Muon Anomalous Magnetic Moment

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Outline

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3. Low Energy $e^+e^- \to hadrons$ and a_μ
4. τ Lepton Decays and CVC
5. Future of a_μ
6. Conclusions

Muon Anomalous Magnetic Moment

$\vec{\mu} = g\frac{e}{2m}\vec{s}$, $g = 2$ for a pointlike particle.

Higher-order effects $\Rightarrow g \neq 2$, $a = (g-2)/2$.

a_e and a_μ are among the best measured quantities:
a_e is known to 3 ppb $(3.3 \cdot 10^{-9})$, a_μ to 0.5 ppm $(5.2 \cdot 10^{-7})$.

$a_{\rm e}$ tests QED only and gives α, while a_μ is much more sensitive to new physics effects: the gain is usually $\sim (m_\mu/m_{\rm e})^2 \approx 4.3 \cdot 10^4$.

Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates new physics beyond the Standard Model.

$$a_\mu^{\rm th} = a_\mu^{\rm SM} + a_\mu^{\rm non-SM}, \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}.$$

New Physics and a_μ

- Supersymmetry ($\tilde{m} \simeq 200$ GeV–1 TeV)
- Cold dark matter and neutralino
- Tachyons ($m^2 < 0$)
- Radiative m_μ generation (new scale $\Lambda \simeq 1 - 2$ TeV)
- Technicolor
- Leptoquarks
- New gauge bosons (Z′ etc.)
- Large extra dimensions ($M > 600$ GeV)
- Lepton flavor violation ($\mu \to e\gamma$, μ edm, $\tau \to \mu\gamma, \ldots$)
- $b \to s\gamma$
- Composite quarks and leptons

Brief History of a_μ Measurements

Values of $a_\mu - 11600000, 10^{-10}$

Lab	Date	Value, 10^{-10}	Sign	$\Delta a_\mu / a_\mu$
CERN	1962	20000 ± 5000	$+$	$4.3 \cdot 10^{-4}$
CERN	1975	59100 ± 110	$+$	$9.4 \cdot 10^{-6}$
CERN	1977	59360 ± 120	$-$	$1.0 \cdot 10^{-5}$
CERN	1979	59230 ± 85	$\pm$	$7.3 \cdot 10^{-6}$
BNL	2000	59191 ± 59	$+$	$5.1 \cdot 10^{-6}$
BNL	2001	$59202 \pm 14 \pm 6$	$+$	$1.3 \cdot 10^{-6}$
BNL	2002	$59204 \pm 7 \pm 5$	$+$	$7.4 \cdot 10^{-7}$
BNL	2004	$59214 \pm 8 \pm 3$	$-$	$7.3 \cdot 10^{-7}$
Mean	2004	59208 ± 6	$\pm$	$5.2 \cdot 10^{-7}$

QED Contribution $a_\mu^{\rm QED}$ – One-loop Term

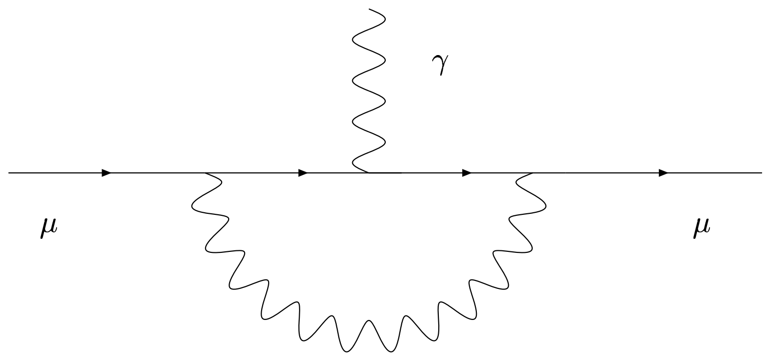
$$a_\mu^{\rm QED} = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau),$$

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A_i^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots.$$

$$\alpha^{-1} = 137.03599911 \pm 0.00000046 \quad (3.4\ \text{ppb}).$$

J.S.Schwinger, 1948:

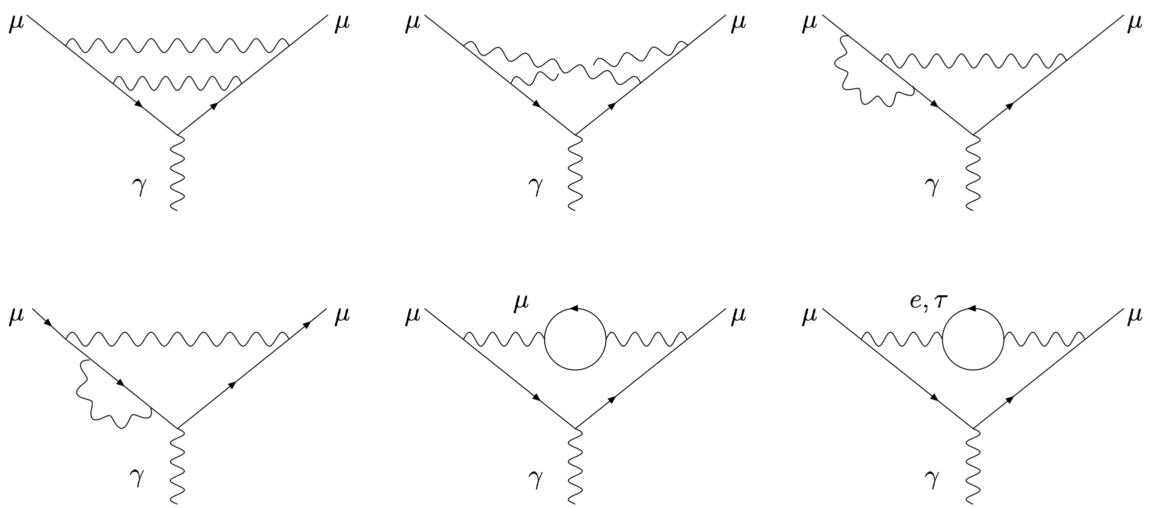
$$A_1^{(2)} = 1/2,\ A_2^{(2)} = A_3^{(2)} = 0.$$



$$C_1 = 1.16141 \cdot 10^{-3},$$

$$a_\mu^{\rm QED} = 1.16585 \cdot 10^{-3}.$$

QED Contribution $a_\mu^{\rm QED}$ – Two-loop Terms

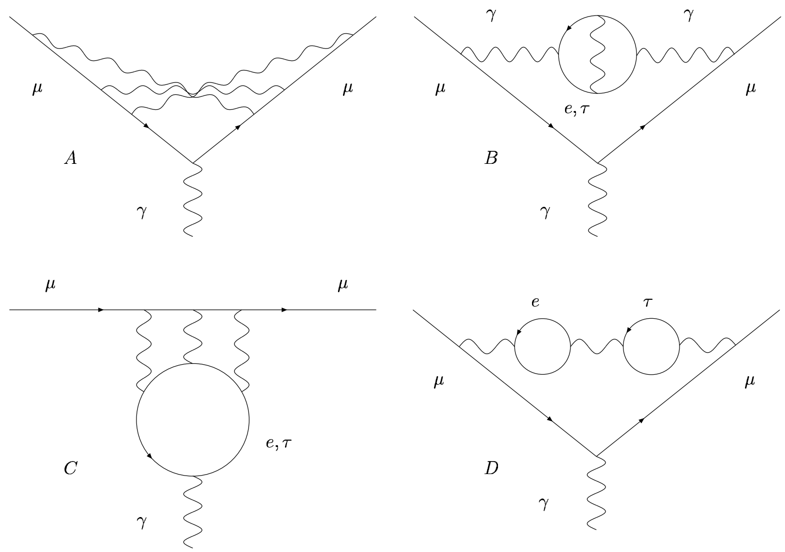


$A_1^{(4)}$ – 7, $A_2^{(4)}(m_\mu/m_e)$ and $A_2^{(4)}(m_\mu/m_\tau)$ – 1 diagram.

C.M. Sommerfeld, 1957; A. Petermann, 1957: $A_1^{(4)} = -0.328478965579\ldots$,
H.H. Ekend, 1966; M. Passera, 2004: $A_2^{(4)}(m_\mu/m_e) = 1.0942583111(84)$,
$A_2^{(4)}(m_\mu/m_\tau) = 0.000078064(25)$, $A_3^{(4)}(m_\mu/m_e, m_\mu/m_\tau) = 0$.

$$C_2 = 0.765857419(27).$$

QED Contribution a_μ^{QED} – Three-loop Terms



$A_1^{(6)}$ – 72, $A_2^{(4)}(m_\mu/m_e)$ and $A_2^{(4)}(m_\mu/m_\tau)$ – 48 diagrams.

E. Remiddi et al., 1996: $A_1^{(6)} = 1.1812414566\ldots$,

S. Laporta and E. Remiddi, 1993; M. Passera, 2004:

$A_2^{(6)}(m_\mu/m_e) = 22.86838002(20)$, $A_2^{(6)}(m_\mu/m_\tau) = 0.00036051(21)$,

$A_3^{(6)}(m_\mu/m_e, m_\mu/m_\tau) = 0.00052766(17)$.

$C_3 = 24.05050964(43)$.

QED Contribution $a_\mu^{\rm QED}$ – Four-loop Terms

More than 1000 diagrams – first evalulated numerically by T. Kinoshita et al. in 1984. Recently revised (correction of mistakes and higher accuracy):

- $A_1^{(8)}$ – 891 diagrams. The value changed from -1.7502(384) to -1.7093(42)
- $A_2^{(8)}(m_\mu/m_e)$ – 469 diagrams. The value changed from 127.50(41) to 132.6823(72)
- $A_2^{(8)}(m_\mu/m_\tau)$ leads to $\mathcal{O}(10^{-13})$ in $a_\mu^{\rm QED}$
- 102 diagrams give $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) == 0.037594(83)$

$$C_4 \approx 131.011(8).$$

QED Contribution $a_\mu^{\rm QED}$ – Five-loop Terms

Only specific contributions were numerically evaluated:

- T. Kinoshita et al., 1990: $C_5 = 570(140)$
- Other groups, 1989-1995: C_5 ranging from -1.3 to 575
- T. Kinoshita and M. Nio: $C_5 \approx A_2^{(10)}(m_\mu/m_e) = 677(40)$ based on 9080 diagrams.

$$C_5 = 677(40)$$

QED Contribution $a_\mu^{\rm QED}$ – Summary

$$
\begin{aligned}
a_\mu^{\rm QED} = \; & 0.5\left(\frac{\alpha}{\pi}\right) \\
+ \; & 0.765857410(27)\left(\frac{\alpha}{\pi}\right)^2 \\
+ \; & 24.05050964(43)\left(\frac{\alpha}{\pi}\right)^3 \\
+ \; & 131.011(8)\left(\frac{\alpha}{\pi}\right)^4 \\
+ \; & 677(40)\left(\frac{\alpha}{\pi}\right)^5 + \ldots \\
= \; & 11658471.88(0.03)(0.04)\cdot 10^{-10},
\end{aligned}
$$

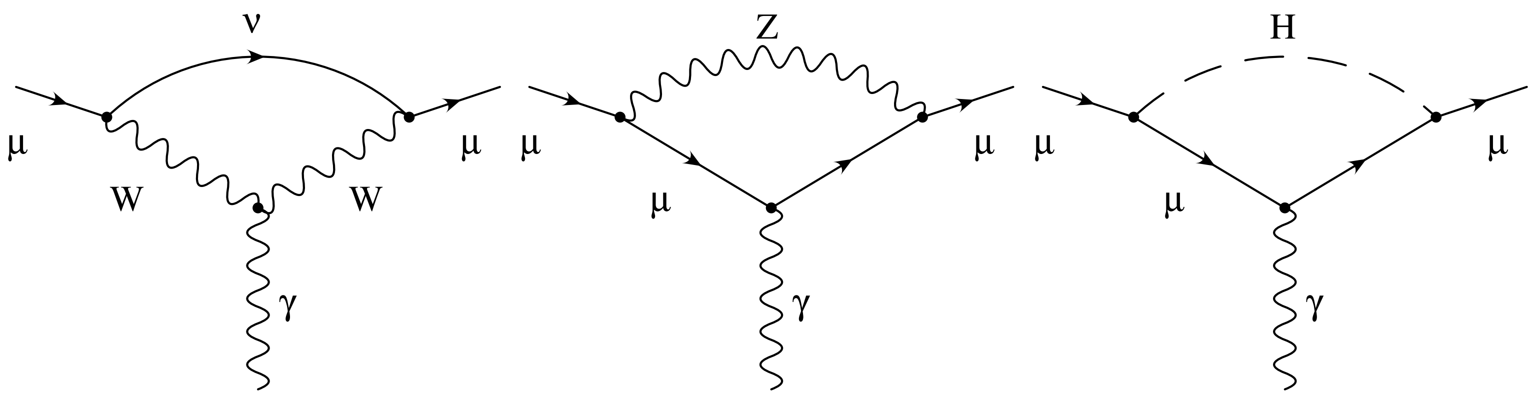
the errors coming from the uncertainties of the $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^4)$ and $\mathcal{O}(\alpha^5)$ terms and that of α.

Comparison with the previous value:

$a_\mu^{\rm QED} = 11658470.6(0.3)\cdot 10^{-10}$ – old value,

$a_\mu^{\rm QED} = 11658471.88(0.03)(0.04)\cdot 10^{-10}$ – new value.

Electroweak Contribution $a_\mu^{\rm EW}$ – One-loop Term



$$a_\mu^{\rm EW}(1-{\rm loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 \ + \ \frac{1}{5}(1-4\sin^2\theta_{\rm W})^2 \ + \ \mathcal{O}\left(\frac{m_\mu^2}{m_{\rm W,H}^2}\right)\right] = 194.8\cdot 10^{-11}.$$

First calculated in 1972 by five independent groups:

G. Altarelli, N. Cabibbo and L. Maiani;

W.A.Bardeen, R. Gastmans and B. Lautrup;

R. Jackiw and S. Weinberg;

I.Bars and M. Yoshimura;

K. Fujikawa, B.W. Lee and A.I. Sanda.

Electroweak Contribution a_μ^{EW} – Higher-order Terms

Naively, $a_\mu^{\mathrm{EW}}(1-\mathrm{loop}) \sim \frac{\alpha}{\pi} a_\mu^{\mathrm{EW}}(2-\mathrm{loop})$, but some of 1678 diagrams are enhanced by $\ln(M_{W,Z}/m_f)$, so that $a_\mu^{\mathrm{EW}}(2-\mathrm{loop}) \approx -0.2 a_\mu^{\mathrm{EW}}(1-\mathrm{loop})$.

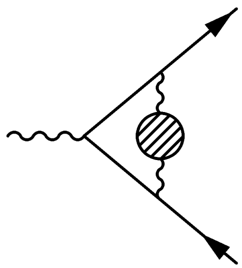
Authors	Date	a_μ^{EW}, 10^{-10}
T.V. Kukhto et al.	1992	15.3 ± 0.5
A. Czarnecki, B. Krause, W.J. Marciano	1996	15.2 ± 0.4
M. Knecht et al.	2002	15.2 ± 0.1
A. Czarnecki, W.J. Marciano, A. Vainshtein	2003	$15.4 \pm 0.1 \pm 0.2$

$$a_\mu^{\mathrm{EW}} = 15.4(0.1)(0.2) \cdot 10^{-10},$$

the errors coming from the hadronic loop uncertainties and the M_H, m_t and unknown three-loop effects.

Hadronic contribution a_μ^{had}

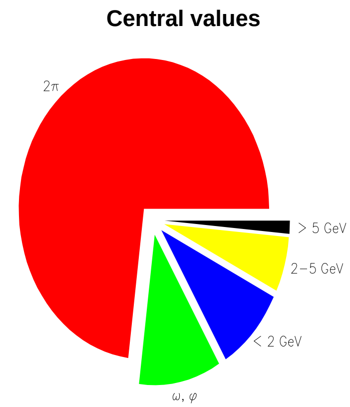
$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$

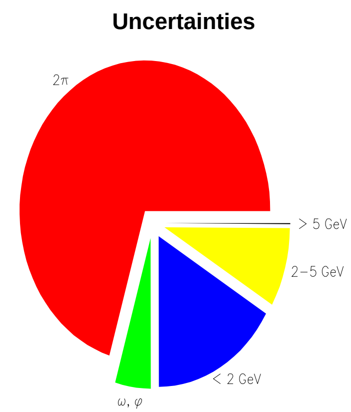


$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{R(s)\,\hat{K}(s)}{s^2},$$

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$, $1/s^2$ emphasizes the role of low energies, particularly important is the reaction $e^+e^- \to \pi^+\pi^-$ with a large cross section below 1 GeV.





How Can $a_\mu^{\mathrm{LO,had}}$ Be Calculated?

There are various approaches:

- Pure theory (perturbative QCD)
- Theory + experiment (resonance regions)
- F_π from theory + experiment
- Add data on τ lepton decays
- Pure data based (straightforward error estimation, statistical errors negligible, systematic errors!)

The latter method is the most conservative one.

Calculations of $a_\mu^{had,LO}$

Authors	Year	$a_\mu^{had,LO}$, 10^{-10}
C.Bouchiat, L.Michel	1961	$\simeq 648$
T.Kinoshita, R.J.Oakes	1967	$\simeq 750$
M.Gourdin, E. de Rafael	1969	650 ± 50
A.Bramon et al.	1972	680 ± 90
V.Barger et al.	1975	660 ± 100
J.Calmet et al.	1977	702 ± 80
T.Kinoshita et al.	1985	707 ± 18
S.Eidelman, F.Jegerlehner	1995	702 ± 15
R.Alemany et al.	1998	701.1 ± 9.4
M.Davier, A.Höcker	1998	692.4 ± 6.2

Measurement of R(s) – General

$$R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-),$$

$$\sigma(e^+e^- \to \text{hadrons}) = N/\int Ldt\epsilon(1+\delta),$$

- N – number of selected events
- $\int Ldt$ – integrated luminosity
- ϵ – detection efficiency (acceptance)
- δ – radiative correction (RC)

RC includes effects of initial and final state radiation (ISR and FSR) as well as vacuum polarization (VP).

Below $\sqrt{s} \sim 2$ GeV exclusive measurements $(\pi^+\pi^-, \pi^+\pi^-\pi^0, \ldots, K\bar{K}, \ldots)$, above – total R (all multihadronic events).

Measurement of R(s) – FSR

For pointlike pions

J. Schwinger, 1970:

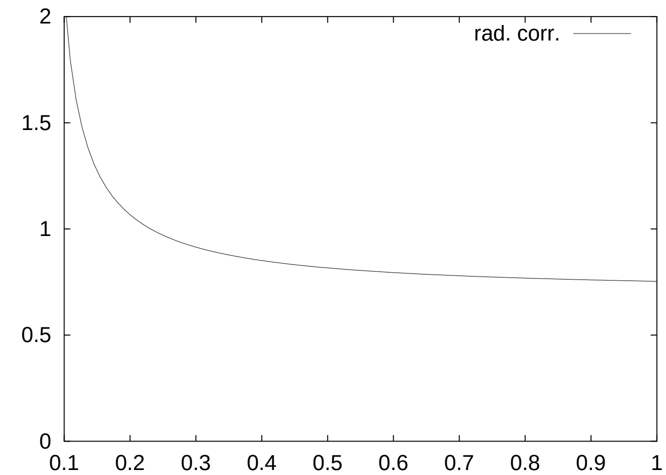
$$\sigma(e^+e^- \to \pi^+\pi^-)$$

$$= \frac{\pi\alpha^2\beta^3}{s}|F_\pi|^2(1+\frac{\alpha}{\pi}\mathcal{F}(\beta)),$$

At $\beta \to 0 \quad \mathcal{F}(\beta) \to \pi\alpha^2/2\beta - 2,$

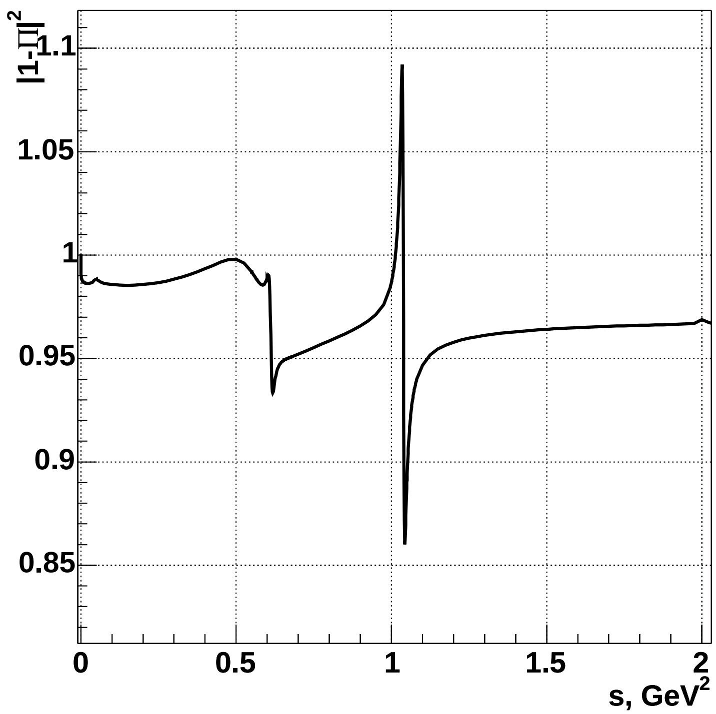
at $\beta \to 1 \quad \mathcal{F}(\beta) \to 3.$

Near $\rho \quad \delta_{\rm FSR} \approx 0.8\%.$



Measurement of R(s) – VP

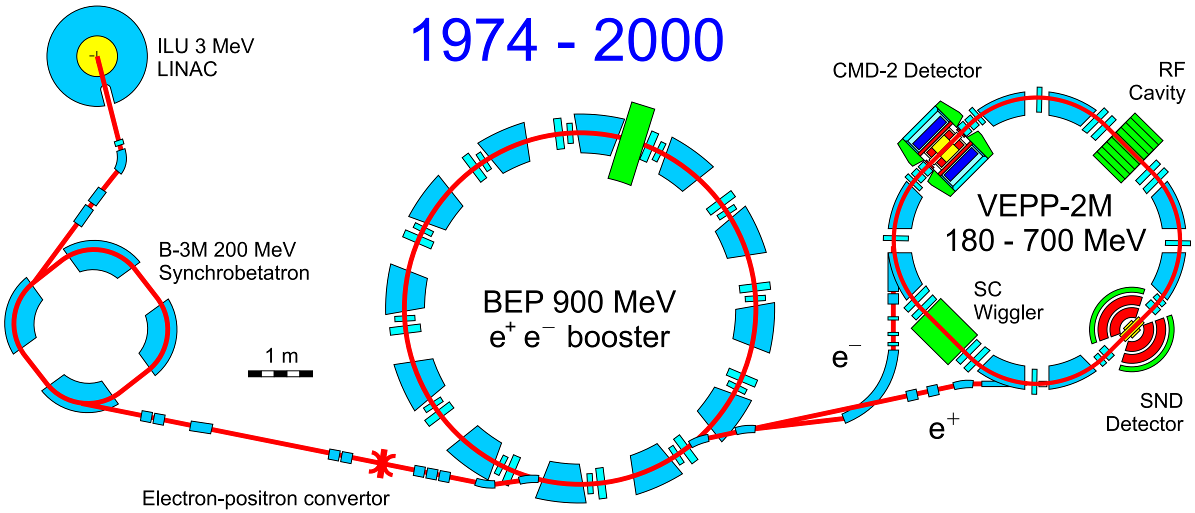
$$\sigma_{\text{bare}} = \sigma_{\text{dressed}} |1 - \Pi(s)|^2,$$
$$\Pi(s) = \Pi_{\text{lep}}(s) + \Pi_{\text{had}}(s).$$



Measurement of R(s) – Special Features

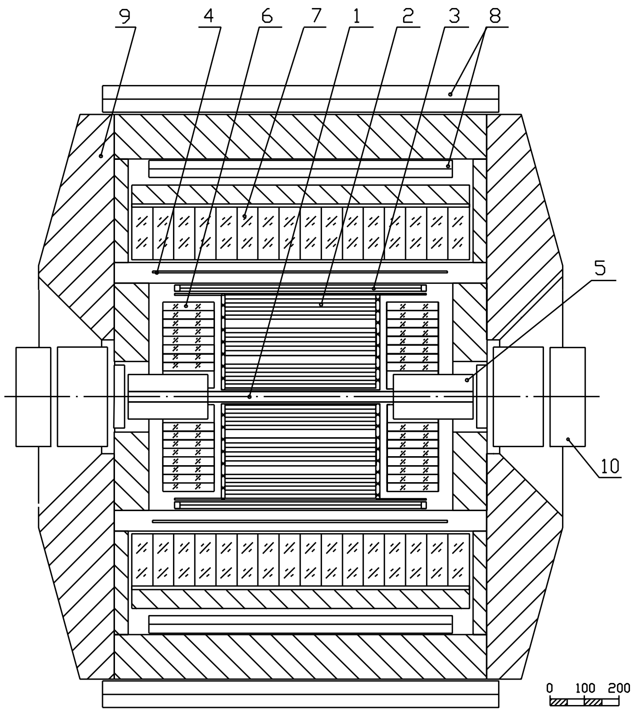
1. Exclusive approach
   - Small background (complete event reeconstruction)
   - Exact production model - ϵ well known!
   - Possibly missing (small σ, undetected) final states
   - FSR unknown for most of the final states
   - For old experiments some corrections missing ($\delta_{\rm VP}^{\rm had}$)
2. Total R
   - Background of $\gamma\gamma \to$ hadrons and $\tau^+\tau^-$
   - FSR automatically included
   - Approximate production model (LUND, PYTHIA, LUARW) – ϵ?
   - $\delta_{\rm VP}^{\rm had}$ also often missing

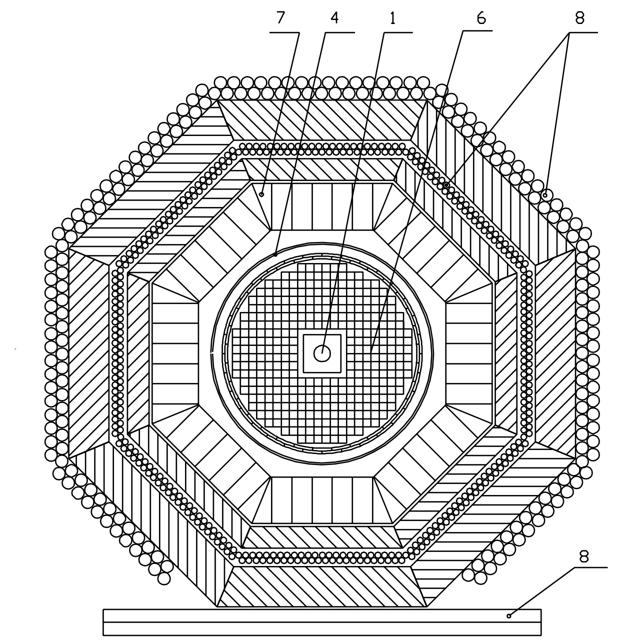
VEPP-2M Collider in Novosibirsk



- Experiments: 1974 – 2000
- Energy range: $0.36 < \sqrt{s} < 1.40$ GeV
- Peak luminosity: $L_{\mathrm{peak}} = 3 \cdot 10^{30}$ cm^{-2} s^{-1}
- Integrated luminosity ≈ 100 pb^{-1} in Novosibirsk below 1.4 GeV compared to ≈ 6 pb^{-1} in Orsay and Frascati at $1.4 < \sqrt{s} < 3.0$ GeV!

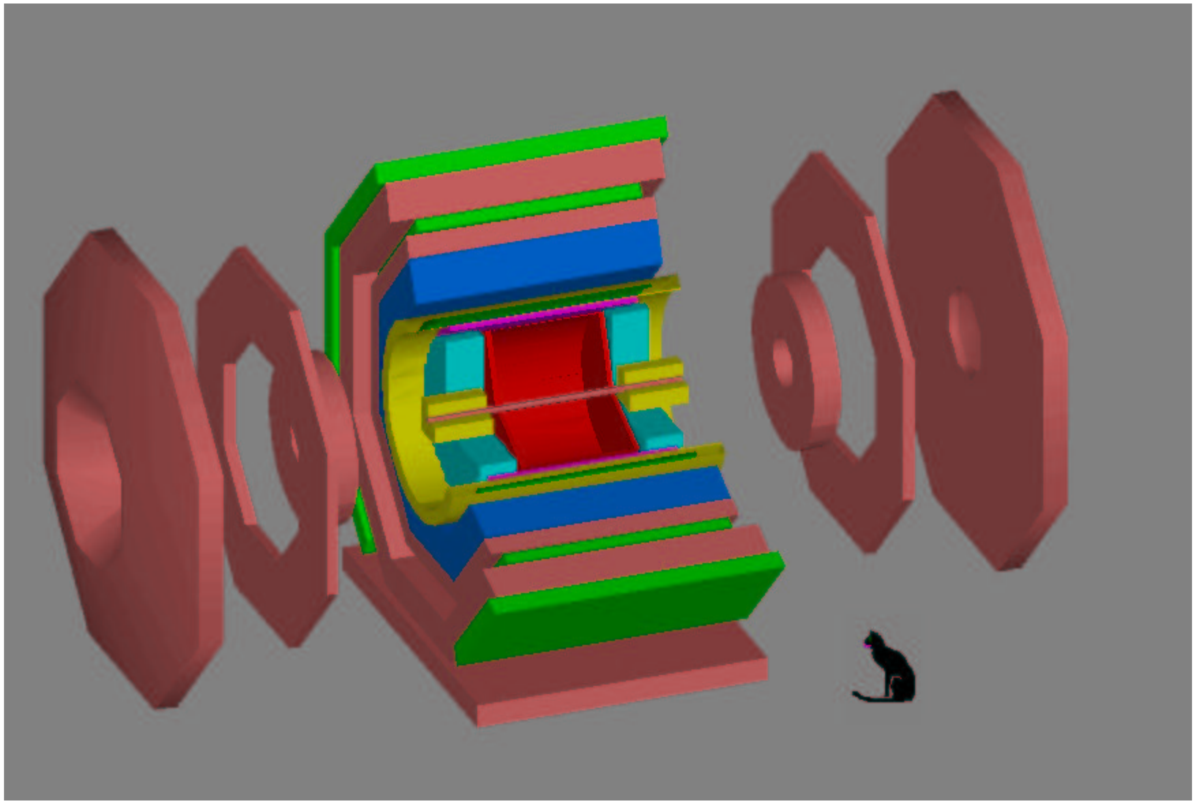
CMD-2 Detector



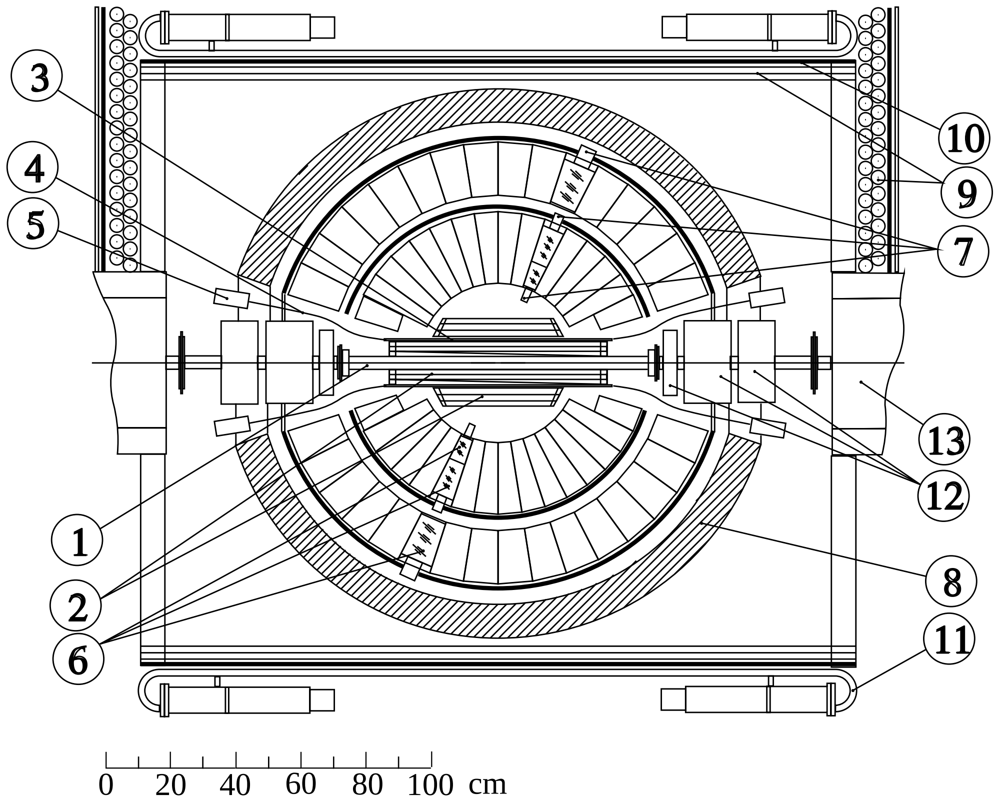


1 – beam pipe; 2 – drift chamber; 3 – Z-chamber; 4 – superconducting solenoid; 5 – compensating solenoid; 6 – BGO endcap calorimeter; 7 – CsI barrel calorimeter; 8 – muon system; 9 – yoke; 10 – quadrupoles.

CMD-2 Detector



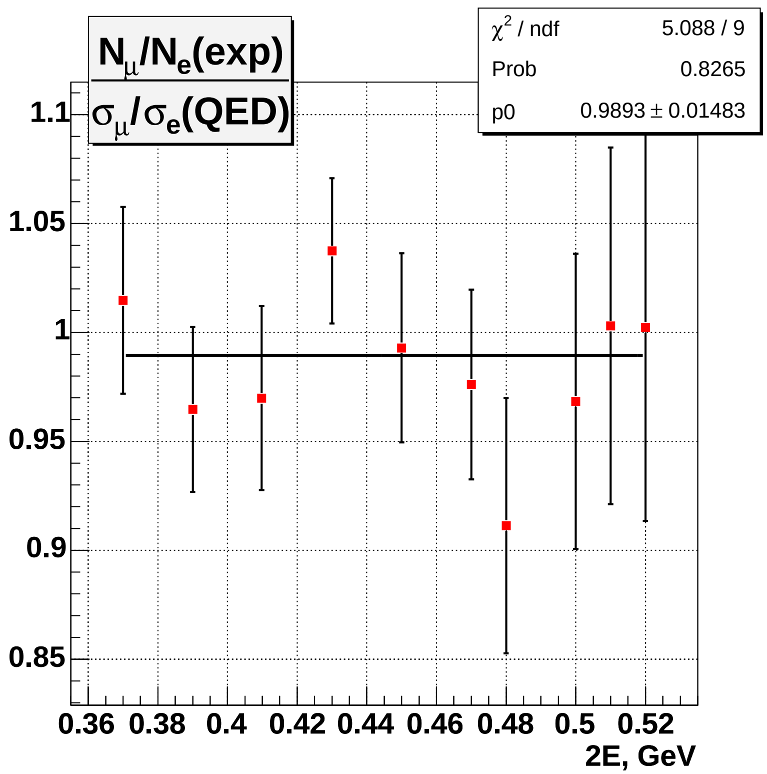
SND Detector

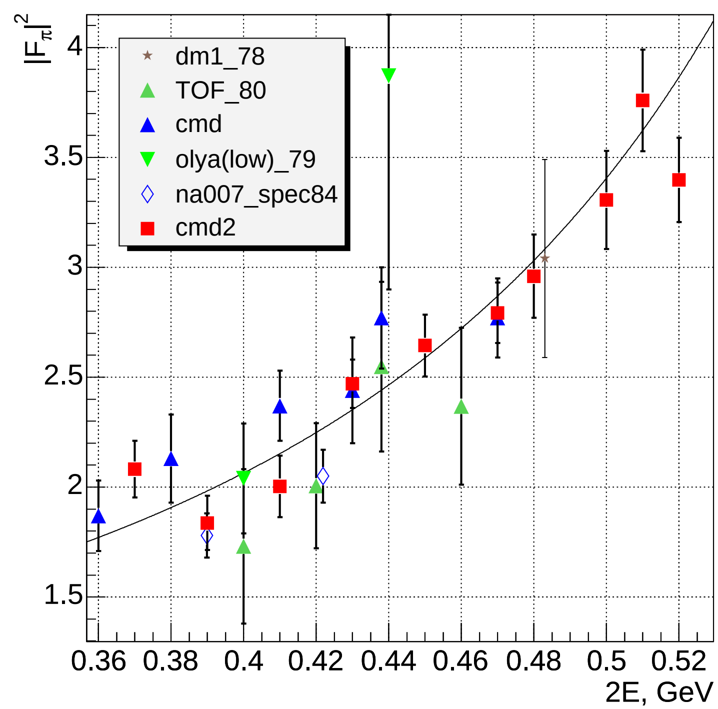


1 – beam pipe; 2 – drift chambers; 3 – scintillation counters; 4 – lightguides; 5 – PMT's; 6 – NaI(Tl) crystals; 7 – vacuum phototriodes; 8 – iron absorber; 9 – streamer tubes; 10 – iron plates; 11 - scintillation counters; 12 – collider magnets.

$e^+e^- \to \pi^+\pi^-$ at CMD-2. 370 MeV $< \sqrt{s} <$ 600 MeV

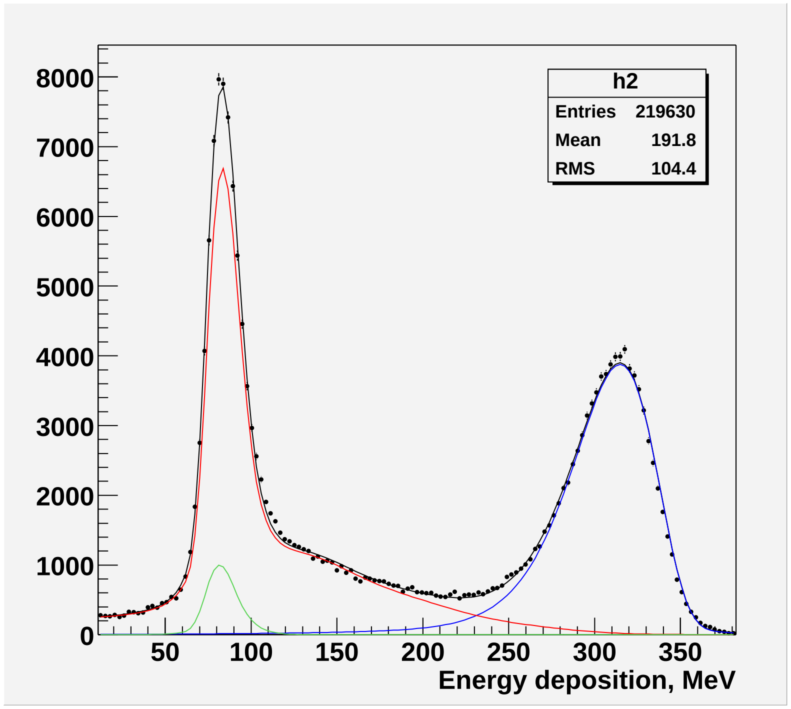
$N_{\rm ev} = 4000$, $e/\mu/\pi$ separation by the momentum in DC

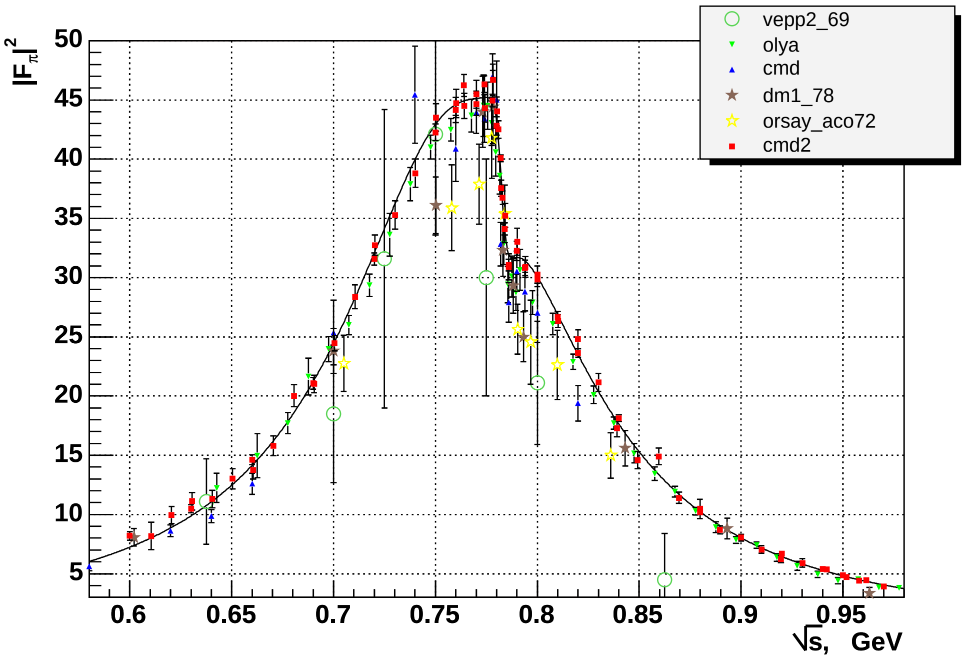




$e^+e^- \to \pi^+\pi^-$ at CMD-2. 610 MeV $< \sqrt{s} <$ 960 MeV

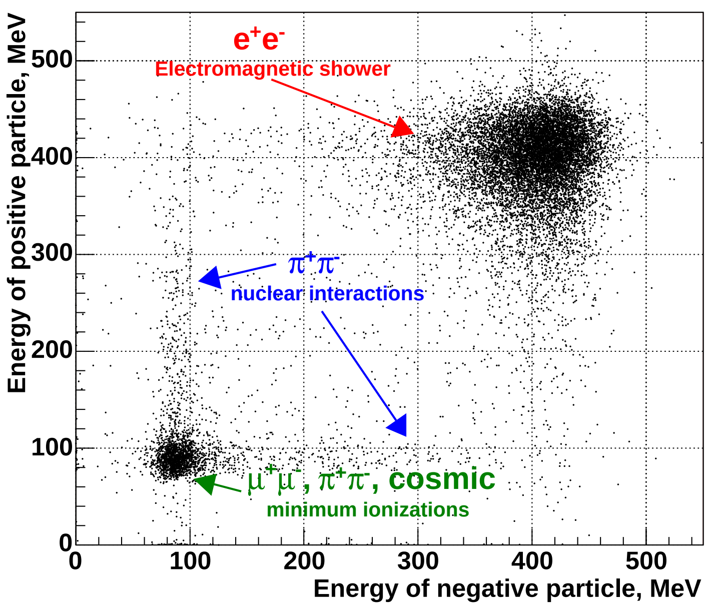
$N_{\rm ev} \approx 630 \cdot 10^3$, $e, \mu/\pi$ separation by energy deposition in CsI

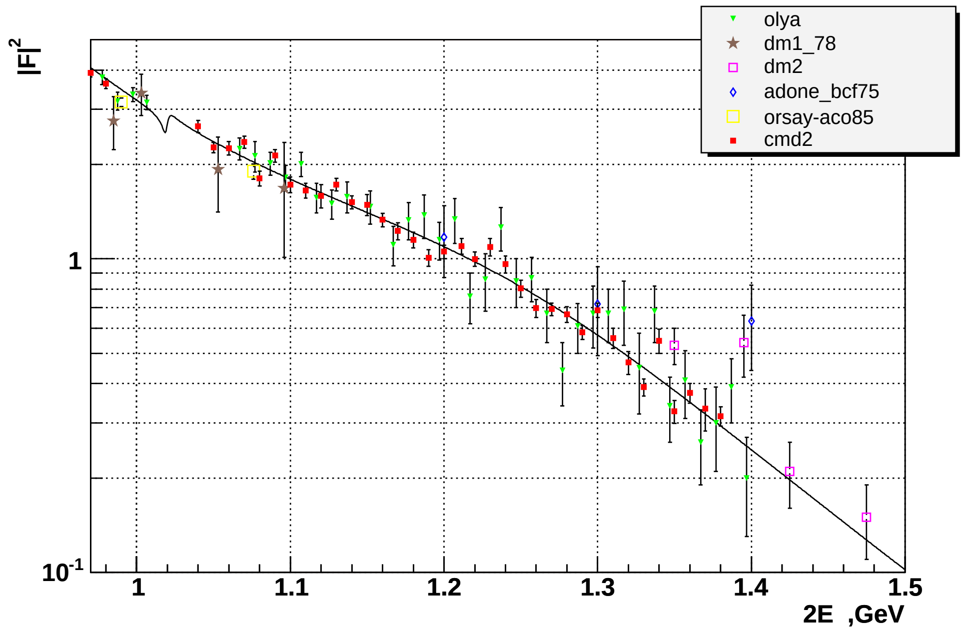




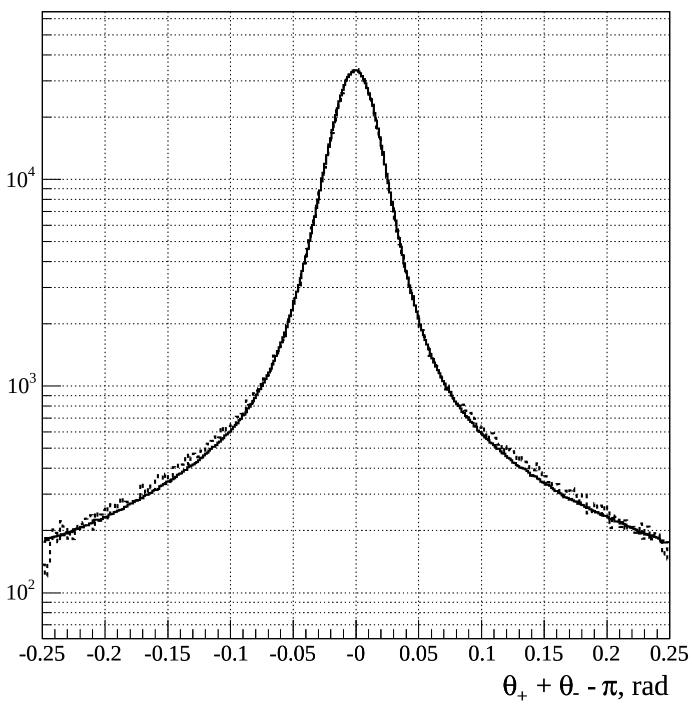
$e^+e^- \to \pi^+\pi^-$ at CMD-2. 1040 MeV $< \sqrt{s} <$ 1380 MeV

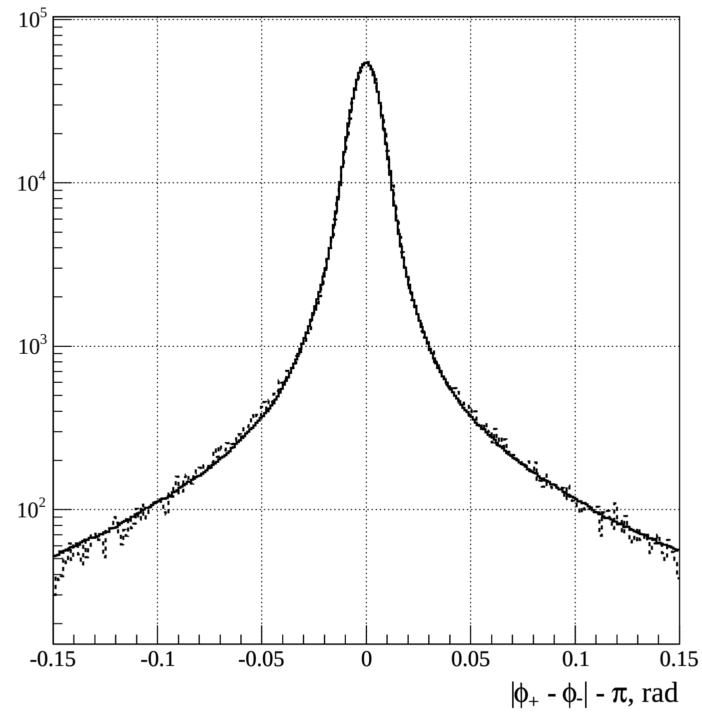
$N_{\rm ev} = 33 \cdot 10^3$, $e, \mu/\pi$ separation by energy deposition in CsI





Radiative corrections



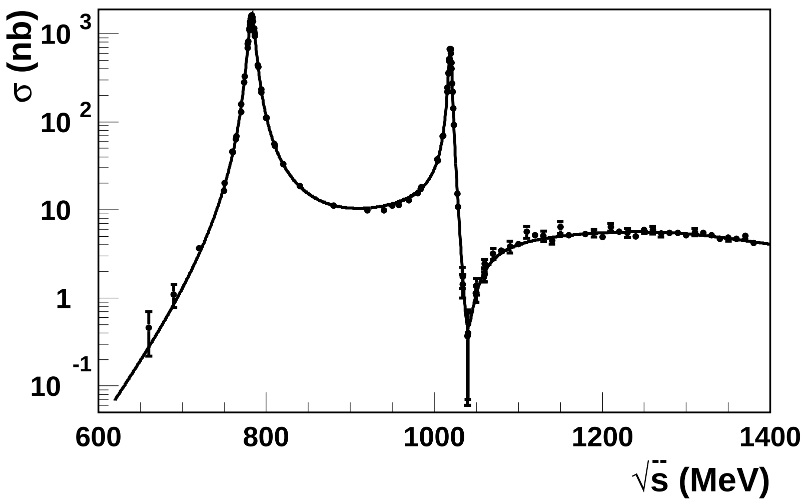


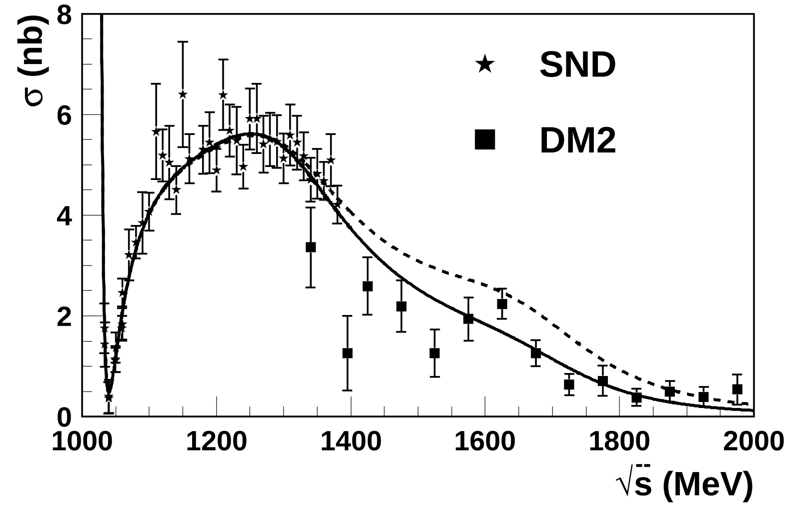
CMD-2 is using a MC generator based on A. Arbuzov et al., 1997; its accuracy $\sim 0.2\%$; agrees with BHWIDE and BABAYAGA within claimed accuracy.

Budget of $e^+e^- \to \pi^+\pi^-$

Source / $\sqrt{s}$, GeV	0.37÷0.52	0.6÷0.96		1.04÷1.38
$N_{\pi\pi}$, 10^3 / Number of points	4/10	114/43	520/29	33/36
Stat. error/point, %	6.0	4.0	1.5	5.0 ÷ 13.0
Fiducial volume, %	0.2	0.2		0.2÷0.5
Detection efficiency, %	0.3	0.2	0.9	0.5÷2.0
Pion losses, %	0.2	0.2		0.2
Radiative corrections, %	0.3	0.4		0.5÷2.0
Background events, %	< 0.1	< 0.1		0.6÷1.6
Beam energy calibration, %	0.3	0.1	0.3	0.7÷1.1
Event separation, %	1.0	0.2		0.5÷3.5
Total systematic error, %	1.2	0.6	1.1	1.3÷5.0

Study of $e^+e^- \to \pi^+\pi^-\pi^0$ at SND

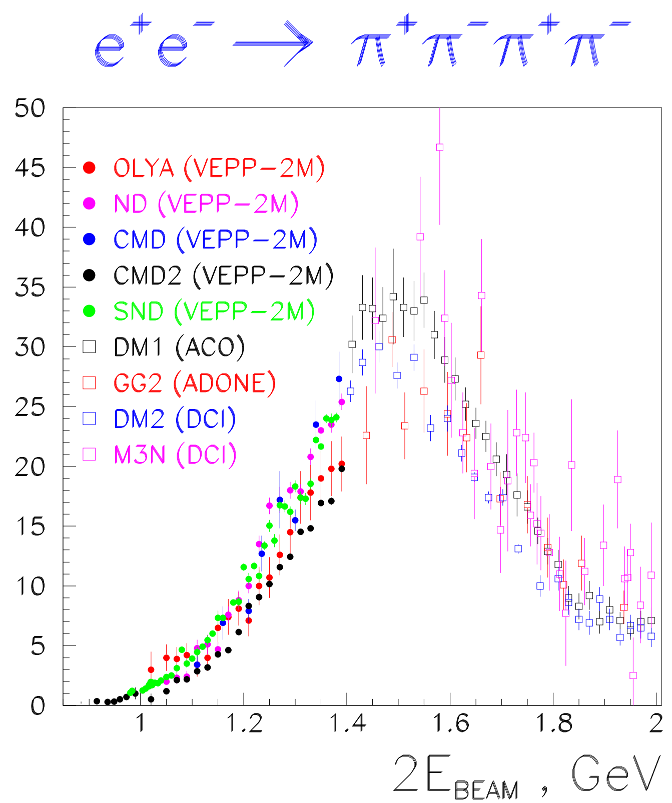


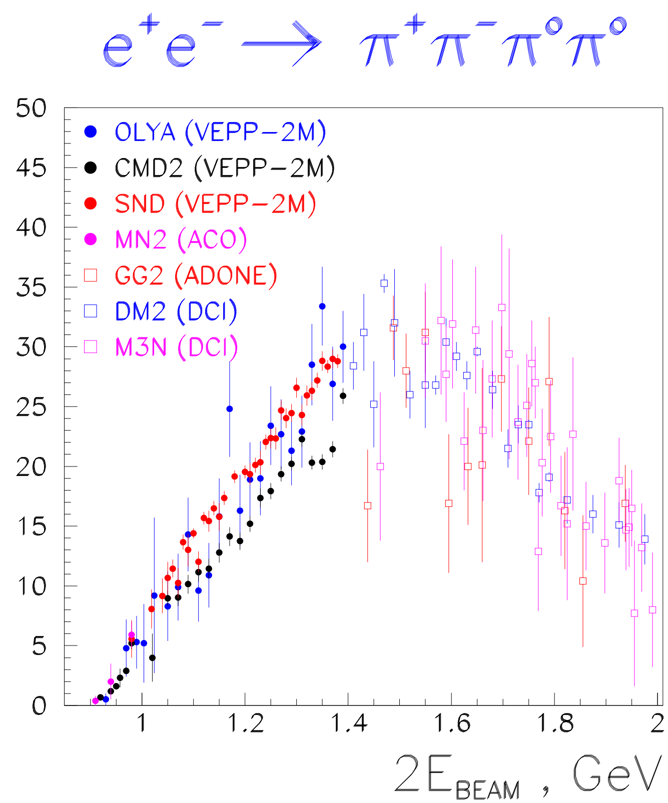


About $1.7 \cdot 10^6$ detected events. The systematic error is $\approx 5\%$.

DM2 data are too low (by a factor of 1.6 or larger)!

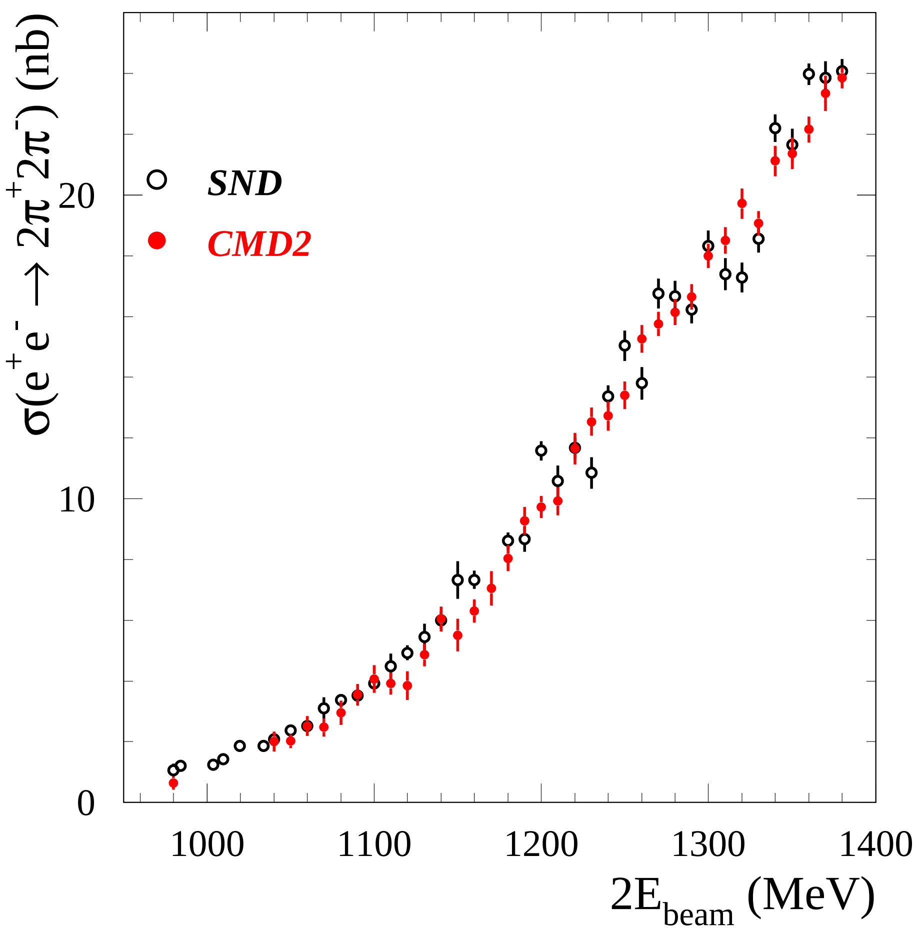
Study of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$

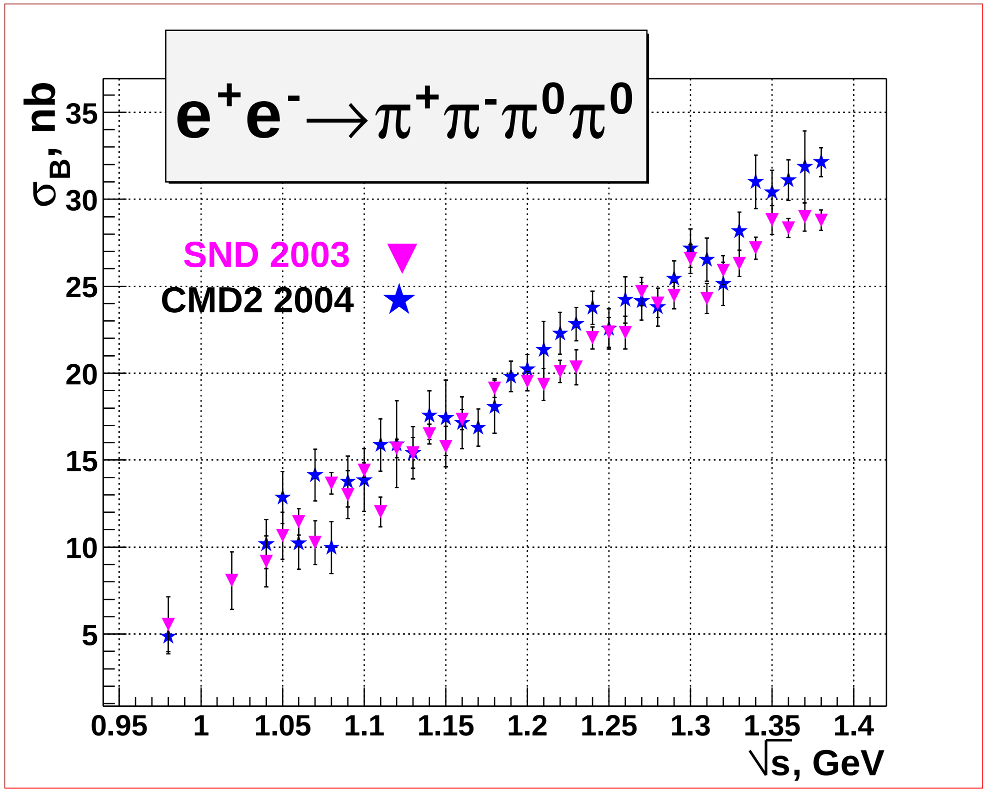




Large data scatter above 1.4 GeV!

Study of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ with CMD-2 and SND

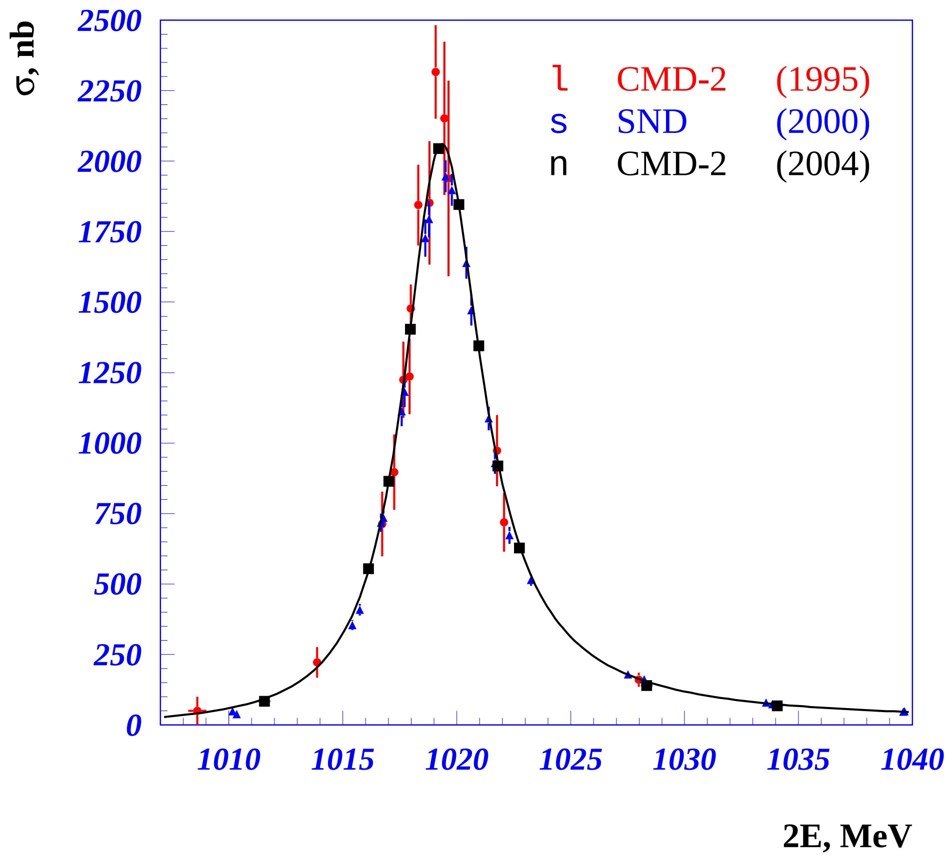




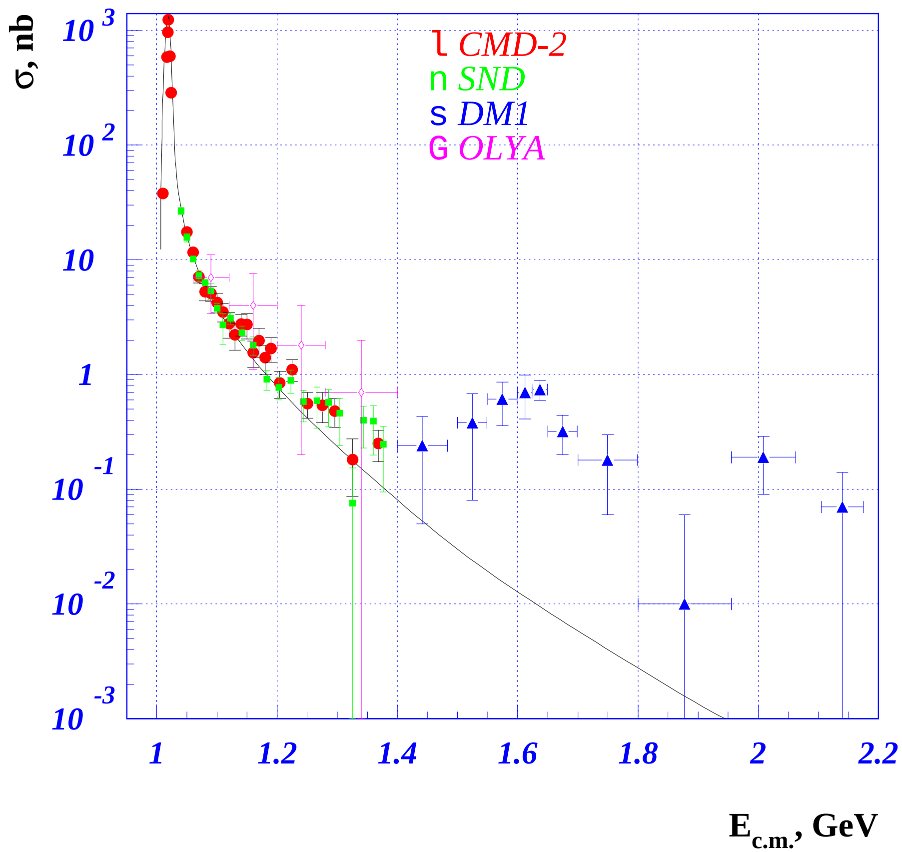
CMD-2: $38 \cdot 10^3$ ev., (5–7)% syst.
SND: $41 \cdot 10^3$ ev., 7% syst.

CMD-2: $10 \cdot 10^3$ ev.,, 6% syst.
SND: $54 \cdot 10^3$ ev., 8% syst.

Studies of $e^+e^- \to \phi \to K^+K^-$ at VEPP-2M

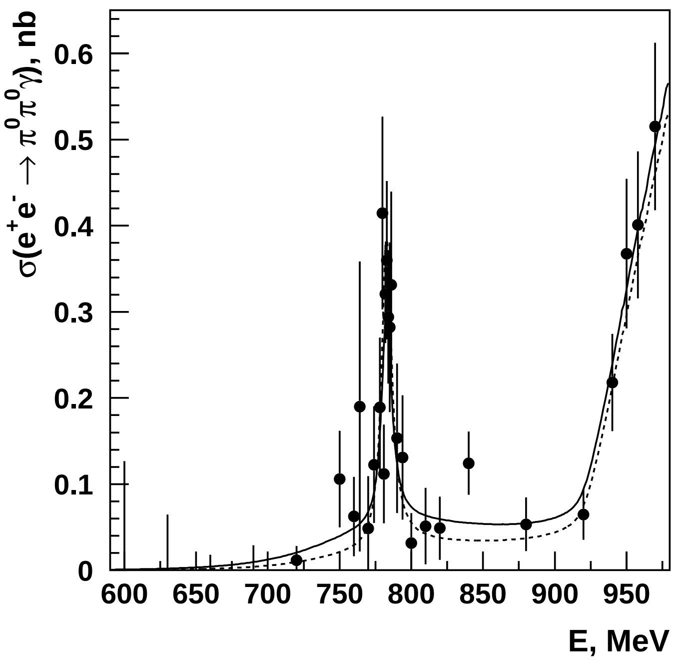


Studies of $e^+e^- \to \phi \to K^0_S K^0_L$ at VEPP-2M



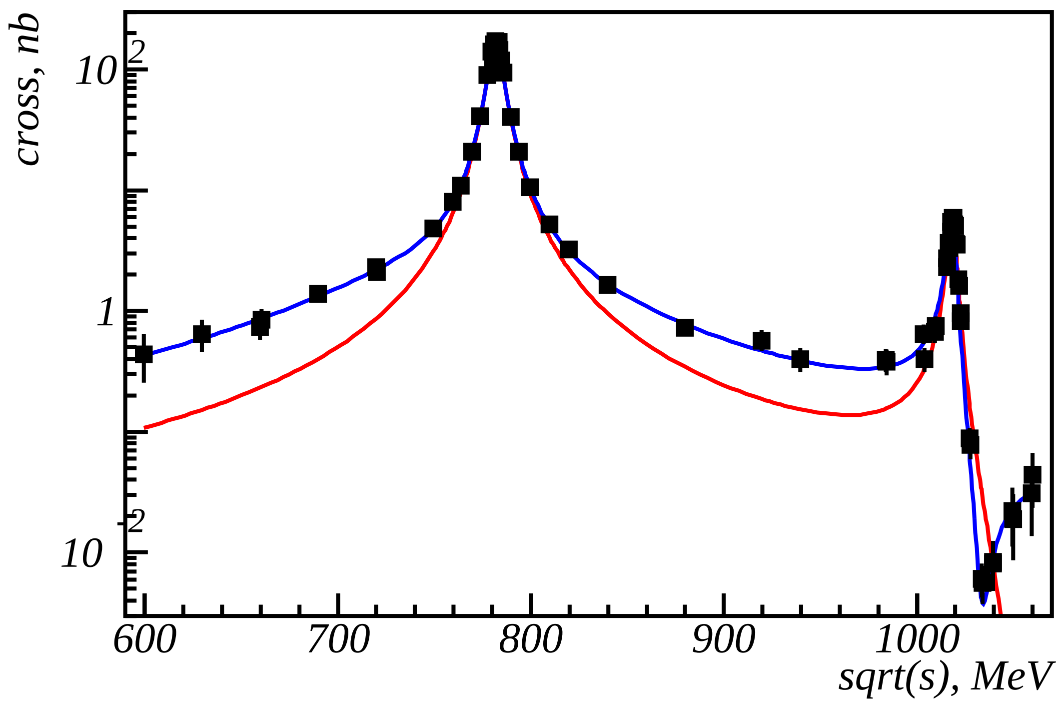
Study of Neutral Final States at VEPP-2M

- $e^+e^- \to \pi^0\gamma$ (SND, CMD-2)
- $e^+e^- \to \eta\gamma$ (SND, CMD-2)
- $e^+e^- \to \pi^0\pi^0\gamma$ (SND, CMD-2)
- $e^+e^- \to \eta\pi^0\gamma$ (CMD-2)



$\rho-, \omega-, \phi-$mesons dominate the cross sections.
From upper limits on nonresonant cross sections $a_\mu^{\rm rad,LO} < 0.7 \cdot 10^{-10}$.

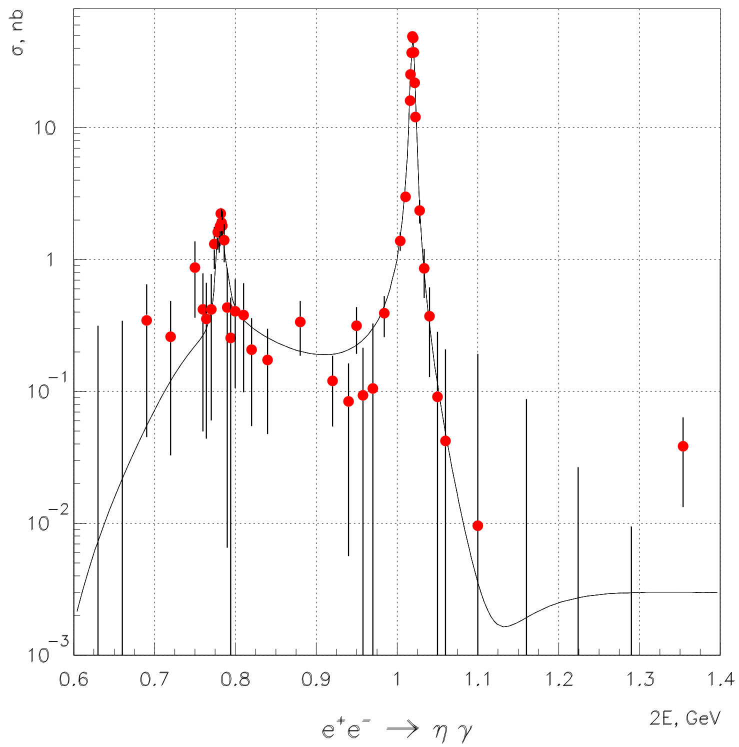
Neutral Final States. $e^+e^- \to \pi^0\gamma \to 3\gamma$ at SND



About $94 \cdot 10^3$ detected events.

The systematic error is 2.0–2.5% at the ω and 5% at the ϕ.

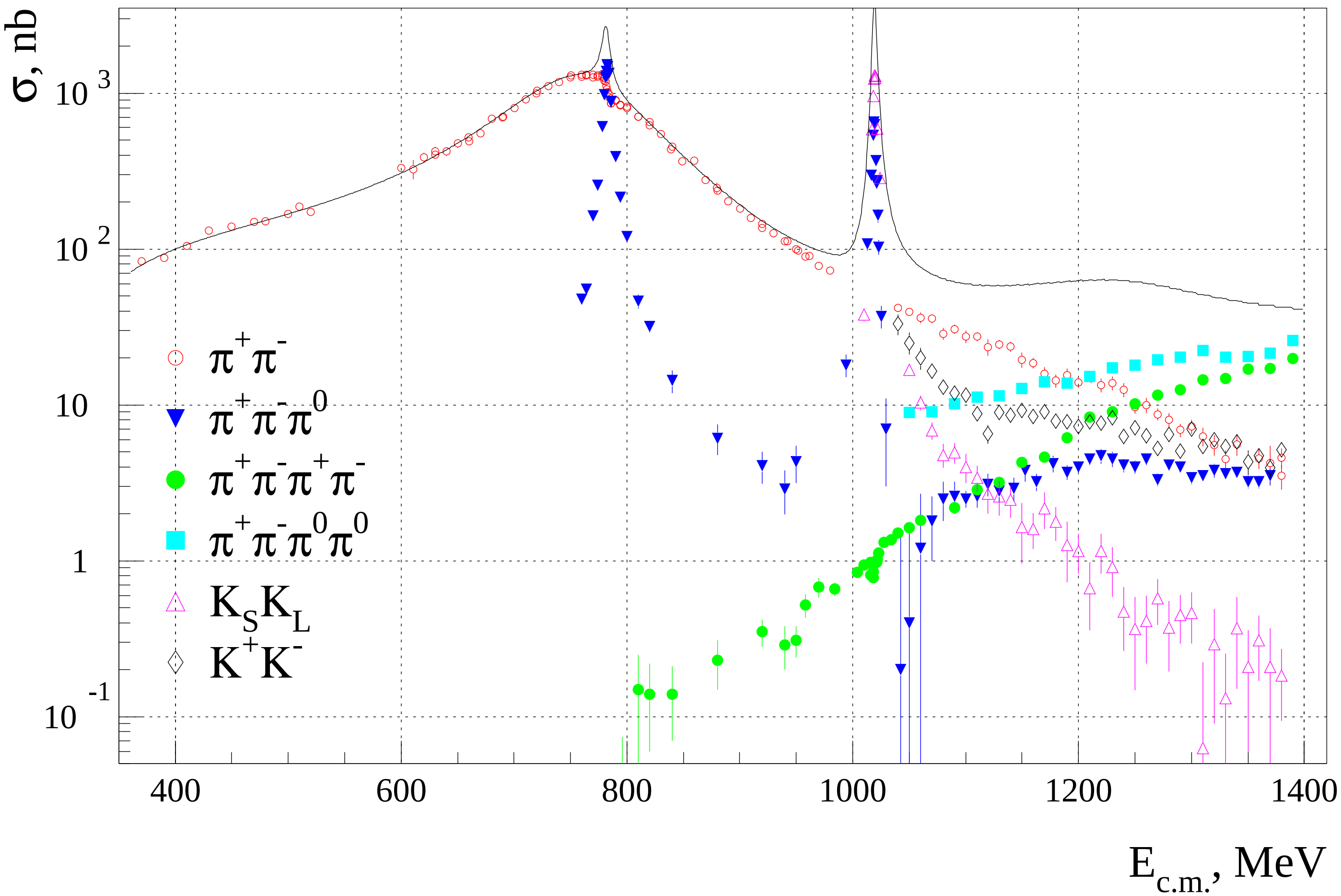
Neutral Final States. $e^+e^- \to \eta\gamma \to 3\pi^0\gamma$ at CMD-2



About $25 \cdot 10^3$ detected events.

The systematic error is 6% at the ω and 4% at the ϕ.

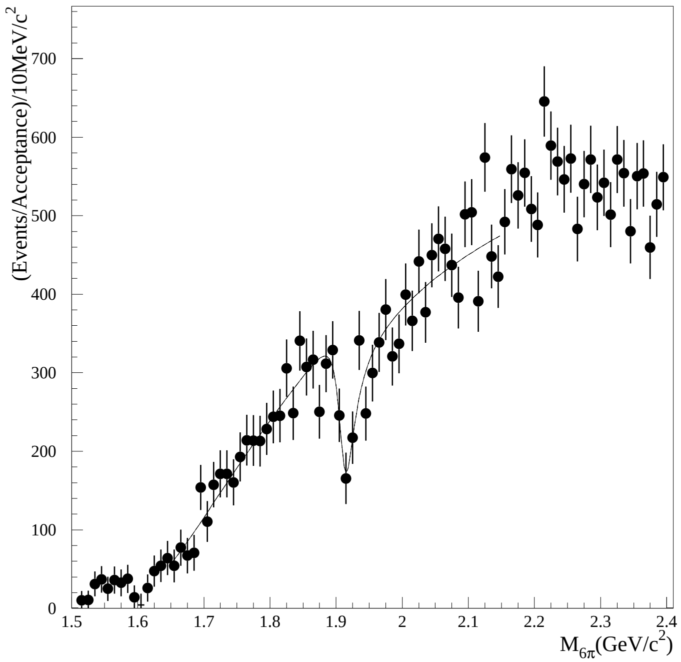
Hadronic Cross Sections at CMD-2



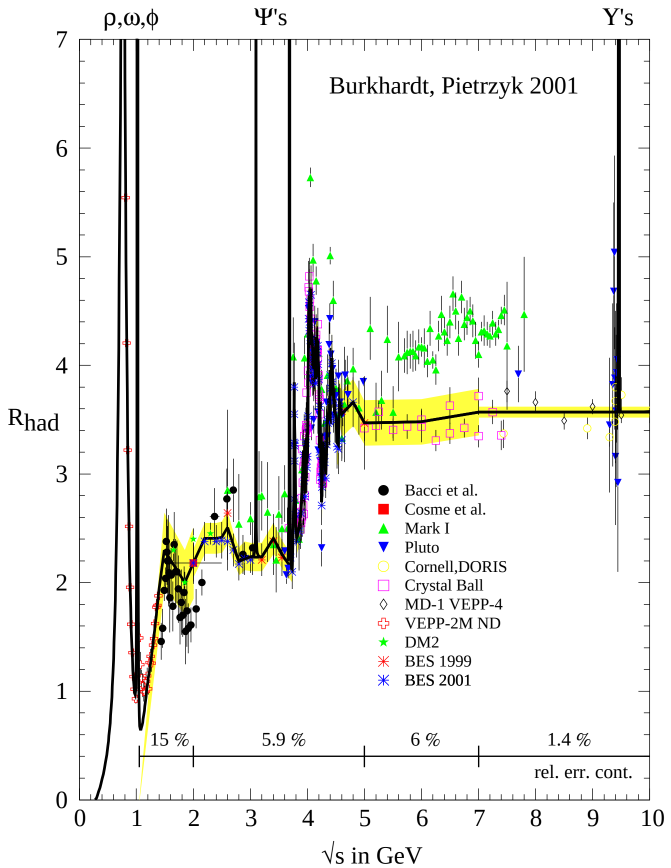
Measurements at 1.4 GeV $< \sqrt{s} <$ 2 GeV

- 5 resonances ($2\rho'$,$2\omega'$,ϕ') with badly known properties
- Mixing of $q\bar{q}$ with hybrids?
- In 2001 E687 (FNAL) observed a narrow dip in $\gamma p \to 3\pi^+3\pi^- p$, M$= 1911 \pm 4 \pm 1$ MeV, $\Gamma = 29 \pm 11 \pm 4$ MeV
- Earlier observed in e^+e^-: DM2 (1988) - $e^+e^- \to 6\pi$, FENICE (1996) - $e^+e^- \to$ hadrons
- A hybrid or $N\bar{N}$ state?

$\gamma p \to 3\pi^+3\pi^- p$



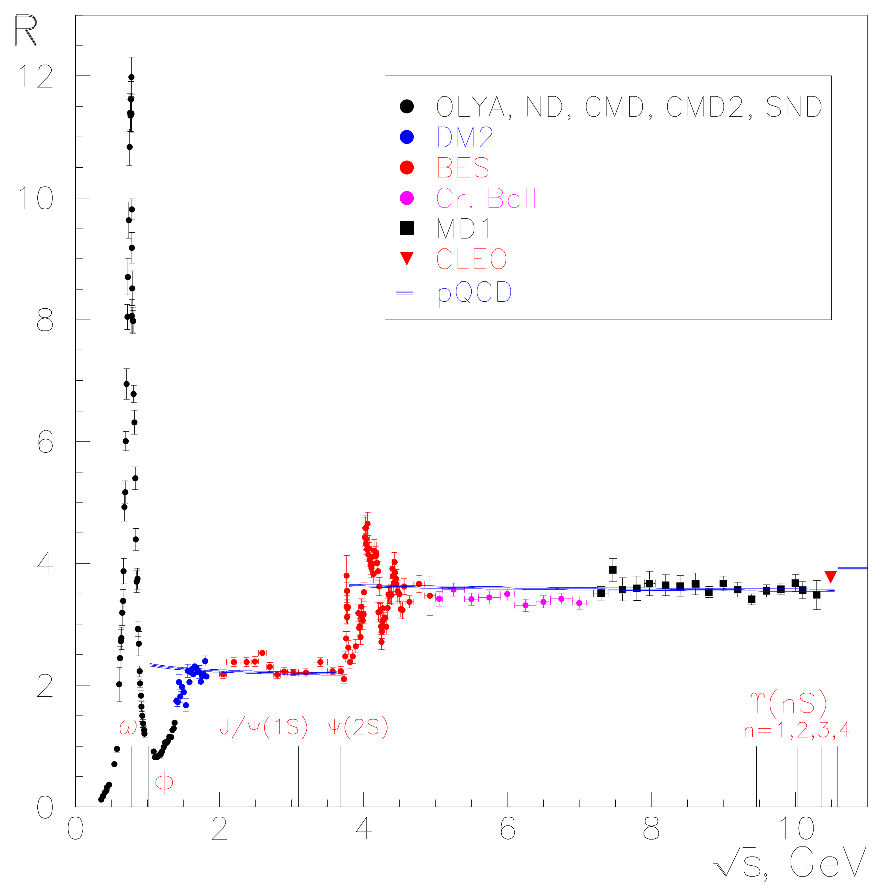
R Measurements at $\sqrt{s} < 10$ GeV



$\gamma\gamma2$ vs. BES

Detector	$\gamma\gamma2$	BES
$\sqrt{s}$, GeV	2.0-3.1	2.0-3.0
Acceptance, %	19-23	50-68
Syst. error, %	21	5.9-8.4
$\int Ldt$, nb^{-1}	130	990
Data sample	920	18500

R Measurements at $\sqrt{s} < 10$ GeV

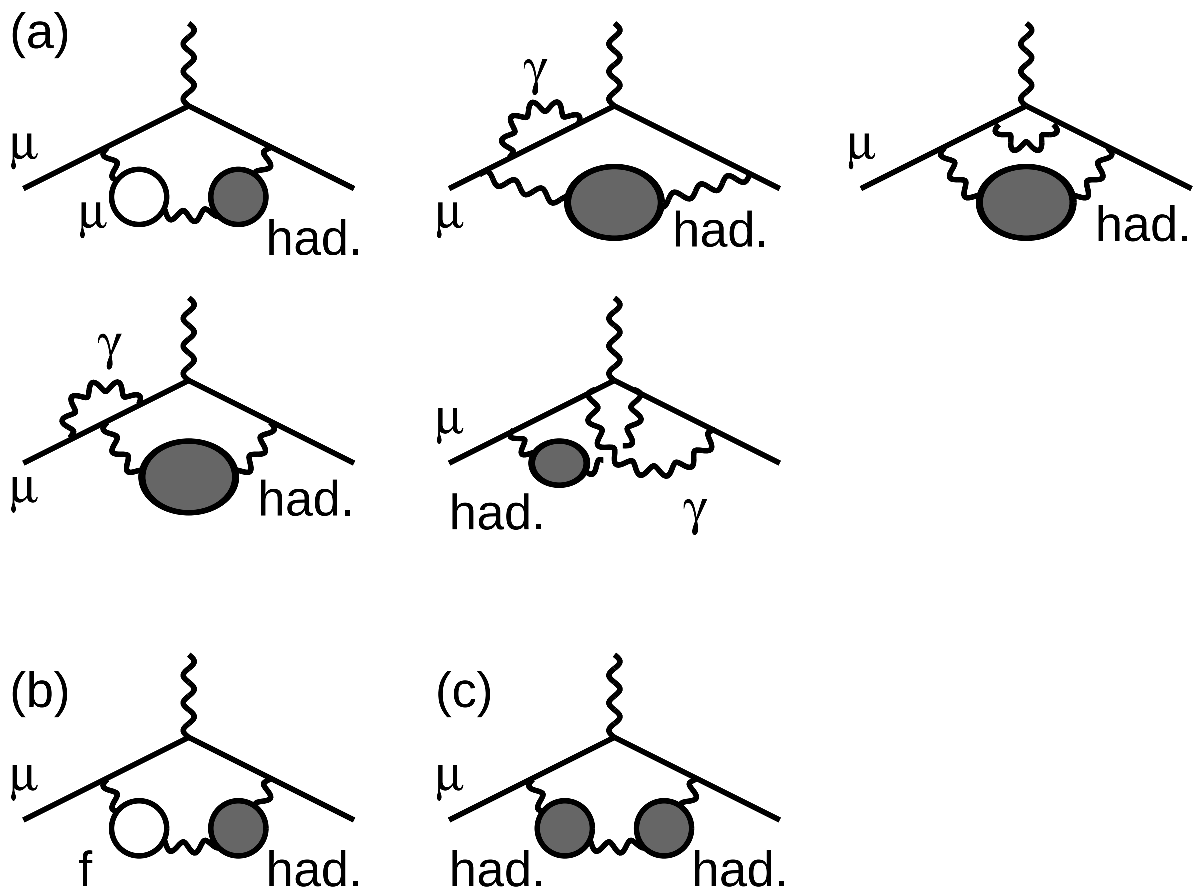


New e^+e^- Data Based Calculation of $a_\mu^{\text{had,LO}}$

$\sqrt{s}$, GeV	$a_\mu^{\text{had,LO}}$, 10^{-10}	$a_\mu^{\text{had,LO}}$, %
2π	$508.20 \pm 5.18 \pm 2.74$	72.99
ω	$37.96 \pm 1.02 \pm 0.31$	5.45
ϕ	$35.71 \pm 0.84 \pm 0.20$	5.13
$0.6 - 2.0$	$63.18 \pm 2.19 \pm 0.86$	9.07
$2.0 - 5.0$	$33.92 \pm 1.72 \pm 0.03$	4.87
$J/\psi, \psi'$	$7.44 \pm 0.38 \pm 0.00$	1.07
> 5.0	$9.88 \pm 0.11 \pm 0.00$	1.42
Total	$696.3 \pm 6.2 \pm 3.6$	100.0

Higher accuracy of e^+e^- data makes the $a_\mu^{\text{had,LO}}$ error 2 times smaller!

Higher-Order Hadronic Contributions $a_\mu^{\rm had,HO}$ – I



There are 14 graphs of the type a (muon loops only involved), 2 – of the type b (e and τ loops) and 1 – of the type c.

Higher-Order Hadronic Contributions $a_\mu^{\mathrm{had,HO}}$ – II

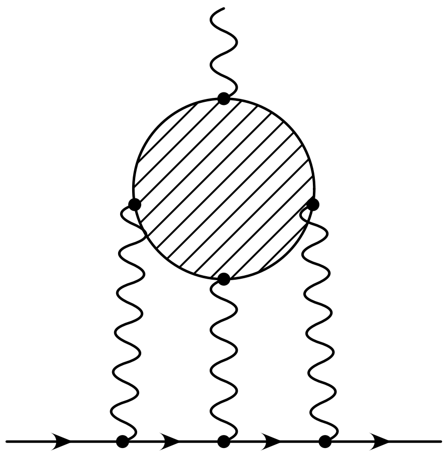
Authors	Year	a	b	c	Total, 10^{-10}
J.Calmet et al.	1976	-20.7 $\pm$ 3.0	11.0 $\pm$ 1.4	0.2 $\pm$ 0.1	-9.5 $\pm$ 3.2
T.Kinoshita et al.	1985	-19.9 $\pm$ 0.4	10.7 $\pm$ 0.3	0.2 $\pm$ 0.1	-9.0 $\pm$ 0.5
B.Krause	1997	-21.1 $\pm$ 0.5	10.7 $\pm$ 0.2	0.3 $\pm$ 0.1	-10.1 $\pm$ 0.6
R.Alemany et al.	1998	-20.9 $\pm$ 0.4	10.6 $\pm$ 0.2	0.3 $\pm$ 0.1	-10.0 $\pm$ 0.6
K.Hagiwara et al.,	2003	-20.7 $\pm$ 0.2	10.6 $\pm$ 0.1	0.3 $\pm$ 0.1	-9.8 $\pm$ 0.1

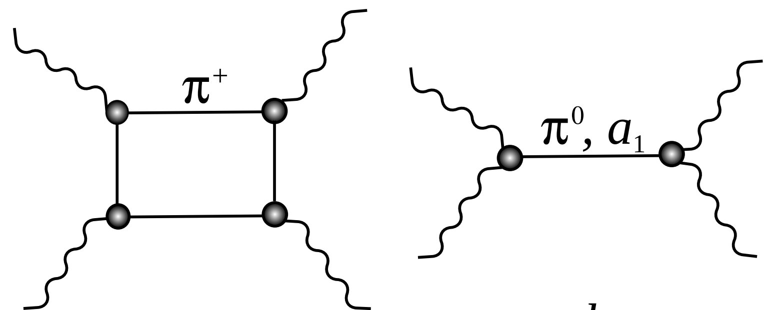
The contributions of all 3 graphs can be calculated in terms of the $\int R(s)G(s)ds/s^{2(3)}$, where $G(s)$ is a smooth function of s, so that the low energy range again dominates the integral. Several calculations agree. The uncertainty of this term is negligible.

Light-by-light Hadronic Contributions – I

This term is very unstable:

large variations of the magnitude; the sign changed 3 times!





a *b*

M. Knecht and A. Nyffeler, 2001:

the correct sign of the pseudoscalar and axial contributions.

Light-by-light Hadronic Contributions - II

Authors	Date	$a_\mu^{\text{lbl}}, 10^{-10}$	Approach
J. Calmet et al.	1976	-26 ± 10	Quark loops
T. Kinoshita et al.	1984	$+6.0 \pm 0.4$	Quark loops
T. Kinoshita et al.	1984	$+4.9 \pm 0.5$	VDM, hadrons
M. Hayakawa et al.	1995	-3.6 ± 1.6	HLS
J. Bijnens et al.	1995	-9.2 ± 3.2	NJL Model
M. Hayakawa et al.	1996	-5.2 ± 1.8	π^0
M. Hayakawa and T. Kinoshita	1998	-7.9 ± 1.5	$\gamma\gamma \to \pi^0, \eta, \eta'$
M. Hayakawa and T. Kinoshita	2001	$+9.0 \pm 1.5$	π^0 and a_1 sign
J. Bijnens et al.	2001	$+8.3 \pm 3.2$	π^0 and a_1 sign
K. Melnikov and A. Vainshtein	2003	$+13.6 \pm 2.5$	QCD

Light-by-light Hadronic Contributions - III

Separate contributions to $a_\mu^{\rm lbl}, 10^{-10}$

Source	T.Kinoshita	J.Bijnens	A.Vainshtein
Pseudoscalar	8.3 ± 0.6	8.5 ± 1.3	11.4 ± 1.0
Scalar	0	-0.7 ± 0.2	0
Axial	0.2 ± 0.2	0.2 ± 0.1	2.2 ± 0.5
$\pi^\pm, \mathrm{K}^\pm$	-0.4 ± 0.8	-1.9 ± 0.3	0 ± 1
Quarks	1.0 ± 1.1	2.1 ± 1.3	0
Total	9.0 ± 1.5	8.3 ± 3.2	13.6 ± 2.5

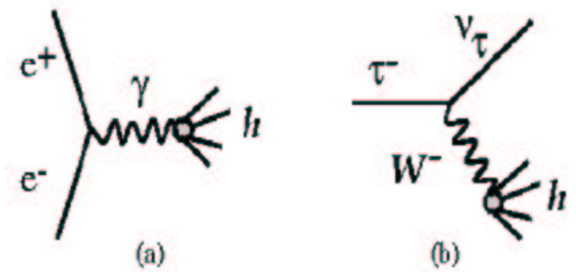
Theory vs Experiment (January 2004)

Contribution	$a_\mu, 10^{-10}$
Experiment	11659208 ± 6
QED	11658470.6 ± 0.3
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	694.9 ± 7.9
Theory	11659180.9 ± 8.0
Exp.–Theory	$27.1 \pm 10.0\ (2.7\sigma)$

Recent theory progress: $a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (20.8 \pm 9.7) \cdot 10^{-10}\ (2.1\sigma)$

How can the theoretical error be improved?

CVC. $e^+e^- \to X^0$ and $\tau^- \to \nu_\tau X^-$



Allowed $I^G J^P = 1^+1^-$:
$X^- = \pi^-\pi^0, (4\pi)^-, \omega\pi^-,$
$\eta\pi^-\pi^0, K^-K^0, (6\pi)^-, \ldots$

$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ud}|^2 S_{EW}}{32\pi^2 m_\tau^3} f_{kin} v_1(q^2)$ with
$v_1(q^2) = \frac{q^2 \sigma^{I=1}_{e^+e^-}(q^2)}{4\pi\alpha^2}$.

CVC tests showed good agreement of the τ branchings predicted from e^+e^- with τ data (N. Kawamoto and A. Sanda, 1978, F. Gilman and D. Miller, 1978, S. Eidelman and V. Ivanchenko, 1991, 1997).

The very first application of τ data to $a_\mu^{\rm had,LO}$ improved the accuracy by a factor of 1.5 (R. Alemany, M. Davier, A. Höcker, 1998)!

Branchings of $\tau^- \to X^- \nu_\tau$ Decay,%

Hadronic State X	Experiment, 2002	CVC Prediction	$\mathcal{B}_{\rm exp} - \mathcal{B}_{\rm CVC}$
$\pi^-\pi^0$	25.31 ± 0.18	24.76 ± 0.25	0.55 ± 0.31
$\pi^-3\pi^0$	1.08 ± 0.10	1.07 ± 0.05	0.01 ± 0.11
$2\pi^-\pi^+\pi^0$	4.19 ± 0.23	3.84 ± 0.17	0.35 ± 0.29
$\omega\pi^-$	1.94 ± 0.07	1.82 ± 0.07	0.12 ± 0.10
Total	31.59 ± 0.31	30.28 ± 0.34	1.31 ± 0.46

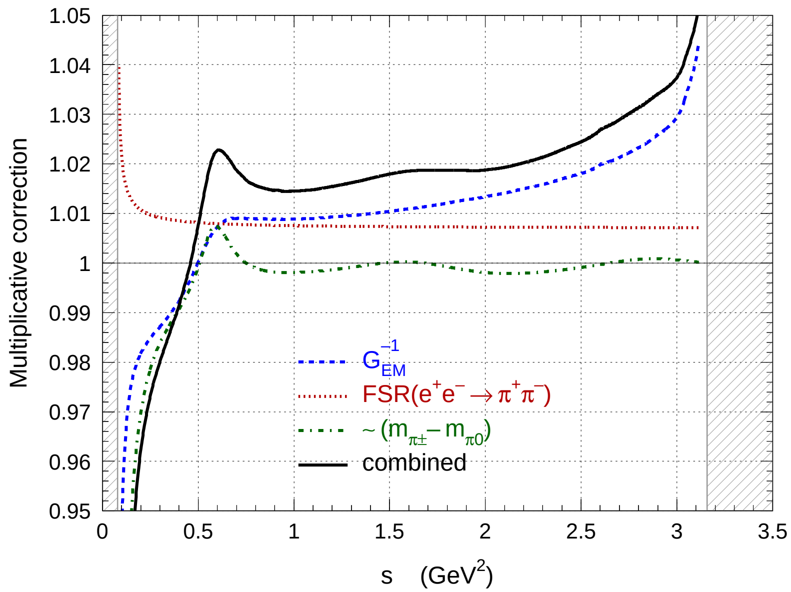
With more accurate data some deviations have been observed.

Corrections to the τ Spectral Functions

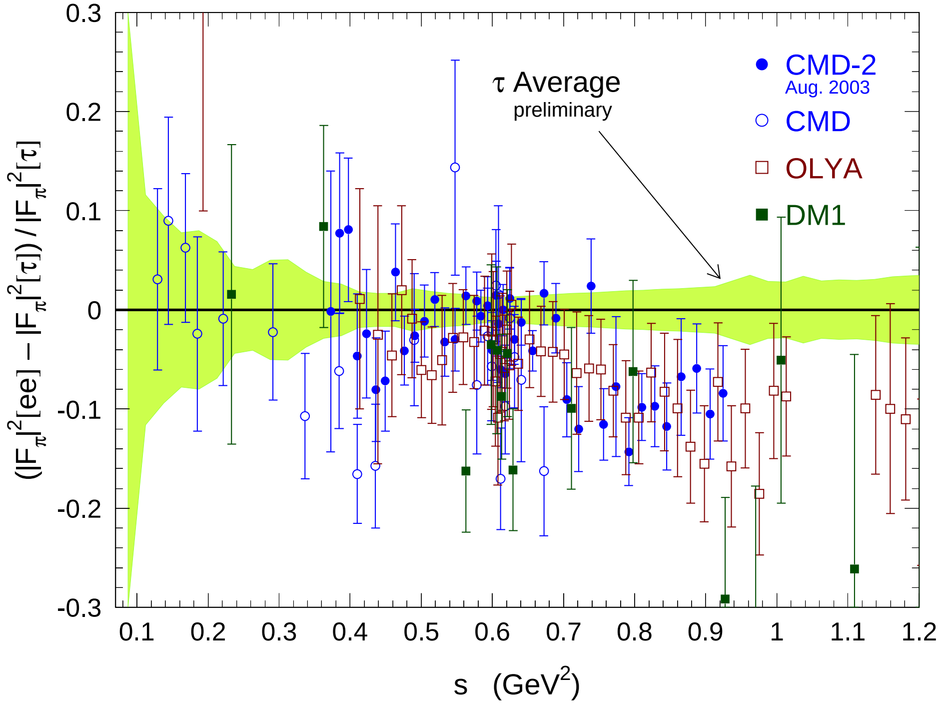
- $S_{EW} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $m_{\pi^\pm} \neq m_{\pi^0}$ (phase space, Γ_ρ)
- $m_{\rho^\pm} \neq m_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays ($\pi\pi\gamma, \pi(\eta)\gamma, l^+l^-$)
- $m_u \neq m_d$ and 2 class currents

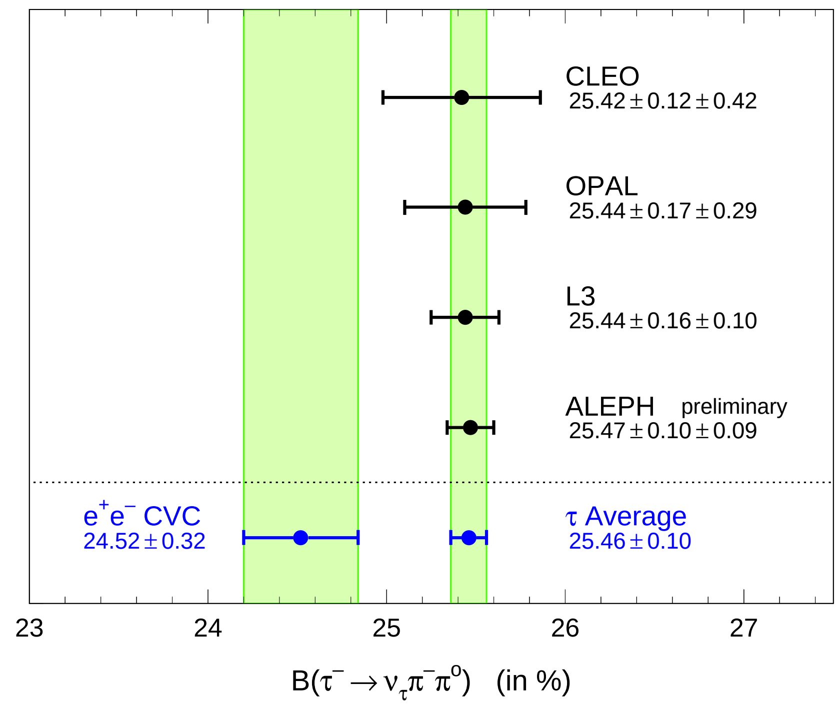
V. Cirigliano, G. Ecker, H. Neufeld, 2002

M. Davier, S. Eidelman, A. Höcker, Z. Zhang, 2002



CVC in the 2π Channel. e^+e^- vs. τ

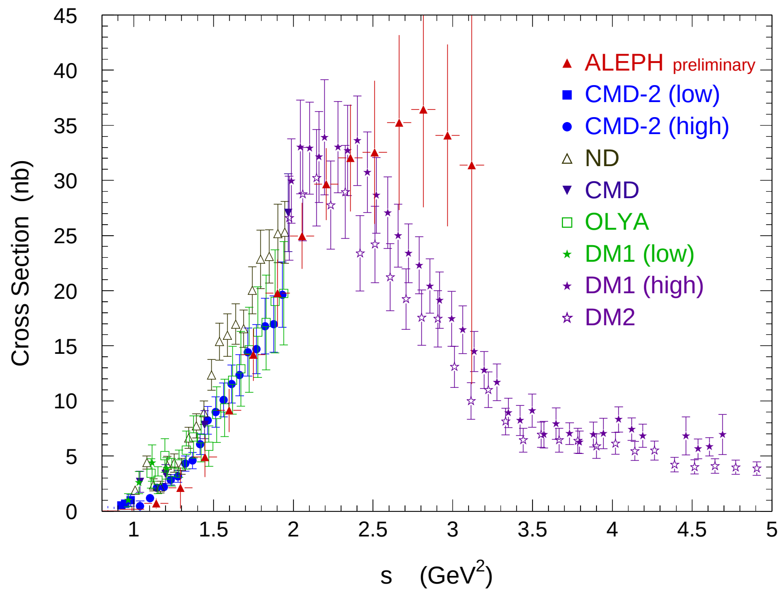




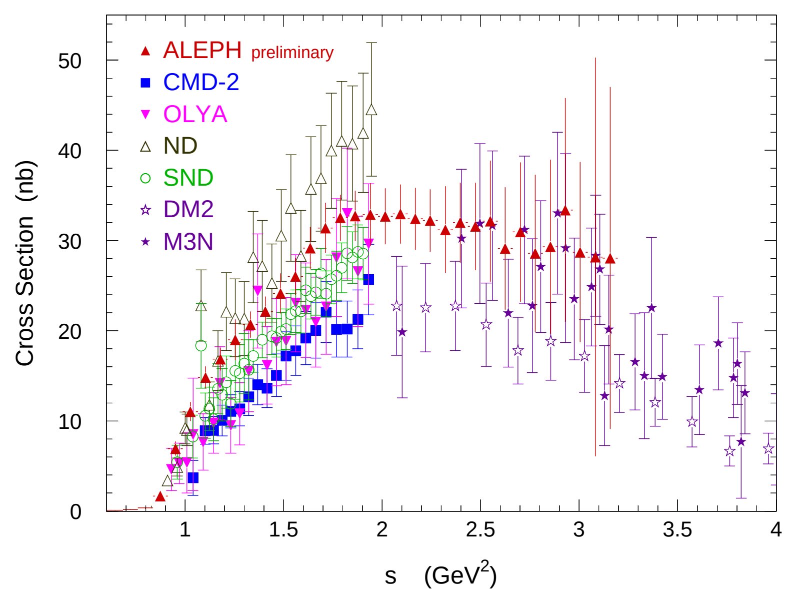
The branching from all groups is systematically higher than the CVC prediction: $\mathcal{B}_\tau - \mathcal{B}_{ee} = (0.94 \pm 0.32)\%$!

CVC in the 4π Channel. e^+e^- vs. τ

$2\pi^+2\pi^-$



$\pi^+\pi^-2\pi^0$



	$2\pi^+2\pi^-$	$\pi^+\pi^-2\pi^0$
$\mathcal{B}(\tau), \%$	1.01 ± 0.08	4.54 ± 0.13
$\mathcal{B}(\text{CVC}), \%$	1.09 ± 0.08	3.63 ± 0.21
$\Delta\mathcal{B}, \%$	-0.08 ± 0.11	$+0.91 \pm 0.25$

Contributions to $a_\mu^{\text{had,LO}}$ from e^+e^- and τ, 10^{-10}

Mode	e^+e^-	τ	$\Delta(e^+e^- - \tau)$
$\pi^+\pi^-$	$508.20 \pm 5.18 \pm 2.74$	$520.06 \pm 3.36 \pm 2.62$	-11.9 ± 6.9
$\pi^+\pi^-2\pi^0$	$16.76 \pm 1.31 \pm 0.20$	$21.45 \pm 1.33 \pm 0.60$	-4.7 ± 1.8
$2\pi^+2\pi^-$	$14.21 \pm 0.87 \pm 0.23$	$12.35 \pm 0.96 \pm 0.40$	$+1.9 \pm 2.0$
Total	$539.17 \pm 5.41 \pm 3.17$	$553.86 \pm 3.74 \pm 3.02$	-14.7 $\pm$ 7.9

The difference of 1.86σ makes averaging meaningless: a scale factor of 1.82 makes a final error equal to $6.94 \cdot 10^{-10}$ ($7.20 \cdot 10^{-10}$ from e^+e^- only), i.e. no gain at all!

Why are e^+e^- and τ Spectral Functions Different?

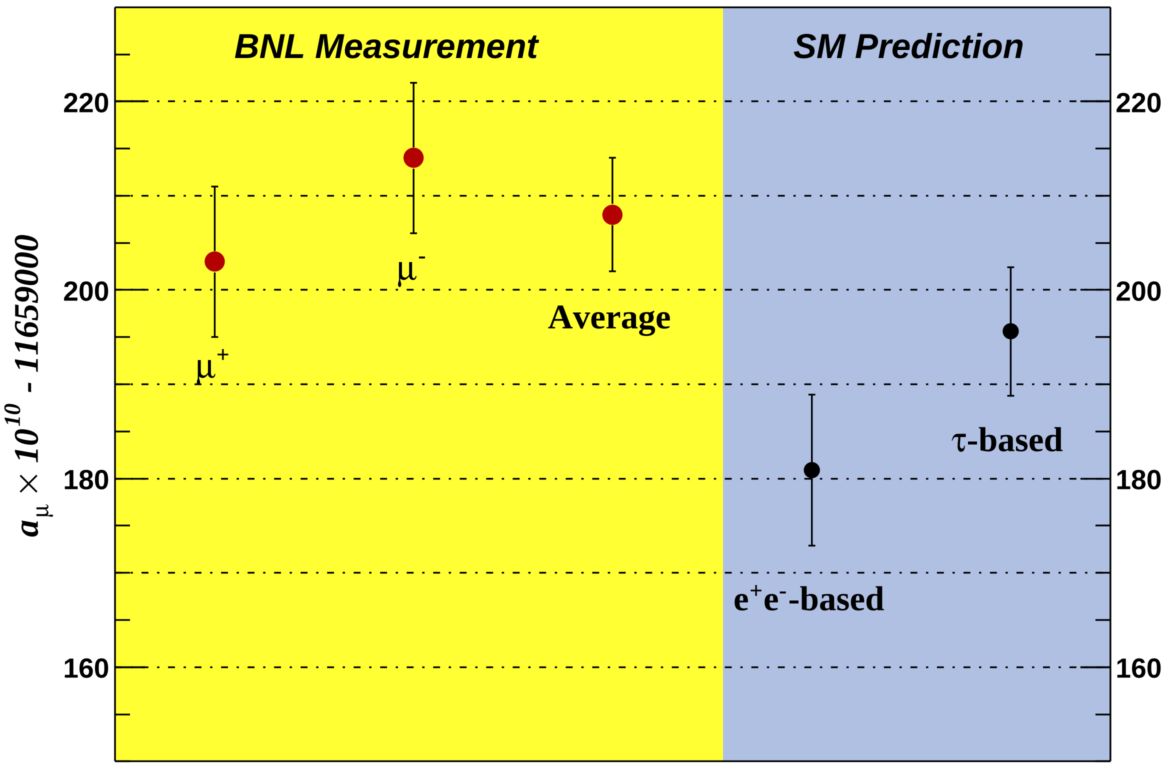
- Problems with data: underestimated systematics, normalization, rad. corr.
- Problems with SU(2) breaking corrections; Is ChPT reliable?
- Non (V-A) contribution to e/w interactions (M.Chizhov, 2003) inspired by problems in $\pi^+ \to e^+\nu_{\rm e}\gamma$ (E.Frlez et al., 2003)
- Effect of charged Higgs propagator in τ decay
- $m_{\rho^\pm} > m_{\rho^0}$ by a few MeV (S.Ghozzi and F.Jegerlehner, 2003, M.Davier, 2003). Current experiments indicate equality within a few MeV.

Recent Calculations of $a_\mu^{\rm had,LO}$

Authors	Data	$a_\mu^{\rm had,LO}$, 10^{-10}
M. Davier et al.	e^+e^-	$696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad}$
K. Hagiwara et al.	e^+e^-	$692.4 \pm 5.9_{\rm exp} \pm 2.4_{\rm rad}$
S. Ghozzi and F. Jegerlehner	e^+e^-	694.8 ± 8.6
V. Ezhela et al.	e^+e^-	$699.6 \pm 8.5_{\rm exp} \pm 1.9_{\rm rad} \pm 2.0_{\rm proc}$
M. Davier et al.	τ	$711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{\rm SU(2)}$

All e^+e^- based calculations agree!

Theory vs Experiment



The e^+e^- and τ based predictions are below the experimental value by 2.7σ (2.1σ) and 0.7σ, respectively!

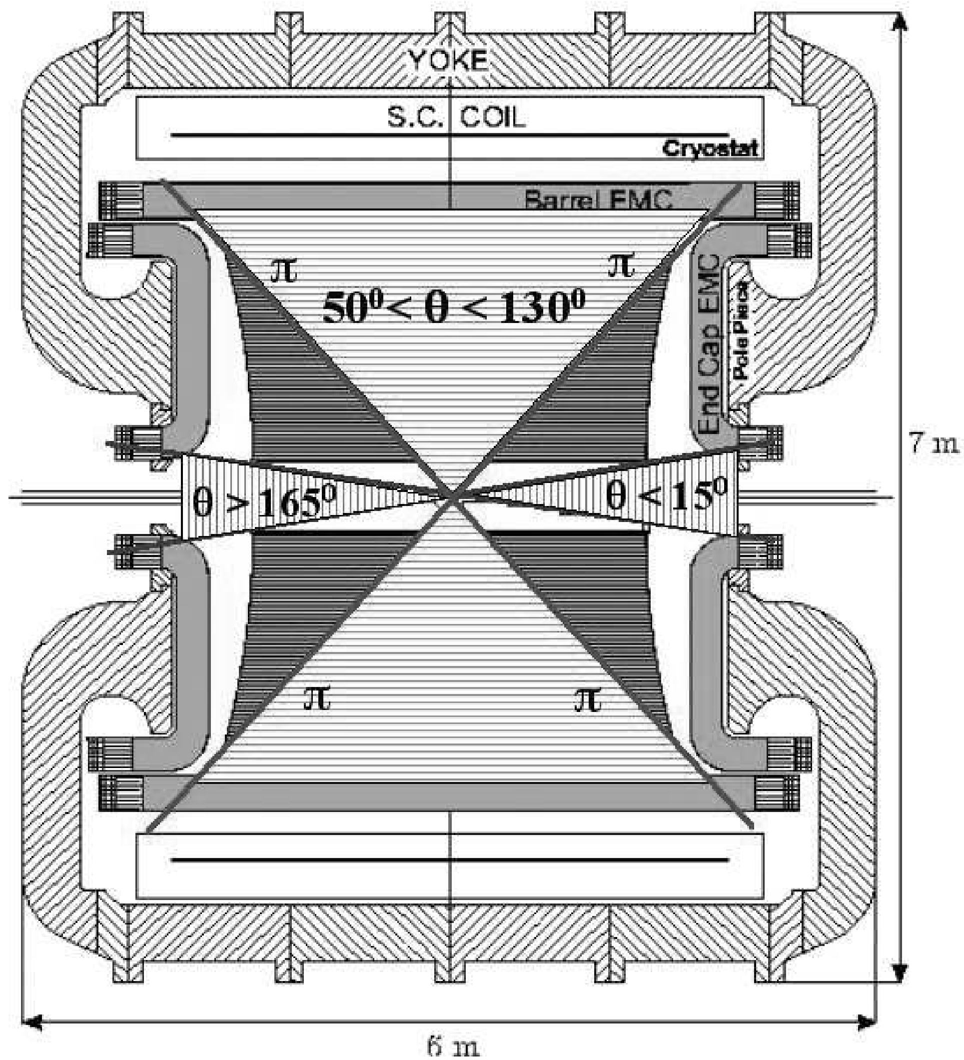
Radiative Return (ISR)

The idea: a photon with E_γ energy emitted by initial $e^\pm$ allows a study of hadron production at smaller energy: $2E' = 2E\sqrt{1 - E_\gamma/E}$. A smaller cross section is compensated by a much higher luminosity.

Already today there is a large data sample with small systematic errors:
KLOE – 11k/pb^{-1} vs. 360k/pb^{-1} with CMD-2, but 1.5M $\pi^+\pi^-$ events in total!
BaBar – $150 \cdot 10^3$ exclusive events between 1 and 3 GeV per 100 fb^{-1}.
Belle – starting.

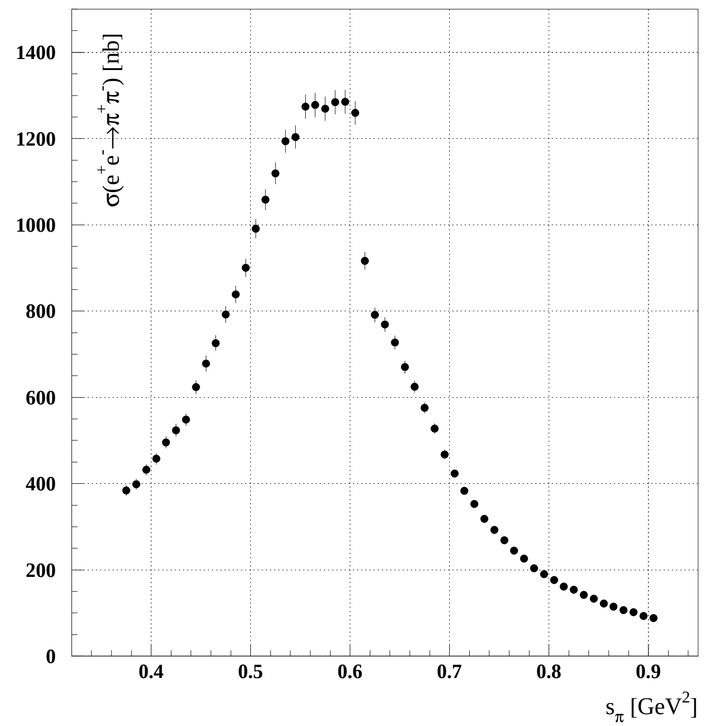
ISR is an independent source of high precision R measurements in the energy range $2m_\pi < \sqrt{s} < 3$ GeV, important for $a_\mu^{\rm had}$ and $\alpha(M_Z^2)$;
it also provides an invaluable input for hadronic spectroscopy and QCD.

KLOE Detector in Frascati



ISR at KLOE

KLOE studied the ρ meson by detecting the process $e^+e^- \to \pi^+\pi^-\gamma$ at ϕ (1.02 GeV). Results for $0.35 < s < 0.95$ GeV^2 are compatible with CMD-2. Already 1.555M events (141.4 pb^{-1}) analyzed and much more on tape! The systematic error is 1.3%

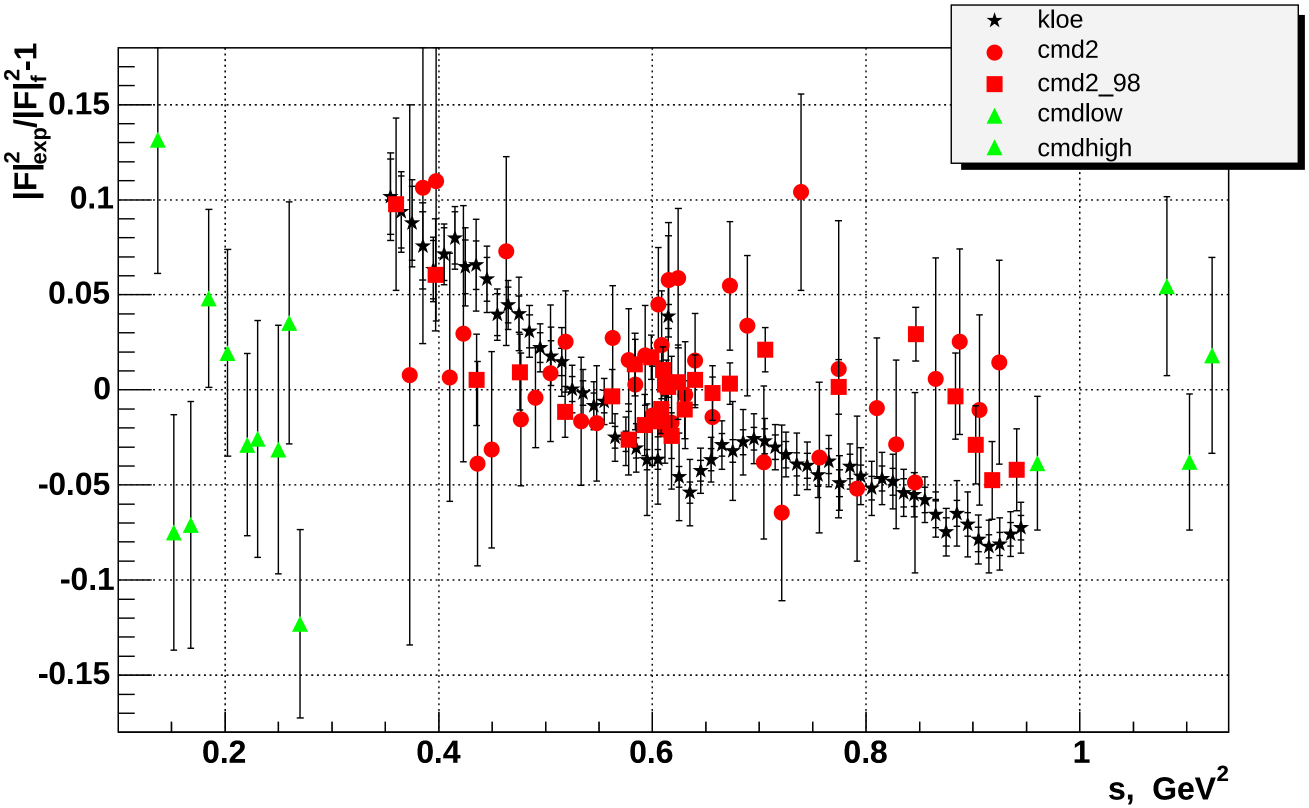


Cross section of $e^+e^- \to \pi^+\pi^-$

KLOE: $a_\mu^{\rm had,LO} = (375.6 \pm 0.8 \pm 4.9) \times 10^{-10}$

VEPP-2M: $a_\mu^{\rm had,LO} = (378.6 \pm 2.7 \pm 2.3) \times 10^{-10}$

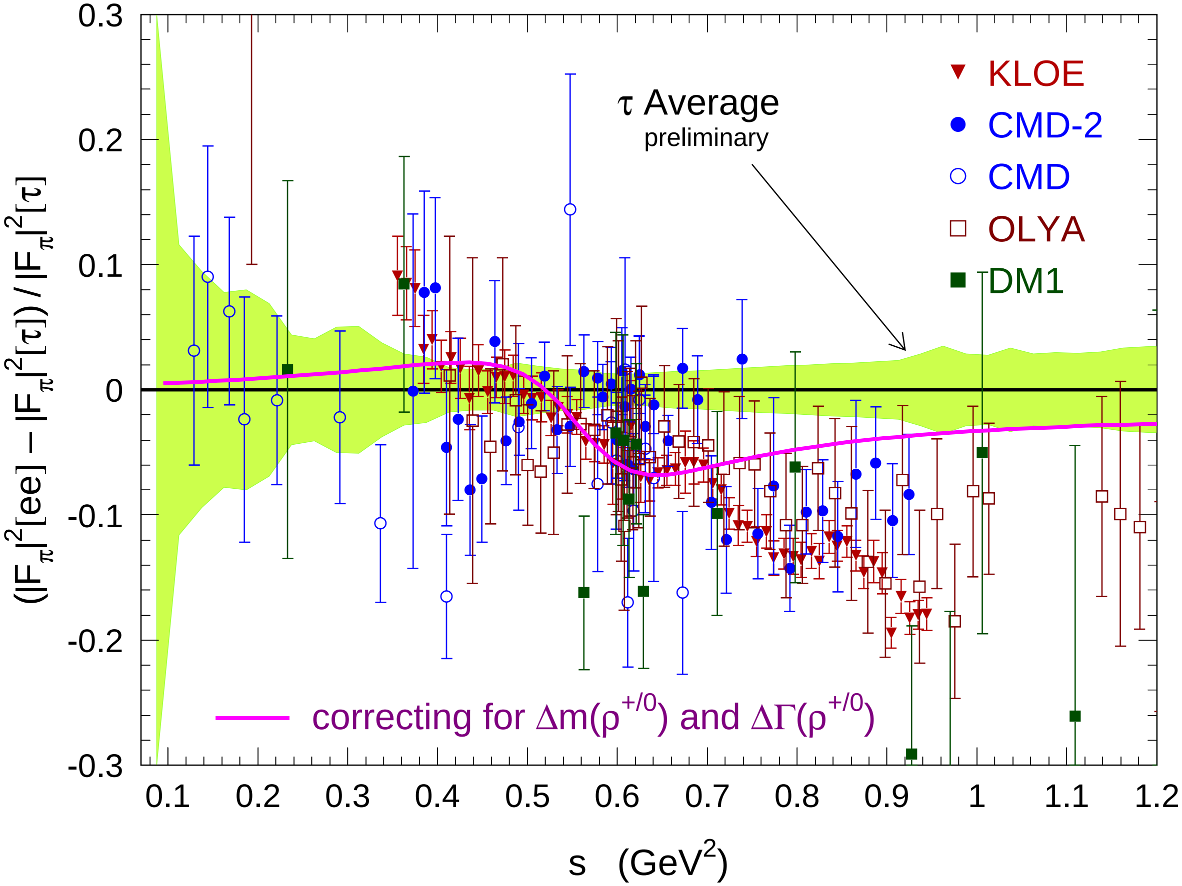
Comparison of KLOE and CMD-2



Combining KLOE and CMD-2 in the LO hadronic part and using recent QED and EW update:

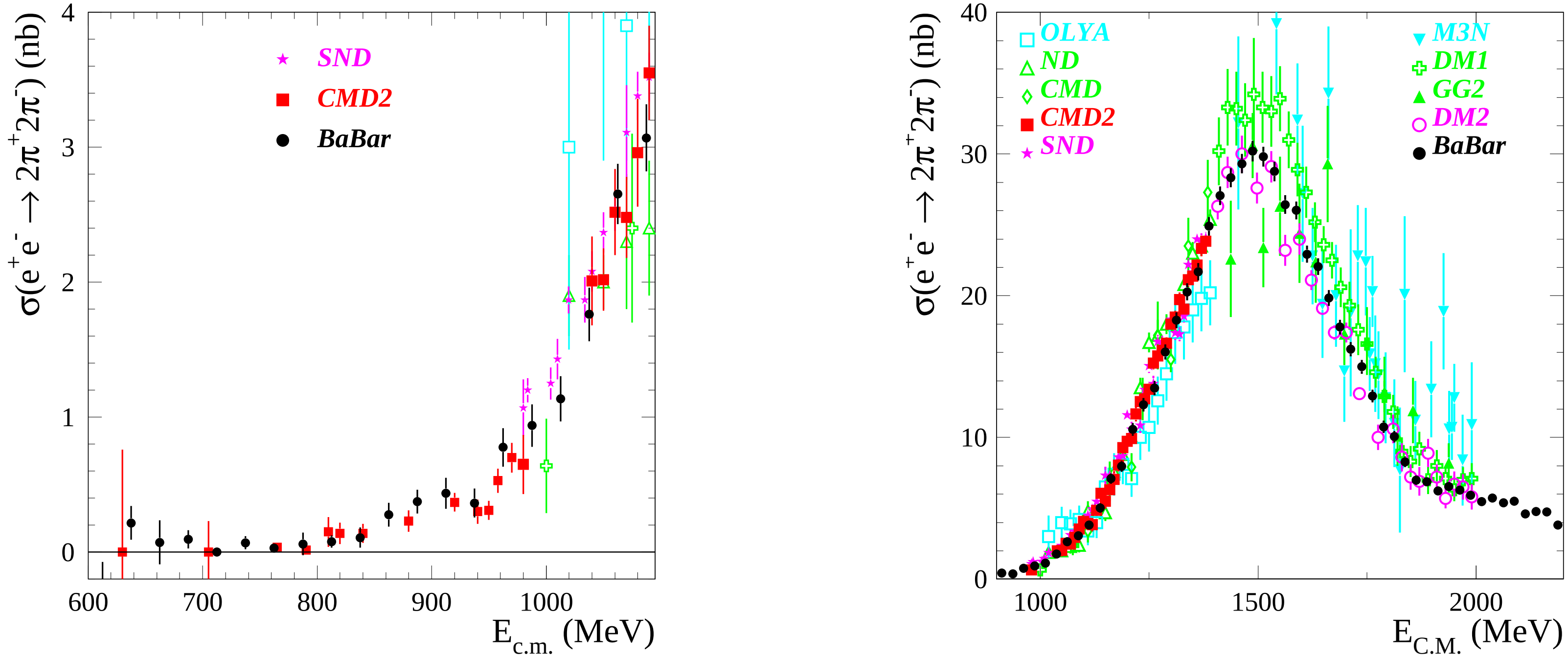
$$a_\mu^{\rm exp} - a_\mu^{\rm th} = (25.3 \pm 9.4) \cdot 10^{-10} \quad (2.7\sigma).$$

τ vs. e^+e^- after KLOE



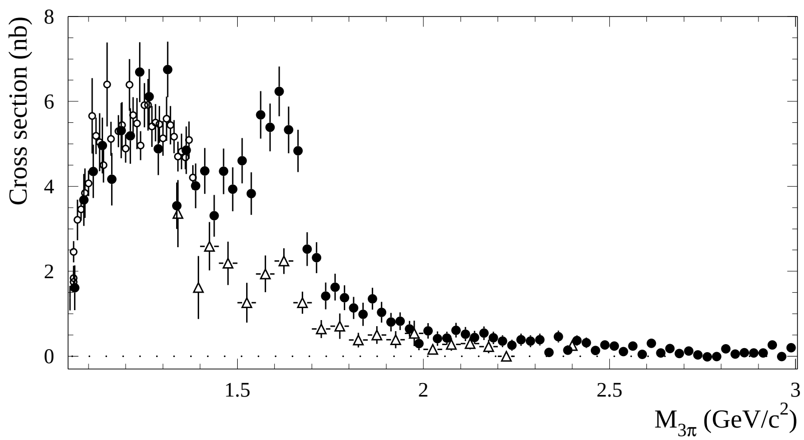
ISR at BaBar – I

BaBar studies exclusive processes $e^+e^- \to n\pi, K\bar{K} + n\pi$ at $\sqrt{s} < 4$ GeV running at $\Upsilon(4S)$ (10.58 GeV) at PEPII

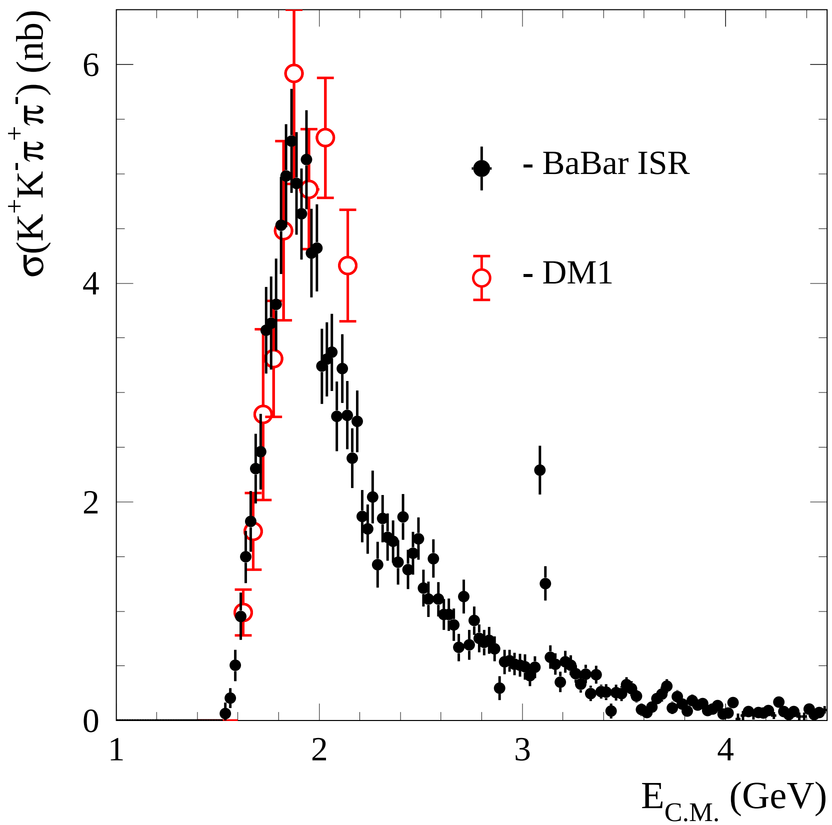


Cross section of $e^+e^- \to 2\pi^+2\pi^-$

ISR at BaBar – II



$$e^+e^- \to \pi^+\pi^-\pi^0$$



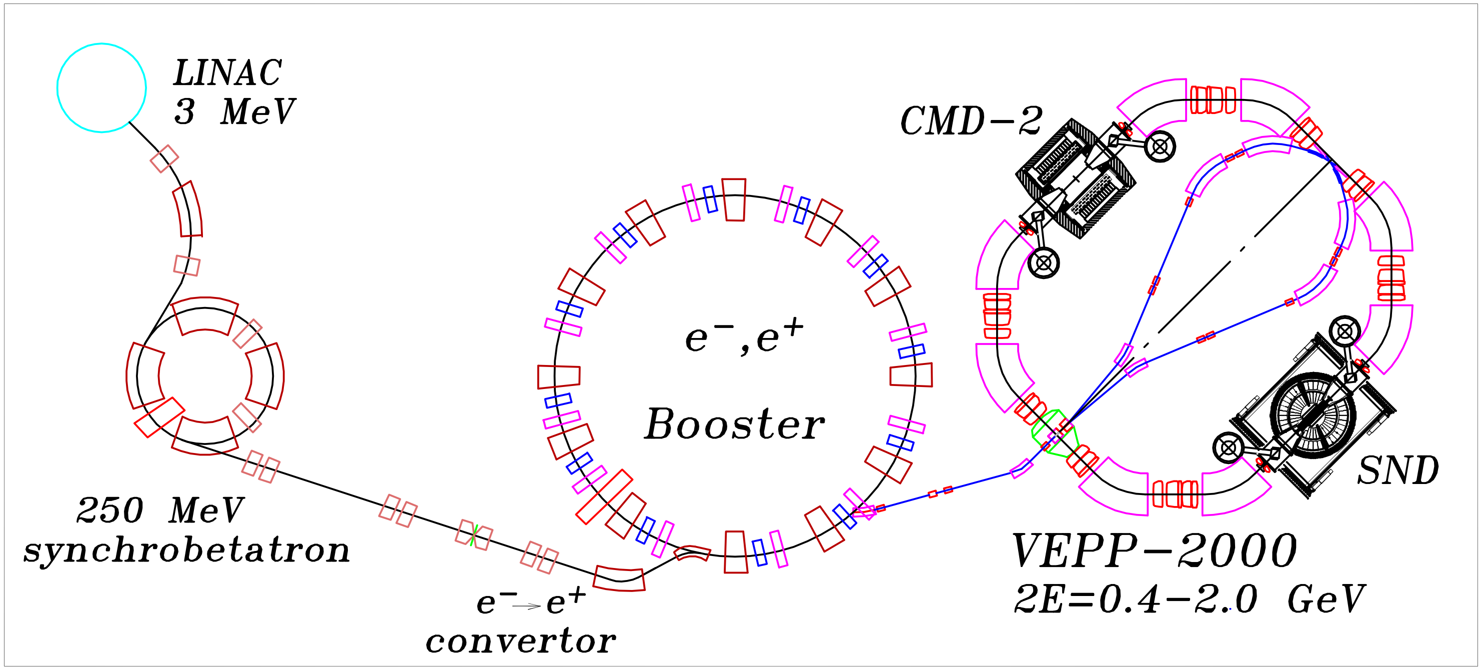
$$e^+e^- \to \pi^+\pi^- K^+K^-$$

Improvement of $a_\mu^{\mathrm{had,LO}}$ in Close Future

1. KLOE in Frascati
   - ISR: $e^+e^- \to \pi^+\pi^-$ at $590 < \sqrt{s} < 975$ MeV
   - $\Gamma(\phi \to l^+l^-) = 1.320 \pm 0.023$ keV vs. 1.27 ± 0.04 keV before
2. CMD-2 in Novosibirsk
   - $e^+e^- \to \pi^+\pi^-$ at $340 < \sqrt{s} < 1380$ MeV
   - $e^+e^- \to 2\pi^+2\pi^-, \pi^+\pi^-2\pi^0$ at $980 < \sqrt{s} < 1380$ MeV
3. BaBar in Stanford
   - ISR: $\Gamma(J/\psi \to l^+l^-) = 5.61 \pm 0.20$ keV vs. 5.26 ± 0.37 keV before
   - ISR: $e^+e^- \to \pi^+\pi^-\pi^0$ at $700 < \sqrt{s} < 2000$ MeV (deviation from DM2)
   - ISR: $e^+e^- \to 2\pi^+2\pi^-$ at $600 < \sqrt{s} < 2000$ MeV
     ($a_\mu^{\mathrm{had,LO}}(4\pi^\pm) \cdot 10^{10} = 12.95 \pm 0.64 \pm 0.13$ vs. $14.21 \pm 0.87 \pm 0.23$)

VEPP-2000

Layout of the VEPP−2000 complex



With $\mathcal{L} = 10^{32}$ cm^{-2}s^{-1} and $\int Ldt \approx 1-2$ fb^{-1} (3–5 years)

$\Delta a_\mu^{\rm had}/a_\mu^{\rm had}$ can be improved by a factor of 2!

Possible Progress for $a_\mu^{\mathrm{LO,had}}$

Experiments are planned at the new machine VEPP-2000 (VEPP-2M upgrade) with 2 detectors (CMD-3 and SND) up to $\sqrt{s}$=2 GeV with $L_{\max} = 10^{32}$ cm^{-2}s^{-1}.

A similar machine (DAΦNE-II) is discussed in Frascati.

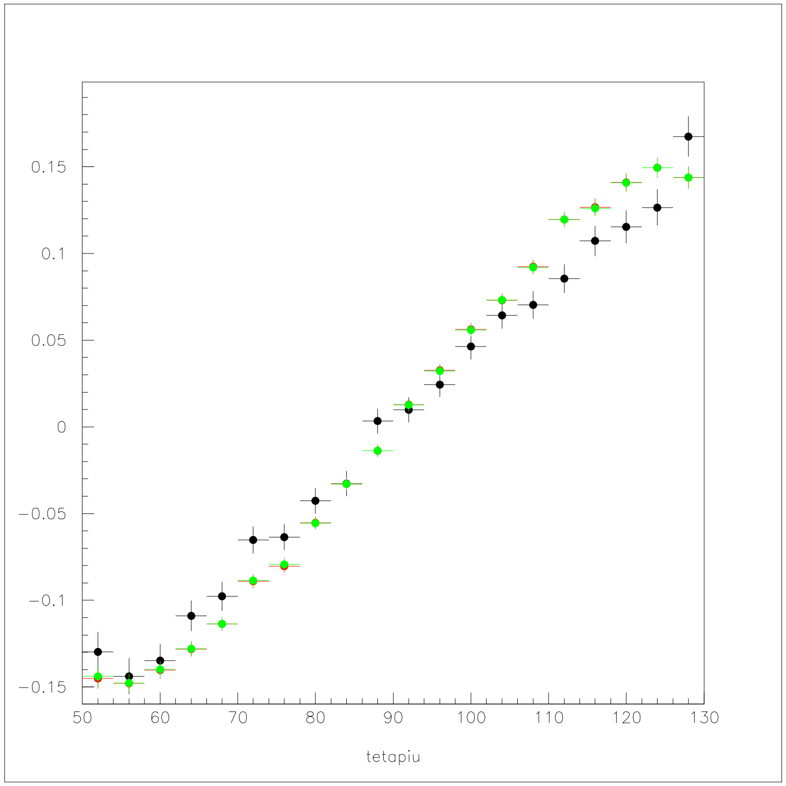
$\sqrt{s}$, GeV	$a_\mu^{\mathrm{LO,had}}$ in 2004 (2007), 10^{-10}	2007 (pes./opt.), 10^{-10}
$2\pi, < 2$	508.4 ± 5.5 (4.0)	3.2/2.0
ω	38.0 ± 1.1 (0.4)	0.4/0.4
ϕ	35.7 ± 0.9 (0.4)	0.3/0.2
0.6–2.0	62.9 ± 2.5 (1.3)	1.2/0.7
Total	645.0 ± 6.2 (4.2)	3.5/2.2

The total error of $a_\mu^{\mathrm{had,LO}}$ falls from $7.2 \cdot 10^{-10}$ to $3.9(2.8) \cdot 10^{-10}$. Other measurements are welcome (DAΦNE-II, ISR)!

Problems in e^+e^- Sector

- Radiative corrections – do we really know them that well?
- Final state radiation from hadrons - validity of scalar QED
- Correlations in the exclusive approach
  (between groups, summing in one group, ...)
- Efficiency for total R measurements:
  is the model adequate?

How Good is Scalar QED Assumption for FSR?



MC using scalar QED well describes KLOE data on charge asymmetry $(N_+ - N_-)/(N_+ + N_-)$ for $e^+e^- \to \pi^+\pi^-\gamma$ with 140 fb^{-1}.

Future of $(g_\mu - 2)/2$

1. Experiment
   - Today a_μ is known with a $5 \cdot 10^{-7}$ relative accuracy.
   - If funded, the BNL group can reach $2.5 \cdot 10^{-7}$ at BNL and $6 \cdot 10^{-8} - 1 \cdot 10^{-7}$ at J-PARC.
2. Theory
   - Today: QED, EW – $2.5 \cdot 10^{-8}$, Hadr. LO – $6.7 \cdot 10^{-7}$
   - We'll badly need at least one order of magnitude improvement in the hadronic contribution accuracy
   - It is equivalent to measuring R (τ) to a 10^{-3} accuracy (???)
   - Or $a_\mu^{\rm had}$ calculation from 1st principles (QCD, Lattice). Recently from Lattice: $a_\mu^{\rm had} = (446 \pm 23) \cdot 10^{-10}, \ (545 \pm 65) \cdot 10^{-10}$.

Conclusions

- BNL success stimulated significant progress of e^+e^- experiments and related theory
- Improvement of e^+e^- data (VEPP-2M, DAΦNE and BEPC) decreased an error of $a_\mu^{\text{had,LO}}$ by a factor of more than 2, but the experimental accuracy of a_μ is still better
- τ data could further improve the accuracy by 1.5 but a serious yet unexplained failure of CVC relations for e^+e^- and τ is observed
- Further improvement in $a_\mu^{\text{had,LO}}$ by a factor of 2–3 will be possible after VEPP-2000, DAΦNE-II, CESRc and $(c-\tau)$ factory as well as with ISR at DAΦNE and B-factories
- $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ differs from 0 by 2.7 $\sigma \Rightarrow$ SM is still alive!