# One-loop Renormalization of Resonance Chiral Theory: Scalar and Pseudoscalar Resonances

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[1] I.Rosell, J.J.Sanz-Cillero, A.Pich. “Quantum loops in the Resonance Chiral Theory: the vector form factor”, JHEP 0408,042(2004) [arXiv: hep-ph/ 0407240]

[2] I.Rosell, P.D.Ruiz-Femenía, J.Portolés. “One-loop Renormalization of Resonance Chiral Theory: Scalar and Pseudoscalar Resonances”, work in progress.

# OUTLINE

- i) QCD at low energies: Effective Field Theories
- ii) QCD at $M_V \leq E \leq 2\ \mathrm{GeV}$
  - i) Problems
  - ii) $1/N_C$ Expansion
  - iii) Resonance Chiral Theory (RChT)
- iii) One-loop Renormalization
  - i) RChT with scalar and pseudoscalar resonances
  - ii) Background field method
  - iii) Heat Kernel: divergences
- iv) Conclusions

# i) QCD at low energies: Effective Field Theories

$\mathcal{L}_{\mathrm{QCD}}$ → running of $\alpha_s$ ($\beta_{\mathrm{QCD}} < 0$)

- Asymptotic Freedom: pQCD
- Confinement: non-pQCD

a SOLUTION: Effective Field Theories → PROBLEM !!!

- Low-energy couplings and symmetries from the high-energy dynamics
- Integration of the heavy particles

$$e^{i\Gamma_{\mathrm{eff}}[\Phi_l]} = \int [d\Phi_h]\, e^{iS[\Phi_h,\Phi_l]}$$

- Suitable degrees of freedom
- Local operators with light particles
- Worked as renormalizable: order by order

# ii) QCD at $M_V \leq E \leq 2$ GeV

1st Problem

Many resonances and no mass gap

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No FORMAL EFT approach

2nd Problem

No natural expansion parameter

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$1/N_C$ Expansion

large $N_C$ limit → tree diagrams of an effective lagrangian

infinite tower of resonances

# ii) QCD at $M_V \leq E \leq 2\ \mathrm{GeV}$ : Resonance Chiral Theory

i) Regime: intermediate energies

ii) Degrees of freedom: pseudo-Goldstone bosons and resonances

iii) Parameter: $1/N_C$ Expansion

iv) Matching:

  i) very-low energies: ChPT

  ii) high energies: OPE and Brodsky-Lepage

APPROXIMATION:
cut in the tower of resonances

supported by the phenomenology

model dependent

# iii) One-loop Renormalization: RChT with scalar and pseudoscalar resonances

a) QCD symmetries

b) Matching with ChPT: chiral symmetry

c) Operators up to *RR*

d) QCD rules:

  a) $\langle \chi^{(2)} \rangle, \langle R\chi^{(2)} \rangle, \langle RR\chi^{(2)} \rangle$

  b) short-distance + large $N_C$: relations between couplings

e) Single Resonance Approximation with only S and P

f) Large $N_C$ : *U(3)* multiplets

# Lagrangian

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}(\phi,S,P)=\mathcal{L}_{\chi}^{(2)}+\mathcal{L}_{\mathrm{kin}}(S,P)+\mathcal{L}_{2}(S)+\mathcal{L}_{2}(P)+\mathcal{L}_{2}(S,P)$$

$$\mathcal{L}_{\chi}^{(2)}=\frac{F^{2}}{4}\left\langle u_{\mu}u^{\mu}+\chi_{+}\right\rangle$$

$$\mathcal{L}_{\mathrm{kin}}(S,P)=\frac{1}{2}\sum_{R=S,P}\left\langle\nabla^{\mu}R\,\nabla_{\mu}R-M_{R}^{2}R^{2}\right\rangle$$

$$\mathcal{L}_{2}(S)=c_{d}\left\langle Su_{\mu}u^{\mu}\right\rangle+c_{m}\left\langle S\chi_{+}\right\rangle+\lambda_{1}^{SS}\left\langle SSu_{\mu}u^{\mu}\right\rangle+\lambda_{2}^{SS}\left\langle Su_{\mu}Su^{\mu}\right\rangle+\lambda_{3}^{SS}\left\langle SS\chi_{+}\right\rangle$$

$$\mathcal{L}_{2}(P)=id_{m}\left\langle P\chi_{-}\right\rangle+\lambda_{1}^{PP}\left\langle PPu_{\mu}u^{\mu}\right\rangle+\lambda_{2}^{PP}\left\langle Pu_{\mu}Pu^{\mu}\right\rangle+\lambda_{3}^{PP}\left\langle PP\chi_{+}\right\rangle$$

$$\mathcal{L}_{2}(S,P)=\lambda_{1}^{SP}\left\langle\left\{\nabla_{\mu}S,P\right\}u^{\mu}\right\rangle+i\lambda_{2}^{SP}\left\langle\left\{S,P\right\}\chi_{-}\right\rangle$$

Why quantum loops
in the RChT?

- Improvement of predictions
- Improvement of non-perturbative QCD

What have we learned
from the VFF at NLO?

- No problem to renormalize RChT
- Successful resonance saturation at NLO
- Problems in the short-distance behaviour

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terms with several resonances

# One-loop Renormalization: Bakground Field Method

Generating functional

$$e^{iZ[J]} = \frac{1}{N}\int [\mathrm{d}\psi]\, e^{iS[\psi,J]}$$

Background field method

$$\psi = \psi_{\mathrm{cl}} + \Delta\psi$$

EOM

$$Z[J]_{L=1} = S[\psi_{\mathrm{cl}}, J] - i\,\log\left[\mathrm{N}'\int[\mathrm{d}\Delta\psi]\exp\left(\int \mathrm{d}^4x_1 \left.\frac{\delta S[\psi,J]}{\delta\psi(x_1)}\right|_{\psi_{\mathrm{cl}}} \Delta\psi(x_1) + \frac{1}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2 \left.\frac{\delta^2 S[\psi,J]}{\delta\psi(x_1)\,\delta\psi(x_2)}\right|_{\psi_{\mathrm{cl}}} \Delta\psi(x_1)\Delta\psi(x_2)\right)\right]$$

$$Z[J]_{L=1} = S[\psi_{\mathrm{cl}}, J] + \frac{i}{2}\log\det\left(\mathcal{D}^{-1}(\psi_{\mathrm{cl}})\right)$$

$$S_1[\psi_{\mathrm{cl}}, J; \mu] = -\frac{i}{2}\log\det\left(\mathcal{D}^{-1}(\psi_{\mathrm{cl}})\right)\Big|_{\mathrm{divergent}}$$

# One-loop Renormalization: Heat Kernel

- In our case…

*27x27* matrices

$$S_1\left[u_{\rm cl}, S_{\rm cl}, P_{\rm cl}, J; \mu\right] = -\frac{i}{2}\log\det\left(\Sigma_\mu \Sigma^\mu + \Lambda\right)\Big|_{\rm divergent},$$

$$\Sigma^\mu_{ij} = \delta_{ij}\partial^\mu + Y^\mu_{ij}$$

- Now using heat kernel techniques…

$$S_1\left[u_{\rm cl}, S_{\rm cl}, P_{\rm cl}, J; \mu\right] = -\frac{1}{(4\pi)^2}\frac{1}{D-4}\int d^4x\ {\rm Tr}\left(\frac{1}{12}Y^{\mu\upsilon}Y_{\mu\nu} + \frac{1}{2}\Lambda^2\right),$$

$$Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu + \left[Y_\mu, Y_\nu\right]$$

# iv) Conclusions

One-loop renormalization of RChT with scalar and pseudoscalar resonances

One is able to work at NLO

Running of the couplings

Possible continuations:

i) Extension to RChT with $V(1^{--})$ and $A(1^{++})$

ii) Finite part of the generating functional