CP
in K and B decays

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LNF Spring School, 12 May 2000

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1. Introduction

The origin of CP violation, to my mind, is one of the two most important questions to be understood in particle physics (the other one being the origin of mass). In the meantime we are finally getting proof - after 51 years of hard work - that $\not{C}\not{P}$ belongs to the weak interaction with 6 quarks and a unitary mixing matrix. Last June 1999, "kaon physicists" had a celebratory get together in Chicago. Many of the comments in these lectures reflect the communal reassessments and cogitations from that workshop. It is clear that a complete experimental and theoretical *albeit* phenomenological solution of the CP violation problem will affect in a most profound way the fabric of particle physics.

2. Historical background

It is of interest, at this junction, to sketch with broad strokes this evolution. With hindsight, one is impressed by how the K mesons are responsible for many of the ideas which today we take for granted.

1. Strangeness which led to quarks and the *flavor* concept.
2. The τ–θ puzzle led to the discovery of parity violation.
3. The ΔI =1/2 rule in non leptonic decays, approximately valid in kaon and all strange particle decays, still not quite understood.
4. The $\Delta S = \Delta Q$ rule in semileptonic decays, fundamental to quark mixing.
5. Flavor changing neutral current suppression which led to 4 quark mixing - GIM mechanism, charm.
6. CP violation, which requires 6 quarks - KM, beauty and top.

2.1 K mesons and strangeness

K mesons were possibly discovered in 1944 in cosmic radiation[(1)] and their decays were first observed in 1947.[(2)] A pair of two old cloud chamber pictures of their decay is on the website http://hepweb.rl.ac.uk/ppUKpics/pr_971217.html
demontrating that they come both in neutral and charged versions. The two pictures are shown in fig. 1.

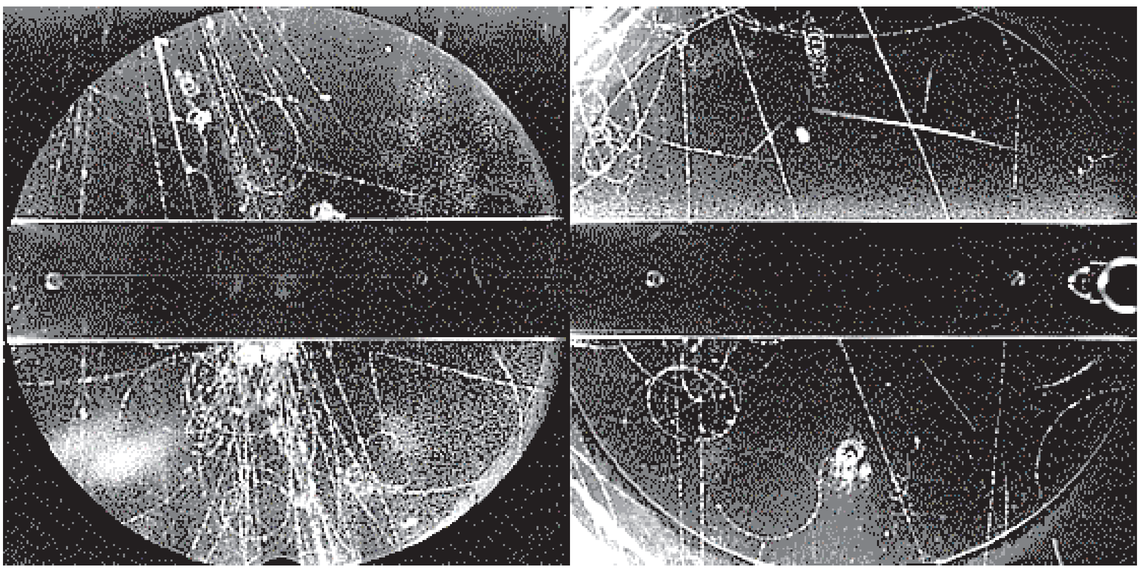


Fig. 1. K discovery

On December 1947 Rochester and Butler (Nature **106**, 885 (1947)) published Wilson chamber pictures showing evidence for what we now call $K^0 \to \pi^+\pi^-$ and $K^+ \to \pi^+\pi^0$.

2.1.1 The Strange Problem

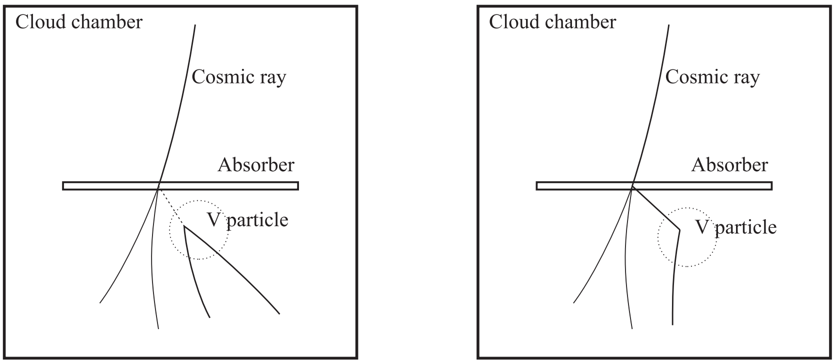


Fig. 2. Production and decay of V particles.

In few triggered pictures, ~1000 nuclear interactions, a few particles which decay in few cm were observed. A typical strong interaction cross section is $(1\ \text{fm})^2 = 10^{-26}\ \text{cm}^2$, corresponding to the production in a 1 g/cm^2 plate of:

$$N_{\text{events}} = N_{\text{in}} \times \sigma \times \frac{\text{nucleons}}{\text{cm}^2} = 10^3 \times 10^{-26} \times 1 \times 6 \times 10^{23} = 6$$

Assuming the V-particles travel a few cm with $\gamma\beta \sim 3$, their lifetime is $\mathcal{O}(10^{-10}\ \text{s})$, typical of weak interactions. We conclude that the decay of V-particles is weak while the production is strong, strange indeed since pions and nucleons appear at the beginning and at the end!! This strange property of K mesons and other particles, the hyperons, led to the introduction of a new quantum number, the strangeness, S.(3) Strangeness is conserved in strong interactions, while 12 first order weak interaction can induce transitions in which strangeness is changed by one unit.

Today we describe these properties in terms of quarks with different "flavors", first suggested in 1964 independently by Gell-Mann and Zweig,(4) reformulating the $SU(3)$ flavor, approximate, global symmetry. The "normal particles" are bound states of quarks: $q\bar{q}$, the mesons, or qqq, baryons, where

$$q = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}.$$

K's, hyperons and hypernuclei contain a strange quark, s:

$$\begin{array}{ll} K^0 = d\bar{s} & \overline{K}^0 = \bar{d}s \\ K^+ = u\bar{s} & K^- = \bar{u}s \\ S = +1 & S = -1. \end{array}$$

The assignment of negative strangeness to the s quark is arbitrary but maintains today the original assignment of positive strangeness for K^0, K^+ and negative for the Λ and Σ hyperons and for $\overline{K}^0$ and K^-. Or, mysteriously, calling negative the charge of the electron.

An important consequence of the fact that K mesons carry strangeness, a new additive quantum number, is that *the neutral K and anti neutral K meson are distinct particles!!!*

$$C|\,K^0\,\rangle = |\,\overline{K}^0\,\rangle,\ \ S|\,K^0\,\rangle = |\,K^0\,\rangle,\ \ S|\,\overline{K}^0\,\rangle = -|\,\overline{K}^0\,\rangle$$

An apocryphal story says that upon hearing of this hypothesis, Fermi challenged Gell-Mann to devise an experiment which shows an observable difference between the K^0 and the $\overline{K}^0$. Today we know that it is trivial to do so. For example, the process $p\bar{p} \to \pi^- K^+ \overline{K}^0$, produces $\overline{K}^0$'s which in turn can produce Λ hyperons while the K^0's produced in $p\bar{p} \to \pi^+ K^- K^0$ cannot.

Another question of Fermi was:
if you observe a $K \to 2\pi$ decay, how do you tell whether it is a K^0 or a $\overline{K}^0$?
Since the '50's K mesons have been produced at accelerators, first amongst them was the Cosmotron.

2.2 Parity Violation

Parity violation, $\not\!P$, was first observed through the θ-τ decay modes of K mesons.

Incidentally, the τ there is not the heavy lepton of today, but is a charged particle which decays into three pions, $K^+ \to \pi^+\pi^+\pi^-$ in todays language.

The θ there refers to a neutral particle which decays into a pair of charged pions, $K^0 \to \pi^+\pi^-$.

The studies of those days were done mostly in nuclear emulsions and JLF contributed also long strands of her hair to make the reference marks between emulsion plates, to enable tracking across plates... The burning question was whether these two particle were the same particle with two decay modes, or two different ones. And if they were the same particle, how could the two different final states have opposite parity?

This puzzle was originally not so apparent until Dalitz advanced an argument which says that one could determine the spin of τ by looking at the decay distribution of the three pions in a "Dalitz" (what he calls phase space) plot, which was in fact consistent with J=0.

The spin of the θ was inferred to be zero because it did not like to decay into a pion and a photon (a photon cannot be emitted in a $0 \to 0$ transition).

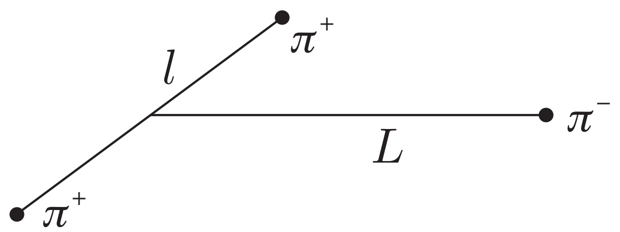


Fig. 3. Definition of l and L for three pion decays of τ^+.

For neutral K's one of the principal decay modes are two or three pions.

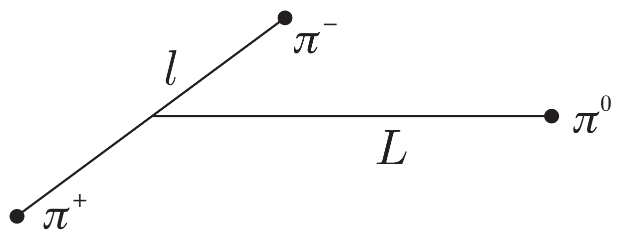


Fig. 4. Definition of l and L for $K^0 \to \pi^+\pi^-\pi^0$.

The relevant properties of the neutral two and three pion systems with zero total angular momentum are given below.

1. $\ell = L = 0,\ 1,\ 2\ldots$
2. $\pi^+\pi^-$, $\pi^0\pi^0$: $P = +1,\ C = +1,\ CP = +1$.
3. $\pi^+\pi^-\pi^-$: $P = -1,\ C = (-1)^l,\ CP = \pm 1$, where l is the angular momentum of the charged pions in their center of mass. States with $l > 0$ are suppressed by the angular momentum barrier.
4. $\pi^0\pi^0\pi^0$: $P = -1,\ C = +1,\ CP = -1$. Bose statistics requires that l for any identical pion pair be even in this case.

Note that the two pion and three pion states have opposite parity.

2.3 Mass and CP eigenstates

While the strong interactions conserve strangeness, the weak interactions do not. In fact, not only do they violate S with $\Delta S=1$, they also violate charge conjugation, C, and parity, P, as we have just seen. However, at the end of the 50's, the weak interaction does not manifestly violate the combined CP symmetry. For now let's assume that CP is a symmetry of the world. We define an arbitrary, unmeasurable phase by:

$$CP|\,K^0\,\rangle = |\,\overline{K}^0\,\rangle$$

Then the simultaneous mass and CP eigenstates are:[5]

$$|\,K_1\,\rangle \equiv \frac{|\,K^0\,\rangle + |\,\overline{K}^0\,\rangle}{\sqrt{2}} \qquad |\,K_2\,\rangle \equiv \frac{|\,K^0\,\rangle - |\,\overline{K}^0\,\rangle}{\sqrt{2}}, \tag{2.1}$$

where K_1 has CP=+1 and K_2 has CP=−1.

While K^0 and $\overline{K}^0$ are degenerate states in mass, as required by CPT invariance, the weak interactions, which induces to second order $K^0 \leftrightarrow \overline{K}^0$ transitions, induces a small mass difference between K_1 and K_2, Δm. We expect that $\Delta m \sim \Gamma$, at least as long as real and imaginary parts of the amplitudes of fig. 5 are about equal, since the decay rate is proportional to the imaginary part and the real part contributes to the mass difference. Dimensionally, $\Gamma = \Delta m = G^2 m_\pi^5 = 5.3\times 10^{-15}$ GeV, in good agreement with measurements. The K_1 mass is the expectation value

$$\langle K_1|H|\,K_1\,\rangle.$$

With $K_1 = (K^0 + \overline{K}^0)/\sqrt{2}$ and anlogously for K_2, we find

$$m_1 - m_2 = \langle K^0|H|\,\overline{K}^0\,\rangle + \langle \overline{K}^0|H|\,K^0\,\rangle,$$

δm is due to $K^0 \leftrightarrow \overline{K}^0$ transitions induced by a ΔS=2 interaction.

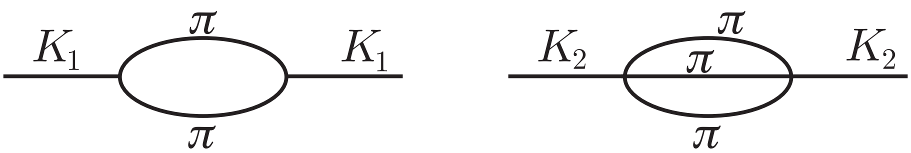


Fig. 5. Contributions to the K_1-K_2 mass difference.

2.4 K_1 and K_2 lifetimes and mass difference

If the total Hamiltonian conserves CP, *i.e.* $[H, CP]=0$, the decays of K_1's and K_2's must conserve CP. Thus the K_1's with $CP=1$, must decay into two pions (and three pions in an $L=\ell=1$ state, surmounting an angular momentum barrier - $\sim(kr)^2(KR)^2 \sim 1/100$ and suppressed by phase space, $\sim 1/1000$), while the K_2's with $CP=-1$, must decay into three pion final states.

Phase space for 3 pion decay is smaller by $32\pi^2$ plus some, since the energy available in 2π decay is ∼220 MeV, while for three πs decay is ∼90 MeV, the lifetime of the K_1 is much much shorter than that of the K_2.

Lederman *et al.*[6] observed long lived neutral kaons in 1956, in a diffusion cloud chamber at the Cosmotron.

Today we have $\tau_1 = (0.8959 \pm 0.0006) \times 10^{-10}$ s and :[†]

$$\begin{aligned} \Gamma_1 &= (1.1162 \pm 0.0007) \times 10^{10}\ \mathrm{s}^{-1} \\ \Gamma_2 &= (1.72 \pm 0.02\times)10^{-3} \times \Gamma_1 \\ \Delta m = m(K_2) - m(K_1) &= (0.5296 \pm 0.0010) \times 10^{10}\ \mathrm{s}^{-1} \\ \Delta m/(\Gamma_1 + \Gamma_2) &= 0.4736 \pm 0.0009. \end{aligned} \tag{2.2}$$

[†]We use natural units, *i.e.* $\hbar = c = 1$. Conversion is found using $\hbar c$=197.3... MeV×fm.

Unit Conversion

To convert from	to	multiply by
1/MeV	s	6.58×10^{-22}
1/MeV	fm	197
$1/\mathrm{GeV}^2$	mb	0.389

2.5 Strangeness oscillations

The mass eigenstates K_1 and K_2 evolve in vacuum and in their rest frame according to

$$|\,K_{1,2},\ t\,\rangle = |\,K_{1,2},\ t=0\,\rangle e^{-i\,m_{1,2}\,t - t\,\Gamma_{1,2}/2} \tag{2.3}$$

If the initial state has definite strangeness, say it is a K^0 as from the production process $\pi^- p \to K^0\Lambda^0$, it must first be rewritten in terms of the mass eigenstates K_1 and K_2 which then evolve in time as above. Since the K_1 and K_2 amplitudes change phase differently in time, the pure S=1 state at t=0 acquires an S=−1 component at $t > 0$. From (2.1) the wave function at time t is:

$$\begin{aligned}\Psi(t) = \sqrt{1/2}[e^{(i\,m_1-\Gamma_1/2)t}|\,K_1\,\rangle + e^{(i\,m_2-\Gamma_1/2)t}|\,K_2\,\rangle] = \\ 1/2[(e^{(i\,m_1-\Gamma_1/2)t} + e^{(i\,m_2-\Gamma_2/2)t})|\,K^0\,\rangle + \\ (e^{(i\,m_1-\Gamma_1/2)t} - e^{(i\,m_2-\Gamma_2/2)t})|\,\bar{K}^0\,\rangle].\end{aligned}$$

The intensity of K^0 ($\bar{K}^0$) at time t is given by:

$$\begin{aligned}I(K^0\ (\bar{K}^0), t) = |\langle K^0\ (\bar{K}^0)|\,\Psi(t)\,\rangle|^2 = \\ \frac{1}{4}[e^{-t\Gamma_1} + e^{-t\Gamma_2} + (-)2e^{-t(\Gamma_1+\Gamma_2)/2}\cos\Delta m\,t]\end{aligned}$$

which exhibits oscillations whose frequency depends on the mass difference, see fig. 6.

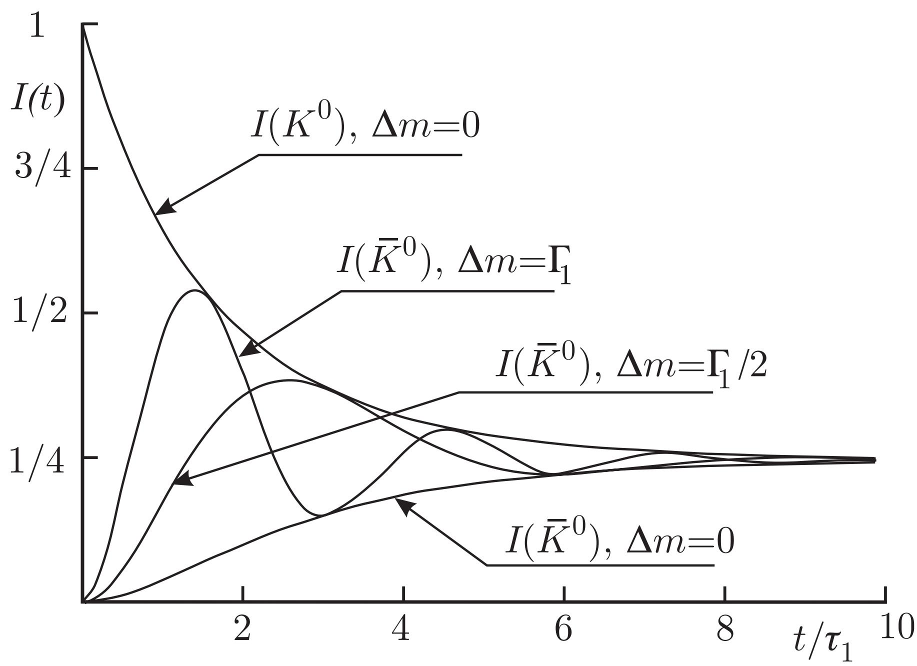


Fig. 6. Evolution in time of a pure S=1 state at time t=0

The appearance of $\bar{K}^0$'s from an initially pure K^0 beam can detected by the production of hyperons, according to the reactions:

$$\begin{aligned}\bar{K}^0 p \to \pi^+\Lambda^0, \quad \to \pi^+\Sigma^+, \quad \to \pi^0\Sigma^+, \\ \bar{K}^0 n \to \pi^0\Lambda^0, \quad \to \pi^0\Sigma^0, \quad \to \pi^-\Sigma^-.\end{aligned}$$

The K_L-K_S mass difference can therefore be obtained from the oscillation frequency.

2.6 Regeneration

Another interesting, and extremely useful phenomenon, is that it is possible to regenerate K_1's by placing a piece of material in the path of a K_2 beam. Let's take our standard reaction,

$$\pi^- p \to K^0\Lambda^0,$$

the initial state wave function of the K^0's is

$$\Psi(t=0) \equiv |\,K^0\,\rangle = \frac{|\,K_1\,\rangle + |\,K_2\,\rangle}{\sqrt{2}}.$$

Note that it is composed equally of K_1's and K_2's. The K_1 component decays away quickly via the two pion decay modes, leaving a virtually pure K_2 beam.

A K_2 beam has equal K^0 and $\overline{K}^0$ components, which interact differently in matter. For example, the K^0's undergo elastic scattering, charge exchange etc. whereas the $\overline{K}^0$'s also produce hyperons via strangeness conserving transitions. Thus we have an apparent rebirth of K_1's emerging from a piece of material placed in the path of a K_2 beam! See fig. 7.

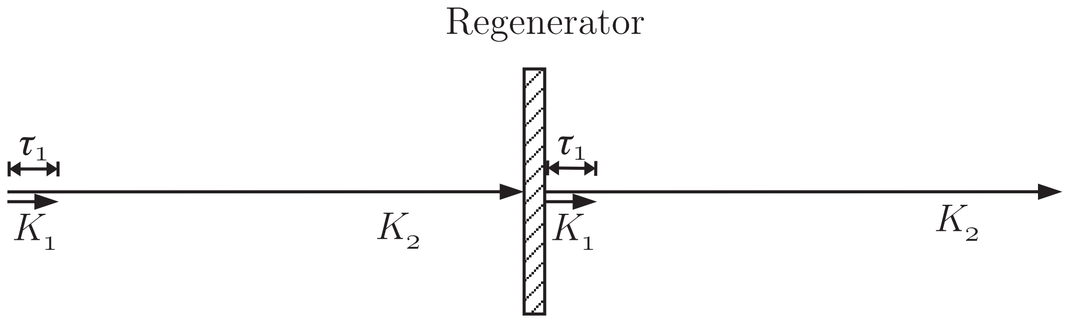


Fig. 7. K_1 regeneration

Virtually all past and present experiments, with the exception of a couple which will be mentioned explicitly, use this method to obtain a source of K_1's (or K_S's, as we shall see later).

Denoting the amplitudes for K^0 and $\overline{K}^0$ scattering on nuclei by f and $\bar{f}$ respectively, the scattered amplitude for an initial K_2 state is given by:

$$\begin{aligned}\sqrt{1/2}(f|\,K^0\,\rangle - \bar{f}|\,\overline{K}^0\,\rangle) &= \frac{f+\bar{f}}{2\sqrt{2}}(|\,K^0\,\rangle - |\,\overline{K}^0\,\rangle) + \frac{f-\bar{f}}{2\sqrt{2}}(|\,K^0\,\rangle + |\,\overline{K}^0\,\rangle)\\ &= 1/2(f+\bar{f})|\,K_2\,\rangle + 1/2(f-\bar{f})|\,K_1\,\rangle.\end{aligned}$$

The so called regeneration amplitude for $K_2 \to K_1$, f_{21} is given by $1/2(f-\bar{f})$ which of course would be 0 if $f=\bar{f}$, which is true at infinite energy.

Another important property of regeneration is that when the K_1 is produced at non-zero angle to the incident K_2 beam, regeneration on different nuclei in a regenerator is incoherent, while at zero degree the amplitudes from different nuclei add up coherently.

The intensity for coherent regeneration depends on the K_1, K_2 mass difference. Precision mass measurements have been performed by measuring the ratio of coherent to diffraction regeneration. The interference of K_1 waves from two or more regenerators has also allowed us to determine that the K_2 meson is heavier than the K_1 meson. This perhaps could be expected, but it is nice to have it measured.

Finally we note that the K_1 and K_2 amplitudes after regeneration are coherent and can interfere if CP is violated.

3. CP Violation in Two Pion Decay Modes

3.1 Discovery

For some years after the discovery that C and P are violated in the weak interactions, it was thought that CP might still be conserved. CP violation was discovered in '64,[7] through the observation of the unexpected decay $K_2 \to \pi^+\pi^-$. This beautiful experiment is conceptually very simple, see fig. 8.

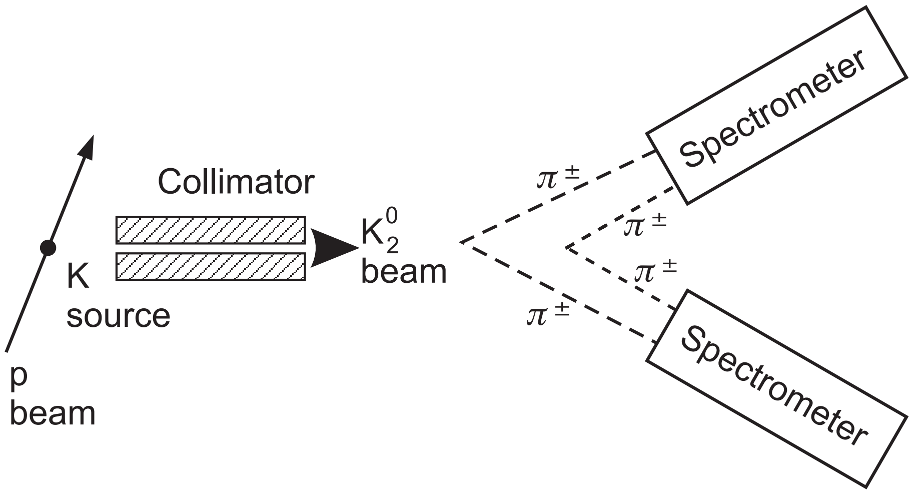


Fig. 8. The setup of the experiment of Christenson *et al.*.

Let a K beam pass through a long collimator and decay in an empty space (actually a big helium bag) in front of two spectrometers. We have mace a K_2 beam. The K_2 decay products are viewed by spark chambers and scintillator hodoscopes in the spectrometers placed on either side of the beam.

Two pion decay modes are distinguished from three pion and leptonics decay modes by the reconstructed invariant mass $M_{\pi\pi}$, and the direction θ of their resultant momentum vector relative to the beam. In the mass interval 494-504 MeV an excess of 45 events collinear with the beam ($\cos\theta > 0.99997$) is observed. For the intervals 484-494 and 504-514 there is no excess, establishing that K_2s decay into two pions, with a branching ratio of the order of 2×10^{-3}.

CP is therefore shown to be violated! The CP violating decay $K_L \to \pi^0\pi^0$ has also been observed.

3.2 K^0 Decays with CP Violation

Since CP is violated in K decays, the mass eigenstates are no more CP eigenstate and can be written, assuming CPT invariance, as:

$$K_S = \Big((1+\epsilon)|\,K^0\,\rangle + (1-\epsilon)|\,\overline{K}^0\,\rangle\Big)/\sqrt{2(1+|\epsilon|^2)}$$
$$K_L = \Big((1+\epsilon)|\,K^0\,\rangle - (1-\epsilon)|\,\overline{K}^0\,\rangle\Big)/\sqrt{2(1+|\epsilon|^2)}$$

Another equivalent form, in terms of the CP eigenstate K_1 and K_2 is:

$$|\,K_S\,\rangle = \frac{|\,K_1\,\rangle + \epsilon|\,K_2\,\rangle}{\sqrt{1+|\epsilon|^2}} \qquad |\,K_L\,\rangle = \frac{|\,K_2\,\rangle + \epsilon|\,K_1\,\rangle}{\sqrt{1+|\epsilon|^2}} \tag{3.1}$$

with $|\epsilon| = (2.259 \pm 0.018) \times 10^{-3}$ from experiment. Note that the K_S and K_L states are not orthogonal states, contrary to the case of K_1 and K_2. If we describe an arbitrary state $a|\,K^0\,\rangle + b|\,\overline{K}^0\,\rangle$ as

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix}.$$

its time evolution is given by

$$i\frac{\mathrm{d}}{\mathrm{d}t}\psi = (\mathbf{M} - i\mathbf{\Gamma}/2)\psi$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$ are 2×2 hermitian matrices which can be called the mass and decay matrix. CPT invariance requires $M_{11} = M_{22}$, *i.e.* $M(K^0) = M(\overline{K}^0)$, and $\Gamma_{11} = \Gamma_{22}$. CP invariance requires arg(Γ_{12}/M_{12})=0. The relation between ϵ and $\mathbf{M}$, $\mathbf{\Gamma}$ is:

$$\frac{1+\epsilon}{1-\epsilon}=\sqrt{\frac{M_{12}-\Gamma_{12}/2}{M^*_{12}-\Gamma^*_{12}/2}}.$$

K_S and K_L satisfy

$$(\mathbf{M}-i\mathbf{\Gamma})|\,K_{S,L}\,\rangle=(M_{S,L}-i\Gamma_{S,L})|\,K_{S,L}\,\rangle$$

where $M_{S,L}$ and $\Gamma_{S,L}$ are the mass and width of the physical neutral kaons, with values given earlier for the K_1 and K_2 states.

Equation (2.3) is rewritten rewritten as:

$$|\,K_{S,L},\ t\,\rangle=|\,K_{S,L},\ t=0\,\rangle e^{-i\,M_{S,L}\,t-\Gamma_{S,L}/2\,t}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}|\,K_{S,L}\,\rangle=-i\mathcal{M}_{S,L}|\,K_{S,L}\,\rangle$$

with

$$\mathcal{M}_{S,L}=M_{S,L}-i\Gamma_{S,L}/2$$

and the values of masses and decay widths given in eq. (2.2) belong to K_S and K_L, rather than to K_1 and K_2. We further introduce the so called superweak phase $\phi_{\rm SW}$ as:

$$\phi_{\rm SW}=\mathrm{Arg}(\epsilon)=\tan^{-1}\frac{2(M_{K_L}-M_{K_S})}{\Gamma_{K_S}-\Gamma_{K_L}}=43.63^\circ\pm0.08^\circ.$$

A superweak theory, is a theory with a ΔS=2 interaction, whose sole effect is to induce a CP impurity ϵ in the mass eigenstates.

Since 1964 we have been asking the question: is CP violated directly in K^0 decays, *i.e.* is the $|\Delta S|$=1 amplitude $\langle\pi\pi|\,K_2\,\rangle\neq0$ or the only manifestation of $\not{C}\!\not{P}$ is to introduce a small impurity of K_1 in the K_L state, via $K^0\leftrightarrow\overline{K}^0$, $|\Delta S|$=2 transitions?

Wu and Yang[8] have analyzed the two pion decays of K_S, K_L in term of the isospin amplitudes:

$$A(K^0\to2\pi,I)=A_Ie^{i\delta_I}$$
$$A(\overline{K}^0\to2\pi,I)=A_I^*e^{i\delta_I}$$

where δ_I are the $\pi\pi$ scattering phase shifts in the I=0, 2 states. W-Y chose an arbitrary phase, by defining A_0 real. They also introduce the ratios of the amplitudes for K decay to a final state f_i, $\eta_i=A(K_L\to f_i)/A(K_S\to f_i)$:

$$\eta_{+-}\equiv|\eta_{+-}|e^{-i\phi_{+-}}=\frac{\langle\pi^+\pi^-|\,K_L\,\rangle}{\langle\pi^+\pi^-|\,K_S\,\rangle}=\epsilon+\epsilon'$$
$$\eta_{00}\equiv|\eta_{00}|e^{-i\phi_{00}}=\frac{\langle\pi^0\pi^0|\,K_L\,\rangle}{\langle\pi^0\pi^0|\,K_S\,\rangle}=\epsilon-2\epsilon',$$

with

$$\epsilon'=\frac{i}{2\sqrt{2}}\,e^{i(\delta_2-\delta_0)}\,\frac{\Re A_2}{A_0}\,\frac{\Im A_2}{\Re A_2}$$

Since $\delta_2-\delta_0{\sim}45^\circ$, Arg$(\epsilon')\sim135^\circ$ *i.e.* ϵ' is orthogonal to ϵ. Therefore, in principle, only two real quantities need to be measured: $\Re\epsilon$ and $\Re(\epsilon'/\epsilon)$, *with sign*.

In terms of the measurable amplitude ratios, η, ϵ andϵ' are given by:

$$\epsilon=(2\eta_{+-}+\eta_{00})/3$$
$$\epsilon'=(\eta_{+-}-\eta_{00})/3$$
$$\mathrm{Arg}(\epsilon)=\phi_{+-}+(\phi_{+-}-\phi_{00})/3.$$

ϵ' is a measure of direct CP violation and its magnitude is $\mathcal{O}(A(K_2 \to \pi\pi)/A(K_1 \to \pi\pi))$.

Our question above is then the same as: is $\epsilon' \neq 0$? Since 1964, experiments searching for a difference in η_{+-} and η_{00} have been going on.

If $\eta_{+-} \neq \eta_{00}$ the ratios of branching ratios for $K_{L,S} \to \pi^+\pi^-$ and $\pi^0\pi^0$ are different.

The first measurement of BR$(K_L \to \pi^0\pi^0)$, *i.e.* of $|\eta_{00}|^2$ was announced by Cronin in 1965.......

Most experiments measure the quantity $\mathcal{R}$, the so called double ratio of the four rates for $K_{L,S} \to \pi^0\pi^0$, $\pi^+\pi^-$, which is given, to lowest order in ϵ and ϵ' by:

$$\mathcal{R} \equiv \frac{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)} \equiv \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = 1 - 6\Re(\epsilon'/\epsilon).$$

Observation of $\mathcal{R} \neq 0$ is proof that $\Re(\epsilon'/\epsilon) \neq 0$ and therefore of "direct" CP violation, *i.e.* that the amplitude for $|\Delta S|=1$, CP violating transitions

$$A(K_2 \to 2\pi) \neq 0.$$

All present observations of CP violation, $\not{C}\not{P}$, *i.e.* the decays $K_L \to 2\pi$, $\pi^+\pi^-\gamma$ and the charge asymmetries in $K_{\ell 3}$ decays are examples of so called "indirect" violation, due to $|\Delta S|=2$ $K^0 \leftrightarrow \overline{K}^0$ transitions introducing a small CP impurity in the mass eigenstates K_S and K_L.

Because of the smallness of ϵ (and ϵ'), most results and parameter values given earlier for K_1 and K_2 remain valid after the substitution $K_1 \to K_S$ and $K_2 \to K_L$.

3.3 Experimental Status

We have been enjoying a roller coaster ride on the last round of CP violation precision experiments.

One of the two, NA31, was performed at CERN and reported a tantalizing non-zero result:[(9)]

$$\Re(\epsilon'/\epsilon) = (23 \pm 6.5) \times 10^{-4}.$$

NA31 alternated K_S and K_L data taking by the insertion of a K_S regenerator in the K_L beam every other run, while the detector collected both charged and neutral two pion decay modes simultaneously.

The other experiment, E731 at Fermilab, was consistent with no or very small direct $\not{C}\not{P}$:[(10)]

$$\Re(\epsilon'/\epsilon) = (7.4 \pm 5.9) \times 10^{-4}.$$

E731 had a fixed K_S regenerator in front of one of the two parallel K_L beams which entered the detector which, however, collected alternately the neutral and charged two pion decay modes.

Both collaborations have completely redesigned their experiments. Both experiments can now observe both pion modes for K_S and K_L simultaneously. Preliminary results indicate that in fact the answer to the above question is a resounding NO!!!

In fact, the great news in HEP for 1999 is that, combining the results of few years ago and the new ones, the value of ϵ' is $(21.2 \pm 4.6) \times 10^{-4}$, which means that there definitely is direct CP violation. The observed value is 4.7σ away from zero.

Attention is now turning to a precise determination of ϵ'.

4. CP Violation at a ϕ–factory

4.1 $e^+e^- \to \phi$, $\phi \to K^0\overline{K}^0$, $(\Upsilon''' \to B^0\overline{B}^0)$

The cross section for production of a bound $q\bar{q}$ pair of mass M and total width Γ with $J^{PC} = 1^{--}$, a so called vector meson V, (ϕ in the following and the $\Upsilon(4S)$ later) in e^+e^- annihilation, see fig. 9, is given by:

$$\sigma_{q\bar{q},\text{res}} = \frac{12\pi}{s}\frac{\Gamma_{ee}\Gamma M^2}{(M^2-s)^2 + M^2\Gamma^2} = \frac{12\pi}{s}B_{ee}B_{q\bar{q}}\frac{M^2\Gamma^2}{(M^2-s)^2 + M^2\Gamma^2}$$

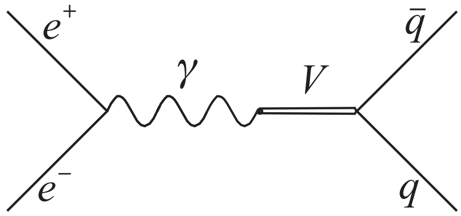


Fig. 9. Amplitude for production of a bound $q\bar{q}$ pair

The ϕ meson is an $s\bar{s}$ $^3\mathrm{S}_1$ bound state with J^{PC}=1^{--}, just as a photon and the cross section for its production in e^+e^- annihilations at 1020 MeV is

$$\sigma_{s\bar{s}}(s = (1.02)^2\ \mathrm{GeV}^2) \sim \frac{12\pi}{s} B_{ee}$$

$$= 36.2 \times (1.37/4430) = 0.011\ \mathrm{GeV}^{-2} \sim 4000\ \mathrm{nb},$$

compared to a total hadronic cross section of ~(5/3) ×87~100 nb.

The production cross section for the $\Upsilon(4S)$ at W=10,400 MeV is ~1 nb, over a background of ~2.6 nb.

The Frascati ϕ–factory, DAΦNE, will have a luminosity $\mathcal{L} = 10^{33}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1} = 1\ \mathrm{nb}^{-1}\mathrm{s}^{-1}$. Integrating over one year, taken as 10^7 s or one third of a calendar year, we find

$$\int_{1\ \mathrm{y}} \mathcal{L}\mathrm{d}t = 10^7\ \mathrm{nb}^{-1},$$

corresponding to the production at DAΦNE of $\sim 4000 \times 10^7 = 4 \times 10^{10}$ ϕ meson per year or approximately 1.3×10^{10} K^0, $\overline{K}^0$ pairs, a large number indeed.

One of the advantages of studying K mesons at a ϕ–factory, is that they are produced in a well defined quantum state. Neutral K mesons are produced as collinear pairs, with $J^{PC} = 1^{--}$ and a momentum of about 110 MeV/c, thus detection of one K announces the presence of the other and gives its direction.

Since in the reaction:

$$e^+e^- \to \text{“}\gamma\text{”} \to \phi \to K^0\overline{K}^0$$

we have

$$C(K^0\overline{K}^0) = C(\phi) = C(\gamma) = -1.$$

we can immediately write the 2-K state. Define $|\,i\,\rangle$=$|\,K\overline{K},\ t = 0,\ C = -1\,\rangle$. Then $|\,i\,\rangle$ must have the form:

$$|\,i\,\rangle = \frac{|\,K^0, \mathbf{p}\,\rangle|\,\overline{K}^0, -\mathbf{p}\,\rangle - |\,\overline{K}^0, \mathbf{p}\,\rangle|\,K^0, -\mathbf{p}\,\rangle}{\sqrt{2}}$$

From eq. (3.1), the relations between K_S, K_L and K^0, $\overline{K}^0$, to lowest order in ϵ, we find:

$$|\,K_S\ (K_L)\,\rangle = \frac{(1+\epsilon)|\,K^0\,\rangle + (-)(1-\epsilon)|\,\overline{K}^0\,\rangle}{\sqrt{2}}.$$

$$|\,K^0\ (\overline{K}^0)\,\rangle = \frac{|\,K_S\,\rangle + (-)|\,K_L\,\rangle}{(1+(-)\epsilon)\sqrt{2}}$$

from which

$$|\,i\,\rangle = \frac{1}{\sqrt{2}}(|\,K_S, -\mathbf{p}\,\rangle|\,K_L, \mathbf{p}\,\rangle - |\,K_S, \mathbf{p}\,\rangle|\,K_L, -\mathbf{p}\,\rangle)$$

so that the neutral kaon pair produced in e^+e^- annihilations is a pure K^0, $\bar{K}^0$ as well as a pure K_S, K_L for *all times,* in vacuum. What this means, is that if at some time t a K_S (K_L, K^0, $\bar{K}^0$) is recognized, the other kaon, if still alive, is a K_L (K_S, $\bar{K}^0$, K^0).

The result above is correct to all orders in ϵ, apart from a normalization constant, and holds even without assuming CPT invariance.

4.2 Correlations in K_S, K_L decays

To obtain the amplitude for decay of $K(\mathbf{p})$ into a final state f_1 at time t_1 and of $K(-\mathbf{p})$ to f_2 at time t_2, see the diagram below, we time evolve the initial state in the usual way:

$$|\,t_1,\ \mathbf{p};\ t_2,\ -\mathbf{p}\,\rangle = \frac{1+|\epsilon^2|}{(1-\epsilon^2)\sqrt{2}}\times \Big(|\,K_S(-\mathbf{p})\,\rangle|\,K_L(\mathbf{p})\,\rangle e^{-i(\mathcal{M}_S t_2+\mathcal{M}_L t_1)} - |\,K_S(\mathbf{p})\,\rangle|\,K_L(-\mathbf{p})\,\rangle e^{-i(\mathcal{M}_S t_1+\mathcal{M}_L t_2)}\Big)$$

Fig. 10. $\phi\to K_L,\ K_S\to f_1,\ f_2$.

where $\mathcal{M}_{S,L} = M_{S,L} - i\Gamma_{S,L}/2$ are the complex K_S, K_L masses.

In terms of the previously mentioned ratios $\eta_i = \langle\, f_i|\,K_L\,\rangle/\langle\, f_i|\,K_S\,\rangle$ and defining $\Delta t = t_2 - t_1$, $t = t_1 + t_2$, $\Delta\mathcal{M} = \mathcal{M}_L - \mathcal{M}_S$ and $\mathcal{M} = \mathcal{M}_L + \mathcal{M}_S$ we get the amplitude for decay to states 1 and 2:

$$A(f_1, f_2, t_1, t_2) = \langle\, f_1|\,K_S\,\rangle\langle\, f_2|\,K_S\,\rangle e^{-i\mathcal{M}t/2} \times \Big(\eta_1 e^{i\Delta\mathcal{M}\Delta t/2} - \eta_2 e^{-i\Delta\mathcal{M}\Delta t/2}\Big)/\sqrt{2}. \tag{4.1}$$

This implies $A(e^+e^- \to \phi \to K^0\bar{K}^0 \to f_1 f_2) = 0$ for $t_1 = t_2$ and $f_1 = f_2$ (Bose statistics).

For $t_1 = t_2$, $f_1 = \pi^+\pi^-$ and $f_2 = \pi^0\pi^0$ instead, $A \propto \eta_{+-} - \eta_{00} = 3\times\epsilon'$ which suggest a (unrealistic) way to measure ϵ'.

The intensity for decay to final states f_1 and f_2 at times t_1 and t_2 obtained taking the modulus squared of eq. (4.1) depends on magnitude and argument of η_1 and η_2 as well as on $\Gamma_{L,S}$ and ΔM. The intensity is given by

$$I(f_1, f_2, t_1, t_2) = |\langle\, f_1|\,K_S\,\rangle|^2|\langle\, f_2|\,K_S\,\rangle|^2 e^{-\Gamma_S\,t/2}\times (|\eta_1|^2 e^{\Gamma_S\Delta t/2} + |\eta_2|^2 e^{-\Gamma_S\Delta t/2} - 2|\eta_1||\eta_2|\cos(\Delta m\,t + \phi_1 - \phi_2))$$

where we have everywhere neglected Γ_L with respect to Γ_S.

Thus the study of the decay of K pairs at a ϕ–factory offers the unique possibility of observing interference pattern in time, or space, in the intensity observed at two different points in space.

This fact is the source of endless excitement and frustration to some people.

Rather than studying the intensity above, which is a function of two times or distances, it is more convenient to consider the once integrated distribution. In particular one can integrate the intensity over all times t_1 and t_2 for fixed time difference $\Delta t = t_1 - t_2$, to obtain the intensity as a function of Δt. Performing the integrations yields, for $\Delta t > 0$,

$$I(f_1,\ f_2;\ \Delta t) = \frac{1}{2\Gamma}|\langle f_1|\,K_S\,\rangle\langle f_2|\,K_S\,\rangle|^2 \times \Big(|\eta_1|^2 e^{-\Gamma_L\Delta t} + |\eta_2|^2 e^{-\Gamma_S\Delta t} - 2|\eta_1||\eta_2|e^{-\Gamma\Delta t/2}\cos(\Delta m\Delta t + \phi_1 - \phi_2)\Big)$$

and a similar espression is obtained for $\Delta t < 0$. The interference pattern is quite different according to the choice of f_1 and f_2 as illustrated in fig. 11.

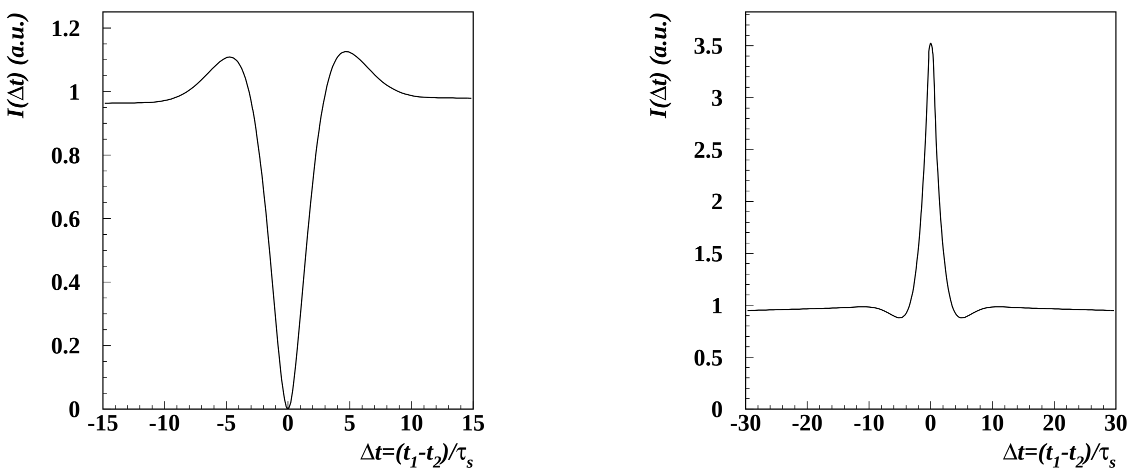


Fig. 11. Interference pattern for $f_{1,2}$=$\pi^+\pi^-$, $\pi^0\pi^0$and ℓ^-, ℓ^+.

The strong destructive interference at zero time difference is due to the antisymmetry of the initial KK state, decay amplitude phases being identical.

The destructive interference at zero time difference becomes constructive since the amplitude for $K^0 \to \ell^-$ has opposite sign to that for $\bar{K}^0 \to \ell^+$ thus making the overall amplitude symmetric.

One can thus perform a whole spectrum of precision "kaon-interferometry" experiments at DAΦNE by measuring the above decay intensity distributions for appropriate choices of the final states f_1, f_2. Four examples are listed below.

1. With f_1=f_2 one measures Γ_S, Γ_L and Δm, since all phases cancel. Rates can be measured with a $\times 10$ improvement in accuracy and Δm to $\sim \times 2$.

2. With f_1=$\pi^+\pi^-$, f_2=$\pi^0\pi^0$, one measures $\Re(\epsilon'/\epsilon)$ at large time differences, and $\Im(\epsilon'/\epsilon)$ for $|\Delta t| \leq 5\tau_s$. Fig. 11 shows the interference pattern for this case.

3. With $f_1 = \pi^+\ell^-\nu$ and $f_2 = \pi^-\ell^+\nu$, one can measure the CPT–violation parameter δ, see our discussion later concerning tests of CPT. Again the real part of δ is measured at large time differences and the imaginary part for $|\Delta t| \leq 10\tau_s$. Fig. 11 shows the interference pattern

4. For $f_1 = 2\pi$, $f_2 = \pi^+\ell^-\nu$ or $\pi^-\ell^+\nu$ small time differences yield Δm, $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$, while at large time differences, the asymmetry in K_L semileptonic decays provides tests of T and CPT. The *vacuum regeneration* interference is shown in fig. 12.

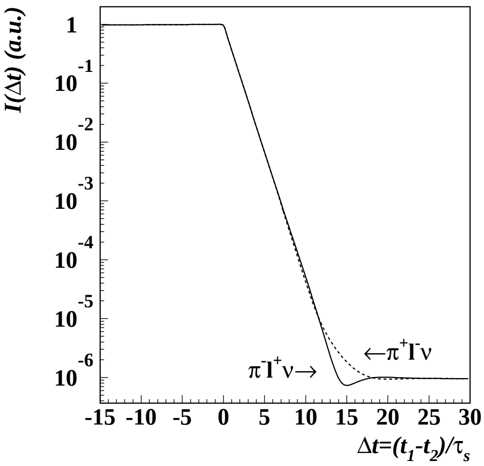


Fig. 12. Interference pattern for $f_1 = 2\pi, f_2 = \ell^\pm$

5. CP Violation in Other Modes

5.1 Semileptonic decays

K-mesons also decay semileptonically, into a hadron with charge Q and strangeness zero, and a pair of lepton-neutrino. These decays at quark levels are due to the elementary processes

$$s \to W^- u \to \ell^- \bar{\nu} u$$
$$\bar{s} \to W^+ \bar{u} \to \ell^+ \nu \bar{u}.$$

Physical K-mesons could decay as:

$$\begin{aligned} K^0 &\to \pi^- \ell^+ \nu,\ \Delta S = -1,\ \Delta Q = -1 \\ \overline{K}^0 &\to \pi^+ \ell^- \bar{\nu},\ \Delta S = +1,\ \Delta Q = +1 \\ \overline{K}^0 &\to \pi^- \ell^+ \nu,\ \Delta S = +1,\ \Delta Q = -1 \\ K^0 &\to \pi^+ \ell^- \bar{\nu},\ \Delta S = -1,\ \Delta Q = +1. \end{aligned}$$

In the standard model, SM, K^0 decay only to ℓ^- and $\overline{K}^0$ to ℓ^+. This is commonly referred to as the $\Delta S = \Delta Q$ rule, experimentally established in the very early days of strange particle studies. Semileptonic decays enable one to know the strangeness of the decaying meson - and for the case of pair production to “tag” the strangeness of the other meson of the pair.

Assuming the validity of the $\Delta S = \Delta Q$ rule, the leptonic asymmetry

$$\mathcal{A}_\ell = \frac{\ell^- - \ell^+}{\ell^- + \ell^+}$$

in K_L or K_S decays is

$$2\Re\epsilon \simeq \sqrt{2}|\epsilon| = (3.30 \pm 0.03) \times 10^{-3}.$$

The measured value of $\mathcal{A}_\ell$ for K_L decays is $(0.327 \pm 0.012)\%$, in good agreement with the above expectation, a proof that CP violation is, mostly, in the mass term.

In strong interactions strangeness is conserved. The strangeness of neutral K-mesons can be tagged by the sign of the charge kaon (pion) in the reaction

$$p + \bar{p} \to K^0(\overline{K}^0) + K^{-(+)} + \pi^{+(-)}.$$

5.2 CP VIOLATION IN K_S DECAYS

CP violation has only been seen in K_L decays ($K_L \to \pi\pi$ and semileptonic decays). This is because, while it is easy to prepare an intense, pure K_L beam, thus far it has not been possible to prepare a pure K_S beam.

However, if the picture of $\not{C}\not{P}$ we have developed so far is correct, we can predict quite accurately the values of some branching ratios and the leptonic asymmetry.

It is quite important to check experimentally such predictions especially since the effects being so small, they could be easily perturbed by new physics outside the standard model.

5.2.1 $K_S \to \pi^0\pi^0\pi^0$

At a ϕ–factory such as DAΦNE, where $\mathcal{O}(10^{10})$ tagged K_S/y will be available, one can look for the $\not{C}\not{P}$ decay $K_S \to \pi^0\pi^0\pi^0$, the counterpart to $K_L \to \pi\pi$.

The branching ratio for this process is proportional to $|\epsilon + \epsilon'_{000}|^2$ where ϵ'_{000} is a quantity similar to ϵ', signalling direct CP violation. While ϵ'_{000}/ϵ might not be as suppressed as the ϵ'/ϵ, we can neglect it to an overall accuracy of a few %. Then $K_S \to \pi^+\pi^-\pi^0$ is due to the K_L impurity in K_S and the expected BR is 2×10^{-9}. The signal at DAΦNE is at the 30 event level. There is here the possibility of observing the CP impurity of K_S, never seen before.

The current limit on BR($K_S \to \pi^+\pi^-\pi^0$) is 3.7×10^{-5}.

5.2.2 BR($K_S \to \pi^\pm\ell^\mp\nu$) and $\mathcal{A}_\ell(K_S)$

The branching ratio for $K_S \to \pi^\pm\ell^\mp\nu$ can be predicted quite accurately from that of K_L and the K_S-K_L lifetimes ratio, since the two amplitudes are equal assuming CPT invariance. In this way we find

$$\mathrm{BR}(K_S \to \pi^\pm e^\mp \nu) = (6.70 \pm 0.07) \times 10^{-4}$$
$$\mathrm{BR}(K_S \to \pi^\pm \mu^\mp \nu) = (4.69 \pm 0.06) \times 10^{-4}$$

The leptonic asymmetry in K_S (as for K_L) decays is $2\Re\epsilon = (3.30 \pm 0.03) \times 10^{-3}$.

Some tens of leptonic decays of K_S have been seen recently by CMD-2 at Novosibirsk resulting in a value of BR of 30% accuracy, not in disagreement with expectation. The leptonic asymmetry $\mathcal{A}_\ell$ in K_S decays is not known. At DAΦNE an accuracy of $\sim 2.5 \times 10^{-4}$ can be obtained. The accuracy on BR would be vastly improved.

This is again only a measurement of ϵ, not ϵ', but the observation for the first time of CP violation in two new channels of K_S decay would be nonetheless of considerable interest.

5.3 CP VIOLATION IN CHARGED K DECAYS

Evidence for direct CP violation can be also be obtained from the decays of charged K mesons. CP invariance requires equality of the partial rates for $K^\pm \to \pi^\pm\pi^+\pi^-$ ($\tau^\pm$) and for $K^\pm \to \pi^\pm\pi^0\pi^0$ ($\tau'^\pm$).

With the luminosities obtainable at DAΦNE one can improve the present rate asymmetry measurements by two orders of magnitude, although *alas* the expected effects are predicted from standard calculations to be woefully small.

One can also search for differences in the Dalitz plot distributions for K^+ and K^- decays in both the τ and τ' modes and reach sensitivities of $\sim 10^{-4}$. Finally, differences in rates in the radiative two pion decays of $K^\pm$, $K^\pm \to \pi^\pm\pi^0\gamma$, are also proof of direct CP violation. Again, except for unorthodox computations, the effects are expected to be very small.

6. Determinations of Neutral Kaon Properties

6.1 CPLEAR

The CPLEAR experiment[11] studies neutral K mesons produced in equal numbers in protonantiproton annihilations at rest:

$$p\bar{p} \to K^- \pi^+ K^0 \quad \text{BR}{=}2\times 10^{-3}$$
$$\to K^+ \pi^- \overline{K}^0 \quad \text{BR}{=}2\times 10^{-3}$$

The charge of $K^\pm(\pi^\pm)$ tags the strangeness S of the neutral K at t=0.

CPLear fig.

They have presented several results(12,13)from studying $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$ and $\pi^\pm\ell^\mp\bar{\nu}(\nu)$ final states. Of particular interest is their measurement of the K_L–K_S mass difference Δm because it is independent of the value of ϕ_{+-}, unlike in most other experiments.

They also obtain improved limits on the possible violation of the $\Delta S = \Delta Q$ rule, although still far from the expected SM value of about 10^{-7} arising at higher order.

The data require small corrections for background asymmetry $\sim$1%, differences in tagging efficiency, $\varepsilon(K^+\pi^-) - \varepsilon(K^-\pi^+)\sim 10^{-3}$ and in detection, $\varepsilon(\pi^+e^-) - \varepsilon(\pi^-e^+)\sim 3\times 10^{-3}$. Corrections for some regeneration in the detector are also needed.

6.1.1 $K^0(\overline{K}^0) \to e^+(e^-)$

Of particular interest are the study of the decays $K^0(\overline{K}^0) \to e^+(e^-)$. One can define the four decay intensities:

$$\left.\begin{array}{lll} I^+(t) & \text{for} & K^0 \to e^+ \\ \overline{I}^-(t) & \text{for} & \overline{K}^0 \to e^- \end{array}\right\} \Delta S = 0$$

$$\left.\begin{array}{lll} \overline{I}^+(t) & \text{for} & \overline{K}^0 \to e^+ \\ I^-(t) & \text{for} & K^0 \to e^- \end{array}\right\} |\Delta S| = 2$$

where $\Delta S = 0$ or 2 means that the strangeness of the decaying K is the same as it was at t=0 or has changed by 2, because of $K^0 \leftrightarrow \overline{K}^0$ transitions.

One can then define four asymmetries:

$$A_1(t) = \frac{I^+(t) + \overline{I}^-(t) - (\overline{I}^+(t) + I^-(t))}{I^+(t) + \overline{I}^-(t) + \overline{I}^+(t) + I^-(t)}$$
$$A_2(t) = \frac{\overline{I}^-(t) + \overline{I}^+(t) - (I^+(t) + I^-(t))}{\overline{I}^-(t) + \overline{I}^+(t) + I^+(t) + I^-(t)}$$

$$A_T(t) = \frac{\overline{I}^+(t) - I^-(t)}{\overline{I}^+(t) + I^-(t)}, \qquad A_{CPT}(t) = \frac{\overline{I}^-(t) - I^+(t)}{\overline{I}^-(t) + I^+(t)}$$

From the time dependence of A_1 they obtain: $\Delta m = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10}\ \text{s}^{-1}$, and $\Delta S = \Delta Q$ is valid to an accuracy of

$$(12.4 \pm 11.9 \pm 6.9) \times 10^{-3}.$$

Measurements of A_T, which they insist in calling a direct test of the validity of T but for me is just a test of CP invariance or lack of it, involves comparing T "conjugate" processes (which in fact are just CP conjugate) is now hailed as a direct measurement of T violation.

The expected value for A_T is $4\times\Re\epsilon$=6.52×10^{-3}. The CPLEAR result is $A_T = (6.6 \pm 1.3 \pm 1.6) \times 10^{-3}$.

In other words, just as expected from the CP impurity of Ks.

6.1.2 $\pi^+\pi^-$ *Final State*

From an analysis of 1.6×10^7 $\pi^+\pi^-$ decays of K^0 and $\overline{K}^0$ they determine $|\eta_{+-}| = (2.312 \pm 0.043 \pm 0.03 \pm 0.011_{\tau_S}) \times 10^{-3}$ and $\phi_{+-} = 42.6° \pm 0.9° \pm 0.6° \pm 0.9°_{\Delta m}$.

Fig. 13 shows the decay intensities of K^0 and $\overline{K}^0$.

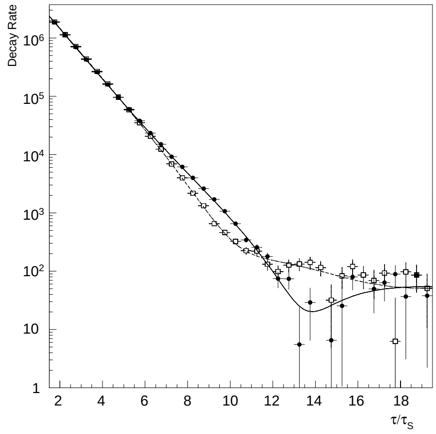


Fig. 13. Decay distributions for K^0 and $\overline{K}^0$

Fig. 14 is a plot of the time dependent asymmetry $A_{+-} = (I(\overline{K}^0 \to \pi^+\pi^-) - \alpha I(K^0 \to \pi^+\pi^-))/(I(\overline{K}^0 \to \pi^+\pi^-) + \alpha I(K^0 \to \pi^+\pi^-))$.

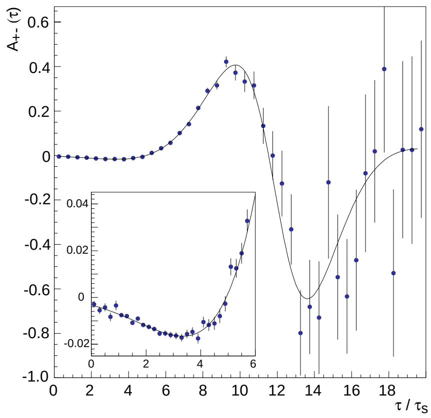


Fig. 14. Difference of decay distributions for $K^0$and $\overline{K}^0$

6.2 E773 at FNAL

E773 is a modified E731 setup, with a downstream regenerator added. Results have been obtained on Δm, τ_S, $\phi_{00} - \phi_{+-}$ and ϕ_{+-} from a study of $K{\to}\pi^+\pi^-$ and $\pi^0\pi^0$ decays![14]

E773 figure

6.2.1 *Two Pion Final States*

This study of $K{\to}\pi\pi$ is a classic experiment where one beats the amplitude $A(K_L \to \pi\pi]_i){=}\eta_i A(K_S \to \pi\pi)$ with the coherently regenerated $K_S{\to}\pi\pi$ amplitude $\rho A(K_S \to \pi\pi)$, resulting in the decay intensity

$$I(t) = |\rho|^2 e^{-\Gamma_S t} + |\eta|^2 e^{-\Gamma_L t} + 2|\rho||\eta| e^{-\Gamma t}\cos(\Delta m t + \phi_\rho - \phi_{+-})$$

Measurements of the time dependence of I for the $\pi^+\pi^-$ final state yields Γ_S, Γ_L, Δm and ϕ_{+-}. They give: $\tau_S = (0.8941 \pm 0.0014 \pm 0.009) \times 10^{-10}$ s.

Setting $\phi_{+-} = \phi_{SW} = \tan^{-1} 2\Delta m/\Delta\Gamma$ and floating Δm; they get:

$$\Delta m = (0.5297 \pm 0.0030 \pm 0.0022) \times 10^{10}\ s^{-1}.$$

Including the uncertainties on Δm and τ_S and the correlations in their measurements they obtain: $\phi_{+-} = 43.53° \pm 0.97°$

From a simultaneous fit to the $\pi^+\pi^-$ and $\pi^0\pi^0$ data they obtain $\Delta\phi = \phi_{00} - \phi_{+-} = 0.62° \pm 0.71° \pm 0.75°$, which combined with the E731 result gives $\Delta\phi = -0.3° \pm 0.88°$.

6.2.2 $K^0 \to \pi^+\pi^-\gamma$

From a study of $\pi^+\pi^-\gamma$ final states $|\eta_{+-\gamma}|$ and $\phi_{+-\gamma}$ are obtained. The time dependence of the this decay, like that for two pion case, allows extraction of the corresponding parameters $|\eta_{+-\gamma}|$ and $\phi_{+-\gamma}$. The elegant point of this measurement is that because interference is observed (which vanishes between orthogonal states) one truly measures the ratio

$$\eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma,\ \not{C}\not{P}\ )}{A(K_S \to \pi^+\pi^-\gamma,\ CP\ \mathrm{OK}\ )}$$

which is dominated by E1, inner bremsstrahlung transitions. Thus again one is measuring the CP impurity of K_L. Direct CP could contribute via E1, direct photon emission K_L decays, but it is not observed within the sensitivity of the measurement.

The results obtained are:(15) $|\eta_{+-\gamma}| = (2.362 \pm 0.064 \pm 0.04) \times 10^{-3}$ and $\phi_{+-\gamma} = 43.6° \pm 3.4° \pm 1.9°$. Comparison with $|\eta_{+-}| \sim |\epsilon| \sim 2.3$, $\phi_{+-} \sim 43°$ gives excellent agreement. This implies that the decay is dominated by radiative contribution and that all one sees is the CP impurity of the K states.

6.3 Combining Results for Δm and ϕ_{+-} from Different Experiments

The CPLEAR collaboration(16) has performed an analysis for obtaining the best value for Δm and ϕ_{+-}, taking properly into account the fact that different experiments have different correlations between the two variables. The data(12,13,14,17–23) with their correlations are shown in fig. 15.

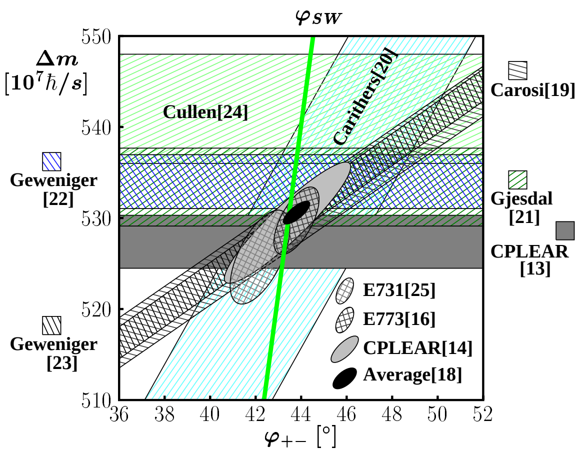


Fig. 15. A compilation of Δm and ϕ_{+-} results, from ref. 16

A maximum likelihood analysis of all data gives

$$\Delta m = (530.6 \pm 1.3) \times 10^7 s^{-1}$$

$$\phi_{+-} = 43.75° \pm 0.6°.$$

Note that ϕ_{+-} is very close to the *superweak phase* ϕ_{SW}=43.44°±0.09°.

6.4 Tests of CPT Invariance

In local field theory, CPT invariance is a consequence of quantum mechanics and Lorentz invariance. Experimental evidence that CPT invariance might be violated would therefore invalidate our belief in either or both QM and L-invariance. We might not be so ready to abandon them, although recent ideas,[24] such as distortions of the metric at the Planck mass scale or the loss of coherence due to the properties of black holes might make the acceptance somewhat more palatable. Very sensitive tests of CPT invariance, or lack thereof, can be carried out investigating the neutral K system at a ϕ–factory.

In general, CPT requires

$$M_{11} - M_{22} = M(K^0) - M(\bar{K}^0) = 0.$$

CPLEAR recently used a very complex analysis to obtain that this mass difference is $1.5 \pm 2.0 \pm \times 10^{-18}$.

KTEV, using a combined values of the τ_s, Δm, ϕ_{SW}, and $\Delta\phi = (-0.01 \pm 0.40)$ obtained the bound that $(M(K^0) - M(\bar{K}^0))/\langle M \rangle = (4.5 \pm 3) \times 10^{-19}$, with some simplifying assumptions.

If we note that m_K^2/M_{Planck} is approximately a few times 10^{-20} it is clear that we are probing near that region, and future experiments, especially at a ϕ–factory is very welcome for confirmation.

7. Three Precision CP Violation Experiments

Three new experiments: NA48[25] in CERN, KTEV[26] at FNAL and KLOE[27] at LNF, have begun taking data, with the primary aim to reach an ultimate error in $\Re(\epsilon'/\epsilon)$ of $\mathcal{O}(10^{-4})$.

The sophistication of these experiments takes advantage of our experience of two decades of fixed target and e^+e^- collider physics. Fundamental in KLOE is the possibility of continuous self-calibration while running, via processes like Bhabha scattering, three pion and charged K decays.

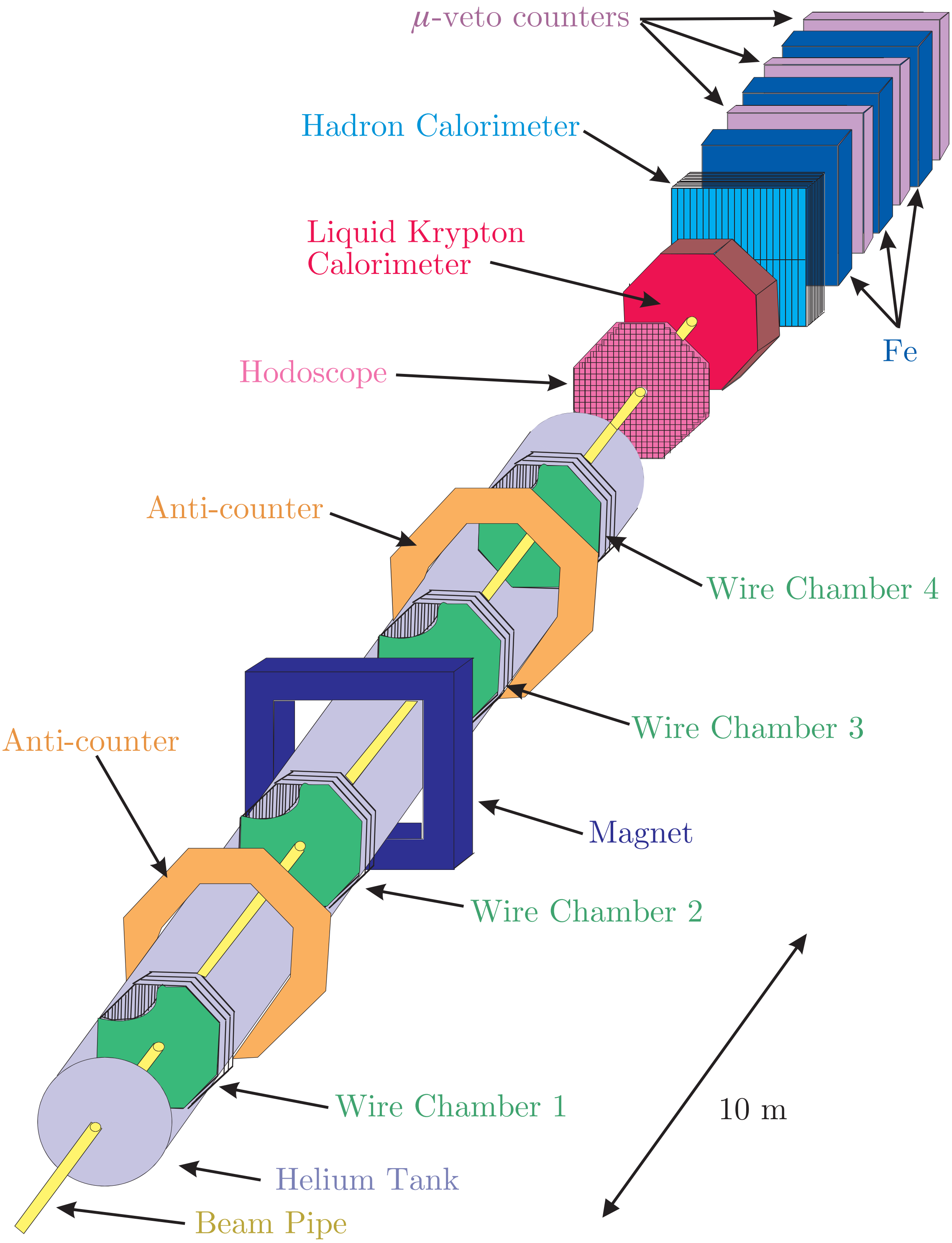


Fig. 16. The NA48 experiment at CERN

7.2 KTEV

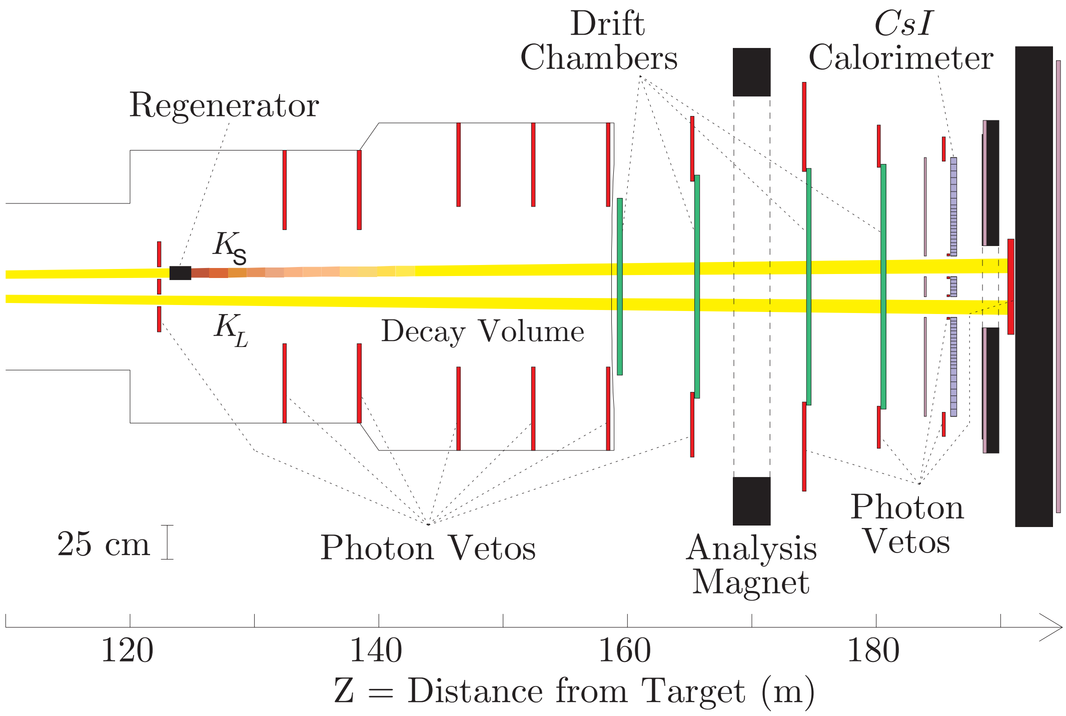


Fig. 17. Plan view of the KTeV experiment. Note the different scales.
$\Re(\epsilon'/\epsilon) = 0.00280 \pm\ 0.00041$ (Feb 1999)

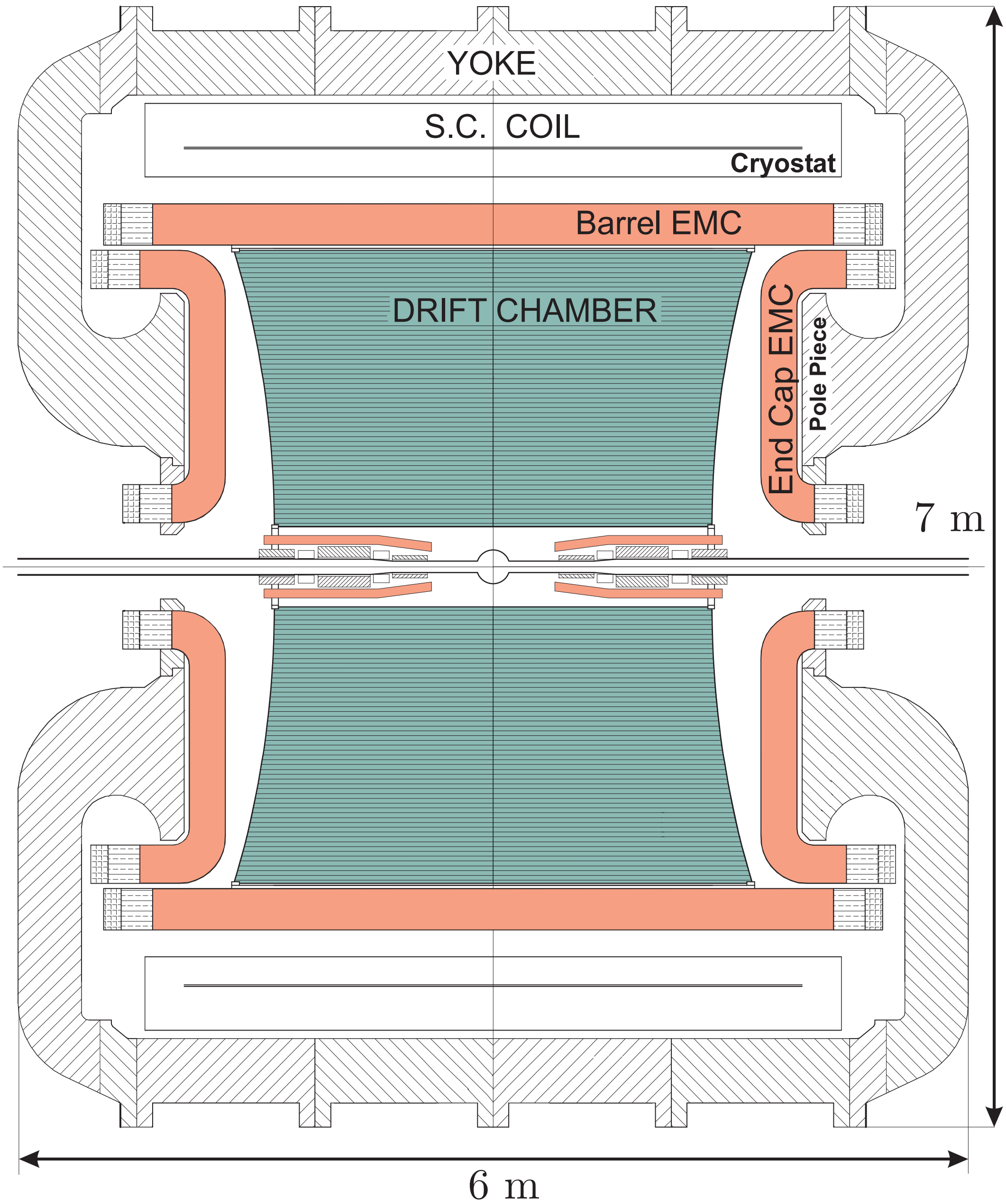
7.3 KLOE

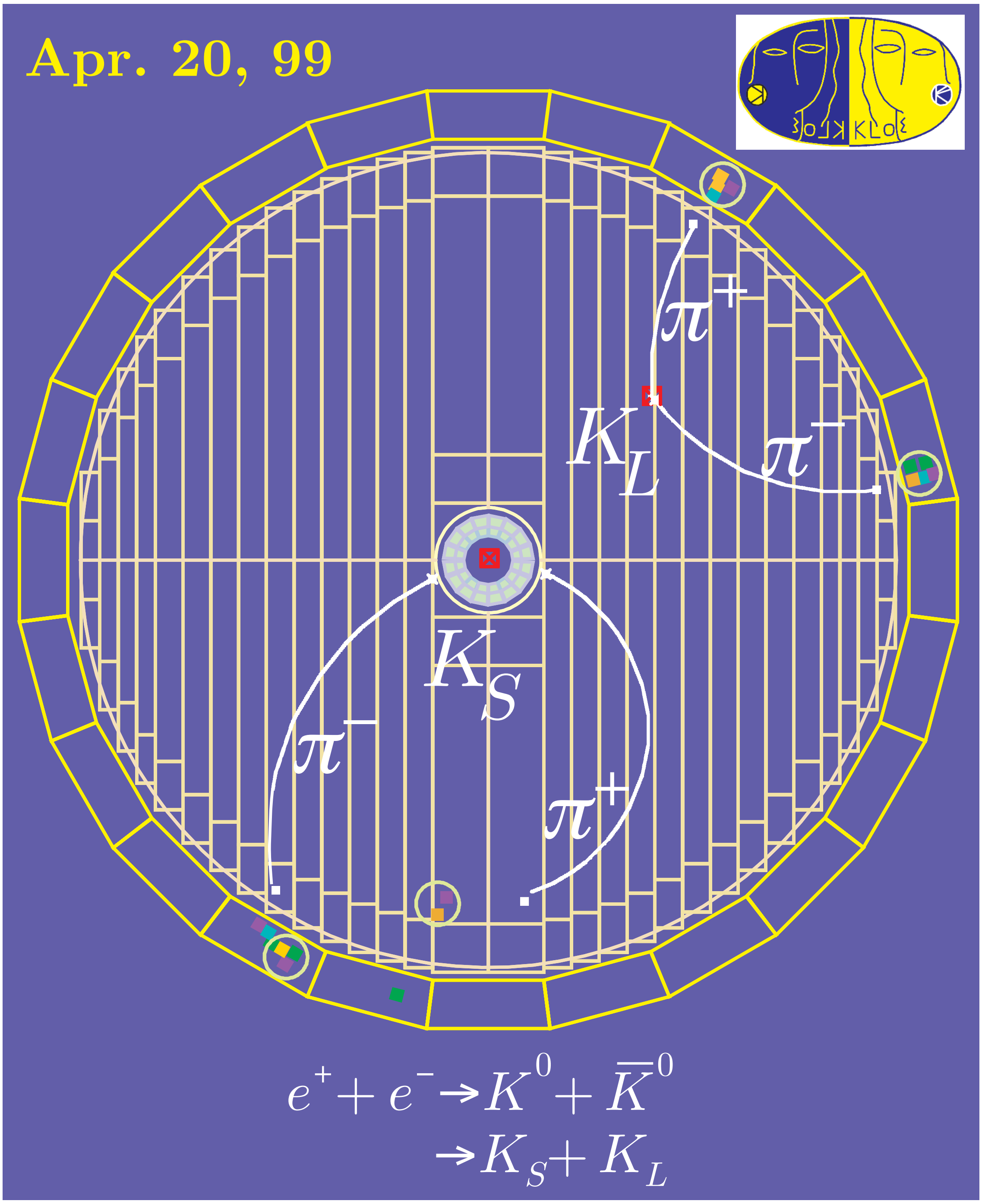
The KLOE detector,[28] designed by the KLOE collaboration and under construction by the collaboration at the Laboratori Nazionali di Frascati, is shown in cross section in fig. 18. The KLOE detector looks very much like a collider detector and will be operated at the DAΦNE collider recently completed at the Laboratori Nazionali di Frascati, LNF.

The main motivation behind the whole KLOE venture is the observation of direct CP violation from a measurement of $\Re(\epsilon'/\epsilon)$ to a sensitivity of 10^{-4}. A pure K_S beam is unique of ϕ–factory.[29,30]

A result from KLOE would be quite welcome in the present somewhat confused situation.

KLOE is at the moment waiting for DAΦNE to deliver some luminosity.



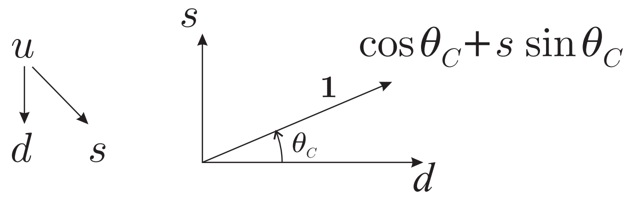


8. The CKM Mixing Matrix

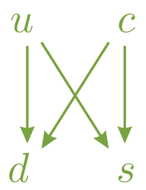
The Standard Model has a *natural* place for CP violation (Cabibbo, Kobayashi and Maskawa). In fact, it is the discovery of CP violation which inspired KM[31] to expand the original Cabbibo[32] -GIM[33] 2×2 quark mixing matrix, to a 3×3 one, which allows for a phase and therefore for CP violation. This also implied an additional generation of quarks, now known as the b and t, matching the τ in the SM. According to KM the six quarks charged current is:

$$J_\mu^+ = (\bar{u}\ \bar{c}\ \bar{t})\gamma_\mu(1-\gamma_5)\mathbf{M}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where M is a 3×3 unitary matrix: $\mathbf{M}^\dagger\mathbf{M}$=1. Since the relative phases of the 6 quarks are arbitrary, **M** contains 3 real parameter, the Euler angles, plus a phase factor, allowing for $\not{C}\not{P}$.



$$\Gamma(s \to d) \propto \sin^2\theta_C$$ - Strange part. decays

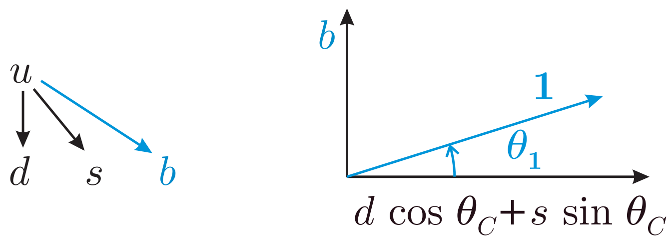


GIM, neutral currents,
2 by 2 unitary matrix,
calculable loops

$$\mathbf{V_{C/GIM}} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_c \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

$$J_\mu^+(udcs) = \bar{u}(\cos\theta_C\, d + \sin\theta_C\, s) + \bar{c}(-\sin\theta_C\, d + \cos\theta_C\, s)$$

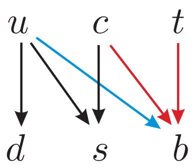
$J_\mu^0 = \bar{d}d + \bar{s}s$ – No FCNC: $K^0 \to \mu\mu$ suppression.



$$\Gamma(b \to u) \propto \sin^2\theta_1 \qquad B \to \pi, \rho \ldots + \ldots$$

$$\mathbf{V_{1\,-\,1,2,3}} = (\cos\theta_C \cos\theta_1 \quad \sin\theta_C \cos\theta_1 \quad \sin\theta_1)$$

$$J_\mu^+(u,dsb) = \bar{u}[..]_\mu(\cos\theta_C \cos\theta_1\, d + \sin\theta_C \cos\theta_1\, s + \sin\theta_1\, b)$$



$$\Gamma(b \to c) \propto sin^2\theta_2$$
$$B \to D, \;\; D^* ... + ...$$

$$\mathbf{V}_{\rm CKM} = \begin{pmatrix} \cos\theta_C \cos\theta_1 & \sin\theta_C \cos\theta_1 & \sin\theta_1 \\ & & \sin\theta_2 \cos\theta_1 \\ & & \cos\theta_2 \cos\theta_1 \end{pmatrix}$$

The complete form of the matrix, in the Maiani notation, is:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & c_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{12} = \cos\theta_{12} = \cos\theta_C$, etc.

While a phase can be introduced in the unitary matrix **V** which mixes the quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

the theory does not predict the magnitude of the effect. The constraint that the mixing matrix be unitary corresponds to the desire of having a universal weak interaction.

Our present knowledge of the magnitude of the V_{ij} elements is given below.

$$\begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.047 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - .9993 \end{pmatrix}$$

The diagonal elements are close but definitely not equal to unity. If such were the case there could be no CP violation.

However, if the violation of CP which results in $\epsilon \neq 0$ is explained in this way then, in general, we expect $\epsilon' \neq 0$. For *technical* reasons, it is difficult to compute the value of ϵ'. Predictions are $\epsilon'/\epsilon \leq 10^{-3}$, but cancellations can occur, depending on the value of the top mass and the values of appropriate matrix elements, mostly connected with understanding the light hadron structure.

A fundamental task of experimental physics today is the determination of the four parameters of the CKM mixing matrix, including the phase which results in CP. A knowledge of all parameters is required to confront experiments. Rather, many experiments are necessary to complete our knowledge of the parameters and prove the uniqueness of the model or maybe finally break beyond it.

8.1 Wolfenstein parametrization

Nature seems to have chosen a special set of values for the elements of the mixing matrix: $|V_{ud}|\sim 1$, $|V_{us}|=\lambda$, $|V_{cb}|\sim\lambda^2$ and $|V_{ub}|\sim\lambda^3$. On this basis Wolfenstein found it convenient to parameterize the mixing matrix in a way which reflects more immediately our present knowledge of the value of some of the elements and has the CP violating phase appearing in only two off-diagonal elements, to lowest order in $\lambda=\sin\theta_{\rm Cabibb}$ a real number describing mixing of s and d quarks.

The Wolfenstein[34] *approximate* parameterization up λ^3 terms is:

$$\mathbf{V}=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

A, also real, is close to one, $A{\sim}0.84{\pm}0.06$ and $|\rho-i\eta|{\sim}0.3$. CP violation requires $\eta\neq 0$. η and ρ are not very well known. Likewise there is no $\not{C}\not{P}$ if the diagonal elements are unity.

The Wolfenstein matrix is not exactly unitary: $V^\dagger V=1+\mathcal{O}(\lambda^4)$.

The phases of the elements of $\mathbf{V}$ to $\mathcal{O}(\lambda^2)$ are:

$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

which defines the angles β and γ.

Several constraints on η and ρ can be obtained from measurements. ϵ can be calculated from the $\Delta S{=}2$ amplitude of fig. 19, the so called box diagram. At the quark level the calculations is straightforward, but complications arise in estimating the matrix element between K^0 and $\overline{K}^0$. Apart from this uncertainties ϵ depends on η and ρ as $|\epsilon|=a\eta+b\eta\rho$ a hyperbola in the η,ρ plane as shown in figure 23.

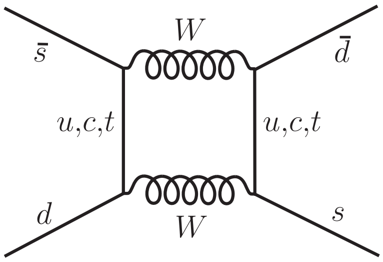


Fig. 19. Box diagram for $K^0\to\overline{K}^0$

The calculation of ϵ' is more complicated. There are three $\Delta S{=}1$ amplitudes that contribute to $K\to\pi\pi$ decays, given below to lowest order in λ for the real and imaginary parts:

$$A(s\to u\bar{u}d)\propto V_{us}V^*_{ud}\sim\lambda \tag{8.1}$$

$$A(s\to c\bar{c}d)\propto V_{cs}V^*_{cd}\sim-\lambda+i\eta A^2\lambda^5 \tag{8.2}$$

$$A(s\to t\bar{t}d)\propto V_{ts}V^*_{td}\sim-A^2\lambda^5(1-\rho+i\eta) \tag{8.3}$$

where the amplitude (8.1) correspond to the natural way for computing $K\to\pi\pi$ in the standard model and the amplitudes (8.2), (8.3) account for direct $\not{C}\not{P}$. If the latter amplitudes were zero there would be no direct CP violation in the standard model. The flavor changing neutral current (FCNC) diagram of fig. 20 called the penguin diagram, contributes to the amplitudes (8.2), (8.3). Estimates of $\Re(\epsilon'/\epsilon)$ range from few$\times10^{-3}$ to 10^{-4}.

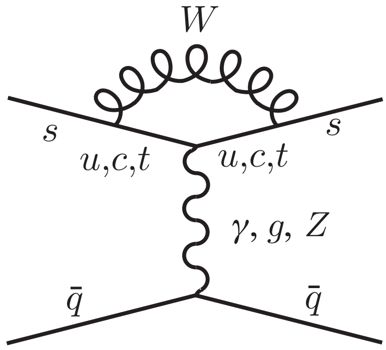


Fig. 20. Penguin diagram, a flavor changing neutral current effective operator

9. Unitary triangles

We have been practically inundated lately by very graphical presentations of the fact that the CKM matrix is unitary, ensuring the renormalizability of the $SU(2)\otimes U(1)$ electroweak theory. The unitarity condition

$$V^\dagger V = 1$$

contains the relations

$$\sum_i V_{ij}^* V_{ik} = \sum_i V_{ji}^* V_{ki} = \delta_{jk}$$

which means that if we take the products, term by term of any one column (row) element with the complex conjugate of another (different) column (row) element their sum is equal to 0. Geometrically it means the three terms are sides of a triangle. Two examples are shown below. The second one has the term $V_{cd}V_{cb}^*$ pulled out, and many of you will recognize it as a common figure used when discussing measuring CP violation in the B system.

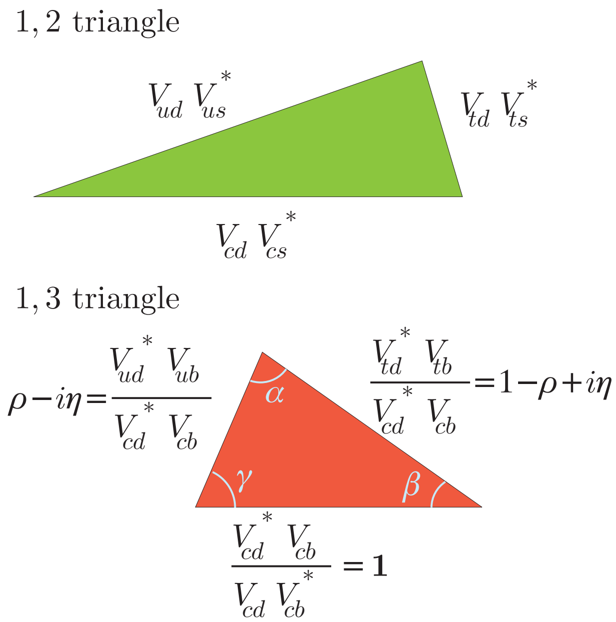


Fig. 21. The (1,2) and (1,3) Unitarity triangles

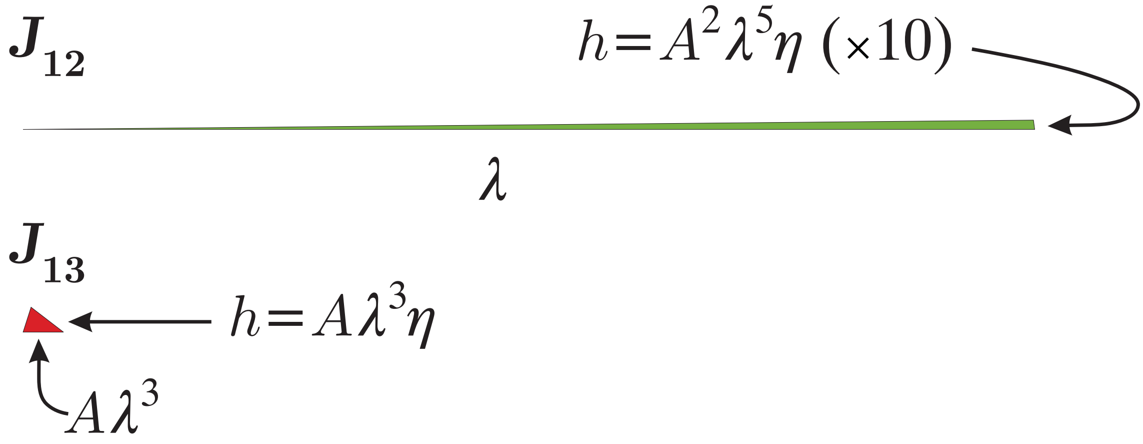


Fig. 22. The B and K Unitarity triangles

Cecilia Jarlskog in 1984 observed that any direct CP violation is proportional to twice the area which she named J (for Jarlskog ?) of these unitary triangles, whose areas are of course are equal, independently of which rows/columns one used to form them. In terms of the Wolfenstein parameters,

$$J \simeq A^2\lambda^6\eta$$

which according to present knowledge is $(2.7 \pm 1.1) \times 10^{-5}$, very small indeed! This number has been called the price of $\not{C}\not{P}$.

Its smallness explains why the ϵ' experiments are so hard to do, and also why B factories have to be built in order to study CP violation in the B system, despite the large value of the angles in the B unitary triangle. Measuring the various J's to high precision, to check for deviations amongst them, is a sensitive way to probe for new physics!

10. Rare K Decays

Rare K decays offer several interesting possibilities, which could ultimately open a window beyond the standard model. The connection with ρ and η is shown in fig. 23.

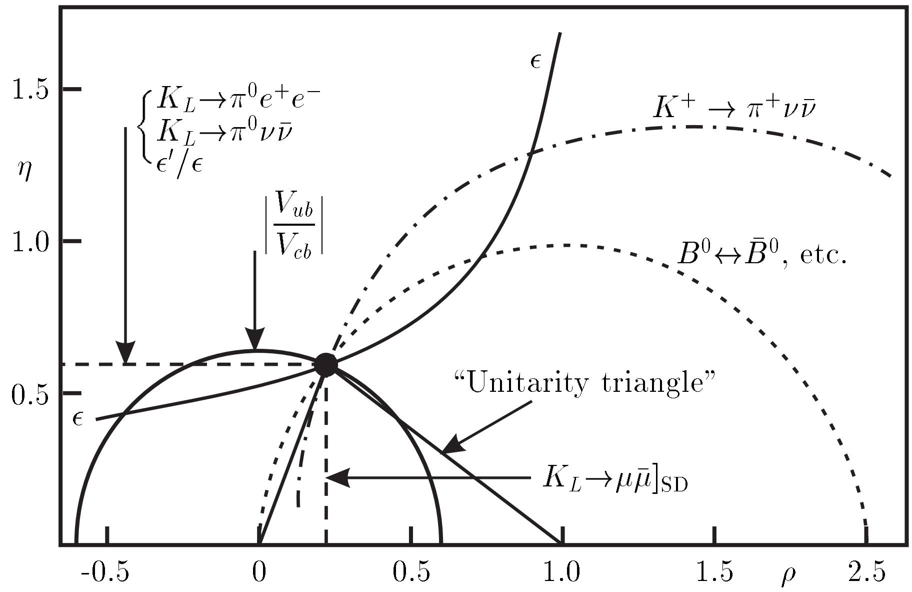


Fig. 23. Constraints on η and ρ from measurements of ϵ, ϵ', rare decays and B meson properties.

Rare decays also permit the verification of conservation laws which are not strictly required in the standard model, for instance by searching for $K^0 \to \mu e$ decays.

The connection between ϵ' and η is particularly unsatisfactory because of the uncertainties in the calculation of the hadronic matrix elements.

This is not the case for some rare decays.

A classifications of measurable quantities according to increasing uncertainties in the calculation of the hadronic matrix elements is given by Buras[35] as:

1. $\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})$,
2. $\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$,
3. $\mathrm{BR}(K_L \to \pi^0 e^+ e^-)$, ϵ_K,
4. ϵ'_K, $\mathrm{BR}(K_L \to \mu\bar{\mu}]_{\mathrm{SD}})$, where SD=*short distance* contributions.

The observation $\epsilon' \neq 0$ remains a unique proof of direct $\not{C}\not{P}$. Measurements of 1 through 3, plus present knowledge, over determine the CKM matrix.

Rare K decay experiments are not easy. Typical expectations for some BR's are:

$$BR(K_L \to \pi^0 e^+ e^-,\ \not{C}\not{P}]_{\mathrm{dir}}) \sim (5 \pm 2) \times 10^{-12}$$
$$BR(K_L \to \pi^0 \nu\bar{\nu}) \sim (3 \pm 1.2) \times 10^{-11}$$
$$BR(K^+ \to \pi^+ \nu\bar{\nu}) \sim (1 \pm .4) \times 10^{-10}$$

Note that the incertainties above reflect our ignorance of the mixing matrix parameters, not uncertainties on the hadronic matrix element which essentially can be "taken" from $K_{\ell 3}$ decays.

The most extensive program in this field has been ongoing for a long time at BNL and large statistics have been collected recently and are under analysis. Sensitivities of the order of 10^{-11} will be reached, although $10^{-(12 \text{ or } 13)}$ is really necessary. Experiments with high energy kaon beams have been making excellent progress toward observing rare decays.

10.1 Search for $K^+ \to \pi^+ \nu\bar{\nu}$

This decay, CP allowed, is best for determining V_{td}. At present after analyzing half of their data, E781-BNL obtains BR is about 2.4×10^{-10}. This estimate is based on ONE event which surfaced in 1995 from about 2.55×10^{12} stopped kaons. The SM expectation is about half that value. Some 100 such decays are enough for a first V_{td} measurements.

10.2 $K_L \to \pi^0 \nu\bar{\nu}$

This process is a "pure" direct $\not{C}\not{P}$ signal. The $\nu\bar{\nu}$ pair is an eigenstate of CP with value +1. Thus CP is manifestly violated.

It is theoretically particularly "pristine", with only about 1-2% uncertainty, since the *hadronic* matrix element need not be calculated, but is directly obtained from the measured $K_{\ell 3}$ decays. Geometrically we see it as being the altitude of the J_{12} triangle.

$$J_{12} = \lambda(1 - \lambda^2/2)\Im(V_{td}V^*_{ts}) \approx 5.6[B(K_L \to \pi^0 \nu\bar{\nu})]^{1/2}$$

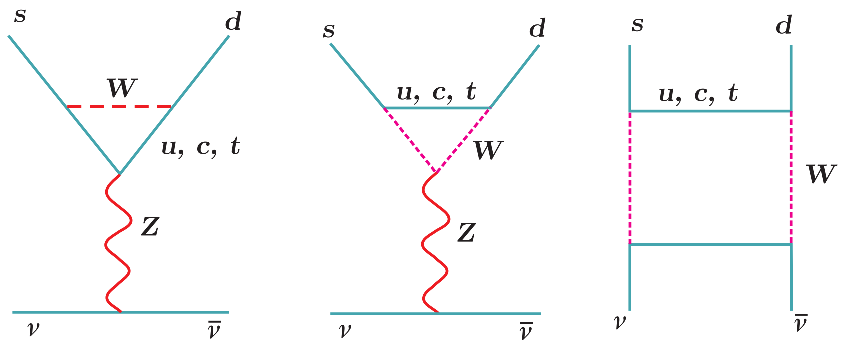


Fig. 24. Feyman Diagrams for $K_L \to \pi^0 \nu\bar{\nu}$

The experimental signature is just a single unbalanced π^0 in a hermetic detector. The difficulty of the experiment is seen in the present experimental limit from E799-I, BR<pt5.8,-5,. The sensitivities claimed for E799-II and at KEK are around 10^{-9}, thus another factor of 100 improvement is necessary.

The new FNAL and BNL proposals at the main injector are very ambitious. There is "hope" to make this measurement a reality early in the third millenium.

11. *B* decays

11.1 Introduction

The discovered at Fermilab in 1977 of the Υ, with mass of ~10 GeV, was immediately taken as proof of the existense of the b quark, heralded by KM and already so christened: b for beauty or bottom. The b quark has B flavor B=-1. The fourth Υ is barely above threshold for decayng into a $B\bar{B}$ pair, where the B meson are $b\bar{u}$, $b\bar{d}$ and their charge conjugate states, in complete analogy to charged and neutral kaons. B^0 and $\overline{B}^0$ are not self conjugate states. That the $\Upsilon(4S)$ decays only to $B^0\overline{B}^0$ pairs was demonstrated by CUSB searching (and not finding) low energy photons from B^* decays. $B^0\overline{B}^0$'s are produced in a C-odd state as $K^0\overline{K}^0$ and can be used for CP studies in the B system, once the short B lifetime problem is overcome.

CUSB also determined the thresholds for B^*B, B^*B^*, $B_s\bar{B}_s$ etc. production.[36]

11.2 B semileptonic decays

Because of their massiveness, B's can decay - weakly - into many more channels than the K's. We might recall that we owe the long lifetime of the K_L to the smallness of the phase space for 3 body decays. The average particle multiplicity in the decay of B and $\bar{B}$ is about six. The leptonic modes have a branching ratio of about 25%, with a unique signature, namely a lepton with energy up to half that of the parent Υ. It was infact through the observation of the sudden appearance of high energy electrons that the existence of the b quark was unambiguously proved in 1980, since the Υ after all has B=0. At quark level beauty decays are: $b \to c\ell^-\bar{\nu}$ and $b \to u\ell^-\bar{\nu}$ with the selection rule ΔB=ΔQ. $\bar{b}$ decay to positive leptons.

The endpoint of the lepton spectrum and its shape depend on the flavor of the hadronic system appearing in the final state. We define as X_c a hadronic system with charm $C = \pm1$ and U=0 where U is the *uppityness*. Likewise X_u has $U = \pm1$ and C=0. The leptonic decays are:

$$B \to \ell^{\pm} + \nu(\bar{\nu}) + X_c$$

$$B \to \ell^{\pm} + \nu(\bar{\nu}) + X_u$$

where X_c=D, D^* ... with $\overline{M}(X_c)$~2 GeV and X_u=π, ρ.. with $\overline{M}(X_u)$~0.7 GeV. The expected lepton spectra are shown in figure.

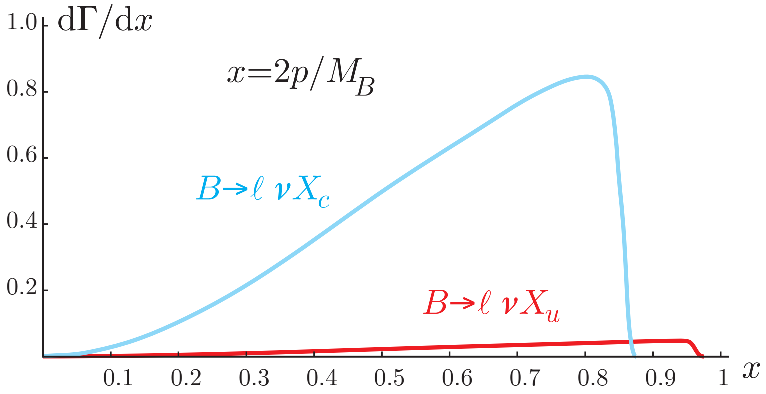


Fig. 25. Leptonic Spectrum in B semileptonic decays.

Total decay rates, *i.e.* the inverse of the lifetimes, and branching ratios of B mesons provide the determination of $|V_{cb}|$ and $|V_{ub}|$. A preferred way is to measure the semileptonic branching ratio by integrating over the whole spectrum. An early determination by CUSB already indicated that $|V_{ub}/V_{cb}|$ is very small, less than 0.06. However, uncertainties in the calculation of the hadronic matrix elements and the shape of the spectrum near the end pont introduce errors in the extraction of $|V_{ub}/V_{cb}|$. Methods (HQET) have been developed to make use of exclusive channels, a good twenty years has been spent in refining such measurements, which still need to be improved!

11.3 Mixing

11.3.1 discovery

Just as with K-mesons, neutral B mesons are not C eigenstates and can mix, i.e, transitions $B^0 \leftrightarrow \overline{B}^0$ are possible. The first observation of mixing was reported by Argus at the DESY DORIS collider running on the $\Upsilon(4S)$.

They observed mixing by comparing the $\ell^+\ell^+$ and $\ell^-\ell^-$ decay rates from $B\overline{B}$ pairs. Defining the ratio

$$r = \frac{\ell^+\ell^+ + \ell^-\ell^-}{\ell^+\ell^- + \ell^-\ell^+ + \ell^+\ell^+ + \ell^-\ell^-}$$

$r \neq 0$ is proof of mixing, not however of $\not{C}\not{P}$. Today, instead of r, the χ_d parameter, which is a measure of the time-intergrated B^0-$\overline{B}^0$ mixing probability that a produced $B^0(\overline{B}^0)$ decays as as $\overline{B}^0(B^0)$, is used. They are related simply by $r = \chi_d/(1-\chi_d)$. The present value of χ_d is 0.172 ± 0.01.

11.3.2 Formalism

We define, analogously to the $K^0\overline{K}^0$ system,

$$B_{\rm L} = p\,|\,B^0\,\rangle + q\,|\,\overline{B}^0\,\rangle$$
$$B_{\rm H} = p\,|\,B^0\,\rangle - q\,|\,\overline{B}^0\,\rangle$$

with $p^2 + q^2$=1 Here L, H stand for light and heavy. The B_d's also have different masses but very similar decay widths.

Mixing is calculated in the SM by evaluating the standard "box" diagrams with intermediate u, c, t and W states. We define:

$$\Delta M = M_H - M_L, \quad \Delta\Gamma = \Gamma_H - \Gamma_L$$

note that ΔM is positive by definition. The ratio q/p is given by:

$$q/p = (\Delta M - i/2\Delta\Gamma)/2(M_{12} - i/2\Gamma_{12}) = \\ 2(M_{12}^* - i/2\Gamma_{12}^*)/(\Delta M - i/2\Delta\Gamma)$$

where

$$\Gamma_{12} \propto [V_{ub}V_{ud}^* + V_{cb}V_{cd}^*]^2 m_b^2 = (V_{tb}V_{td}^*)^2 m_b^2$$

and $M_{12} \propto (V_{tb}V_{td}^*)^2 m_t^2$, so they have almost the same phase. x and y, for B_d and B_s mesons are:

$$x_{d,s} = \Delta M_{d,s}/\Gamma_{d,s}, \qquad y_{d,s} = \Delta\Gamma_{d,s}/\Gamma_{d,s}$$

y_d is less than 10^{-2}, and x_d is about 0.7, and if we ignore the width difference between the two B_d states,

$$\left.\frac{q}{p}\right|_{B_d} \approx \frac{(V_{tb}^*V_{td})}{(V_{tb}V_{td}^*)} = e^{-2i\beta}$$

Therefore $|q/p|_d$ is very close to 1 and since $2\Re\epsilon_{B_d} \approx 1 - |q/p|_d$, ϵ_{B_d} is imaginary. y_s is about 0.2, and x_s theoretically could be as large as 20, so far only lower bounds are quoted.

χ_d as defined before, in terms of q, p, x, y is

$$\chi_d = \frac{1}{2}\left|\frac{q}{p}\right|^2 \frac{x_d^2 - y_d^2/4}{(1 + x_d^2)(1 - y_d^2/4)}$$

which reduces to a good approximation:

$$\chi_d = \frac{x_d^2}{2(1 + x_d^2)},$$

from which one obtains that $x_d = 0.723 \pm 0.032$.

In summary, from evaluating the box diagrams, one finds:

$$x_l \propto \; m_t^2 \tau_{B_l} m_{B_l} |V_{tl}V_{tb}^*|^2.$$

where the subscript l refers to the light meson partner which makes up the B meson, i.e. $l = s$ or d.[37]

An amusing historical note. The surprisingly large amount of mixing seen required that the top mass be larger than the then acceptable value of about 20 GeV. Stretching beyond reason the limits for $|V_{ub}|$ and the value of r than known, a lower limit $M_{\rm top} \geq 40$ GeV was obtained. The first CUSB limit on $|V_{ub}|$ already implied $M_{\rm top}$ >120 MeV.

Using the top mass today known, and ΔM measured from the $B^0\overline{B}^0$ oscillation frequency from experiments at FNAL and LEP, one obtains the estimates $|V_{td}| = (8.4 \pm 1.4) \times 10^{-3}$ and $\Im(V_{td}V_{tb}^*) = (1.33 \pm 0.30) \times 10^{-4}$.

From a fit, Parodi *et al.*,[38] obtain

$$\sin 2\beta = 0.71 \pm 0.13\,, \qquad \sin 2\gamma = 0.85 \pm 0.15.$$

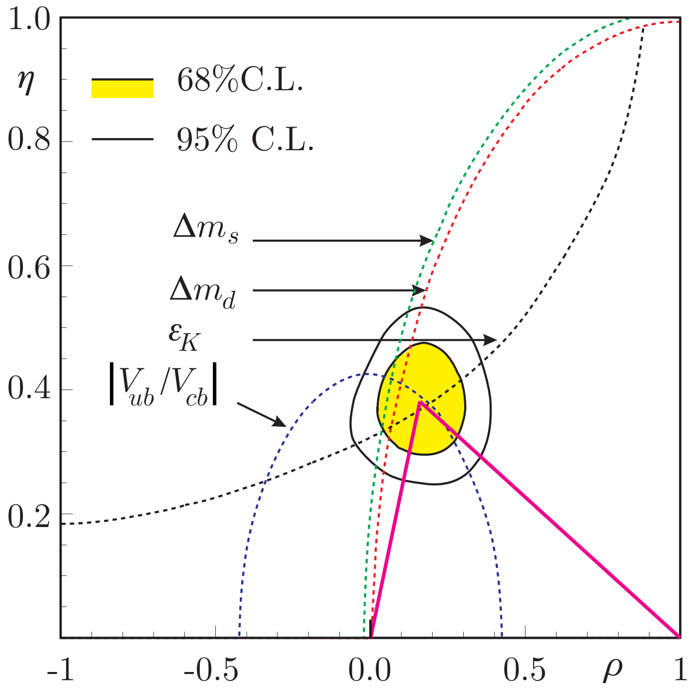


Fig. 38. Fit to data in the η-ρ plane.

Of course the whole point of the exercise is to measure directly η and ρ and then verify the uniqueness of the mixing matrix.

11.4 CP Violation

Semileptonic decays of Bs allow, in principle, to observe $\not{C}\!\not{P}$ by studying the dilepton and total lepton charge asymmetries. This however has turned to be rather difficult because of the huge background and so far yielded no evidence for $\not{C}\!\not{P}$ in B.

We can estimate the magnitude of the leptonic asymmetry from

$$4\Re\epsilon_B = \Im\left(\frac{\Gamma_{12}}{M_{12}}\right) = \frac{|\Gamma_{12}|}{|M_{12}|}\mathrm{Arg}\left(\frac{\Gamma_{12}}{M_{12}}\right)$$

or approximately

$$\frac{m_b^2}{m_t^2} \times \frac{m_c^2}{m_b^2}$$

which is $\mathcal{O}(10^{-4})$.

11.4.1 α, β and γ

Sensitivity to CP violation in the B system is usually discussed in terms of the 3 interior angles of the U_{13} triangle.

$$\alpha = \mathrm{Arg}\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \beta = \mathrm{Arg}\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \gamma = \mathrm{Arg}\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

The favorite measurements are asymmetries in decays of neutral B decays to CP eigenstates. f_{CP}, in particular $J/\psi(1S)K_S$ and possibly $\pi\pi$, which allow a clean connection to the CKM parameters. The asymmetry is due to interference of the amplitude A for $B^0 \to J/\psi\, K_S$ with the amplitude $\bar{A}$ for $B^0 \to \overline{B}^0 \to J/\psi\, K_S$.

The time-dependent CP asymmetry is:

$$a_{f_{CP}}(t) \equiv \frac{I(B^0(t)\to J/\psi\,K_S)-I(\overline{B}^0(t)\to J/\psi\,K_S)}{I(B^0(t)\to J/\psi\,K_S)+I(\overline{B}^0(t)\to J/\psi\,K_S)}$$
$$= \frac{(1-|\lambda_{f_{CP}}|^2)\cos(\Delta Mt)-2\Im(\lambda_{f_{CP}})\sin(\Delta Mt)}{1+|\lambda_{f_{CP}}|^2}$$
$$= \Im\lambda_{f_{CP}}\sin(\Delta Mt)$$

with

$$\lambda_{f_{CP}} \equiv (q/p)(\bar{A}_{f_{CP}}/A_{f_{CP}}), \quad |\lambda_{f_{CP}}|=1.$$

In the above, $B^0(\overline{B}^0)$ is a state tagged as such at time t, for instance by the sign of the decay lepton of the other meson in the pair.

The time integrated asymmetry, which vanishes at a B-factory because the $B^0\overline{B}^0$ pair is in a C-odd state is given by

$$a_{f_{CP}} = \frac{I(B^0\to J/\psi\,K_S)-I(\overline{B}^0\to J/\psi\,K_S)}{I(B^0\to J/\psi\,K_S)+I(\overline{B}^0\to J/\psi\,K_S)} = \frac{x}{1+x^2}\Im\lambda_{f_{CP}}$$

Staring at box diagrams, with a little poetic license one concludes

$$a_{f_{CP}} \propto \Im\lambda_{f_{CP}} = \sin 2\beta.$$

or

$$a_{f_{CP}} \approx 0.5\sin 2\beta \sim 1$$

The license involves ignoring penguins, which is probably OK for the decay to $J/\psi\,K_S$, presumably a few % correction.

For the $\pi\pi$ final state, the argument is essentially the same. However the branching ratio for $B\to\pi\pi$ is extraordinarily small and penguins *are* important. The asymmetry is otherwise proportional to $\sin(2\beta+2\gamma)=-\sin 2\alpha$. Here we assume $\alpha+\beta+\gamma=\pi$, which is something that we would instead like to prove. The angle γ can be obtained from asymmetries in B_s decays and from mixing, measurable with very fast strange Bs.

11.5 CDF and DØ

CDF at the Tevatron is the first to profit from the idea suggested first by Toni Sanda, to study asymmetries in the decay of tagged B^0 and of $\overline{B}^0$ to a final state which is a CP eigenstate. They find

$$\sin 2\beta = 0.79^{+0.41}_{-0.44}, \quad 0.0 \le \sin 2\beta < 1 \quad \text{at 93\% CL}$$

Their very lucky central value agrees with the aforementioned SM fit, but there is at least a two fold ambiguity in the determination of β which they can not differentiate with their present errors. In the coming Tevatron runs, CDF not only expect to improve the determinations of $\sin 2\beta$ by a factor of four, so $\delta\sin 2\beta\approx 0.1$, but to measure $\sin(2\alpha)$ from using the asymmetry resulting from $B^0\to\pi^+\pi^-$ interfering with $\overline{B}^0\to\pi^+\pi^-$ to a similar accuracy. By being very optimistic, they hope to get a first measurement of $\sin(\gamma)$ by using $B^0_s/\bar{B}^0_s\to D^\pm_s K^\mp$ from about 700 signal events.

DØwill have te same sensitivity

11.6 LHC

The success of CDF shows that LHCB, BTeV, ATLAS and CMS will attain, some day, ten times better accuracy than the almost running ones (including Belle and Babar), so bear serious watching.

11.7 B FACTORIES

In order to overcome the short B lifetime problem, and still profit from the coherent state property of B's produced on the $\Upsilon(4S)$, two asymmetric e^+e^- colliders have been built, PEP-II and KEKB. The two colliders both use a high ≈9 GeV beam colliding against a ≈ 3.1 GeV beam, so that the center of mass energy of the system is at the $\Upsilon(4S)$ energy, but the B's are boosted in the laboratory, so they travel detectable distances before their demise. In order to produce the large number of $B^0\overline{B}^0$ pairs, the accelerators must have luminosities on the order of 3×10^{33} cm^{-2}s^{-1}, about one orders of magnitude that of CESR. Both factories have infact achieved luminosities of 2×10^{33} cm^{-2} s^{-1} and in SLAC Babar collects 120 pb^{-1} per day while Belle gets around 90. Both experiments will have 25% mearuments of β by next summers and reach higher accuracy if and when the colliders will achieve an surpass luminosities of 10^{34}.

12. CP, kaons and B-mesons: the future

CP violation, $\not{C}\not{P}$ in the following, was discovered in neutral K decays about 36 years ago, in 1964![7]

Two important observations about $\not{C}\not{P}$ were to follow in the next 10 years.

In 1967 Andrej Sakharov[39] gave the necessary conditions for baryogenesis: baryon number violation, C and CP violation, thermal non-equilibrium.

Finally in 1973, Kobayashi and Maskawa[31] extended the 1963 mixing idea of Nicola Cabibbo[32] and the GIM[33] four quark idea to three quark families, noting that $\not{C}\not{P}$ becomes possible in the standard model.

Since those days $\not{C}\not{P}$ has been somewhat of a mystery, only observed in kaon decays. While the so-called CKM mixing matrix allows for the introduction of a phase and thus $\not{C}\not{P}$, the standard model does not predict its parameters. It has taken 35 years to be able to prove experimentally that $\Re(\epsilon'/\epsilon) \neq 0$ and quite a long time also to learn how to compute the same quantity from the CKM parameters.

In the last few years calculations[40] had led to values of $\Re(\epsilon'/\epsilon)$ of $\mathcal{O}((4\text{-}8) \times 10^{-4})$ with errors estimated to be of the order of the value itself. Experimental results are in the range (12-27) $\times 10^{-4}$ with errors around 3-4 in the same units.

This is considered a big discrepancy by some authors. More recently it has been claimed[41] that $\Re(\epsilon'/\epsilon)$ could well reach values greater than 20×10^{-4}.

I would like to discuss in more general terms the question of how to test whether the standard model is consistent with $\not{C}\not{P}$ observations independently of the value of $\Re(\epsilon'/\epsilon)$ and where can we expect to make the most accurate measurements in the future.

Quite some time ago by Bjorken introduced the unitarity triangles.

Much propaganda has been made about the "B-triangle", together with claims that closure of the triangle could be best achieved at B-factories in a short time. This has proved to be over optimistic, because hadronic complications are in fact present here as well. Cecilia Jarlskog[42] has observed that $\not{C}\not{P}$ effects are proportional to a factor J which is twice the area of any of the unitarity triangles. J, called the price of $\not{C}\not{P}$ by Fred Gilmann, does not depend on the representation of the mixing matrix we use.

In the Wolfenstein approximate parametrization $J = \lambda^6 A^2 \eta$, as easily verified. The K or 1,2 and B or 1,3 triangles have been shown. The 2,3 triangle is also interesting.

In the above λ=0.2215±0.0015 is the sine of the Cabibbo angle (up to $V_{ub}^2 \sim 10^{-5}$ corrections), a real number describing mixing of s and d quarks, measured in K decays since the early days. A, also real, is obtained from the $B \to D\ldots$ together with the B-meson lifetime and is close to one, A∼0.84±0.06. From $b \to u$ transitions $|\rho - i\eta|$∼0.3.

Present knowledge about J is poor, $J = 2.7 \pm 1.1 \times 10^{-5}$, *i.e.* ∼40% accuracy. J is a small number and as such subject to effects beyond present physics.

The important question is where and when can we expect to get more precise results. Lets call J_{mn} the area of the triangles corresponding to the m$^{\rm th}$ and n$^{\rm th}$ columns of $\mathbf{V}$. J_{12} is measured in K decays,[43] including λ^2 corrections:

$$J_{12} = \lambda(1 - \lambda^2/2)\Im V_{td}V^*_{ts}$$

where the first piece is 0.219 ± 0.002 and the last is equal to $25.6\times\sqrt{{\rm BR}(K^0 \to \pi^0\nu\bar{\nu})}$. There are no uncertainties in the hadronic matrix element which is taken from K_{l3} decays.

While the branching ratio above is small, 3×10^{-11}, it is the most direct and clean measure of η, the imaginary part of V_{td} and V_{ub}. 100 events give J_{12} to 5% accuracy! Then the SM can be double checked *e.g.* comparing with ϵ, and $K^+ \to \pi^+\nu\bar{\nu}$, as shown in fig. 28 and $\Re(\epsilon'/\epsilon)$ will be measured and computed to better accuracy.

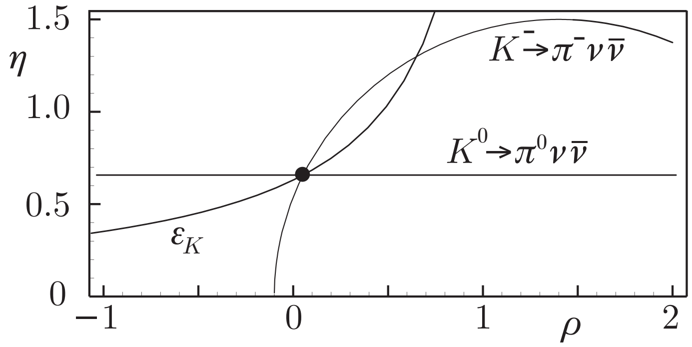


Fig. 28. Constraints in K decays.

B decays give J_{13} from $|V_{cb}|$, $|V_{ub}|$, $B - \bar{B}$ mixing, $B \to J/\psi K$, $B \to \pi\pi$.

Long terms goals are here 2-3 % accuracy in $|V_{cb}|$, 10% for $|V_{ub}|$, sin2β to 0.06 and sin2α to 0.1.

CDF, who has already measured sin2β to 50%, and DØ at FNAL[44] offer the best promise for sin2β.

B-factories will also contribute, in particular to the measurements of $|V_{cb}|$ and $|V_{ub}|$. It will take a long time to reach 15% for J_{13}. LHC, with good sensitivity to B_s-$\bar{B}_s$ mixing, will reach 10% and perhaps 5%.

The branching ratio for $K^0 \to \pi^0\nu\bar{\nu}$ is not presently known. The experience for performing such a measurement is however fully in hand. The uniquely precise way by which this ratio determines η, makes it one of the first priorities of particle physics, at this time. Compared to the very large investments in the study of the B system, it is a most competitive way of obtaining fundamental and precise results at a small cost.

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