involved: M_K & M_W

"double" EQFT approach to simplify the problem

1) Construction of the *partonic* effective $|\Delta S|=1$ Hamiltonian

Purpose: elimination of non−local structures
resummation of the large *logs* generated in pQCD



G. Altarelli, L. Maiani, *'74*
M.K. Gaillard, B.W. Lee, *'74*
⋮
A.J. Buras *et al.*, *'93−'94*
M. Ciuchini *et al.*, *'93−'94*

large *logs* of the type $\alpha_s \cdot \log\left[(p_i p_j)/M_W^2\right]$
($p_i =$ external momenta)

$$H_{eff}^{|\Delta S|=1} = \sum_i C_i(\mu) Q_i \quad + \quad O(\mu^2/M_W^2)$$

μ = renormalization scale (unphysical)

Wilson coefficients

$\alpha_s^n(\mu)\cdot\log(\mu/M_W^2)^n,$

$\alpha_s^{n+1}(\mu)\cdot\log(\mu/M_W^2)^n, \ldots$

resummed by means of RGE

$\mu \ll M_W$ → max. resummation of the large logs

$\mu \gg \Lambda_{QCD}$ → pQCD still applicable

↓

$\mu \gtrsim m_c$

Four−quark operators

$\sim \bar{s}^\alpha \Gamma d^\beta \;\; \bar{u}^\beta \Gamma u^\alpha$

$$A(K \to \pi\pi) = \sum_i C_i(\mu) \; \langle \pi\pi | Q_i | K \rangle(\mu)$$

μ independent !

pQCD effects — non−perturbative dynamics

The problem has been reduced to the evaluation of hadronic matrix elements of a finite set of four−quark operators (11 for $m_c < \mu < m_b$)

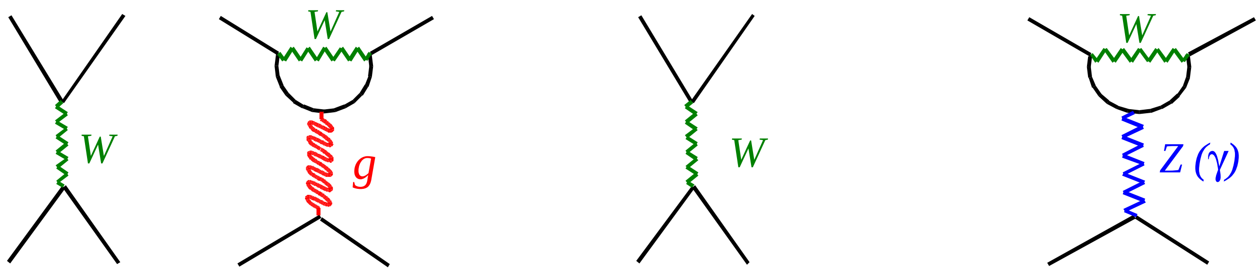
2) Construction of the *chiral* $|\Delta S|=1$ non−leptonic lagrangian

$$H_{eff}^{|\Delta S|=1} = \sum_i C_i(\mu) Q_i$$

↓

transforming linearly under $SU(3)_L \times SU(3)_R$

$(8_L, 1_R)$ $(27_L, 1_R)$ $(8_L, 8_R)$



first non−vanishing chiral realization at $O(p^2)$ (unique)

first non−vanishing chiral realization at $O(p^2)$ (unique)

first non−vanishing chiral realization at $O(p^0)$ (unique), (suppressed by $O(e^2)$ relevant to $\not{CP}$ only)

N.B.: Chiral symmetry alone does not help to evaluate hadronic matrix elements of 4−quark operators (new unknown couplings), it only helps to relate each other the matrix elements of a given operator in different processes

Lowest−order non−leptonic weak Lagrangian

$$L_W^{(2)} = G_8 F^4 tr\left(\hat{\lambda}\, L_\mu L^\mu\right)$$
$$+G_{27} F^4 \left[(L_\mu)_{23}(L^\mu)_{11} + \frac{2}{3}(L_\mu)_{21}(L^\mu)_{13}\right]$$
$$+G_{\underline{8}} F^6 tr\left(\hat{\lambda}\, U^+ Q U\right) + h.c.$$

$$L_\mu = (L_\mu)^+ = i U^+ D_\mu U = u^+ u_\mu u \qquad \hat{\lambda} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

G_i = new unknown couplings ($\sim E^{-2}$) to be determined by experimental data

Im (G_i) $\Rightarrow$ $\not{CP}$

Naive factorization ansatz

$$(\bar{q}_L^i \gamma^\mu q_L^j) \times (\bar{q}_L^j \gamma^\mu q_L^k) \rightarrow \frac{F}{4}(L_\mu)_{ji}(L^\mu)_{jk}$$

$\Rightarrow$ $G_8, G_{27} \sim \sin\vartheta_c G_F$
$G_{\underline{8}} \sim \alpha_{em} \sin\vartheta_c G_F$

From the experimental data on $A(K \rightarrow 2\pi)$ $\Rightarrow$
$|G_8| \simeq 9.1 \times 10^{-6}$ GeV
$|G_{27}/G_8|$ 5.7×10^{-2}

[exercise n.5]: evaluate G_8 and G_{27} in the factorization approach for the 2 dominant operators $Q_\pm = \bar{s}_L \gamma^\mu u_L \bar{u}_L \gamma^\mu d_L \pm \bar{s}_L \gamma^\mu d_L \bar{u}_L \gamma^\mu u_L$

$O(p^2)$ predictions with $L_W^{(2)}$

I) $K \to 3\pi$ decays

$$K^\pm \to \pi^\pm \pi^\pm \pi^\mp \quad [I=1,2] \qquad K^0(\bar{K}^0) \to \pi^0 \pi^0 \pi^0 \quad [I=1]$$
$$K^\pm \to \pi^0 \pi^0 \pi^\mp \quad [I=1,2] \qquad K^0(\bar{K}^0) \to \pi^\pm \pi^\mp \pi^0 \quad [I=0,1,2]$$

$$|A(K \to 3\pi)| \propto 1 + gY + jX + hY^2 + kX^2 + \ldots$$

$$\begin{cases} Y = (s_3 - s_0)/M_\pi^2 \\ X = (s_1 - s_2)/M_\pi^2 \end{cases}$$

Dalitz−Plot variables

rapidly convergent expansion due to the smallness of the 3π phase space

$$s_i = (p_K - p_{\pi_i})^2 \qquad s_0 = \sum_i s_i = \frac{1}{3}\left(M_K^2 + \sum_i M_{\pi_i}^2\right)$$

ampl.	$O(p^2)$	exp. Value
α_1	+74.0	$+91.71 \pm 0.32$
β_1	−16.5	-25.68 ± 0.27
ζ_1	0	-0.47 ± 0.15
ξ_1	0	-1.51 ± 0.30
α_3	−4.1	-7.36 ± 0.47
β_3	−1.0	-2.43 ± 0.41
γ_3	+1.8	$+2.26 \pm 0.23$
ζ_3	0	-0.21 ± 0.08
ξ_3	0	-0.12 ± 0.17
ξ'_3	0	-0.21 ± 0.51

20%−30% errors in the dominant terms as expected by naïve power counting ($M_K^2/\Lambda_\chi^2 \sim 0.3$)

quadratic terms in X and Y require $O(p^4)$ operators

II) $K \to \pi\pi\gamma$ decays

$$K^0(\bar{K}^0) \to \pi^+\pi^-\gamma \qquad K^\pm \to \pi^\pm \pi^0 \gamma$$
$$K^0(\bar{K}^0) \to \pi^0\pi^0\gamma$$

General structure dictated by Lorentz and gauge invariance:

$$\begin{aligned} A(K\to\pi\pi\gamma) \;=\; & E(z_i)\epsilon_\mu(qp_1 p_2^\mu - qp_2 p_1^\mu)/M_K^3 + \\ & M(z_i)\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu p_{1\nu} p_{2\rho} q_\sigma / M_K^3 \end{aligned}$$

$$z_i = p_i q / M_K^2 \qquad z_3 = z_1 + z_2 = p_K q / M_K^2$$

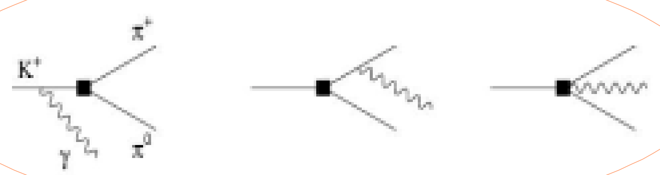
$$E(z_i) \;=\; E_{IB}(z_i) \;+\; E_{DE}(z_i)$$

$$E_{IB}(z_i) = \frac{e A(K\to\pi_1\pi_2)}{M_K z_3}\left(\frac{Q_2}{z_2} - \frac{Q_1}{z_1}\right)$$

Bremsstrahlung amplitude

predicted with negligible error at $O(p^2)$ if G_8 and G_{27} are fitted from $K\to\pi\pi$

[exercise n.1]: compute E_{IB} with $L_W^{(2)}$



$$\begin{cases} E_{DE}(z_i) = E_1 + E_2(z_2 - z_1) + \ldots \\ M(z_i) = M_1 + M_2(z_2 - z_1) + \ldots \end{cases}$$

Direct−emission amplitudes: require higher−derivatives operators

Non−leptonic weak interactions at NLO

$$e^{i Z(l,r,s,p)} = \int DU(\phi)\; e^{\int d^4x \left[L_S^{(2)} + L_W^{(2)} + L_S^{(4)} + L_W^{(4)} \right]}$$

$O(G_F^0)$ $\quad$ $O(G_F^1)$

$L_W^{(4)}$ = most general next−to−leading order structure transforming as $(8_L, 1_R) + (27_L, 1_R) + (8_L, 8_R)$

$O(e^0 p^4)$

regularization of the divergences generated by

$$L_W^{(2)}(G_{8,} G_{27}) \times L_S^{(2)}$$

$O(e^2 p^2)$

not available yet...

J. Kambor, J. Missimer, D. Wyler, *Nucl. Phys. B346 (90) 17*
G. Ecker, J. Kambor, D. Wyler, *Nucl. Phys. B394 (93) 101*

37 new operators considering only the (dominant) $(8_L, 1_R)$ sector !

The theory certainly has less predictive power...
... but a lot of interesting & precise predictions can still be done

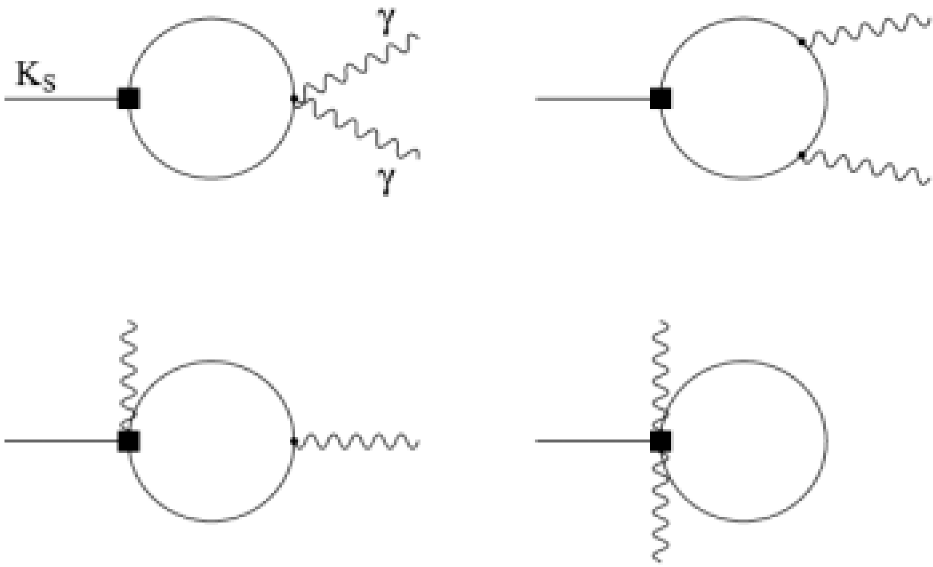
A particular case: $K_S \to \gamma\gamma$

Vanishing amplitude at $O(p^2)$

No counterterm of the type $K^0 F^{\mu\nu} F_{\mu\nu}$ allowed at $O(p^4)$

The $O(p^4)$ one−loop amplitude must be finite

G. D'Ambrosio, D. Espriu, *Phys. Lett. B175 (86) 237*



$$BR(K_S \to \gamma\gamma)^{(4)} = 2.1 \times 10^{-6}$$

$$BR(K_S \to \gamma\gamma)^{\exp} = (2.4 \pm 0.9) \times 10^{-6}$$

One of the most interesting non−trivial tests of CHPT in the sector of non−leptonic weak decays

Classification of the $O(p^4)$ operators

$$L^{(4)}_{W\,(8_L,1_R)} = G_8 F^2 \sum_{i=1}^{37} N_i W_i^{(4)}$$

37 $[N_1 - N_{37}]$

→ 35

→ 2 contact terms of external fields

35 → 13 operators with an explicit W field → $O(G_F^2)$ corrections to semilept. Decays

35 → 22 genuine non−leptonic operators

22 → 13 operators without explicit γ field

- 4 4−deriv. $[N_1 - N_4]$: K→ 3π (quad. slopes)
- 9 ≤2−deriv. $[N_5 - N_{13}]$: K→ 2π, K→ 3π

22 → 9 operators with an explicit $F^{\mu\nu}$ tensor

- 4 elect. (1γ) $[N_{14} - N_{17}]$: K→ 2πγ, K→ 3πγ, K→ πll
- 4 mag. (1γ) $[N_{28} - N_{31}]$: K→ 2πγ, K→ 3πγ
- 1 (2γ) $[N_{18}]$: K→ πγγ, K→ ππγγ

Non−trivial chiral tests at NLO

I) $K \to 3\pi$ decays

$(8_L, 1_R)$ & $(27_L, 1_R)$ operators with two or less derivatives ($W_5^{(4)} - W_{13}^{(4)}$) provide a common "renormalization" in $K \to 2\pi$ and $K \to 3\pi$ amplitudes of both G_8 & G_{27}

Only the (few) four−derivative operators modify the $O(p^2)$ relations between $K \to 2\pi$ and $K \to 3\pi$ amplitudes

→ Tables

II) Radiative processes

High−degree of predictivity (e.g. the combination $[N_{14} - N_{15} - N_{16} - N_{17}]$ determine more than five distinct observables....) still to be tested

→ Table

Beyond $O(p^4)$

Another particular example: $K_L \to \pi^0 \gamma\gamma$

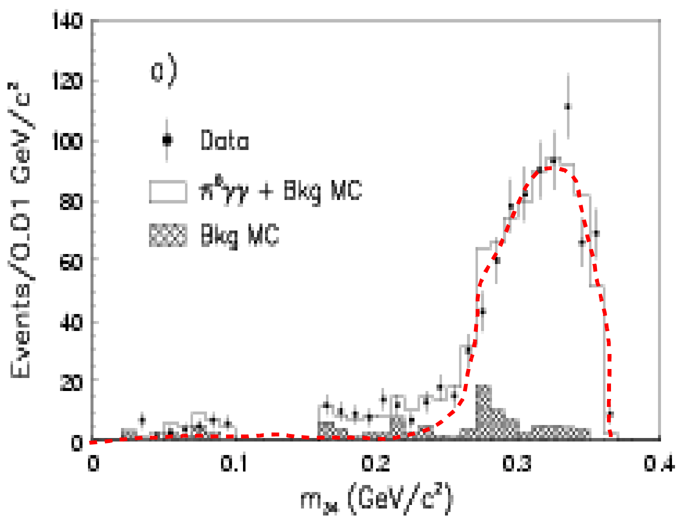
Vanishing amplitude at $O(p^2)$

No counterterm allowed at $O(p^4)$

The $O(p^4)$ one−loop amplitude is finite

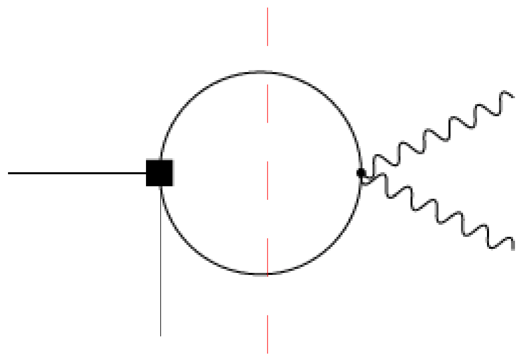
$$BR(K_L \to \pi^0 \gamma\gamma)^{(4)} = (0.6)\times 10^{-6}$$
$$BR(K_L \to \pi^0 \gamma\gamma)^{(\exp)} = (1.7\pm 0.1)\times 10^{-6}$$

Serious discrepancy in the BR
good agreement in the di−photon spectrum → large $O(p^6)$ corrections



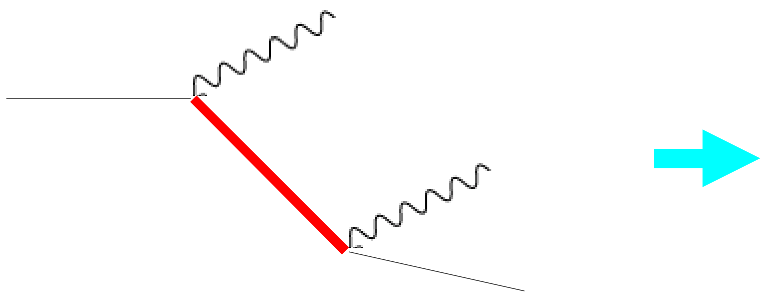
Why $K_L \to \pi^0 \gamma\gamma$ is so different from $K_S \to \gamma\gamma$?

I) Unitarity corrections



The one−loop diagram does not provide the correct imaginary part if G_8 is fitted from $K \to 2\pi$

II) Vector−meson contributions



large $O(p^6)$ local term

Contrary to the $K_S \to \gamma\gamma$ case in $K_L \to \pi^0 \gamma\gamma$ we can have sizable contributions induced by spin−1 resonances

Conclusions

CHPT is an EQFT that provides a low−energy realization of the Standard Model in terms of the light pseudoscalar degrees of freedom

It is not only consistent and predictive, but also very useful to extract precise information about fundamental parameters of the SM