Quantum Loops in Resonance Theory :

The Vector Form-Factor

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Outline

I.) The $1/N_C$ counting and RχT at LO

II.) RχT lagrangian at NLO in $1/N_C$

III.) Low/High energy: → From RχT to χPT

→ From RχT to pQCD

The Problem

Pertubative

$Q^2 \gg 1\,\text{GeV}^2$ pQCD + OPE

Expansion in α_S

Non-perturbative

$Q^2 \sim 1\,\text{GeV}^2$ RESONANCES

QFT?

Non-perturbative

$Q^2 \ll 1\,\text{GeV}^2$ χPT

χ Expansion

Perturbative

QCD at $N_c \to \infty$

(Minimal Hadronical Ansatz)

$1/N_C$ Counting and RχT

['t Hooft 74]
[Witten 79]

$1/N_C$ Suppression of $q\bar{q}$ loops →

- $1/N_C$ Suppression of meson loops
- LO given by tree-level

LO in $1/N_C$

RχT

[Ecker *et al* '89]

- Resonances (R=V,A,S,P) + Goldstones (pNGB=π, K, η_8, η_1)
- Chiral symmetry
- General lagrangian for R + pNGB: $\langle R\, \mathcal{O}(p^2) \rangle$
- Minimal Hadronic Ansatz: *Long/short distances* √

RχT Lagrangian

at LO in $1/N_C$

$$\mathcal{L}_{2\chi}[\pi] = \frac{F^2}{4}\left\langle u^\mu u_\mu + \chi_+ \right\rangle$$

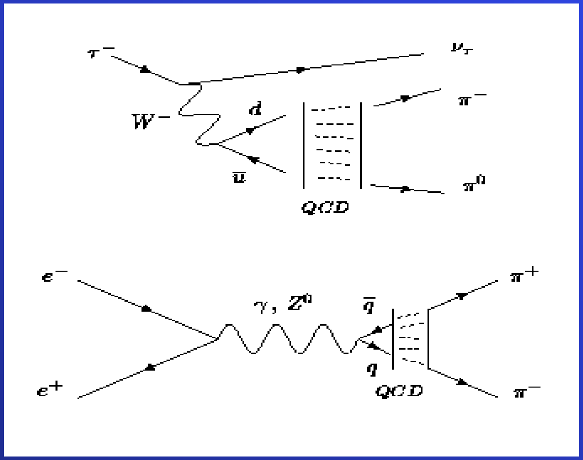
$$\mathcal{L}_{2V}[\pi, V] = -\frac{1}{2}\left\langle \nabla_\lambda V^{\lambda\mu} \nabla^\nu V_{\nu\mu} - \frac{1}{2} M_V^2 V_{\mu\nu} V^{\mu\nu} \right\rangle + \frac{F_V}{2\sqrt{2}}\left\langle V_{\mu\nu} f_+^{\mu\nu} \right\rangle + \frac{iG_V}{2\sqrt{2}}\left\langle V_{\mu\nu}\left[u^\mu, u^\nu\right]\right\rangle$$

$$\mathcal{L}_{2S}[\pi, S] = c_d \left\langle S u^\mu u_\mu \right\rangle + \ldots$$

$$\mathcal{L}_{2A}[\pi, A], \quad \mathcal{L}_{2P}[\pi, P]$$

QCD – Long/Short Distance

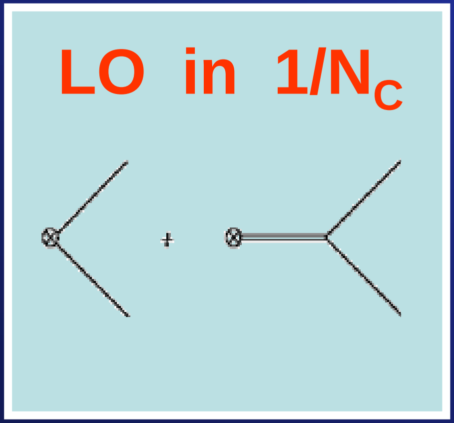
- -Vector FF
- -Axial FF
- -Scalar FF
- -2 point Green-Funcs.
- -Scattering Ampls.



VFF at LO in $1/N_C$:

Tree-level amplitudes

$$\langle \pi^+\pi^- | \left(\frac{1}{2}\bar{u}\gamma^\mu u - \frac{1}{2}\bar{d}\gamma^\mu d \right) | 0 \rangle = \mathcal{F}(q^2)\ (p_{\pi^+} - p_{\pi^-})^\mu$$

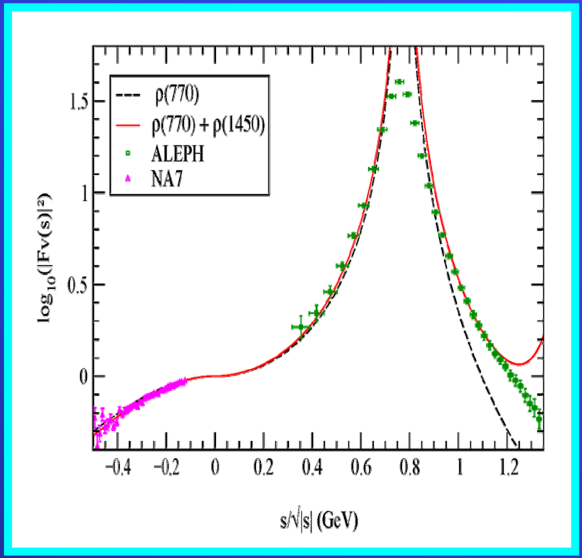


QCD

Short Dist.

$$\mathcal{F}(q^2) = 1 + \underbrace{\frac{F_V G_V}{F^2}} \frac{q^2}{M_V^2 - q^2} \xrightarrow{q^2 \to \infty} 0$$

$F_V G_V = F^2$



QCD at NLO in $1/N_c$:

Quantum loops

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$\mathcal{L}_{R\chi T}$ at LO in $1/N_C$ + QCD short distance

* Couplings fixed by QCD-short distances:

$$\frac{F_V}{\sqrt{2}} = \sqrt{2}\, G_V = 2\, c_d = F \sim \mathcal{O}(\sqrt{N_C})$$

* There are also mass relations

$$M_A^2 = 2\, M_V^2 ,\; M_P^2 = 2\, M_S^2 \left(1 - \delta\right) \sim \mathcal{O}(1)$$

$$\delta \approx 3\pi\alpha_S F^2/M_S^2 \sim 0.08\alpha_S$$

WELL DEFINED

$1/N_C$ counting

with LO vertices

$$\sum_{n_{loops}} \left(\frac{1}{(4\pi)^2} \frac{p^2}{F^2} \right)^{n_{loops}}$$

NLO Lagrangian

RχT at LO [Ecker et al'89]

One Loop Diagrams ⟶ NLO Contributions

However, Loops=Divergencies $\mathcal{O}(p^4)$!!

New NLO pieces (NLO couplings)

Minimal Extension: Pieces to renormalize

Perturbative calculation:

Next-to-leading order in $1/N_C$

Simplifications:

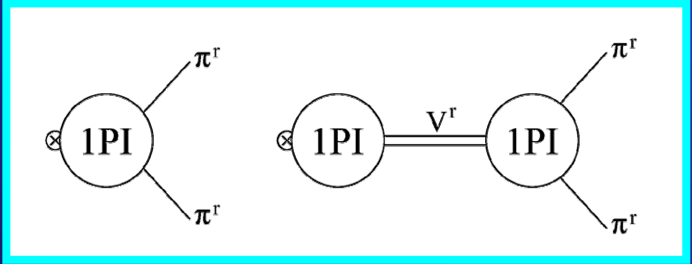
- $m_q=0$
- $U(2)_L \times U(2)_R$

NLO in $1/N_C$:

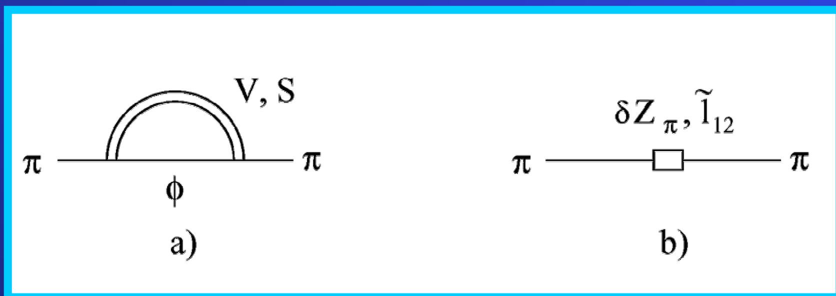
- One loop → LO vertices
- Tree-level → LO vertices and one NLO vertex

Renormalization procedure:

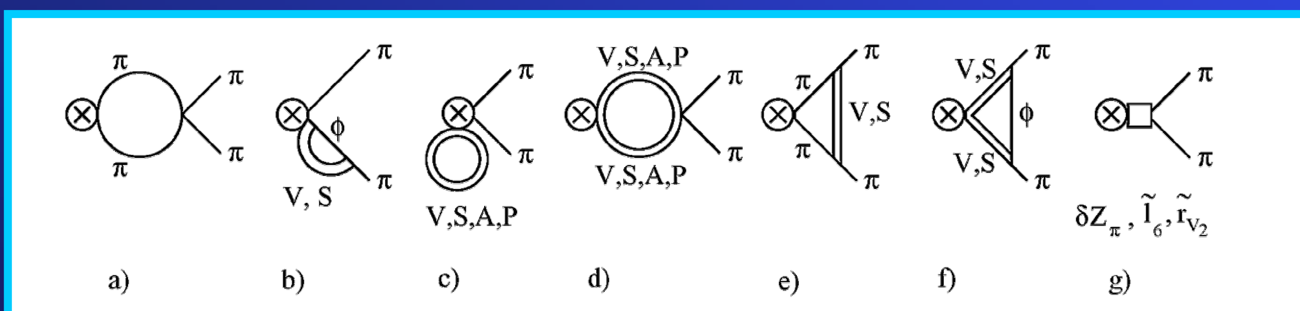
Renormalized propagators (self-energy) ⇒ Renormalized vertex-functions (1PI) ⇒ Renormalized VFF



- **Only External Pions**



$$-\tilde{\ell}_{12}\left\langle \nabla^{\mu}u_{\mu}\nabla^{\nu}u_{\nu}\right\rangle$$

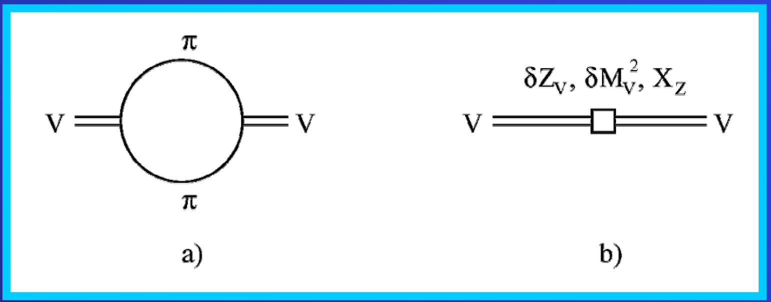


$$\frac{i\,\tilde{\ell}_{6}}{4}\left\langle f_{+}^{\mu\nu}\left[u_{\mu},u_{\nu}\right]\right\rangle + i\,\tilde{c}_{51}\left\langle \nabla^{\rho}f_{+}^{\mu\nu}\left[h_{\mu\rho},u_{\nu}\right]\right\rangle + \ldots$$

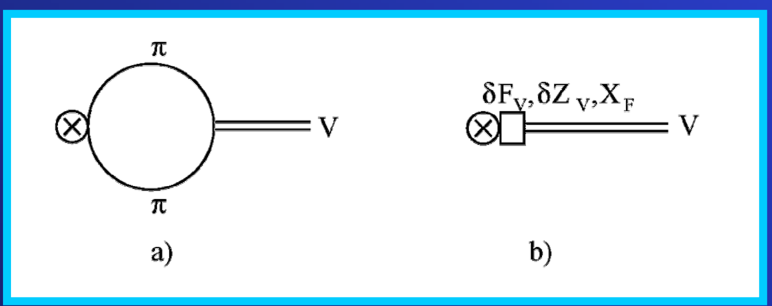
$$\left.\right\} \quad \mathcal{L}_{2\chi} + \tilde{\mathcal{L}}_{4\chi} + \tilde{\mathcal{L}}_{6\chi}$$

Other processes → Remaining operators and $\tilde{\ell}_i$ and $\tilde{c}_j$ couplings

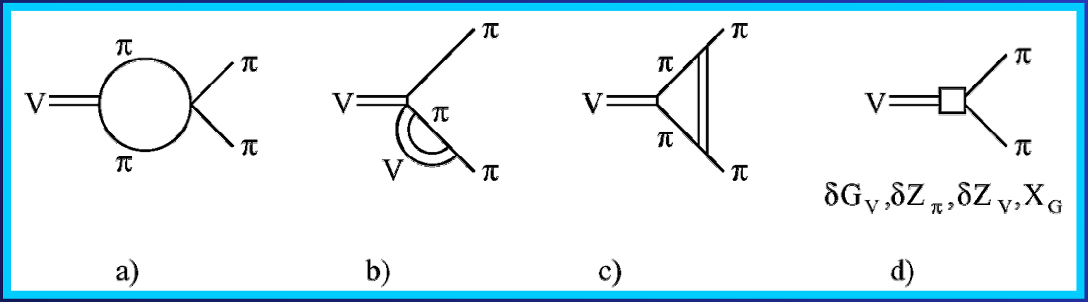
• External Pions and Vectors



$$\frac{X_{Z_1}}{2}\left\langle \nabla^2 V^{\mu\nu}\{\nabla_\nu, \nabla^\rho\} V_{\mu\rho}\right\rangle + \ldots$$



$$X_{F_1}\left\langle V_{\mu\nu}\nabla^2 f_+^{\mu\nu}\right\rangle + \ldots$$



$$i\, X_{G_1}\left\langle\{\nabla^\rho, \nabla_\mu\} V^{\mu\nu}[u_\nu, u_\rho]\right\rangle + \ldots$$

$$\mathcal{L}_{2V} + \mathcal{L}_{4V}$$

Equations of Motion

$$\ell_6^{eff} = \ell_6 + 2\, X_Z F_V G_V - 2\sqrt{2}\, X_F G_V - 4\sqrt{2}\, X_G F_V$$

$$r_{V_1}^{eff} = r_{V_1} = 4F^2 (c_{51} - c_{53})$$

$$F_V^{eff} = F_V + 2X_Z F_V M_V^2 - 2\sqrt{2} X_F M_V^2$$

$$G_V^{eff} = G_V + 2X_Z G_V M_V^2 - 4\sqrt{2} X_G M_V^2$$

$$M_V^{eff\,2} = M_V^2 + 2X_Z M_V^4$$

- EOM from the LO lagrangian

- Full reduction of $\mathcal{L}_{4V}$ to $(\mathcal{L}_\chi + \mathcal{L}_{2V})^{eff}$

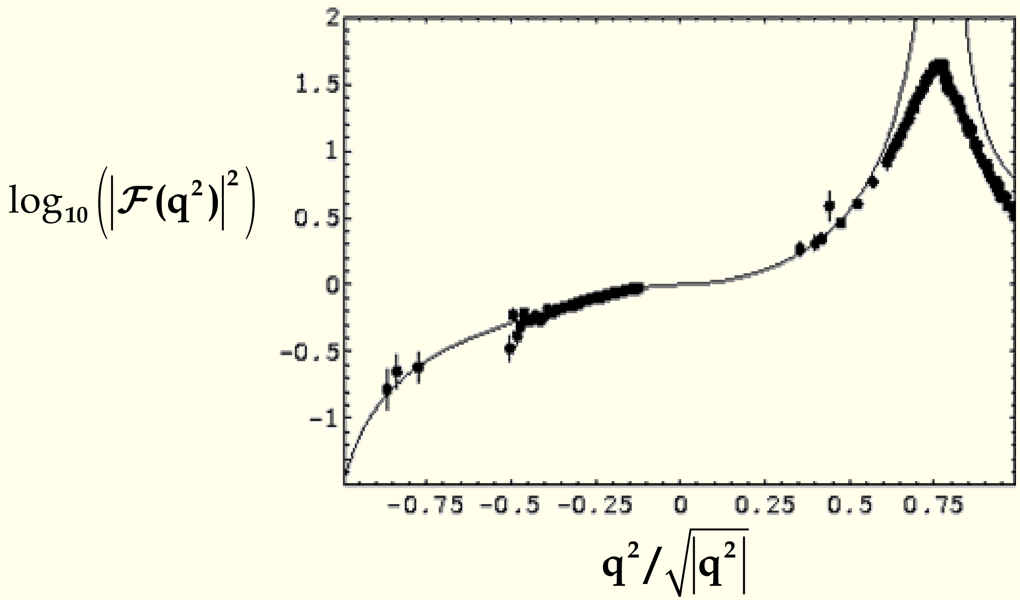
$$\mathcal{F}(q^2) = \mathcal{F}(\ell_6^{eff}, r_{V_1}^{eff}, F_V^{eff}, G_V^{eff}, M_V^{eff})$$

Experimental Results

and

High/Low Energy Limits

Space-like form-factor



$\mu = 770\,\text{MeV}$

$$\frac{F_V^{eff} G_V^{eff}}{F^2} = 1.1$$

$$\frac{2G_V^{eff\,2}}{F^2} = 1$$

$$M_V^{eff} = 776\,\text{MeV}$$

$$M_A^2 = 2M_V^2$$

$$2c_d = F$$

$$M_S = 1\,\text{GeV}$$

$$M_P^2 = 2\,M_S^2$$

$$\ell_6^{eff} = 0, \quad r_{V_1}^{eff} = 0$$

Radiative corrections:

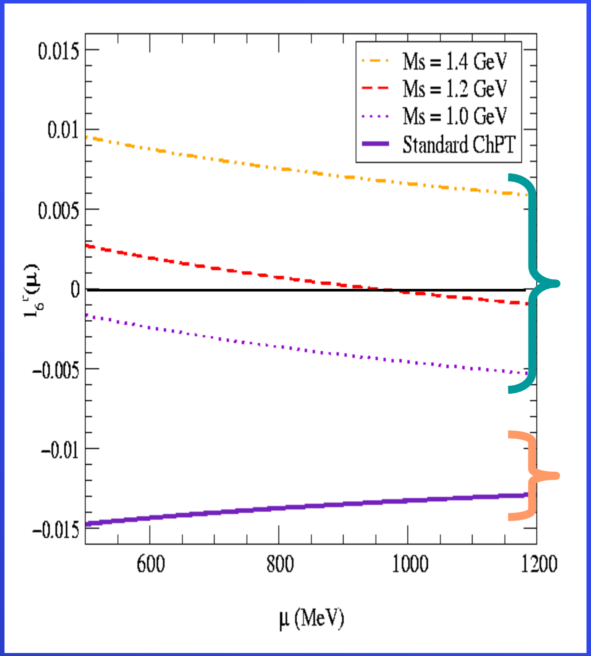
UNDER CONTROL !

From RχT to χPT

RχT $\left(q^2 \ll M_R^2, \Lambda_\chi^2\right)$

$$\mathcal{F}(q^2) = 1 + \left[\frac{F_V^{eff}(\mu)\, G_V^{eff}(\mu)}{M_V^{eff\,2}(\mu)} - \frac{\tilde{\ell}_6^{eff}(\mu)}{F^2} - \frac{1}{12\pi^2} \ln\left(\frac{M_V^2}{\mu^2}\right) + \dots \right] q^2 + $$

$$- \frac{1}{96\pi^2 F^2} q^2 \ln\left(-\frac{q^2}{\mu^2}\right) + \dots$$



$\mathcal{O}(p^4)$ χPT

$$\mathcal{F}(q^2) = 1 - \frac{\ell_6^{\chi PT}(\mu)}{F^2} q^2 - \frac{1}{96\pi^2 F^2} q^2 \ln\left(-\frac{q^2}{\mu^2}\right) + \dots$$

$\left(\begin{array}{c} \ell_6 \text{ in } n_f=2 \\ L_9 \text{ in } n_f=3 \end{array} \right)$

* Phenomenological suppression $\tilde{\ell}_6^{eff}(\mu) \ll \ell_6^{\chi PT}(\mu)$

$\mathcal{O}(1)$ $\mathcal{O}(N_C)$

* Similar with $\mathcal{O}(p^6)$ χPT

From RχT to pQCD

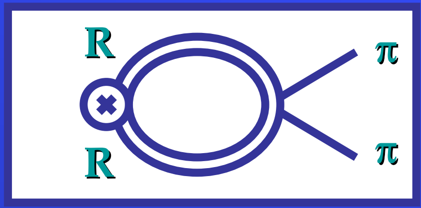
pQCD $Q^2 \to \infty$:

$$\mathcal{F}(Q^2) \xrightarrow{Q^2\to\infty} 0$$

RχT $Q^2 \to \infty$:

Intermediate Resonance states:

Constant Resonance form-factors !



$$\mathcal{F}(Q^2) = a_{(4)}\frac{Q^4}{F^2} + a_{(2)}\frac{Q^2}{F^2} + a_{(0)}\frac{M_R^2}{F^2} + \mathcal{O}\left(\frac{M_R^4}{Q^2F^2}\right)$$

constants
& $\ln(Q^2)$

If $a_{(j)}=0$ then
RENORMALIZATION
through LO lagrangian

Conclusions

- QCD at large N_C : *Minimal Hadronical Ansatz*

- RχT at NLO in $1/N_C$:

  - ✓ Quantum loops: Well defined counting and renormalization
  - ✓ Radiative corrections: Small within the range $|q^2| \leq 1\ \mathrm{GeV}^2$
  - ✓ Low energy limit: $R\chi T \xrightarrow{q^2 \ll \Lambda_\chi^2} \chi PT$
  - ? High energy limit:
    - Wrong behaviour (wrong resonance FF)
    - Need for extra operators with 2, 3... Resonance fields, e.g. $\langle V\, A\, \mathcal{O}(p^2) \rangle$