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η' effects in radiative Kaon decays

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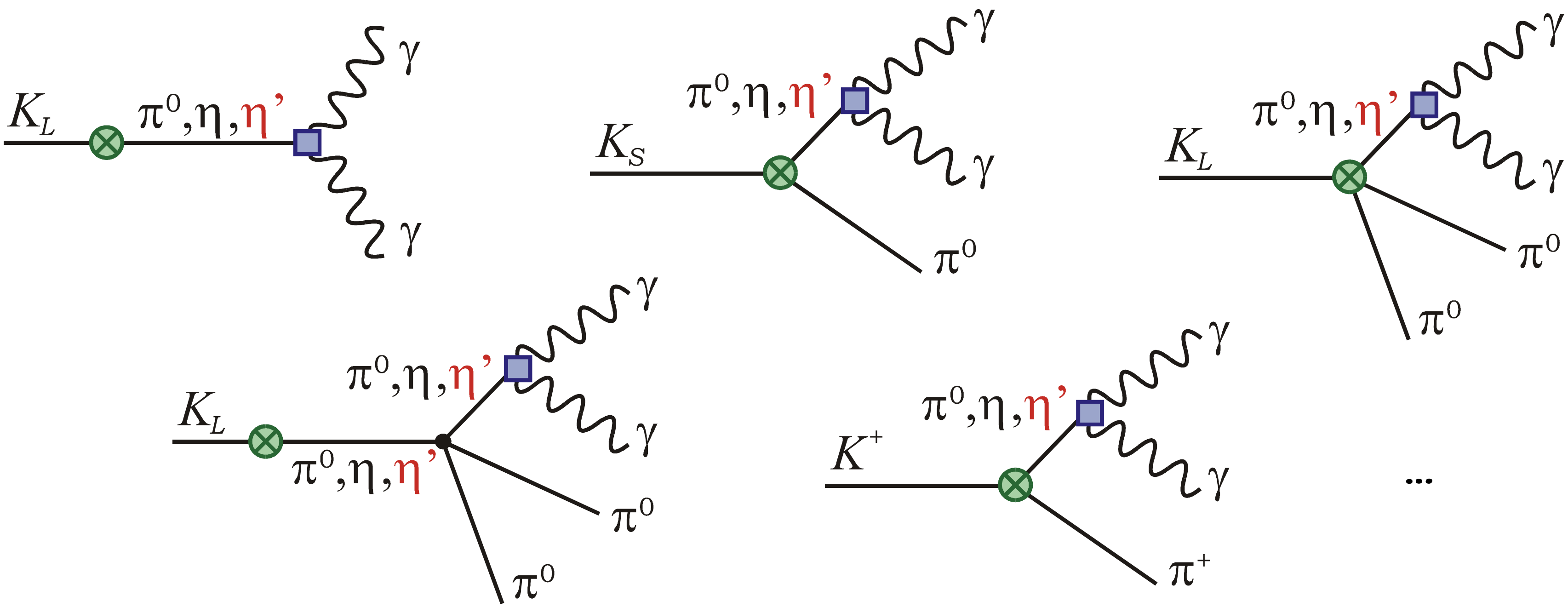
Motivation

The η_0 meson has a potentially important effect in some radiative K decays through its anomalous coupling to photons. For example:



$SU(3)$ chiral expansion → local, undetermined counterterms.
What can be learned from an extension to $U(3)$, with propagating η_0 ?

Outline

1. $SU(3)$ and $U(3)$ chiral expansions
2. The decay $K_L^0 \to \gamma\gamma$
3. The decay $K_S^0 \to \pi^0\gamma\gamma$
4. The decay $K^+ \to \pi^+\gamma\gamma$
5. Conclusion

1. $SU(3)$ and $U(3)$ chiral expansions

QCD with $m_q = 0$	QCD with $m_q = 0$, $N_C \to \infty$
$SU(3)_L \times SU(3)_R \to SU(3)_V$ 8 G.B.: $\Phi = \frac{\lambda^a}{\sqrt{2}}\pi^a, \quad a = 0,\ldots,8$ $L_{eff}(\Phi) = L^{(p^2)} + L^{(p^4)} + \ldots$	$U(3)_L \times U(3)_R \to U(3)_V$ 9 G.B.: $\Phi = \frac{\lambda^a}{\sqrt{2}}\pi^a + \frac{1}{\sqrt{3}}\eta_0$ $L_{eff}(\Phi) = \cancel{L_\infty^{(p^0)}} + L_\infty^{(p^2)} + L_\infty^{(p^4)} \ldots + L_{1/N_C}^{(p^0)} + L_{1/N_C}^{(p^2)} + \ldots$

Leading eff. Lagr. in the $U(3)$ framework:

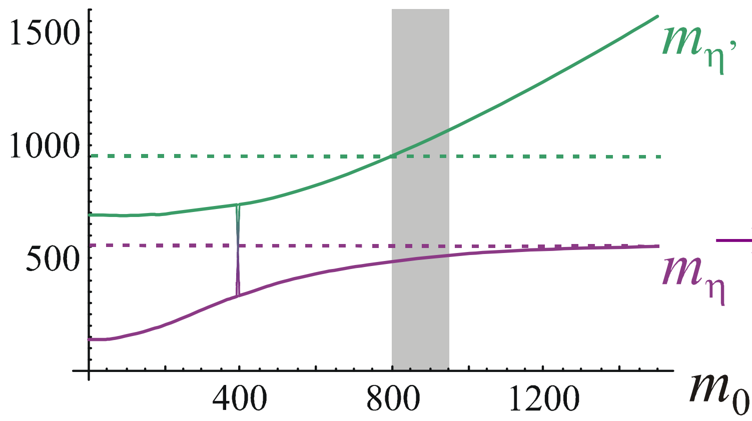
$$L_{eff} = \frac{F_\pi^2}{4} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_\pi^2}{4} r Tr\left(mU^\dagger + Um\right) + \frac{F_\pi^2}{16}\frac{m_0^2}{N_C}\left(\overbrace{Tr\ln U - Tr\ln U^\dagger}^{\sim \eta_0}\right)^2$$

$$U \equiv \exp\left(\frac{i\sqrt{2}}{F_\pi}\Phi\right)$$

Mixing η_8-η_0:

$$M_{80}^2 = \begin{bmatrix} \underbrace{(4m_K^2 - m_\pi^2)/3} & -2\sqrt{2}(m_K^2 - m_\pi^2)/3 \\ -2\sqrt{2}(m_K^2 - m_\pi^2)/3 & (2m_K^2 + m_\pi^2)/3 + m_0^2 \end{bmatrix}$$

Graphically:



The leading $SU(3)$ mass relation (GMO) is recovered in the limit $m_0 \to \infty$.

Agreement at the 10% level for $m_0 \simeq 850\, MeV$.

η-η' anomalous radiative decays are also well reproduced (20% level for $m_0 \simeq 850\, MeV$):

	exp	th
$Br(\eta \to \gamma\gamma)$	(39.43±0.26)%	45.1%
$Br(\eta' \to \gamma\gamma)$	(2.12±0.14)%	2.6%

What about η-η' pole contributions to radiative Kaon decays ?

$\to$ Leading $\Delta S = 1$ weak operators:

$$L_8 = 4 G_8 \sum_q J_\alpha^{dq} J^{\alpha qs}$$

$$L_{8s} = -4 G_{8s} J_\alpha^{ds}\, Tr(J^\alpha) \sim K^0 \eta_0$$

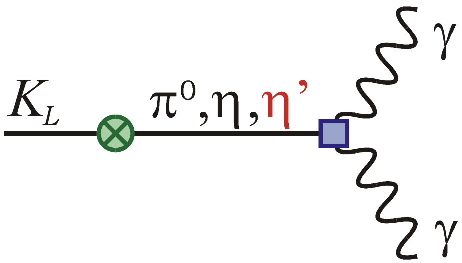
Penguin operators Q_3 and Q_5

$$J^\alpha \equiv -i \frac{F_\pi^2}{2} U \partial^\alpha U^\dagger$$

$$L_{27} = 4 G_{27} \left[\tfrac{2}{3} J_\alpha^{du} J^{\alpha us} + J_\alpha^{ds} J^{\alpha uu} - \tfrac{1}{3} J_\alpha^{ds}\, Tr(J^\alpha) \right]$$

2. The decay $K_L^0 \to \gamma\gamma$

CP conservation $\to$ $A(K_L^0 \to \gamma\gamma) = A\,\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}k_{1\rho}k_{2\sigma}$



- SD contribution small
- LD: leading contribution from poles (>< $K_S^0 \to \gamma\gamma$)

$SU(3)$ framework:

$$A_{8,LO}^{SU(3)} \propto \frac{1}{m_K^2 - m_\pi^2} + \sqrt{1/3}\,\frac{1}{m_K^2 - m_{\eta_8}^2}\,\sqrt{1/3} \underset{GMO}{=} 0$$

Extension to $U(3)$:

$$A_{8,LO}^{U(3)} \propto \frac{1}{\Delta}\begin{pmatrix} 1 & \sqrt{1/3} & -2\sqrt{2/3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{-3m_0^2 + \Delta}{m_0^2 - 3\Delta} & \frac{-2\sqrt{2}}{m_0^2 - 3\Delta} \\ 0 & \frac{-2\sqrt{2}}{m_0^2 - 3\Delta} & \frac{-1}{m_0^2 - 3\Delta} \end{pmatrix} \begin{pmatrix} 1 \\ \sqrt{1/3} \\ 2\sqrt{2/3} \end{pmatrix} = 0 \quad !!$$

$$\Delta \equiv m_K^2 - m_\pi^2$$

extension of GMO implicitly used ($m_{phys} \to m_{th}$)

- Usually, corrections to GMO ($\leftrightarrow$ including leading η_8-η_0 mixing effects) are invoked to account for the decay. So what happens?

  Saturation of ***both*** the strong ***and*** weak $SU(3)$ counterterms by the η_0 resonance gives, at leading order in $1/N_C$:

$$A^{SU(3)}_{8,p^6} \propto G_8(\not{loops} + 16L_7 + 8\not{L_8} - \not{N_6} + 2\not{N_{12}} + 2N_{13}) = 0$$

$$L_7 = -\frac{N_C F_\pi^2}{144 m_0^2}, \quad N_{13} = 8\frac{N_C F_\pi^2}{144 m_0^2}$$

- What is responsible for the decay? In the $U(3)$ framework, at $O(p^4)$:

  $(SU(3) \;\; \to O(p^6))$

  - 27 : $A_{27} / A_{\exp} \simeq 0.17$
  - 27+8$_S$: *OK for* $G_{8S} \simeq -0.2 G_8$
  - NLO *corrections* $(p^6, 1/N_C)$?

  $m_{th} \to m_{phys}$ *before evaluation:*
  The η *pole must be at the right position.*

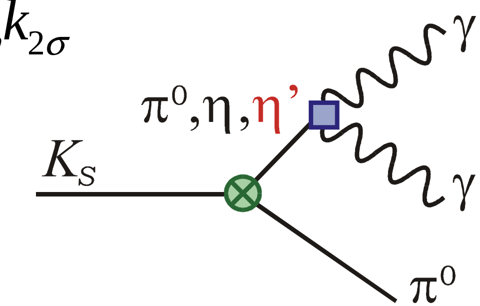
Rem: Working first with m_{th} instead of m_{phys} has allowed us to gain some insight. Also, the weak mass term $(mU^\dagger + Um)^{ds}$ would give a non-zero contribution at LO if the physical masses were used

3. The decay $K_S^0 \to \pi^0\gamma\gamma$

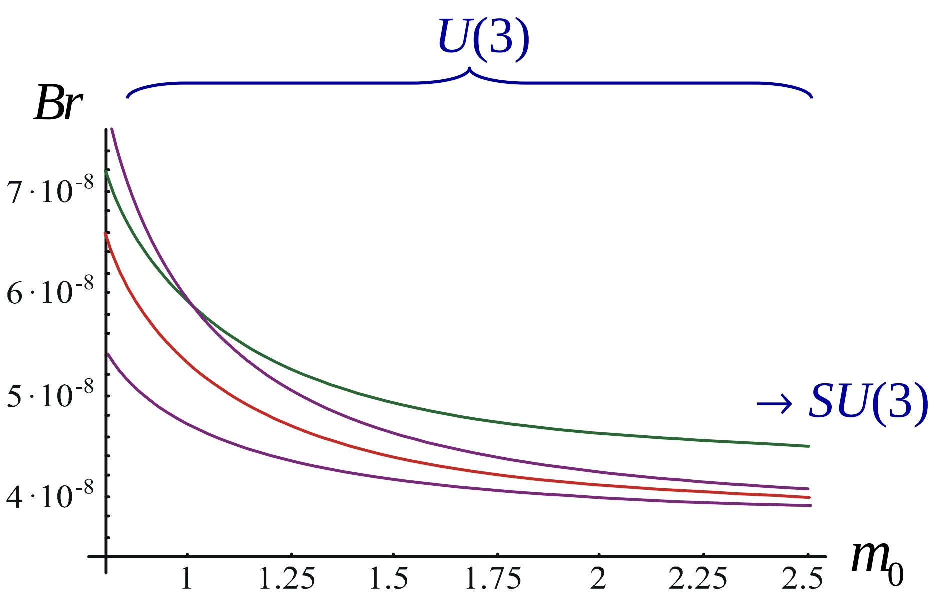
CP conservation $\to$ $A(K_S^0 \to \pi^0\gamma\gamma) = A\,\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}k_{1\rho}k_{2\sigma}$

Again, the leading contribution comes from poles

$(>< K_L^0 \to \pi^0\gamma\gamma)$



First order prediction:



— : 8

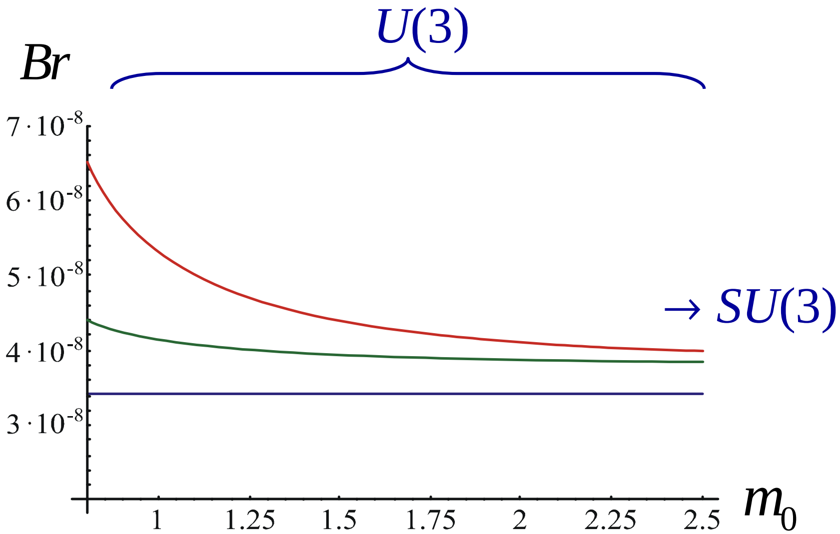
— : 8+27

— : 8+27+8s $(G_{8s} = \pm G_8/3)$

$Br(K_S^0 \to \pi^0\gamma\gamma)_{z>0.2}$

$U(3)$ 8+27 m_0= 850 MeV	$6.1\ 10^{-8}$
$SU(3)$ 8+27	$3.8\ 10^{-8}$
NA48 (2004)	$4.9\pm1.8\ 10^{-8}$

3. The decay $K_S^0 \to \pi^0\gamma\gamma$

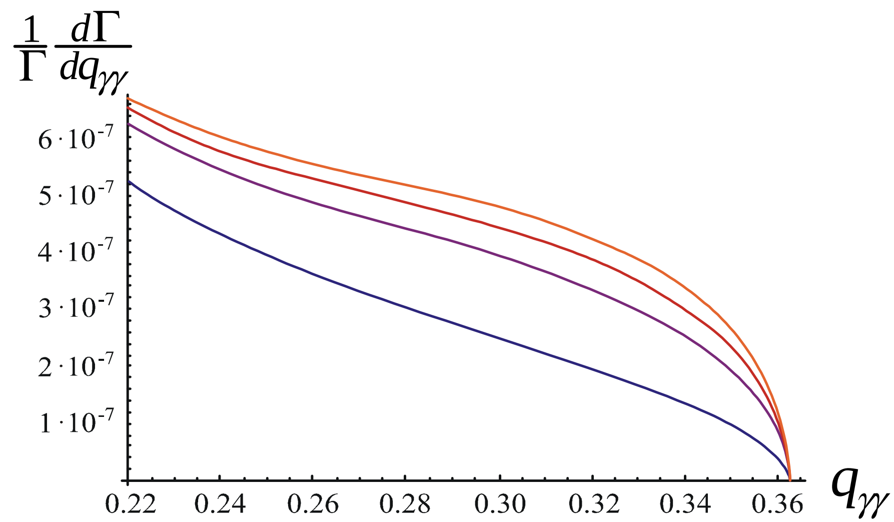


π^0, η_8, η_0 contributions to the 8+27 prediction

- : $\pi^0 + \eta_8 + \eta_0$
- : $\pi^0 + \eta_8$
- : π^0

Spectrum for various values of m_0 (8+27 weak op.)

- : m_0= 800 MeV
- : m_0= 850 MeV
- : m_0= 900 MeV
- : $SU(3)$ $limit$



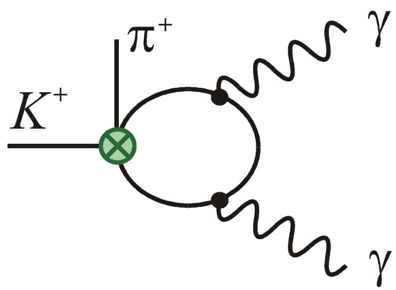
CCL: Enhancement of $Br(K_S^0 \to \pi^0\gamma\gamma)$ due to the η_0 pole.
However, a more careful treatment of the η pole is needed.

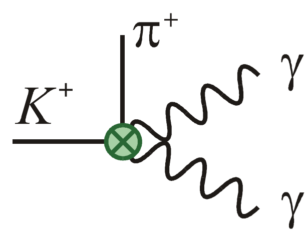
4. The decay $K^+ \to \pi^+\gamma\gamma$

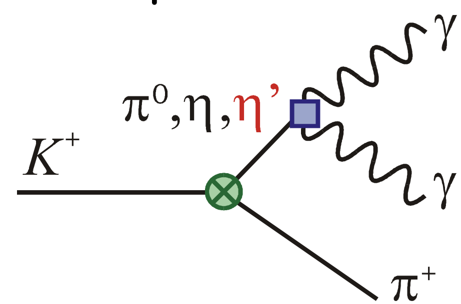
$$A(K^+ \to \pi^+\gamma\gamma) = A_L\ [(k_1k_2)(\varepsilon_1^*\varepsilon_2^*) - (k_1\varepsilon_2^*)(k_2\varepsilon_1^*)] + A_P\ \varepsilon^{\mu\nu\rho\sigma}\varepsilon_{1\mu}^*\varepsilon_{2\nu}^* k_{1\rho}k_{2\sigma}$$

loops, counterterms

poles







A_L and A_P do not interfere. An effect of the η_0 pole in A_P is thus easily converted into a constraint on the $SU(3)$ counterterms in A_L :

$$Br(K^+ \to \pi^+\gamma\gamma)_{z>0.2}^{Full} = \underbrace{Br(K^+ \to \pi^+\gamma\gamma)_{z>0.2}^{Loops}}_{(5.79+1.65\hat{c}+0.29\hat{c}^2)10^{-7}} + Br(K^+ \to \pi^+\gamma\gamma)_{z>0.2}^{Poles}$$

In this case, however, the η_0 effect is small.

5. Conclusion

The effect of the η_0 meson in various radiative K decays has been investigated in the framework of large N_C ChPT.

- The recourse to $U(3)$ chiral symmetry has allowed us to identify important cancellations between the different parts of $A(K_L^0 \to \gamma\gamma)$. The 27 and $8s$ (penguin) operators were proposed as a possible mecanism for the decay.
- An enhancement of $Br(K_S^0 \to \pi^0\gamma\gamma)$ has been displayed
- Small effect in $K^+ \to \pi^+\gamma\gamma$
- Other modes currently being analysed

However, the proximity of m_η to m_K calls for a better control over NLO corrections.