N_f - DEPENDENCE OF $\langle\bar{q}q\rangle$

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in preparation

N_f - DEPENDENCE OF χS PARAMETERS

$\Updownarrow$

EFFECT OF LIGHT QUARK LOOPS

WEAK

- ZWEIG RULE
- RESULTS OF $1/N_c$ EXPANSION
- QUENCHED APPROX. IN LATTICE QCD

STRONG

- pQCD β - FUNCTION
  DECONFINEMENT FOR LARGE N_f
- SOME LATTICE SIMULATIONS OBSERVE STRONG N_f - DEP OF χS
  [R.D. MAWHINNEY]
- $1/N_c$ - EXPANSION NOT VERY EFFICIENT IN 0^{++} CHANNEL
- SUM RULE

$$\frac{\langle\bar{q}q\rangle(3)}{\langle\bar{q}q\rangle(2)} = 1 - 0.54 \pm 0.27$$

[B. MOUSSALLAM]

CHIRAL LIMIT $SU_L(N_f) \times SU_R(N_f)$

N_f QUARKS MASS $m \to 0$

OTHER QUARKS MASSES $m_j \neq 0$ FIXED

$$\langle \mathcal{O} \rangle_{N_f} = \lim_{m \to 0} \int [dG]\, e^{-S_{YM}[G]} \underbrace{[\det(i\not{D}+m)]^{N_f}}_{\text{SUPPRESSION FACTOR}} \prod_j \det(i\not{D}+m_j)\, \mathcal{O}$$

FOR χS PARAMETERS

ACTS AS IF "< 1"

IN PARTICULAR, QUARK CONDENSATE $\Sigma(N_f) = -\langle \bar{q}q \rangle^{(N_f)}$

PS DECAY CONSTANT $F(N_f)$

$$\Sigma(N_f+1) < \Sigma(N_f) \quad \leftarrow \text{STRONG SUPPR.}$$

$$F(N_f+1) < F(N_f) \quad \leftarrow \text{WEAKER SUPPR.}$$

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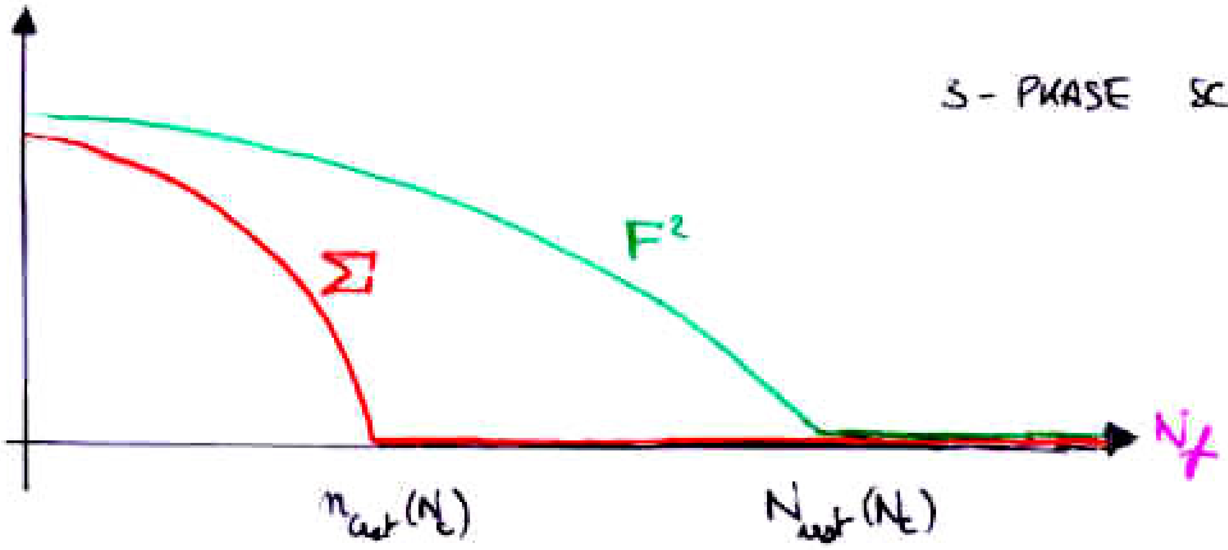
3 - PHASE SCENARIO



~~XS~~	~~XS~~	XS
$\Sigma \neq 0$	$\Sigma = 0$	
$\langle \bar{q}q \rangle$	$\langle \bar{q}q \rangle$	

$\rightarrow$ $N_f \sim 2 \div 3$ CLOSE TO $n_{crit}(N_c)$?

COMPARISON

$\Sigma(2)$ & $\Sigma(3)$

$m_u = m_d \rightarrow 0$, $m_s \neq 0$ $m_u = m_d = m_s \rightarrow 0$

?

$\Sigma(2)$ IS A FUNCTION OF m_s

$$\frac{\partial \Sigma(2)}{\partial m_s} = \lim_{m \to 0} i \int d^4x \, \langle 0 | T[\bar{u}u(x) \quad \bar{s}s(0)] | 0 \rangle$$
$$\equiv \Pi_Z(m_s)$$

- ORDER PARAMETER $N_f = 2$ & 3
- ZWEIG RULE VIOLATION IN 0^{++} (VACUUM CHANNEL)

$$\Sigma(2) = \Sigma(3) + \int_0^{m_s} d\mu \, \Pi_Z(\mu)$$

BECAUSE $\Sigma(2) \xrightarrow[m_s \to 0]{} \Sigma(3)$

$$\Sigma(2) = \Sigma(3) + m_s \, Z^s_{eff}(m_s) + O(m_s^2 \ln m_s)$$

$\downarrow$ SUPPRESSED (DENSITY OF STATES)

$\downarrow$ ENHANCED (FLUCTUATIONS)

NEAR $n_{crit}(N_c)$

WHAT DO WE KNOW ABOUT $\Sigma(2)$ & $\Sigma(3)$ FROM M_π, M_K, M_η ?

$$F_\pi^2 M_\pi^2 = 2m\,\Sigma(3) + O(m_q^2)$$

- $O(m_q^2)$ CAN BE NEGL. (HIGHER ORDER CORR) IF $X(3) = \dfrac{2m\,\Sigma(3)}{F_\pi^2 M_\pi^2} \sim 1$
- NEAR $n_{crit}(N_c)$, $\Sigma(N_f)$ SUPPRESSED !

$$F_\pi^2 M_\pi^2 = 2m\left[\Sigma(3) + (m_s + 2m)\, Z^s_{eff}(m_s)\right] + 4m^2 A_{eff} + F_\pi^2 \delta M_\pi^2$$

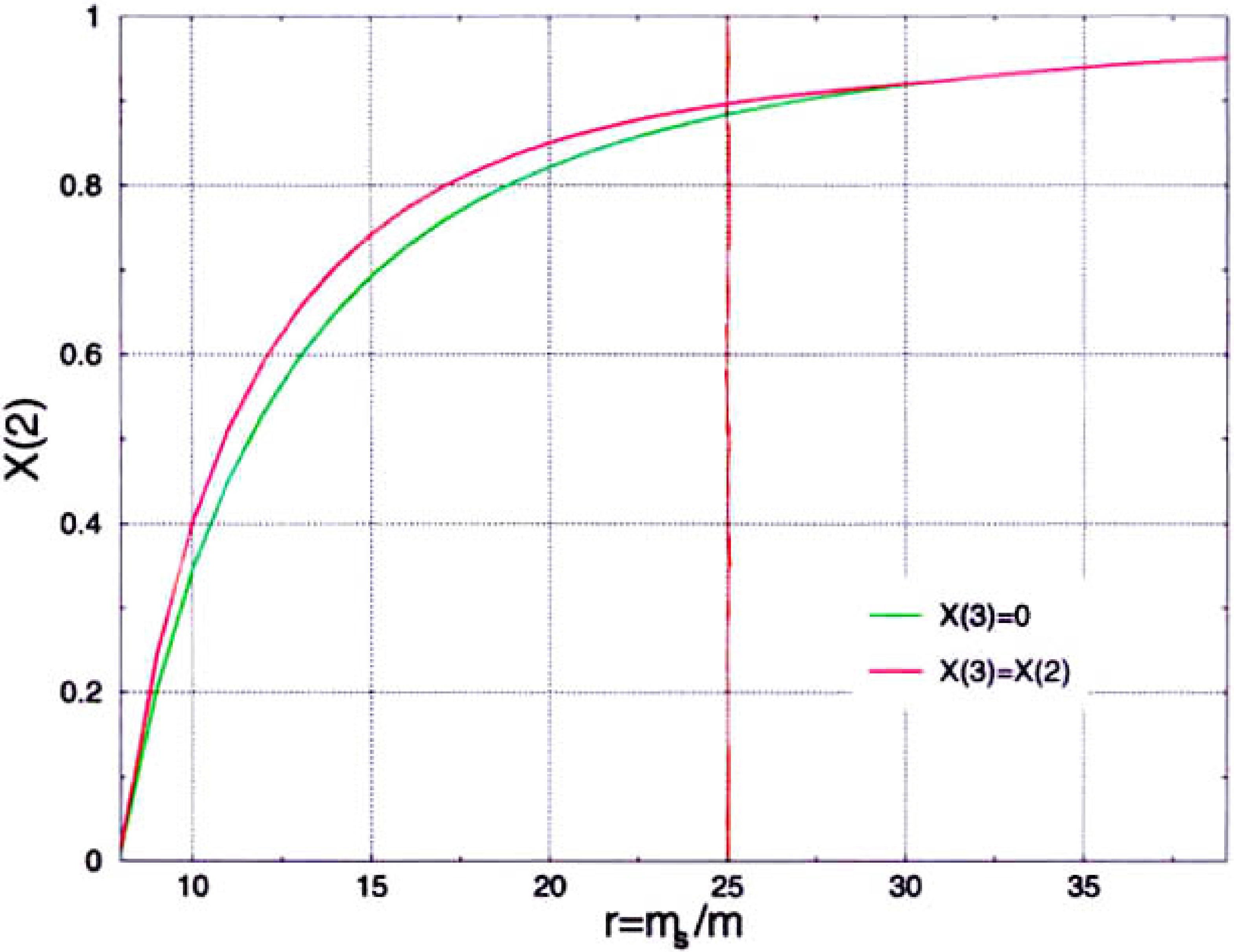
Z^s_{eff}, A_{eff}, δM_π^2 DEFINED SO THAT

→ FOR $X(3)$ LARGE, 1 - LOOP STANDARD χPT

→ FOR ANY $X(3)$, TREE - LEVEL GENERALIZED χPT [INCLUDING m_q^2 CORR.]

➡ WITH THIS EXPANSION, WE CAN DEAL WITH BOTH CASES $n_{crit}(N_c)$ NEAR 3 & $n_{crit}(N_c) \gg 3$

$$X(N_f) = \frac{2m\,\Sigma(N_f)}{M_\pi^2 F_\pi^2}$$



→ ZR MAXI

→ NO ZR

r & $X(2)$ [$\leftrightarrow \pi\pi$ SCATT] STRONGLY CORRELATED

BUT $X(3)$ [FINE TUNING] HARD TO DETERMINE !

LET US TRY TO EVALUATE $\dfrac{\partial Z(2)}{\partial m_s} = \Pi_Z(m_s)$

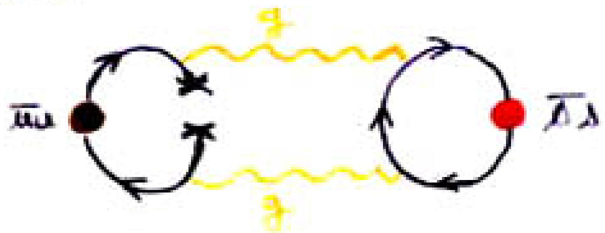
USING DATA (AVAILABLE IN THE 0^+ SECTOR)

IN A CHIRAL SUM RULE

[B. MOUSSALLAM hep-ph/9909292]

$$\hat{\Pi}(q^2) = \lim_{m\to 0}\; i\int d^4x\; e^{iqx}\langle 0|T[\bar{u}u(x)\,\bar{s}s(0)]|0\rangle$$

OPERATOR PRODUCT EXPANSION :



$$\underset{Q^2\to\infty}{\sim}\; \frac{C\, m_s \langle \bar{u}u\rangle}{Q^2[\ln Q^2]^{\frac{24}{27}}}$$

SUPERCONVERGENT

$$\Pi_Z(m_s) = \hat{\Pi}(0) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\,\hat{\Pi}(s)}{s}\, ds$$

- $0 \le \sqrt{s} \le \sqrt{s_1} \sim 1.2 \div 1.4$ GeV

  EXPLICIT EVALUATION OF $\mathrm{Im}\,\hat{\Pi}(s)$

- $s \geq s_1$ USE OF WEINBERG SR $\int_0^\infty \mathrm{Im}\,\hat{\Pi}(s) = 0$

  TO ESTIMATE $\int_{s_1}^\infty \mathrm{Im}\,\hat{\Pi}(s)\, ds$

  AND THEN $\int_{s_1}^\infty \frac{1}{s}\,\mathrm{Im}\,\hat{\Pi}(s)\, ds$

$$\mathrm{Im}\,\hat{\Pi}(q^2) = \frac{1}{2} \sum_n (2\pi)^4\, \delta(q - P_n)\, \langle 0|\bar{u}u|n\rangle\langle n|\bar{s}s|0\rangle$$

FOR $\sqrt{s} \leq \sqrt{s_1} = 1.2 \div 1.4$ GeV $\begin{cases} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ \cancel{|4\pi\rangle} \text{ EXPERIMENTALLY} \end{cases}$

→ DETERMINATION OF THE SCALAR FORM FACTORS

$\langle 0|\bar{u}u|\pi\pi\rangle$ $\quad\quad$ $\langle 0|\bar{s}s|\pi\pi\rangle$

$\langle 0|\bar{u}u|K\bar{K}\rangle$ $\quad\quad$ $\langle 0|\bar{s}s|\pi\pi\rangle$

VALUES AT 0 : $\left(\frac{\partial M^2_{\pi,K}}{\partial m}\right)_{m=0}, \left(\frac{\partial M^2_K}{\partial m_s}\right)_{m=0}$ [XPT]

+

T – MATRIX MODEL CONSTRAINED BY 0^{++} DATA

↓ OMNES – MUSKHELISHVILI EQUATIONS FOR 2 COUPLED CHANNELS (NUMERICAL RESOLUTION)

FORM FACTORS & SPECTRAL FUNCTION

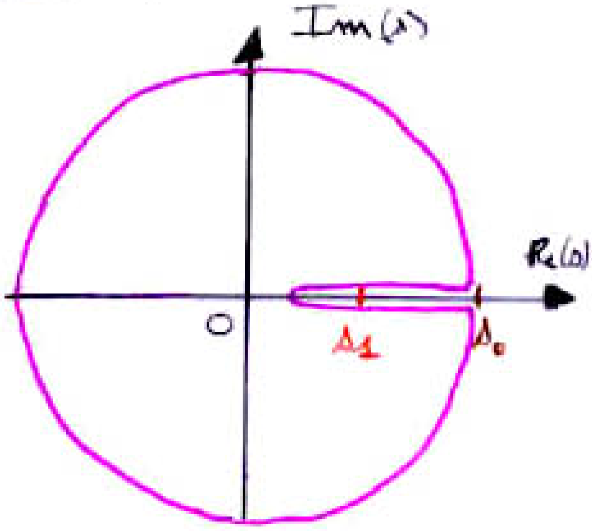
DIRECT ↙ $\quad\quad$ ↘ WEINBERG S.R. ESTIMATE

$$\Pi_Z(m_s) = \frac{1}{\pi}\int_0^{s_1} \frac{\mathrm{Im}\,\hat{\Pi}(s)}{s}\,ds + \frac{1}{\pi}\int_{s_1}^{\infty} \frac{\mathrm{Im}\,\hat{\Pi}(s)}{s}\,ds$$

WITH 1-LOOP STANDARD XPT [X(2) & X(3) = O(1)]

$$\frac{X(3)}{X(2)} = 1 - 0.5 \pm 0.27 \longrightarrow \;!$$

HOW TO IMPROVE THIS RESULT ?



$$\Pi_Z(m_s) = \frac{\partial \Sigma(2)}{\partial m_s}$$

$$= \underbrace{\frac{1}{\pi}\int_0^{s_1} ds\ \mathrm{Im}\,\hat{\Pi}(s)\ \frac{1}{s}\left(1-\frac{s}{s_0}\right)}$$

- 3 MODELS OF T-MATRIX
- VALUES AT 0 $\left(\frac{\partial M^2_{\pi,K}}{\partial m}\right)_{m=0}$, $\left(\frac{\partial M^2_K}{\partial m_s}\right)_{m=0}$ IN GENERALIZED χPT

[$X(3)$ FREE PARAMETER]

$$+ \underbrace{\frac{1}{\pi}\int_{s_1}^{s_0} ds\ \mathrm{Im}\,\hat{\Pi}(s)\ \frac{1}{s}\left(1-\frac{s}{s_0}\right)}$$

WEINBERG S.R. ESTIMATE

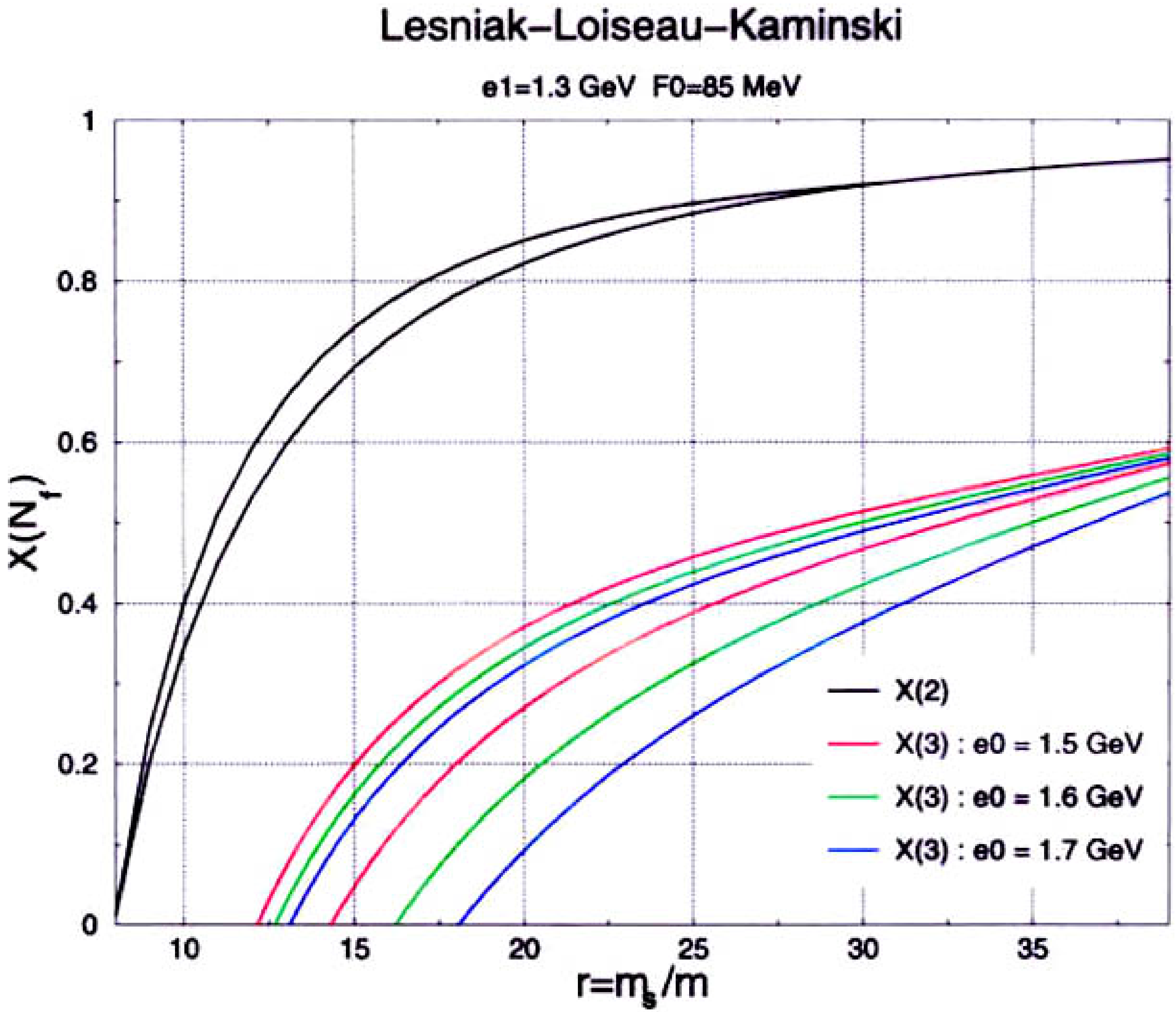
$$+ \underbrace{\frac{1}{2\pi i}\int_{|s|=s_0} ds\ \hat{\Pi}(s)\ \frac{1}{s}\left(1-\frac{s}{s_0}\right)}$$

CONTRIBUTION FROM 1^{st} TERM OF OPERATOR PROD EXP $\sim \alpha_s^2 \frac{m_s^2}{s_0} X(2)$ [SMALL PIECE : DUALITY ARISES LATELY]

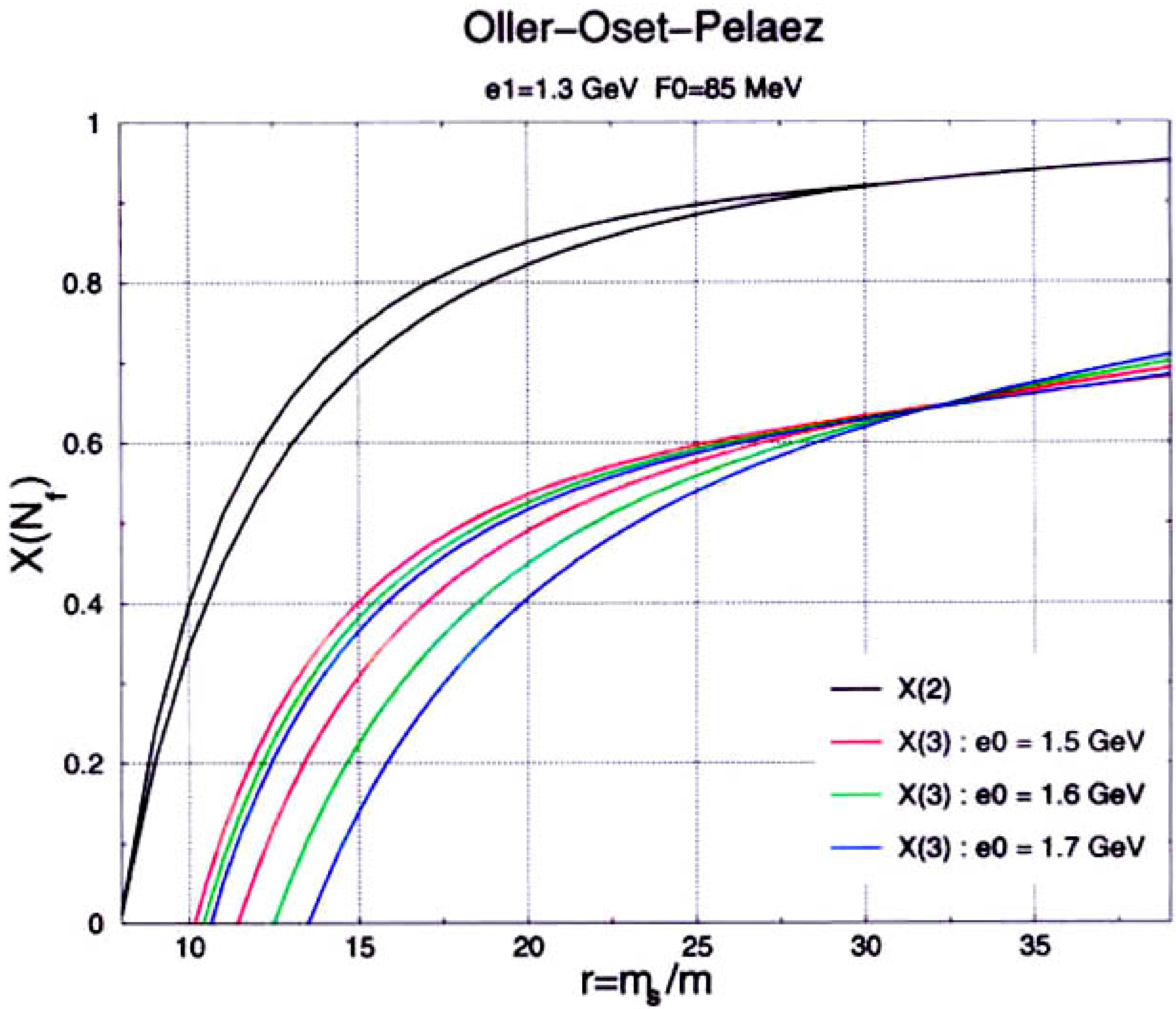
$M_\pi^2, M_K^2, M_\eta^2, F_\pi^2, F_K^2$ + THIS SUM RULE

$\downarrow$

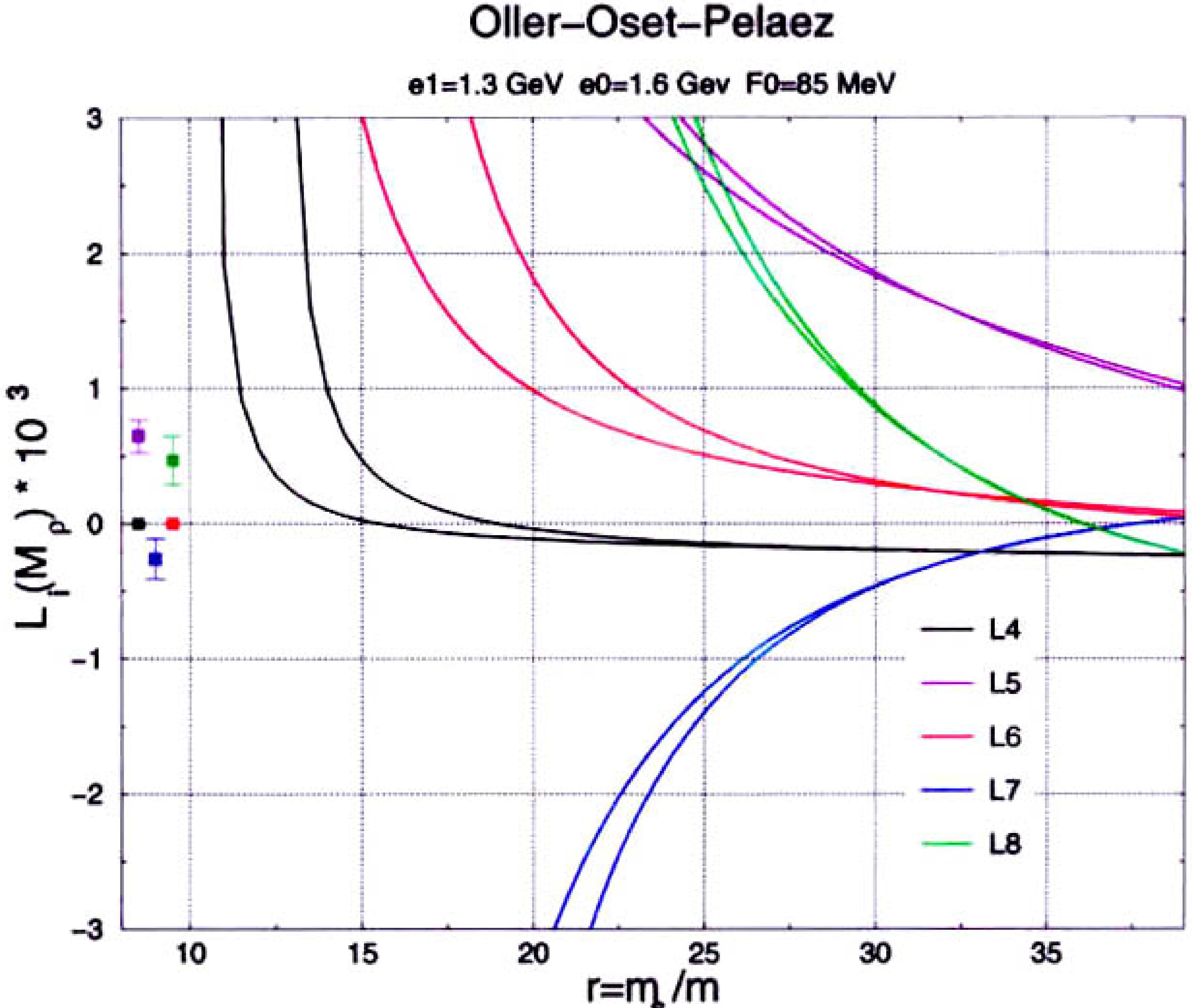
$X(3)$ & L.E.C FUNCTIONS OF r & F_0

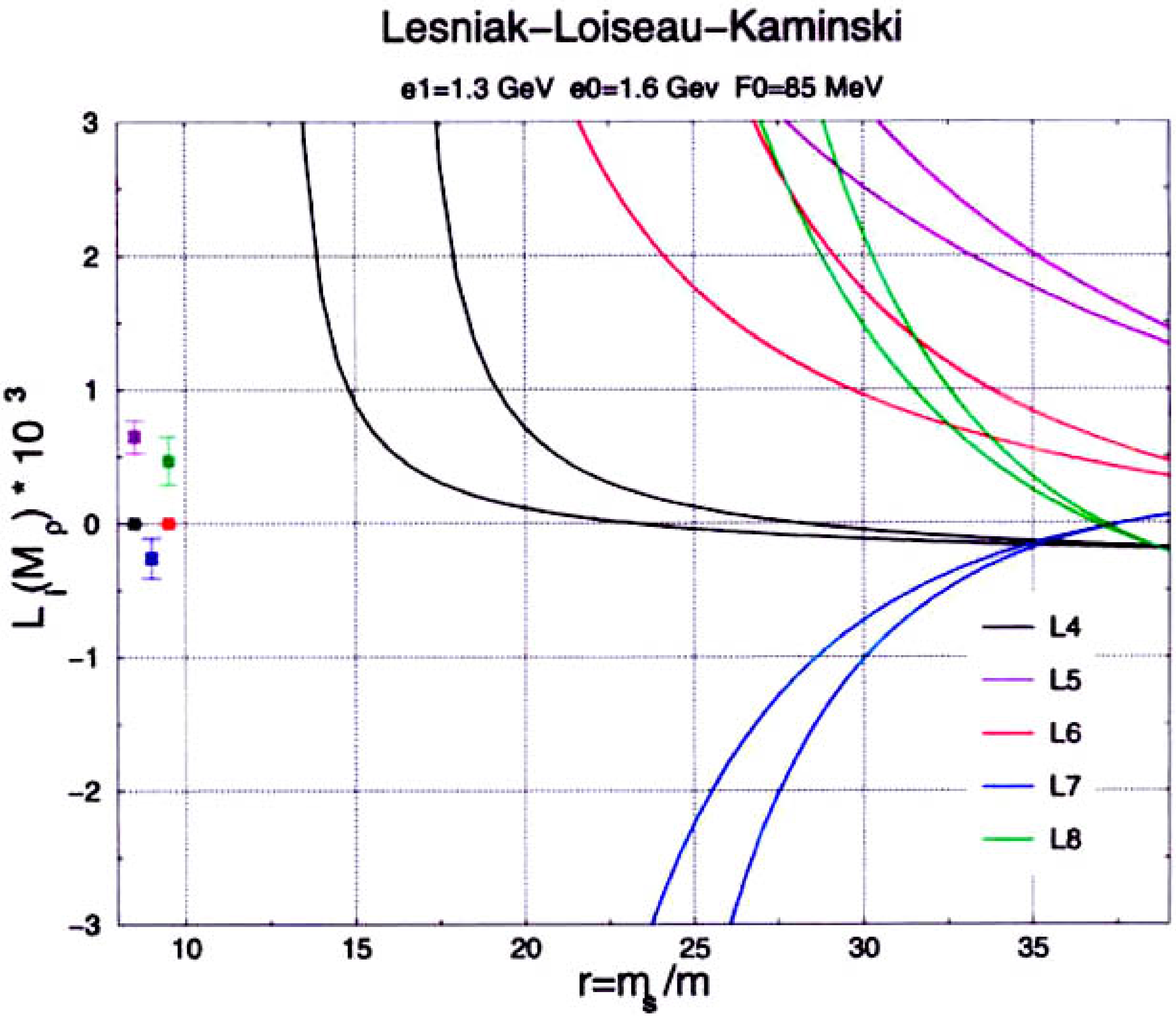


Lesniak-Loiseau-Kaminski
e1=1.3 GeV F0=85 MeV
X(N_f)
r=m_s/m
0
0.2
0.4
0.6
0.8
1
10
15
20
25
30
35
X(2)
X(3) : e0 = 1.5 GeV
X(3) : e0 = 1.6 GeV
X(3) : e0 = 1.7 GeV



Oller–Oset–Pelaez
e1=1.3 GeV F0=85 MeV
X(N_f)
r=m_s/m
0
0.2
0.4
0.6
0.8
1
10
15
20
25
30
35
X(2)
X(3) : e0 = 1.5 GeV
X(3) : e0 = 1.6 GeV
X(3) : e0 = 1.7 GeV


Oller-Oset-Pelaez
e1=1.3 GeV e0=1.6 Gev F0=85 MeV
L4
L5
L6
L7
L8
r=m_s /m
L_i(M_ρ) * 10^3
3
2
1
0
-1
-2
-3
10
15
20
25
30
35



Lesniak–Loiseau–Kaminski
e1=1.3 GeV e0=1.6 Gev F0=85 MeV
L4
L5
L6
L7
L8
r=m_s/m
L_i(M_ρ) * 10^3

CONCLUDING REMARKS

- FROM THE 0^{++} CHANNEL, $\Sigma(3)$ MUCH SMALLER THAN $\Sigma(2)$

  → PROXIMITY OF A CHIRAL PHASE TRANS ?

- WE SHOULD BE MORE CAREFUL WITH ARGUMENTS RELYING ON ZWEIG RULE !

  → CONNECTION SU(2) / SU(3) χPT

- WE NEED A BETTER EXPERIMENTAL KNOWLEDGE OF THE 0^{+} CHANNEL

  → $f_0(980)$ MAIN CONTRIBUTION TO S.R.

  T-MATRIX MODELS WITH RATHER $\neq$ SHAPES IN SPECTRAL FUNCTION

- SEVERAL CHIRAL PHASES WHEN $N_f \nearrow$

  → TECHNICOLOR RICHER THAN "SCALED QCD"