ϕ RADIATIVE DECAYS AND η-η' MIXING

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- Introduction
- η-η' mixing
- ϕ radiative decays
- couplings of η and η' to pseudoscalar current
- " " " " axial vector current
- conclusions

Based on work in collaboration with M.R. PENNINGTON

REASONS FOR STUDYING RADIATIVE ϕ DECAYS

THEORETICAL

- They give information on structure and properties of low-mass resonances (for example $f_0(980)$)
- They can provide insights in the problem of η-η' mixing (a still debated subject)
- They probe the strange content of light pseudoscalars

EXPERIMENTAL

- KLOE experiment at the DAΦNE ϕ-factory is starting its data-taking (some results have already been provided by Novosibirsk's group)

$\eta - \eta'$ MIXING

- OCTET - SINGLET BASIS

$$\langle 0 | J^i_{5\mu} | P(p) \rangle = i f^i_P p_\mu \qquad i = 8, 0 \qquad P = \eta, \eta'$$

where:

$$J^8_{5\mu} = \frac{1}{\sqrt{6}} \left(\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2 \bar{s} \gamma_\mu \gamma_5 s \right)$$

$$J^0_{5\mu} = \frac{1}{\sqrt{3}} \left(\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s \right)$$

Two mixing angles are required to describe the mixing consistently (Leutwyler)

$$f^8_\eta = f_8 \cos\theta_8 \qquad f^0_\eta = - f_0 \sin\theta_0$$

$$f^8_{\eta'} = f_8 \sin\theta_8 \qquad f^0_{\eta'} = f_0 \cos\theta_0$$

- FLAVOUR BASIS (Feldmann et al)

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}} \left(\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d \right)$$

$$J^s_{\mu 5} = \bar{s} \gamma_\mu \gamma_5 s$$

$$f^q_\eta = f_q \cos\phi_q \qquad f^s_\eta = - f_s \sin\phi_s$$

$$f^q_{\eta'} = f_q \sin\phi_q \qquad f^s_{\eta'} = f_s \cos\phi_s$$

if (Feldmann) $\left| \frac{\phi_q - \phi_s}{\phi_q + \phi_s} \right| \ll 1 \quad \Rightarrow \quad \phi_q = \phi_s = \phi$

RELATIONS BETWEEN THE TWO MIXING SCHEMES

$$\text{tg}\,\theta_8 = \frac{f_q \sin\phi_q - \sqrt{2}\, f_s \cos\phi_s}{f_q \cos\phi_q + \sqrt{2}\, f_s \sin\phi_s}$$

which becomes, for $\phi_q = \phi_s = \phi$

$$\theta_8 = \phi - \text{arctg}\left(\sqrt{2}\,\frac{f_s}{f_q}\right)$$

$$\text{tg}\,\theta_0 = \frac{f_s \sin\phi_s - \sqrt{2}\, f_q \cos\phi_q}{f_s \cos\phi_s + \sqrt{2}\, f_q \sin\phi_q}$$

for $\phi_q = \phi_s = \phi$:

$$\theta_0 = \phi - \text{arctg}\left(\frac{\sqrt{2}\, f_q}{f_s}\right)$$

AND

$$f_8^2 = \frac{1}{3} f_q^2 + \frac{2}{3} f_s^2 + \frac{\sqrt{2}}{3} f_q f_s \cos(\phi_s - \phi_q)$$

$$f_0^2 = \frac{2}{3} f_q^2 + \frac{1}{3} f_s^2 + \frac{\sqrt{2}}{3} f_q f_s \cos(\phi_s - \phi_q)$$

$\phi \to \eta\gamma$ and $\phi \to \eta'\gamma$

$$\langle \eta(q_2) | \bar{s}\gamma^\nu s | \phi(q_1,\epsilon_1)\rangle = F(q^2)\, \epsilon^{\nu\alpha\beta\delta} (q_1)_\alpha (q_2)_\beta (\epsilon_1)_\delta \qquad q = q_1 - q_2$$

$$\Gamma(\phi\to\eta\gamma) = \frac{\alpha g^2}{24}\left(\frac{m_\phi^2 - m_\eta^2}{m_\phi}\right)^3$$

where: $g = \frac{F(0)}{3}$

QCD SUM RULE approach:

$$\Pi_{\mu\nu}(q_1^2, q_2^2, q^2) = i^2 \int d^4x\, d^4y\, e^{-iq_1\cdot x + iq_2\cdot y} \langle 0| T[J_5^s(y) J_\nu(0) J_\mu(x)]|0\rangle =$$

$$= \Pi(q_1^2, q_2^2, q^2)\, \epsilon_{\mu\nu\alpha\beta} (q_1)^\alpha (q_2)^\beta$$

with: $J_5^s = \bar{s} i\gamma_5 s$ $\quad J_\nu = \bar{s}\gamma_\nu s$

• 1st step: writing a dispersion relation for Π

$$\Pi(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2}\int ds_1 \int ds_2 \frac{\rho(s_1,s_2,q^2)}{(s_1-q_1^2)(s_2-q_2^2)} + \text{subtraction}$$

for low values of s_1, s_2 ρ contains a double δ-function corresponding to the transition $\phi\to\eta$

$$\Pi^{HAD}(q_1^2,q_2^2,q^2) = \frac{A\, F(q^2)\, m_\phi f_\phi}{(m_\phi^2-q_1^2)(m_\eta^2-q_2^2)} + \frac{1}{\pi^2}\int_{s_{1,0}}^{\infty} ds_1 \int_{s_{2,0}}^{\infty} ds_2 \frac{\rho^{had}(s_1,s_2,q^2)}{(s_1-q_1^2)(s_2-q_2^2)}$$

$A = \langle 0|\bar{s} i\gamma_5 s|\eta\rangle \to$ should be computed within the same QCD S.R. approach

$s_{1,0}$ and $s_{2,0}$ are effective thresholds for higher resonances and continuum contributions

- 2nd step: QCD computation

↳ Expansion of the T-product in the three point function by OPE
a perturbative contribution + non perturbative terms (condensates)

$$\Pi^{QCD}(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2} \int_{4m_s^2}^{\infty} ds_1 \int_{4m_s^2}^{\infty} ds_2 \, \frac{\rho^{QCD}(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} + C_3 \langle \bar{s}s \rangle + C_5 \langle \bar{s} g \sigma G q \rangle + \cdots$$

Quark-Hadron global duality $\Rightarrow$ $\rho^{had} \equiv \rho^{QCD}$

SUM RULE: $\Pi^{HAD} \equiv \Pi^{QCD}$

- Further improvement: Borel transform

$$\mathcal{B}\left[\frac{1}{s+Q^2}\right]^n = \frac{e^{-s/M^2}}{(n-1)!\,(M^2)^n} \qquad M^2 \equiv \text{Borel parameter}$$

- improves the convergence of the series in the OPE by factorials
- enhances the contribution of low lying states
- allows to neglect subtraction terms (which are polynomials: the B transf. vanishes)

FINAL SUM RULE

$$A\, F(q^2)\, m_\phi f_\phi = e^{m_\phi^2/M_1^2}\, e^{m_\eta^2/M_2^2} \left\{ \int ds_1 \int ds_2 \, e^{-s_1/M_1^2}\, e^{-s_2/M_2^2} \, \frac{3 m_s}{\pi^2 \sqrt{\lambda(s_1, s_2, q^2)}} + \right.$$

$$+ e^{-m_s^2/M_1^2}\, e^{-m_s^2/M_2^2} \left[\langle \bar{s}s \rangle \left(2 - \frac{m_s^2}{M_1^2} - \frac{m_s^2}{M_2^2} + \frac{m_s^4}{M_1^4} + \frac{m_s^4}{M_2^4} + \frac{m_s^2 (2 m_s^2 - q^2)}{M_1^2 M_2^2} \right) + \right.$$

$$\left.\left. + \langle \bar{s} g \sigma G s \rangle \left(\frac{1}{6 M_1^2} + \frac{2}{3 M_2^2} - \frac{m_s^2}{2 M_1^4} - \frac{m_s^2}{2 M_2^4} + \frac{2 q^2 - 3 m_s^2}{3 M_1^2 M_2^2} \right) \right] \right\}$$

Preliminary computation of

$$\langle 0 | \bar{s} i \gamma_5 s | \eta \rangle = A$$

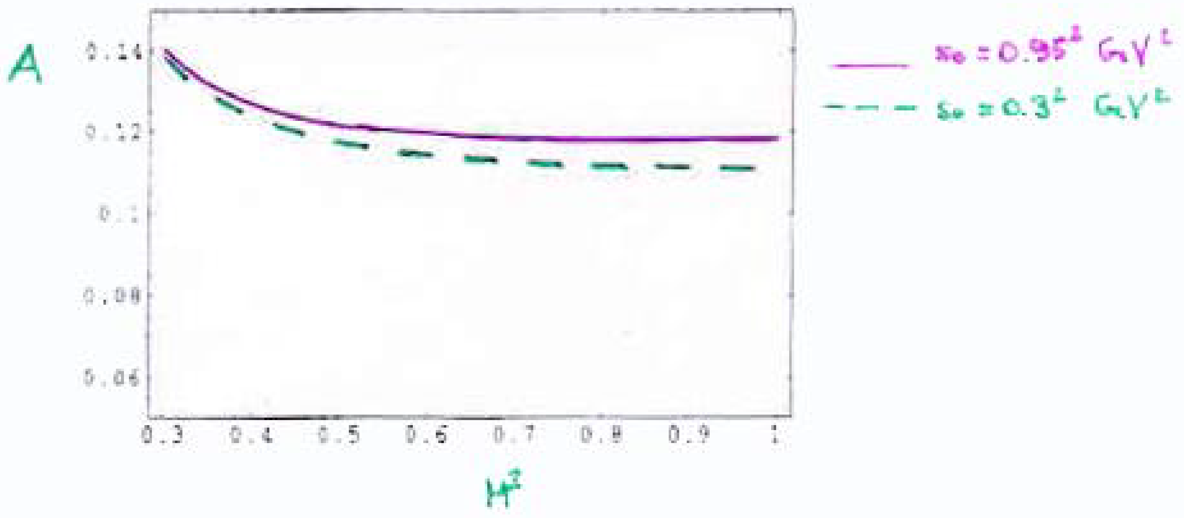
Two point function:

$$\Pi_{ab}(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T \left[J_5^a(x) J_5^{b\dagger}(0) \right] | 0 \rangle$$

sum rule:

$$A^2 e^{-m_\eta^2/M^2} = \frac{3}{8\pi^2} \int_{4m_s^2}^{s_0} ds \; s \sqrt{1 - \frac{4m_s^2}{s}} \, e^{-s/M^2} +$$

$$- m_s e^{-m_s^2/M^2} \left[\langle \bar{s}s \rangle \left(1 - \frac{m_s^2}{M^2} + \frac{m_s^4}{M^4} \right) + \frac{1}{M^2} \langle \bar{s} g \sigma G s \rangle \left(1 - \frac{m_s^2}{2M^2} \right) \right]$$



Requirements on M^2:

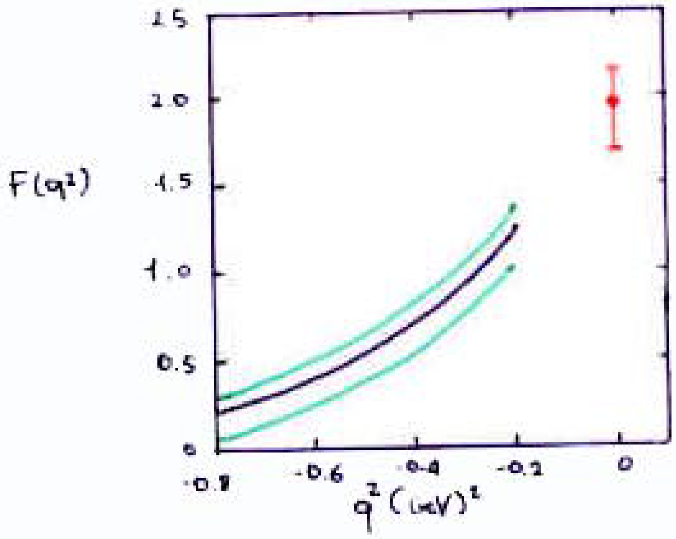
- perturbative contribution $\geq$ 20% continuum → upper bound on M^2
- perturbative term $\geq$ non perturbative terms → lower bound on M^2
- independence of the result on M^2 → selection of a STABILITY WINDOW

RESULT: $A = (0.115 \pm 0.004)$ GeV^2

An analogous procedure for $A' = \langle 0 | \bar{s} i \gamma_5 s | \eta' \rangle$ gives:

$$A' = (0.151 \pm 0.015) \text{ GeV}^2$$

RESULT OF THE SUM RULE OBTAINED VARYING ALL THE INPUT PARAMETERS



RESULTS

$$|g| = \frac{F(0)}{3} = (0.66 \pm 0.06)\ \mathrm{GeV}^{-1}$$

$$BR(\phi \to \eta\gamma) = (1.15 \pm 0.2)\ \%$$ (independent of any mixing scheme!)

to be compared with: $BR(\phi \to \eta\gamma)_{exp} = (1.18 \pm 0.03 \pm 0.06)\ \%$

Novosibirsk VEPP-2M groups
PL B460 (99) 242

Repeating the same analysis for the η' channel:

$$|g'| = \frac{F'(0)}{3} = (1.0 \pm 0.2)\ \mathrm{GeV}^{-1}$$

$$BR(\phi \to \eta'\gamma) = (1.18 \pm 0.4)\ 10^{-4}$$

to be compared with: $BR(\phi \to \eta'\gamma)_{exp} = \left(0.82^{+0.21}_{-0.19} \pm 0.11\right) 10^{-4}$

hep-ex/9911036

In the flavour basis mixing scheme:

$$R = \frac{BR(\phi \to \eta\gamma)}{BR(\phi \to \eta'\gamma)} = \left(\frac{m^2_\phi - m^2_\eta}{m^2_\phi - m^2_{\eta'}}\right)^2 tg^2\phi_s$$

$$\Rightarrow \qquad \phi_s = 58.5° \pm 6.5°$$

and $\quad \phi_s^{exp} = 63.3° \pm 6°$

η - η' COUPLINGS TO AXIAL VECTOR CURRENTS

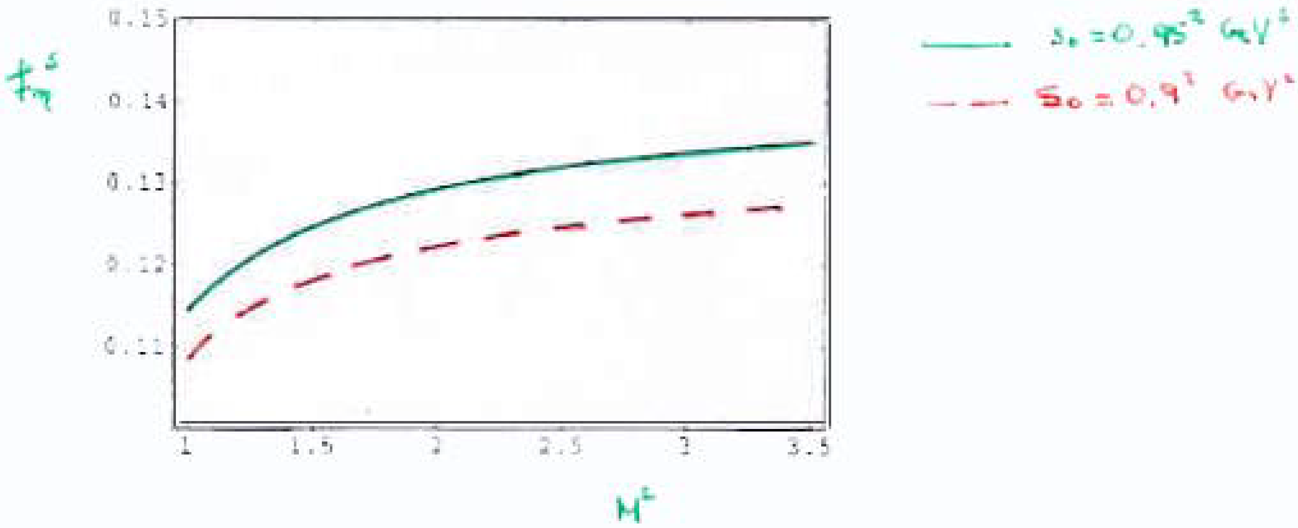
$$\Pi_{\mu\nu}(q^2) = i\int d^4x\; e^{iqx} \langle 0|T[J^s_{5\mu}(x)\, J^s_{5\nu}(0)]|0\rangle$$

Sum rule:

$$(f_\eta^s)^2 = e^{m_\eta^2/M^2}\left[\frac{1}{\pi}\int_{4m_s^2}^{s_0} ds\; e^{-s/M^2}\; \frac{1}{4\pi}\sqrt{1-\frac{4m_s^2}{s}}\;\frac{2m_s^2+s}{s} + \frac{2m_s}{M^2}\langle\bar{s}s\rangle\, e^{-m_s^2/M^2}\right]$$

RESULT : $f_\eta^s = (0.130 \pm 0.01)$ GeV

idem : $f_{\eta'}^s = (0.122 \pm 0.02)$ GeV



Analogously, using $J^q_{\mu 5}$:

$f_\eta^q = (0.144 \pm 0.004)$ GeV

$f_{\eta'}^q = (0.125 \pm 0.015)$ GeV

Collecting the results:

$$f_\eta^s = (0.130 \pm 0.01)\ \text{GeV}$$

$$f_{\eta'}^s = (0.122 \pm 0.02)\ \text{GeV}$$

$$f_\eta^q = (0.144 \pm 0.004)\ \text{GeV}$$

$$f_{\eta'}^q = (0.125 \pm 0.015)\ \text{GeV}$$

Using:

$$\frac{f_\eta^s}{f_{\eta'}^s} = tg\,\phi_s \qquad \frac{f_{\eta'}^q}{f_\eta^q} = tg\,\phi_q$$

we get:

$$\phi_s = 46.6^\circ \pm 7^\circ$$

$$\phi_q = 41^\circ \pm 4^\circ$$

$$\Rightarrow \qquad \left|\frac{\phi_s - \phi_q}{\phi_s + \phi_q}\right| = 0.065 \ll 1$$

MATRIX ELEMENT OF THE DIVERGENCE OF THE AXIAL VECTOR CURRENT

$$\langle 0|\partial^\mu J^s_{5\mu}|\eta\rangle = m^2_\eta f^s_\eta$$

it contains the axial-vector anomaly:

$$\partial^\mu J^s_{5\mu} = \partial^\mu(\bar{s}\gamma_\mu\gamma_5 s) = 2m_s\bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi}G\tilde{G}$$

$$\Rightarrow \quad 2m_s\langle 0|\bar{s}i\gamma_5 s|\eta^{(\prime)}\rangle = f^s_{\eta^{(\prime)}}\, m^2_{\eta^{(\prime)}} - \langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta^{(\prime)}\rangle$$

Using the previous results for $A = \langle 0|\bar{s}i\gamma_5 s|\eta\rangle$ $A' = \langle 0|\bar{s}i\gamma_5 s|\eta'\rangle$ f^s_η, $f^s_{\eta'}$

we get:

$$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta\rangle = (0.0082 \pm 0.004)\ \text{GeV}^3$$

$$\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle = (0.072 \pm 0.025)\ \text{GeV}^3$$

close to naive quark model expectations

CONCLUSIONS

The method of QCD sum rules gives:

$$BR(\phi \to \eta\gamma) = (1.15 \pm 0.2)\,\%$$

$$BR(\phi \to \eta'\gamma) = (1.13 \pm 0.4)\,10^{-4}$$

These estimates DO NOT rely on any assumption about the mixing pattern in the η-η' system.

Experimental data (Novosibirsk groups)

$$BR(\phi \to \eta\gamma)_{exp} = (1.18 \pm 0.03 \pm 0.06)\,\%$$

$$BR(\phi \to \eta'\gamma)_{exp} = \left(0.82\,^{+0.21}_{-0.19} \pm 0.11\right) 10^{-4}$$

A set of couplings and the mixing angles in the flavour basis mixing scheme have also been computed.

Controversial estimates exist in the literature

$\Longrightarrow$ WAITING FOR NEW EXPERIMENTAL RESULTS (e.g. from DAΦNE)