K_{e4} and Low Energy Constants *

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- Definitions. Form-Factors
- CHPT. Calculation to 2-loops
- Checks.
- LEC fit. Errors.
- Masses, decay constants, form-factors to 2-loops.
- π-π scattering parameters.
- Notes and Summary.

* "K_{e4} Form-Factors and π-π Scattering"
hep-ph/0003258

G Amoros

before starting...

the 2-loop K_{e4} calculation is one of the next steps in CHPT

for selfconsistent checks

to compare with π-π scatt.

to obtain a first 3 flavour LEC fit (how do higher orders modify their values?)

to be updated with the new measurements (form-factors, ...)

$$K^+(p) \longrightarrow \pi^+(p_+)\pi^-(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$K^+ \longrightarrow \pi^0 \quad \pi^0 \quad \ell^+ \quad \nu_\ell$$

$$K^0 \longrightarrow \pi^- \quad \pi^0 \quad \ell^+ \quad \nu_\ell$$

$$T^{+-} = \frac{G_F}{\sqrt{2}} V_{us}^* \bar{u}\gamma_\mu(1-\gamma_5)v\left(V^\mu - A^\mu\right)$$

$$V_\mu = -\frac{H}{m_K^3}\varepsilon_{\mu\nu\rho\sigma}(p_\ell+p_\nu)^\nu(p_+ + p_-)^\rho(p_+ - p_-)^\sigma$$

$$A_\mu = -\frac{i}{m_K}\left[(p_+ + p_-)_\mu F + (p_+ - p_-)_\mu G + (p_\ell + p_\nu)_\mu R\right]$$

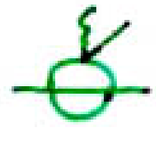
- $m_e^2 R \simeq 0$
- H to $\mathcal{O}(p^6)$: 1-loop
- F and G to $\mathcal{O}(p^6)$: 2-loops

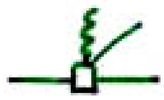
$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

$$F = \frac{m_K}{\sqrt{2}F_\pi}\left[1 + \frac{F_{1loop}}{F_\pi^2} + \frac{F_{LL} + F_V + F_{CT}}{F_\pi^4} + \cdots\right]$$

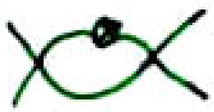
$$G = \frac{m_K}{\sqrt{2}F_\pi}\left[1 + \frac{G_{1loop}}{F_\pi^2} + \frac{G_{LL} + G_V + G_{CT}}{F_\pi^4} + \cdots\right]$$

$(F, G)_{1loop}$: 1-loop diagrams + $O(p^4)$ CT

$(F, G)_V$: $\mathcal{L}_2 \times \mathcal{L}_2 \times \mathcal{L}_2$ $O(p^6)$ 

$(F, G)_{CT}$: $\mathcal{L}_6$ CT $O(p^6)$ 

$(F, G)_{LL}$: $\mathcal{L}_4 \times \mathcal{L}_2$, $\mathcal{L}_2 \times \mathcal{L}_2^{*}$ $O(p^6)$



* $O(p^6)$ contribution from the rewriting with physical quantities

What to worry about?

- to reproduce the 1-loop result. ✓
- isospin relations: $T^{+-} = \frac{T^{-0}}{\sqrt{2}} + T^{00}$ ✓
- to renormalize the wave function ✓
- to consider physical [2-loop] masses and decay constants.* ✓
- to cancel the divergences
  - polynomial (with CT) ✓
  - spurious, non-local ✓
- to use or not to use the GMO relation (the difference is $O(p^8)$). WE USE. ✓

- values for the CT:

  $O(p^4)$ CT $\rightarrow$ FIT* [OUR PROJECT]

  $O(p^6)$ $\rightarrow$ saturation from the nearest resonances: vector, axial-vector, scalars and the η' (η-η' mixing). ✓

We have a huge 2-loop result.
What and how do we fit?

Theoretical side:

$B_0\hat{m}$, $B_0 m_s$, F_0, $L_1 - L_{10}$ parameters from the CHPT Lagrangian : 13

- L_{10} gives not contribution
- L_9 appears with $s_\ell = (p_e + p_\nu)^2$. The FIT is done with $s_\ell = 0$.
- L_4 and L_6 from the large-N_c limit. $(\simeq 0)$.
- Note that

$$\left.\begin{aligned} F_0 &= F_0(F_\pi, L_i, m_i) \\ (B_0\hat{m}) &= (B_0\hat{m})(m_\pi^2, L_i, \dots)^* \end{aligned}\right\} \text{THEY ARE OBTAINED AFTER THE FIT!}$$

Finally,

$$\left.\underbrace{\frac{m_s}{\hat{m}}, \frac{F_K}{F_\pi}}_{\text{written as masses and } L_i.}, L_1, L_2, L_3, L_5, L_7, L_8\right\} \underbrace{\boxed{L_1, L_2, L_3, L_5, L_7, L_8}}_{\text{parameters to fit}}$$

* $m_\pi^2 = 2B_0\hat{m} + (m^2)^{(4)} + (m^2)^{(6)}$

with 'physical' quantities

Experimental side:

- masses:

$$F_K / F_\pi \;{}^*$$

$$\left.\frac{m_s}{\hat{m}}\right|_1 = \frac{2 m_{0K}^2 - m_{0\pi}^2}{m_{0\pi}^2} \;{}^*$$

$$\|$$

$$\left.\frac{m_s}{\hat{m}}\right|_2 = \frac{3 m_{0\eta}^2 - m_{0\pi}^2}{2 m_{0\pi}^2} \;{}^*$$

$$\|$$

$$24 \qquad \boxed{\text{FIXED}}$$

* $(m^2)_{phys} = m_0^2 + (m^2)^{(4)} + (m^2)^{(6)}.$

- $f_s(0), g_s(0), \lambda_f \cdot$ [K_{e4} FORM-FACTORS]

↳ NEXT SLIDE

$$\boxed{F_K/F_\pi ,\ \left.\frac{m_s}{\hat{m}}\right|_1 ,\ \left.\frac{m_s}{\hat{m}}\right|_2 ,\ f_s(0),\ g_s(0),\ \lambda_f}$$ Input

Rosselet et al. (77)

$$f_s(q^2) = f_s(0)\,(1 + \lambda_f\, q^2)$$

$$g_p(q^2) = g_p(0)\,(1 + \lambda_g\, q^2)$$

$$q^2 = \frac{s_\pi - 4m_\pi^2}{4m_\pi^2}$$

$$f_s(0) = 5.59 \pm 0.14$$

$$g_p(0) = 4.77 \pm 0.27$$

$$\lambda_f = \lambda_g = 0.08 \pm 0.02$$

Relation with the F and G form-factors:

$$F = f_s\, e^{i\delta_s} + f_p\, e^{i\delta_p} \cos\theta_\pi + \text{d-wave} + \dots$$

$$G = g_p\, e^{i\delta_p} + \text{d-wave} + \dots$$

$$f_s(0) = F(s_\pi \simeq 4m_\pi^2,\ s_e = 0,\ \cos\theta_\pi = 0)$$

$$g_p(0) = G(s_\pi \simeq 4m_\pi^2,\ s_e = 0,\ \cos\theta_\pi = 0)$$

Table 1: $L_4^r = 0, L_6^r = 0$ and $L_9^r = 6.90\ 10^{-3}$. All $L_i^r(\mu)$ values quoted have been brought to the scale $\mu = 0.77$ GeV. The standard values are $m_s/\hat{m} = 24$, $\sqrt{s_\pi^r} = 0.336$ GeV and $s_\ell = 0$. [1] Gasser and Leutwyler, Nucl. Phys. B250 (1985) 465; [2] Bijnens, Colangelo and Gasser, Nucl. Phys. B427 (1994) 427

	Main Fit	[1],[2]	$\mathcal{O}(p^4)$	fit 2	fit 3	fit 4	fit 5	fit 6	fit 7	fit 8	fit 9
$10^3\ L_1^r$	**0.52±0.23**	0.37±0.23	0.46	0.53	0.50	0.49	0.52	0.45	0.44	0.63	0.65
$10^3\ L_2^r$	**0.72±0.24**	1.35±0.23	1.49	0.73	0.67	0.74	0.80	0.51	1.04	0.73	0.85
$10^3\ L_3^r$	**−2.70±0.99**	−3.5 ±0.85	−3.18	−2.71	−2.58	−2.73	−2.75	−2.16	−2.95	−2.67	−3.27
$10^3\ L_5^r$	**0.65±0.12**	1.4 ±0.5	1.46	0.62	0.65	0.64	0.82	0.67	1.00	0.51	0.60
$10^3\ L_7^r$	**−0.26±0.15**	−0.4 ±0.2	−0.49	−0.20	−0.26	−0.26	−0.30	−0.26	−0.23	−0.25	−0.26
$10^3\ L_8^r$	**0.47±0.18**	0.9 ±0.3	1.08	0.35	0.47	0.47	0.58	0.46	0.52	0.44	0.48
changed			$\mathcal{O}(p^4)$	$m_s/\hat{m}$	$\sqrt{s_\ell}$	$\sqrt{s_\pi^r}$	$L_4^r; L_6^r$	V_μ, η'	μ	μ	$g(0)$
quantity				26	0.1	0.293	−0.3; −0.2	only	0.5	1.0	4.93
Unit					GeV	GeV	10^{-3}		GeV	GeV	

Figure 2: The F and G form-factor at $s_\ell = 0$ and $\cos\theta_\pi = 0$.The shaded band is the result of Rosselet et al. Shown are the full result for the absolute value, and the real part to lowest-order, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$. We also show the contribution from the $\mathcal{O}(p^6)$ Lagrangian; this is dominated by the vector contribution.

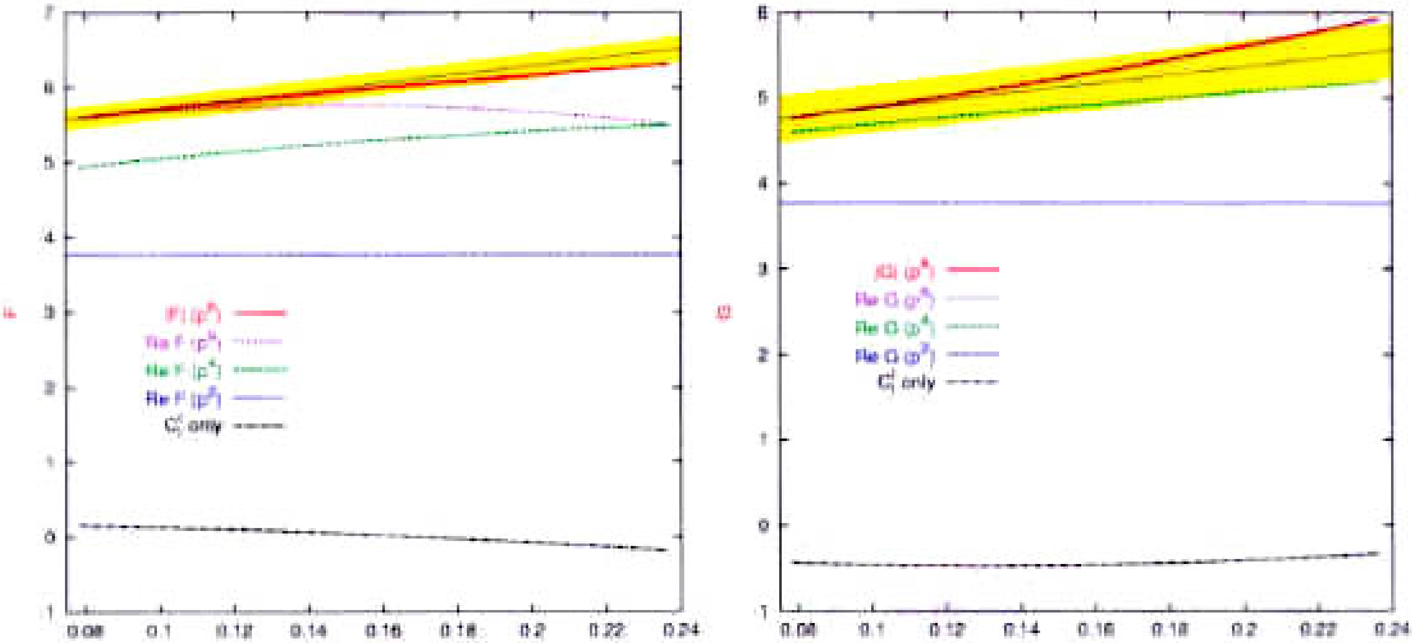
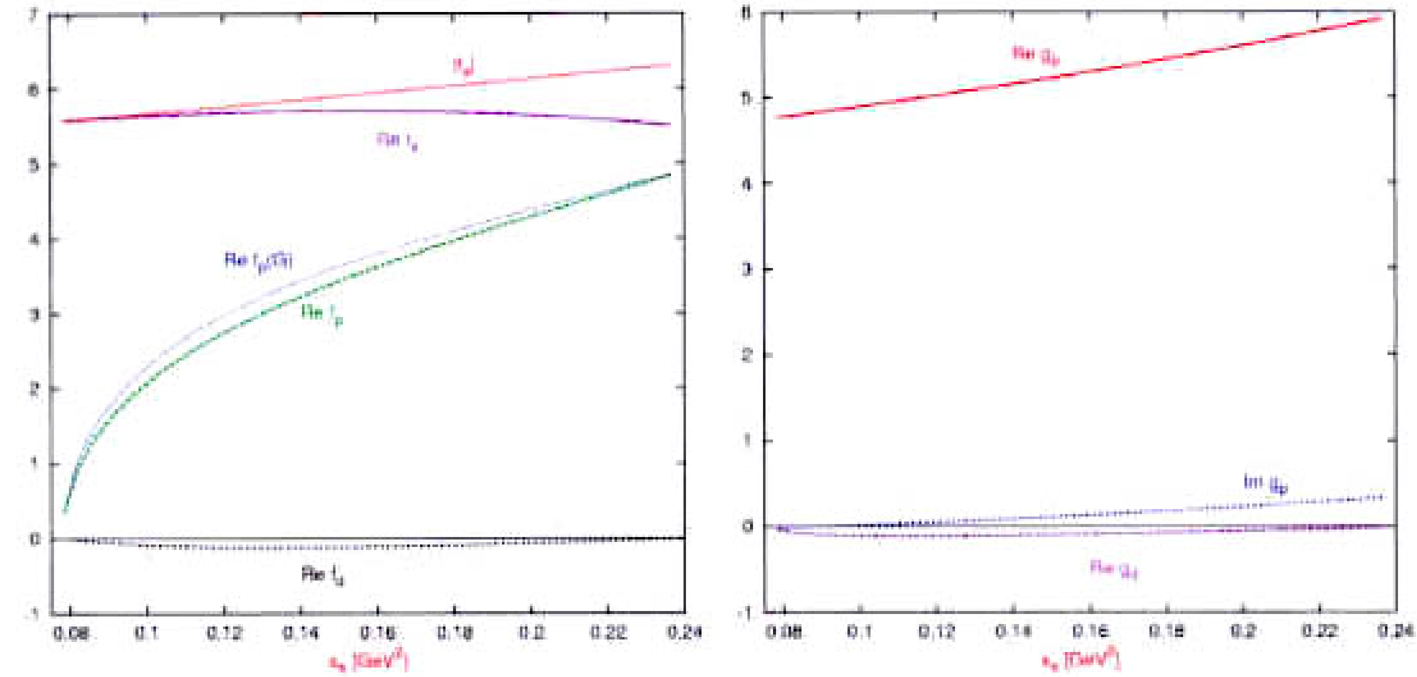


Figure 3: (a) The partial wave expansion of the combination $\tilde{F} = F + \sigma_\pi PL\cos\theta_\pi G/X$. Plotted are the absolute value of f_s and the real part of f_s, f_p and f_d. We also show the f_p contribution from the second term in $\tilde{F}$. (b) The partial wave expansion of G. Plotted are the real part of g_p and g_d and the imaginary part of g_p.



(a) (b)

π-π scattering parameters.

- π-π is known to 2-loops in two-flavour CHPT : ℓ_i
- K_{e4} : three-flavour CHPT: L_i

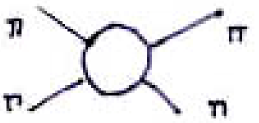
The matching $\ell_i(SU(2)) \leftrightarrow L_i(SU(3))$ is known to 1-loop, then

the ℓ_i can have $O(p^6)$ corrections.

A 2-loops matching has to be done!?

Table 2: Threshold parameters in units of m_{π^+}. Set I is $\bar{l}_1 = -1.7$ and $\bar{l}_2 = 6.1$ while Set II is $\bar{l}_1 = -1.5$ and $\bar{l}_2 = 4.5$. Both sets have $\bar{l}_3 = 2.9$ and $\bar{l}_4 = 4.3$. Experimental values are taken from Dumbrajs et al. Errors are from the 68% confidence level ranges only.

	Main Fit	$\mathcal{O}(p^2)$	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$ Set I	$\mathcal{O}(p^6)$ Set II	experiment
a_0^0	0.219 ± 0.005	0.159	0.203	0.222	0.212	0.26 ± 0.05
b_0^0	0.279 ± 0.011	0.182	0.254	0.282	0.256	0.25 ± 0.03
$-10a_0^2$	0.420 ± 0.010	0.454	0.423	0.420	0.450	0.28 ± 0.12
$-10b_0^2$	0.756 ± 0.021	0.908	0.755	0.729	0.81	0.82 ± 0.08
$10a_1^1$	0.378 ± 0.021	0.303	0.356	0.404	0.38	0.38 ± 0.02
$10^2 b_1^1$	0.59 ± 0.12	0	0.31	0.83	0.56	–
$10^2 a_2^0$	0.22 ± 0.04	0	0.16	0.28	input	0.17 ± 0.03
$-10^3 b_2^0$	0.32 ± 0.10	0	0.40	0.24	0.27	–
$10^3 a_2^2$	0.29 ± 0.10	0	0.32	0.24	input	0.13 ± 0.30
$-10^3 b_2^2$	0.36 ± 0.9	0	0.23	0.33	0.22	–
$10^4 a_3^1$	0.62 ± 0.11	0	0.20	0.65	0.48	–
$-10^4 b_3^1$	0.36 ± 0.02	0	0.15	0.38	0.37	–



Isospin amplitudes:

$$T^I(s,t) = 32\pi \sum_{I=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\, t_\ell^I(s).$$

$$t_\ell^I(s) = \sqrt{\frac{s}{s-4m_\pi^2}}\ \frac{1}{2i}\left[e^{2i\delta_\ell^I} - 1\right].$$

$$\mathrm{Re}\left[t_\ell^I(s)\right] = q^{2\ell}\left[a_\ell^I + q^2 b_\ell^I + O(q^4)\right].$$

$s = 4(m_\pi^2 + q^2)$

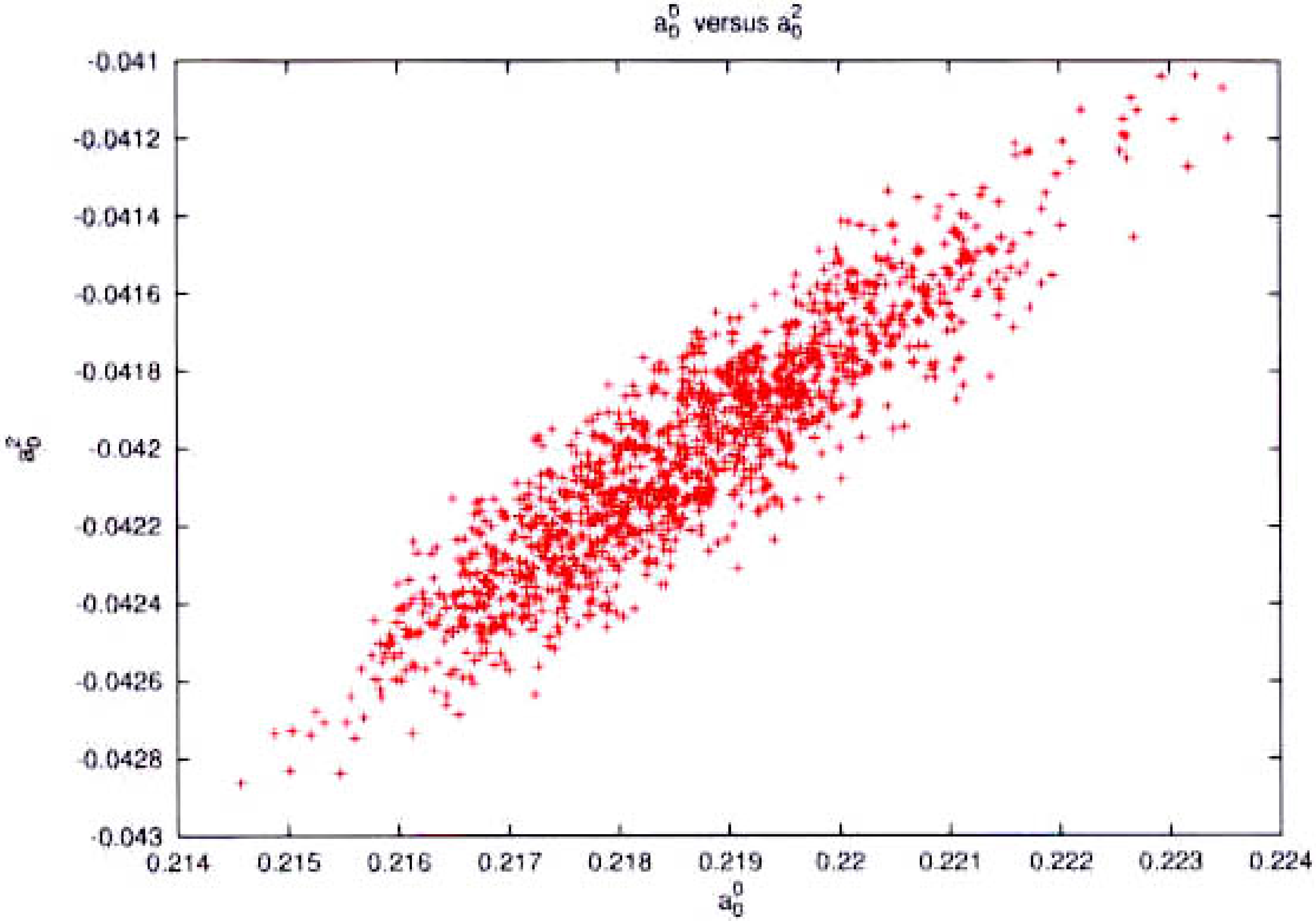


Figure 4: The prediction for the a_0^0 and a_0^2 scattering lengths for the set of L_i^r distributed according to the correlations of our main fit. Only points within the 68% confidence level domain are plotted.

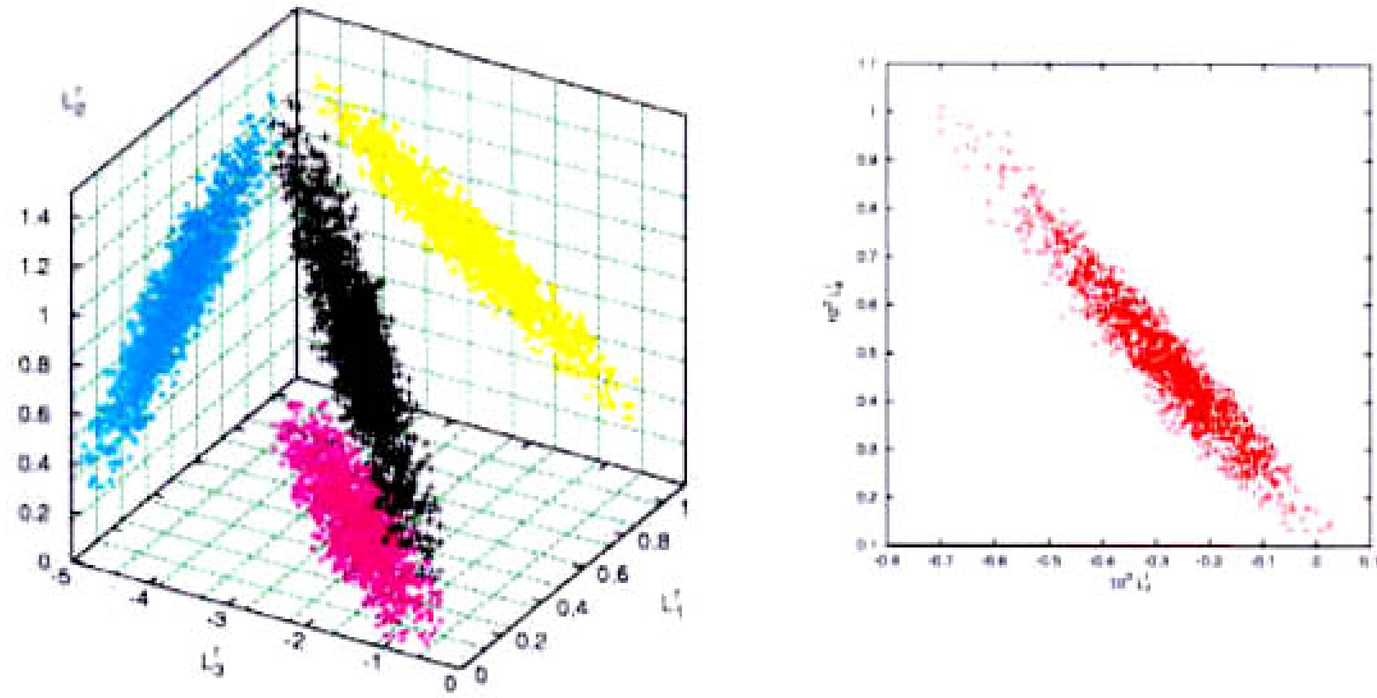
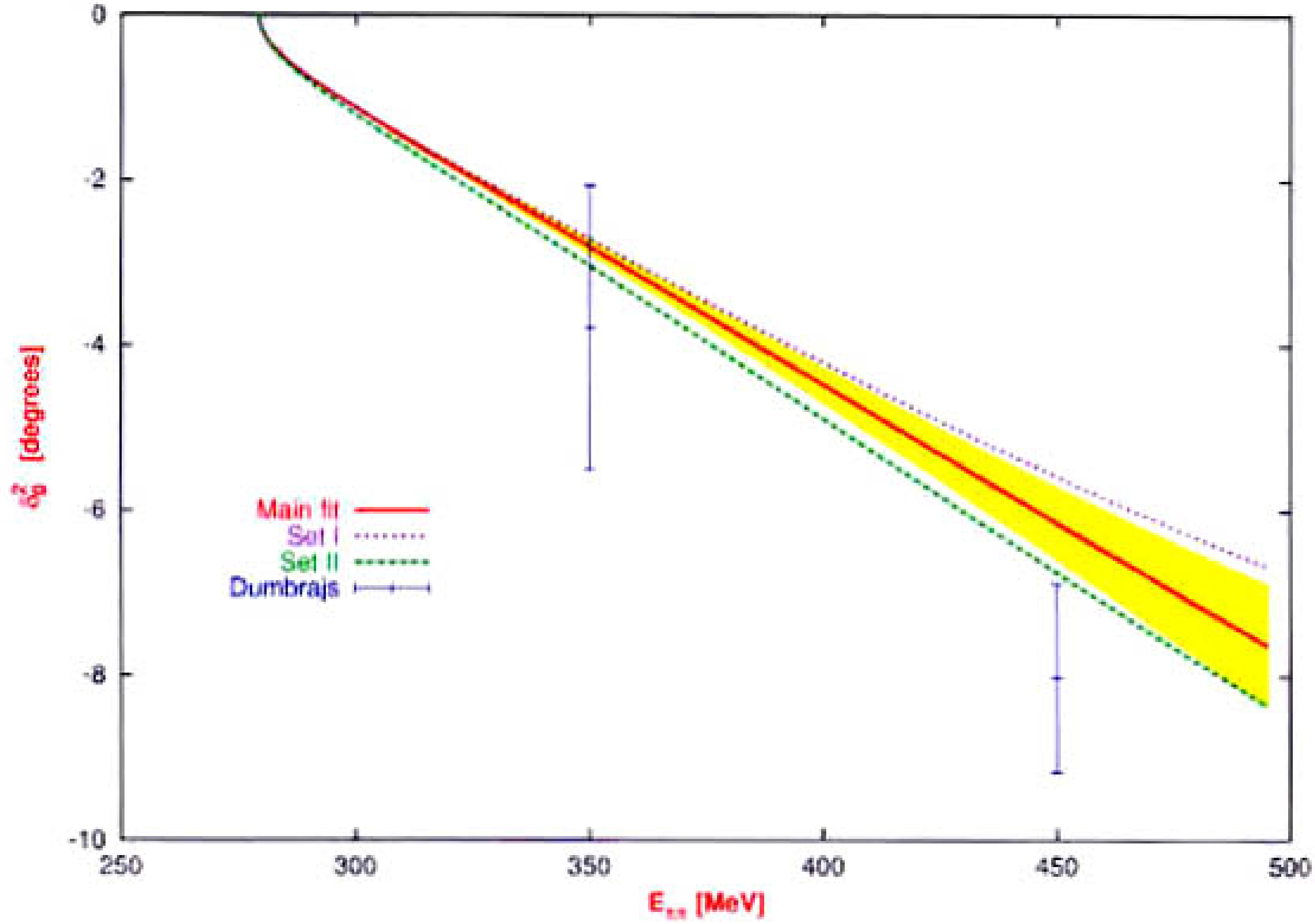
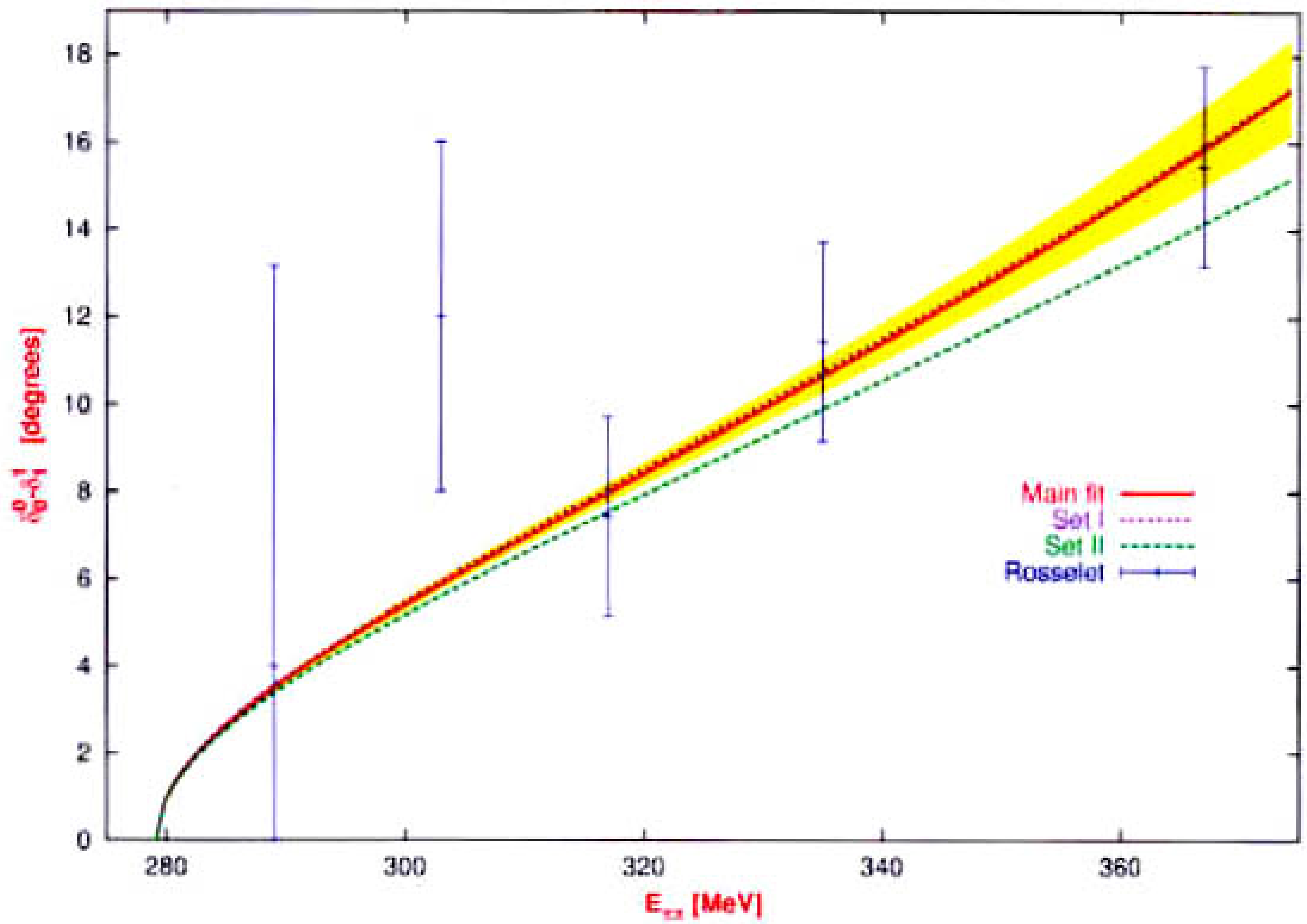


Figure 1: The sets of L_i^r within a 68% confidence level range of χ^2. In the first plot we show + the values of the first three L_i^r and $\times$ the projections on the coordinate planes. The second plot shows the L_7^r–L_8^r correlations.



Phase-shift δ_0^2 as function of the center of mass energy $E_{\pi\pi}$. Data points are taken from Hoogland et al. The shaded band indicates the 68% confidence level range.



Difference of phase-shifts $\delta_0^0 - \delta_1^1$ plotted as a function of the center of mass energy $E_{\pi\pi}$. Data points are taken from Rosselet et al. The shaded band is the 68% confidence level range.

masses and decay constants

$$
\begin{array}{lcccc}
 & & & p^4 & p^6 \\
m_\pi^2 / m_{\pi\,\mathrm{phys}}^2 & = & 0.746 & + \; 0.007 & + \; 0.247 \\
m_K^2 / m_{K\,\mathrm{phys}}^2 & = & 0.695 & + \; 0.019 & + \; 0.286 \\
m_\eta^2 / m_{\eta\,\mathrm{phys}}^2 & = & 0.742 & - \; 0.040 & + \; 0.298
\end{array}
$$

+ there is a cancellation to $O(p^4)$
  (contributions with different sign)
+ large (?) $O(p^6)$
  (contributions with the same sign)

$$
\begin{array}{lcccc}
F_\pi / F_0 & = & 1 & + \; 0.136 & - \; 0.076 \\
F_K / F_\pi & = & 1 & + \; 0.134 & + \; 0.086 \\
F_\eta / F_\pi & = & 1 & + \; 0.202 & + \; 0.069
\end{array}
$$

Why is $O(p^6)/O(p^4)$ so big?

- $m_s/\hat{m} = 20, 30.$
- resonance saturation.
- $L_4 \neq 0$, $L_6 \neq 0$.
- Is it an accident?

Summary

- NEW FIT FOR L_i. SOME OF THEM WITH APPRECIABLE CHANGE.
- THE FORM-FACTORS IN AGREEMENT WITH THE EXPERIMENT. Rosselet
- 'STRANGE' BEHAVIOUR (?) FOR THE MASSES.
- RECALCULATION OF THE π-π Scattering param. [WARNING: matching to one-loop!].