Introduction to CHPT

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Lectures plan:

Motivations and lowest−order Lagrangians (G.I.)

Renormalization (G.C.)

Unitarity (G.C.)

Non−leptonic weak−interactions (G.I.)

Lecture n. 1

(Motivations and lowest−order lagrangians)

Motivations

Effective Quantum Field Theories

Chiral Symmetry

Non−linear realizations

Lowest−order (strong) Lagrangian

Determination of F & B

Quark mass ratios & isospin breaking

Example of a tree−level process ($K \to \pi l \nu$)

Motivations

At low energies ($E \ll 1$ GeV) QCD is in a highly non−perturbative regime: this makes very difficult any description of the hadronic world in terms of partonic degrees of freedom.

The hadronic spectrum is very simple at low energies: only 8 pseudoscalar fields (π, K, η).

The interactions among the pseudoscalar mesons become weak in the limit $E \to 0$.

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Reasonable to expect that QCD can be treated perturbatively even at low energies with a suitable choice of degrees of freedom.

(q, G)		(π, K, η)
pQCD	⟷	CHPT
pert. at high E		*pert. at low E*

Effective Quantum Field Theories

CHPT is a typical example of EQFT, i.e. of a QFT which has an intrinsic limitation in the (energy) range of validity ($E < \Lambda$).

All QFT we know can be considered as EQFT, i.e. as low energy approximations of "more fundamental" theories.

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Basic principle of EQFT:

Only few degrees of freedoms are relevant in a given energy range: the heavy ones can be integrated out.

Do we need renormalizability (in the "classical" sense) within EQFT ?

Renormalizable theories are a particular subset of EQFT with

- the virtue of being very predictive (only a finite set of couplings need independently of the energy range)
- the disadvantage of non containing explicit indications about their validity range

General consistency conditions for EQFT:

The operators needed to regularize the theory are expanded in a power series

- for any $n \geq 0$ there is only a finite set of operators contributing at $O(E^n)$ to the physical amplitudes
- the coefficients of these operators, $C_i^{(n)}$, scale according to $C_i^{(n+k)} / C_j^{(n)} \sim O(1/\Lambda^k)$

⬇

Finite set of couplings needed to describe the physics at $E < \Lambda$ with arbitrary precision

Chiral Symmetry

$$L_{QCD}^{[0]} = \sum_{i=0}^{N_f} \bar{q}^i iD_\mu q^i - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + L_{heavy\,quarks}$$

Global symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

N_f = light quarks

$U(1)_A \Rightarrow$ broken the quantum level by the axial anomaly

$U(1)_V \Rightarrow$ realized

$SU(N_f)_L \times SU(N_f)_R \Rightarrow$ not manifest in the hadronic spectrum

[chiral group G] (only an approximate $SU(3)_V$ is realized)

(well motivated) assumptions of CHPT ($N_f = 3$)

G is spontaneously broken into $H = SU(3)_{L+R}$

(π, K, η) → Goldstone bosons of G/H (correct quantum numbers, vanishing interactions for $E \to 0$)

the explicit breaking terms $O(m_q)$ can be treated as small perturbations around the chiral limit

$M_\pi^2 / M_\rho^2 \sim 0.03 \qquad M_K^2 / M_\rho^2 \sim 0.4$

We must construct the most general $L(\Phi,\Psi)$ invariant under G and add to it the breaking terms (transforming linearly under G)

- Φ = Goldstone boson fields parametrizing G/H

  [Φ transform non linearly under G]

- Ψ = heavy (non−dynamical) matters fields

  (nucleons, vector mesons, etc...)

  [Ψ transform linearly only under H]

S. Weinberg, *Phys. Rev. Lett. 18 (67) 188*

↓

General procedure to construct a non−linear realization of a spontaneously broken symmetry

S. Coleman, J. Wess, B. Zumino, *Phys. Rev. D177 (69) 2239*

C.G. Callan, S. Coleman, J. Wess, B. Zumino, *Phys. Rev. D177 (69) 2239*

Non−linear realizations

V_i = generators of H $\quad [V_i, V_j] \sim V_k$

A_i = remaining generators of G $\quad [A_i, A_j] \sim V_k \quad [A_i, V_j] \sim A_k$

$\forall g \in G$ can be written as $g = e^{\xi \cdot A} e^{\eta \cdot V}$

$g_0 \in G \qquad g_0\, e^{\xi \cdot A} = e^{\xi'(\xi, g_0) \cdot A}\, e^{\eta'(\xi, g_0) \cdot V}$

$h_0 \in H \qquad \psi \to h_0\, \psi\, h_0^{-1}$ (ex. of linear realization ⇒ adj. representation)

$$\xi \to \xi'(\xi, g_0) \qquad \psi \to e^{\eta'(\xi, g_0) \cdot V}\, \psi\, e^{-\eta'(\xi, g_0) \cdot V}$$

provide a non−linear realization of G which becomes linear if restricted to the subgroup H

Proof [exercise n.1]:

a) $g: \xi \to \xi' \qquad g_1: \xi' \to \xi'' \qquad g_1 g_0: \xi \to \xi''$

$g: \psi \to \psi' \qquad g_1: \psi' \to \psi'' \qquad g_1 g_0: \psi \to \psi''$

b) $h_0 \in H \qquad h_0\, e^{\xi \cdot A} = (h_0\, e^{\xi \cdot A} h_0^{-1})\, h^0 = e^{\xi' \cdot A} h_0$

$e^{\xi \cdot A} \to h_0\, e^{\xi \cdot A} h_0^{-1} \qquad \psi \to h_0\, \psi\, h_0^{-1}$

Specifying the discussion to the chiral group:

$$g_L e^{\xi\cdot A} = e^{\xi'\cdot A} e^{\eta'\cdot V}$$
$$g_R e^{-\xi\cdot A} = e^{-\xi'\cdot A} e^{\eta'\cdot V}$$

parity transformation

$$u(\xi) = e^{\xi\cdot A}$$
$$h(g,\xi) = e^{\eta'\cdot V}$$

$$u(\xi) \to g_L u(\xi) h^{-1}(g,\xi) = h(g,\xi) u(\xi) g_R^{-1}$$
$$u(\xi)^+ \to g_L u(\xi)^+ h^{-1}(g,\xi) = h(g,\xi) u(\xi)^+ g_R^{-1}$$

basic tools to construct objects transforming linearly under G

$$U = u^2 \to g_L U g_R^{-1}$$
$$U^+ = (u^+)^2 \to g_R U^+ g_L^{-1}$$
$$u \psi u \to g_L (u \psi u) g_R^{-1}$$
$$u \psi u^+ \to g_L (u \psi u^+) g_R^{-1}$$
$$\vdots$$

↓

Identification of ξ with the pseudoscalar meson field:

$$u(\xi) = e^{i\sqrt{2}\phi/F}$$

F = unknown dimensional coupling ($\sim E$)

the identification is not unique, but all possible formulations leads to equivalent physical results

$$\phi = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\eta\sqrt{2/3} \end{bmatrix}$$

If we do not include heavy matter fields and symmetry breaking terms, the first non−trivial operator invariant under *G* arises at *O*(p^2) and is unique:

$$L^{(2)}_{[0]} = \frac{F^2}{4}\, tr\left(\partial_\mu U^+ \partial^\mu U\right) + O(p^4)$$

$$= \frac{1}{2}\, tr\left(\partial_\mu \phi \partial^\mu \phi\right)$$

$$+ \frac{1}{6F^2}\, tr\left([\partial_\mu \phi, \phi][\partial^\mu \phi, \phi]\right) + \ldots$$

Coupling fixed to reproduce the kinetic term

∞ series of interactions terms ruled by a single unknown coupling

[exercise n.2]: show that $A(\pi^+\pi^0 \to \pi^+\pi^0) = (p'_+ - p_+)^2/F^2$

Lowest−order Lagrangian

How to introduce the symmetry−breaking terms ?

Systematic approach by means of external (non−dynamical sources)

J. Gasser, H. Leutwyler, *Ann. Phys, 158 (84) 142;*
Nucl. Phys. B250 (85) 465

$$L_{QCD}(l^\mu, r^\mu, s, p) = L_{QCD}^{[0]} + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R$$
$$- \bar{q}_R (s + ip) q_L - \bar{q}_L (s - ip) q_R$$

$L_{QCD}(l^\mu, r^\mu, s, p)$ is still invariant under G if

$$\begin{aligned}
l_\mu &\rightarrow g_L l_\mu g_L^{-1} && + && i(g_L \partial_\mu g_L^{-1}) \\
r_\mu &\rightarrow g_R r_\mu g_R^{-1} && + && i(g_R \partial_\mu g_R^{-1}) \\
(s - ip) &\rightarrow g_L (s - ip) g_R^{-1} \\
(s + ip) &\rightarrow g_R (s + ip) g_L^{-1}
\end{aligned}$$

↑

Needed if we want to promote G (or some of its subgroups) to a local symmetry

Some of the advantages obtained by introducing the external fields:

quark mass terms recovered for

$$s = diag(m_u, m_d, m_s) \qquad p = 0$$

couplings to external *W*, Z or photon fields obtained for

$$r_\mu = -e Q A_\mu + O(Z_\mu)$$

$$l_\mu = -e Q A_\mu - \frac{e}{\sqrt{2}\sin(\vartheta_W)}(T_+ W_\mu^+ + h.c.) + O(Z_\mu)$$

$$Q = \frac{1}{3}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T_+ = \begin{bmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

CKM matrix elements

[exercise n.3]: compute explicitly the $O(Z_\mu)$ terms

For later convenience we also define

$$\chi = 2B(s + ip)$$

B = new unknown dimensional coupling ($\sim E$)

The lowest−order chiral realization of $L_{QCD}(l^\mu, r^\mu, s, p)$ is given by:

$$L^{(2)}(l^\mu, r^\mu, s, p) = \frac{F^2}{4} \, tr\left(D_\mu U^+ D^\mu U + \chi^+ U + U^+ \chi \right)$$

Completely determined in terms of two couplings: F & B

$$D_\mu U = \partial_\mu U + i U l_\mu - i r_\mu U$$

Power−counting rules for the external fields:

$l_\mu \;,\; r_\mu \sim O(p)$ natural consequence of local invariance ($D_\mu U \sim O(p)$)

$\chi \;,\; \chi^+ \sim O(p^2)$ necessary if we want $M_\pi^2 \sim O(p^2)$

→ unique expansion in terms of masses and momenta of the pseudoscalar mesons: well justified a posteriori but not necessarily the most general case.

$$L^{(2)}(l^\mu, r^\mu, s, p) \sim O(p^2)$$

Determination of F & B

F can be determined by the pion (kaon) decay constant measured from $\pi \to l\,\nu_l \quad (K \to l\,\nu_l)$

$$\frac{\partial L_{QCD}(l^\mu)}{\partial (l_\mu)_{ij}} = q_L^i \gamma^\mu q_L^j$$

$$\Updownarrow$$

$$\frac{\partial L^{(2)}(\phi; l^\mu)}{\partial (l_\mu)_{ij}} = \frac{iF^2}{2}(D_\mu U^+ U)_{ji} = \frac{F}{\sqrt{2}} D_\mu(\phi)_{ji} + O(\phi^2)$$

$$\langle 0 | \bar{u}_L \gamma^\mu d_L | \pi^-(p) \rangle \doteq -\frac{i}{\sqrt{2}} F_\pi p^\mu$$

$F_\pi = 93$ MeV

$(F_K / F_\pi \sim 1.2)$

$$\Downarrow$$

$$F_\pi = F\left[1 + O(m_q)\right]$$

higher order terms responsible for the difference between F_K and F_π

Similarly, by comparing the vacuum expectation values of the scalar currents, it follows that:

$$\langle 0 | \bar{q} q | 0 \rangle \; = \; - B F^2$$

in practice B always appears multiplied by the quark masses and this product can easily be determined in terms of the pseudoscalar meson masses:

$$M^2_{\pi^+} \; = \; M^2_{\pi^0} \; = \; B(m_d + m_u)$$
$$M^2_{K^+} \; = \; B(m_s + m_u)$$
$$M^2_{K^0} \; = \; B(m_s + m_d)$$
$$M^2_{\eta^8} \; = \; B(4 m_s + m_d + m_u)/3$$

Octet component of the η meson

1 parameter−free prediction: $3 M^2_{\eta^8} = 4 M^2_K - M^2_\pi$

(Gell−Mann−Okubo mass formula)

$M_{\eta^8}^{\text{(Gell−Mann−Okubo)}}$ = 567 MeV $M_\eta^{\text{(exp)}}$ = 547 MeV

good *a posteriori* check of the assumption $\chi \;\sim\; O(p^2)$
i.e. that $O(m_q^2)$ corrections are small

Quark mass ratios & isospin−breaking

Using the relations

$$M^2_{\pi^0} = B(m_d+m_u) + O(m_q^2)$$
$$M^2_{K^0} = B(m_s+m_d) + O(m_q^2)$$
$$M^2_{\pi^+} = B(m_d+m_u) + \Delta_{E.M.} + O(m_q^2)$$
$$M^2_{K^+} = B(m_s+m_u) + \Delta_{E.M.} + O(m_q^2)$$

correction due to electromagnetic interactions

we cannot determine the absolute values of quark masses (B is unknown) but we can determine their ratios with reasonable accuracy:

$$\frac{m_d-m_u}{m_d+m_u} = \frac{(M_{K^0}-M_{K^+})-(M^2_{\pi^0}-M^2_{\pi^+})}{M^2_{\pi^0}} = 0.22+0.07 = 0.29$$

$$\frac{2m_s}{m_d+m_u} = \frac{(M_{K^+}-M_{\pi^+})+(M^2_{K^0}-M^2_{\pi^0})}{M^2_{\pi^0}} = 24.9$$

⇓

These results indicate large isospin−breaking effects and huge *SU(3)* violations in the mass terms of the QCD lagrangian.

Why these effects are not manifest in most of the hadronic spectrum ?

$p(938.3)$ $n(939.6)$ $\Lambda(1116)$

$\rho(770)$ $\omega(782)$ $K^*(892)$

Light−quark masses are small with respect to the scale of chiral−symmetry breaking.

The extremely accurate *SU(2)* isospin symmetry of the hadronic world is an "accidental" consequence of the smallness of *u* and *d* masses !

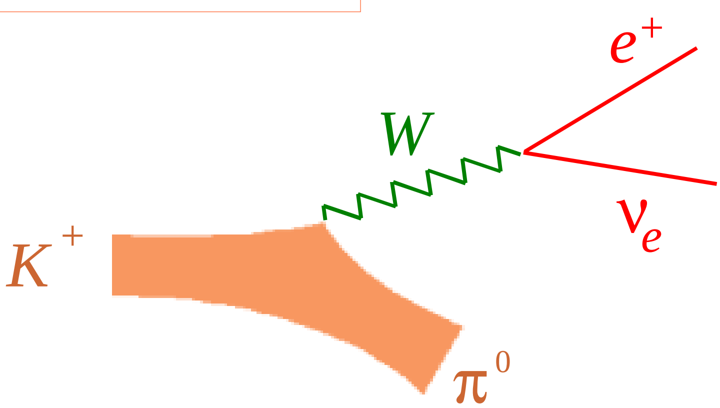
↓

$$\Lambda_\chi \sim 10^3 \text{ MeV}$$
$$m_s \sim 10^2 \text{ MeV}$$
$$m_d, m_u \sim 10^1 \text{ MeV}$$

A posteriori confirmation that the explicit chiral−symmetry breaking terms can be treated as small perturbations around the chiral limit

Explicit example of a tree−level process

$K^+ \to \pi^0 e^+ \nu_e$



the *W* boson can be considered as an external field from the point of view of QCD (or CHPT)

$$l_\mu \to -\frac{eV_{us}^*}{\sqrt{2}\sin\vartheta_W} W_\mu^- \delta_{31}$$

$$L_{1-l_\mu}^{(2)} = \frac{iF^2}{2}\, tr\left(l^\mu \partial_\mu U^+ U\right)$$

$$= -\frac{ieF^2 V_{us}^*}{2\sqrt{2}\sin\vartheta_W} W^\mu\ (\partial_\mu U^+ U)_{13} + \ldots$$

$$= -\frac{ieV_{us}^*}{4\sin\vartheta_W} W^\mu\ (\partial_\mu \pi^0 K^+ - \partial_\mu K^+ \pi^0) + \ldots$$

$$\Downarrow$$

$$A(K^+ \to \pi^0 e^+ \nu_e)^{(2)} = \frac{iG_F V_{us}^*}{2} (p_K + p_\pi)_\mu \bar{u}(p_\nu) \gamma^\mu (1-\gamma_5) v(p_e)$$

[exercise n.4]: evaluate $A(K \to 2\pi l \nu_l)$ at $O(p^2)$