

One-loop Renormalization of Resonance Chiral Theory: Scalar and Pseudoscalar Resonances

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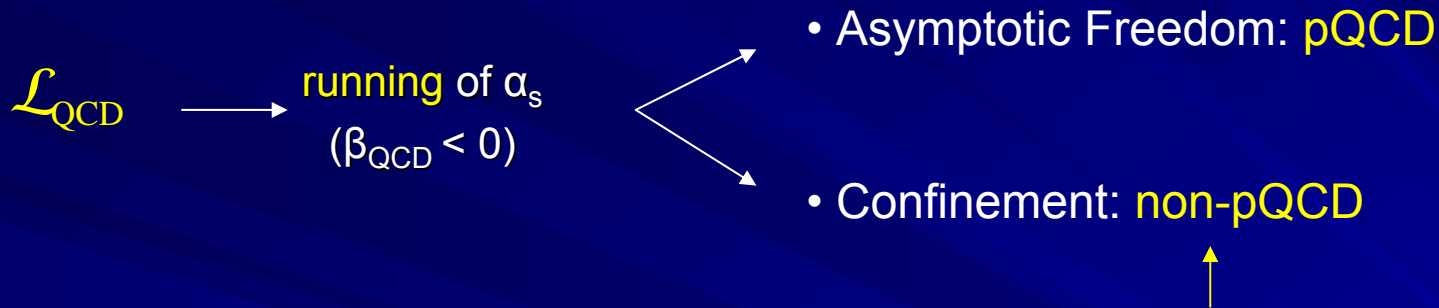
[1] I.Rosell, J.J.Sanz-Cillero, A.Pich. “Quantum loops in the Resonance Chiral Theory: the vector form factor”, JHEP 0408,042(2004) [arXiv: hep-ph/ 0407240]

[2] I.Rosell, P.D.Ruiz-Femenía, J.Portolés. “One-loop Renormalization of Resonance Chiral Theory: Scalar and Pseudoscalar Resonances”, work in progress.

OUTLINE

- i) QCD at low energies: Effective Field Theories
- ii) QCD at $M_V \leq E \leq 2 \text{ GeV}$
 - i) Problems
 - ii) $1/N_C$ Expansion
 - iii) Resonance Chiral Theory (RChT)
- iii) One-loop Renormalization
 - i) RChT with scalar and pseudoscalar resonances
 - ii) Background field method
 - iii) Heat Kernel: divergences
- iv) Conclusions

i) QCD at low energies: Effective Field Theories



a SOLUTION: Effective Field Theories

PROBLEM !!!

- Low-energy **couplings** and **symmetries** from the high-energy dynamics
- Integration of the **heavy particles**

$$e^{i\Gamma_{\text{eff}}[\Phi_1]} = \int [d\Phi_h] e^{iS[\Phi_h, \Phi_1]}$$

- Suitable **degrees of freedom**
- Local operators with **light particles**
- Worked as **renormalizable**: order by order

ii) QCD at $M_\nu \leq E \leq 2 \text{ GeV}$

1st Problem

Many resonances and **no mass gap**



No FORMAL EFT approach

2nd Problem

No natural expansion parameter



$1/N_C$ Expansion

large N_C limit



tree diagrams of an **effective lagrangian**
infinite tower of resonances

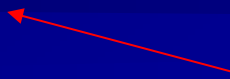
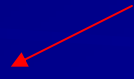
ii) QCD at $M_V \leq E \leq 2 \text{ GeV}$: Resonance Chiral Theory

- i) Regime: intermediate energies
- ii) Degrees of freedom: pseudo-Goldstone bosons and resonances
- iii) Parameter: $1/N_C$ Expansion
- iv) Matching:
 - i) very-low energies: ChPT
 - ii) high energies: OPE and Brodsky-Lepage

APPROXIMATION:
cut in the tower of resonances

supported by the phenomenology

model dependent



iii) One-loop Renormalization: RChT with scalar and pseudoscalar resonances

- a) QCD symmetries
- b) Matching with ChPT: chiral symmetry
- c) Operators up to RR

d) QCD rules:

a) $\langle \chi^{(2)} \rangle, \langle R\chi^{(2)} \rangle, \langle RR\chi^{(2)} \rangle$

b) short-distance + large N_C : relations between couplings

- e) Single Resonance Approximation with only S and P
- f) Large N_C : $U(3)$ multiplets

Lagrangian

$$\mathcal{L}_{R\chi T}(\phi, S, P) = \mathcal{L}_\chi^{(2)} + \mathcal{L}_{\text{kin}}(S, P) + \mathcal{L}_2(S) + \mathcal{L}_2(P) + \mathcal{L}_2(S, P)$$

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

$$\mathcal{L}_{\text{kin}}(S, P) = \frac{1}{2} \sum_{R=S, P} \langle \nabla^\mu R \nabla_\mu R - M_R^2 R^2 \rangle$$

$$\mathcal{L}_2(S) = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle + \lambda_1^{SS} \langle S S u_\mu u^\mu \rangle + \lambda_2^{SS} \langle S u_\mu S u^\mu \rangle + \lambda_3^{SS} \langle S S \chi_+ \rangle$$

$$\mathcal{L}_2(P) = i d_m \langle P \chi_- \rangle + \lambda_1^{PP} \langle P P u_\mu u^\mu \rangle + \lambda_2^{PP} \langle P u_\mu P u^\mu \rangle + \lambda_3^{PP} \langle P P \chi_+ \rangle$$

$$\mathcal{L}_2(S, P) = \lambda_1^{SP} \langle \{ \nabla_\mu S, P \} u^\mu \rangle + i \lambda_2^{SP} \langle \{ S, P \} \chi_- \rangle$$

Why **quantum loops**
in the **RChT**?

- Improvement of **predictions**
- Improvement of **non-perturbative QCD**

What have we learned
from the **VFF at NLO**?

- No problem to **renormalize RChT**
- Successful **resonance saturation** at NLO
- Problems in the **short-distance behaviour**



terms with **several resonances**

One-loop Renormalization: Background Field Method

Generating functional
$$e^{iZ[J]} = \frac{1}{N} \int [d\psi] e^{iS[\psi, J]}$$

Background field method

$$\psi = \psi_{cl} + \Delta\psi$$

EOM

$$Z[J]_{L=1} = S[\psi_{cl}, J] - i \log \left[N' \int [d\Delta\psi] \exp \left(\int d^4x_1 \frac{\delta S[\psi, J]}{\delta \psi(x_1)} \Big|_{\psi_{cl}} \Delta\psi(x_1) + \frac{1}{2} \int d^4x_1 d^4x_2 \frac{\delta^2 S[\psi, J]}{\delta \psi(x_1) \delta \psi(x_2)} \Big|_{\psi_{cl}} \Delta\psi(x_1) \Delta\psi(x_2) \right) \right]$$

$$Z[J]_{L=1} = S[\psi_{cl}, J] + \frac{i}{2} \log \det(\mathcal{D}^{-1}(\psi_{cl}))$$

$$S_1[\psi_{cl}, J; \mu] = -\frac{i}{2} \log \det(\mathcal{D}^{-1}(\psi_{cl})) \Big|_{\text{divergent}}$$

One-loop Renormalization: Heat Kernel

- In our case...

$$S_1[u_{\text{cl}}, S_{\text{cl}}, P_{\text{cl}}, J; \mu] = -\frac{i}{2} \log \det \left(\Sigma_\mu \Sigma^\mu + \Lambda \right) \Big|_{\text{divergent}},$$

27x27 matrices

$$\Sigma_{ij}^\mu = \delta_{ij} \partial^\mu + Y_{ij}^\mu$$

- Now using heat kernel techniques...

$$S_1[u_{\text{cl}}, S_{\text{cl}}, P_{\text{cl}}, J; \mu] = -\frac{1}{(4\pi)^2} \frac{1}{D-4} \int d^4x \text{Tr} \left(\frac{1}{12} Y^{\mu\nu} Y_{\mu\nu} + \frac{1}{2} \Lambda^2 \right),$$
$$Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu + [Y_\mu, Y_\nu]$$

iv) Conclusions

One-loop renormalization of RChT with
scalar and pseudoscalar resonances

One is able to work
at NLO

Running of the couplings

Possible continuations:

- i) Extension to RChT with $V(1^-)$ and $A(1^{++})$
- ii) Finite part of the generating functional