The CKM Paradigm and Beyond

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Literature

Buchalla, AJB, Lautenbacher

Rev. Mod. Phys. 68 (1996) 1125

AJB, Fleischer

in Heavy Flavours II (World Scientific) (1998) (hep-ph / 9704376)

AJB

Les Houches Lectures (1997) (hep-ph / 9806471)
Erice Lectures (2000) (hep-ph / 0101336)

Y. Nir

SLAC Summer Institute on Particle Physics (hep-ph / 9911321)
Scottish Universities Summer School (hep-ph / 0109090)

The BABAR Physics Book

B-Physics at the LHC (hep-ph / 0003238)

Books: Branco, Lavoura, Silva;
Bigi, Sanda

B Physics at the Tevatron (Run II and Beyond) (hep-ph/0201071)

Fleischer:

Physics Reports (hep-ph/0207108)

1.

Grand View

The Standard Model

Quarks

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad \begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \end{matrix} \qquad \begin{matrix} +2/3 \\ -1/3 \end{matrix}$$

+ Leptons

Fundamental Forces

Gauge Theory : $SU(3) \otimes SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{QED}$

QCD	
Strong Interactions	Electroweak Interactions
(Gluons)	$(W^{\pm}, Z^0, \gamma)$

Mesons

$$K^0 = (d\bar{s}) \quad K^+ = (u\bar{s}) \quad K^- = (\bar{u}s)$$

$$\pi^+ = (u\bar{d}) \quad \pi^0 = (\bar{u}u - \bar{d}d)/\sqrt{2} \quad \pi^- = (\bar{u}d)$$

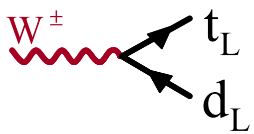
$$B_d^0 = (d\bar{b}) \quad \bar{B}_d^0 = (\bar{d}b) \quad B^+ = (u\bar{b})$$

$$B_s^0 = (s\bar{b}) \quad \bar{B}_s^0 = (\bar{s}b) \quad B^- = (\bar{u}b)$$

$q\bar{q}$ Bound States

Four Basic Properties in the SM

1. Charged Current Interactions only between left-handed Quarks



$$\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)\cdot V_{td}$$

2. Quark Mixing

$\{$ Weak Eigenstates $\} \neq \{$Mass Eigenstates $\}$

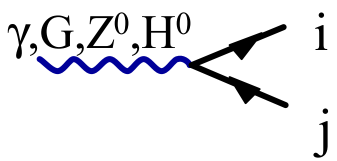
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{ub} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

(Weak Eigenstates) (Unitarity CKM-Matrix) (Mass Eigenstates)

3. GIM Mechanism

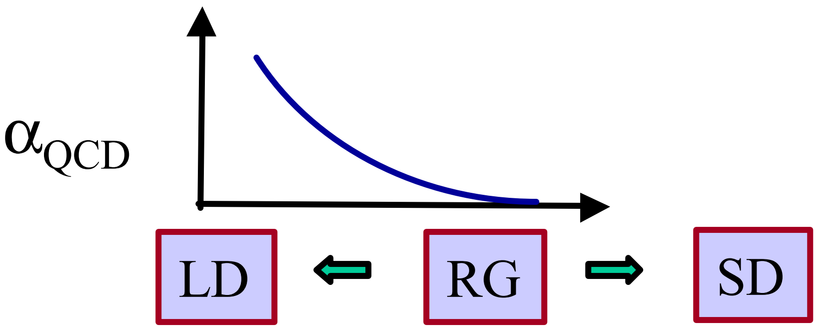
Natural suppression of FCNC

$\{$

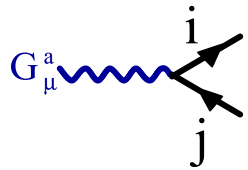


$= 0 \}$ ⇒ {Loop Induced Decays, sensitive to short distance flavour dynamics}

4. Asymptotic Freedom



Q



$g_{QCD}\, T^a_{ij} \gamma_\mu$

$$\alpha_{QCD} = \frac{g^2_{QCD}}{4\pi}$$

$$\alpha_{QCD}(Q) = \frac{4\pi}{\beta_0 \ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)}\left[1 - \frac{\beta_1}{\beta_0^2}\,\frac{\ln\,\ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)}{\ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)} + \ldots\right]$$

$$\Lambda^{(5)}_{\overline{MS}} = 225 \pm 40 \text{ MeV} \qquad \alpha^{(5)}_{\overline{MS}}(M_Z) = 0.118 \pm 0.003$$

SD = Short Distances (Perturbation Theory)

RG = Renormalization Group Effects

LD = Long Distances (Non-Perturbative Physics)

Kobayashi-Maskawa Picture of CP Violation

CP Violation arises from a single phase δ
in $W^{\pm}$ interactions of Quarks

ud $c_{12}c_{13}$	us $s_{12}c_{13}$	ub $s_{13}e^{-i\delta}$
cd $-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}$	cs $c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}$	cb $s_{23}c_{13}$
td $s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}$	ts $-s_{23}c_{12}-s_{12}s_{23}s_{13}e^{i\delta}$	tb $c_{23}c_{13}$

Four Parameters: $(\theta_{12} \approx \theta_{cabibbo})$

$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta$

$c_{ij} \equiv \cos\theta_{ij}$; $s_{ij} \equiv \sin\theta_{ij}$; $c_{13} \cong c_{23} \cong 1$

Wolfenstein Parametrization

Parameters: λ, A, ρ, η

	d	s	b
u	$1-\frac{\lambda^2}{2}$	λ	V_{ub}
c	$-\lambda$	$1-\frac{\lambda^2}{2}$	V_{cb}
t	V_{td}	V_{ts}	1

$\lambda=0.22$

$V_{us}=\lambda+0(\lambda^7)$

$V_{cb}=A\lambda^2+0(\lambda^8)$

$V_{ts}=-A\lambda^2+0(\lambda^4)$

$(A=0.83\pm0.02)$

$$V_{ub} \equiv A\lambda^3(\rho - i\eta) \qquad V_{td} = A\lambda^3(1-\bar{\rho}-i\bar{\eta})$$

$$\bar{\rho} = \rho\left(1-\frac{\lambda^2}{2}\right) \qquad \bar{\eta} = \eta\left(1-\frac{\lambda^2}{2}\right)$$

(AJB, Lautenbacher, Ostermaier, 94)

$$R_b \equiv \sqrt{\bar{\rho}^2+\bar{\eta}^2} = \left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

Circle around $(\bar{\rho},\bar{\eta}) = (0,0)$

$$R_t \equiv \sqrt{(1-\bar{\rho})^2+\bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

Circle around $(\bar{\rho},\bar{\eta}) = (1,0)$

Particular Definition of λ, A, ρ, η

$$s_{12} \equiv \lambda$$
$$s_{23} \equiv A\,\lambda^2$$
$$s_{13}e^{i\delta} \equiv A\,\lambda^3(\rho - i\eta)$$

BLO: Phys.Rev. (94); (Schmidtler, Schubert)

At $0(\lambda^5)$ equivalent to (Branco, Lavoura, 88)

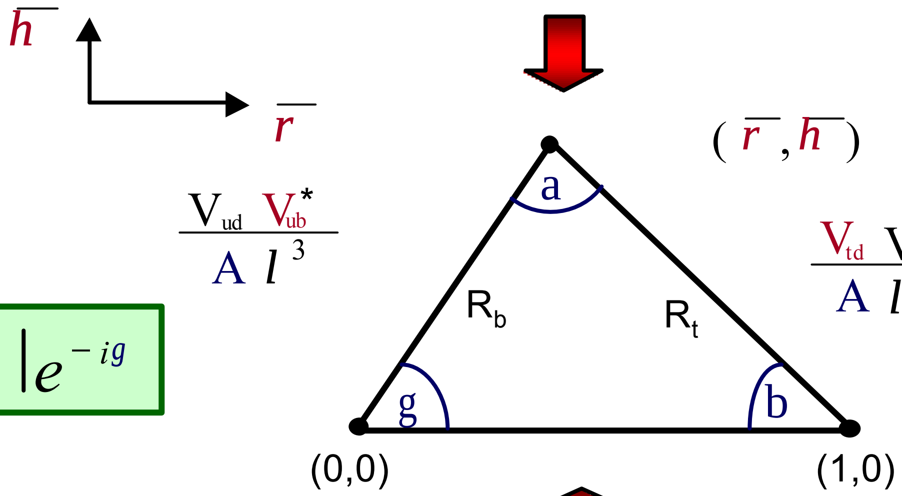
Basic Virtues of this Definition:

$$V_{us} = \lambda + 0\left(\lambda^7\right)$$
$$V_{ub} = A\lambda^3\left(\rho - i\eta\right)$$
$$V_{cb} = A\lambda^2 + 0\left(\lambda^8\right)$$
$$V_{td} = A\lambda^3\left(1 - \bar{\rho} - i\bar{\eta}\right)$$

The apex of UT given by $\left(\bar{\rho}, \bar{\eta}\right)$ (BLO)

Unitarity Triangle

$$V_{ud} V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td} V_{tb}^{*} = 0$$

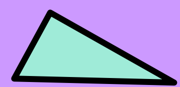


$\bar{h}$ ¹ 0 Signals CP Violation

$$V_{ub} = |V_{ub}| e^{-ig}$$

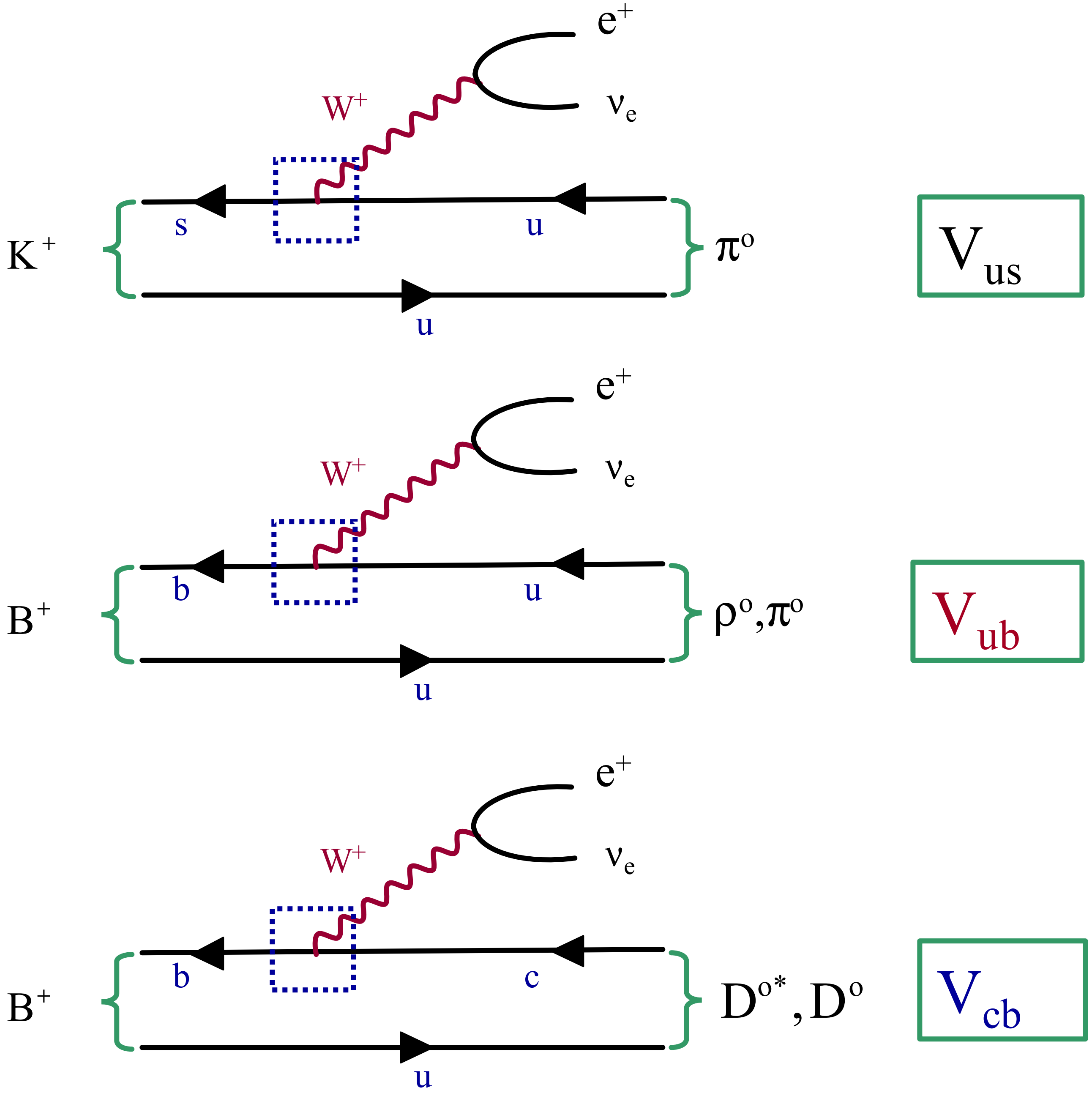
$$V_{td} = |V_{td}| e^{-ib}$$

An Important Target of Particle Physics

$$J_{CP} = \lambda^{2} |V_{cb}|^{2} \bar{\eta} = 2 \cdot$$ 

Area of unrescaled UT

Tree Level Decays

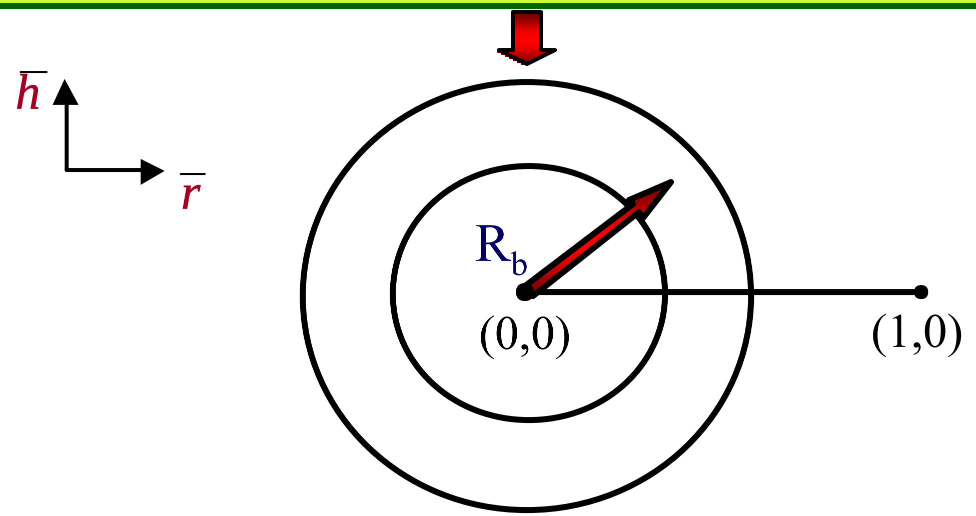


Information from Tree Level Decays

$$|V_{us}| = 0.2240 \pm 0.0036 = \lambda$$

$$|V_{cb}| = (41.5 \pm 0.8) \cdot 10^{-3} \qquad (A = 0.83 \pm 0.02)$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.086 \pm 0.008 \qquad (R_b = 0.37 \pm 0.04)$$



Apex of Unitarity Triangle somewhere on this Band

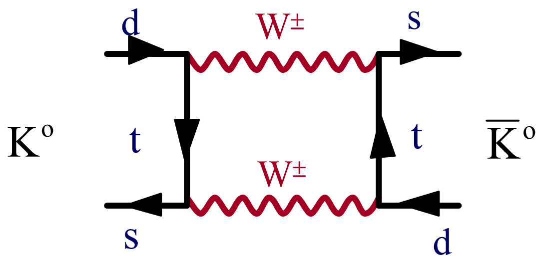
To find it **GO TO**

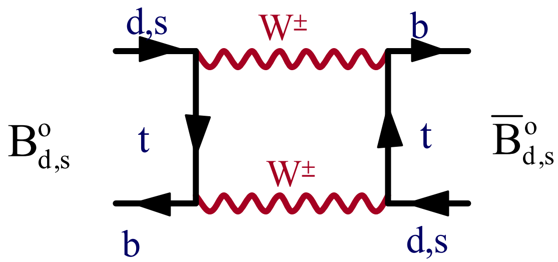
Loop Induced Decays

CP-Violation in K-Decays

CP-Violation in B-Decays

View at Short Distance Scales



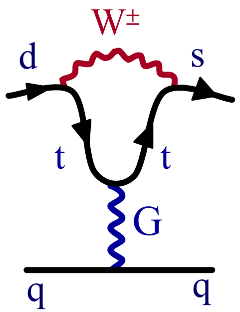


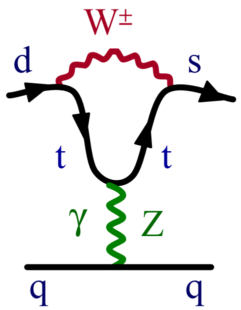
~~C~~P ε_K-Parameter

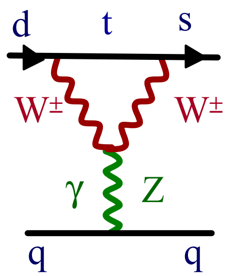
$\Delta M\ (K_L\text{-}K_S)$

$B^o_d - \overline{B}^o_d$ Mixing

ε'

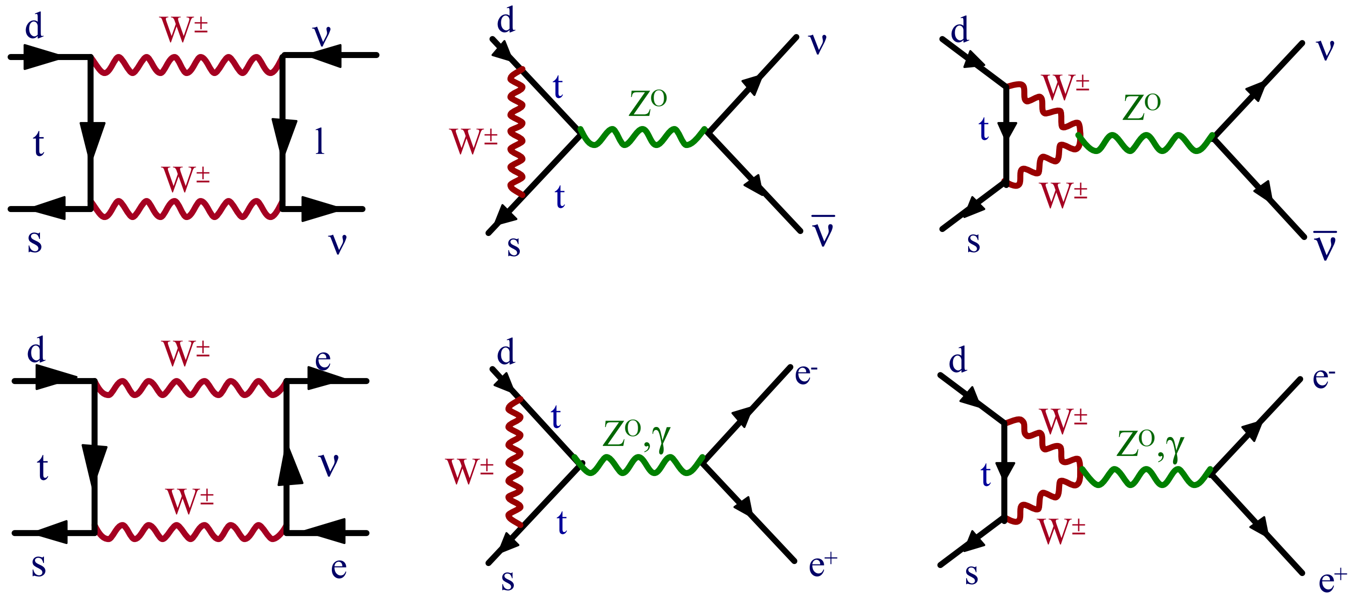




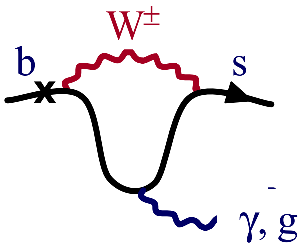


View at Short Distance Scales

★ $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$ ★ $K_L \to \mu\bar{\mu}$, $B \to \mu\bar{\mu}$, $B \to X_S \nu\bar{\nu}$



$K_L \to \pi^0 e^+ e^-$ $B \to X_S e^+ e^-,\ X_S \mu\bar{\mu}$

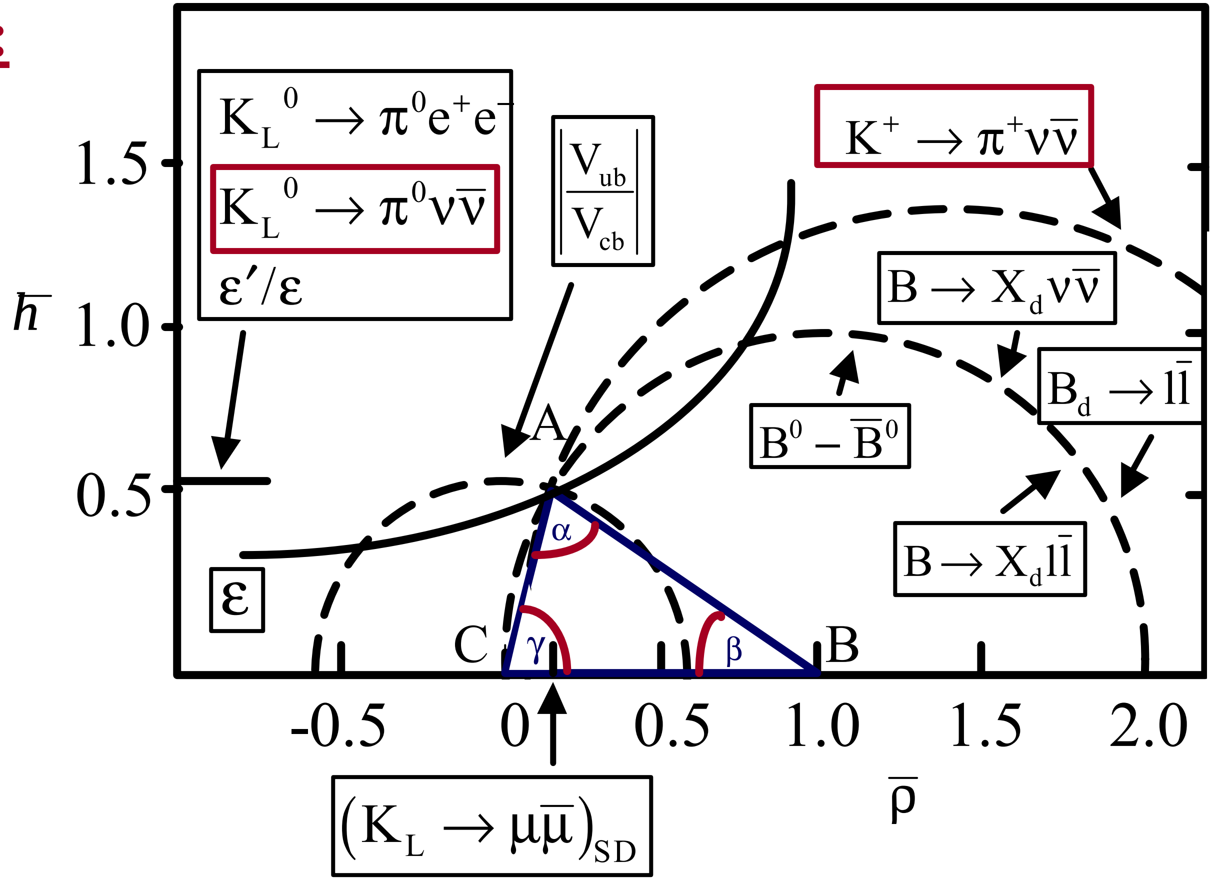


$B \to X_S \gamma$ $B \to K^* \gamma$ ★

$B \to X_d \gamma$ $b \to s$ gluon

Hunting Δ with Rare and ~~CP~~ Decays

2011:



★ **Quark Mixing and CP Violation closely related in the St. Model**

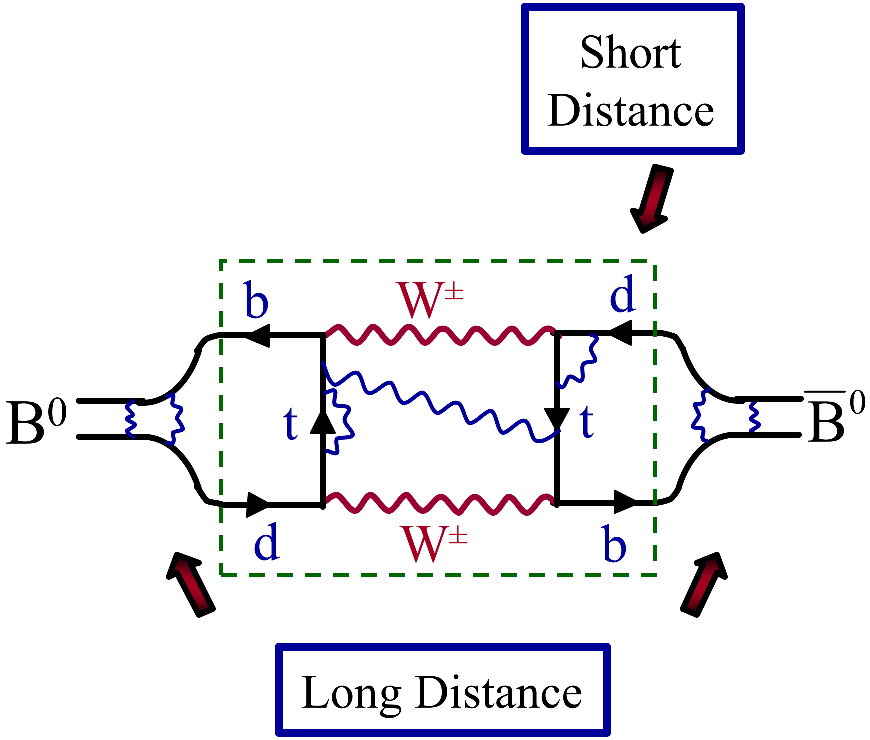
★ $\left\{\begin{matrix} \text{CP Asymmetries} \\ \text{in} \\ \text{B-Decays} \end{matrix}\right\}$ ⇒ $\left\{\begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}\right\}$

2.

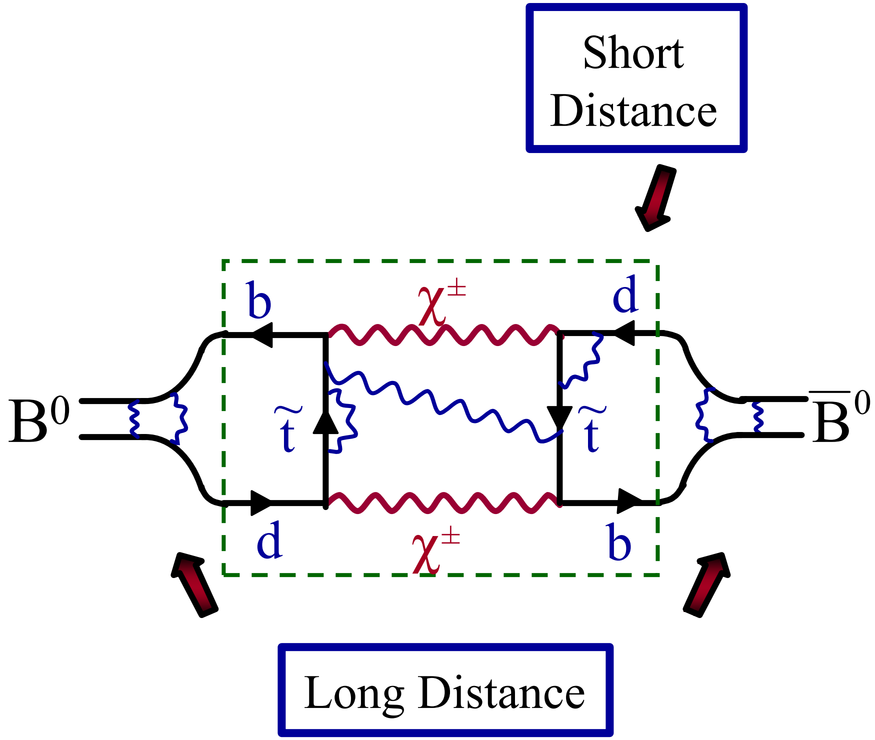
Theoretical Framework

The Problem of Strong Interactions

$B_d^0 - \overline{B}_d^0$ Mixing (SM)



$B_d^0 - \overline{B}_d^0$ Mixing (MSSM)



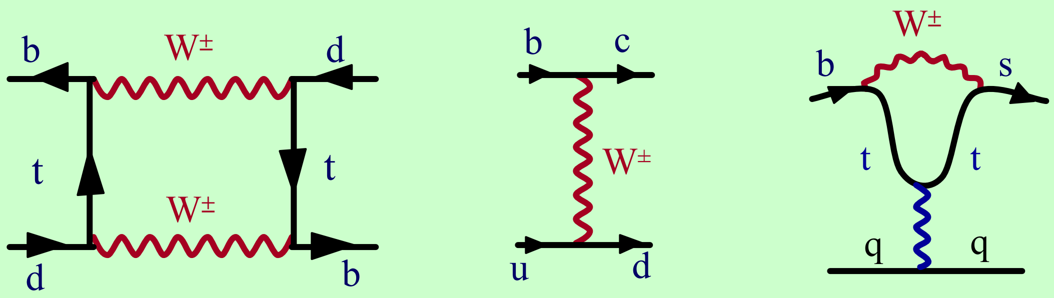
SD : Perturbative (Asymptotic Freedom)

LD : Non-Perturbative (Confinement)

Effective Field Theory

Full Theory

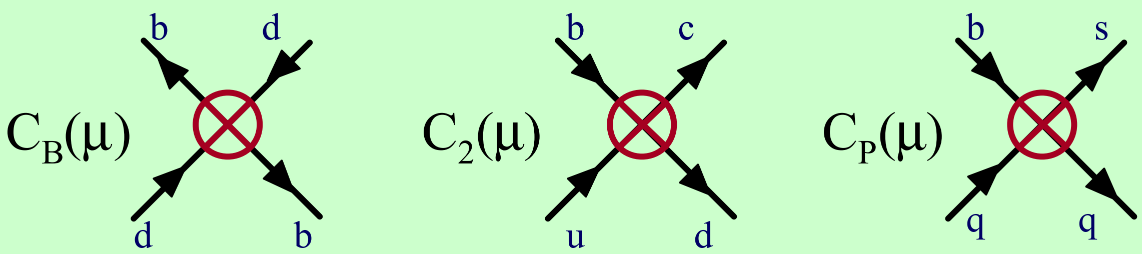
($W^{\pm}$, Z^0, G, γ, t, H^0, b, u, d, s, c, l)



$\mu \geq M_W$

Effective Theory

(G, γ, b, u, d, s, c, l)



$\mu \cong 0(m_b)$

"Generalized Fermi Theory" with calculable "couplings" $C_B(\mu)$, $C_2(\mu)$,...

Operator Product Expansion

$\left\{\begin{array}{l}\text{Wilson}\\\text{Coefficients}\end{array}\right\}$ $\left\{\begin{array}{l}\text{Local}\\\text{Operators}\end{array}\right\}$

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\, Q_i$$

Q_i ⟺

Four Quark Interaction Vertex $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$

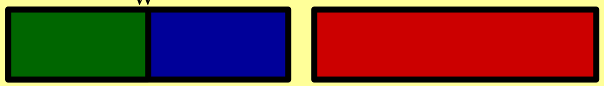
$C_i(\mu)$ ⟺ **Coupling Constants** $C(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{23}}$

{K, B, D,...}

$$A(M \to F) = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle F | Q_i(\mu) | M \rangle$$

$\left\{\begin{array}{l}\pi\pi,\ \pi\nu\bar{\nu}\\ \mu\bar{\mu},\ K^*\gamma,\ldots\end{array}\right\}$

M_W μ=0(1 GeV, m_b) 0



Short RG Long Distance

$\left\{\begin{array}{l}\text{Top}\\\text{SUSY}\\H^{\pm}\ldots\end{array}\right\}$ $\left\{\begin{array}{l}\text{Renormalization Group}\\ \sum \left(\alpha_s \log \frac{M_W}{\mu}\right)^n\end{array}\right\}$ $\left\{\begin{array}{l}\text{Lattice, 1/N}\\\text{HQET, QCDS}\\\text{ChPTh}\end{array}\right\}$

$$\langle \bar{K}^0 | (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} | K^0 \rangle = \frac{8}{3} \hat{B}_K F_K^2 m_K^2 [\alpha_s(\mu)]^{2/9}$$

Penguin-Box Expansion (SM)

Buchalla, AJB, Harlander (90)

$$x_t = \frac{m_t^2}{M_W^2}$$

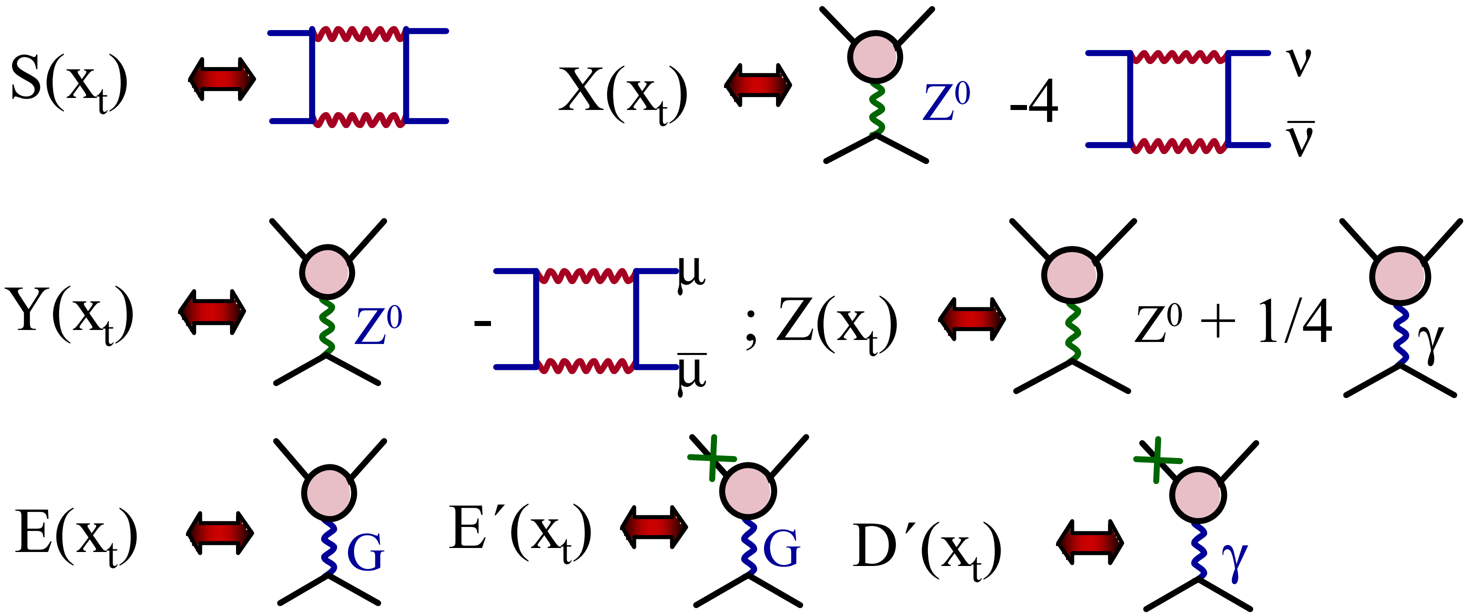
The m_t dependence of all K and B Decays resides in 7 Basic Universal Functions $F_i(x_t)$

$$A(\text{Decay}) = \sum_i B_i \eta^i_{QCD} V^i_{CKM} F_i(x_t)$$

F_i : S, X, Y, Z, E, E', D'

(Gauge Invariant set of functions)

Decay	Contributing Functions
$B_d^0-\overline{B}_d^0,\ B_s^0-\overline{B}_s^0,\ \varepsilon$	$S(x_t)$
$K\to\pi\nu\bar{\nu},\ B\to X_s\nu\bar{\nu}$	$X(x_t)$
$K_L\to\mu\bar{\mu},\ B\to l\bar{l}$	$Y(x_t)$
ε'	$Z(x_t),\ X(x_t),\ Y(x_t),\ E(x_t)$
$K_L\to\pi^0e^+e^-$	$Y(x_t),\ Z(x_t),\ E(x_t)$
$B\to X_s e^+e^-$	$Y(x_t),\ Z(x_t),\ D'(x_t), E'(x_t), E(x_t)$
$B\to X_s\gamma$	$D'(x_t),\ E'(x_t)$



Status of NLO

Review: Buchalla, AJB, Lautenbacher (Rev. Mod. Phys. 96)

Decay	Authors
$\Delta F=1$ Hamiltonians (Current – Current)	Altarelli, Curci, Martinelli, Petrarca; AJB + Weisz
NLO Corrections to B_{SL}	ACMP, Buchalla; Bagan, Ball, Braun, Gosdzinsky; Lenz, Nierste, Ostermaier
ΔM (K_L-K_S)	Herrlich, Nierste (η_1)
$B^0_{d,s} - \bar{B}^0_{d,s}$ Mixing	AJB, Jamin, Weisz (η_2^B), see also Urban, Krauss, Jentschura, Soff
ε_K	AJB, Jamin, Weisz (η_2^K) Herrlich, Nierste (η_3^K)
$\Delta S=1$, $\Delta B=1$ Hamiltonians with QCD and EW Penguins ε'/ε	AJB, Jamin, Lautenbacher, Weisz Ciuchini, Franco, Martinelli, Reina
$K_L \to \pi^0 e^+ e^-$	AJB, Lautenbacher, Misiak, Münz
$B \to X_{s,d}\, \gamma$ $B \to X_{s,d}\, g$	Chetyrkin, Misiak, Münz; Greub, Hurth, Wyler; AJB, Czarnecki, Misiak, Urban; Ali, Greub; Pott; Adel, Yao; Ciuchini, Degrassi, Gambino, Giudice
$B \to X_{s,d}\, l^+ l^-$	Misiak; AJB, Münz
$K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, $B \to \mu\bar{\mu}$ $K_L \to \mu\bar{\mu}$, $B \to X_s \nu\bar{\nu}$, $K^+ \to \pi^+ \mu\bar{\mu}$	Buchalla, AJB (94) Misiak, Urban (98)
Inclusive $\Delta S=1$	Jamin, Pich

Most Recent

$(\Delta\Gamma)_{B_s^0-\bar{B}_s^0}$ $(\Delta\Gamma)_{B_d^0-\bar{B}_d^0}$	Beneke, Buchalla, Greub, Lenz, Nierste
Two Loop $\hat{\gamma}$ for "New" $\Delta F=2$ Operators	Ciuchini, Franco, Lubicz, Martinelli, Scimeni, Silvestrini; AJB, Misiak, Urban
Charmonium Decays	Beneke, Maltoni, Rothstein
SUSY $B\to X_s\gamma$	Ciuchini, Degrassi, Gambino; Bobeth, Misiak, Urban; Giudice
SUSY $B_d^0-\bar{B}_d^0$, ε_K	Ciuchini, Lubicz, Conti, Vladikas; Donini, Franco, Martinelli, Scimeni; Gimenz, Giusti, Masiero, Silvestrini; Talevi
2 HDM $B\to X_s\gamma$	Ciuchini, Degrassi, Gambino, Giudice; Ciafaloni, Romanino; Strumia; Borzumati, Strumia
$B\to D\pi$, $B\to\pi\pi$	Beneke, Buchalla, Neubert, Sachrajda
SUSY $B\to X_s l^+ l^-$	Bobeth, Misiak, Urban, Ewerth
SUSY $K\to\pi\nu\bar{\nu}$, $K_L\to\mu\bar{\mu}$ $B\to X_s\nu\bar{\nu}$, $B\to\mu\bar{\mu}$	Bobeth, AJB, Krüger, Urban

Master Formula for Weak Decays

Non-Perturbative Factors in the SM

QCD RG Factors

Short Distance Loop Functions (Penguins, Boxes)

New Flavour-Changing Parameters

Represent different Dirac and Colour Stuctures

$$A(\text{Decay}) = B_i \; \eta^i_{QCD} \; V^i_{CKM} \left[F^i_{SM} + F^i_{New}\right] + B^{New}_i \left[\eta^i_{QCD}\right]^{New} V^i_{New} \left[G^i_{New}\right]$$

Non-Perturbative Factors beyond SM

Short Distance Loop Functions Penguins, Boxes)

F^i_{SM}, F^i_{New}, G^i_{New} : **Fully calculable in Perturbation Theory**

η^i_{QCD}, $\left[\eta^i_{QCD}\right]^{New}$: **Fully calculable in RG improved Perturbation Theory**

B_i, B^{New}_i : **Require Non-Perturbative Methods or can be extracted from leading decays**

(represent $\langle Q_i \rangle$)

Possible Dirac Structures in $K^0 - \bar{K}^0$ and $B^0_{d,s} - \bar{B}^0_{d,s}$

SM:

$$\gamma_\mu (1-\gamma_5) \otimes \gamma^\mu (1-\gamma_5)$$

Beyond SM:

$$\gamma_\mu (1-\gamma_5) \otimes \gamma^\mu (1+\gamma_5)$$

$$(1-\gamma_5) \otimes (1+\gamma_5)$$

$$(1-\gamma_5) \otimes (1-\gamma_5)$$

$$\sigma_{\mu\nu} (1-\gamma_5) \otimes \sigma^{\mu\nu} (1-\gamma_5)$$

MSSM with large tanβ
General Supersymmetric Models
Models with complicated Higgs System

NLO $\left[\eta^i_{QCD}\right]^{New}$: Ciuchini, Franco, Lubicz, Martinelli, Scimemi, Silvestrini

AJB, Misiak, Urban, Jäger

General Structure in Models with Minimal Flavour Violation

AJB, Gambino, Gorbahn, Jäger, Silvestrini;
D'Ambrosio, Giudice, Isidori, Strumia (different formulation)

- **No new Operators** (Dirac and Colour Structures) beyond those present in the SM
- Flavour Changing Transitions governed by CKM. **No new complex phases** beyond those present in the SM

$$A(\text{Decay}) = B_i\, \eta^i_{QCD}\, V^i_{CKM} \Big[\underbrace{F^i_{SM} + F^i_{New}}_{\text{real}} \Big]$$

Examples: SM

MSSM at not too large $\tan\beta = \frac{v_2}{v_1}$

Main Targets of $\not{CP}$

Useful for CKM and △

(Rather clean)

Standard Analysis of △

e_K, $|V_{ub}/V_{cb}|$, $\Delta M_d(B_d^0 - \overline{B}_d^0)$, $\Delta M_s(B_s^0 - \overline{B}_s^0)$

(Mixture of K- and B-Physics)

CP-Violation in Rare K-Decays

$K_L \to \pi^0 \nu\bar{\nu}$ $(K^+ \to \pi^+ \nu\bar{\nu})$

$(K_L \to \pi^0 e^+ e^-)$

$(\sin 2\beta)$
(η)
$(|V_{td}|)$

CP-Violation in B-Decays
(Asymmetries and other Strategies)

(α, β, γ)

Important Tests of $\not{CP}$, $\not{T}$:
ε'/ε, Electric Dipole Moments
$\not{CP}$ in Hyperon Decays
$\not{CP}$ in D-Decays

Large Hadronic Uncertainties

3.

Particle Mixing and Various Types of CP Violation

Modern Classification of CP Violation

We have:

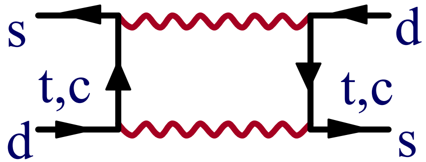
Particle-Antiparticle Mixing

and

Decay

1. CP Violation in Mixing
2. CP Violation in Decay
3. CP Violation in the Interference of Mixing and Decay

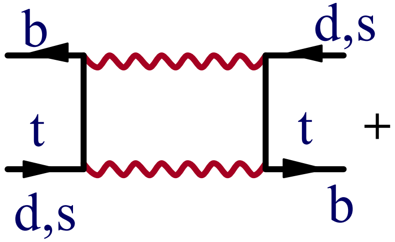
Indirect CP in $K_L \to \pi\pi$

{ Im 

+ **QCD** } ⇒ ε_K ⇐ {Experiment}

exp: $\varepsilon_K = (2.280 \pm 0.013) \cdot 10^{-3} e^{i\frac{\pi}{4}}$

$B^0_{d,s} - \overline{B}^0_{d,s}$ Mixing

{ 

+ **QCD** } ⇒ $(\Delta M)_{d,s}$ ⇐ { CLEO, ARGUS, 4∗LEP,SLD, BaBar, Belle }

$$(\Delta M)_{d,s} \equiv M(B^0_H)_{d,s} - M(B^0_L)_{d,s}$$

Mass Eigenstates

exp: ★

$(\Delta M)_d = (0.503 \pm 0.006)/\text{ps}$

$(\Delta M)_s > 14.4/\text{ps}$ (95% C.L.) (LEP/SLD)

Classification of $\not{CP}$ in B- and K-Decays

(Nir 99),...

1. CP Violation in Mixing

$$B_{H,L} = p|B^0\rangle \pm q|\overline{B}^0\rangle$$

Mass Eigenstates

$\not{CP}$: $|q/p| \neq 1$ ⇒ (Not CP Eigenstates)

$$a_{SL} = \frac{\Gamma(\overline{B}^0(t) \to l^+ \nu X) - \Gamma(B^0(t) \to l^- \nu X)}{\Gamma(\overline{B}^0(t) \to l^+ \nu X) + \Gamma(B^0(t) \to l^- \nu X)}$$

$$a_{SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = \mathrm{Im}\frac{\Gamma_{12}}{M_{12}}$$

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2}$$

Observed in K-system: $\mathrm{Re}\,\varepsilon_K \neq 0$

$\overline{B}^0 \to B^0 \to l^+ \nu X$

⇕ (Phase Difference)

$B^0 \to \overline{B}^0 \to l^- \nu X$

"wrong charge" leptons

Hadronic Uncertainties in Γ_{12}, M_{12}

2. CP Violation in Decay

$$A_f = \langle f | H^{weak} | B \rangle \quad \bar{A}_{\bar{f}} = \langle \bar{f} | H^{weak} | \bar{B} \rangle$$

$$\not{CP}: \; \left| \bar{A}_{\bar{f}} / A_f \right| \neq 1 \qquad f \xrightarrow{CP} \bar{f}$$

$$a_{f^\pm}^{Decay} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} = \frac{1 - \left| \bar{A}_{f^-} / A_{f^+} \right|^2}{1 + \left| \bar{A}_{f^-} / A_{f^+} \right|^2}$$

Requires at least two different contibutions with different <u>weak</u> (φ_i) and <u>strong</u> (δ_i) phases

$$A_f = \sum_i A_i e^{i(\delta_i + \varphi_i)} \qquad \bar{A}_{\bar{f}} = \sum_i A_i e^{i(\delta_i - \varphi_i)} \qquad (A_2 \ll A_1) \qquad r \equiv \frac{A_2}{A_1} \ll 1$$

$$i = 1,\; 2$$

$$a_{f^\pm}^{Decay} \approx -2r \sin(\delta_2 - \delta_1) \sin(\varphi_2 - \varphi_1)$$

Observed in K-system: $\mathrm{Re}\, \varepsilon'_K \neq 0$

Hadronic Uncertainties in A_i , δ_i

B^0-Decays into CP-Eigenstate

$$B^0 \rightarrow \overline{B}^0 \rightarrow f, \quad B^0 \rightarrow B^0 \rightarrow f$$

ΔM = Difference between Mass Eigenstates in $(B^0, \overline{B}^0)$ System

$f \equiv f_{CP}$ = CP eigenstate
η_f = CP-parity = ± 1

<u>Time-dependent asymmetry:</u>

$$a_{CP}(t,f) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\overline{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\overline{B}^0(t) \rightarrow f)}$$

$$a_{CP}(t,f) = a_{CP}^{Decay} \cos(\Delta M t) + a_{CP}^{"mix-ind"} \sin(\Delta M t)$$

$$a_{CP}^{Decay} = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2} \equiv C_f \qquad a_{CP}^{"mix-ind"} = \frac{2 \operatorname{Im} \xi_f}{1 + |\xi_f|^2} \equiv -S_f$$

$$\xi_f = \underbrace{\exp[i 2\varphi_M]}_{\text{Mixing}} \frac{\overline{A}_f(\overline{B}^0 \rightarrow f)}{A_f(B^0 \rightarrow f)}$$

← Decay Amplitudes

For a **single** decay contribution or sum of contributions with **the same weak phase**

$$\xi_f = -\eta_f \exp[i 2\varphi_M] \cdot \exp[-i 2\varphi_D]$$

$|\xi_f|^2 = 1 \qquad \varphi_D$: weak phase in the B^0 decay

⇒ ξ_f = given only in terms of CKM phase

$$a_{CP}^{decay} = 0$$

Dominance of a single CKM Amplitude

A_{Tree}, A_P - hadronic matrix elements
δ_T, δ_P - final state interaction phases
φ_T, φ_P - weak CKM phases

$$\frac{\overline{A}_f(\overline{B}^0 \to f)}{A_f(B^0 \to f)} = -\eta_f \left[\frac{A_{Tree} e^{i(\delta_T - \varphi_T)} + A_P e^{i(\delta_P - \varphi_P)}}{A_{Tree} e^{i(\delta_T + \varphi_T)} + A_P e^{i(\delta_P + \varphi_P)}} \right]$$

Tree Dominance

$$\frac{\overline{A}_f(\overline{B}^0 \to f)}{A_f(B^0 \to f)} = -\eta_f e^{-i2\varphi_T}$$

(Pure Phase)
Very Clean !

Penguin Dominance

$$\frac{\overline{A}_f(\overline{B}^0 \to f)}{A_f(B^0 \to f)} = -\eta_f e^{-i2\varphi_P}$$

(Pure Phase)
Very Clean !

Also pure phase if $\varphi_T = \varphi_P$!! (Example: $B_d^0 \to J/\psi K_S$)

3. CP Violation in the Interference of Mixing and Decay

Misnomer: ("Mixing induced CP-Violation")

$$a_{CP}(t,f) = \mathrm{Im}\,\xi_f \sin(\Delta M t)$$

$$\mathrm{Im}\,\xi_f = \eta_f \sin(2\varphi_D - 2\varphi_M) \equiv -S_f$$

Very clean TH

Measures the difference between the phases of B^0-$\overline{B}^0$ mixing ($2\varphi_M$) and of decay amplitude ($2\varphi_D$)

Examples:

$B_d^0 \to \psi K_S$: $\varphi_D = 0 \quad \varphi_M = -\beta \quad \eta_f = -1$

$$\mathrm{Im}\,\xi_{\psi K_S} = -\sin 2\beta$$

$B_d^0 \to \pi^+\pi^-$: $\varphi_D = \gamma \quad \varphi_M = -\beta \quad \eta_f = +1$

$$\mathrm{Im}\,\xi_{\pi\pi} = \sin\big(2(\gamma+\beta)\big) = -\sin 2\alpha$$

$K_L \to \pi^0 \nu\bar{\nu}$: Measures the difference between the phases in K^0-$\overline{K}^0$ mixing and $\bar{s} \to \bar{d}\nu\bar{\nu}$ amplitude

B^0-Decays into CP Eigenstates

$$\left(\text{Two Contributions} \quad r = \frac{A_2}{A_1} \ll 1 \right)$$

$$a_{CP}(t,f) = C_f \cos(\Delta M t) - S_f \sin(\Delta M t)$$

$$C_f = -2r \sin(\varphi_1 - \varphi_2) \sin(\delta_1 - \delta_2)$$

$$S_f = -\eta_f \left[\sin 2(\varphi_1 - \varphi_M) + 2r \cos 2(\varphi_1 - \varphi_M) \sin(\varphi_1 - \varphi_2) \cos(\delta_1 - \delta_2) \right]$$

φ_i = weak phases $\qquad$ δ_i = strong phases

$\{r = 0\}$ ⇒ $C_f = 0 \qquad S_f = -\eta_f \sin 2(\varphi_1 - \varphi_M)$

Classification of CP Violation

$\not{CP}$ in	Examples	Old Terminology
Mixing	$\mathrm{Re}(\varepsilon_K)$, $a_{SL}(K)$, $a_{SL}(B)$	Indirect $\not{CP}$
Decay	ε'/ε, $a_{CP}(B^{\pm})$	Direct $\not{CP}$
Interference of Mixing and Decay	$K_L \to \pi^0 \nu\bar{\nu}$, $a_{CP}(\psi K_S)$ $\mathrm{Im}(\varepsilon_K)$	*)

*) In order to find out the presence of $\not{CP}$ in Decay (direct $\not{CP}$) at least two processes, asymmetries have to be measured

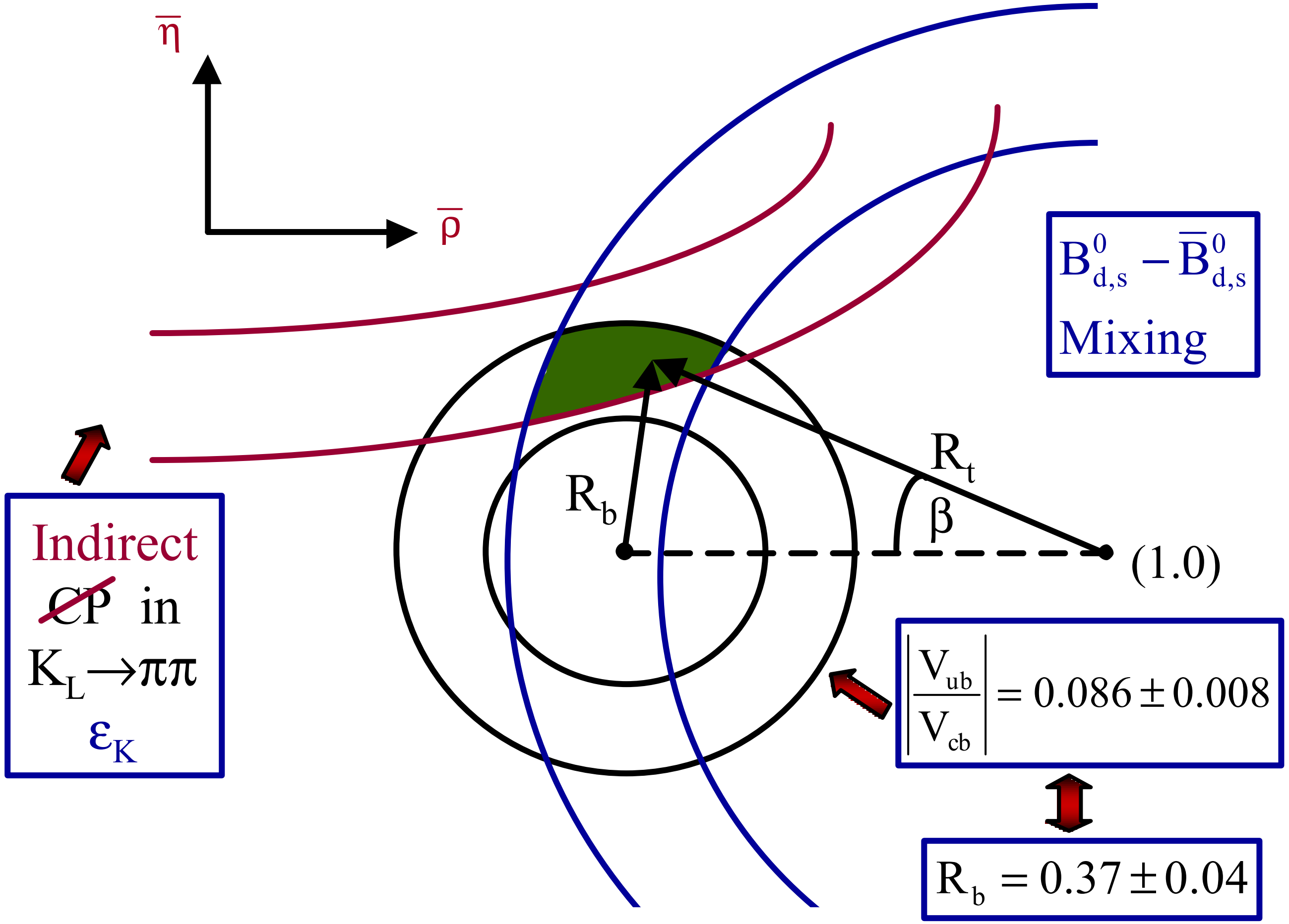
Comparison of Two-Languages

CP violation in mixing	≡	Manifestation of indirect $\not{CP}$
CP violation in decay	≡	Manifestation of direct $\not{CP}$
CP violation in interference of mixing and decay	≡	With a single decay it is impossible to state whether $\not{CP}$ in mixing or decay. But $\mathrm{Im}\,\xi_{f_1} \neq \mathrm{Im}\,\xi_{f_2}$ signals CP violation in decay (Direct $\not{CP}$)

4.

Standard Analysis of Unitarity Triangle

Standard Analysis of UT



Relevant Parameters

$\hat{B}_K,\ F_{B_d}\sqrt{\hat{B}_{B_d}},\ \ \xi = F_{B_s}\sqrt{\hat{B}_{B_s}} / F_{B_d}\sqrt{\hat{B}_{B_d}}$ ⟺ ε_K ΔM_d $\Delta M_s / \Delta M_d$

Basic Formulae

1. ε_K - Hyperbola

$$\bar{\eta}\left[\left(1-\bar{\rho}\right)A^2F_{tt}\eta^{tt}_{QCD}+P_c\left(\varepsilon\right)\right]A^2\hat{B}_K=0.213$$

$\eta^{tt}_{QCD}=0.57\pm0.01;\quad P_C\left(\varepsilon\right)=0.28\pm0.05;\quad F_{tt}=2.39\pm0.12$

$\left(F_{tt}\equiv S\left(x_t\right)\right)$

2. $B^0_d-\bar{B}^0_d$ Mixing Constraint

$$R_t=0.86\left[\frac{0.041}{\left|V_{cb}\right|}\right]\sqrt{\frac{2.34}{F_{tt}}}\sqrt{\frac{\Delta M_d}{0.50/ps}}\left[\frac{230MeV}{\sqrt{\hat{B}_d}F_{B_d}}\right]\sqrt{\frac{0.55}{\eta^{QCD}_B}}$$

$\left|V_{cb}\right|=0.041\pm0.001;\quad \Delta M_d=\left(0.503\pm0.006\right)/ps;\quad \eta^{QCD}_B=0.55\pm0.01$

3. $B^0_s-\bar{B}^0_s$ Mixing Constraint $(\Delta M_d/\Delta M_s)$

$$R_t=0.90\sqrt{\frac{\Delta M_d}{0.50/ps}}\sqrt{\frac{18.4/ps}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]$$

$$\xi=\frac{\sqrt{\hat{B}_s}F_{B_s}}{\sqrt{\hat{B}_d}F_{B_d}}$$

$\Delta M_s>14.4\ /\ ps$ (95% C.L.) LEP (SLD)

4. $\sin 2\beta$ from $A_{CP}(\psi K_S)$

$$A_{CP}(\psi K_S) \equiv -a_{\psi K_S} \sin(\Delta M_d t)$$

$$a_{\psi K_S} = \sin 2\beta \qquad \text{(SM)}$$

$$\sin 2\beta_{\psi K_S} = \begin{cases} 0.79 \pm {}^{0.41}_{0.44} & \text{(CDF)} \\ 0.741 \pm \underset{\text{(stat)}}{0.067} \pm \underset{\text{(syst)}}{0.033} & \text{(BaBar)} \\ 0.719 \pm 0.074 \pm 0.035 & \text{(Belle)} \end{cases}$$

$$\left(\text{ALEPH}: 0.84\ {}^{+0.82}_{-1.04} \pm 0.16\right)$$

⇓

(Nir) $\sin 2\beta = 0.734 \pm 0.054$ $\left(a_{\psi K_S}\right)$

⇓

$$\beta = \begin{cases} (23.6 \pm 2.2)^\circ \\ (66.4 \pm 2.2)^\circ & \text{(excluded in the SM)} \end{cases} \qquad (\sin\beta = 0.400 \pm 0.035)$$

Crucial Parameters in SM and Beyond

CERN
CKM Workshop

$\lvert V_{us}\rvert = \lambda$	0.2240 ± 0.0036	
$\lvert V_{ub}\rvert$	$(3.57 \pm 0.31)\cdot 10^{-3}$	
$\lvert V_{cb}\rvert$	$(41.5 \pm 0.8)\cdot 10^{-3}$	★
$\left\lvert \frac{V_{ub}}{V_{cb}} \right\rvert$	0.086 ± 0.008	★
$m_t\,(m_t)$	(167 ± 5) GeV	
$\hat{B}_K$	0.86 ± 0.15	(ε_K)
$\sqrt{\hat{B}_d} F_{Bd}$	$(235\ ^{+33}_{-41})$ MeV	(ΔM_d)
$\xi = \frac{\sqrt{\hat{B}_s} F_{Bs}}{\sqrt{\hat{B}_d} F_{Bd}}$	1.24 ± 0.08 (1.22 ± 0.07)*	$\left(\frac{\Delta M_s}{\Delta M_d}\right)$

Lellouch,
Bećirević
(Amsterdam)

Valid for all extensions of SM !!

*Bećirević et al.

Different Treatments of Errors

Particle Data Group

Gilman, Kleinknecht, Renk

"Gaussian" Approach

Ali + London; Mele, ...

Bayesian Approach

Ciuchini, D´Agostini, Franco, Lubicz, Martinelli, Parodi, Roudeau, Stocchi

Frequentist Approach

Höcker, Lacker, Laplace, Diberder

95% CL Scan Method

Plaszczynski, Shune; BaBar

Naive Scanning

Rosner; Stone; AJB

→ **Bayesian**

Universal Unitarity Triangle

AJB, Gambino, Gorbahn, Jäger, Silvestrini (00)

In the full class of MFV-models it is possible to construct quantities that depend on CKM parameters but in which the dependence on new physics parameters cancels out

↓

CKM Matrix determined without "New Physics Pollution"

↓

Universal Unitarity Triangle

Examples

$$R_t = 0.90\sqrt{\frac{\Delta M_d}{0.50/ps}}\sqrt{\frac{18.4/ps}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]$$

$$a_{\psi K_s} = \sin 2\beta$$

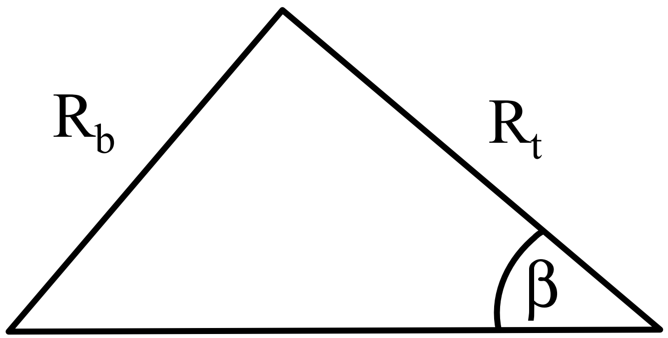
Universal Unitarity Triangle 2002

AJB, Parodi, Stocchi

Use only quantities that are independent of parameters specific to a given Minimal Flavour Violation model

$$\left|\frac{V_{ub}}{V_{cb}}\right| \Rightarrow R_b = \frac{\left(1-\lambda^2/2\right)}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

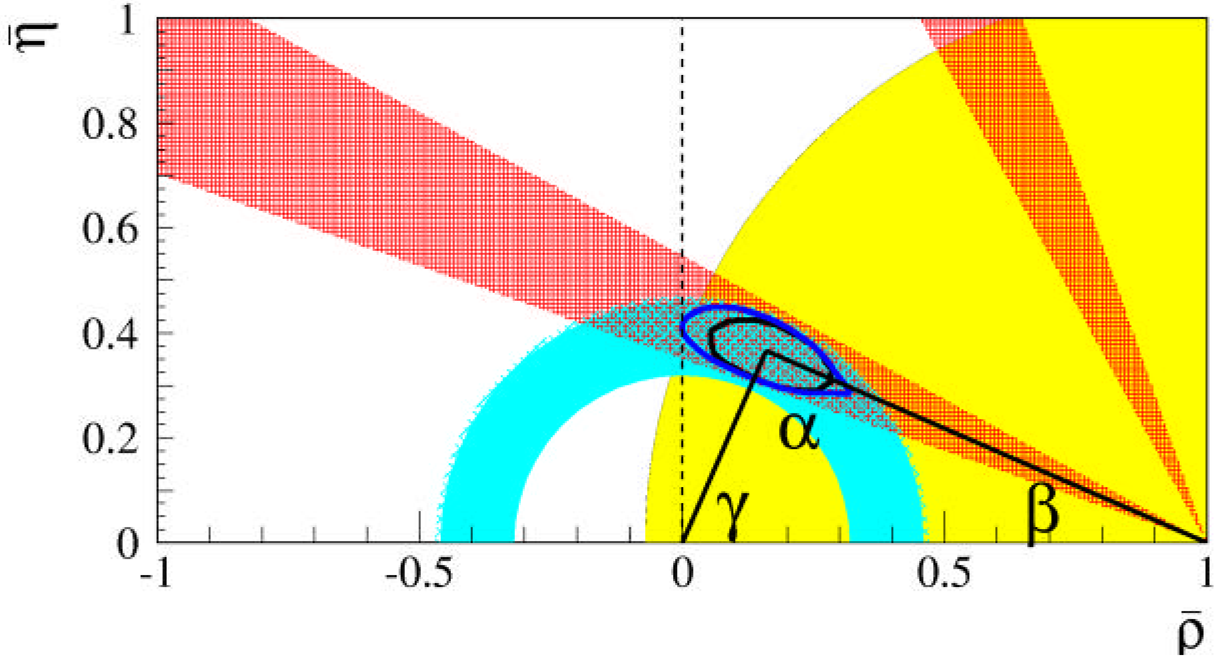
$$\frac{\Delta M_d}{\Delta M_s} \Rightarrow R_t = \frac{\xi_{th}}{\lambda}\sqrt{\frac{\Delta M_d}{\Delta M_s}} \qquad a_{\psi K_s} \Rightarrow \sin 2\beta$$



$$\xi_{th} = \frac{\sqrt{\hat{B}_s}F_{B_s}}{\sqrt{\hat{B}_d}F_{B_d}}$$

Unitarity Triangle 2002 (SM and MFV Models)

(AJB, Parodi, Stocchi)
(95% C.L. ranges)
(AJB, hep-ph/0210291)



$$\sin 2\beta = \begin{cases} 0.734 \pm 0.054 & (a_{\psi K_s}) \\ 0.715^{+0.055}_{-0.045} & (\text{UT without } a_{\psi K_s}) \end{cases}$$

Perfect Agreement

$$(\sin 2\beta)_{\substack{\text{World}\\\text{Average}}} = 0.725 \pm 0.033$$

$$\beta = (23.2 \pm 1.4)^\circ$$

Bayesian Output (November 2002)

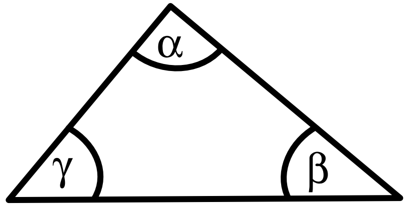
AJB, Parodi, Stocchi hep-ph/0207101

	SM	UUT
$\bar{\eta}$	0.357±0.027	0.369±0.032
$\bar{\rho}$	0.173±0.046	0.151±0.057
$\sin 2\beta$	0.725±0.033	0.725±0.034
$\sin 2\alpha$	-0.09±0.25	0.05±0.31
γ	$(63.5\pm7.0)^0$	$(67.5\pm9.0)^0$
R_b	0.400±0.022	0.404±0.023
R_t	0.900±0.050	0.927±0.061
$\lvert V_{td}\rvert/10^{-3}$	8.15±0.41	8.36±0.55
$\lvert \mathrm{Im}\lambda_t\rvert/10^{-4}$	1.31±0.09	1.35±0.12
$\lvert V_{td}\rvert/\lvert V_{ts}\rvert$	0.205±0.011	0.209±0.014
$\Delta M_s\,(\mathrm{ps}^{-1})$	$18.0^{+1.7}_{-1.5}$	$17.3^{+2.2}_{-1.3}$

$(\lambda_t = V_{ts}^* V_{td})$

5.

α, β, γ from B-Decays



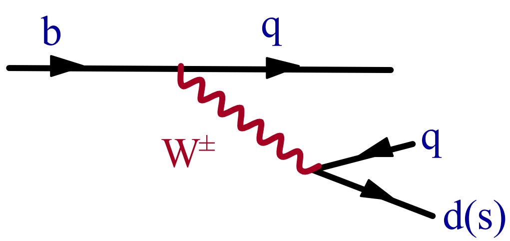
$$V_{td} = |V_{td}| e^{-i\beta}$$

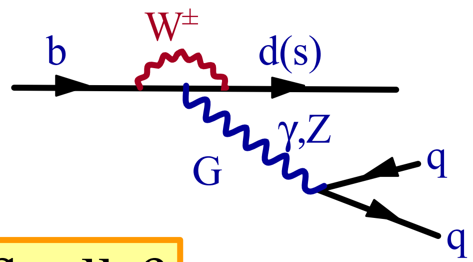
$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

Basic Contributions

Class I — Decays with Trees and Penguins

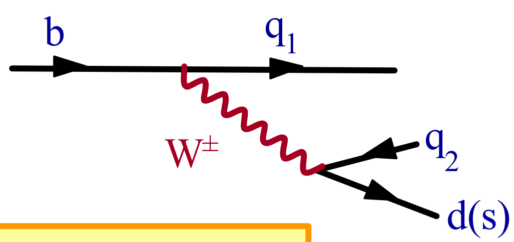




$q = u, c$

$b \to c\bar{c}s$
$b \to c\bar{c}d$
$b \to u\bar{u}s$
$b \to u\bar{u}d$

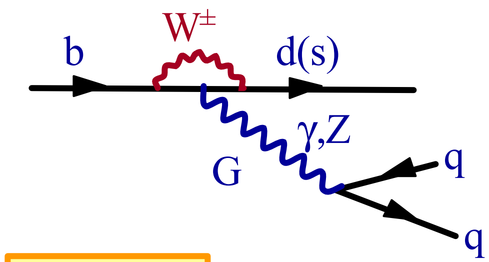
Class II — Trees only



$q_1 \neq q_2 \in \{u, c\}$

$b \to c\bar{u}s$
$b \to c\bar{u}d$
$b \to u\bar{c}s$
$b \to u\bar{c}d$

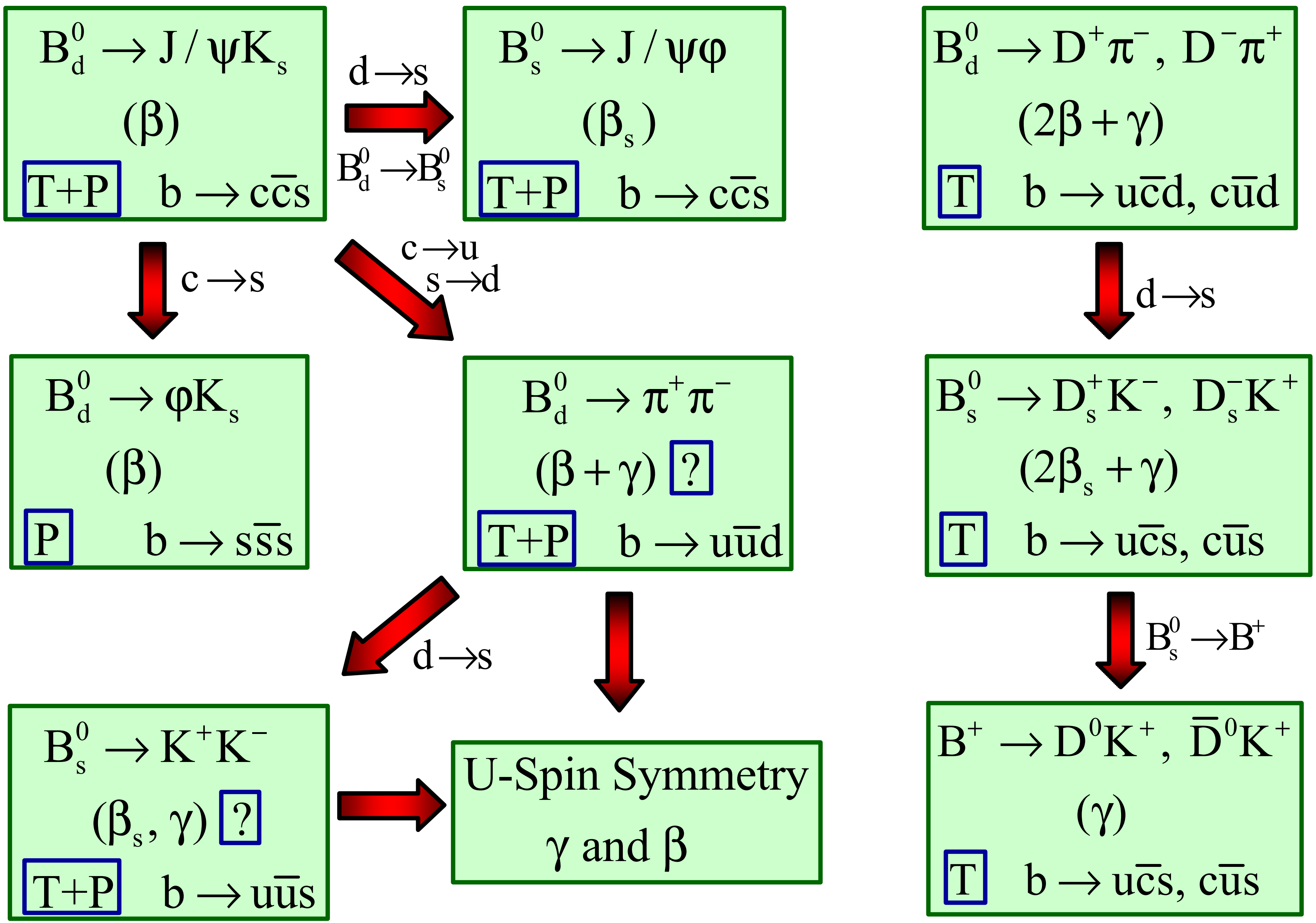
Class III — Penguins only



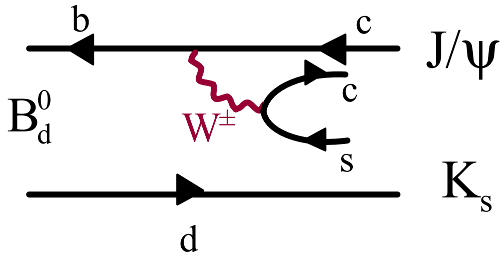
$q = d, s$

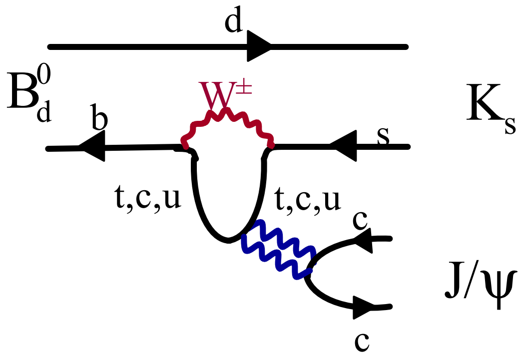
$b \to s\bar{s}s$
$b \to s\bar{s}d$
$b \to d\bar{d}s$
$b \to d\bar{d}d$

α, β, γ from B-Decays



$B_d^0 \to J/\psi K_S$ and β





$$V_{td} = |V_{td}| e^{-i\beta}$$

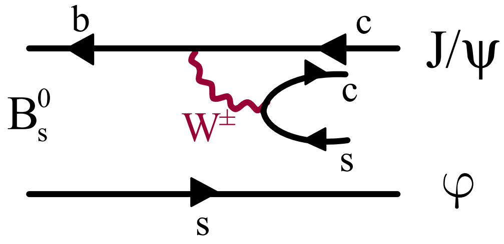
$$V_{cs}V_{cb}^* \cong A\lambda^2$$
$$V_{us}V_{ub}^* \cong A\lambda^4 R_b e^{i\gamma}$$
$$V_{ts}V_{tb}^* = -V_{cs}V_{cb}^* - V_{us}V_{ub}^*$$

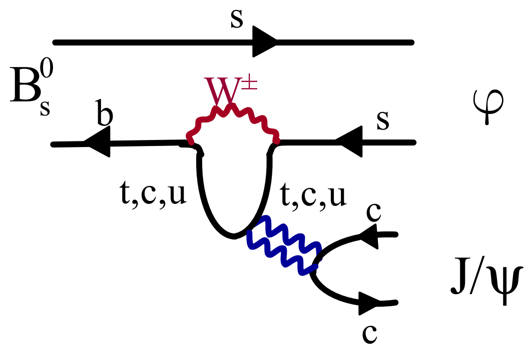
$$A\left(B_d^0 \to J/\psi K_S\right) = V_{cs}V_{cb}^*\left(A_T + P_c\right) + V_{us}V_{ub}^* P_u + V_{ts}V_{tb}^* P_t$$
$$= V_{cs}V_{cb}^*\left(A_T + P_c - P_t\right) + V_{us}V_{ub}^*\left(P_u - P_t\right)$$

(Dominance of a single phase)

$$\left\{ \begin{array}{l} \left|\dfrac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right| \leq 0.02 \\ \dfrac{P_u - P_t}{A_t + P_c - P_t} \ll 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \varphi_D = 0 \\ \varphi_M = -\beta \\ |\xi_{\psi K_S}| = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a_{CP}^{mix}(\psi K_S) = \eta_{\psi K_S} \sin 2(\varphi_D - \varphi_M) = -\sin 2\beta \\ a_{CP}^{dir}(\psi K_S) = 0 \qquad a_{CP}(\psi K^+) \simeq 0 \\ C_{\psi K_S} = 0 \qquad S_{\psi K_S} = \sin 2\beta \end{array} \right\}$$

$B_s^0 \to J/\psi\varphi$ and β_s





$$V_{ts} = |V_{ts}| e^{-i\beta_S}$$

Differs from

$B_d^0 \to J/\psi K_S$ only by

"spectator" quark $d \to s$

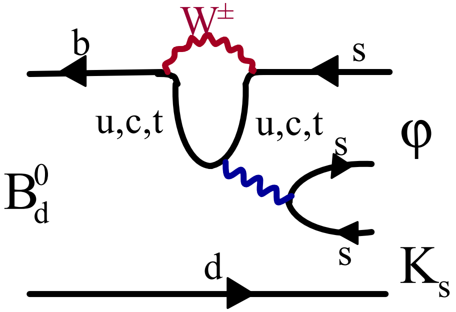
$(\varphi_D = 0)$

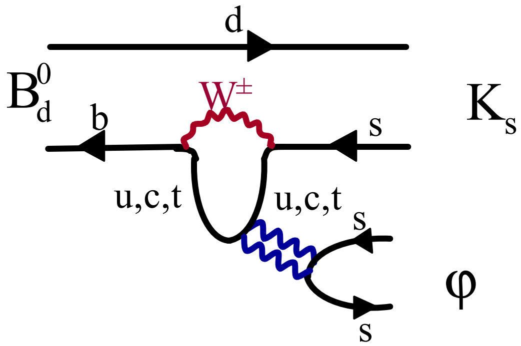
Complication: $(J/\psi\varphi)$ admixture of CP = + and CP = −

(Can be resolved: see Page 40: "B-Decays at the LHC ")

$$\begin{cases} \varphi_D = 0 \\ \varphi_M = -\beta_S \simeq -\lambda^2\eta \\ |\xi_{\psi\varphi}| = 1 \end{cases} \Rightarrow \begin{cases} a_{CP}^{mix} = \sin 2(\varphi_D - \varphi_M) \simeq \underbrace{2\lambda^2\eta}_{2\beta_S} \simeq 0.03 \\ a_{CP}^{dir} \simeq 0 \\ \text{A lot of room for New Physics!} \end{cases}$$

$B_d^0 \to \varphi K_S$ and β (Pure Peguin Decay)





$$V_{cs}V_{cb}^* \simeq A\lambda^2$$
$$V_{us}V_{ub}^* \simeq A\lambda^4 R_b e^{i\gamma}$$
$$V_{ts}V_{tb}^* = -V_{cs}V_{cb}^* - V_{us}V_{ub}^*$$

$$A\left(B_d^0 \to \varphi K_S\right) = V_{cs}V_{cb}^* P_c + V_{us}V_{ub}^* P_u + V_{ts}V_{tb}^* P_t$$
$$= V_{cs}V_{cb}^*\left(P_c - P_t\right) + V_{us}V_{ub}^*\left(P_u - P_t\right)$$

(Dominance of a single phase)

$$\left\{\begin{array}{l} \left|\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right| \le 0.02 \\ \frac{P_u - P_t}{P_c - P_t} \approx 0(1) \end{array}\right\} \Rightarrow \left(\text{neglecting } V_{us}V_{ub}^*\right) \left\{\begin{array}{l} a_{CP}^{mix}(\varphi K_S) = -\sin 2\beta = a_{CP}^{mix}(\psi K_S) \\ C_{\varphi K_S} \approx 0 \qquad S_{\psi K_S} = S_{\varphi K_S} = \sin 2\beta \\ \left|S_{\psi K_S} - S_{\varphi K_S}\right| \le 0.04 \ (\text{SM}) \end{array}\right\}$$

Grossman, Isidori, Worah, London, Soni

First Results for $B_d^0 \to \varphi K_S$

$$(\sin 2\beta)_{\phi K_S} = \begin{cases} +0.45 \pm 0.43 \pm 0.07 & \text{(BaBar)} \\ -0.96 \pm 0.50 \,{}^{+0.11}_{-0.09} & \text{(Belle)} \end{cases}$$

World Averages

$$S_{\phi K_S} = -0.05 \pm 0.24$$

$$C_{\psi K_S} = -0.15 \pm 0.33$$

(Belle)

$$S_{\eta' K_S} = 0.76 \pm 0.36$$

$$C_{\eta' K_S} = -0.26 \pm 0.22$$

(BaBar)

$$S_{\eta' K_S} = 0.02 \pm 0.035$$

$$\left| S_{\varphi K_S} - S_{\psi K_S} \right| \simeq 0.88 \pm 0.34$$

(Violation of SM by 2.6σ)

(fully consistent with SM)

but $S_{\varphi K_S} \neq S_{\eta' K_S}$ possible
as non-leading terms
could be different

Grossman, Isidori Worah

Ciuchini Silvestrini

New Physics:

Enhanced QCD Penguins
Z^0 Penguins, ..

Hiller, Raidal, Ciuchini + Silvestrini
Fleischer, Mannel

Decays to CP non-eigenstates and γ

$\overset{(-)}{B}{}^0_d \to D^{\pm}\pi^{\mp}$

(Dunietz+Sachs)

$d \to s$

$\overset{(-)}{B}{}^0_s \to D^{\pm}_s K^{\mp}$

Aleksan, Dunietz, Kayser

- $B^0_d\left(B^0_s\right)$ and $\overline{B}^0_d\left(\overline{B}^0_s\right)$ can decay to the same final state
- Requires full time-dependent analysis: 4 time dependent rates

  $B^0_{d,s}(t) \to f,\quad \overline{B}^0_{d,s}(t) \to f,$

  $B^0_{d,s}(t) \to \bar{f},\quad \overline{B}^0_{d,s}(t) \to \bar{f},$
- Tree diagrams only

$$\xi_f = e^{i2\varphi_M}\frac{A\left(\overline{B}^0 \to f\right)}{A\left(B^0 \to f\right)}$$

$$\xi_{\bar{f}} = e^{i2\varphi_M}\frac{A\left(\overline{B}^0 \to \bar{f}\right)}{A\left(B^0 \to \bar{f}\right)}$$

$\overline{B}^0 \searrow$, $\updownarrow$, $B^0 \nearrow$ f

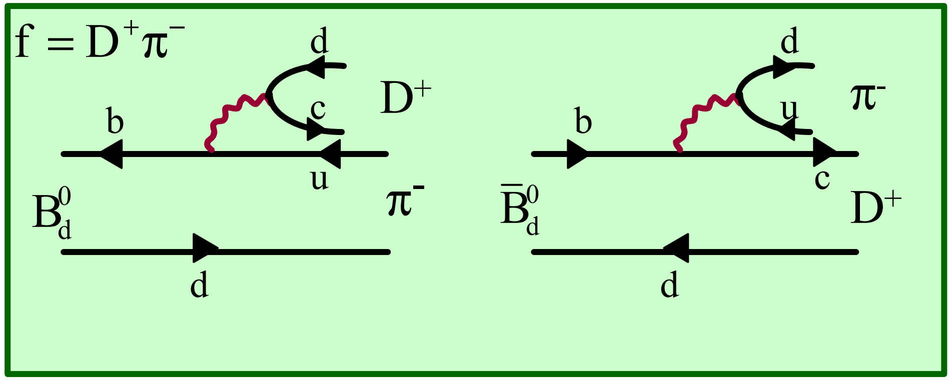
$\overline{B}^0 \searrow$, $\updownarrow$, $B^0 \nearrow$ $\bar{f}$

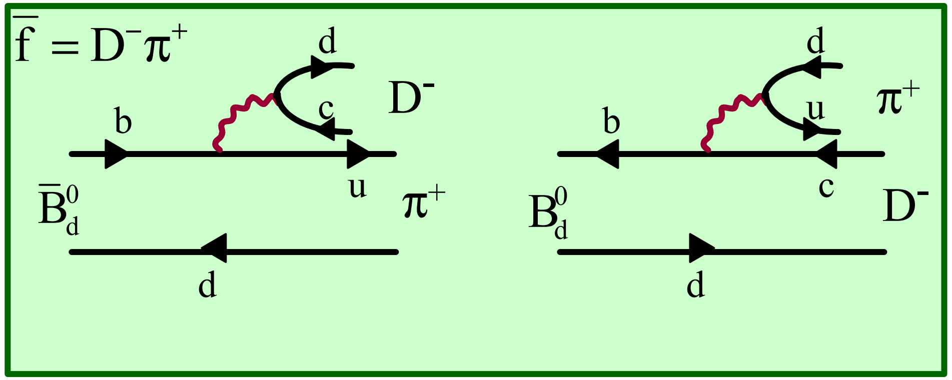
$$\varphi_M = \begin{cases} -\beta & B^0_d \\ -\beta_s & B^0_s \end{cases}$$

$$\xi_f \cdot \xi_{\bar{f}} = F\left(\gamma, \beta_{(s)}\right)$$

(Dunietz, Sachs)

$B_d^0 \to D^{\pm}\pi^{\mp},\ \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$ and γ





$\left(M_f A\lambda^4 R_b e^{i\gamma}\right)$ $\left(\bar{M}_f A\lambda^2\right)$ $\left(\bar{M}_{\bar{f}} A\lambda^4 R_b e^{-i\gamma}\right)$ $\left(M_{\bar{f}} A\lambda^2\right)$

$$\xi_f^{(d)} = e^{-i2\beta}\frac{A\left(\bar{B}_d^0 \to f\right)}{A\left(B_d^0 \to f\right)} = e^{-i(2\beta+\gamma)}\frac{1}{\lambda^2 R_b}\frac{\bar{M}_f}{M_f}$$

$$\xi_{\bar{f}}^{(d)} = e^{-i2\beta}\frac{A\left(\bar{B}_d^0 \to \bar{f}\right)}{A\left(B_d^0 \to \bar{f}\right)} = e^{-i(2\beta+\gamma)}\lambda^2 R_b \frac{\bar{M}_{\bar{f}}}{M_{\bar{f}}}$$

$$\bar{M}_f = M_{\bar{f}} \qquad M_f = \bar{M}_{\bar{f}}$$

Hadronic Matrix Elements

$$\xi_f^{(d)} \cdot \xi_{\bar{f}}^{(d)} = e^{-i2(2\beta+\gamma)}$$

$2\beta+\gamma$ without hadronic uncertainties

(β known)

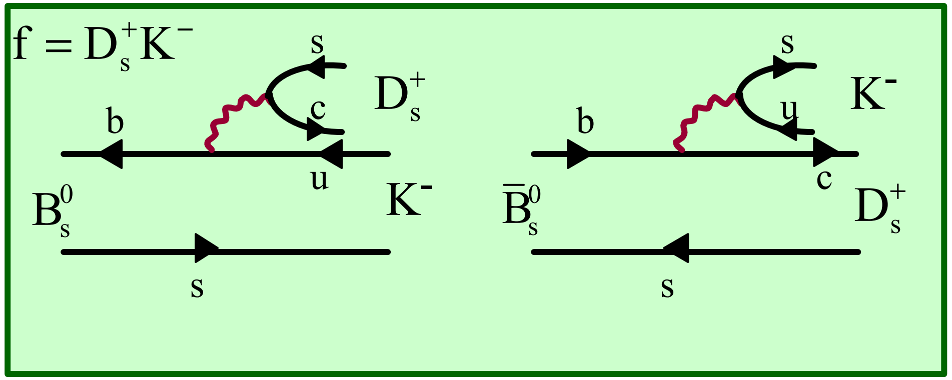
γ

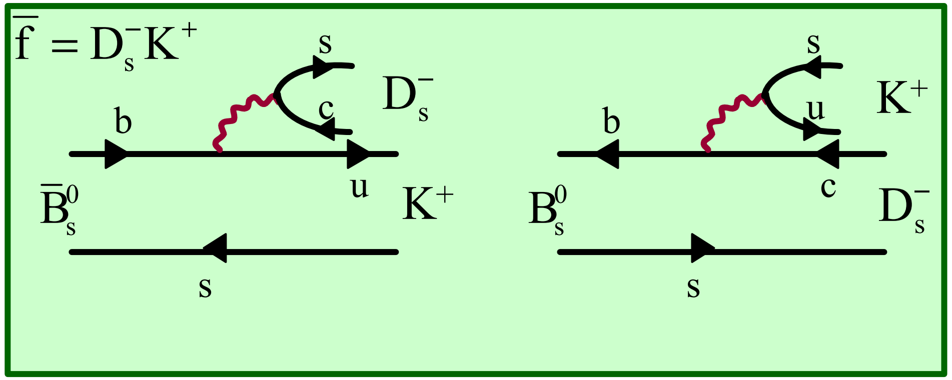
Small Interference: difficult exp. task

Aleksan
Dunietz
Kayser

$B_s^0 \to D_s^\pm K^\mp,\ \bar{B}_s^0 \to D_s^\pm K^\mp$ and γ

Directly obtained from $B_d^0,\ \bar{B}_d^0 \to D^\pm \pi^\mp$ through $d \to s$





$\left(M_f A\lambda^3 R_b e^{i\gamma}\right)$ $\left(\bar{M}_f A\lambda^3\right)$ $\left(\bar{M}_{\bar{f}} A\lambda^3 R_b e^{-i\gamma}\right)$ $\left(M_{\bar{f}} A\lambda^3\right)$

In analogy to $B_d^0,\ \bar{B}_d^0 \to D^\pm \pi^\mp$

$$\xi_f^{(s)} \cdot \xi_{\bar{f}}^{(s)} = e^{-i2(2\beta_s+\gamma)}$$

➡ $2\beta_s + \gamma$ without hadronic uncertainties ➡ γ

β_s from $B_s^0 \to \varphi\psi$

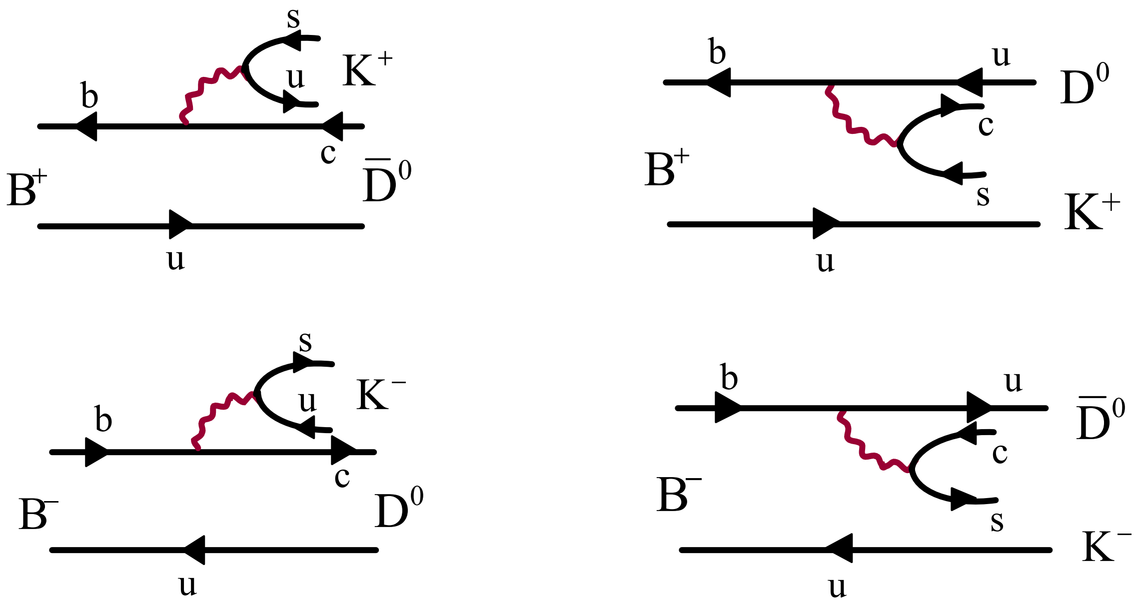
β_s – phase in $B_s^0 - \bar{B}_s^0$

Much bigger interference than in $B_d^0,\ \bar{B}_d^0 \to D^\pm \pi^\mp$

$B^{\pm} \to D^0 K^{\pm},\ \bar{D}^0 K^{\pm}$ and γ

(Gronau + Wyler)

Directly obtained from $B_s^0,\ \bar{B}_s^0 \to D_s^{\pm} K^{\pm}$ through $B_s \to B^{\pm}$



$K^+\bar{D}^0 \neq K^+D^0$

Need

$B^+ \to D_+^0 K^+$

$D_+^0 = \frac{1}{\sqrt{2}}\left(D^0 + \bar{D}^0\right)$

To each process only single diagram contributes

$A\left(B^+ \to \bar{D}^0 K^+\right) = A\left(B^- \to D^0 K^-\right)$

$A\left(B^+ \to D^0 K^+\right) = A\left(B^- \to \bar{D}^0 K^-\right) e^{2i\gamma}$

$0\left(A\lambda^3\right)$

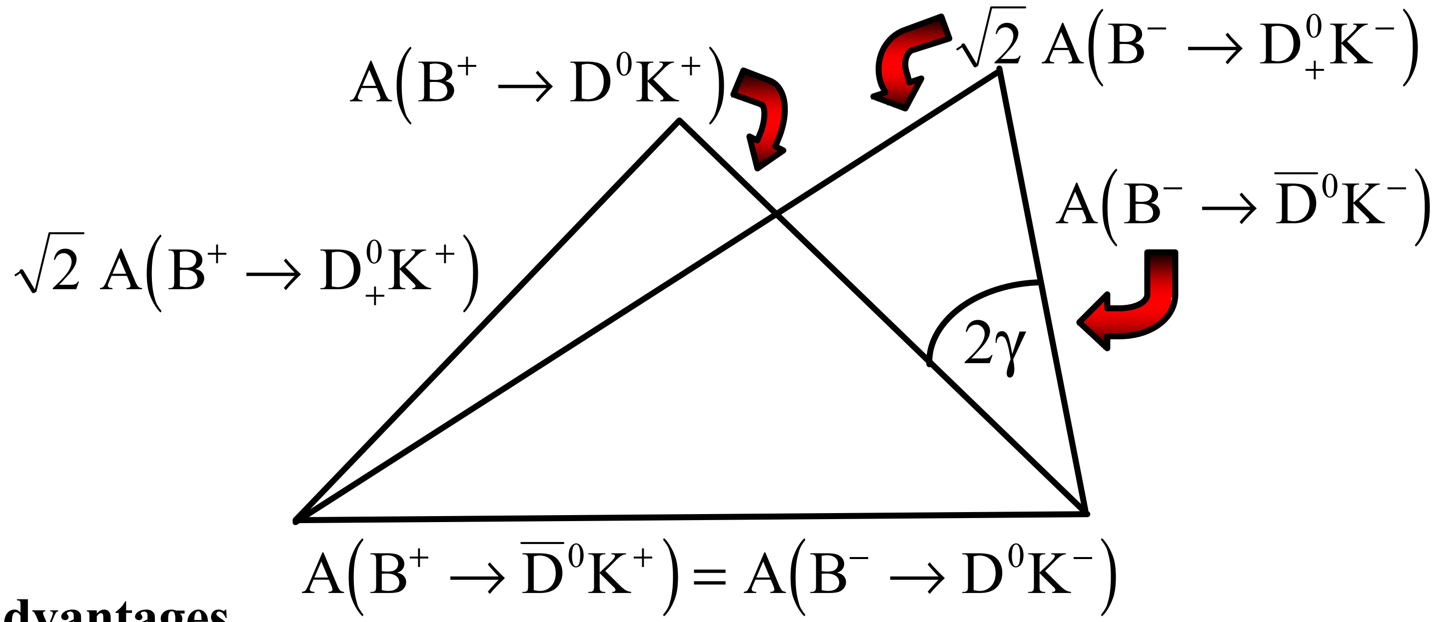
$0\left(A\lambda^3 R_b\right)$ Colour suppressed

Gronau-Wyler Method for γ

$$\sqrt{2}\,A(B^+ \to D^0_+ K^+) = A(B^+ \to D^0 K^+) + A(B^+ \to \overline{D}^0 K^+)$$

$$\sqrt{2}\,A(B^- \to D^0_+ K^-) = A(B^- \to \overline{D}^0 K^-) + A(B^- \to D^0 K^-)$$

$$D^0_+ = \frac{1}{2}\left(|D^0\rangle + |\overline{D}^0\rangle\right) \qquad CP = +$$



Advantages

- Pure Trees
- No tagging
- No time dependent measurements
- Only rates

Disadvantages

- $Br(B^+ \to D^0 K^+) \sim 0(10^{-6})$
- $Br(B^+ \to \overline{D}^0 K^+) \sim 0(10^{-4})$
- Detection of D^0_+

Other clean Strategies for γ and β

Gronau + London; Fleischer

Analogous arguments as in:

$B_d^0 \to D^{\pm}\pi^{\mp},\ \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$ $(2\beta+\gamma)$

$B_s^0 \to D_s^{\pm}K^{\mp},\ \bar{B}_s^0 \to D_s^{\pm}K^{\mp}$ $(2\beta_s+\gamma)$

$B^{\pm} \to D^0K^{\pm},\ \bar{D}^0K^{\pm}$ (γ)

$B_d^0 \to K_sD^0,\ K_s\bar{D}^0$ $(2\beta+\gamma),\ \gamma$

$B_d^0 \to \pi^0D^0,\ \pi^0\bar{D}^0$ $(2\beta+\gamma),\ \gamma$

$B_s^0 \to \varphi D^0,\ \varphi\bar{D}^0$ $(2\beta_s+\gamma),\ \gamma$

$B_s^0 \to K_sD^0,\ K_s\bar{D}^0$ $(2\beta_s+\gamma),\ \gamma$

$B^{\pm} \to D^0\pi^{\pm},\ \bar{D}^0\pi^{\pm}$ (γ)

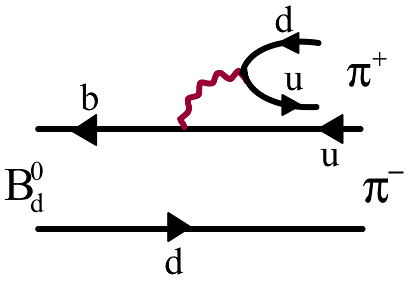
$B_c^{\pm} \to D^0D_s^{\pm},\ \bar{D}^0D_s^{\pm}$ (γ)

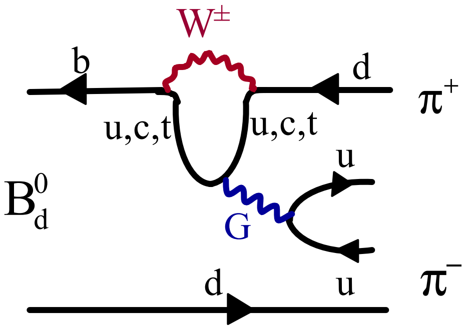
$B_c^{\pm} \to D^0D^{\pm},\ \bar{D}^0D^{\pm}$ (γ)

$(2\beta+\gamma)$, $(2\beta_s+\gamma)$: Time dependence tagging

γ : Rates only

$B_d^0 \to \pi^+\pi^-$ and α





$$V_{ub}^* V_{ud} = A\lambda^3 R_b e^{i\gamma}$$
$$V_{cb}^* V_{cd} = A\lambda^3$$
$$V_{tb}^* V_{td} = -V_{ub}^* V_{ud} - V_{cb}^* V_{cd}$$

$$A\left(B_d^0 \to \pi^+\pi^-\right) = V_{ub}^* V_{ud}\left(A_T + P_u\right) + V_{cb}^* V_{cd} P_c + V_{tb}^* V_{td} P_t$$
$$= V_{ub}^* V_{ud}\left(A_T + P_u - P_t\right) + V_{cb}^* V_{cd}\left(P_c - P_t\right)$$

$$\left|\frac{V_{cb}^* V_{cd}}{V_{ub}^* V_{ud}}\right| = \frac{1}{R_b} \approx 0(2)$$
$$\frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P_{\pi\pi}}{T_{\pi\pi}} \quad ?$$

Dominance of a single amplitude uncertain

Assuming

$$\frac{P_{\pi\pi}}{T_{\pi\pi}} \ll 1$$

$$\varphi_D = \gamma \qquad \varphi_M = -\beta \qquad \left|\xi_{\pi\pi}\right| = 1$$

$$a_{CP}^{mix} = \eta_{\pi\pi} \sin 2\left(\varphi_D - \varphi_M\right) = \sin 2\left(\gamma + \beta\right) = -\sin 2\alpha$$

$$a_{CP}^{dir} = 0 \qquad C_{\pi\pi} = 0 \qquad S_{\pi\pi} = \sin 2\alpha$$

First Results for $B_d^0 \to \pi^+\pi^-$

$$C_{\pi\pi} = \begin{cases} -0.19 \pm 0.19 \pm 0.05 & (\text{BaBar}) \\ -0.77 \pm 0.27 \pm 0.08 & (\text{Belle}) \end{cases}$$

Consistent with 0

$\not{C}\not{P}$

$$S_{\pi\pi} = \begin{cases} -0.40 \pm 0.22 \pm 0.03 & (\text{BaBar}) \\ -1.23 \pm 0.41 \pm 0.07 & (\text{Belle}) \end{cases}$$

Consistent with 0

$\not{C}\not{P}$

World Average: $C_{\pi\pi} = -0.38 \pm 0.16 \qquad S_{\pi\pi} = -0.58 \pm 0.20$

Isospin analysis (Gronau + London)
Model independent determination of α

Model independent upper bound
(Grossman, Quinn; Charles)

$$\sin^2(\alpha_{eff} - \alpha) \le \frac{Br(B^0 \to \pi^0\pi^0)}{Br(B^+ \to \pi^+\pi^0)}$$

$$\sin 2\alpha_{eff} \equiv \frac{\mathrm{Im}\,\xi_{\pi\pi}}{|\xi_{\pi\pi}|}$$

Model dependent determination of α using $(P_{\pi\pi}/T_{\pi\pi})_{TH}$

Beneke, Buchalla, Neubert, Sachrajda: small $C_{\pi\pi}$

Keum,Li,Sanda: large $C_{\pi\pi}$

Most recent:
Buchalla, Safir;
AJB, Fleischer, Recksiegel, Schwab
Gronau, Rosner et al.
Ali, Lunghi, Parkhomenko

U-Spin Strategies

(d↔s)

Fleischer:

$$\left\{\begin{array}{l} B_d^0 \to \pi^+\pi^- \\ B_s^0 \to K^+K^- \end{array}\right\} \Rightarrow \beta, \gamma$$

$$\left\{\begin{array}{l} B_d^0 \to J/\psi K_s \\ B_s^0 \to J/\psi K_s \end{array}\right\} \Rightarrow \gamma$$

$$\left\{\begin{array}{l} B_d^0 \to D^+D^- \\ B_s^0 \to D_s^+D_s^- \end{array}\right\} \Rightarrow \gamma$$

Uncertainty from U-Spin breaking

Gronau + Rosner; Chiang Wolfenstein:

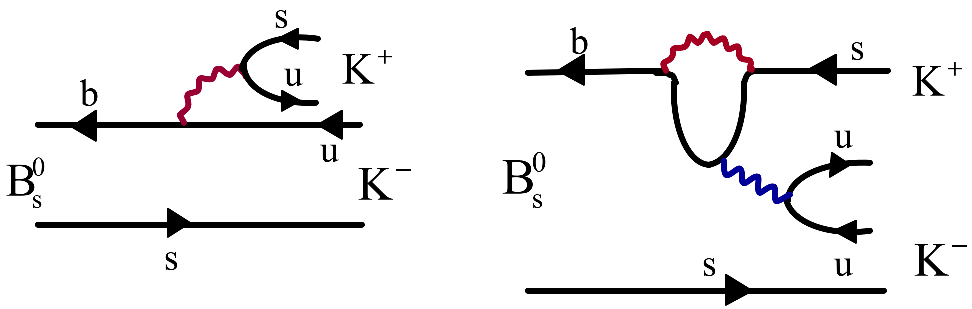
$$\left\{\begin{array}{l} B_d^0 \to \pi^-K^+ \\ B_s^0 \to \pi^+K^- \end{array}\right\} \Rightarrow \gamma$$

Uncertainty from U-Spin breaking, rescattering, colour suppressed EW-Penguins

$B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ (β and γ)

(Fleischer)

$\left\{\text{Replace in } B_d^0 \to \pi^+\pi^- : \; d \to s\right\}$



$$V_{ub}^* V_{us} = A\lambda^4 e^{i\gamma} R_b$$
$$V_{cb}^* V_{cs} = A\lambda^2$$
$$V_{tb}^* V_{ts} = -V_{ub}^* V_{us} - V_{cb}^* V_{cs}$$

$$A\left(B_s^0 \to K^+K^-\right) = V_{ub}^* V_{us}\left(A'_T + P'_u - P'_t\right) + V_{cb}^* V_{cs}\left(P'_c - P'_t\right)$$

U-Spin Symmetry:

$$\frac{P_{\pi\pi}}{T_{\pi\pi}} = \frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P'_c - P'_t}{A'_T + P'_u - P'_t} = \frac{P_{KK}}{T_{KK}} \equiv d e^{i\delta}$$

← strong phase

$a_{CP}^{mix}\left(B_d^0 \to \pi^+\pi^-\right)$ $\quad a_{CP}^{mix}\left(B_s^0 \to K^+K^-\right)$

$a_{CP}^{dir}\left(B_d^0 \to \pi^+\pi^-\right)$ $\quad a_{CP}^{dir}\left(B_s^0 \to K^+K^-\right)$

⇒

d, δ, β, γ
subject to U-Spin
breaking corrections

$(\beta_s \text{ from } B_s \to J/\psi\varphi)$

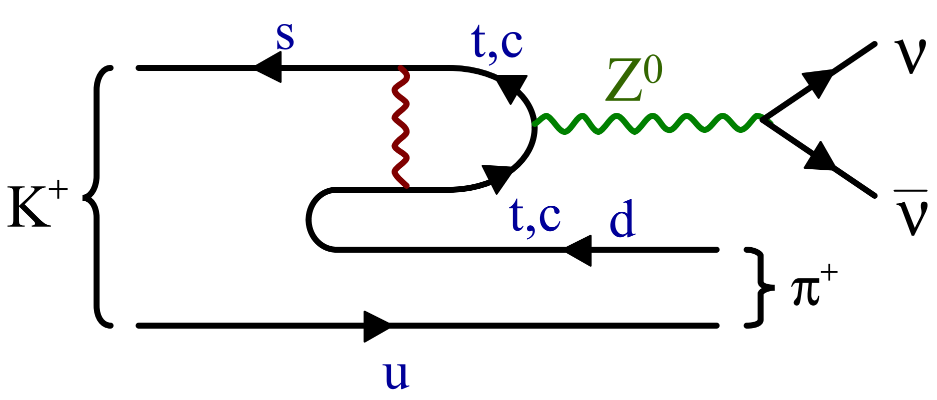
β present in $B_d^0 - \bar{B}_d^0$ mixing

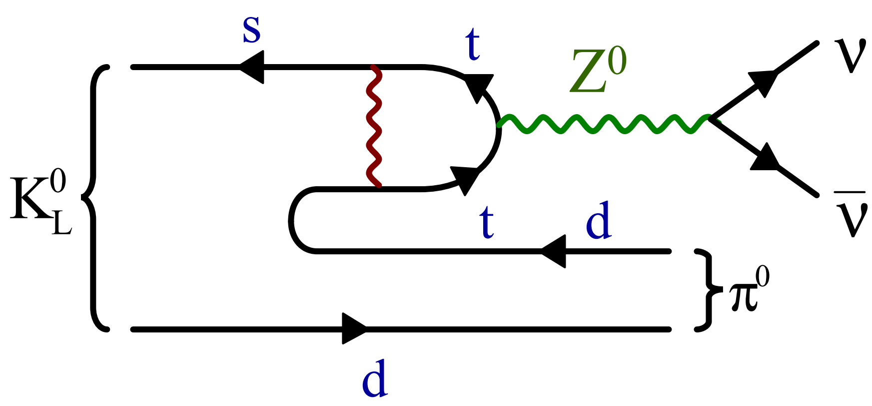
6.

$$K_L \to \pi^0 \nu \bar{\nu}$$

$$K^+ \to \pi^+ \nu \bar{\nu}$$

Decays $K \to \pi \nu \bar{\nu}$





Isospin Symmetry

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$$\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

Leading Decay: $K^+ \to \pi^0 e^+ \nu$

Isospin Breaking:

Marciano, Parsa

$K^+(10\%)$ $K_L(5\%)$

Long Distance: $\leq$1-2%

Rein + Seghal
Hagelin + Littenberg
Lu + Wise; Fajfer
Geng, Hsu, Lin
Buchalla, Isidori
Falk, Lewandowski, Petrov

Waiting for Precise Measurements of $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$

AJB, Schwab, Uhlig (04)

1. Present Status within SM (TH and Parametric Uncertainties)

2. Impact of Present and Future Measurements of $K \to \pi \nu \bar{\nu}$ on CKM

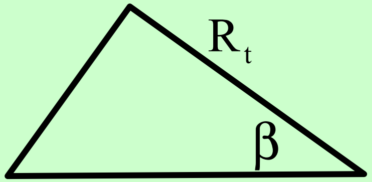
3. $K \to \pi \nu \bar{\nu}$ in Scenarios with New Complex Phases in EWP and $B_d^0 - \bar{B}_d^0$ Mixing

4. Interplay of $K \to \pi \nu \bar{\nu}$ with
   - ~~CP~~ in B Decays
   - $\Delta M_d / \Delta M_s$, Rare Decays

Basic Formulae for $K^+ \to \pi^+ \nu\bar{\nu}$ (SM)

$X \equiv X(m_t)$

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = 4.8 \cdot 10^{-11}\left[A^4 R_t^2 X^2 + 2 P_c A^2 R_t X \cos\beta + P_c^2\right]$$



$$= 10^{-11}[\ \underset{\text{(top)}}{4.1} + \underset{\text{(top-charm)}}{3.0} + \underset{\text{(charm)}}{0.7}\ ]$$

$$A = \frac{|V_{cb}|}{\lambda^2} \simeq 0.83$$

Buchalla AJB (94) NLO*)

$$P_c = 0.389 \pm \underbrace{0.033}_{\Delta m_c = 50\,\text{MeV}} \pm \underbrace{0.045}_{\text{scale } \mu_c} \pm \underbrace{0.010}_{\alpha_s} \simeq 0.39 \pm 0.07$$

AJB Schwab Uhlig (04)

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = \left[7.8 \pm \underbrace{0.8}_{P_c} \pm \underbrace{0.9}_{CKM}\right]10^{-11} \simeq (7.8 \pm 1.2)10^{-11}$$

(Isidori)

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = \left[14.7\ {}^{+13.0}_{-8.9}\right]10^{-11}$$

E787 (2)
E949 (1)
3 Events

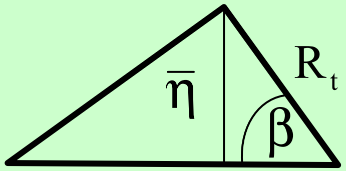
*) NNLO : AJB, Gorbahn, Haisch (04 Summer ?)

(Direct $\not{CP}$)

Basic Formulae for $K_L \to \pi^0 \nu\bar{\nu}$

(SM) Buchalla AJB (NLO)

$$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) = 3.0 \cdot 10^{-11} \left[\frac{\bar{\eta}}{0.35}\right]^2 \left[\frac{|V_{cb}|}{41.5 \cdot 10^{-3}}\right]^4 \left[\frac{X}{1.53}\right]^2$$



$$= \left(3.0 \pm \overbrace{0.6}^{\text{CKM}}\right) \cdot 10^{-11}$$

(AJB Schwab Uhlig)

KTeV : $Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) < 5.9 \cdot 10^{-7}$

Future: E391a, KOPIO, JHF

Model independent bound (Grossman, Nir)

$$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) \leq 4.4\, Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \leq 2 \cdot 10^{-9} \ (90\%\text{C.L.})$$

E391a could get the first non-trivial upper bound.

$$\bar{\eta} = R_t \sin\beta$$

KOPIO: ~ 50 Events

E391 (JHF): ~ 1000 Events

Theoretically clean Relations

D´Ambrosio + Isidori (02)

$$Br\left(K^{+}\to\pi^{+}\nu\bar{\nu}\right)=\bar{\kappa}_{+}\left|V_{cb}\right|^{4}X^{2}\left[R_t^2\sin^2\beta+\left(R_t\cos^2\beta+\frac{\lambda^4 P_c}{\left|V_{cb}\right|^2 X}\right)^2\right]$$

$$R_t\sim\xi\frac{\sqrt{\Delta M_d}}{\sqrt{\Delta M_s}}\qquad \bar{\kappa}_{+}=7.64\cdot 10^{-6}\qquad P_c=0.39\pm 0.07$$

AJB, Schwab, Uhlig (04)

$$Br\left(K^{+}\to\pi^{+}\nu\bar{\nu}\right)=\bar{\kappa}_{+}\left|V_{cb}\right|^{4}X^{2}\left[T_1^2+\left(T_2+\frac{\lambda^4 P_c}{\left|V_{cb}\right|^2 X}\right)^2\right]$$

$$T_1=\frac{\sin\beta\sin\gamma}{\sin(\beta+\gamma)}\qquad T_2=\frac{\cos\beta\sin\gamma}{\sin(\beta+\gamma)}$$

Golden Relations

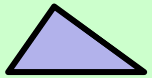
(All involving $K_L \to \pi^0 \nu\bar{\nu}$)

Buchalla, AJB (94)

$$\cot\beta = \frac{\sqrt{B_1 - B_2} - P_c}{\sqrt{B_2}} \quad \Rightarrow \quad (\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$$

$$B_1 \sim Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)$$

$$B_2 \sim Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)$$

$J_{CP} \sim$  $\sim \dfrac{\sqrt{Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)}}{X(m_t)}$

AJB, Schwab, Uhlig (04)

$$\frac{\sin\beta \sin\gamma}{\sin(\beta+\gamma)} = 0.35 \left[\frac{1.53}{X(m_t)}\right]\left[\frac{0.0415}{|V_{cb}|}\right]^2 \sqrt{\frac{Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)}{3 \cdot 10^{-11}}}$$

β from $a_{\psi K_S}$

γ from $\begin{matrix} B_s^0 \to D_s^{\pm} K^{\mp} \\ B_d^0 \to D^{\pm} \pi^{\mp} \end{matrix}$

Anatomy of $|V_{td}|$ from $K^+ \to \pi^+ \nu\bar{\nu}$

AJB
Schwab
Uhlig

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = 0.39\frac{\sigma(P_c)}{P_c} + 0.70\frac{\sigma(Br(K^+))}{Br(K^+)} + \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

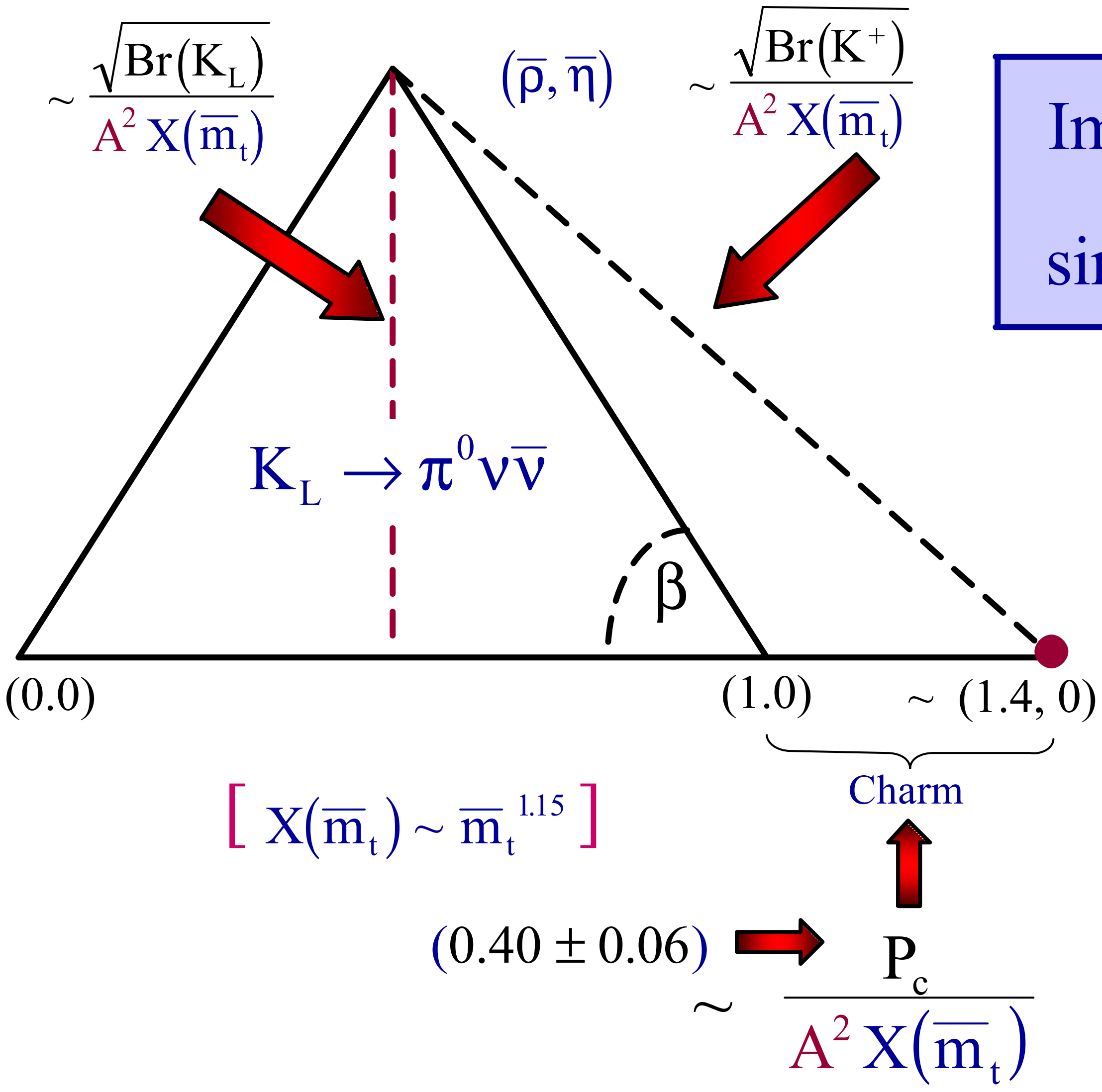
Present:	± 7%	± (Very Large)	± 2%	
$\sigma(Br(K^+)) = 10\%$, $\sigma(P_c) = 0.03$	± 3%	± 7%	± 1.4%	(Scenario I)
$\sigma(Br(K^+)) = 5\%$, $\sigma(P_c) = 0.02$	± 2%	± 3.5%	± 1%	(Scenario II)

Determination
at 4-5% possible

$|V_{td}|$ from UT fit:
6-12% dependently
on the error analysis

UT from $K \rightarrow \pi\nu\bar{\nu}$

Buchalla
AJB



$$\mathrm{Im}\,\lambda_t = F_1(\overline{m}_t, Br(K_L))$$

$$\sin 2\beta = F_2(P_c, Br(K_L), Br(K^+))$$

$$\lambda_t = V_{ts}^* V_{td}$$

$\sin 2\beta$ $(K \rightarrow \pi\nu\bar{\nu})$ ⟷ $\sin 2\beta$ $(B \rightarrow J/\psi, K_s)$ $\rightarrow \varphi K_s$

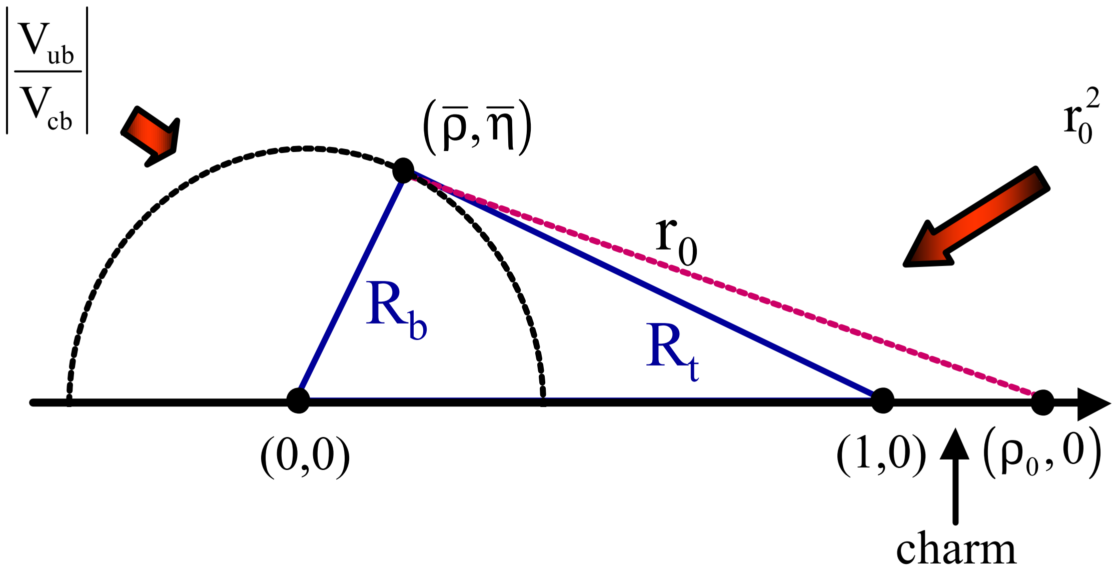
K-Physics ⟷ B - Physics

Test of SM and Beyond

$K^+ \to \pi^+ \nu\bar{\nu}$ in the $(\bar{\rho}, \bar{\eta})$ Plane

$$Br\left(K^+ \to \pi^+\nu\bar{\nu}\right) = 4.31\cdot 10^{-11} A^4 X^2(m_t) \frac{1}{\sigma}\left[(\sigma\bar{\eta})^2 + (\rho_0 - \bar{\rho})^2\right]$$

$$\sigma = \frac{1}{\left(1-\lambda^2/2\right)^2} \qquad \rho_0 = 1 + \frac{P_c}{A^2 X(m_t)} \approx 1.4$$



$$r_0^2 = \frac{1}{A^4 X^2(m_t)}\left[\frac{\sigma Br\left(K^+ \to \pi^+\nu\bar{\nu}\right)}{4.31\cdot 10^{-11}}\right]$$

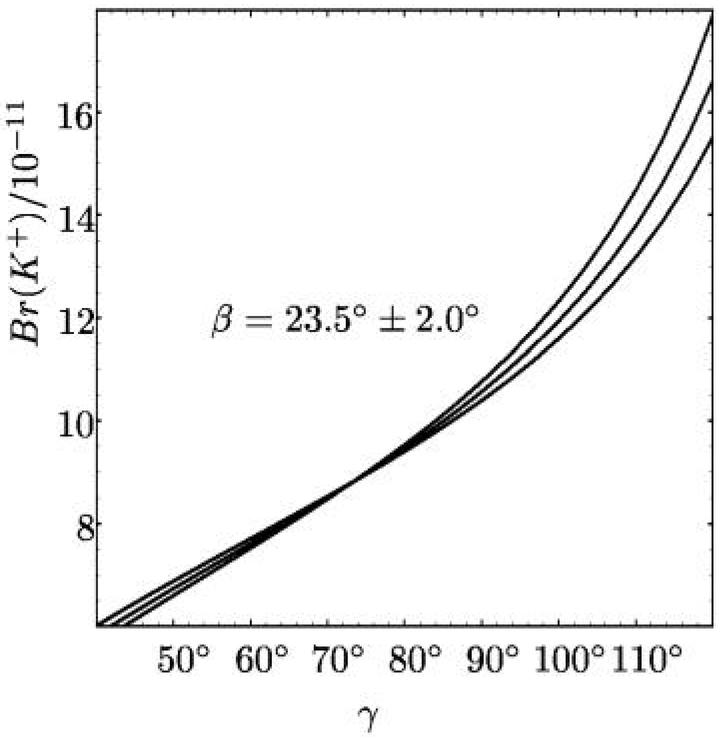
$$R_t = 1 + R_b^2 - 2\bar{\rho}$$

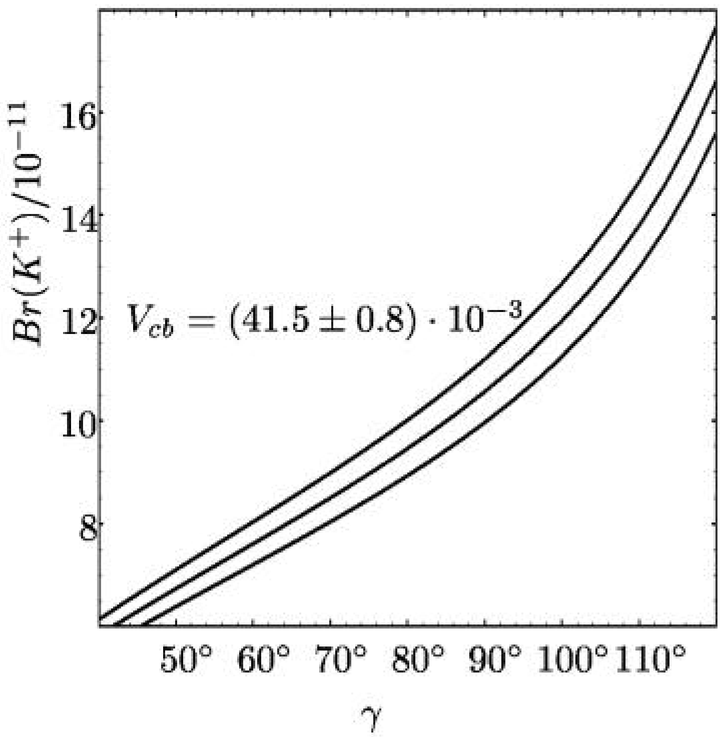
$$V_{td} = A\lambda^3\left(1-\bar{\rho}-i\bar{\eta}\right)$$

$$|V_{td}| = \lambda |V_{cb}| R_t$$

β and $|V_{cb}|$ Dependence of $Br(K^+ \to \pi^+ \nu \bar{\nu})$

BSU (04)





The Angle β from $K \to \pi\nu\bar{\nu}$

Buchalla, AJB (94)
AJB, Schwab, Uhlig (04)

BSU:

$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = 0.31\frac{\sigma(P_c)}{P_c} + 0.55\frac{\sigma(Br(K^+))}{Br(K^+)} \pm 0.39\frac{\sigma(Br(K_L))}{Br(K_L)}$$

$\sigma(\sin 2\beta) =$	$\pm\, 0.041$	$\pm\, ?$	$\pm\, ?$	(Present)
$\sigma(\sin 2\beta) =$	0.017	$\pm\, 0.039$	$\pm\, 0.028$	(Scenario I) Br′s at 10%
$\sigma(\sin 2\beta) =$	0.011	$\pm\, 0.020$	$\pm\, 0.014$	(Scenario II) Br′s at 5%

TH very clean

$\sigma(\sin 2\beta) \approx 0.02 - 0.03$ requires $\sigma(Br's) \leq 5\%$

The Angle γ from $K \to \pi\nu\bar{\nu}$

AJB, Schwab, Uhlig (04)

$$\frac{\sigma(\gamma)}{\gamma} = 0.75\frac{\sigma(P_c)}{P_c} + 1.32\frac{\sigma(Br(K^+))}{Br(K^+)} + 0.07\frac{\sigma(Br(K_L))}{Br(K_L)} + 4.1\frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

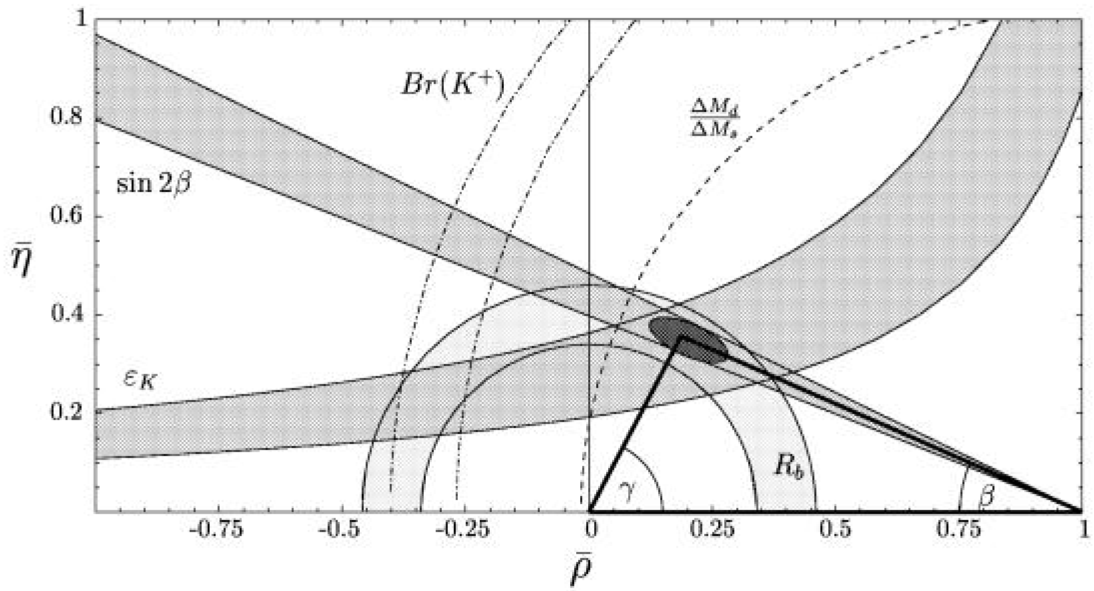
$\sigma(\gamma) =$	$\pm 8.3^\circ$	$\pm ?$	$\pm ?$	$\pm 4.9^\circ$	(Present)
$\sigma(\gamma) =$	$\pm 3.7^\circ$	$\pm 8.5^\circ$	$\pm 0.4^\circ$	$\pm 3.8^\circ$	(Scenario I) Br's at 10%
$\sigma(\gamma) =$	$\pm 2.5^\circ$	$\pm 4.2^\circ$	$\pm 0.2^\circ$	$\pm 2.5^\circ$	(Scenario II) Br's at 5%

TH very clean

$\sigma(\gamma) \approx \pm 5^\circ$ requires $\sigma(Br(K^+)) \le 5\%$

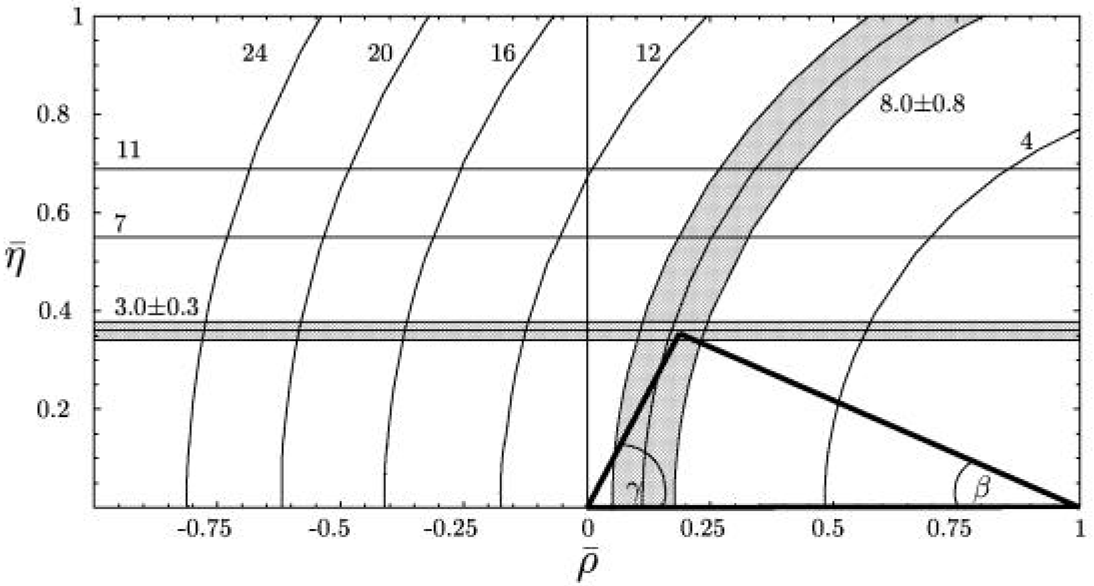
Unitarity Triangle 2004

(AJB, Schwab, Uhlig)



Unitarity Triangle from $K \rightarrow \pi\nu\bar{\nu}$

(2010)



7.

$B \to \pi\pi$ and $B \to \pi K$ Puzzles

Basic Structure

AJB, Fleischer, Recksiegel, Schwab (03,04)

Basic Scenario : New Physics enters dominantly through EW Penguins including new complex phases

AJB, Colangelo, Isidori, Romanino, Silvestrini (00)
Buchalla, Hiller, Isidori (01)

Step 1. : B ® ππ Decays described within SM
(EW –Penguins small)
→ (Isospin Symmetry, Data) → Hadronic Parameters in $d,\ \theta,\ x,\ \Delta$ → Sizable Departures from existing Non-Perturbative Frameworks

Step 2. : $d,\ \theta,\ x,\ \Delta$ + $SU(3)_F$ → Hadronic Parameters in B ® πK → (B ® pK, Data) → Enhanced EWP with large New Complex Phase + γ $(65 \pm 7)^{\circ}$

Step 3. : Correlations between B ® πK, Rare K and B Decays and other Processes ↔ Implications for Rare K and B Decays sensitive to EWP

The B→ππ Puzzles

(10^{-6}) $\mathrm{Br}(B^{\pm} \to \pi^{\pm}\pi^{0}) = 5.3 \pm 0.8$ $\mathrm{Br}(B_d^{0} \to \pi^{+}\pi^{-}) = 4.6 \pm 0.4$ $\mathrm{Br}(B_d \to \pi^{0}\pi^{0}) = 1.9 \pm 0.5$

QCDF/Data: ~ 1 OK ~ 2 (?) ~ 1/6 (?)

TH Puzzle

$$R_{+-}^{\pi\pi} \equiv 2\left[\frac{\mathrm{Br}(B^{\pm} \to \pi^{\pm}\pi^{0})}{\mathrm{Br}(B_d \to \pi^{\pm}\pi^{\mp})}\right]\frac{\tau(B_d^{0})}{\tau(B^{+})} = 2.12 \pm 0.37$$

(QCDF: ~ 1.2)

$$R_{00}^{\pi\pi} \equiv 2\left[\frac{\mathrm{Br}(B_d \to \pi^{0}\pi^{0})}{\mathrm{Br}(B_d \to \pi^{\pm}\pi^{\mp})}\right] = 0.83 \pm 0.23$$

(QCDF: ~ 0.07)

EXP Puzzle

$$A_{CP}^{dir}(\pi^{+}\pi^{-}) \equiv C = \begin{cases} -0.19 \pm 0.19 \pm 0.05 & \text{(BaBar)} \\ -0.77 \pm 0.27 \pm 0.08 & \text{(Belle)} \end{cases}$$

$$A_{CP}^{mix}(\pi^{+}\pi^{-}) \equiv -S = \begin{cases} 0.40 \pm 0.22 \pm 0.03 & \text{(BaBar)} \\ 1.23 \pm 0.41 \, {}^{+0.07}_{-0.08} & \text{(Belle)} \end{cases}$$

⇒ HFAG Averages

-0.38 ± 0.16

$+0.58 \pm 0.20$

$\left\{ \begin{array}{c} \text{Isospin} \\ \text{Symmetry} \end{array} \right\}$

B→ππ Amplitudes

(Tiny EWP neglected)

Hadronic
4 Parameters
(d, θ, x, Δ)

Strong Phases

$$\sqrt{2}A\left(B^+ \to \pi^+\pi^0\right) = -\left[\tilde{T} + \tilde{C}\right] = -\left[T + C\right]$$

$$A\left(B_d^0 \to \pi^+\pi^-\right) = -\left[\tilde{T} + P\right]$$

$$\sqrt{2}A\left(B_d^0 \to \pi^0\pi^0\right) = -\left[\tilde{C} - P\right]$$

$$de^{i\theta} = -\left|\frac{P}{\tilde{T}}\right| e^{i(\delta_P - \delta_{\tilde{T}})}$$

$$xe^{i\Delta} = -\left|\frac{\tilde{C}}{\tilde{T}}\right| e^{i(\delta_{\tilde{C}} - \delta_{\tilde{T}})}$$

$$P = \lambda^3 A\left(P_t - P_c\right) \equiv \lambda^3 A\ P_{tc}$$

$$\tilde{T} = \underbrace{\lambda^3 A\ R_b e^{i\gamma}\left[T_0\right.}_{T} - \left(P_{tu} - \varepsilon\right)\left.\right]$$

$$\tilde{C} = \underbrace{\lambda^3 A\ R_b e^{i\gamma}\left[C_0\right.}_{C} + \left(P_{tu} - \varepsilon\right)\left.\right]$$

AJB, Fleischer (95)
Ciuchini et al (97)
Isola et al (01)

Could be important!!

If $(P_{tu} - \varepsilon)$ important
$\tilde{T}$ ↓ $\tilde{C}$ ↑

BFRS

$Br\left(B_d^0 \to \pi^+\pi^-\right)$ ↓

$Br\left(B_d^0 \to \pi^0\pi^0\right)$ ↑

P_q – QCD Penguins (include annihilation)

T_0 (C_0) – colour-allowed (suppressed) Trees

ε - Exchange topologies

$Br\left(B^\pm \to \pi^+\pi^0\right)$ →

Determination of Hadronic Parameters: d, q, x, D

Input : $\gamma = (65 \pm 7)^{o}$ $\varphi_d = 2\beta = (47 \pm 4)^{o}$

$$A_{CP}^{dir}\left(\pi^+\pi^-\right) = F_1\left(d,\theta,\gamma\right) \overset{exp}{=} -0.38 \pm 0.16$$

$$A_{CP}^{mix}\left(\pi^+\pi^-\right) = F_2\left(d,\theta,\varphi_d,\gamma\right) \overset{exp}{=} -0.58 \pm 0.20$$

$$R_{+-}^{\pi\pi} = F_3\left(d,\theta,x,\Delta,\gamma\right) \overset{exp}{=} 2.12 \pm 0.37$$

$$R_{00}^{\pi\pi} = F_4\left(d,\theta,x,\Delta,\gamma\right) \overset{exp}{=} 0.83 \pm 0.23$$

$$d = 0.48^{+0.35}_{-0.22}$$

$$\theta = \left(138^{+19}_{-23}\right)^{O}$$

$$x = 1.22^{+0.26}_{-0.21}$$

$$\Delta = -\left(71^{+19}_{-26}\right)^{O}$$

What are the values of d, θ, x, Δ telling us ?
How can we use them ?

BFRS:
(0312259)
(0402181)

Output for Hadronic Parameters (ππ)

$$de^{i\theta} = -\left|\frac{P}{\tilde{T}}\right| e^{i(\delta_P - \delta_{\tilde{T}})} = \left(0.48 \begin{array}{l} +0.35 \\ -0.22 \end{array}\right) e^{i\left(138^{+19}_{-23}\right)^{\circ}}$$

$$xe^{i\Delta} = \left|\frac{\tilde{C}}{\tilde{T}}\right| e^{i(\delta_{\tilde{C}} - \delta_{\tilde{T}})} = \left(1.22 \begin{array}{l} +0.26 \\ -0.21 \end{array}\right) e^{-i\left(71^{+19}_{-26}\right)^{\circ}}$$

- ☆ Large **non-factorizable** contributions: (P_{tu} - ε)
- ☆ Large **strong phases**

Pattern
Confirmed by Ali, Lunghi, Parkhomenko (0403275)

The B→πK Puzzle

Already present since 2000 (AJB + Fleischer)

$$R_c \equiv 2\left[\frac{Br(B^+\to\pi^0K^+)+Br(B^-\to\pi^0K^-)}{Br(B^+\to\pi^+K^0)+Br(B^-\to\pi^-\bar{K}^0)}\right]=1.17\pm0.12$$

$$R_n \equiv \frac{1}{2}\left[\frac{Br(B_d^0\to\pi^-K^+)+Br(\bar{B}_d^0\to\pi^+K^-)}{Br(B_d^0\to\pi^0K^0)+Br(\bar{B}_d^0\to\pi^0\bar{K}^0)}\right]=0.76\pm0.10$$

$$R \equiv \left[\frac{Br(B_d^0\to\pi^-K^+)+Br(\bar{B}_d^0\to\pi^+K^-)}{Br(B^+\to\pi^+K^0)+Br(\bar{B}^-\to\pi^-\bar{K}^0)}\right]\frac{\tau(B^+)}{\tau(B_d^0)}=0.91\pm0.07$$

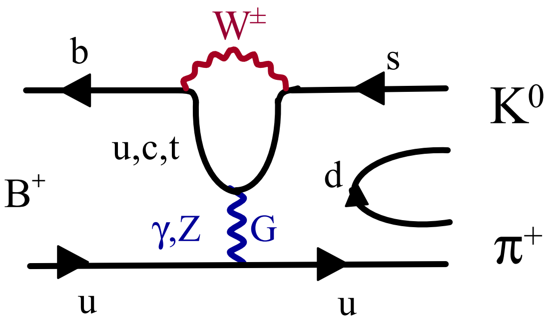
CLEO
Belle
BaBar

R and R_c consistent with expectations within SM !

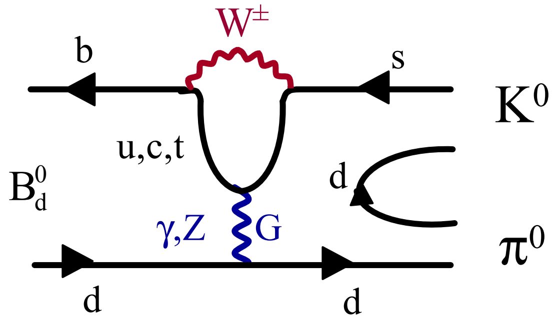
But the pattern $R_c > 1$ and $R_n < 1$ very puzzling!

In most approaches $R_n \approx R_c$ expected.

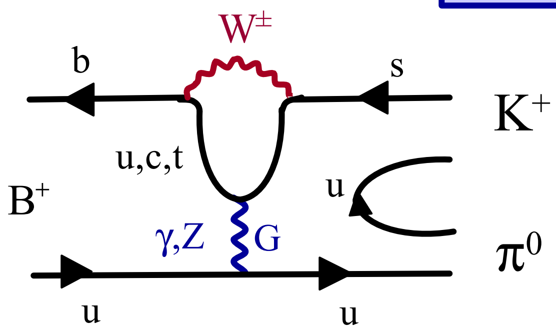
$B^+ \to \pi^+ K^0$

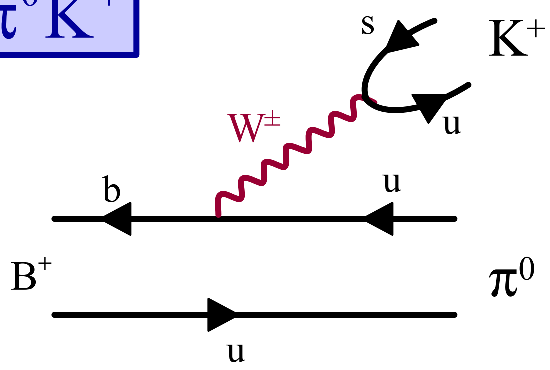


$B_d^0 \to \pi^0 K^0$



$B^+ \to \pi^0 K^+$

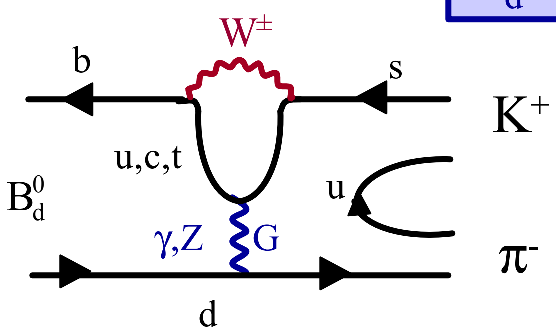


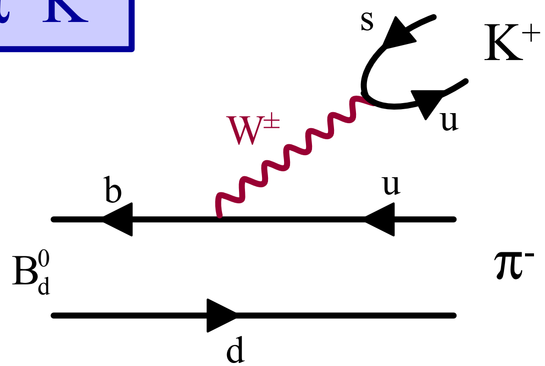


Penguins: λ^2 (c,t)
(P) $\lambda^4 e^{i\gamma}$ (u)

Trees: $\lambda^4 e^{i\gamma}$
(T)

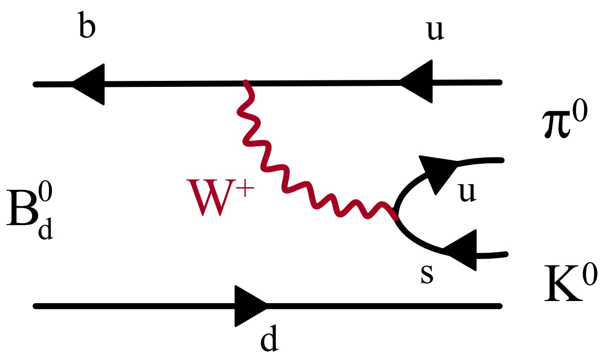
$B_d^0 \to \pi^- K^+$





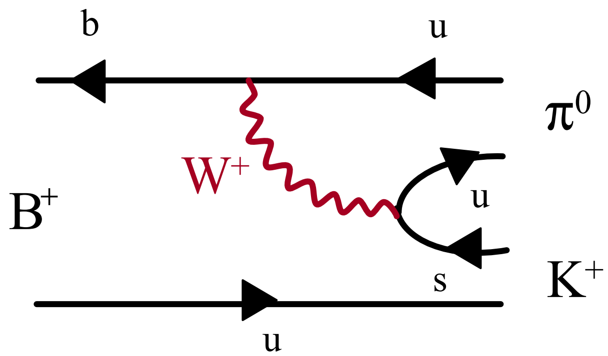
Colour suppressed Tree Topologies (C)

$B_d^0 \to \pi^0 K^0$



$\lambda^4 e^{i\gamma}$

$B^+ \to \pi^0 K^+$



$\lambda^4 e^{i\gamma}$

Determining EWP from $B \to \pi K$: $q\, e^{i\theta} e^{iw}$

AJB, Fleischer
Recksiegel, Schwab

Parametrization of EWP : $q\, e^{i\theta} e^{i\omega}$

θ = New CP-violating phase!

ω = strong phase

SM: $q \simeq 0.69, \quad \theta \simeq 0, \quad \omega \simeq 0$ Neubert Rosner

Step 1 : $B \to \pi\pi$ Analysis $\xrightarrow{SU(3)_F}$ (including factorizable breaking) All hadronic parameters of $B \to \pi K$ system determined

Independent of EWP

Step 2 : $R + A_{CP}^{dir}\left(B_d \to \pi^{\mp} K^{\pm}\right) \overset{exp}{=} 0.095 \pm 0.028$ ⇒ $\gamma = \left(64.7 {}^{+6.3}_{-6.9}\right)^{O}$

In accordance with UT fits

Step 3 :

$$R_c = F_5(q, \theta, w) \overset{exp}{=} 1.17 \pm 0.12$$

$$R_n = F_6(q, \theta, w) \overset{exp}{=} 0.76 \pm 0.10$$

$$A_{CP}^{dir}\left(\pi^0 K^{\pm}\right) = F_7(q, \theta, w) \overset{exp}{=} 0.00 \pm 0.07$$

⇒

$$q = 1.74 {}^{+1.28}_{-0.95}$$

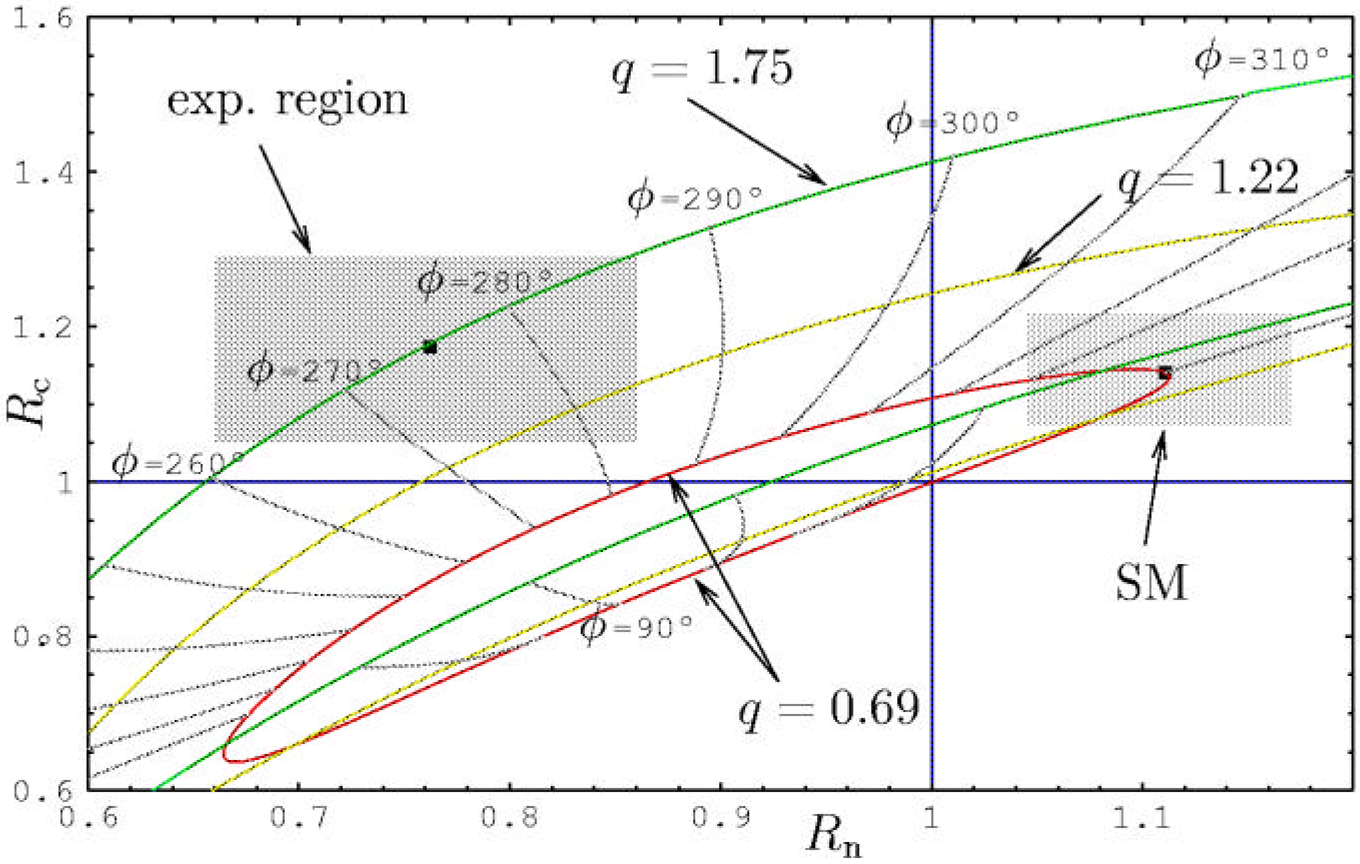
$$\varphi = -\left(85 {}^{+11}_{-15}\right)^{O}$$

$$\omega = -\left(4 {}^{+44}_{-32}\right)^{O}$$

☆

LARGE NEW ~~CP~~ PHASE !!

R_c versus R_n in the Presence of New $\not{CP}$ EWP



$P_{EW} = q\ e^{i\varphi}$ SM: $q = 0.69$ $\varphi = 0$

AJB, Fleischer, Recksiegel, Schwab

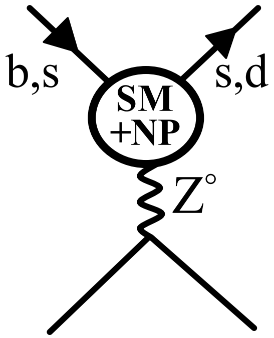
8.

Impact of a New ~~CP~~ Phase on Rare Decays

Relation: $B \to \pi K$ ⟺ Rare K and B Decays

BFRS

Fundamental
Short Distance
Physics of EWP :



$$\sim C(\underbrace{m_t, \tilde{m}, \text{new phases}}_{v})$$

$$B \to \pi K$$
$$(q, \varphi, \omega)$$

$$K \to \pi \nu \bar{\nu},\ B \to X_S \mu^+ \mu^-$$
$$K_L \to \pi^0 e^+ e^-,\ B \to \mu^+ \mu^-$$
$$B \to X_S \nu \bar{\nu}$$
$$\begin{pmatrix} X = C + \text{boxes} \\ Y = C + \text{boxes} \end{pmatrix}$$

Renormalization group :

$$|C(v)| e^{i\theta_C} = 2.35\ q e^{i\varphi} - 0.82$$

SM :

$$0.80 = 2.35 \cdot \underbrace{0.69}_{\text{Neubert + Rosner}} - 0.82$$

Implications for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^o \nu\bar{\nu}$

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (7.8 \pm 1.2)\cdot 10^{-11} \Rightarrow (7.5 \pm 2.1)\cdot 10^{-11}$$

$$Br(K_L \to \pi^o \nu\bar{\nu})_{SM} = (3.0 \pm 0.6)\cdot 10^{-11} \Rightarrow (3.1 \pm 1.0)\cdot 10^{-10}$$

Enhancement of |X| compensated by suppression of "top-charm" interference

Strong Violation of the "Golden" Relation :

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\varphi K_S}$$

$$-\left(0.69 {+0.23 \atop -0.41}\right) \neq 0.74 \pm 0.05$$

Buchalla, AJB (94)

$\sin 2\beta_X$

$$\beta_X = \beta - \theta_X$$

$$X = |X| e^{i\theta_X}$$

Saturation of the model-independent Grossman-Nir bound :

FIG

$$Br(K_L \to \pi^o \nu\bar{\nu}) \leq 4.4 \; Br(K^+ \to \pi^+ \nu\bar{\nu})$$

Here:

$$\frac{Br(K_L \to \pi^o \nu\bar{\nu})}{Br(K^+ \to \pi^+ \nu\bar{\nu})} \approx 4.4 \; \sin^2(\beta_X) \approx 4.2 \pm 0.2$$

Impact of $X = |X| e^{i\theta_X}$ on $K^+ \to \pi^+ \nu\bar{\nu}$

$$Br\left(K^+ \to \pi^+\nu\bar{\nu}\right) = 4.8\cdot 10^{-11}\left[A^4 R_t^2 |X|^2 + 2P_c A^2 R_t |X| \cos(\beta - \theta_X) + P_c^2\right]$$

$$= \quad 10^{-11}\ [4.1 \quad + \quad 3.0 \quad + \quad 0.7] \qquad X \simeq 1.53$$

$$= \quad 10^{-11}\ [8.5 \quad - \quad 1.7 \quad + \quad 0.7] \qquad X = 2.2 e^{-i86^o}$$

(top) (charm-top) (charm) $\beta\text{-}\theta_X \approx 110^o$

$$Br\left(K^+ \to \pi^+\nu\bar{\nu}\right) = (7.8 \pm 1.2)\cdot 10^{-11} \Rightarrow (7.5 \pm 2.1)\cdot 10^{-11}$$

(SM)

Impact of $X = |X| e^{i\theta_X}$ on $K_L \to \pi^o \nu\bar{\nu}$

$$\frac{Br(K_L \to \pi^o \nu\bar{\nu})}{Br(K_L \to \pi^o \nu\bar{\nu})_{SM}} = \underbrace{\left|\frac{X}{X_{SM}}\right|^2}_{\approx 2} \underbrace{\left[\frac{\sin(\beta - \theta_X)}{\sin\beta}\right]^2}_{\approx 5}$$

$\beta\text{-}\theta_X \approx 110^o$

$\beta \simeq 23$

$$Br(K_L \to \pi^o \nu\bar{\nu}) = (3.0 \pm 0.6) \cdot 10^{-11} \Rightarrow (3.1 \pm 1.0) \cdot 10^{-10}$$

(SM)

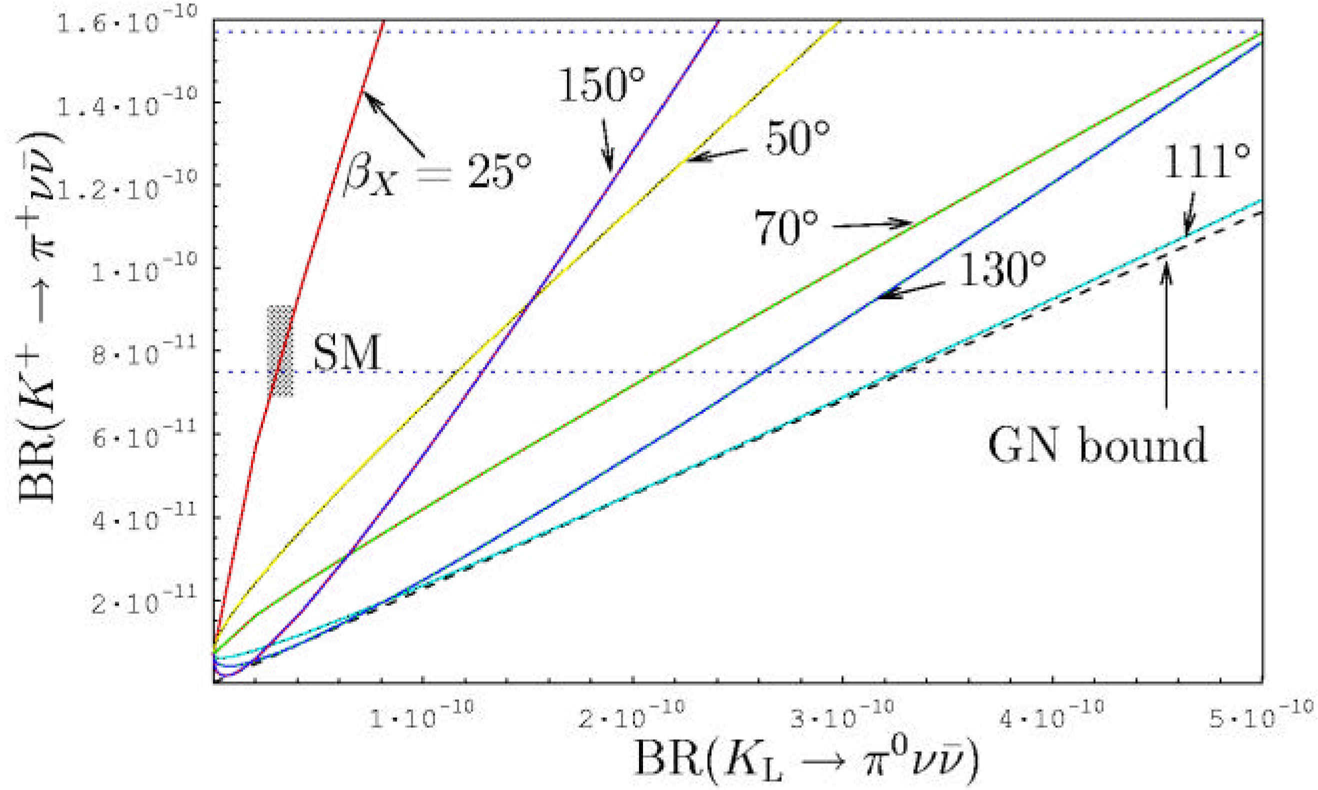
$$\frac{Br(K_L \to \pi^o \nu\bar{\nu})}{Br(K^+ \to \pi^+ \nu\bar{\nu})} = \underset{(SM)}{0.4} \Rightarrow 4.2$$

< 4.4 Grossman-Nir bound

Order of magnitude enhancement !!

$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ with $X=|X|e^{i\theta_x}$

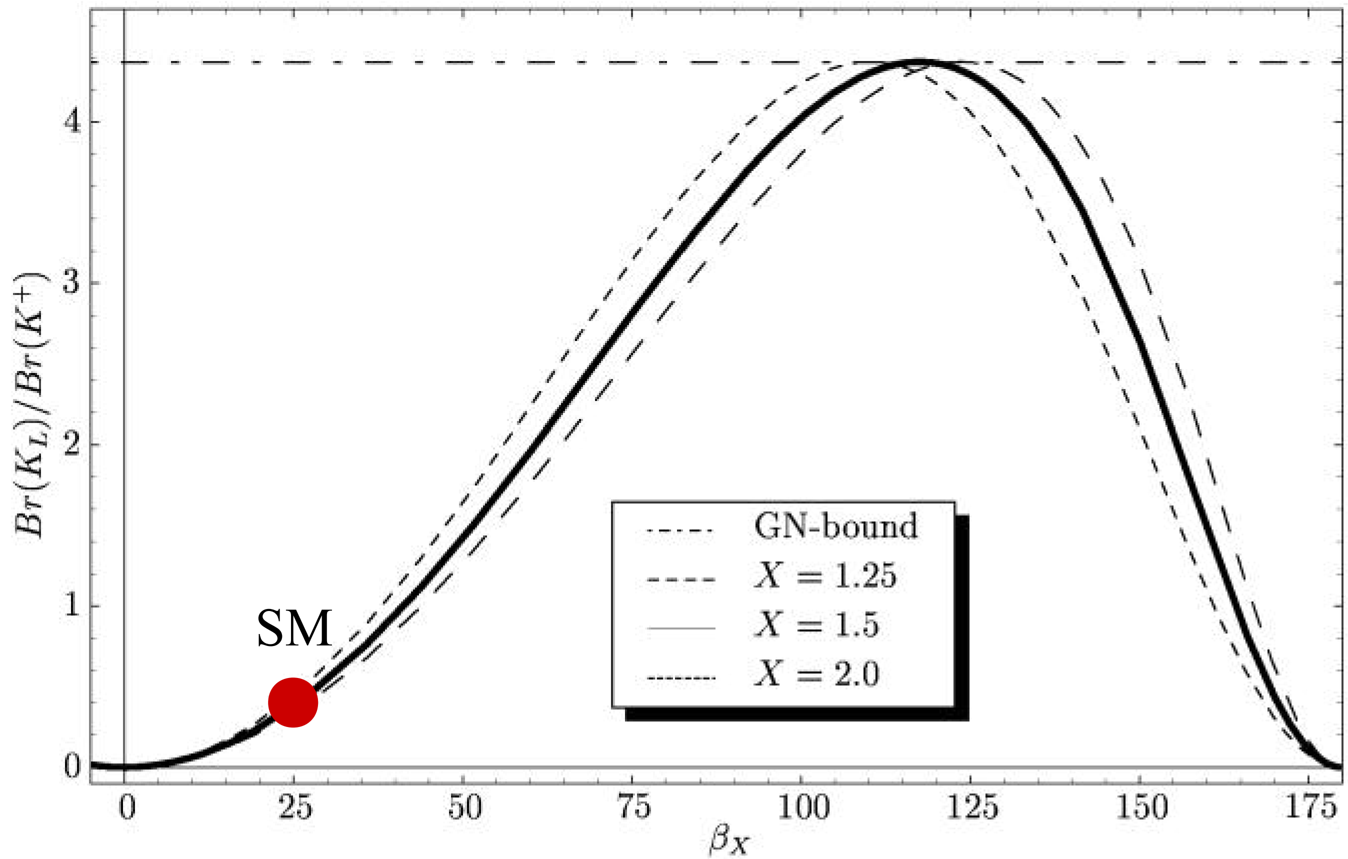
$$\beta_x = \beta - \theta_x$$



AJB, Fleischer, Recksiegel, Schwab

$$\frac{Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)}{Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)} \text{ versus } \beta_X$$

BSU (04)



Other Impacts on Rare K and B Decays

$$\frac{Br\left(B \to X_{s,d}\nu\bar{\nu}\right)}{Br\left(B \to X_{s,d}\nu\bar{\nu}\right)_{SM}} \approx 2.0$$

$$\frac{Br\left(B_{s,d} \to \mu^+\mu^-\right)}{Br\left(B_{s,d} \to \mu^+\mu^-\right)_{SM}} \approx 5.0$$

Spectacular Effects in
FB CP Asymmetry in
$B_d \to K^*\mu^+\mu^-$

Lepton polarization
asymmetries in $b \to s l^+ l^-$
(Choudhury, Gaur, Cornell (04))

BUT: $(\sin 2\beta)_{\varphi K_S} > (\sin 2\beta)_{\psi K_S}$

Consistent with BaBar
but
differs by sign from Belle

$(-0.96 \pm 0.50 \pm 0.10)$

Impact on $K_L \to \pi^o e^+ e^-$ and $K_L \to \pi^o \mu^+ \mu^-$

$$Br\left(K_L \to \pi^o e^+ e^-\right)_{SM} = \left(3.7 \begin{smallmatrix} +1.1 \\ -0.9 \end{smallmatrix}\right) \cdot 10^{-11}$$

Dominated by indirect $\not{CP}$
(Buchalla, D'Ambrosio, Isidori)
(Friot, Greynat, de Rafael)

→

$$\left(9.0 \pm 1.6\right) \cdot 10^{-11}$$

Dominated by direct $\not{CP}$
BFRS

Isidori
Smith
Unterdorfer

$$Br\left(K_L \to \pi^o \mu^+ \mu^-\right) = \left(1.5 \pm 0.3\right) \cdot 10^{-11}$$

Dominated by CP-conserving
+ indirect $\not{CP}$

→

$$\left(4.3 \pm 0.7\right) \cdot 10^{-11}$$

Dominated by direct $\not{CP}$
ISU

KTeV:

$$Br\left(K_L \to \pi^o e^+ e^-\right) < 2.8 \cdot 10^{-10}$$

$$Br\left(K_L \to \pi^o \mu^+ \mu^-\right) < 3.8 \cdot 10^{-10}$$

9.

Brief Look Beyond Standard Model

What is affected by New Physics?

Assume: 3 Generations Unitarity

1

V_{ud}	V_{us}	
V_{cd}	V_{cs}	V_{cb}
		V_{tb}

All determined in tree level processes → Essentially independent of New Physics

2

$$\frac{V_{ts}}{V_{cb}} = -1 + \underbrace{\frac{1}{2}\lambda^2\left[1 - 2(\rho + i\eta)\right]}$$

$(R_b < 0.5)$ → At most 5% effect; Typically: 1% effect } Possible impact of New Physics

$$V_{ts} \cong -V_{cb}$$

Independently of New Physics

3

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

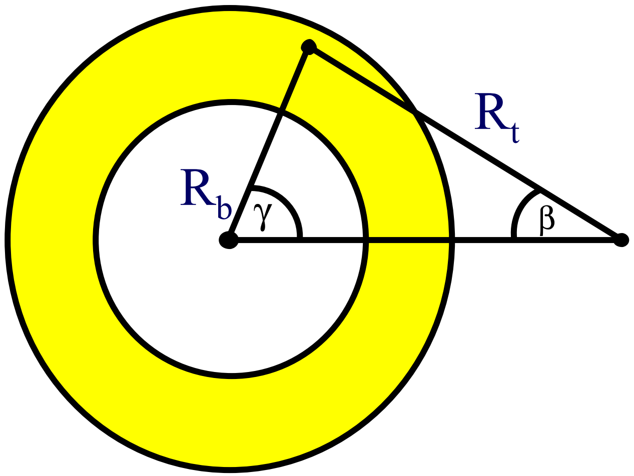
$$V_{td} = |V_{td}| e^{-i\beta}$$

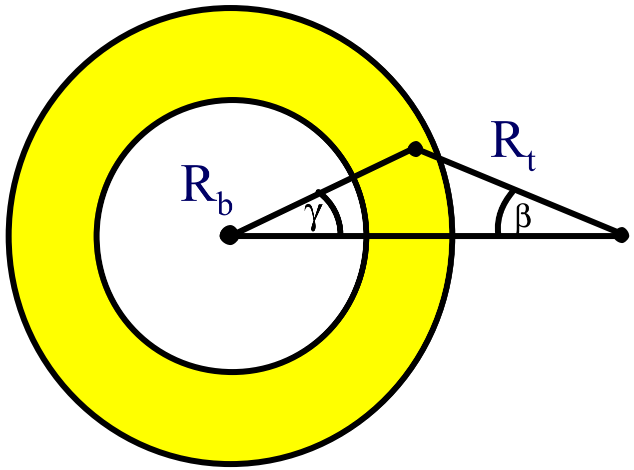
Essentially independent of New Physics ($|V_{ub}|$ and γ)

Can be affected by New Physics ($|V_{td}|$ and β)

Unitarity Connection: $|V_{ub}|e^{-i\gamma} \Leftrightarrow |V_{td}|e^{-i\beta}$

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$





R_b = Independent of New Physics

R_t, β, γ = Can be affected by New Physics

Impact of New Physics

General Comments

- Essentially no impact on tree-decays
- Possible substantial impact on loop induced decays (on short distance physics)

⇓

- No impact on determination of $\lambda \equiv |V_{us}|$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$
- No impact on calculations of non-perturbative parameters (B_i) (except for new B_i's from new operators or new strong forces)
- Possible substantial impact on $\bar{\rho}$, $\bar{\eta}$, $\triangle$, $\not{CP}$, rare decays

Special Features of $\not{C}P$ in SM

1. CP is **<u>explicitly</u>** broken (Yukawa couplings)

2. **Single** complex phase (δ_{KM})

3. $\not{C}P$ only in charged current weak interactions ($W^{\pm}$) of quarks (flavour changing)

⇓

4. CP strongly suppressed in **neutral current** transitions (Z^0,γ,G,H^0) and very strongly suppressed in **flavour diagonal** transitions (electric dipole moments)

5. CP is not an approximate symmetry ($\sin\delta_{KM}$=0(1)).
$\not{C}P$ small only because of small quark mixing angles

Possible Features of ~~CP~~ beyond SM

1. CP is **explicitly** and/or **spontaneously** ($ve^{i\alpha}$) broken
2. **Several** complex phases
3. ~~CP~~ occurs at tree-level in both charged current and **neutral current** (Z^0) interactions of quarks, in lepton interactions, in new sectors beyond SM (extended Higgs, SUSY, ...), in **strong interactions**, in **scalar** interactions, in **flavour diagonal** interactions

⇓

4. Large ~~CP~~ effects in **neutral current** and **flavour diagonal** transitions possible

5. CP is an approximate symmetry (all complex phases small)

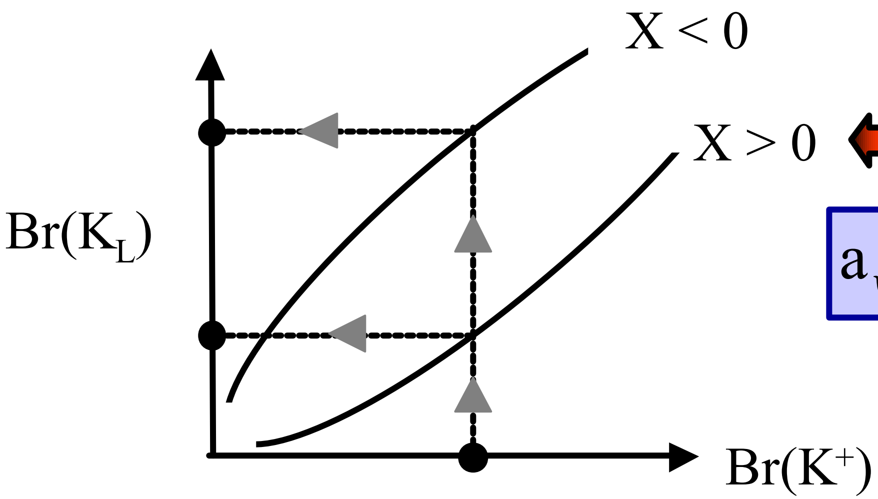
Intriguing Property of Models with Minimal Flavour Violation

AJB, Fleischer (01)

$$Br(K_L) = F\left(Br(K^+),\ a_{\psi K_S},\ sgn(X)\right)$$

TH very clean

Independently of any parameters, for given $Br(K^+)$ and $a_{\psi K_S}$ only <u>two values</u> of $Br(K_L)$ possible.



$a_{\psi K_S}$ = fixed

$$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \le 3.9 \cdot 10^{-10}$$

(90% C.L.)

$a_{\psi K_S} \le 0.8$

$$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) \le \begin{cases} 3.1 \cdot 10^{-10} & X > 0 \\ 4.9 \cdot 10^{-10} & X < 0 \end{cases}$$

KTeV: $\le 5.9 \cdot 10^{-7}$

Results in MSSM

AJB, Gambino, Gorbahn, Jäger, Silvestrini (hep-ph/0007313)

Define: $$T(Q) \equiv \frac{[Q]_{\rm MSSM}}{[Q]_{\rm SM}}$$

$$0.53 \le T(\varepsilon'/\varepsilon) \le 1.07$$

$$0.65 \le T(K^+ \to \pi^+ \nu\bar{\nu}) \le 1.02$$

$$0.41 \le T(K_L \to \pi^0 \nu\bar{\nu}) \le 1.03$$

$$0.48 \le T(K_L \to \pi^0 e^+ e^-) \le 1.10$$

$$0.73 \le T(B \to X_S \nu\bar{\nu}) \le 1.34$$

$$0.68 \le T(B_S \to \mu\bar{\mu}) \le 1.53$$

Constraints on supersymmetric parameters from:

i) $B^0_{d,s} - \bar{B}^0_{d,s}$ mixings, ε, $B \to X_S \gamma$

ii) EW – precision studies

iii) Lower bound on M_{H^0}

Rare K Decays in General SUSY Models

AJB, Colangelo, Isidori, Romanino, Silvestrini (hep 9908371)

Main new effects:

- γ-magnetic Penguins
- Enhanced Z^0-Penguins

Constraints from ΔM_K, ε_K
ε'/ε, $K_L \to \mu\bar{\mu}$ and Renormalization Group

Most probable bounds: (SM)$_{max}$

Most probable bounds	$(SM)_{max}$
$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) \lesssim 1.2 \cdot 10^{-10}$	$0.4 \cdot 10^{-10}$
$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \lesssim 1.7 \cdot 10^{-10}$	$1.1 \cdot 10^{-10}$
$Br\left(K_L \to \pi^0 e^+ e^-\right) \leq 2.0 \cdot 10^{-11}$	$0.7 \cdot 10^{-11}$

Larger values possible, but rather unlikely

Earlier Analyses: Nir, Worah; AJB, Romanino, Silvestrini

10.

Short Outlook

Shopping List 1999-2008

- ★ ε'/ε at $\Delta(\varepsilon'/\varepsilon) = 10^{-4}$
- ★ $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$
- ★ $\sin 2\beta$ from $B \to \psi K_S$, φK_S
- ★ $(\Delta M)_s$ from $B_s^o - \bar{B}_s^o$ Mixing
- ★ $B \to X_{s,d}\mu\bar{\mu}$, $B_{s,d} \to \mu\bar{\mu}$, $B \to X_{s,d}\nu\bar{\nu}$
- ★ $K_{L,S} \to \pi^o e^+ e^-$, $K_L \to \mu e$
- ★ α and γ from B-decays
- ★ Electric Dipole Moment of the Neutron
- ★ Improved Measurements of V_{ub}, V_{cb}, $B \to X_{d,s}\gamma$
- ★ Improved Calculations of Hadronic (Non-Pertubative) Parameters

Parameters in Electroweak Gauge Sector

$$\alpha_{QED},\ G_F,\ \sin^2\theta_W$$

⇓

$$\alpha_{QED},\ G_F,\ M_Z$$

⇓

$$\alpha_{QED},\ M_W,\ M_Z$$

Flavour Sector

Until 2001

$$|V_{us}|,\ |V_{cb}|,\ \bar\rho,\ \bar\eta$$

For the next years

$$|V_{us}|,\ |V_{cb}|,\ R_t,\ \sin 2\beta$$

appears like a better choice.

Or, even better:

$$|V_{us}|,\ |V_{cb}|,\ R_t,\ \beta$$

(AJB
Parodi
Stocchi)

Fundamental Flavour Parameters

AJB, Parodi, Stocchi, hep-ph/0207101 (updated)

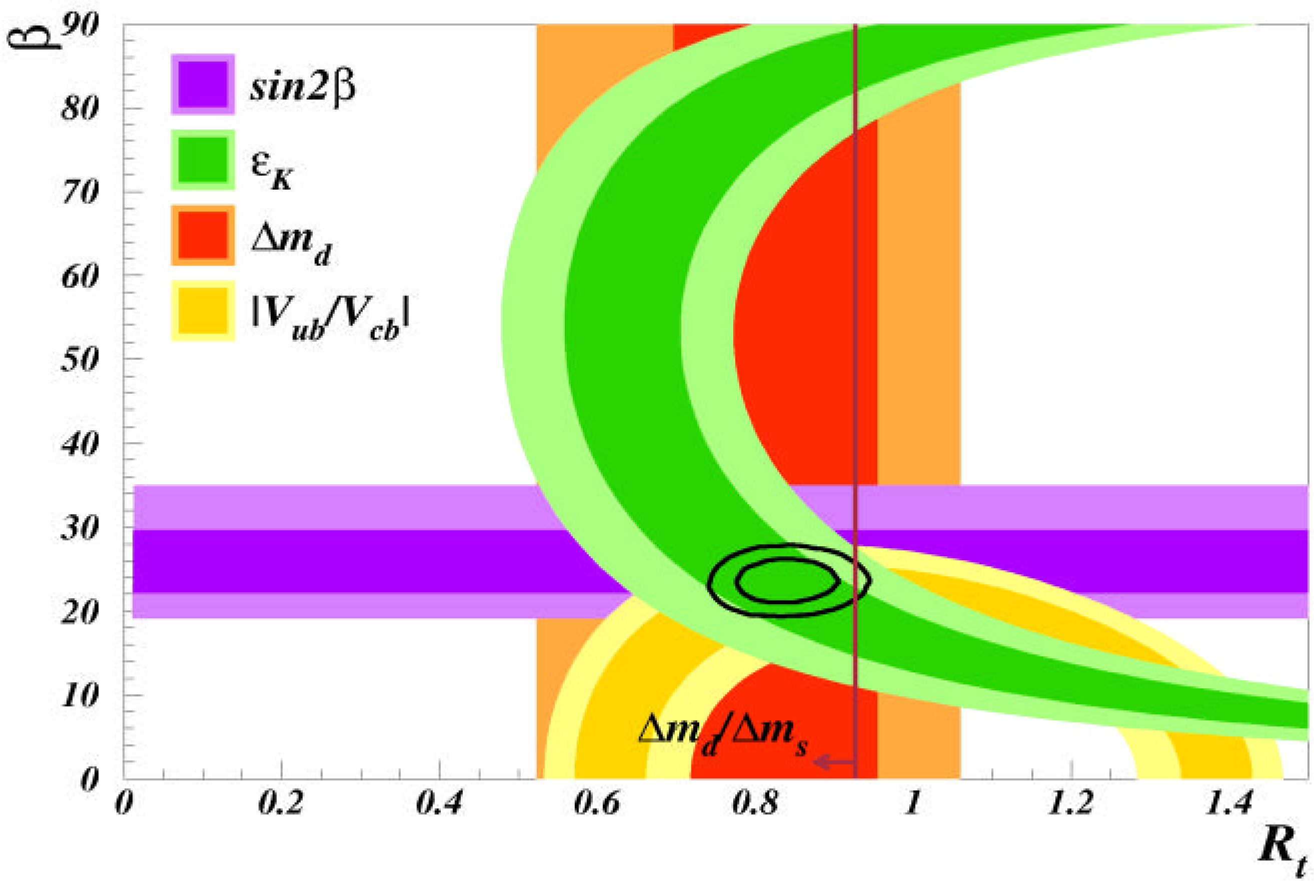
$$\left|V_{us}\right| = 0.2240 \pm 0.0036 \qquad \left|V_{cb}\right| = (41.3 \pm 0.7)\cdot 10^{-3}$$

$$R_t = 0.91 \pm 0.05 \qquad \beta = \begin{cases} (23.6 \pm 2.2)^\circ \; (a_{\psi K_s}) \\ (23.2 \pm 1.4)^\circ \; (\text{total}) \end{cases}$$

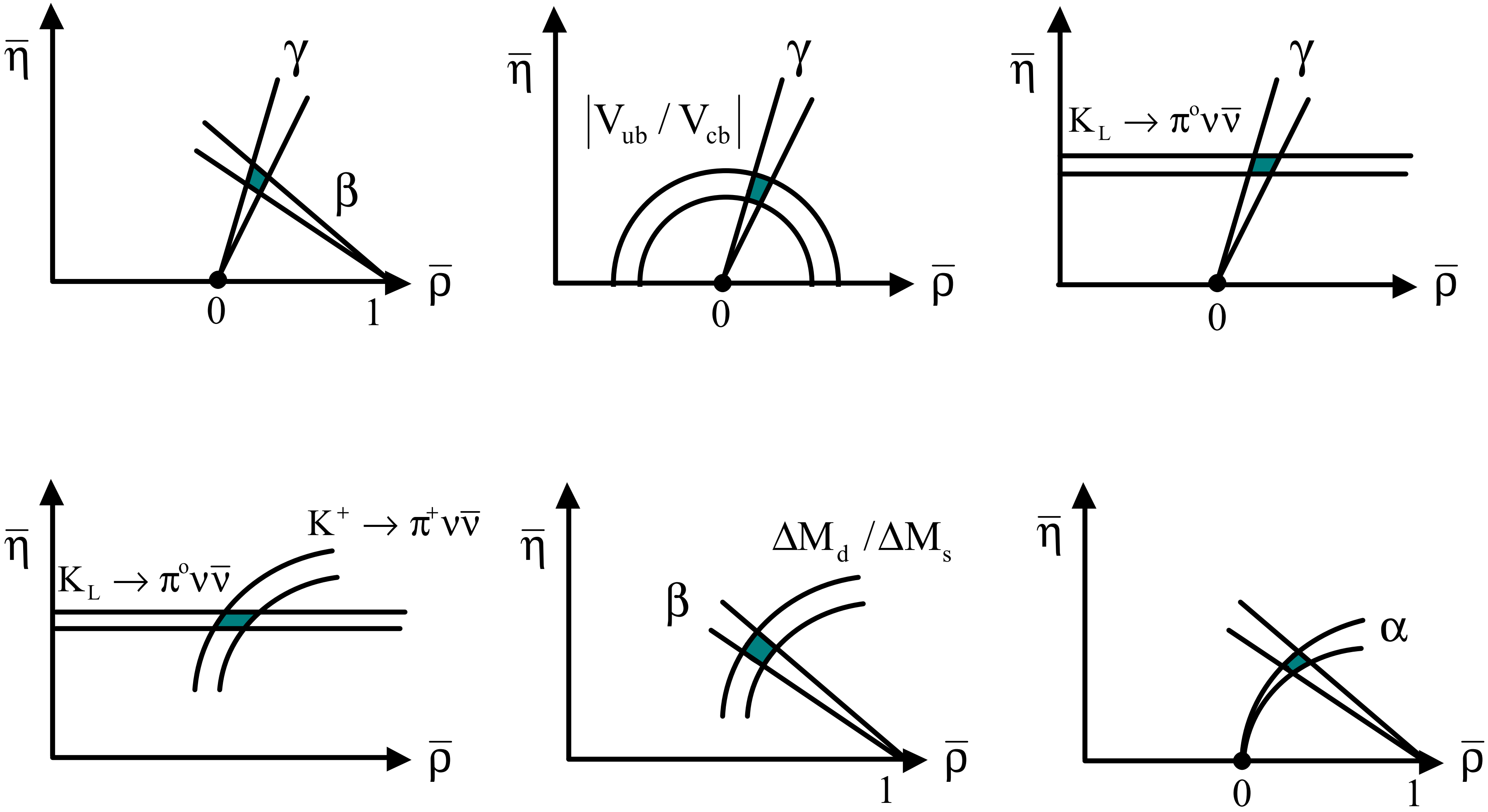
$$\sin 2\beta = \begin{cases} 0.734 \pm 0.054 & (a_{\psi K_s}) \\ 0.725 \pm 0.033 & (\text{total}) \end{cases}$$

(R_t, β) Plot 2002

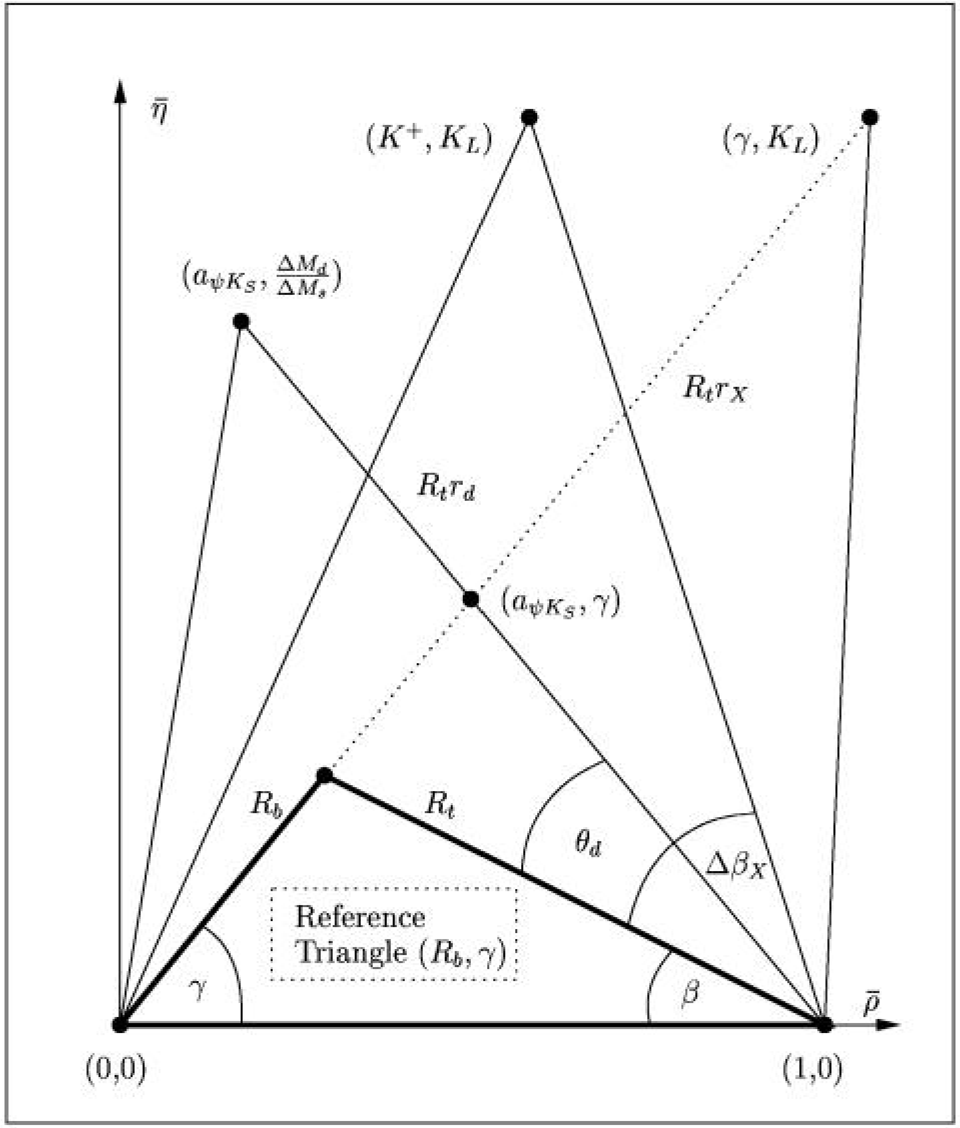
(AJB, Parodi, Stocchi)



Searching for New Physics



The 2011 Vision of the Unitarity Triangle



AJB
Schwab
Uhlig

Shopping List for 2003 - 2004

sin 2β from
$B \to \varphi K_S$
(New Physics?)

Clarification of
CP-Violation in
$B_d^0 \to \pi^+\pi^-$

First Measurements
of ΔM_S and
$B_{s,d} \to \mu\bar{\mu}$ (?)
(Tevatron)

(Improved Calculations
of ξ, $F_{B_d}\sqrt{\hat{B}_d}$, $F_{B_s}\sqrt{\hat{B}_s}$)

Improved
Measurement
of $K^+ \to \pi^+\nu\bar{\nu}$
(Brookhaven)

First Measurements
of γ in
B Decays

Improved Determinations
of $|V_{cb}|$ and $R_b \sim \frac{|V_{ub}|}{|V_{cb}|}$

1989-1999

Electroweak Precision Studies

α_{QED} , G_F , M_Z , m_t , M_W, m_H

($\sin^2\theta_W$)

Spontaneous Symmetry Breakdown

2000-2011

CKM Precision Studies

λ , A , $\bar{\rho}$, $\bar{\eta}$, m_t

with the hope to discover **New Physics**
and learn about **Flavour Dynamics**

The Future until 2011 should be very exciting