# Effective Field Theories

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# Outline

- Introduction
- Dimensional Analysis and Applications (Saves a lot of time)
- Fermi Interaction
- Loops
- Radiative Corrections and Operator Mixing
- Power Counting
- Decoupling of Heavy Particles

# Basic Ideas

Interactions at low energies are local. Interactions at a momentum scale $p$ can be described by interactions which appear local at distance scales $1/p$.

Dynamics at low energies (long distances) does not depend on the dynamics at high energies (short distances).

Low energy dynamics using an effective Lagrangian with only a few degrees of freedom.

Need to make this quantitative.

# Examples

Hydrogen atom energy levels — treated as a non-relativistic bound state problem:

Need to know the charge of the proton. Details of quark substructure, weak interactions, GUTS, . . . irrelevant.

More accurate calculation needs $m_p$, $\mu_p$

Charge radius, . . .

Weak interactions, . . .

# Effect of higher scales in Hydrogen

Typical scales:

Length scale $a_0 = 1/(m_e\alpha)$,
Time scale $1/R = 1/(m_e\alpha^2)$.

Weak interaction correction to the energy levels

$$\left(\frac{m_e\alpha}{M_W}\right)^2 \sim \left(5 \times 10^{-8}\right)^2$$

and is tiny.

However, if one is interested in atomic parity violation, weak interactions are the leading contribution.

Effective theory in terms of some low energy parameters.

In some cases, one can compute these from a more fundamental theory (typically, if it is weakly coupled).

The heavy quark Lagrangian in QCD can be computed in powers of $\alpha_s(m_Q)$ and $1/m_Q$.

Fermi theory from the electroweak theory

Chiral perturbation theory, one has parameters that are fit to experiment.

Standard model — don't know the more fundamental theory

# Dependence on high energy parameters

High energy dynamics irrelevant:

H energy levels do not depend on $m_t$ — but this depends on what is held fixed as $m_t$ is varied.

Usually, one takes low energy parameters such as $m_p$, $m_e$, $\alpha$ from low energy experiments, and then uses them in the Schrödinger equation.

But instead, hold high energy parameters such as $\alpha(\mu)$ and $\alpha_s(\mu)$ fixed at $\mu \gg m_t$.

$$m_t \frac{\mathrm{d}}{\mathrm{d}m_t}\left(\frac{1}{\alpha}\right) = -\frac{1}{3\pi}$$

The proton mass also depends on the top quark mass,

$$m_p \;\propto\; m_t^{2/27}$$

There are other constraints from the symmetry of the high energy theory:

For example, the chiral lagrangian preserves $C$, $P$ and $CP$ because QCD does.

More interesting case: Non-relativistic quantum mechanics satisfies the spin-statistics theorem because of causality in QED.

EFT powerful because one can compute low energy dynamics without knowledge of the high energy Lagrangian.

Corrections typically fall off at least as fast as $(p/M)^2$.

Bad if you want to find new physics!

# Dimensional Analysis

Effective Lagrangian (neglect topological terms)

$$L = \sum c_i O_i = \sum L_D$$

is a sum of local, gauge and Lorentz invariant operators.
The functional integral has

$$e^{iS}$$

so $S$ is dimensionless.

Kinetic terms:

$$S = \int \mathrm{d}^d x \;\bar\psi\; i \not{D}\; \psi, \qquad S = \int \mathrm{d}^d x \;\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$$

so

$$0 = -d + 2\,[\psi] + 1, \qquad 0 = -d + 2\,[\phi] + 2$$

Dimensions given by

$$[\phi] = (d{-}2)/2, \qquad [\psi] = (d{-}1)/2, \qquad [D] = 1, \qquad [gA_\mu] = 1$$

Field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ so $A_\mu$ has the same dimension as a scalar field.

$$[g] = 1 - (d-2)/2 = (4-d)/2$$

In $d = 4$,

$$[\phi] = 1, \qquad [\psi] = 3/2, \qquad [A_\mu] = 1, \qquad [D_\mu] = 1, \qquad [g] = 0$$

Only Lorentz invariant renormalizable interactions (with $D \leq 4$) are

$$\begin{aligned}
D = 0: &\quad 1 \\
D = 1: &\quad \phi \\
D = 2: &\quad \phi^2 \\
D = 3: &\quad \phi^3, \bar{\psi}\psi \\
D = 4: &\quad \phi\bar{\psi}\psi, \phi^4
\end{aligned}$$

and kinetic terms which include gauge interactions.

Renormalizable interactions have coefficients with mass dimension $\geq 0$.
In $d = 2$,

$$[\phi] = 0, \qquad [\psi] = 1/2, \qquad [A_\mu] = 0, \qquad [D_\mu] = 1, \qquad [g] = 1$$

so an arbitrary potential $V(\phi)$ is renormalizable. Also $\left(\bar{\psi}\psi\right)^2$ is renormalizable. In $d = 6$,

$$[\phi] = 2, \qquad [\psi] = 5/2, \qquad [A_\mu] = 2, \qquad [D] = 1, \qquad [g] = -1$$

Only allowed interaction is $\phi^3$.

# What Fields to use for EFT?

The two dimensional Thirring model with a fundamental fermion field

$$L = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}g\left(\bar{\psi}\gamma^{\mu}\psi\right)^{2},$$

is dual to the sine-Gordon model with a fundamental scalar field

$$L = \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi + \frac{\alpha}{\beta^{2}}\cos\beta\phi,$$

where the coupling constants $g$ and $\beta$ are related by

$$\frac{\beta^{2}}{4\pi} = \frac{1}{1+g/\pi}.$$

The fermion of the Thirring model is the sine-Gordon soliton, and the boson of the sine-Gordon model is a fermion-antifermion bound state in the Thirring model.

Strongly coupled sine-Gordon theory with $\beta^2 \sim 4\pi$ is better described as a weakly coupled theory of the soliton $\psi$.

Low energy QCD described in terms of meson fields.

Effective Lagrangian:

$$L_D = \frac{O_D}{M^{D-d}}$$

so in $d = 4$,

$$L_{\text{eft}} = L_{D \leq 4} + \frac{O_5}{M} + \frac{O_6}{M^2} + \ldots$$

An infinite number of terms (and parameters)

# Power Counting

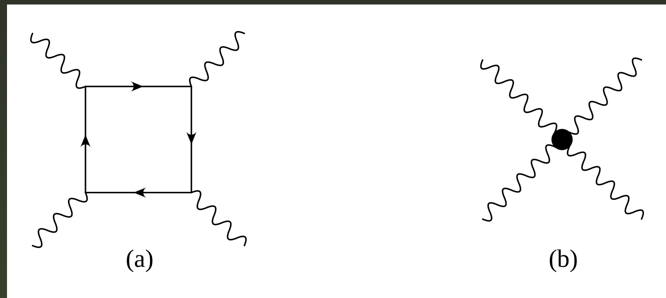
If one works at some typical momentum scale $p$, and neglects terms of dimension $D$ and higher, then the error in the amplitudes is of order

$$\left(\frac{p}{M}\right)^{D-4}$$

A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy.

Usual renormalizable case given by taking $M \to \infty$.

# Photon-Photon Scattering




$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{m_e^4}\left[c_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + c_2\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right].$$

(Terms with only three field strengths are forbidden by charge conjugation symmetry.)

$e^4$ from vertices, and $1/16\pi^2$ from the loop.

An explicit computation gives

$$c_1 = \frac{1}{90}, \qquad c_2 = \frac{7}{90}.$$

Scattering amplitude

$$A \sim \frac{\alpha^2 \omega^4}{m_e^4}$$

and

$$\sigma \sim \left( \frac{\alpha^2 \omega^4}{m_e^4} \right)^2 \frac{1}{\omega^2} \frac{1}{16\pi} \sim \frac{\alpha^4 \omega^6}{16\pi m_e^8} \times \frac{15568}{22275}$$

Even if one did not know the details of the electron photon interaction (e.g. the charge of the electron), one would still get

$$\sigma \propto \frac{\omega^6}{m_e^8}$$

If instead of $\gamma$, we considered $\phi$ scattering, the same analysis would give

$$\sigma \propto \frac{1}{\omega^2}$$

and $\psi$ scattering would give

$$\sigma \propto \frac{\omega^2}{M^4}$$

# Rayleigh Scattering

Scattering of light from atoms

$$L = \psi^\dagger \left( i\partial_t - \frac{p^2}{2M} \right) \psi + a_0^3 \ \psi^\dagger \psi \left( c_1 E^2 + c_2 B^2 \right)$$

Atoms are neutral, so no gauge interactions in the kinetic term

$$A \sim c_i a_0^3 \omega^2$$

$$\sigma \propto a_0^6 \ \omega^4 .$$

# Other examples

Proton decay: Operator is dimension six $qqql$

$$\Gamma \propto \frac{m_p^5}{M^4}$$

SUSY can have dimension 5 operators

$$\Gamma \propto \frac{m_p^3}{M^2}$$

Rate is larger by

$$\left(\frac{M}{m_p}\right)^2 \sim 10^{30}$$

# Other examples

New physics in $K\bar{K}$ mixing: Dimension six $ss\bar{d}\bar{d}$

$$\Delta M \propto \frac{1}{M^2}$$

In the standard model

$$\Delta M \propto \frac{m_c^2}{M_W^4}$$

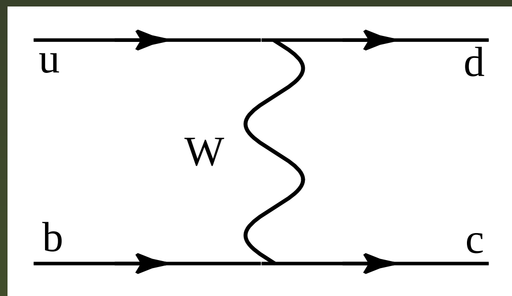
Find

$$M > \frac{M_W^2}{m_c} \sim 5 \text{ TeV}$$

Put in $16\pi^2$, CKM, get $1000$ TeV.

# Low energy weak interactions

$$-\frac{ig}{\sqrt{2}}\; V_{ij}\; \bar{q}_i\, \gamma^\mu\, P_L\, q_j\,,$$




$$A = \left(\frac{ig}{\sqrt{2}}\right)^2 V_{cb} V_{ud}^* \;\; (\bar{c}\,\gamma^\mu\, P_L\, b)\, \left(\bar{d}\,\gamma^\nu\, P_L\, u\right) \left(\frac{-ig_{\mu\nu}}{p^2 - M_W^2}\right),$$

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \dots\right),$$

and retaining only a *finite* number of terms.

$$A = \frac{i}{M_W^2}\left(\frac{ig}{\sqrt{2}}\right)^2 V_{cb}V_{ud}^* \;\left(\bar{c}\,\gamma^\mu\, P_L\, b\right)\left(\bar{d}\,\gamma_\mu\, P_L\, u\right) + \mathcal{O}\left(\frac{1}{M_W^4}\right).$$

$$L = -\frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^* \;\left(\bar{c}\,\gamma^\mu\, P_L\, b\right)\left(\bar{d}\,\gamma_\mu\, P_L\, u\right) + \mathcal{O}\left(\frac{1}{M_W^4}\right),$$

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2}.$$

Effective Lagrangian for $\mu$ decay

$$L = -\frac{4G_F}{\sqrt{2}} \; (\bar{e}\,\gamma^\mu\, P_L\,\nu_e)\,(\bar{\nu}_\mu\,\gamma^\mu\, P_L\,\mu) + \mathcal{O}\left(\frac{1}{M_W^4}\right),$$

Gives the standard result for the muon lifetime at lowest order,

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}.$$

The advantages of EFT show up in higher order calculations

EFT: a theory, need to be able to compute observable quantities which are finite and unambiguous.

Need a renormalization procedure. Also, note that the EFT is a different theory from the full theory, and has a different divergence structure.

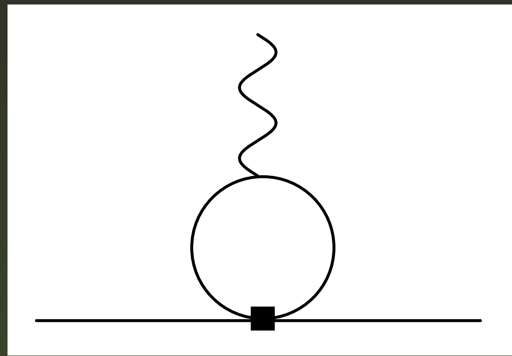
$$\int \mathrm{d}^4p \frac{1}{(p^2 - M_W^2)^2(p^2 - m^2)} \to \int \mathrm{d}^4p \frac{1}{M_W^4(p^2 - m^2)}$$

Power counting scheme: An expansion in powers of $p/M$.

EFT useful to a certain order in $p/M$. If one needs the complete expression, better to use the full theory.

Renormalization procedure needs to be consistent with the power counting scheme.

# Loops



Gives a contribution

$$I \sim \frac{1}{M_W^2} \Lambda^2 \sim \mathcal{O}(1)$$

Similarly, a dimension eight operator has vertex $k^2/M_W^4$, and gives a contribution

$$I' \sim \frac{1}{M_W^4} \int d^4k \; \frac{1}{k^2} k^2 \sim \frac{\Lambda^4}{M_W^4} \sim \mathcal{O}(1)$$

Would need to know the entire effective Lagrangian, since all terms are equally important. The reason for this breakdown is using a cutoff procedure with a dimensionful parameter $\Lambda$.

More generally, need to make sure that dimensionful parameters at the high scale do not occur in the numerator in Feynman diagrams.

In doing weak interactions, one should not have $M_G$ or $M_P$ appear in the numerator.

Need to use a mass independent subtraction scheme such as $\overline{MS}$. In this case, $\mu$ can only occur in logarithms, so

$$
\begin{aligned}
I &= \frac{1}{M_W^2} \int d^4k \; \frac{1}{k^2} \sim \frac{m^2}{M_W^2} \log \mu, \\
I' &= \frac{1}{M_W^4} \int d^4k \; \frac{1}{k^2} k^2 \sim \frac{m^4}{M_W^4} \log \mu,
\end{aligned}
$$

Expanding $1/(p^2 - M_W^2)$ in a power series ensures that there is no pole for $p \sim M_W$, and so $M_W$ cannot appear in the numerator.

Compare

$$\int_0^\infty \mathrm{d}k^2 \frac{(k^2)^a}{(k^2+m_1^2)(k^2+m_2^2)} = \frac{\pi}{\sin \pi a} \frac{(m_1^2)^a - (m_2^2)^a}{m_1^2 - m_2^2}$$

with first expanding:

$$\int_0^\infty \mathrm{d}k^2 \frac{(k^2)^a}{(k^2+m_1^2)} \left[ \frac{1}{k^2} - \frac{m_2^2}{k^4} + \dots \right] = \frac{\pi}{\sin \pi a} \frac{(m_1^2)^a}{m_1^2 - m_2^2}$$

The $(m_2^2)^a$ is non-analytic at the origin when $a \neq 0$.

Pole at $k^2 = -m_2^2$ does not contribute to the integral.

These are terms in the full theory that must be reproduced in the effective theory.

If $a \to 1 + \epsilon$, then the full theory answer is

$$-\frac{1}{\epsilon} - \frac{m_1^2 \ln m_1^2/\mu^2 - m_2^2 \ln m_2^2/\mu^2}{m_1^2 - m_2^2}$$

and the effective field theory answer is

$$-\frac{1}{\epsilon}\frac{m_1^2}{m_1^2 - m_2^2} - \frac{m_1^2 \ln m_1^2/\mu^2}{m_1^2 - m_2^2}$$

Difference of $1/\epsilon$ terms is given by different counterterms in the full and effective theories. Anomalous dimensions are different.

Difference of finite parts is

$$-\frac{m_2^2 \ln m_2^2/\mu^2}{m_1^2 - m_2^2}$$

Taken care of by matching corrections in the effective theory.

Manifest power counting in $p/M$.

Loop graphs consistent with the power counting, since one can never get any $M$'s in the numerator. If the vertices have $1/M^a$, $1/M^b$, etc. then any amplitude (including loops) will have

$$\frac{1}{M^a}\frac{1}{M^b}\ldots = \frac{1}{M^{a+b+\ldots}}$$

Correct dimensions due to factors of the low scale in the numerator.

Only a finite number of terms to any given order.

To order $1/M$: $L_5$ at tree level

To order $1/M^2$: $L_5$ and $L_6$ at tree level,
or loop graphs with at most two insertions of $L_5$.

General power counting result: you can count the powers of $M$. Sometimes, this is written in a fancier form.

$$p^r, \qquad r = \sum_k n_k (k-4)$$

There can also be logarithmic dependence on $M$. So dimension 5 operators can depend on

$$\frac{1}{M} \ln \frac{M}{m}$$

The log's are calculable using the RG.
Dimension four operators can depend on

$$\ln \frac{M}{m}$$

This is the dependence of $\alpha$ and $m_p$ on $m_t$ mentioned earlier.

Renormalizable interaction: like having $M$ in the numerator. Iterations can then only produce positive powers of $M$, so one gets only operators with dimension $\leq d$. There are only a finite number of these.

If one has operators with $D < d$ and $D > d$, then expect $M$ in numerator and denominators. For example $M^2\phi^2$. To have $m \ll M$ is the hierarchy problem.

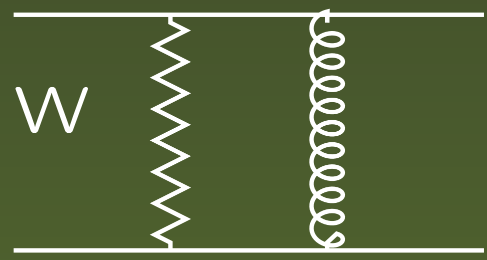
Not a problem for fermion masses because of chiral symmetry. $(\bar{\psi}_L\psi_R)$

# Radiative Corrections

Look at radiative corrections, which can generate

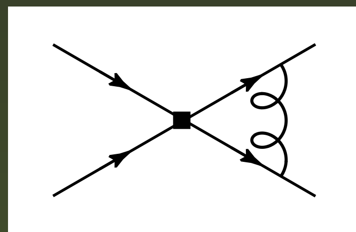
$$\left(\alpha_s \ln \frac{M_W}{m_b}\right)^n$$

which need to be summed.
In the full theory, corrections to Fermi amplitude are box type graphs,



Need to compute complicated graphs (which are finite).

In the EFT, compute



A much simpler, UV divergent diagram that gives an anomalous dimension to the Fermi interaction.

# Composite Operators

How do you renormalize composite operators?

Treat them as terms in the Lagrangian, by adding them as $c_i O_i$. Then compute graphs and add counterterms to subtract the $1/\epsilon$ poles. One then computes RG equations for the coefficients by requiring that

$$\mu \frac{\mathrm{d}c_i^{(0)}}{\mathrm{d}\mu} = 0$$

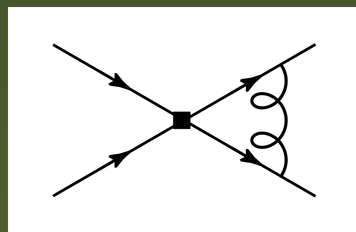
# Operator Mixing

$$L = -c(\mu)\frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^*\ \left(\bar{c}\,\gamma^\mu\,P_L\,b\right)\left(\bar{d}\,\gamma_\mu\,P_L\,u\right)$$

Introduce

$$\begin{aligned} O_1 &= \left(\bar{c}^\alpha\,\gamma^\mu\,P_L\,b_\alpha\right)\left(\bar{d}^\beta\,\gamma_\mu\,P_L\,u_\beta\right) \\ O_2 &= \left(\bar{c}^\alpha\,\gamma^\mu\,P_L\,b_\beta\right)\left(\bar{d}^\beta\,\gamma_\mu\,P_L\,u_\alpha\right) \end{aligned}$$

We need $O_2$ because



where the vertex is $O_1$ gives a graph with the structure of $O_2$.

$$\mu \frac{\mathrm{d} c_i}{\mathrm{d}\mu} = \gamma_{ji} c_j$$

$$\gamma = \frac{g^2}{8\pi^2} \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}$$

$$\mu \frac{\mathrm{d} O_i}{\mathrm{d}\mu} = -\gamma_{ij} O_j$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \sum_i c_i O_i = 0$$

e.g. $$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} m\bar{\psi}\psi = 0$$

Use

$$O_\pm = O_1 \pm O_2$$

Then

$$\mu \frac{\mathrm{d}c_\pm}{\mathrm{d}\mu} = \gamma_\pm c_\pm$$

$$\gamma_+ = \frac{g^2}{4\pi^2}, \qquad \gamma_- = -\frac{g^2}{2\pi^2}$$

The operators are in different representations of flavor.

$$c_\pm = \frac{1}{2}\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{a_\pm}$$

$$a_+ = \frac{6}{33-2n}, \qquad a_- = -\frac{12}{33-2n}$$

No expansion parameter.

$$c_+(m_b) = 0.42, \qquad c_-(m_b) = 0.7$$

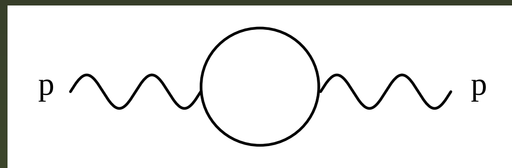
$$c_\pm O_\pm \;=\; \left[c_+ + c_-\right] O_1 + \left[c_+ - c_-\right] O_2$$

Induce new local operators due to RG running. Difficult to see this by doing the full theory calculation.

In this particular case, the full theory computation has no $\ln$ term in the total decay rate. Expanding $c_\pm$ shows that the single log cancels, but the higher order RG terms do not.

# Decoupling

Heavy particles decouple from low energy physics. Not explicit in a mass independent scheme such as $\overline{MS}$.




$$i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)$$
$$\left[\frac{1}{6\epsilon} - \int_0^1 dx\ x(1-x)\ \log\frac{m^2 - p^2 x(1-x)}{\mu^2}\right],$$

Subtracts the value of the graph at a Euclidean momentum point $p^2 = -M^2$,

$$-i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[\int_0^1 dx\ x(1-x)\ \log\frac{m^2 - p^2 x(1-x)}{m^2 + M^2 x(1-x)}\right].$$

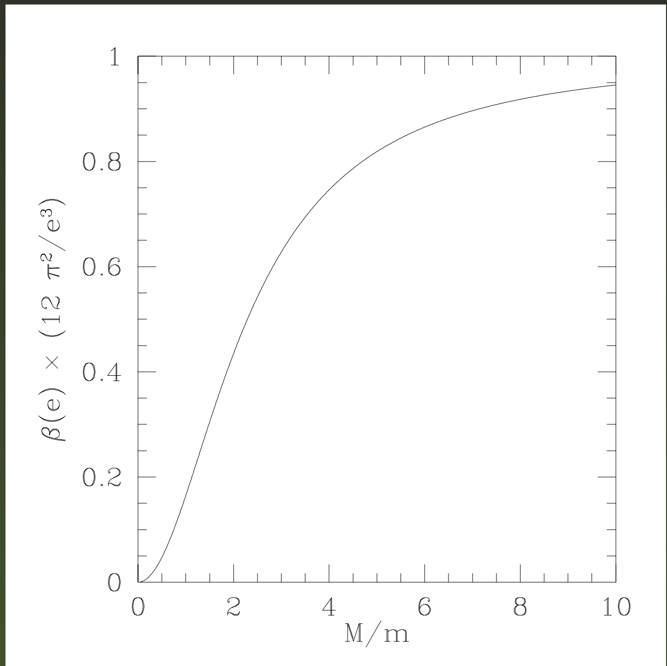
$$\begin{aligned}\beta(e) &= -\tfrac{e}{2} M \tfrac{d}{dM}\tfrac{e^2}{2\pi^2}\left[\int_0^1 dx\ x(1-x)\ \log\tfrac{m^2 - p^2 x(1-x)}{m^2 + M^2 x(1-x)}\right] \\ &= \tfrac{e^3}{2\pi^2}\int_0^1 dx\ x(1-x)\ \tfrac{M^2 x(1-x)}{m^2 + M^2 x(1-x)}.\end{aligned}$$

$m \ll M$:

$$\beta(e) \approx \frac{e^3}{2\pi^2} \int_0^1 dx \;\, x(1-x) = \frac{e^3}{12\pi^2}.$$

$M \ll m$:

$$\beta(e) \approx \frac{e^3}{2\pi^2} \int_0^1 dx \;\, x(1-x) \frac{M^2 x(1-x)}{m^2} = \frac{e^3}{60\pi^2} \frac{M^2}{m^2}.$$


1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
M/m
$\beta(e) \times (12\ \pi^2/e^3)$

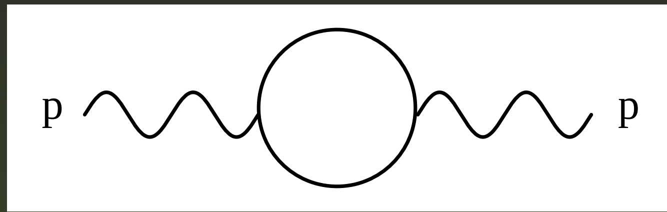
$$-i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[\int_0^1 dx\ \ x(1-x)\log\frac{m^2-p^2x(1-x)}{\mu^2}\right].$$

$$\begin{aligned}\beta\,(e) &= -\frac{e}{2}\mu\frac{d}{d\mu}\frac{e^2}{2\pi^2}\left[\int_0^1 dx\ \ x(1-x)\log\frac{m^2-p^2x(1-x)}{\mu^2}\right]\\ &= \frac{e^3}{2\pi^2}\int_0^1 dx\ \ x(1-x) = \frac{e^3}{12\pi^2},\end{aligned}$$

$$-i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[\int_0^1 dx\ \ x(1-x)\log\frac{m^2}{\mu^2}\right],$$

Large logs cancel the wrong $\beta$-function contributions.

Explicitly integrate out heavy particles.




Present in theory above $m$, but not in theory below $m$. Assume that $p \ll m$, so

$$
\begin{aligned}
& \int_0^1 dx\ \ x(1-x)\ \ \log \frac{m^2 - p^2 x(1-x)}{\mu^2} \\
= \ & \int_0^1 dx\ \ x(1-x)\ \ \left[\log \frac{m^2}{\mu^2} + \frac{p^2 x(1-x)}{m^2} + \ldots\right] \\
= \ & \frac{1}{6} \log \frac{m^2}{\mu^2} + \frac{p^2}{30 m^2} + \ldots
\end{aligned}
$$

So in theory above $m$:

$$i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[\frac{1}{6\epsilon} - \frac{1}{6}\log\frac{m^2}{\mu^2} - \frac{p^2}{30m^2} + \ldots\right] + c.t.$$

Counterterm cancels $1/\epsilon$ term (and also contributes to the $\beta$ function).

$$i\frac{e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[-\frac{1}{6}\log\frac{m^2}{\mu^2} - \frac{p^2}{30m^2} + \ldots\right]$$

The log term gives

$$Z = 1 - \frac{e^2}{12\pi^2} \log \frac{m^2}{\mu^2}$$

so that in the effective theory,

$$\frac{1}{e_L^2(\mu)} = \frac{1}{e_H^2(\mu)} \left[ 1 - \frac{e_H^2(\mu)}{12\pi^2} \log \frac{m^2}{\mu^2} \right]$$

One usually integrates out heavy fermions at $\mu = m$, so that (at one loop), the coupling constant has no matching correction.

The $p^2$ term gives the dimension six operator

$$-\frac{1}{4}\frac{e^2}{2\pi^2}\frac{1}{30m^2}F_{\mu\nu}\partial^2 F^{\mu\nu}$$

and so on.

Even if the structure of the graphs is the same in the full and effective theories, one still needs to compute the difference to compute possible matching corrections, because the integrals need not have the same value.

This difference is independent of IR physics, since both theories have the same IR behavior.

What have we gained?

A much simpler theory to use to discuss the IR physics.

Calculations divided up into pieces, each of which only involves a single scale.

Can sum large logarithms and use RG improved perturbation theory.