Dispersive approach to χPT

LNF Spring school - Bruno Touschek
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Introduction - Chiral perturbation theory

- low energy effective theory (energy compared to $\Lambda \sim 1\,\mathrm{GeV}$)
- basic degrees of freedom : π, K and η = pseudo-Goldstone bosons of the symmetry breaking $\mathbb{SU}(3)_V \times \mathbb{SU}(3)_A \times \mathbb{U}(1)_V$ down to $\mathbb{SU}(3)_V \times \mathbb{U}(1)_V$.

  Can be colected in the matrix (according to our convention):

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  e.g. $\mathcal{L}_2$ contains 2 derivatives or one quark mass (in standard χPT)
- importance of a given diagram - Weinberg power-counting scheme - given by the chiral dimension (written as $O\left(p^D\right)$)

$$D = 2 + \sum_{n=2}^{\infty} (n-2) N_n + 2N_L$$

N_n: # of vertices from $\mathcal{L}_n$

N_L: # of independent loops

Problem - singlet scalar condensate

$$B_0 = -\frac{1}{F_0^2}\langle 0|\bar{q}q|0\rangle$$

- its nonzero value is a sufficient condition for the symmetry breaking (But not necessary!)
- STANDARD power counting believe $B_0 \sim \Lambda$, i.e. $B_0 = O(p^0)$ and so $\chi = M = O(p^2)$ (quark mass matrix)

The $O(p^2)$ Lagrangian of the standard χPT

$$\mathcal{L}_2 = \frac{F_0^2}{4}\left(\mathrm{Tr}\left(D_\mu U\,(D^\mu U)^\dagger\right) + 2B_0\,\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right)$$

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$$+A_0\,\mathrm{Tr}\left(\left(\chi^\dagger U\right)^2 + \left(\chi U^\dagger\right)^2\right) + Z_0^S\left[\mathrm{Tr}\left(\chi^\dagger U + \chi U^\dagger\right)\right]^2$$
$$\left. + Z_0^P\left[\mathrm{Tr}\left(\chi^\dagger U - \chi U^\dagger\right)\right]^2 + 2H_0\,\mathrm{Tr}\left(\chi^\dagger\chi\right)\right)$$

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+ odd orders of Lagrangian in GχPT

Motivation

Which scheme should we use?

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- the determination of the pion scattering length from K_{l4} ruled out the possibility of small condensate
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In the 3 flavour case ($\mathbb{SU}(3)$):

- there is no similar (direct) experimental indication yet
- to extend this work to the $\mathbb{SU}(3)$ case is AIM OF OUR WORK

Our weapons

A

U

C

Our weapons

Analyticity

U

C

Our weapons

Analyticity

Unitarity

C

Our weapons

Analyticity

Unitarity

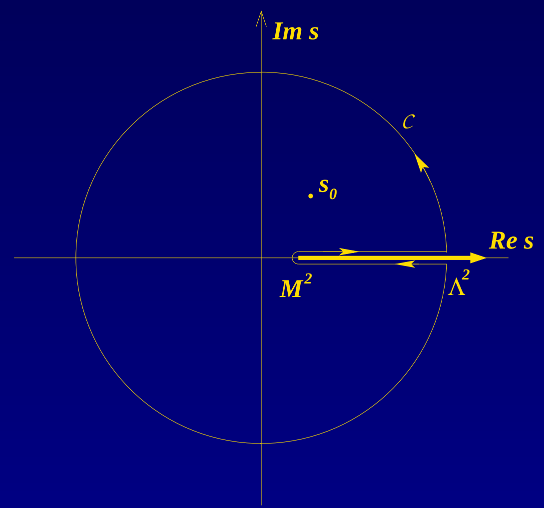
Crossing symmetry

Our weapons

Analyticity of amplitude

At first, we will consider a toy example - with assumptions

- branch cut for real $s > M^2$
- real for real $s < M^2$
- analytic $\forall s$ except the branch cut



The unsubtracted DRs

$$\lim_{\Lambda^2\to\infty} \frac{1}{2\pi i}\oint_{|s|=\Lambda^2} \frac{F(s)}{s-s_0}ds = 0 \qquad \Rightarrow \qquad F(s_0) = \frac{1}{\pi}\int_{M^2}^{\infty} \frac{\operatorname{Im} F(s)}{s-s_0-i\varepsilon}ds$$

m-times subtracted $\left(g(s) = \frac{F(s)-F(0)-\cdots-\frac{s^{m-1}}{(m-1)!}F^{(m-1)}(0)}{s^m} \right)$

$$F(s_0) = P_m(s_0) + \frac{s_0^m}{\pi}\int_{M^2}^{\infty} \frac{ds}{s^m}\frac{\operatorname{Im} F(s)}{(s-s_0-i\varepsilon)}$$

Physical amplitudes have more complicated analytical structure - other branch cuts and poles on the real axis. But the main thought is the same (poles $\to$ contributions to the polynomial).

Unitarity of scattering matrix

- $S_{if} = \delta_{if} + i(2\pi)^4\delta^{(4)}(P_f - P_i)\mathcal{T}_{if}$
- the partial wave decomposition (PWD)

$$\mathcal{T}_{if}(s, \cos\theta) = 32\pi N \sum_l A_l^{i\to f}(s)(2l+1)P_l(\cos\theta)$$

- for two-particle scattering process (assuming time invariance and that the only relevant contributions are from two-particle intermediate states)

$$\mathrm{Im}\, A_l^{i\to f}(s) = \sum_{(1,2)} 2\frac{N'N''}{NS}\frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{s} A_l^{i\to(k_1,k_2)}(s)\left[A_l^{f\to(k_1,k_2)}(s)\right]^*$$

$S = 1(2)$ for (un)distinguishable states k_1, k_2

Crossing symmetry

$$AB \to CD \xrightarrow{\text{T-channel}} A\overline{C} \to \overline{B}D$$

$$S(s,t,u) \xrightarrow{\text{T-channel}} fT(s,t,u)$$

Our choice of the phase factors f: $S(s,t,u) = T(t,s,u) = U(u,t,s)$

"Our" reconstruction theorem

Using **A** and **C** and the Goldstone character of the interacting particles, we have derived theorem for reconstruction of amplitudes (up to $O\left(p^8\right)$) for given processes if we know the imaginary parts of all crossed amplitudes only:

$$S(s,t;u) = R_4(s,t;u) + \Phi_0(s) + [s(t-u) + \Delta_{AB}\Delta_{CD}]\Phi_1(s)$$
$$+ \Psi_0(t) + [t(s-u) + \Delta_{AC}\Delta_{BD}]\Psi_1(t)$$
$$+ \Omega_0(u) + [u(t-s) + \Delta_{AD}\Delta_{BC}]\Omega_1(u) + O\left(p^8\right),$$

where $\Delta_{ij} = m_i^2 - m_j^2$, $R_4(s,t;u)$ is third order polynomial in Mandelstam variables obeying the same symmetry as S and

$$\Phi_0(s) = 32 s^3 \int_\Sigma^{\Lambda^2} \frac{dx}{x^3} \frac{\mathrm{Im}\, S_0(x)}{x-s},$$

$$\Phi_1(s) = 96 s^3 \int_\Sigma^{\Lambda^2} \frac{dx}{x^3} \frac{\mathrm{Im}\, S_1(x)}{(x-s)\lambda_{AB}^{1/2}(x)\lambda_{CD}^{1/2}(x)},$$

$$\lambda_{XY}(x) = \left(x - (m_X + m_Y)^2\right)\left(x - (m_X - m_Y)^2\right)$$

min of squared invar. mass of ∀ intermediate states

The other by cyclic permutation

$(\Phi \mapsto \Psi \mapsto \Omega)$ $(s \mapsto t \mapsto u)$ $(S \mapsto T \mapsto U)$ $(ABCD \mapsto ACBD \mapsto \mathrm{ADCB})$ $(\Sigma \mapsto \tau \mapsto \Upsilon)$

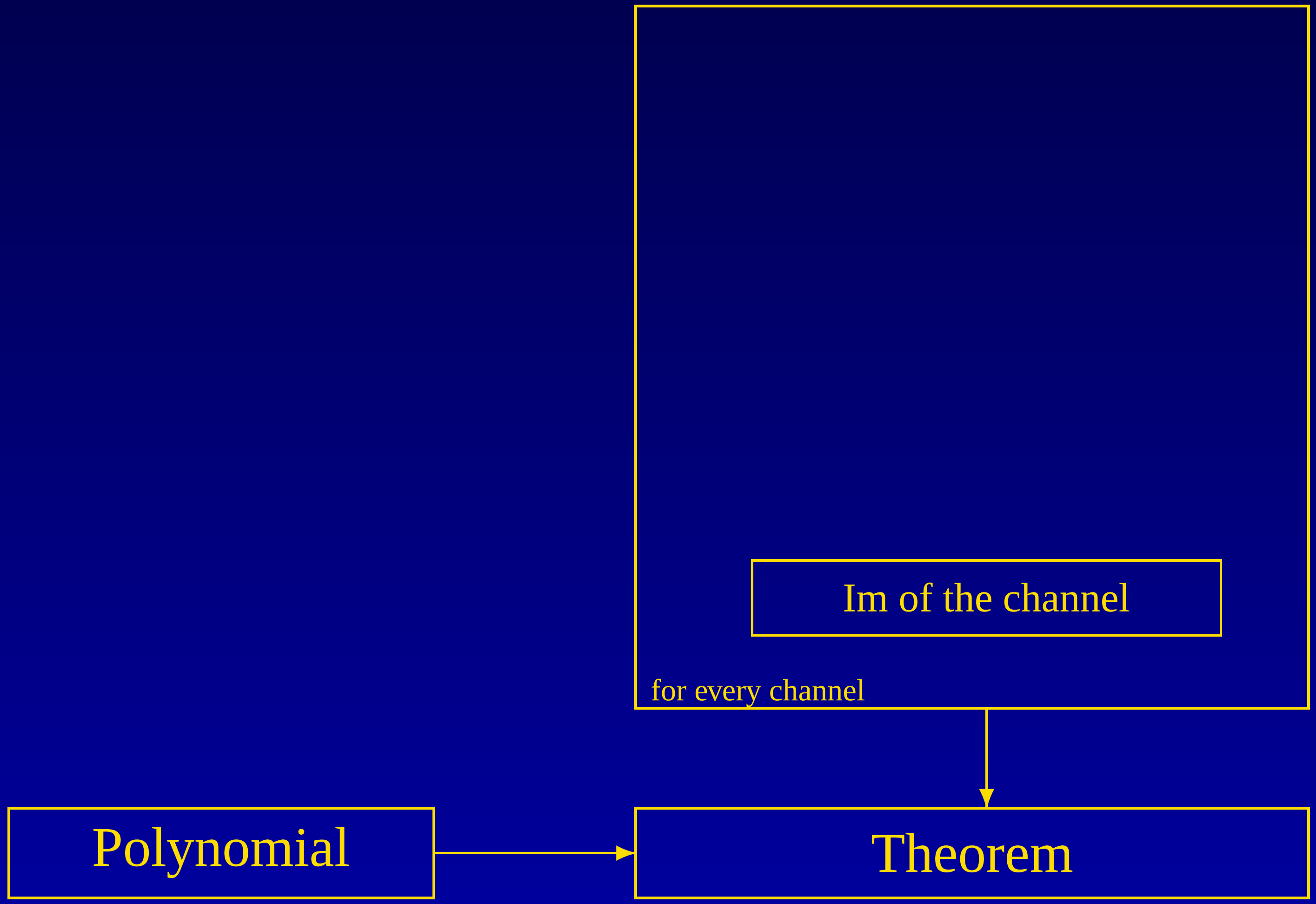
Main steps of its proof

- three times subtracted DRs for the amplitude with fixed u variable - contains polynomial $P_3(s,t;\underline{u})$
- using crossing symmetry and including the high-energy part of the integrals (up to $O(p^8)$) into the polynomial

$$S(s,t;u) = P_4(s,t;\underline{u}) + \frac{s^3}{\pi}\int_{\Sigma}^{\Lambda^2} \frac{dx}{x^3}\frac{\mathrm{Im}\, S(x,\mathcal{M}-x-u;u)}{x-s}$$
$$+\frac{t^3}{\pi}\int_{\tau}^{\Lambda^2} \frac{dx}{x^3}\frac{\mathrm{Im}\, T(x,\mathcal{M}-x-u;u)}{x-t} + O(p^8)\,.$$

- decomposition into the partial waves; the higher partial waves ($l \geq 2$) suppressed to $O(p^8)$
- full symmetry properties, kinematics and including some other terms into the polynomial leads to the theorem
- many of this simplifications are because the lowest order of Lagrangian is $O(p^2)$

Application of the theorem



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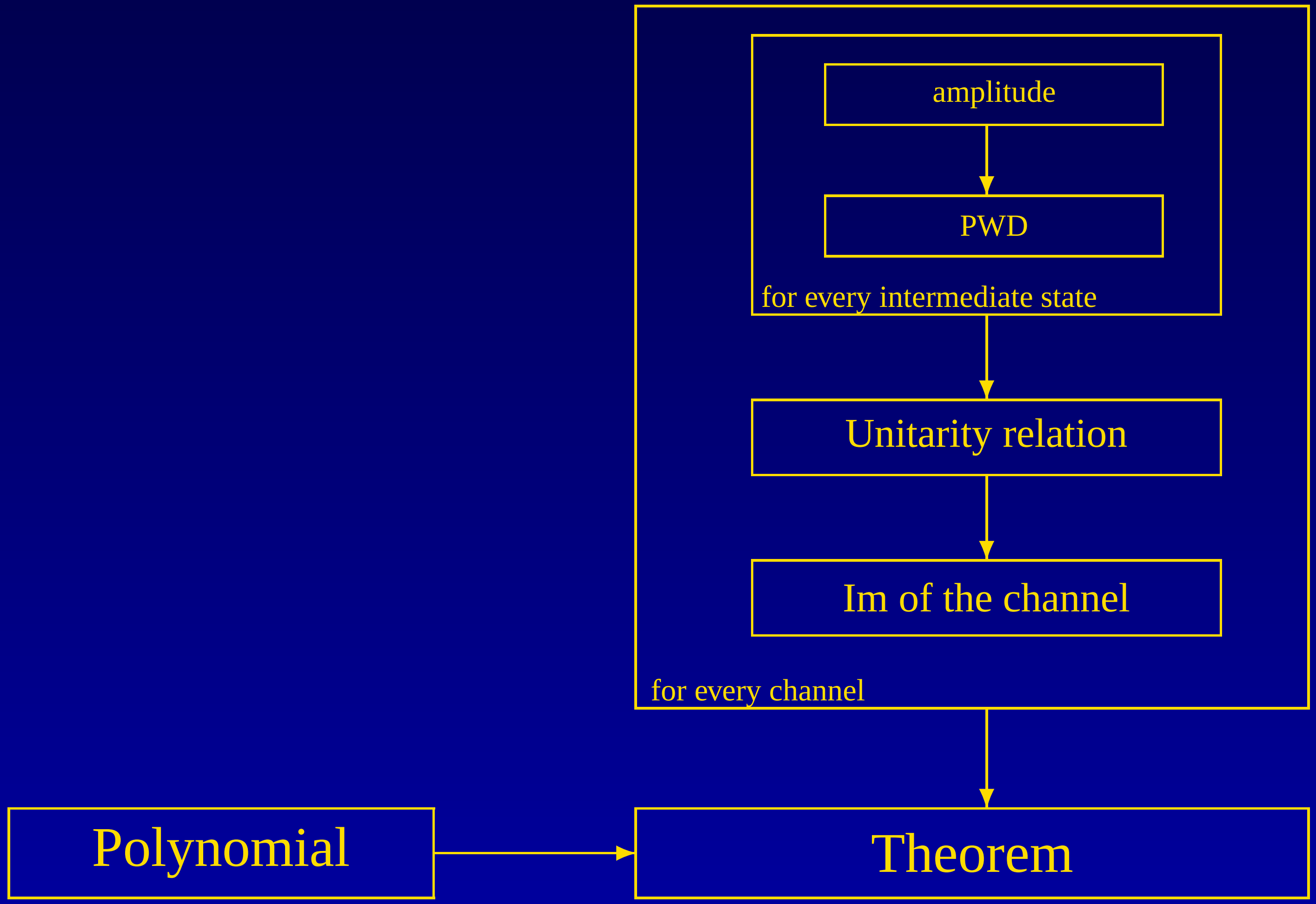

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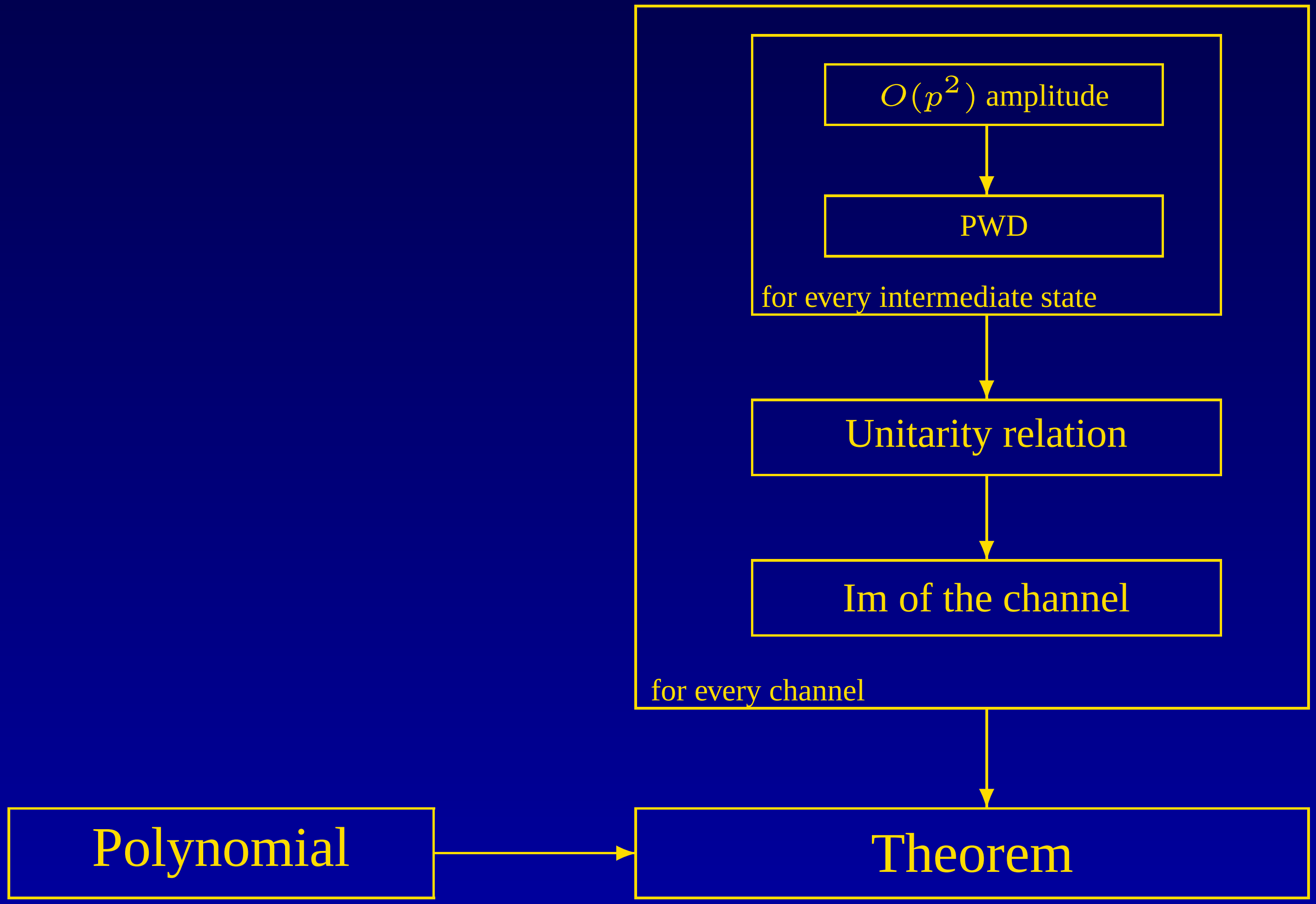

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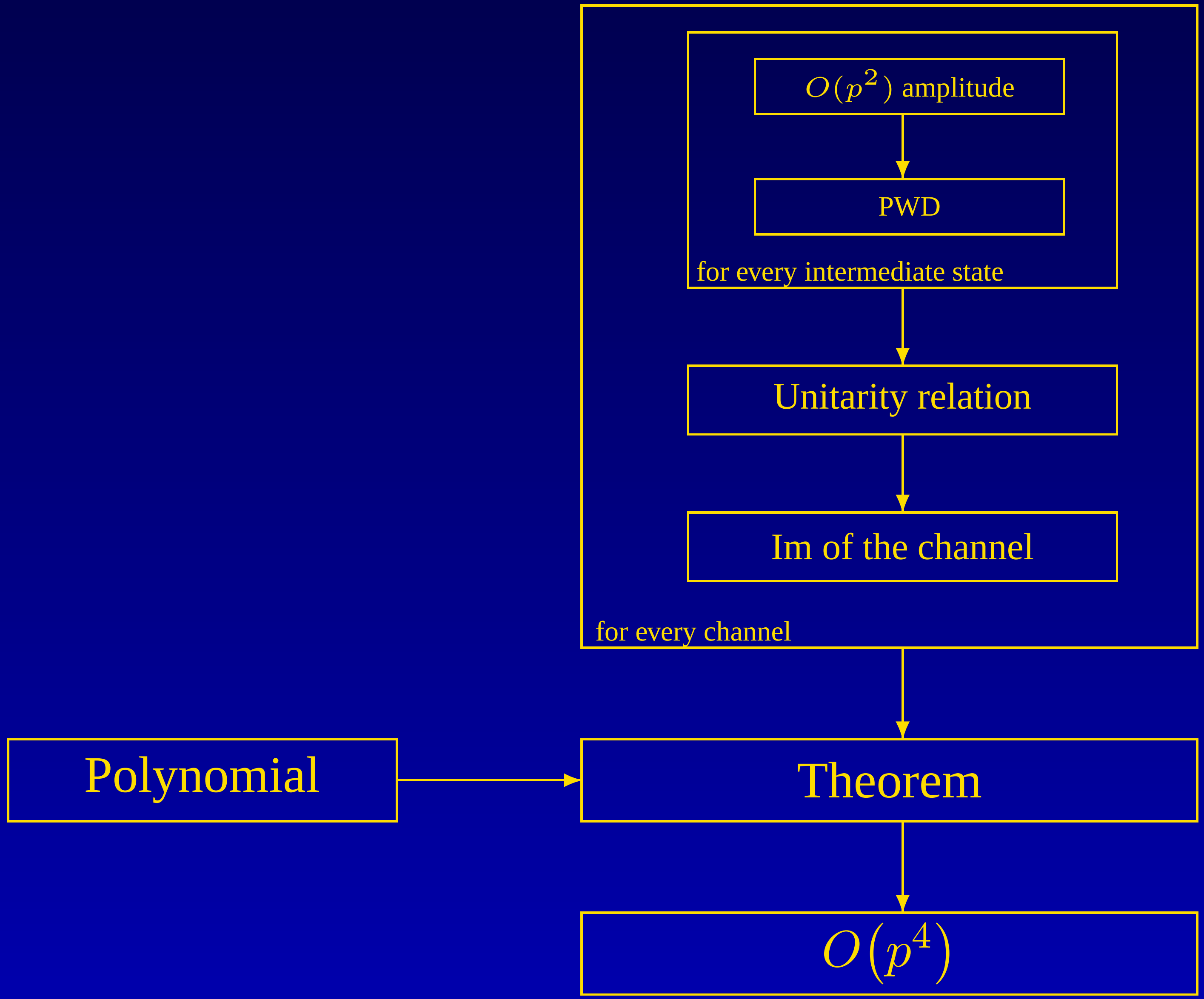

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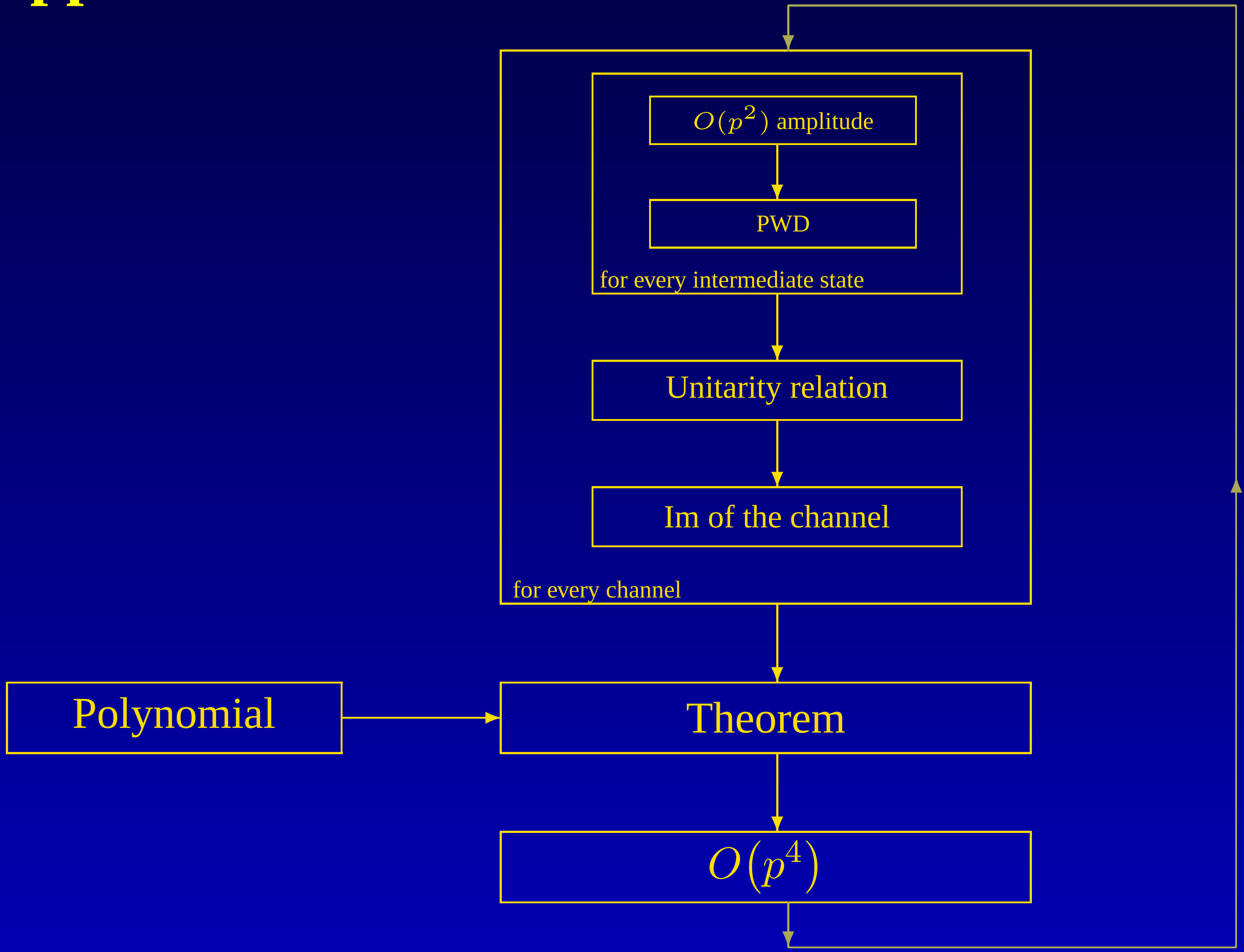

The 4-mesons processes to $O(p^4)$

We will apply the theorem according to the previous scheme to the 2-lightest-mesons scattering processes $2 \to 2$:

- **symmetry** - we work in the strong isospin conservation limit
  In this limit - only 7 independent processes (from the Ward identities)

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Results for the amplitudes to $O(p^4)$

- all the amplitudes expressed in terms of only 30 independent constants (the 'subtraction' polynomial of the second order)
- besides, there appears Mandelstam variables, pseudoscalar meson masses and the decay constant F_π^2
- the perturbative unitarity is encoded in the dispersion integrals
  - once subtracted $\overline{J}_{PQ}(s) = \frac{s}{16\pi^2} \int_\Sigma^\infty \frac{dx}{x} \frac{1}{x-s} \frac{\lambda_{PQ}^{1/2}(x)}{x}$ and twice subtracted $\overline{\overline{J}}_{PQ}(s) = \frac{s^2}{16\pi^2} \int_\Sigma^\infty \frac{dx}{x^2} \frac{1}{x-s} \frac{\lambda_{PQ}^{1/2}(x)}{x}$
- the standard power counting limit of our results corresponds to the ones given in the literature
- the results very voluminous $\Rightarrow$ for illustration amplitude of $\pi\eta \to \pi\eta$:

$$A_{\pi\eta\to\pi\eta} = \frac{1}{3F_\pi^2}\left(\beta_{\pi\eta}(3t - 2M_\eta^2 - 2M_\pi^2) + M_\pi^2\alpha_{\pi\eta}\right) + \frac{1}{3F_\pi^4}\left(\delta_{\pi\eta}(s^2+u^2) + \varepsilon_{\pi\eta}t^2\right)$$

$$+ \frac{1}{72F_\pi^4}\left\{\overline{J}_{KK}(t)\left[9\beta_{\eta K}t - 2(3\beta_{\eta K} + \alpha_{\eta K})M_K^2 + 6(\alpha_{\eta K} - \beta_{\eta K})M_\eta^2\right]\left[3\beta_{\pi K}t + 2(1-\beta_{\pi K})(M_K^2 + M_\pi^2) + 4(\alpha_\pi\right.\right.$$

$$\left.- 4\,\overline{J}_{\eta\eta}(t)\alpha_{\pi\eta}\alpha_{\eta\eta}M_\pi^2\left(M_\pi^2 - 4M_\eta^2\right) + 4\overline{J}_{\pi\pi}(t)\alpha_{\pi\eta}M_\pi^2\left(6\beta_{\pi\pi}t + (5\alpha_{\pi\pi} - 8\beta_{\pi\pi})M_\pi^2\right)\right\}$$

$$+ \left[\frac{1}{9F_\pi^4}\left\{\overline{J}_{\pi\eta}(s)\alpha_{\pi\eta}^2M_\pi^4 + \frac{3}{8}\overline{J}_{KK}(s)\left[3\beta_{\eta\pi K}s - 2(1+\beta_{\eta\pi K})M_K^2 + (1-\alpha_{\eta\pi K} - \beta_{\eta\pi K})M_\pi^2 + (1-\beta_{\eta\pi K})M_\eta^2\right]^2\right.\right.$$

$$+ [s \leftrightarrow u] + O\left(p^5\right).$$

Advantages and disadvantages

Advantages

- this approach is general and methodologically contributive
- we have bindings between the higher order of some amplitude and the lower orders of another ones
  If we had experimental data of all $2 \longrightarrow 2$ mesons scattering, we could proceed order by order fitting only the polynomial.
  If we had some process so exactly that we can fit its amplitude, we could forecasts amplitudes of other processes.

Disadvantages

- there is no simple linkage of the LEC to QCD
- the only properties simplifying LEC are symmetries

Number of LEC to a given order

approach	$O(p^2)$	$O(p^3)$	$O(p^4)$	$O(p^6)$
SχPT	2	2	10	$\sim$40
DAχPT	13	15	30	47
GχPT	5	14	43	$>$100

THE END