Neutrinoless Double Beta Decay

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MANITOP

Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

based on

- W.R., *Neutrinoless Double Beta Decay and Particle Physics*, Int. J. Mod. Phys. **E20** (2011) 1833
- W.R., *Neutrinoless Double Beta Decay and Neutrino Physics*, J. Phys. **G39** (2012) 124008
- H. Päs and W.R., *Neutrinoless Double Beta Decay*, New J. Phys. **17** (2015), 115010

Neutrinoless double beta decay

$$(A, Z) \to (A, Z+2) + 2\, e^- \quad (0\nu\beta\beta)$$

- rare process, not yet observed
- violates lepton number
- best chance to confirm Majorana nature of light oscillating neutrinos
- constraints a lot of BSM models
- observation would have a lot of implications

Upcoming/running experiments: exciting time!!

best limit was from 2001, improved 2012

Name	Isotope	Source = Detector; calorimetric with			Source ≠ Detector
		high $\triangle E$	low $\triangle E$	topology	topology
AMoRE	^{100}Mo	✓	–	–	–
CANDLES	^{48}Ca	–	✓	–	–
COBRA	^{116}Cd (and ^{130}Te)	–	–	✓	–
CUORE	^{130}Te	✓	–	–	–
DCBA/MTD	^{82}Se / ^{150}Nd	–	–	–	✓
EXO	^{136}Xe	–	–	✓	–
GERDA	^{76}Ge	✓	–	–	–
CUPID	^{82}Se / ^{100}Mo / ^{116}Cd / ^{130}Te	✓	–	–	–
KamLAND-Zen	^{136}Xe	–	✓	–	–
LUCIFER	^{82}Se / ^{100}Mo / ^{130}Te	✓	–	–	–
LUMINEU	^{100}Mo	✓	–	–	–
MAJORANA	^{76}Ge	✓	–	–	–
MOON	^{82}Se / ^{100}Mo / ^{150}Nd	–	–	–	✓
NEXT	^{136}Xe	–	–	✓	–
SNO+	^{130}Te	–	✓	–	–
SuperNEMO	^{82}Se / ^{150}Nd	–	–	–	✓
XMASS	^{136}Xe	–	✓	–	–

Why should we probe Lepton Number Violation?

- L and B *accidentally* conserved in SM
- effective theory: $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{L}_{\rm LNV} + \frac{1}{\Lambda^2}\,\mathcal{L}_{\rm LFV,\,BNV,\,LNV} + \ldots$
- baryogenesis: B is violated
- B, L often connected in GUTs
- GUTs have seesaw and Majorana neutrinos
- (chiral anomalies: $\partial_\mu J^\mu_{B,L} = c\, G_{\mu\nu}\, \tilde{G}^{\mu\nu} \neq 0$ with $J_\mu^B = \sum \overline{q_i}\,\gamma_\mu\, q_i$ and $J_\mu^L = \sum \overline{\ell_i}\,\gamma_\mu\, \ell_i$)

$\Rightarrow$ Lepton Number Violation as important as Baryon Number Violation

($0\nu\beta\beta$ is much more than a neutrino mass experiment)

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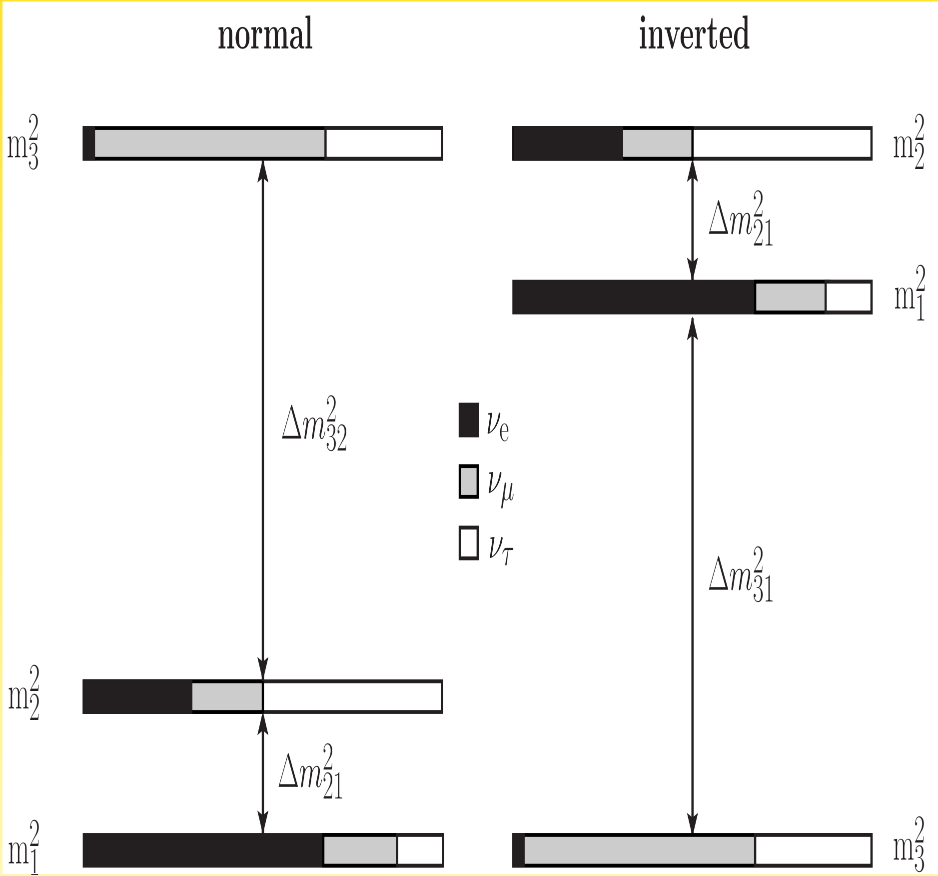
I) Dirac vs. Majorana neutrinos

Neutrinos oscillate $\Rightarrow m_\nu \neq 0$



according to Standard Model, they shouldn't

Limits (later):

- direct (Kurie plot): $m_\beta = \sqrt{\sum |U_{ei}|^2 m_i^2} \leq 2.3$ eV
- neutrinoless double beta decay: $|m_{ee}| = |\sum U_{ei}^2 m_i| \lesssim 0.3$ eV
- cosmology: $\sum m_i \lesssim 0.7$ eV

$\Rightarrow$ "m_ν" less than 1 eV

How can we introduce neutrino mass terms?

I1) Basics

a) Dirac masses

with $L \sim (2,-1)$ and $\Phi \sim (2,1)$, add $\nu_R \sim (1,0)$:

$$\mathcal{L}_D = g_\nu \, \overline{L} \, \tilde{\Phi} \, \nu_R \xrightarrow{\text{SSB}} \frac{v}{\sqrt{2}} \, g_\nu \, \overline{\nu_L} \, \nu_R = m_\nu \, \overline{\nu_L} \, \nu_R$$

But $m_\nu \lesssim$ eV implies $g_\nu \lesssim 10^{-12} \lll g_e$

highly unsatisfactory fine-tuning...

actually, $m_e = 10^{-6}\, m_t$, so WTF?

point is that

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{with} \quad m_u \simeq m_d$$

has to be contrasted with

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{with} \quad m_\nu \simeq 10^{-6}\, m_e$$

what suppresses neutrino mass **for every generation?**

b) Majorana masses

need charge conjugation:

electron e^-: $[\gamma_\mu\,(i\partial^\mu + e\,A^\mu) - m]\,\psi = 0 \quad (1)$

positron e^+: $[\gamma_\mu\,(i\partial^\mu - e\,A^\mu) - m]\,\psi^c = 0 \quad (2)$

Try $\psi^c = S\,\psi^*$, evaluate $(S^*)^{-1}\,(2)^*$ and compare with (1):

$$\Rightarrow S = i\gamma_2$$

and thus

$$\psi^c = i\gamma_2\,\psi^* = i\gamma_2\,\gamma_0\overline{\psi}^T \equiv C\,\overline{\psi}^T$$

flips all charge-like quantum numbers

Properties of C:

$$C^\dagger = C^T = C^{-1} = -C$$

$$C\,\gamma_\mu\,C^{-1} = -\gamma_\mu^T$$

$$C\,\gamma_5\,C^{-1} = \gamma_5^T$$

$$C\,\gamma_\mu\gamma_5\,C^{-1} = (\gamma_\mu\gamma_5)^T$$

properties of charged conjugate spinors:

$$(\psi^c)^c = \psi$$

$$\overline{\psi^c} = \psi^T\,C$$

$$\overline{\psi_1}\,\psi_2^c = \overline{\psi_2^c}\,\psi_1$$

$$\boxed{(\psi_L)^c = (\psi^c)_R}$$

$$\boxed{(\psi_R)^c = (\psi^c)_L}$$

C flips chirality: LH becomes RH

common source of confusion (added here to create or add confusion):

that was actually particle-antiparticle conjugation ($\nu_L \to \overline{\nu}_R$)

charge conjugation leaves chirality untouched ($\nu_L \to \overline{\nu}_L$)

same effect for total state $\nu = \nu_L + \nu_R$, different for chiral states

Fermion mass terms

$$\mathcal{L} = m_\nu \bar{\psi}\psi = m_\nu \overline{\psi_L}\,\psi_R + h.c. = m_\nu\,(\overline{\psi_L}\,\psi_R + \overline{\psi_R}\,\psi_L)$$

both chiralities a must for mass term!

there are two and only two possibilities:

(i) ψ_L independent of ψ_R: **Dirac particle**

(ii) $\psi_L = (\psi_R)^c$: **Majorana particle**

(ii) $\Rightarrow \psi^c = (\psi_L + \psi_R)^c = (\psi_L)^c + (\psi_R)^c = \psi_R + \psi_L = \psi$: $\boxed{\psi^c = \psi}$

$\Rightarrow$ Majorana fermion is identical to its antiparticle, a truly neutral particle

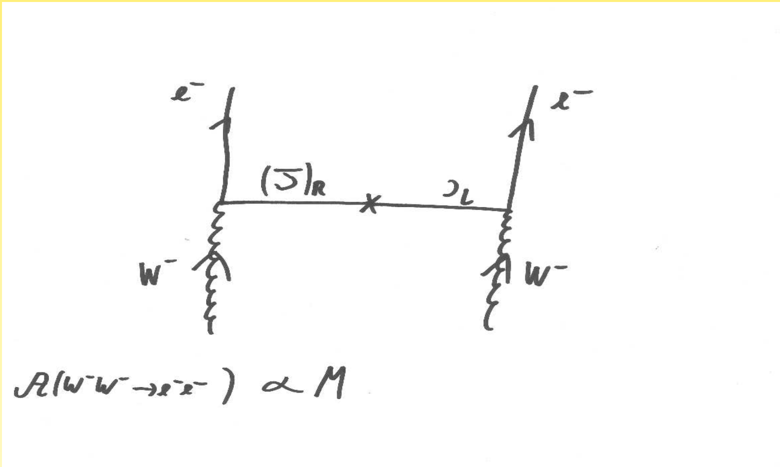
$\Rightarrow$ does not carry conserved additive quantum numbers mass term for Majorana particles:

$$\mathcal{L}_M = \frac{1}{2}\,\overline{\psi}\,M\,\psi = \frac{1}{2}\,\overline{\psi_L + (\psi_L)^c}\,M\,(\psi_L + (\psi_L)^c) = \tfrac{1}{2}\,\overline{\psi_L}\,M\,\psi_L^c + h.c.$$

- Majorana mass term (not allowed by gauge symmetry, never mind for now)
- $\mathcal{L}_M \propto \psi^*\,\psi^\dagger \Rightarrow$ NOT invariant under $\psi \to e^{i\alpha}\,\psi$

  $\Rightarrow$ breaks Lepton Number by 2 units!

in standard approach to $0\nu\beta\beta$: origin of decay



Dirac vs. Majorana

in $V-A$ theories: observable difference **always** suppressed by $(m/E)^2$

- In terms of degrees of freedom (helicity and particle/antiparticle):
  $\nu_\mathrm{D} = (\nu_\uparrow, \nu_\downarrow, \bar{\nu}_\uparrow, \bar{\nu}_\downarrow)$ versus $\nu_\mathrm{M} = (\nu_\uparrow, \nu_\downarrow)$
- weak interactions act on chirality (left-/right-handed)
- chirality is not a good quantum number ("spin flip"): $L = \downarrow + \frac{m}{E} \uparrow$
- Dirac:
  - what we produce from a W^- is $\ell^-(\bar{\nu}_\uparrow + \frac{m}{E}\bar{\nu}_\downarrow)$
  - the $\bar{\nu}_\downarrow$ CANNOT interact with another W^- to generate another ℓ^-: $\Delta L = 0$
- Majorana:
  - what we produce from a W^- is $\ell^-(\nu_\uparrow + \frac{m}{E}\nu_\downarrow)$
  - the $\nu_\downarrow$ CAN interact with another W^- to generate another ℓ^-: $\Delta L = 2$

$\Rightarrow$ amplitude $\propto (m/E) \Rightarrow$ probability $\propto (m/E)^2$

Dirac vs. Majorana

- Z-decay:

$$\frac{\Gamma(Z \to \nu_{\rm D}\nu_{\rm D})}{\Gamma(Z \to \nu_{\rm M}\nu_{\rm M})} \simeq 1 - 3\,\frac{m_\nu^2}{m_Z^2}$$

- Meson decays

$$\mathrm{BR}(K^+ \to \pi^- \, e^+ \, \mu^+) \propto |m_{e\mu}|^2 = \left|\sum U_{ei}\, U_{\mu i}\, m_i\right|^2 \sim 10^{-30} \left(\frac{|m_{e\mu}|}{\mathrm{eV}}\right)^2$$

- neutrino-antineutrino oscillations

$$P(\nu_\alpha \to \bar{\nu}_\beta) = \frac{1}{E^2} \left| \sum_{i,j} U_{\alpha j} U_{\beta j} U^*_{\alpha i} U^*_{\beta i}\, m_i m_j\, e^{-i(E_j - E_i)t} \right|$$

- Majorana only works for neutral fermions $\Rightarrow$ neutrinos only candidate in SM
- $\psi^c = \psi$ makes spinor have two degrees of freedom, not four:

$$\psi_D = \begin{pmatrix} \Phi \\ \xi \end{pmatrix} \longrightarrow \psi_M = \begin{pmatrix} \Phi \\ -i\sigma_2 \Phi^* \end{pmatrix}$$

(Φ, ξ are 2-component Weyl spinors)

- in terms of creators/annihilators, $\Psi = \Psi^c$ happens for

$$\Psi = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{+ipx} \right]$$

where $v_s = C\bar{u}_s^T$ and $u_s = C\bar{v}_s^T$

- mass term $\overline{\nu_L}\, \nu_L^c$ not allowed as ν_L is in $(2, -1)$
- if $N_R \sim (1, 0)$ exists: $\overline{N_R}\, N_R^c$ allowed!

Majorana nature implies new relation for spinors:

$$\sum_s u_s \bar{u}_s = \not{p} + m \quad , \quad \sum_s v_s \bar{v}_s = \not{p} - m$$

$$\sum_s u_s v_s^T = (\not{p} + m) C^T \quad , \quad \sum_s \bar{u}_s^T \bar{v}_s = C^{-1} (\not{p} - m)$$

$$\sum_s \bar{v}_s \bar{u}_s = C^{-1} (\not{p} + m) \quad , \quad \sum_s v_s u_s^T = (\not{p} - m) C^T$$

and new propagators

$$(1) \qquad \langle 0 | T \left\{ \Psi(x) \bar{\Psi}(y) \right\} | 0 \rangle = S(x-y)$$

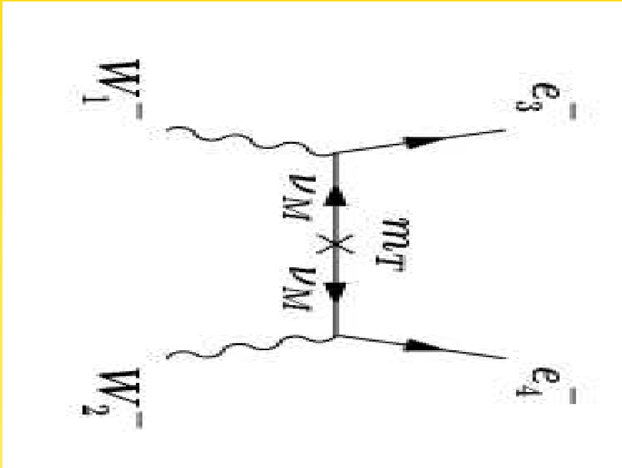
$$(2) \quad \langle 0 | T \left\{ \Psi(x) \Psi(y) \right\} | 0 \rangle = -S(x-y) C$$

$$(3) \quad \langle 0 | T \left\{ \bar{\Psi}(x) \bar{\Psi}(y) \right\} | 0 \rangle = C^{-1} S(x-y)$$

with standard propagator

$$S(x-y) = i \int \frac{d^4 p}{(2\pi)^4} \left(\frac{\not{p} + m}{p^2 - m^2 + i\epsilon} \right) e^{-ip(x-y)}$$

$\Rightarrow$ can create or annihilate two neutrinos ((2) and (3)) consider t-channel $W^-W^- \to e^-e^-$ from $\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu \bar{e} \gamma^\mu (1-\gamma_5)\nu_M$



$$-i\mathcal{M} = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \epsilon_1^\mu \epsilon_2^\nu \left[\bar{u}_3 \gamma_\mu (1-\gamma_5) i \frac{\not{k}+m}{k^2-m^2} (-C) \bar{u}_4 \gamma_\nu (1-\gamma_5)\right]$$

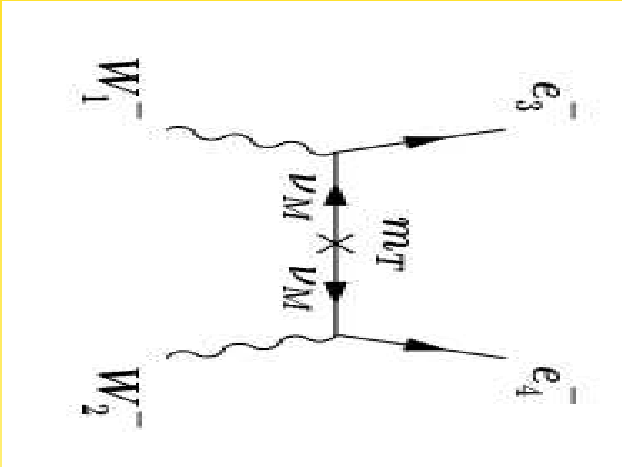
$$\propto (\bar{u}_3 \gamma_\mu (1-\gamma_5)) (\not{k}+m) (-C) (\bar{u}_4 \gamma_\nu (1-\gamma_5))$$

$$= (\bar{u}_3 \gamma_\mu (1-\gamma_5)) (\not{k}+m) (-(1-\gamma_5)\gamma_\nu v_4)$$

$$= m\, \bar{u}_3 \gamma_\mu (1-\gamma_5) \gamma_\nu v_4$$

proportional to (suppressed by) neutrino mass!!

consider t-channel $W^- W^- \to e^- e^-$ from $\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu \bar{e} \gamma^\mu (1 - \gamma_5) \nu_M$

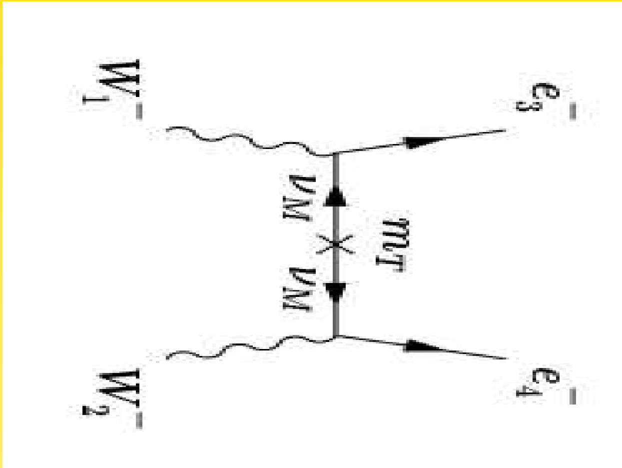


$$-i\mathcal{M} \propto m \, \bar{u}_3 \gamma_\mu (1 - \gamma_5) \gamma_\nu v_4$$

proportional to (suppressed by) neutrino mass!!

same result by replacing one $\bar{e}\gamma^\mu(1-\gamma_5)\nu_M$ with $-\bar{\nu}_M \gamma^\mu (1+\gamma_5) e^c$

consider t-channel $W^-W^- \to e^-e^-$ from $\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu \bar{e} \gamma^\mu (1-\gamma_5)\nu_M$



$$\mathcal{M} \propto \bar{e}_3 \gamma_\mu (1-\gamma_5)\nu \, \bar{e}_4 \gamma_\nu (1-\gamma_5)\nu$$

$$= \bar{e}_3 \gamma_\mu (1-\gamma_5)\nu \, \bar{\nu}^c \gamma_\nu (1+\gamma_5) e_4^c (-1)$$

time-ordered product $\nu\bar{\nu}$ is propagator $\propto (\not{k} + m)$

use $e = f\,u + \bar{f}^\dagger\,v$, $\bar{e} = f^\dagger\,\bar{u} + \bar{f}\,\bar{v}$, $e^c = f^\dagger\,v + \bar{f}\,u \Rightarrow e_4^c$ gets a v_4

how to calculate identities:

$\bar{e}\,\gamma_\mu(1-\gamma_5)\,\nu$	|	start
$= \overline{(e^c)^c}\,\gamma_\mu(1-\gamma_5)\,(\nu^c)^c$	|	$\psi = (\psi^c)^c$
$= -(e^c)^T\,C^{-1}\,\gamma_\mu(1-\gamma_5)\,C\,\overline{\nu^c}^T$	|	$\overline{\psi^c} = -\psi^T\,C^{-1}$ and $\psi^c = C\,\overline{\psi}^T$
$= -(e^c)^T\left(-\gamma_\mu^T - (\gamma_\mu\gamma_5)^T\right)\overline{\nu^c}^T$	|	$C\gamma_\mu C^{-1} = -\gamma_\mu^T,\;\; C\gamma_\mu\gamma_5 C^{-1} = (\gamma_\mu\gamma_5)^T$
$= (e^c)^T\left[\gamma_\mu(1+\gamma_5)\right]^T\overline{\nu^c}^T$	|	collecting signs
$= -\overline{\nu^c}\,\gamma_\mu(1+\gamma_5)\,e^c$	|	is scalar and fermion exchange

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I2) Seesaw Mechanisms

how to generate Majorana neutrino masses...

a) Higher dimensional operators

include $d \geq 5$ operators, gauge and Lorentz invariant, only SM fields:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_5 + \mathcal{L}_6 + \ldots = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{O}_5 + \frac{1}{\Lambda^2}\,\mathcal{O}_6 + \ldots$$

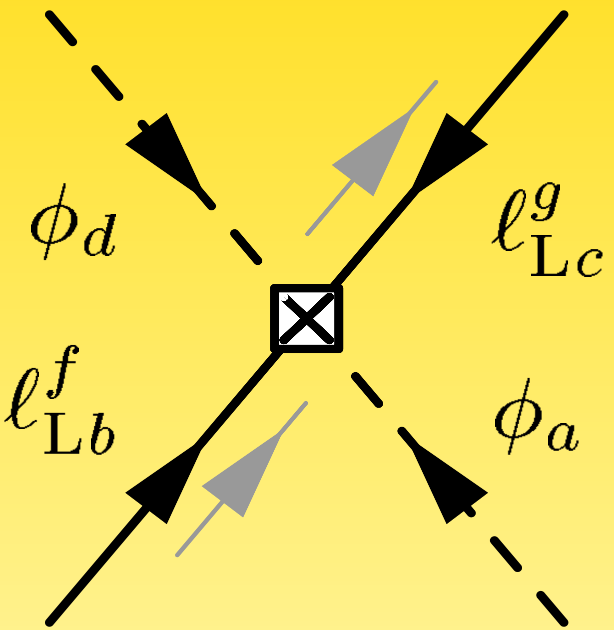
there is only one dimension 5 term! "leading order new physics"

$$\frac{1}{\Lambda}\,\mathcal{O}_5 = \frac{c}{\Lambda}\,\overline{L}\,\tilde{\Phi}\,\tilde{\Phi}^T\,L^c \xrightarrow{\rm SSB} \frac{c\,v^2}{2\,\Lambda}\,\overline{\nu_L}\,\nu_L^c \equiv m_\nu\,\overline{\nu_L}\,\nu_L^c$$

Majorana mass term! $\Delta L = 2$

it follows $\Lambda \gtrsim c\,\left(\frac{0.1~\rm eV}{m_\nu}\right)\,10^{14}$ GeV   Weinberg 1979

Weinberg operator is $LL\Phi\Phi$



seesaw mechanisms are "UV-completions" of this effective operator by integrating out heavy physics

Master formula: $2 \times 2 = 3 + 1$

$SU(2)_L \times U(1)_Y$ with $2 \otimes 2 = 3 \oplus 1$:

$$\overline{L}\,\tilde{\Phi} \sim (2,+1) \otimes (2,-1) = (3,0) \oplus (1,0)$$

to make a singlet, couple to $(1,0)$ or $(3,0)$, because $3 \otimes 3 = 5 \oplus 3 \oplus 1$

Alternatively:

$$\overline{L^c}\,L \sim (2,-1) \otimes (2,-1) = (3,-2) \oplus (1,-2)$$

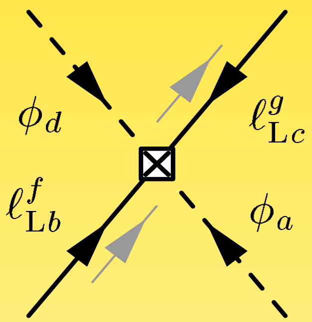
to make a singlet, couple to $(1,+2)$ or $(3,+2)$. However, singlet combination $(1,-2)$ is $\overline{\nu}\,\ell^c - \overline{\ell}\,\nu^c$, which cannot generate neutrino mass term

$$\Longrightarrow \quad (1,0) \quad \text{or} \quad (3,+2) \quad \text{or} \quad (3,0)$$

type I | type II | type III

Origin of small masses

$$\mathcal{L} = \frac{c}{\Lambda}\, \overline{L^c}\, \tilde{\Phi}^*\, \tilde{\Phi}^\dagger\, L$$



has only 3 tree-level realizations

- $N_R \sim (1,0)$ type I seesaw
- $\Delta \sim (3,2)$ type II seesaw
- $\Sigma \sim (3,0)$ type III seesaw

seesaws include **new** representations, new energy scales, new concepts

Type I Seesaw

introduce $N_R \sim (1,0)$ and couple to $g_\nu \overline{L}\,\tilde{\Phi} \sim (1,0)$

becomes $g_\nu\, v/\sqrt{2}\,\overline{\nu_L}\,N_R \equiv m_D\,\overline{\nu_L}\,N_R$

in addition: Majorana mass term for N_R: $\frac{1}{2} M_R \overline{N_R^c} N_R$

using $\overline{\nu_L} m_D N_R = \overline{N_R^c} m_D^T \nu_L^c$:

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \left(\begin{array}{cc} 0 & m_D \\ m_D & M_R \end{array} \right) \left(\begin{array}{c} \nu_L^c \\ N_R \end{array} \right) + h.c.$$
$$\equiv \frac{1}{2}\,\overline{\Psi}\,\mathcal{M}_\nu\,\Psi^c + h.c.$$

Dirac + Majorana mass term is a Majorana mass term!

Diagonalization:

$$\mathcal{L} = \tfrac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \tfrac{1}{2} \quad \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \quad \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathcal{U}^*}_{\mathrm{diag}(m_\nu, M)} \quad \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with

general formula:	if $m_D \ll M_R$:
$\tan 2\theta = \frac{2\,m_D}{M_R - 0}$	$\ll 1$
$m_\nu = \frac{1}{2}\left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4\,m_D^2}\right]$	$\simeq -m_D^2/M_R$
$M = \frac{1}{2}\left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4\,m_D^2}\right]$	$\simeq M_R$

Note: m_D associated with EWSB, part of SM, bounded by $v/\sqrt{2} = 174$ GeV

M_R is SM singlet, does whatever it wants: $\Rightarrow M_R \gg m_D$

Hence, $\theta \simeq m_D/M_R \ll 1$

$\nu = \nu_L \cos\theta - N_R^c \sin\theta \simeq \nu_L$ with mass $m_\nu \simeq -m_D^2/M_R$

$N = N_R \cos\theta + \nu_L^c \sin\theta \simeq N_R$ with mass $M \simeq M_R$

in effective mass terms

$$\mathcal{L} \simeq \frac{1}{2}\, m_\nu\, \overline{\nu_L}\, \nu_L^c + \frac{1}{2}\, M_R\, \overline{N_R^c}\, N_R$$

compare with Weinberg operator:

$$\Lambda = -\frac{c\, v^2}{m_D^2}\, M_R$$

also: integrate N_R away with Euler-Lagrange equation matrix case: block diagonalization

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \frac{1}{2} \quad \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \; \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \mathcal{U}^*}_{\text{diag}(m_\nu, M)} \; \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with 6×6 diagonal matrix

$$\mathcal{U} = \begin{pmatrix} 1 & -\rho \\ \rho^\dagger & 1 \end{pmatrix}, \quad \mathcal{U}^\dagger = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}, \quad \mathcal{U}^* = \begin{pmatrix} 1 & -\rho^* \\ \rho^T & 1 \end{pmatrix}$$

write down individual components:

write down individual components:

$$
\begin{aligned}
m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\
0 &= m_D - \rho\, m_D^T\, \rho^* + \rho\, M_R \\
M &= -\rho^\dagger\, m_D - m_D^T\, \rho^* + M_R
\end{aligned}
$$

now, ρ (aka θ from before) will be of order m_D/M_R:

$$
\begin{aligned}
m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\
0 &\simeq m_D + \rho\, M_R \Rightarrow \rho = -m_D\, M_R^{-1} \\
M &\simeq M_R
\end{aligned}
$$

insert ρ in m_ν to find:

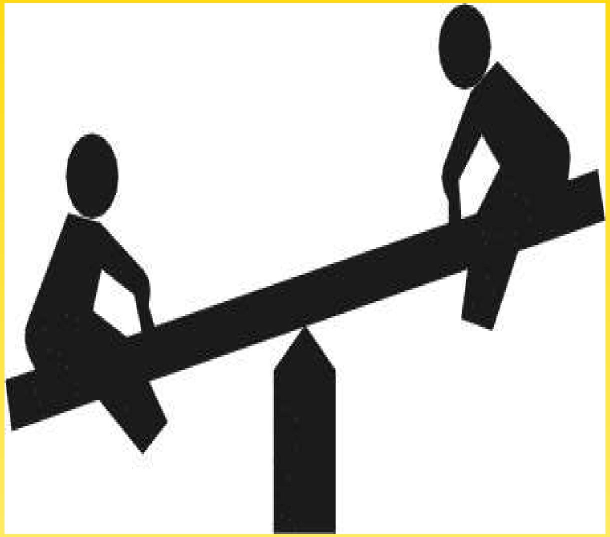
$$
m_\nu = -m_D\, M_R^{-1}\, m_D^T
$$

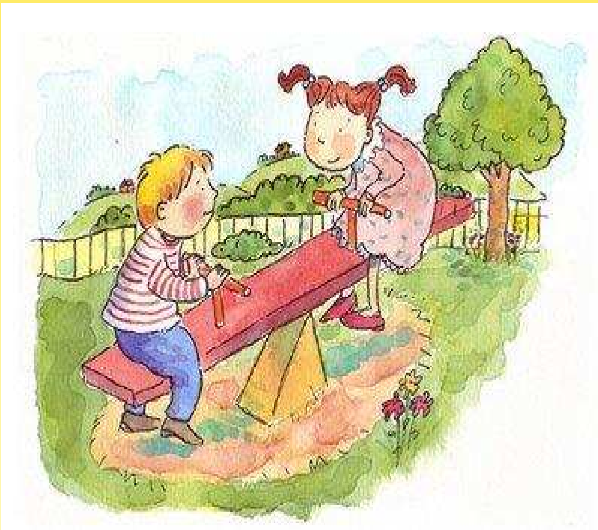
$$m_\nu = \frac{m_D^2}{M_R} = \frac{m_{\rm SM}^2}{M_R} = m_{\rm SM}\,\epsilon$$

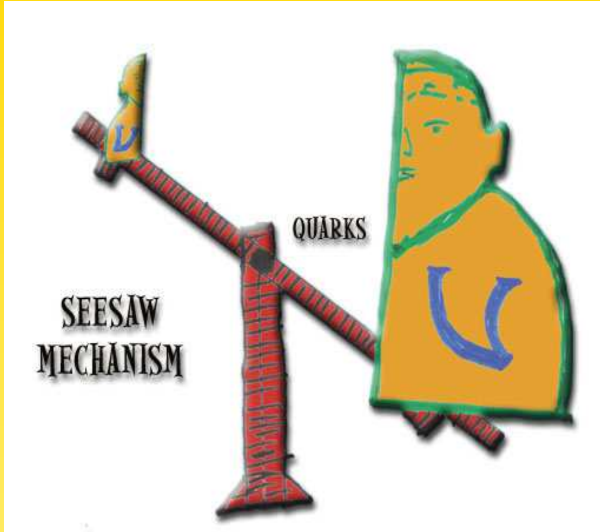
(type I) Seesaw Mechanism

Minkowski; Yanagida; Glashow; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanović (77-80)

we found the requested suppression mechanism!



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SEESAW
MECHANISM


SEE-SAW DON COVAY
DON COVAY SEE-SAW


THE SEESAW
Waiting


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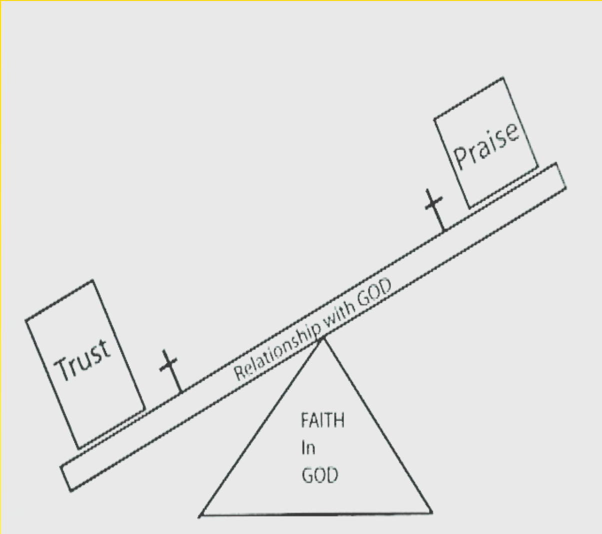
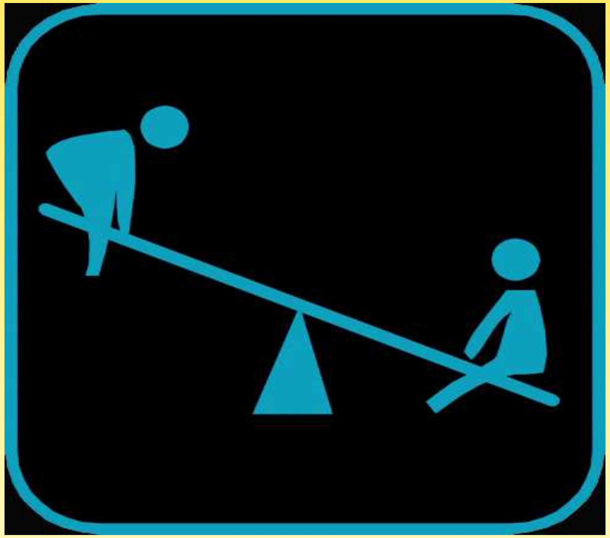

Praise
Trust
Relationship with GOD
FAITH
In
GOD



SEE SAW

Seesaw Formalism

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L, \, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} \stackrel{M_R \gg m_D}{\Longrightarrow} m_\nu = m_D^T M_R^{-1} m_D$$

arbitrary 6×6 (?) matrix

with new aspects:

- fermionic singlets $N_R \sim (1, 0)$
- new energy scale M_R ($\propto 1/m_\nu$)
- lepton number violation

See-Saw Phenomenology

Full mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} = \mathcal{U} \begin{pmatrix} m_\nu^{\rm diag} & 0 \\ 0 & M_R^{\rm diag} \end{pmatrix} \mathcal{U}^T \text{ with } \mathcal{U} = \begin{pmatrix} N & S \\ T & V \end{pmatrix}$$

- N is the PMNS matrix: non-unitary
- $S = m_D^\dagger \, (M_R^*)^{-1}$ describes mixing of heavy neutrinos with SM leptons

Type II Seesaw

$$\mathcal{L} \propto \overline{L^c}\, L \to \nu^T\, \nu$$

has isospin $I_3 = 1$ and transforms as $\sim (3, -2)$

$\Rightarrow$ introduce Higgs triplet $\sim (3, +2)$ with $(I_3 = Q - Y/2)$:

$$\Delta = \begin{pmatrix} \Delta^+ & -\sqrt{2}\,\Delta^{++} \\ \sqrt{2}\,\Delta^0 & -\Delta^+ \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}$$

gives mass matrix:

$$\mathcal{L} = \frac{1}{2}\, y_\nu\, \overline{L^c}\, i\sigma_2\, \Delta\, L \xrightarrow{\text{vev}} \frac{1}{2}\, y_\nu\, v_T\, \overline{\nu_L^c}\, \nu_L \equiv \frac{1}{2}\, m_\nu\, \overline{\nu_L^c}\, \nu_L$$

$\leftrightarrow$ **neutrino mass without right-handed neutrinos!**

$v_T \ll v$ because

$$V = -M_\Delta^2 \, \mathrm{Tr}\left(\Delta^\dagger \, \Delta\right) + \mu \, \Phi^\dagger \Delta \, \tilde{\Phi}$$

with $\frac{\partial V}{\partial \Delta} = 0$ one has $\boxed{v_T = \frac{\mu \, v^2}{M_\Delta^2}}$

v_T can be suppressed by M_Δ and/or μ

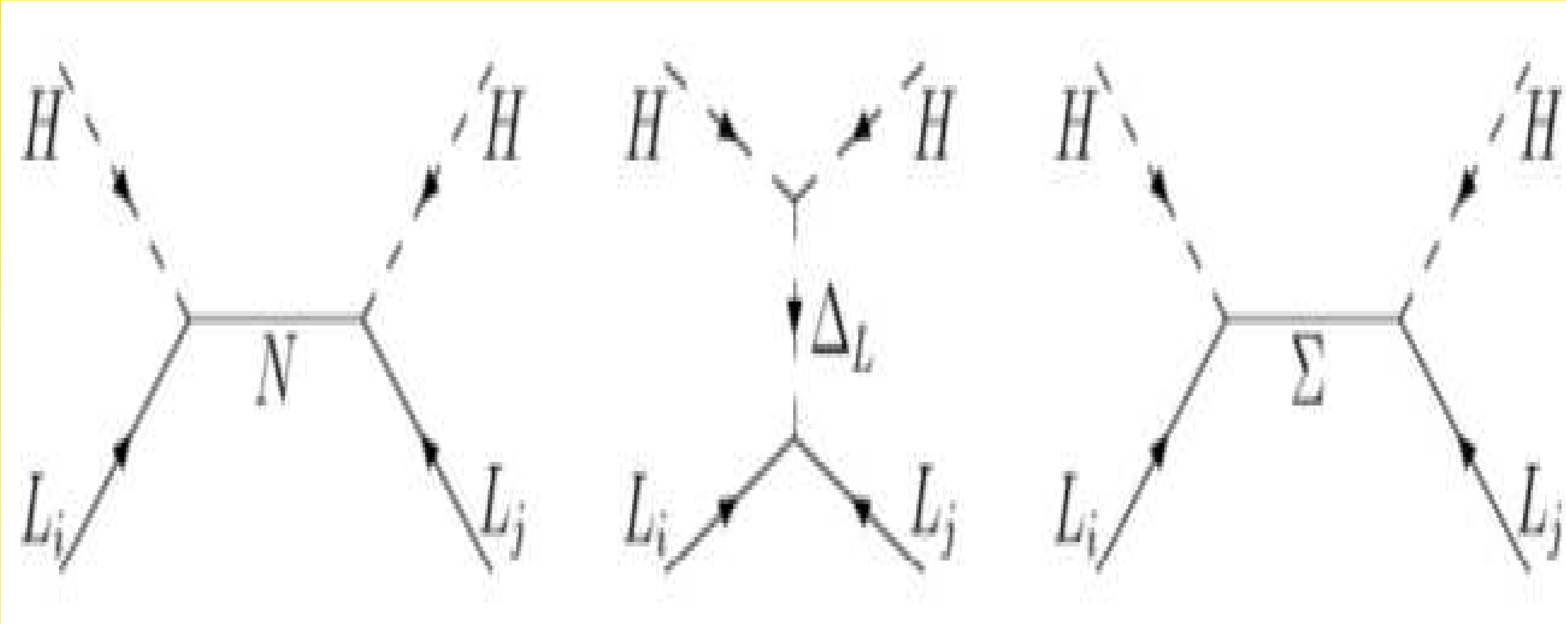
compare with Weinberg operator:

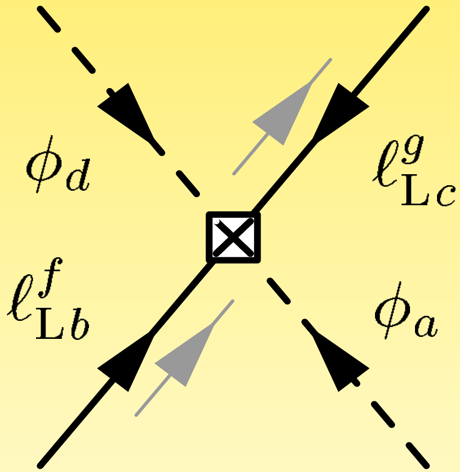
$$\boxed{\Lambda = \frac{c \, M_\Delta^2}{g_\nu \, \mu}}$$

Type II (or Triplet) Seesaw Mechanism

Magg, Wetterich; Mohapatra, Senjanovic; Lazarides, Shafi, Wetterich; Schechter, Valle (80-82)

Seesaw Summary





Paths to Neutrino Mass

approach	ingredient	quantum number of messenger	$\mathcal{L}$	m_ν	scale
"SM" (Dirac mass)	RH ν	$N_R \sim (1,0)$	$h\overline{N_R}\Phi L$	hv	$h = \mathcal{O}(10^{-12})$
"effective" (dim 5 operator)	new scale + LNV	–	$h\,\overline{L^c}\,\Phi\,\Phi\,L$	$\frac{h\,v^2}{\Lambda}$	$\Lambda = 10^{14}$ GeV
"direct" (type II seesaw)	Higgs triplet + LNV	$\Delta \sim (3,2)$	$h\overline{L^c}\Delta L + \mu\Phi\Phi\Delta$	hv_T	$\Lambda = \frac{1}{h\mu} M_\Delta^2$
"indirect 1" (type I seesaw)	RH ν + LNV	$N_R \sim (1,0)$	$h\overline{N_R}\Phi L + \overline{N_R} M_R N_R^c$	$\frac{(hv)^2}{M_R}$	$\Lambda = \frac{1}{h} M_R$
"indirect 2" (type III seesaw)	fermion triplets + LNV	$\Sigma \sim (3,0)$	$h\overline{\Sigma}\,L\Phi + \mathrm{Tr}\overline{\Sigma} M_\Sigma \Sigma$	$\frac{(hv)^2}{M_\Sigma}$	$\Lambda = \frac{1}{h} M_\Sigma$

plus seesaw variants (linear, double, inverse,...)

plus radiative mechanisms

plus extra dimensions

plusplusplus

Contents

I3) Summary neutrino physics

common prediction of all mechanisms: in basis with diagonal charged leptons

$$\mathcal{L} = \frac{1}{2}\,\nu^T\, m_\nu\, \nu \quad \text{with} \quad m_\nu = U\,\text{diag}(m_1, m_2, m_3)\,U^T$$

with PMNS matrix

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \text{diag}(e^{i\alpha}, e^{i\beta}, 1)$ ($\leftrightarrow$ Majorana, lepton number violation)

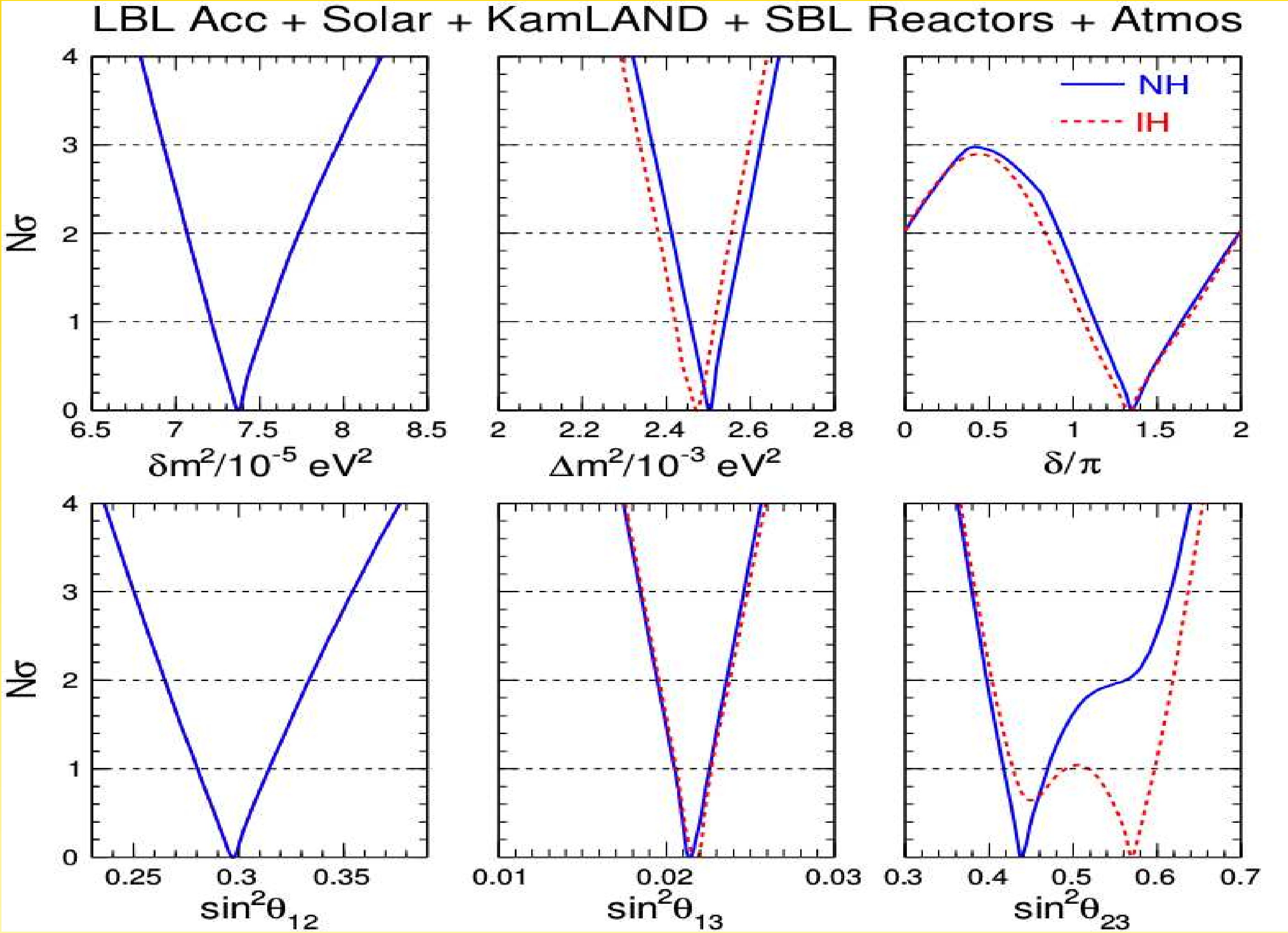
$\Rightarrow$ 3 angles, 3 phases, 3 masses

"three Majorana neutrino paradigm"

Status 2016

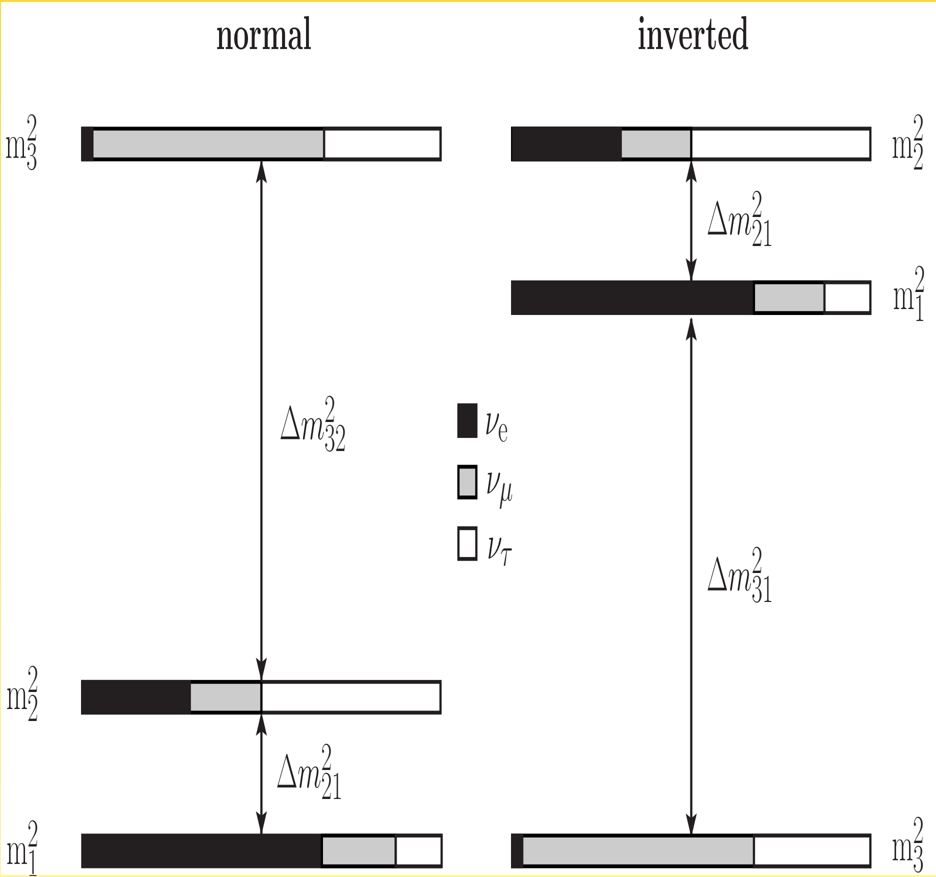
9 physical parameters in m_ν

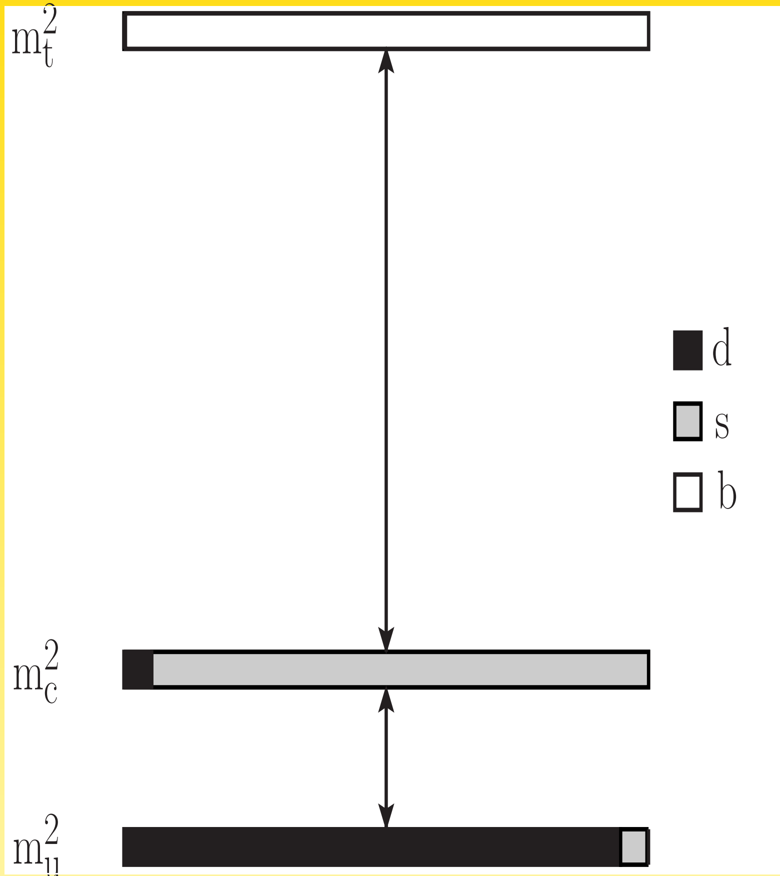
- θ_{12} and $m_2^2 - m_1^2$
- θ_{23} and $|m_3^2 - m_2^2|$
- θ_{13}
- m_1, m_2, m_3
- sgn$(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β



Lisi *et al.*, 1601.07777

(3-flavor, matter effects in solar, atm., LBL expts)





Why so different? ↔ Flavor symmetries!

PMNS-matrix:

$$|U| = \begin{pmatrix} 0.801\ldots0.845 & 0.514\ldots0.580 & 0.137\ldots0.158 \\ 0.225\ldots0.517 & 0.441\ldots0.699 & 0.614\ldots0.793 \\ 0.246\ldots0.529 & 0.464\ldots0.713 & 0.590\ldots0.776 \end{pmatrix}$$

CKM-matrix:

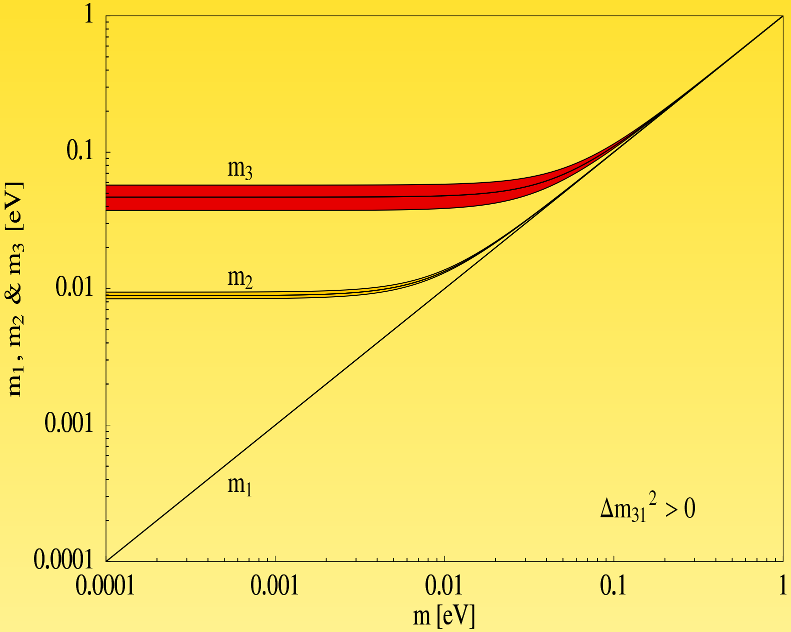
$$|V| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355^{+0.00015}_{-0.00014} \\ 0.22522 \pm 0.00061 & 0.97341 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

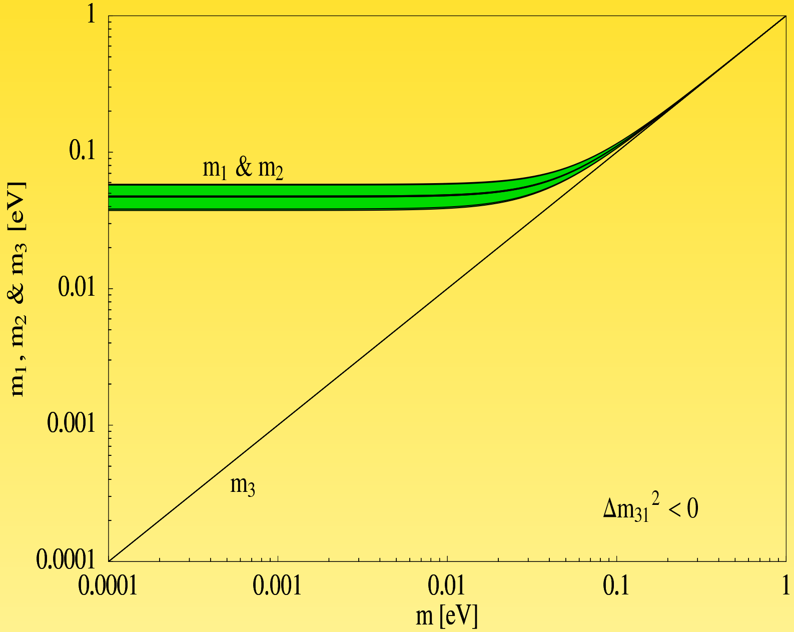
large mixing in PMNS, **small** mixing in CKM

Why so different? $\leftrightarrow$ Flavor symmetries!

Neutrino masses

- neutrino masses $\leftrightarrow$ scale of their origin
- neutrino mass ordering $\leftrightarrow$ form of m_ν





- $m_3^2 \simeq \Delta m_{\rm A}^2 \gg m_2^2 \simeq \Delta m_\odot^2 \gg m_1^2$: normal hierarchy (NH)
- $m_2^2 \simeq |\Delta m_{\rm A}^2| \simeq m_1^2 \gg m_3^2$: inverted hierarchy (IH)
- $m_3^2 \simeq m_2^2 \simeq m_1^2 \equiv m_0^2 \gg \Delta m_{\rm A}^2$: quasi-degeneracy (QD)

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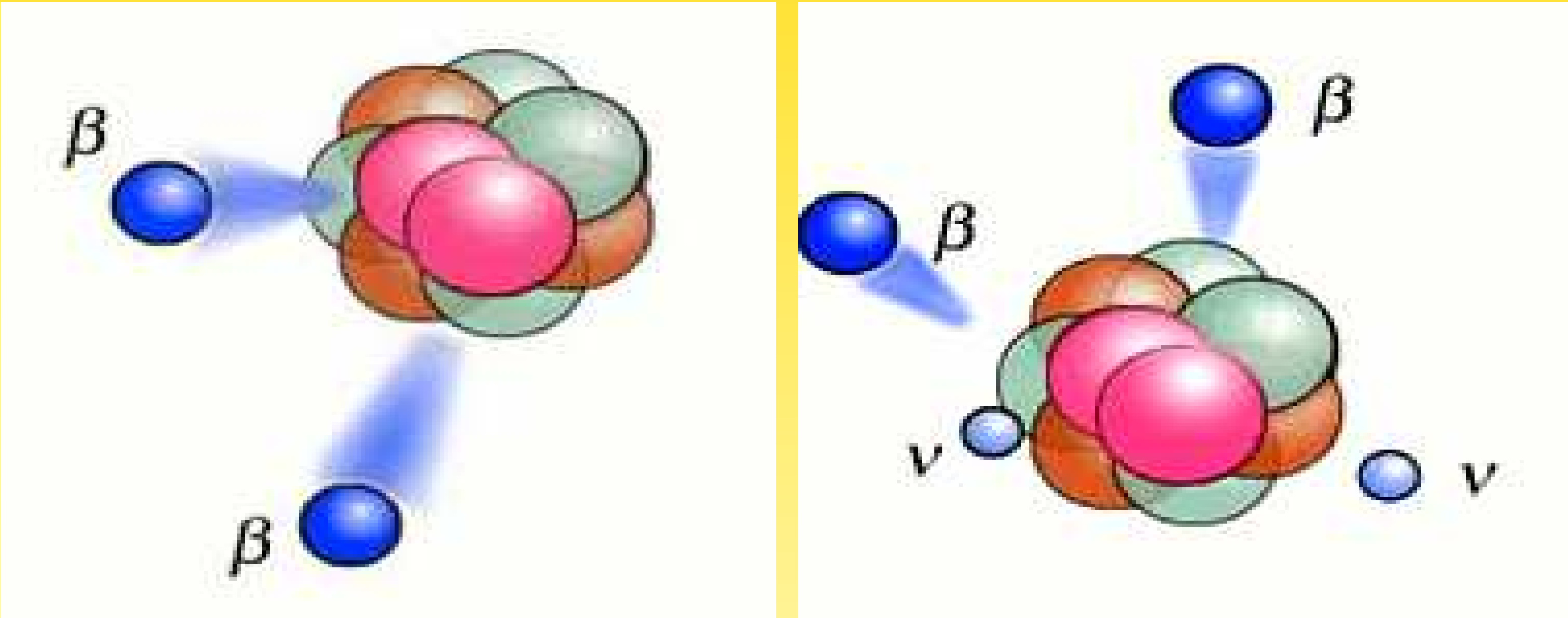
II) Neutrinoless Double Beta Decay: Standard Interpretation

$$(A, Z) \to (A, Z+2) + 2\,e^- \quad (0\nu\beta\beta)$$



- second order in weak interaction: $\Gamma \propto G_F^4 \Rightarrow$ rare!
- not to be confused with $(A, Z) \to (A, Z+2) + 2\,e^- + 2\,\bar{\nu}_e \quad (2\nu\beta\beta)$ (which occurs more often but is still rare)

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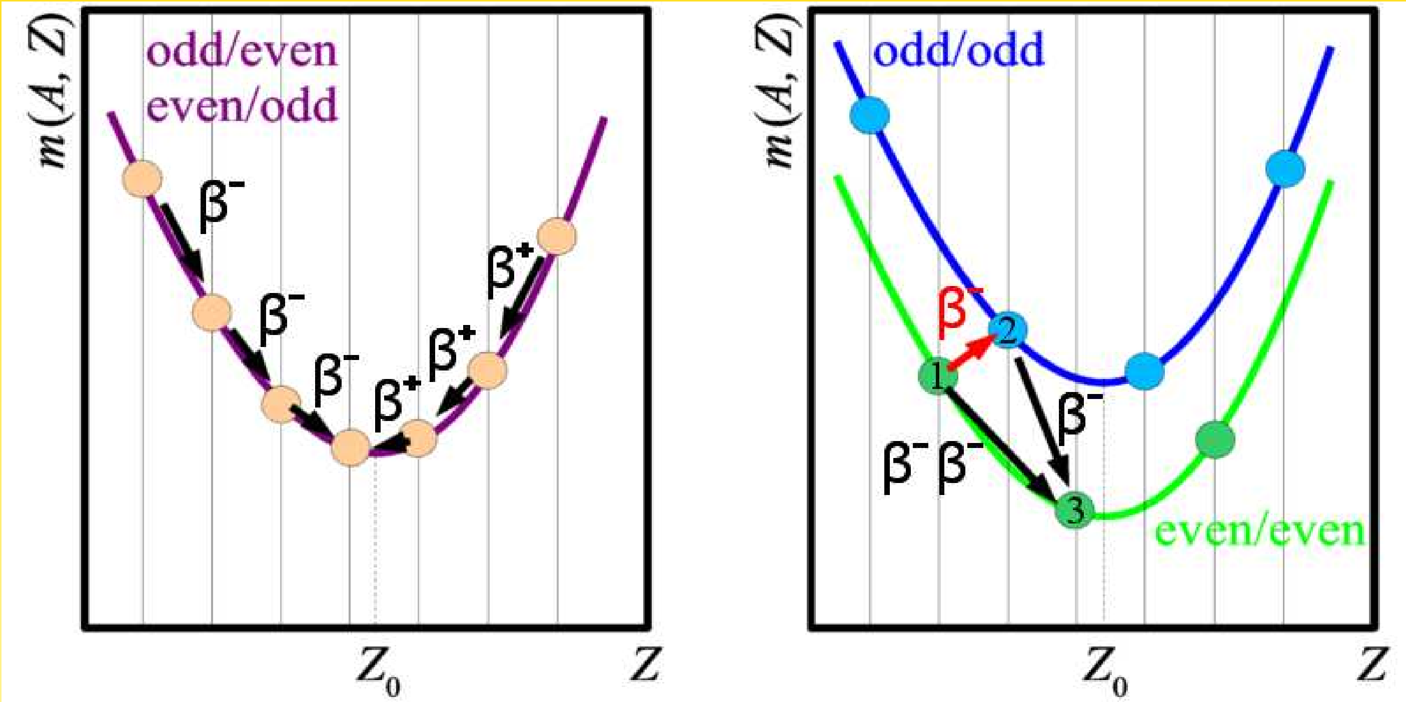
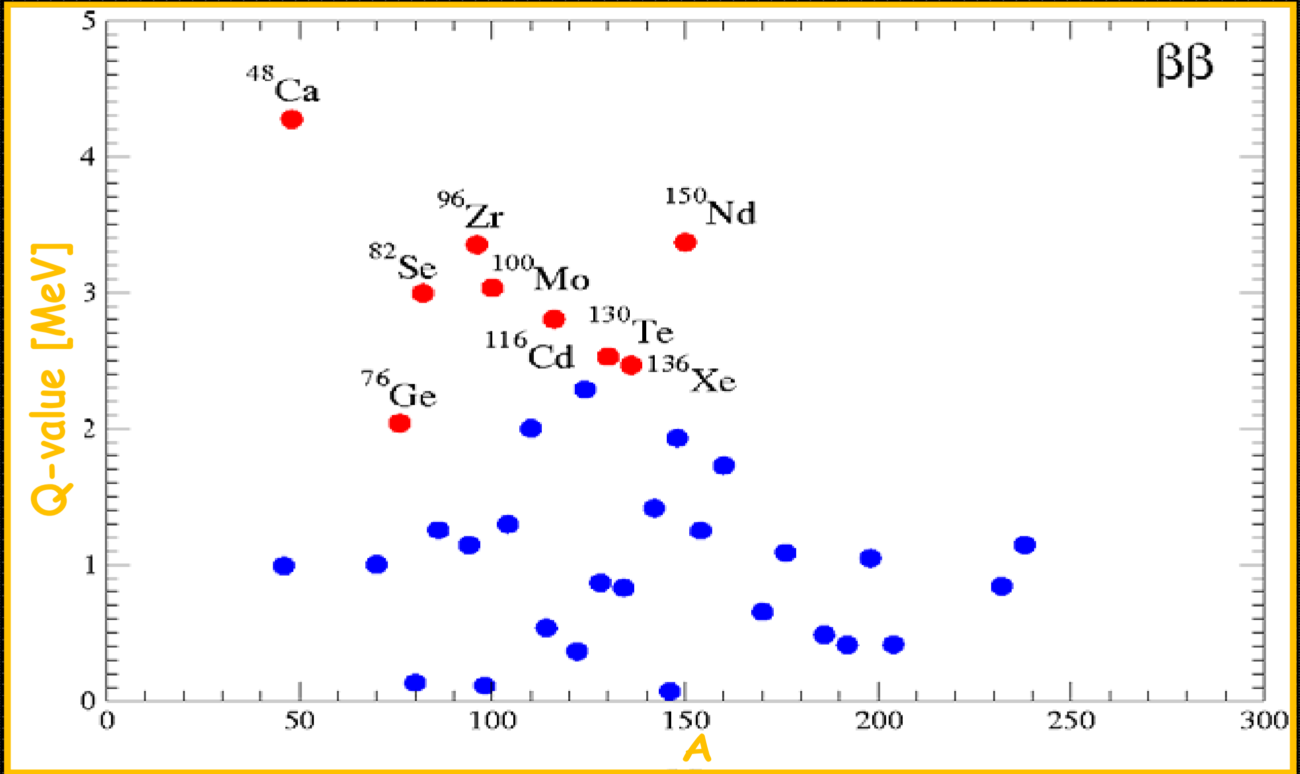
III1) Basics

Need to forbid single β decay:



- $E_{\text{Bindung}} = E_{\text{Volumen}} - E_{\text{Oberfläche}} - E_{\text{Coulomb}} - E_{\text{Symmetrie}} \pm E_{\text{Paarbildung}}$ $\Rightarrow$ even/even $\rightarrow$ even/even
- either direct ($0\nu\beta\beta$) or two simultaneous decays with virtual (energetically forbidden) intermediate state ($2\nu\beta\beta$)

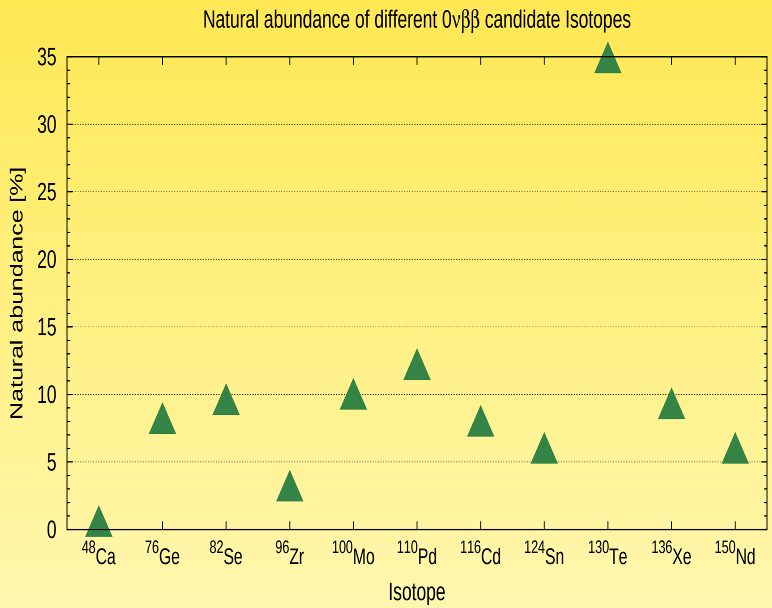
How many nuclei in this condition?

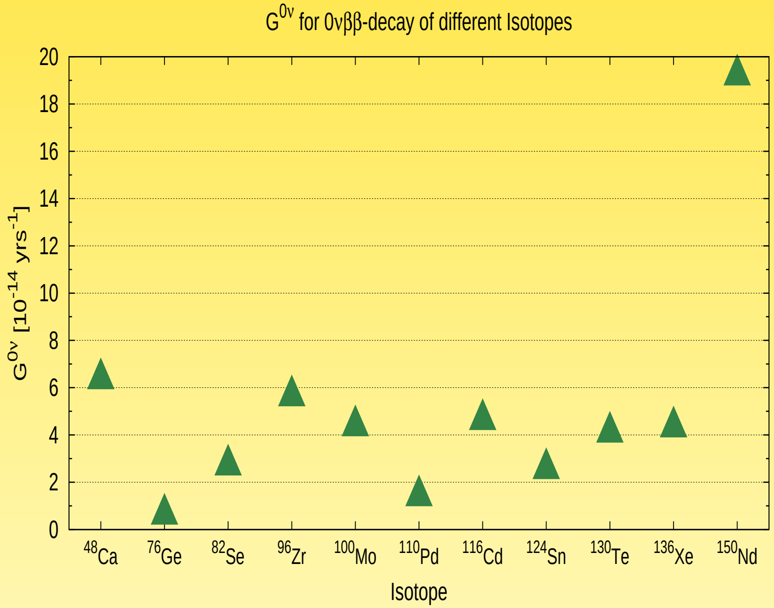


Slide by A. Giuliani

- 35 candidate isotopes
- 9 are interesting: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd
- Q-value vs. natural abundance vs. reasonably priced enrichment vs. association with a well controlled experimental technique vs....

  $\Rightarrow$ no superisotope





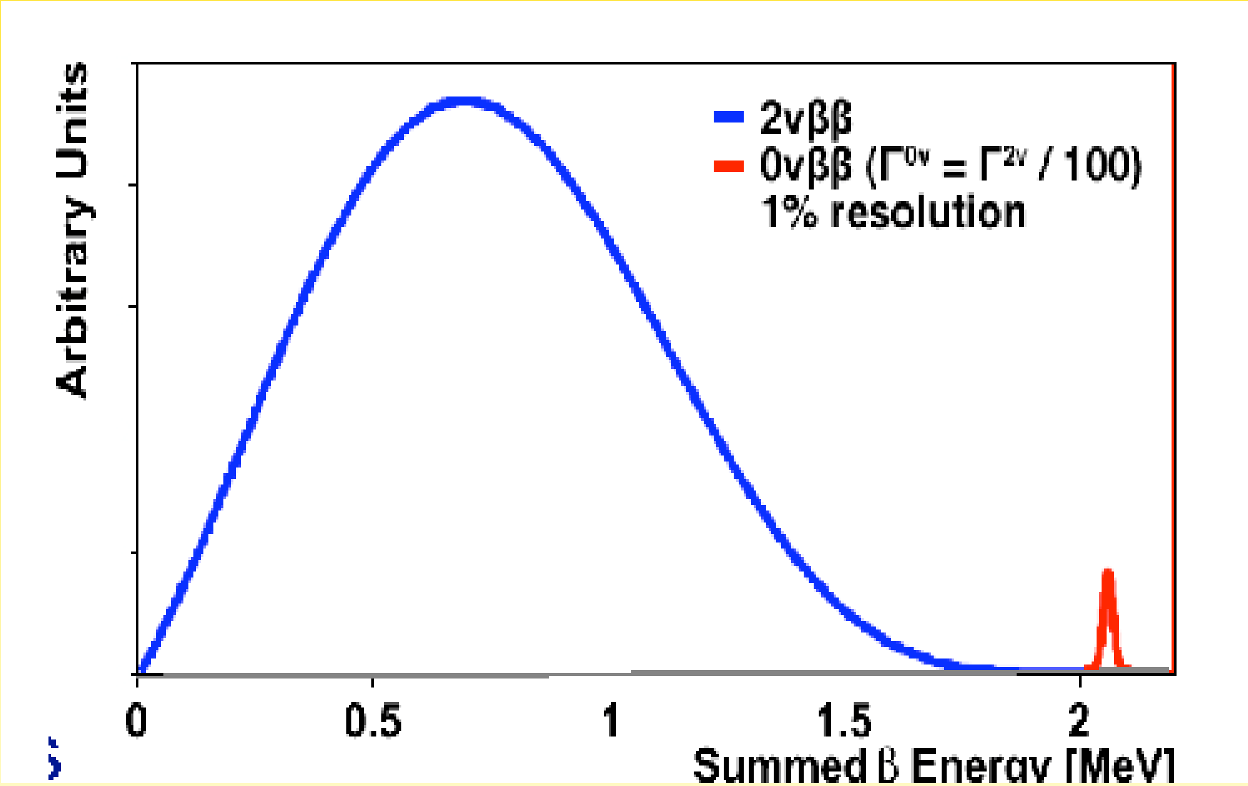
$$T_{1/2}^{0\nu} \propto 1/a \qquad T_{1/2}^{0\nu} \propto Q^{-5}$$

Isotope	G [10^{-14} yrs^{-1}]	Q [keV]	nat. abund. [%]
^{48}Ca	6.35	4273.7	0.187
^{76}Ge	0.623	2039.1	7.8
^{82}Se	2.70	2995.5	9.2
^{96}Zr	5.63	3347.7	2.8
^{100}Mo	4.36	3035.0	9.6
^{110}Pd	1.40	2004.0	11.8
^{116}Cd	4.62	2809.1	7.6
^{124}Sn	2.55	2287.7	5.6
^{130}Te	4.09	2530.3	34.5
^{136}Xe	4.31	2461.9	8.9
^{150}Nd	19.2	3367.3	5.6

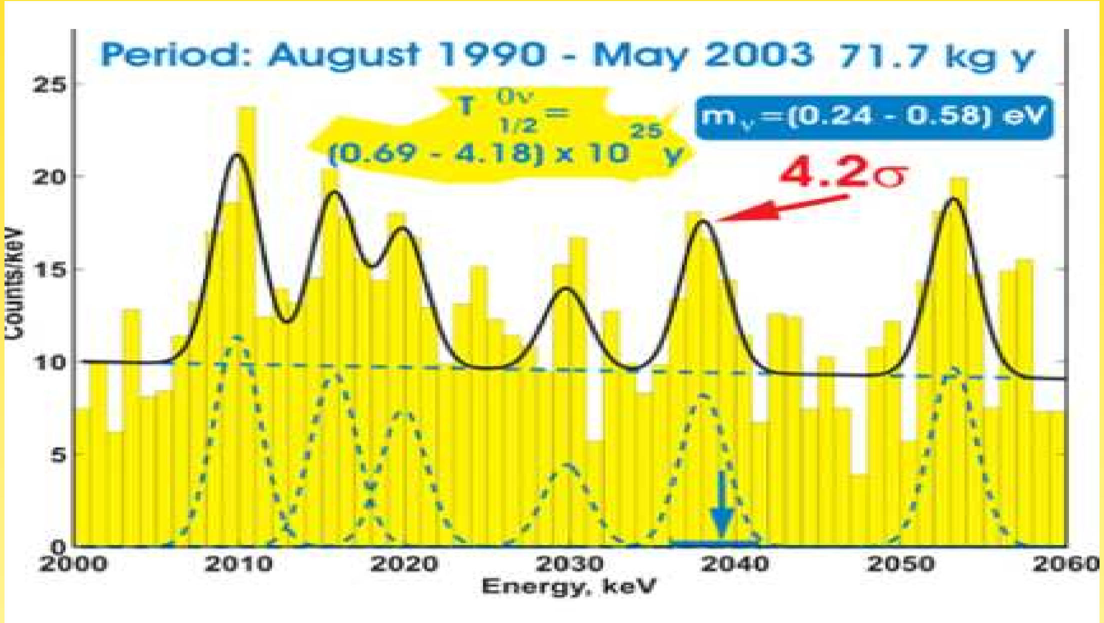
Most mechanisms: $G_x \propto Q^5$

Experimental Aspects

- experimental signature: sum of electron energies $= Q$
  (plus: 2 electrons and daughter isotope)
- background of $2\nu\beta\beta \leftrightarrow$ resolution



if claimed, typical spectrum will look like:



⇒ **first reason for multi-isotope determination**

Experimental Aspects

number of events (measuring time $t \ll T^{0\nu}_{1/2}$ life-time):

$$N = \ln 2 \, a \, M \, t \, N_A \, (T^{0\nu}_{1/2})^{-1}$$

with

- a is abundance of isotope
- M is used mass
- t is time of measurement
- N_A is Avogadro's number

Experimental Aspects

Number of events (measuring time $t \ll T^{0\nu}_{1/2}$ life-time):

$$N = \ln 2 \, a \, M \, t \, N_A \, (T^{0\nu}_{1/2})^{-1}$$

suppose there is no background:

- if you want 10^{26} yrs you need 10^{26} atoms
- 10^{26} atoms are 10^3 mols
- 10^3 mols are 100 kg

From now on you can only loose: efficiency, background, natural abundance,...

Experimental Aspects

$$(T^{0\nu}_{1/2})^{-1} \propto \begin{cases} a\,M\,\varepsilon\,t & \text{without background} \\ a\,\varepsilon\sqrt{\dfrac{M\,t}{B\,\Delta E}} & \text{with background} \end{cases}$$

with

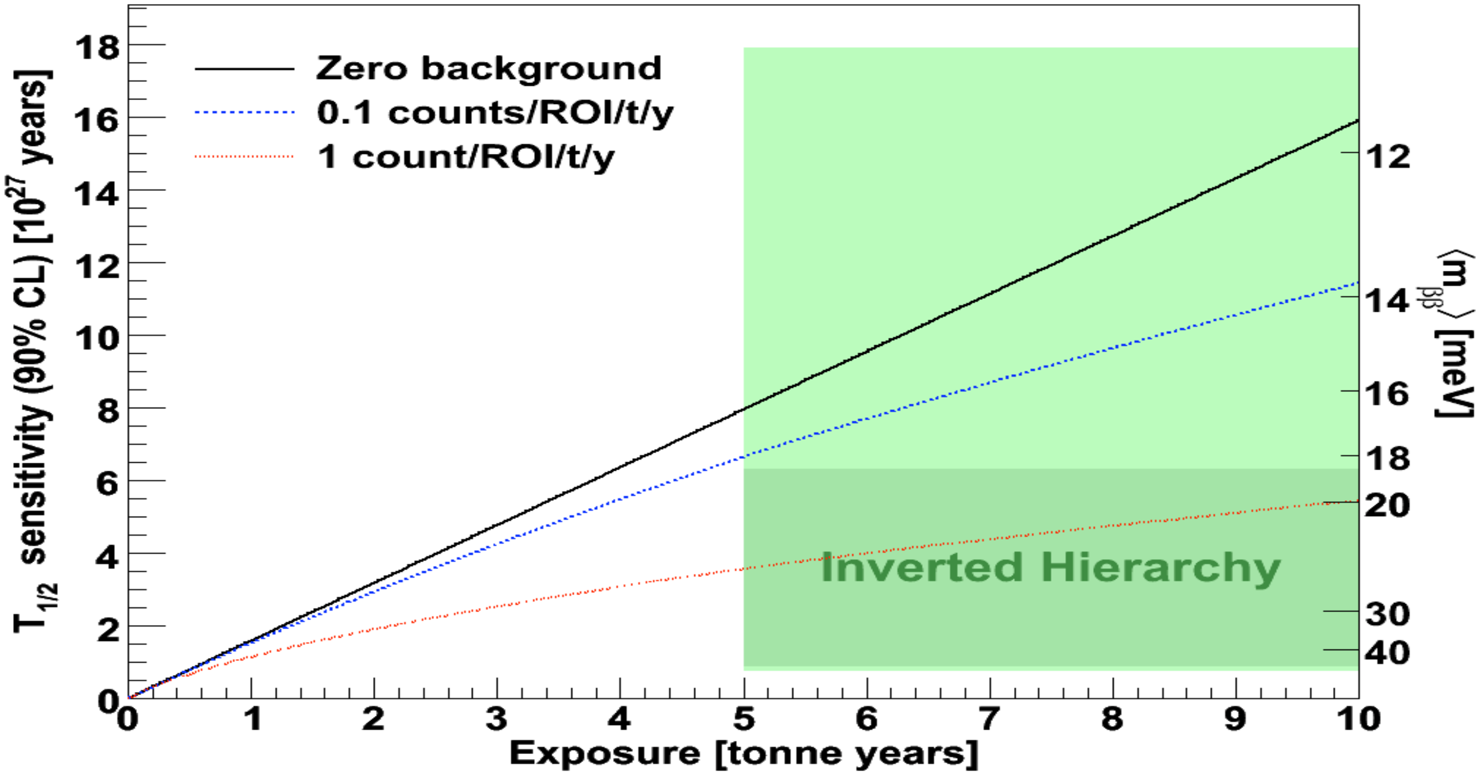
- B is background index in counts/(keV kg yr)
- ΔE is energy resolution
- ϵ is efficiency
- $(T^{0\nu}_{1/2})^{-1} \propto$ (particle physics)^2

Note: factor 2 in particle physics is combined factor of 16 in $M \times t \times B \times \Delta E$

Sensitivity and backgrounds

1-tonne ^{76}Ge Example

$T_{1/2}^{0\nu} = \ln(2)N\varepsilon t/UL(B)$



Neutrinoless Double Beta Decay

Goldhaber-Grodzins-Sunyar Celebration

slide by J.F. Wilkerson

Interpretation of Experiments

Master formula:

$$\Gamma^{0\nu} = G_x(Q,Z)\,|\mathcal{M}_x(A,Z)\,\eta_x|^2$$

- $G_x(Q,Z)$: phase space factor
- $\mathcal{M}_x(A,Z)$: nuclear physics
- η_x: particle physics

Interpretation of Experiments

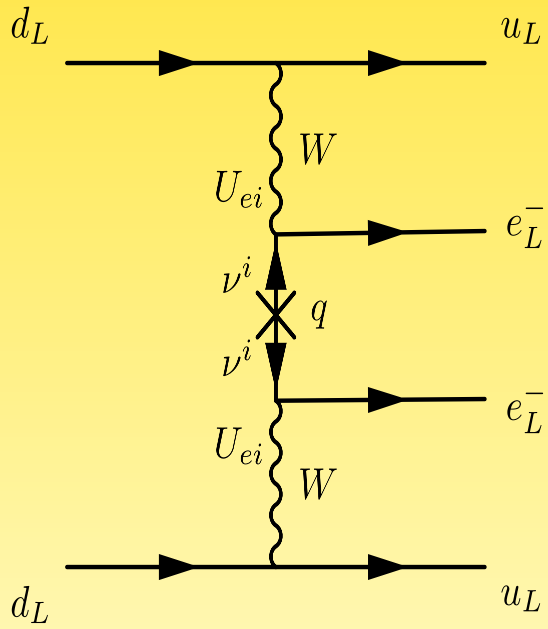
Master formula:

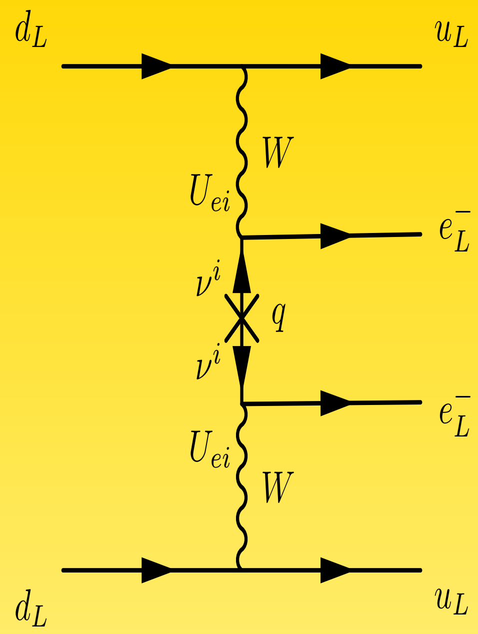
$$\Gamma^{0\nu} = G_x(Q,Z)\,|\mathcal{M}_x(A,Z)\,\eta_x|^2$$

- $G_x(Q,Z)$: phase space factor; calculable
- $\mathcal{M}_x(A,Z)$: nuclear physics; problematic
- η_x: particle physics; interesting

Standard Interpretation

Neutrinoless Double Beta Decay is mediated by light and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to $0\nu\beta\beta$ give negligible or no contribution





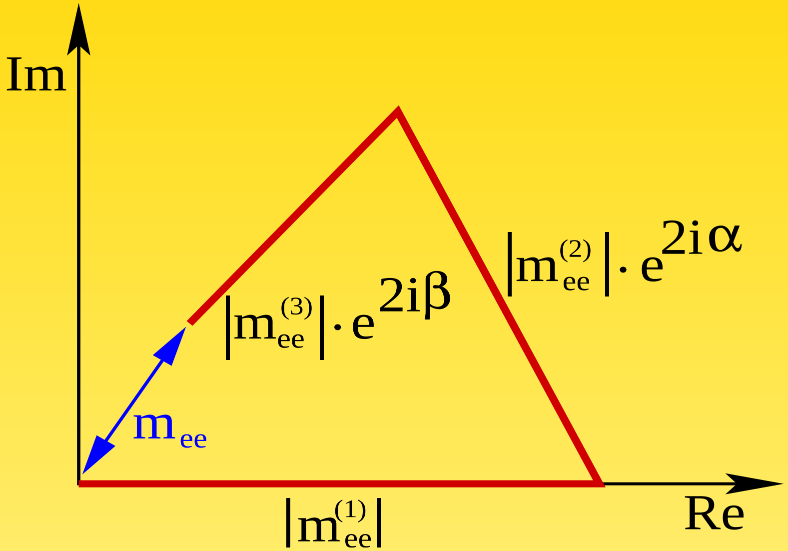
- U_{ei}^2 from charged current
- m_i/E_i from spin-flip and *if neutrinos are Majorana particles*

amplitude proportional to coherent sum **(“effective mass”)**

$$|m_{ee}| = \left|\sum U_{ei}^2 \, m_i\right|$$

$m/E \simeq$ eV/100 MeV is tiny: only N_A can save the day!

The effective mass



amplitude proportional to coherent sum (<u>"effective mass"</u>):

$$|m_{ee}| \equiv \left|\sum U_{ei}^2 \, m_i\right| = \left||U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, m_2 \, e^{2i\alpha} + |U_{e3}|^2 \, m_3 \, e^{2i\beta}\right|$$

$$= f\left(\theta_{12}, |U_{e3}|, m_i, \mathrm{sgn}(\Delta m_{\mathrm{A}}^2), \alpha, \beta\right)$$

7 out of 9 parameters of neutrino physics!

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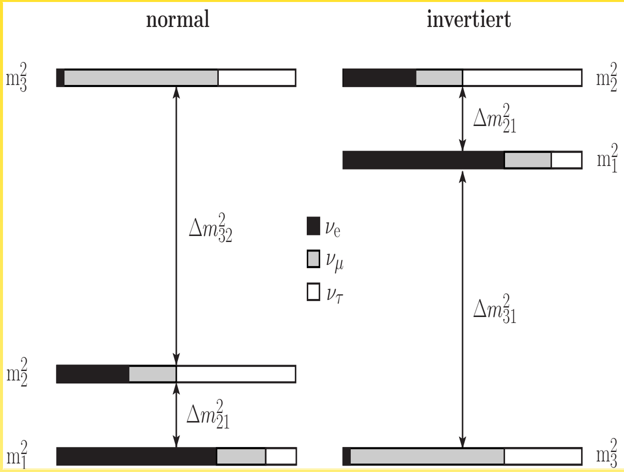
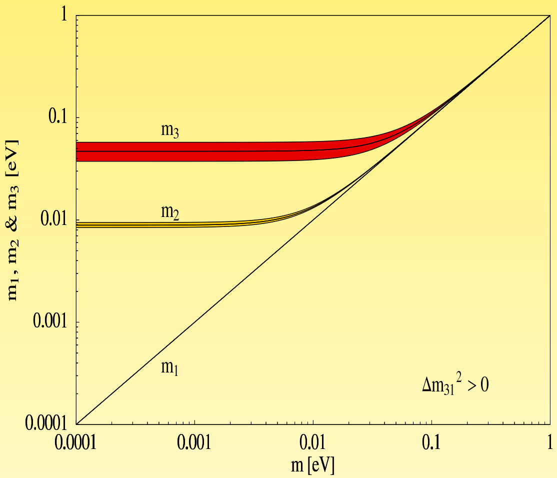
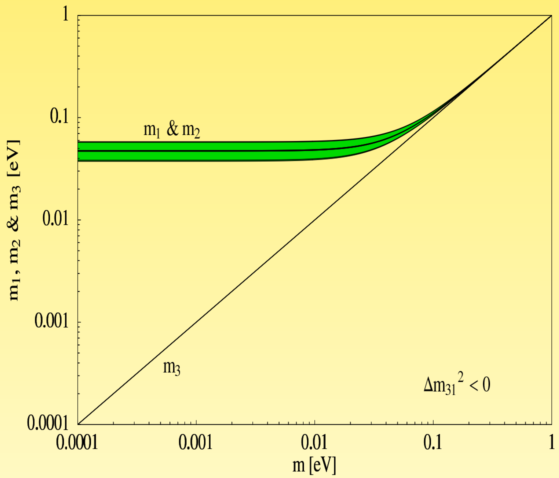
II2) Implications for and from neutrino physics

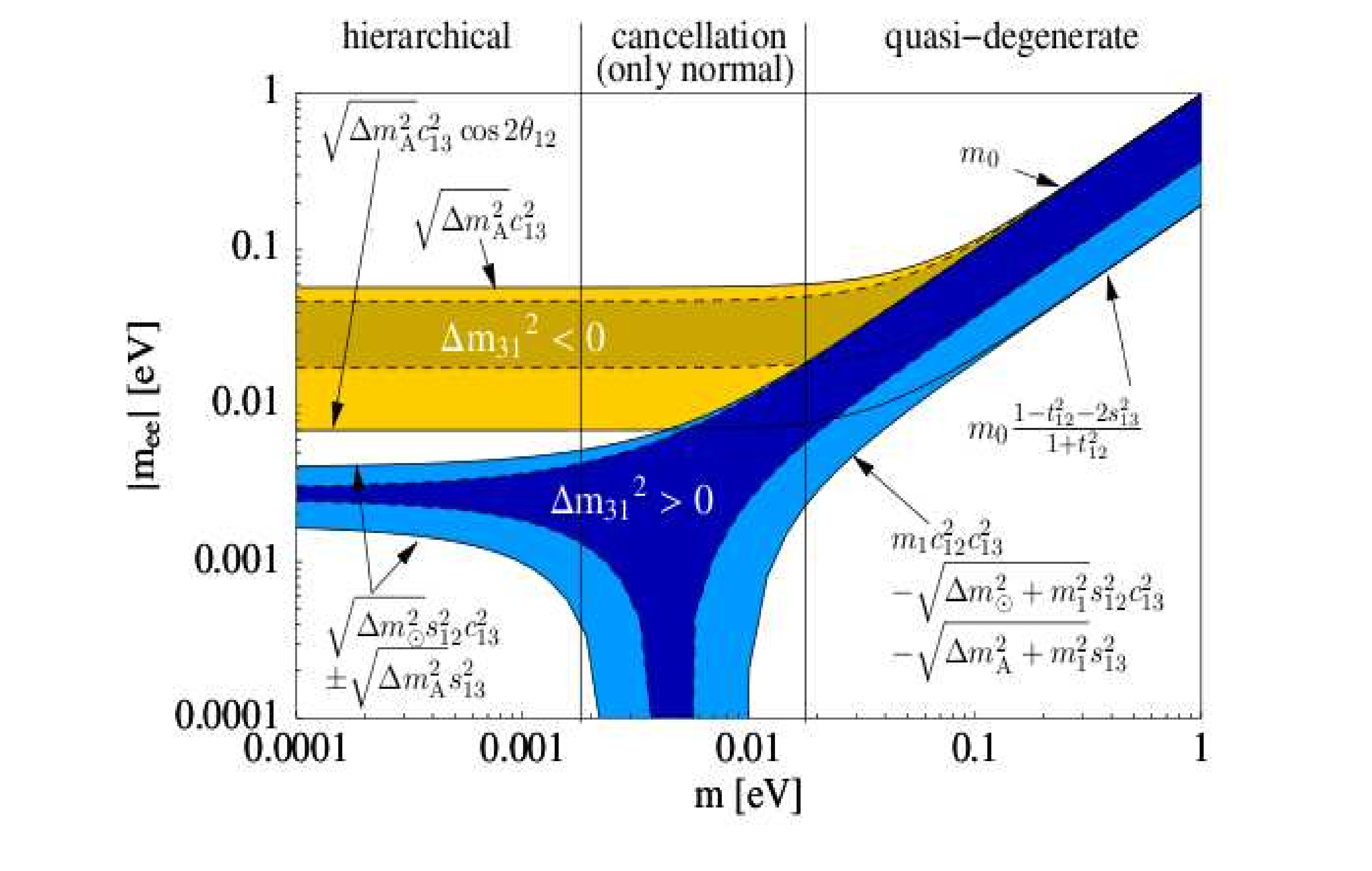
$|m_{ee}| = \left|\sum U_{ei}^2 \, m_i\right|$: fix known things, vary known unknown things, assume no unknown unknowns are present:





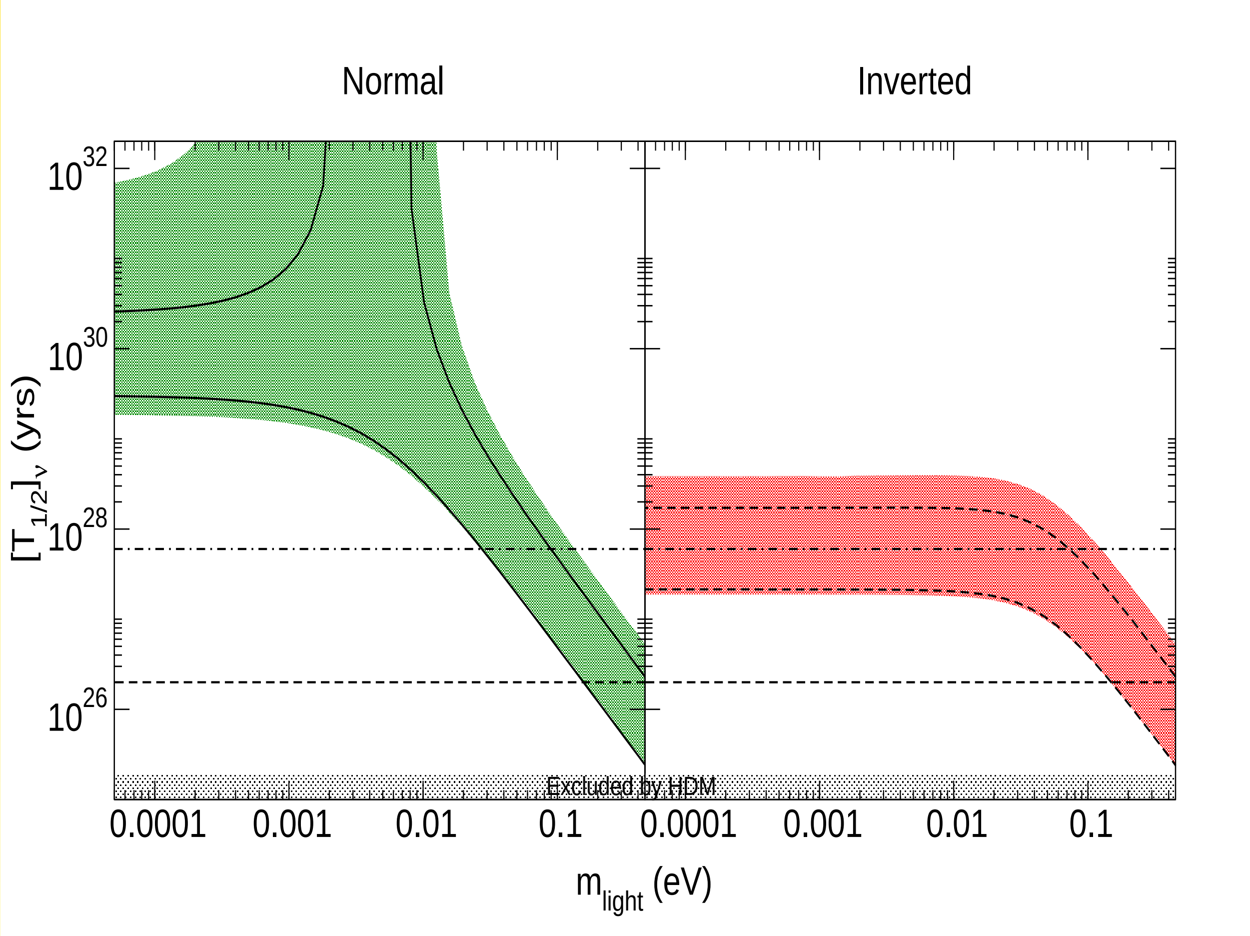


The usual plot



The usual plot: the other way around

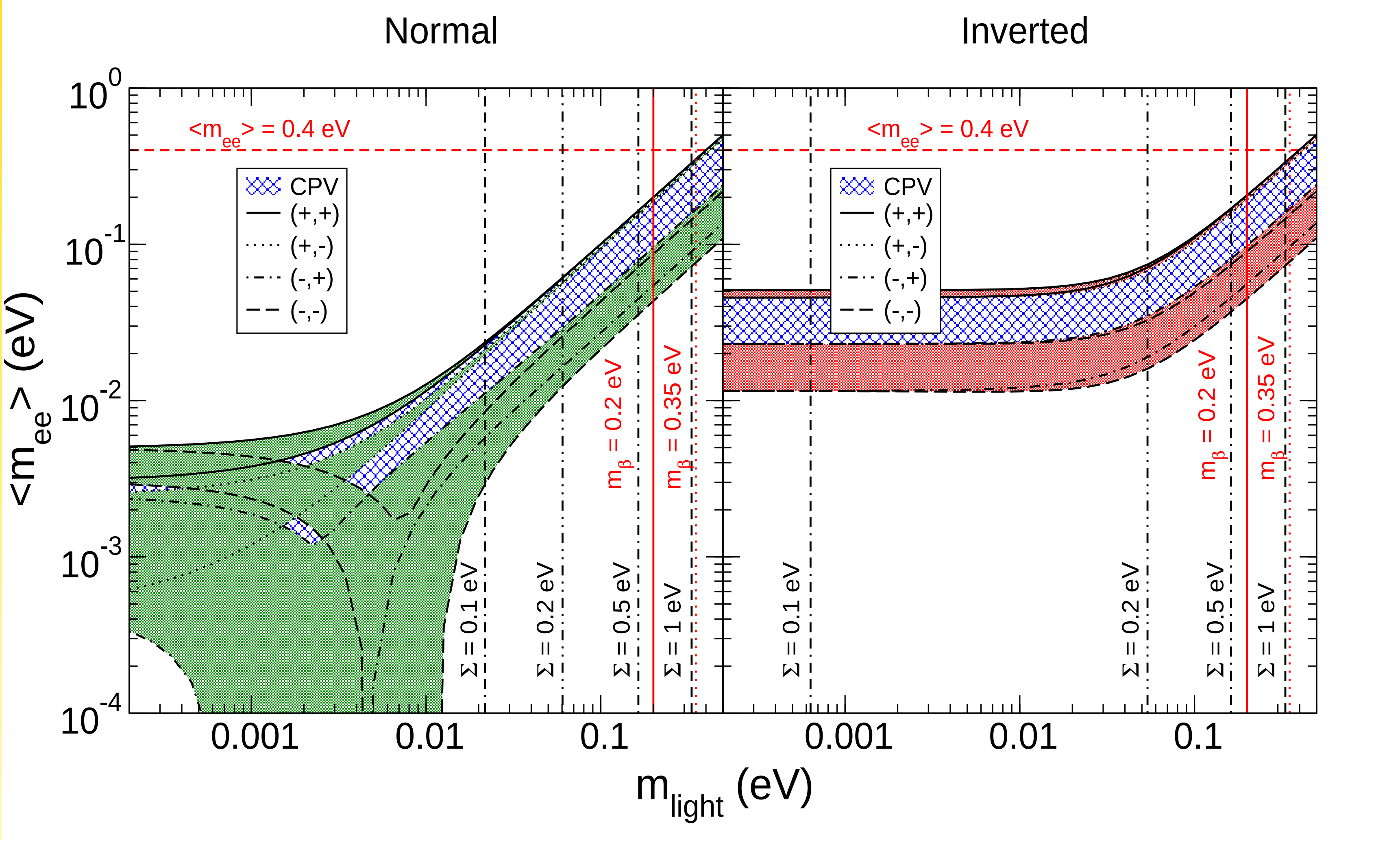
(life-time instead of $|m_{ee}|$)



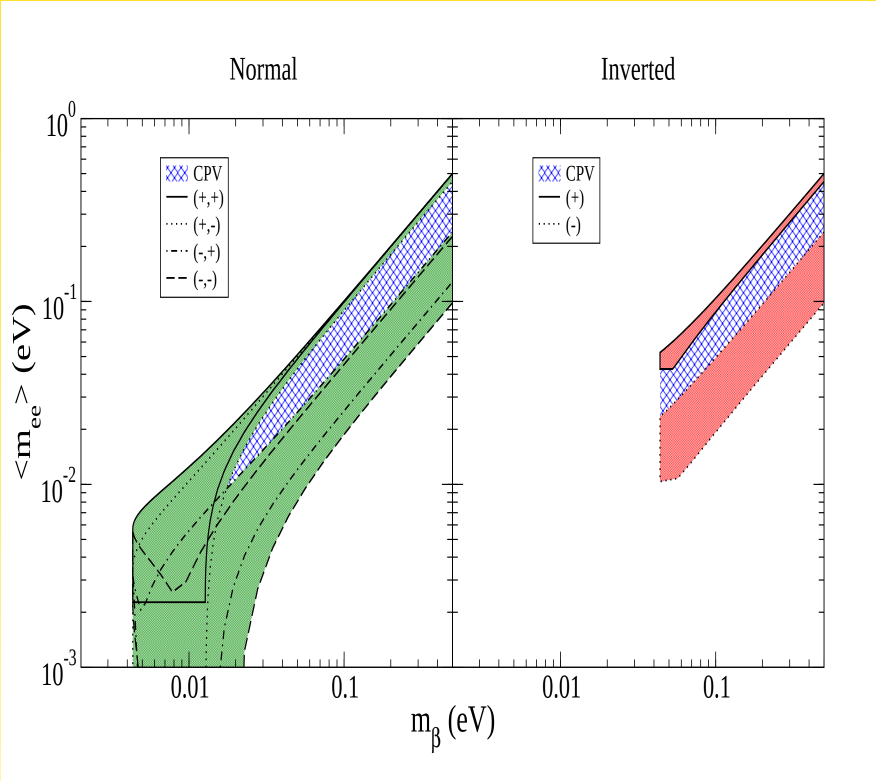
Which mass ordering with which life-time?

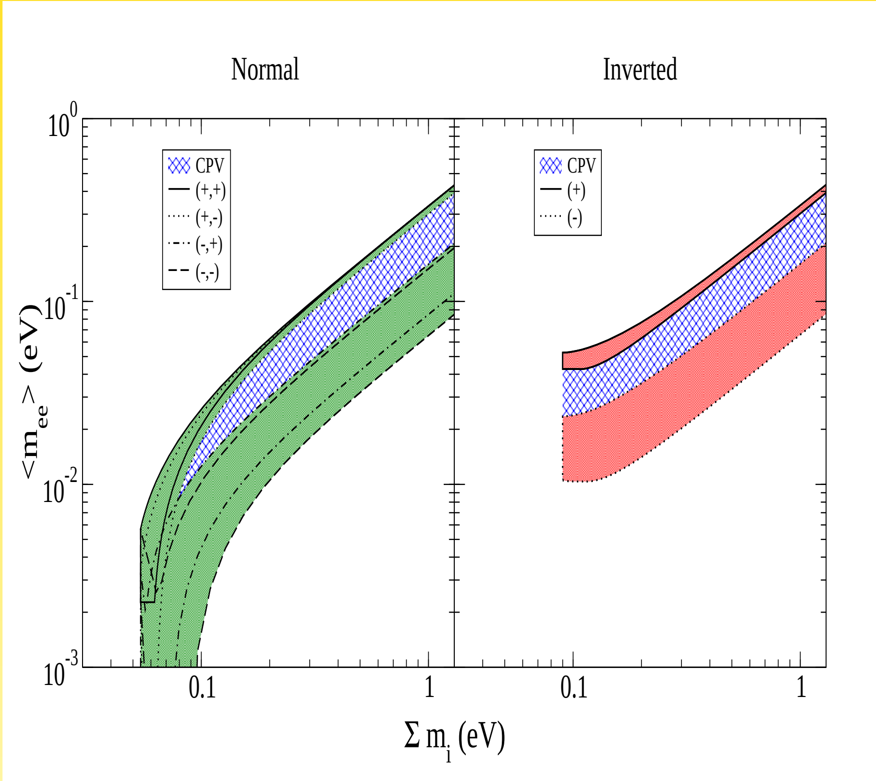
	Σ	m_β	$\lvert m_{ee} \rvert$
NH	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_\odot + \lvert U_{e3}\rvert^2 \Delta m^2_{\rm A}}$	$\left\lvert \sqrt{\Delta m^2_\odot} + \lvert U_{e3}\rvert^2 \sqrt{\Delta m^2_{\rm A}}\, e^{2i(\alpha-\beta)} \right\rvert$
	$\simeq 0.05$ eV	$\simeq 0.01$ eV	~ 0.003 eV $\Rightarrow T^{0\nu}_{1/2} \gtrsim 10^{28-29}$ yrs
IH	$2\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}\sqrt{1-\sin^2 2\theta_{12}\sin^2\alpha}$
	$\simeq 0.1$ eV	$\simeq 0.05$ eV	~ 0.03 eV $\Rightarrow T^{0\nu}_{1/2} \gtrsim 10^{26-27}$ yrs
QD	$3 m_0$	m_0	$m_0\sqrt{1-\sin^2 2\theta_{12}\sin^2\alpha}$
			$\gtrsim 0.1$ eV $\Rightarrow T^{0\nu}_{1/2} \gtrsim 10^{25-26}$ yrs

The usual plot: include other neutrino mass approaches

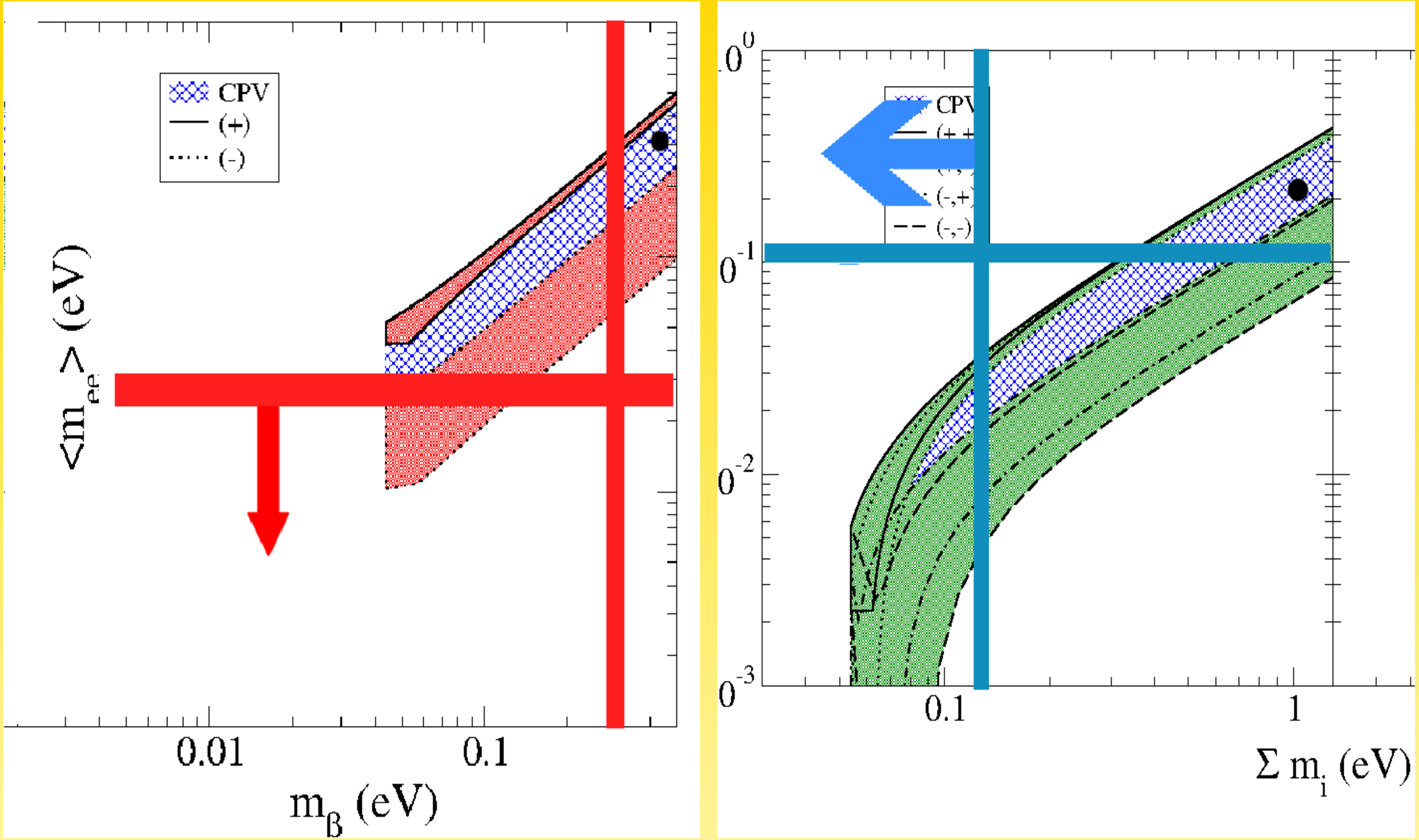


Plot against other observables





Complementarity of $|m_{ee}| = U_{ei}^2\, m_i$, $m_\beta = \sqrt{|U_{ei}|^2\, m_i^2}$ and $\Sigma = \sum m_i$



CP violation!

Dirac neutrinos!

something else does $0\nu\beta\beta$!

Neutrino Mass

$$m(\text{heaviest}) \gtrsim \sqrt{|m_3^2 - m_1^2|} \simeq 0.05 \text{ eV}$$

3 **complementary** methods to measure neutrino mass:

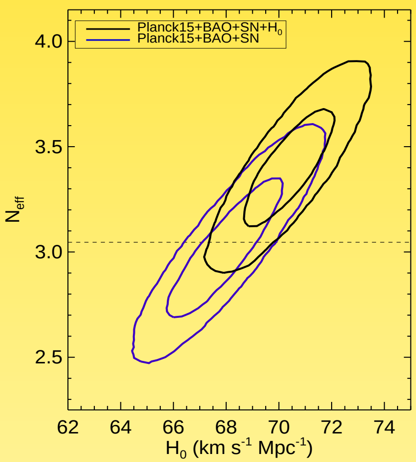
Method	observable	now [eV]	near [eV]	far [eV]	pro	con
Kurie	$\sqrt{\sum \lvert U_{ei}\rvert^2 m_i^2}$	2.3	0.2	0.1	model-indep.; theo. clean	final?; worst
Cosmo.	$\sum m_i$	0.5	0.2	0.05	best; NH/IH	systemat.; model-dep.
$0\nu\beta\beta$	$\lvert\sum U_{ei}^2 m_i\rvert$	0.3	0.1	0.05	fundament.; NH/IH	model-dep.; theo. dirty

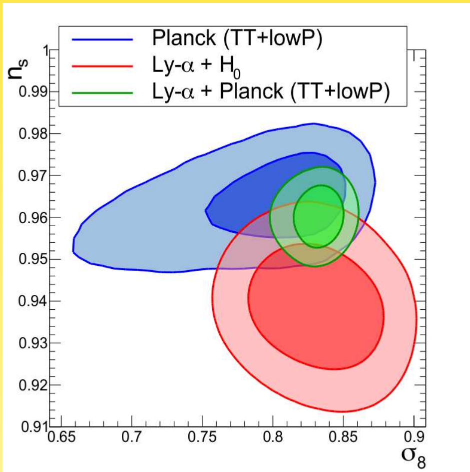
Cosmological Limits

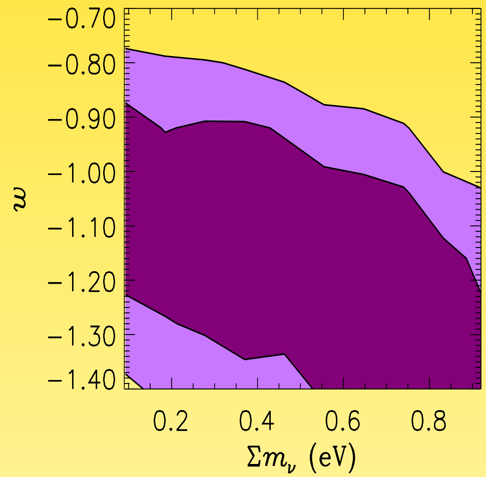
stacking more and more data sets on top of each other, limits become stronger

including more and more parameters, limits become weaker

needed to break degeneracies, but induces systematic issues







`Riess` *et al.*, `1604.01424`

`Palanque-Delabrouille` *et al.*, `1506.05976`

`Hannestad, PRL` **95**

most extreme `(1410.7244)`: at 1σ: $\Sigma m_\nu \leq 0.08$ eV, disfavors inverted ordering...

2 kinds of neutrino masses

1) ee-element of mass matrix

$$\sqrt{\frac{1}{T_{1/2}^{0\nu}}} \propto |(m_\nu)_{ee}| \text{ with } (m_\nu)_{ee} = \frac{h_{ee}\, v^2}{\Lambda} \text{ in } \mathcal{L}_{\rm eff} = \frac{1}{2}\, \frac{h_{\alpha\beta}}{\Lambda} \overline{L_\alpha}\, \tilde{\Phi}\, \tilde{\Phi}^T\, L_\beta^c$$

direct probe of fundamental object in low energy Lagrangian!

2) neutrino mass scale: QD neutrinos

$$|m_{ee}|^{\rm QD} = m_0 \left| c_{12}^2\, c_{13}^2 + s_{12}^2\, c_{13}^2\, e^{2i\alpha} + s_{13}^2\, e^{2i\beta} \right|$$

$$\Rightarrow m_0 \le |m_{ee}|^{\rm exp}_{\rm min} \frac{1 + \tan^2\theta_{12}}{1 - \tan^2\theta_{12} - 2\,|U_{e3}|^2} \simeq 3\,|m_{ee}|^{\rm exp}_{\rm min} \simeq {\rm eV}$$

same order as Mainz/Troitsk!

Alternative processes

$$(A,Z) \to (A,Z+2)^* + 2\,e^- \qquad (0\nu\beta\beta)^*$$

$$(A,Z) \to (A,Z-2) + 2\,e^+ \qquad (0\nu\beta^+\beta^+)$$

$$e_b^- + (A,Z) \to (A,Z-2) + e^+ \qquad (0\nu\beta^+\mathrm{EC})$$

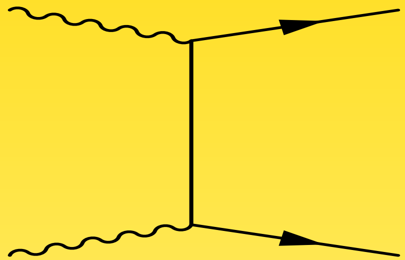
$$2\,e_b^- + (A,Z) \to (A,Z-2)^* \qquad (0\nu\mathrm{ECEC})$$

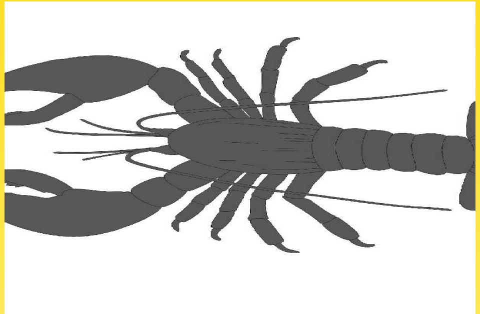
depend on same particle physics parameters, but more difficult to realize/test

BUT: ratio to $0\nu\beta\beta$ is test of NME calculation and mechanism

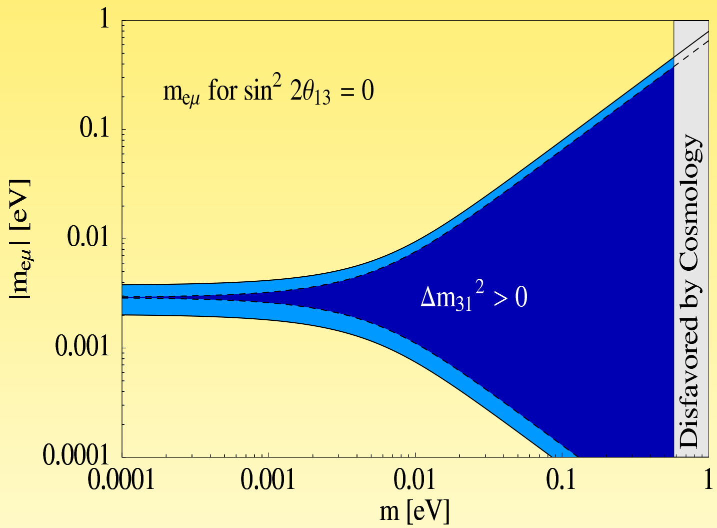
Alternative processes

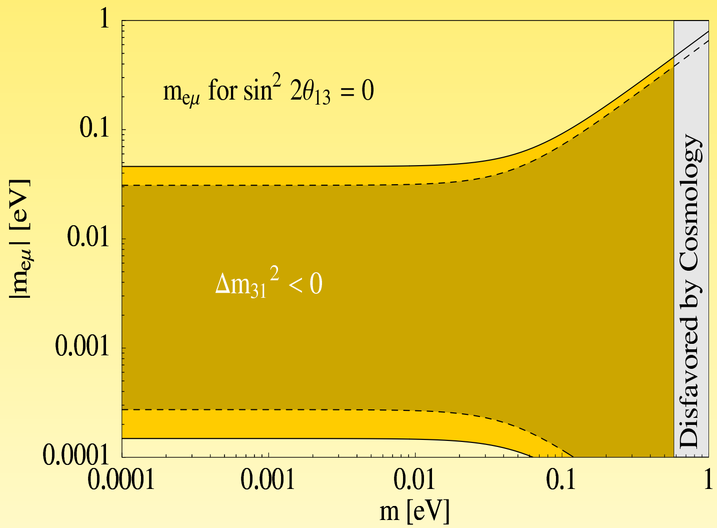
the lobster:





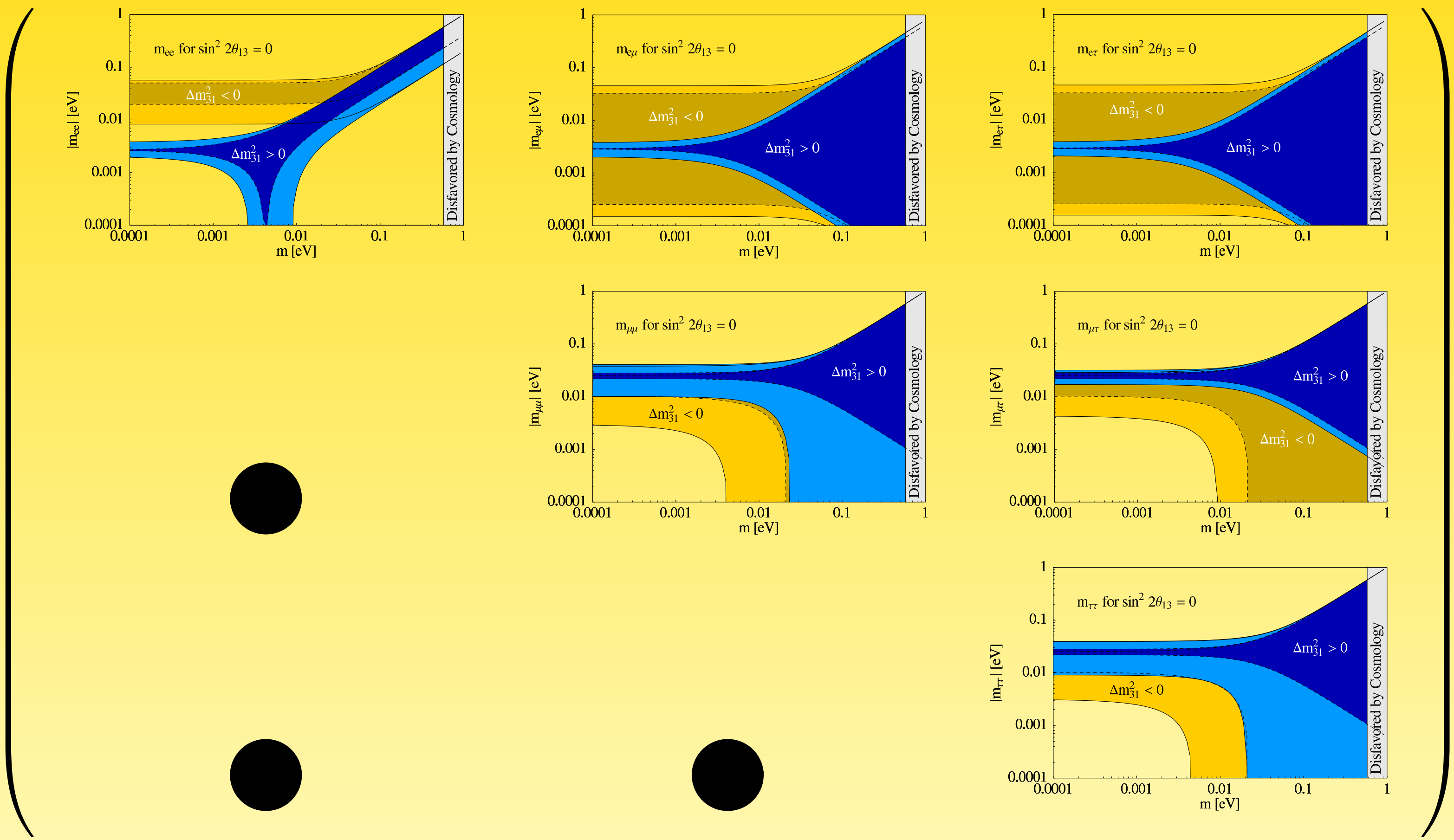
$$\mathrm{BR}(K^+ \to \pi^- \, e^+ \, \mu^+) \propto |m_{e\mu}|^2 = \left| \sum U_{ei} \, U_{\mu i} \, m_i \right|^2 \sim 10^{-30} \left(\frac{|m_{e\mu}|}{\mathrm{eV}} \right)^2$$





Reconstructing m_ν

$$m_\nu =$$



Recent Results

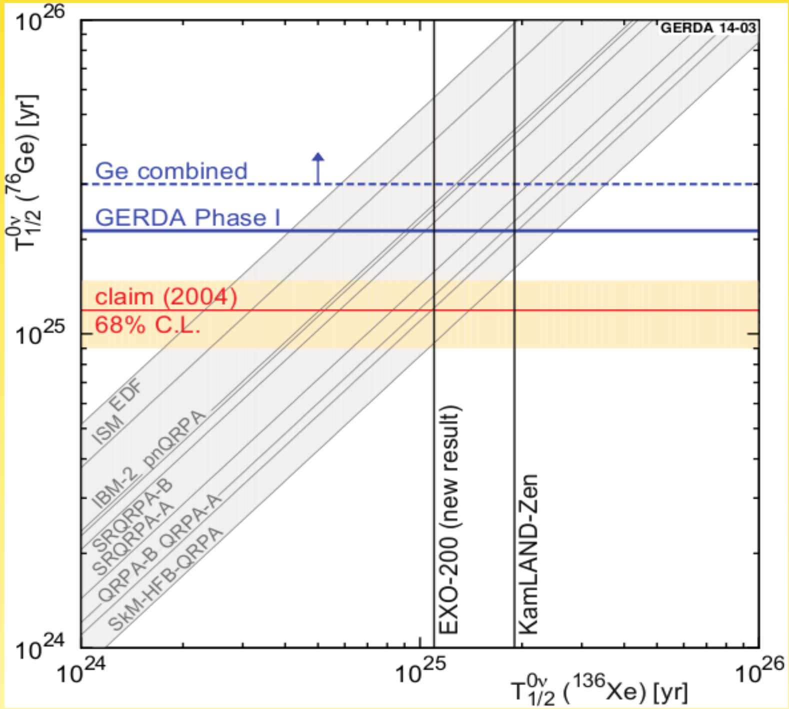
- ^{76}Ge:
  - GERDA: $T_{1/2} > 2.1 \times 10^{25}$ yrs
  - GERDA + IGEX + HDM: $T_{1/2} > 3.0 \times 10^{25}$ yrs
- ^{136}Xe:
  - EXO-200: $T_{1/2} > 1.1 \times 10^{25}$ yrs **(first run with less exposure:** $T_{1/2} > 1.6 \times 10^{25}$ **yrs...)**
  - KamLAND-Zen: $T_{1/2} > 2.6 \times 10^{25}$ yrs

Xe-limit is stronger than Ge-limit when:

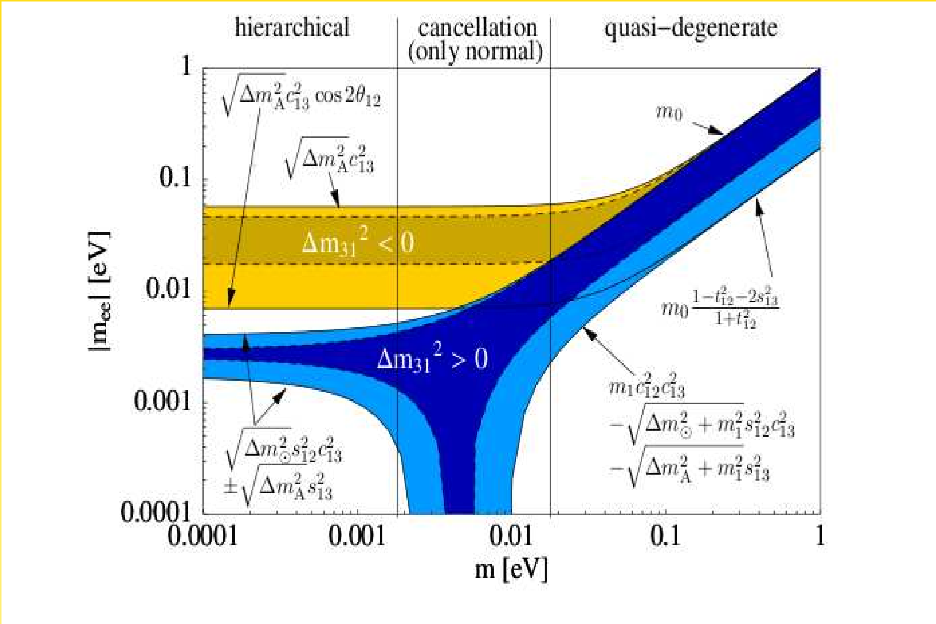
$$T_{\rm Xe} > T_{\rm Ge} \, \frac{G_{\rm Ge}}{G_{\rm Xe}} \left| \frac{\mathcal{M}_{\rm Ge}}{\mathcal{M}_{\rm Xe}} \right|^2 \text{ yrs}$$

Current Limits on $|m_{ee}|$

NME	^{76}Ge		^{136}Xe	
	GERDA	comb	KLZ	comb
EDF(U)	**0.32**	**0.27**	**0.13**	–
ISM(U)	**0.52**	**0.44**	**0.24**	–
IBM-2	**0.27**	**0.23**	**0.16**	–
pnQRPA(U)	**0.28**	**0.24**	**0.17**	–
SRQRPA-A	**0.31**	**0.26**	**0.23**	–
QRPA-A	*0.28*	*0.24*	*0.25*	–
SkM-HFB-QRPA	*0.29*	*0.24*	*0.28*	–



Inverted Ordering



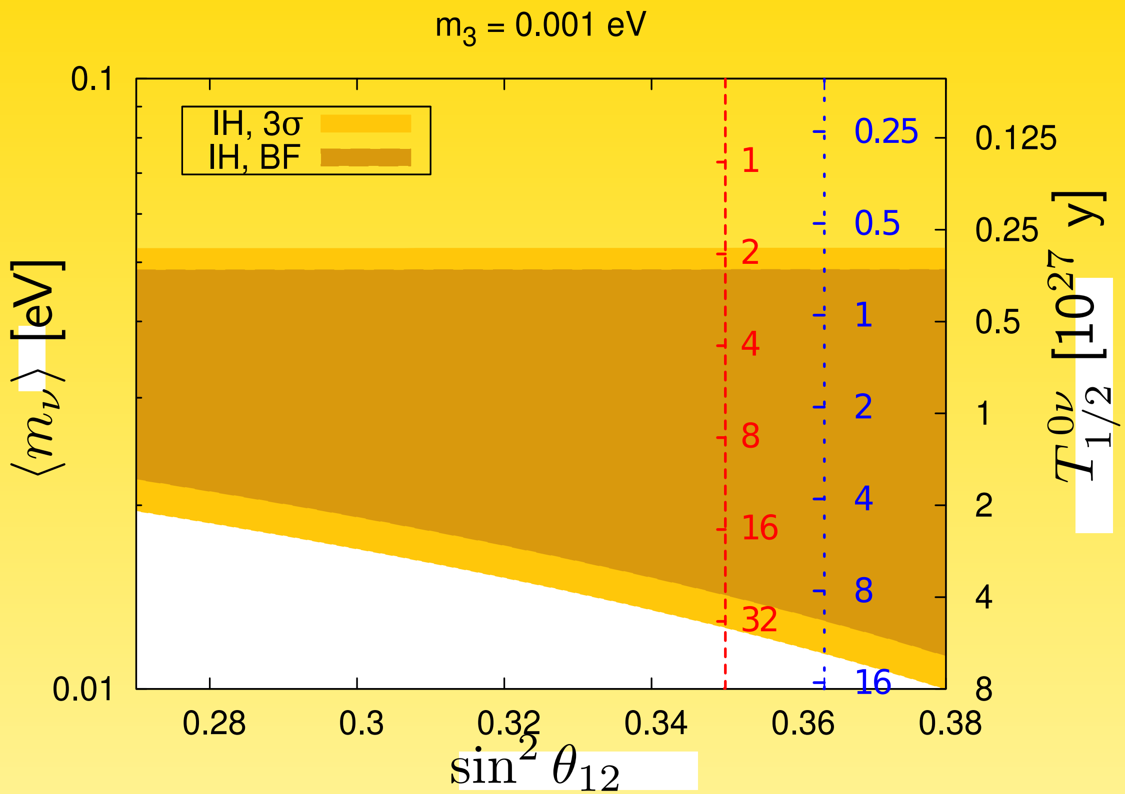
Nature provides 2 scales:

$$|m_{ee}|^{\text{IH}}_{\text{max}} \simeq c_{13}^2 \sqrt{\Delta m_A^2} \quad \text{and} \quad |m_{ee}|^{\text{IH}}_{\text{min}} \simeq c_{13}^2 \sqrt{\Delta m_A^2} \cos 2\theta_{12}$$

requires $\mathcal{O}(10^{26} \ldots 10^{27})$ yrs

is the lower limit $|m_{ee}|^{\text{IH}}_{\text{min}}$ fixed?

Inverted Hierarchy

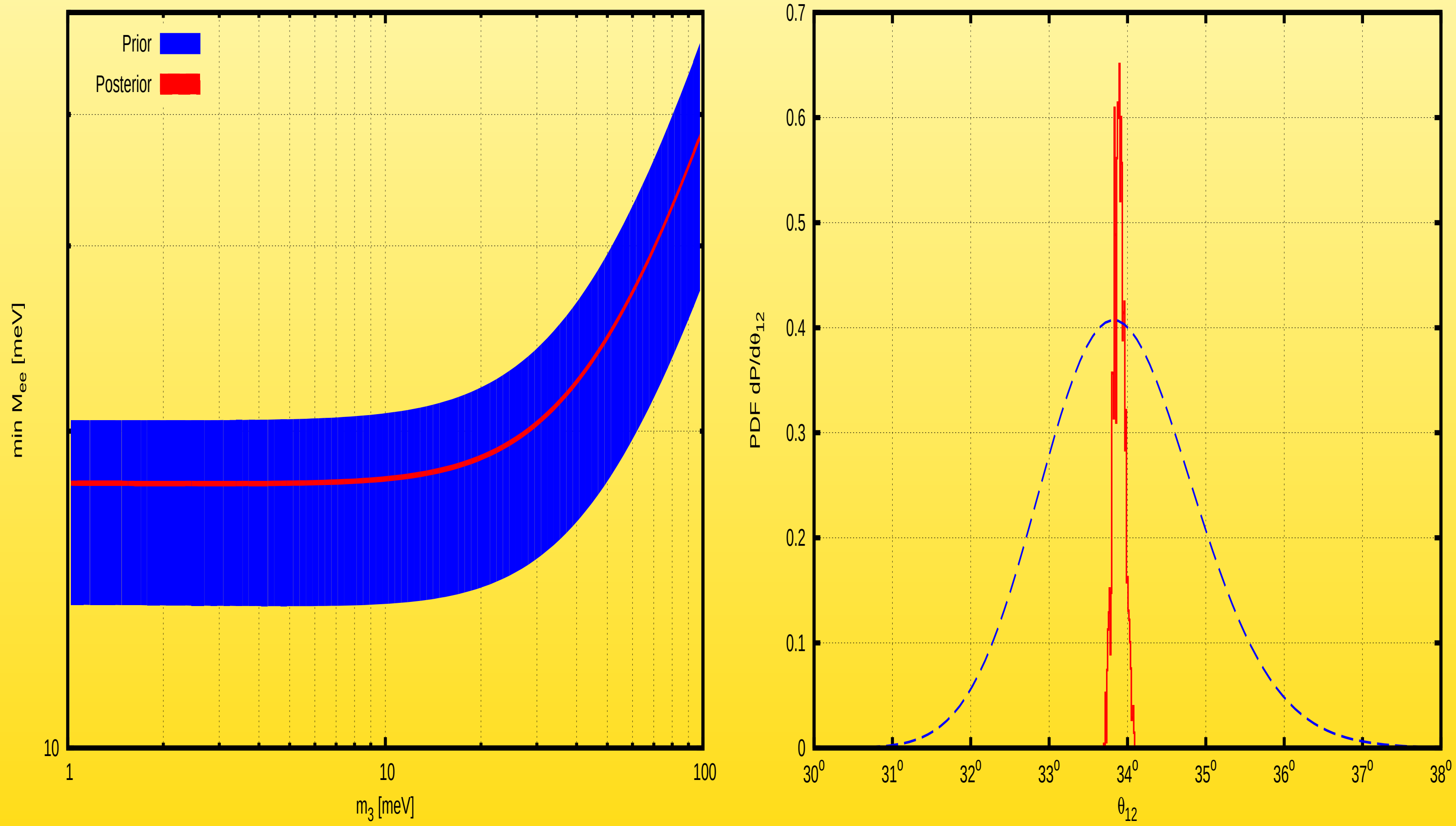


Current 3σ range of $\sin^2\theta_{12}$ gives factor of ~ 2 uncertainty for $|m_{ee}|^{\rm IH}_{\rm min}$

$\Rightarrow$ combined factor of ~ 16 in $M \times t \times B \times \Delta E$

$\Rightarrow$ need precision determination of θ_{12}! $\leftrightarrow$ JUNO

Ge, W.R., PRD **92**; http://nupro.hepforge.org



CP Violation?

Majorana phases: consider IH spectrum

$$|m_{ee}| \propto \left|\cos^2\theta_{12} + e^{2i\alpha}\,\sin^2\theta_{12}\right| = \sqrt{1-\sin^2 2\theta_{12}\,\sin^2\alpha}$$

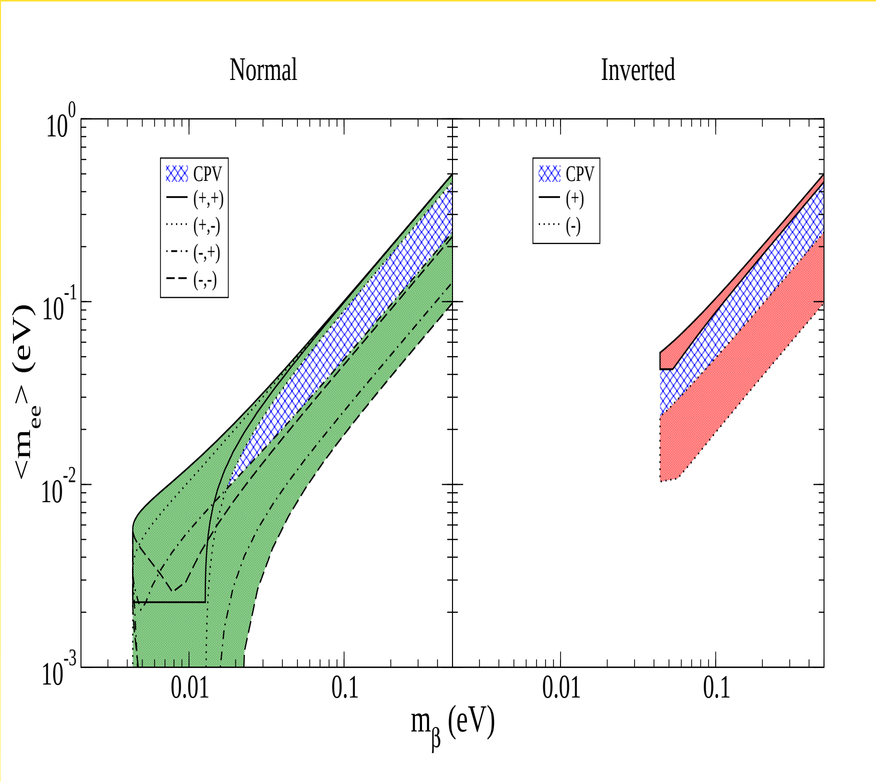
α can be probed if

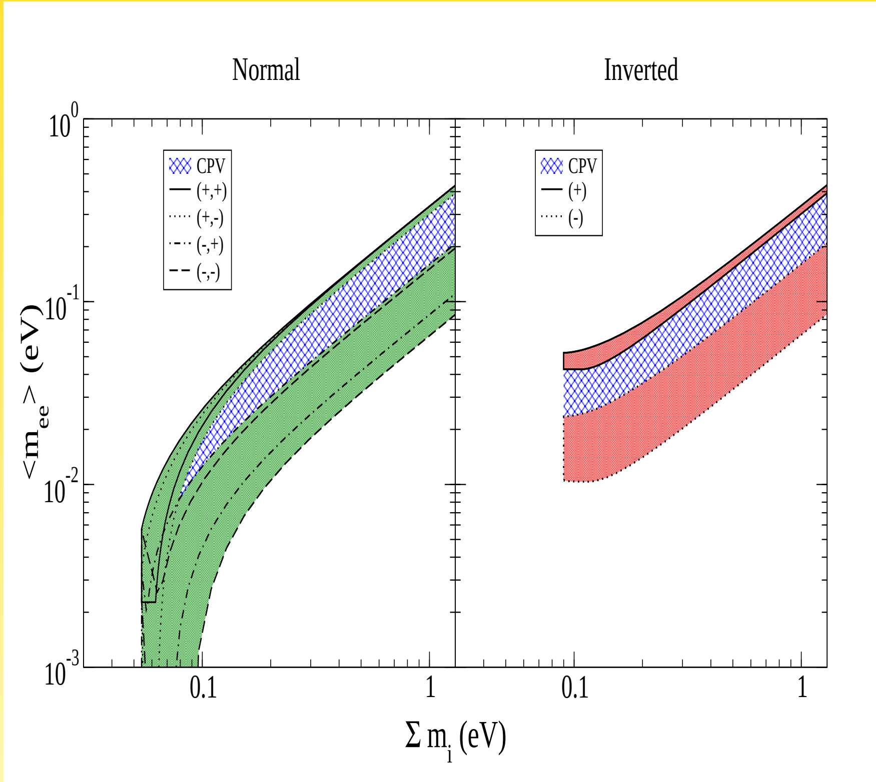
- uncertainties on $|m_{ee}|$ from NME smaller than 2
- $\sigma(|m_{ee}|) \lesssim 15\%$
- $\sigma(\Delta m^2_{\rm A}) \lesssim 10\%$ (IH) or $\sigma(m_0) \lesssim 10\%$ (QD)
- $\sin^2\theta_{12} \gtrsim 0.29$
- $2\alpha \in [\pi/4, 3\pi/4]$ or $[5\pi/4, 7\pi/4]$

Pascoli, Petcov, W.R., PLB **549**

No to "no-go" from Barger *et al.*, PLB **540**

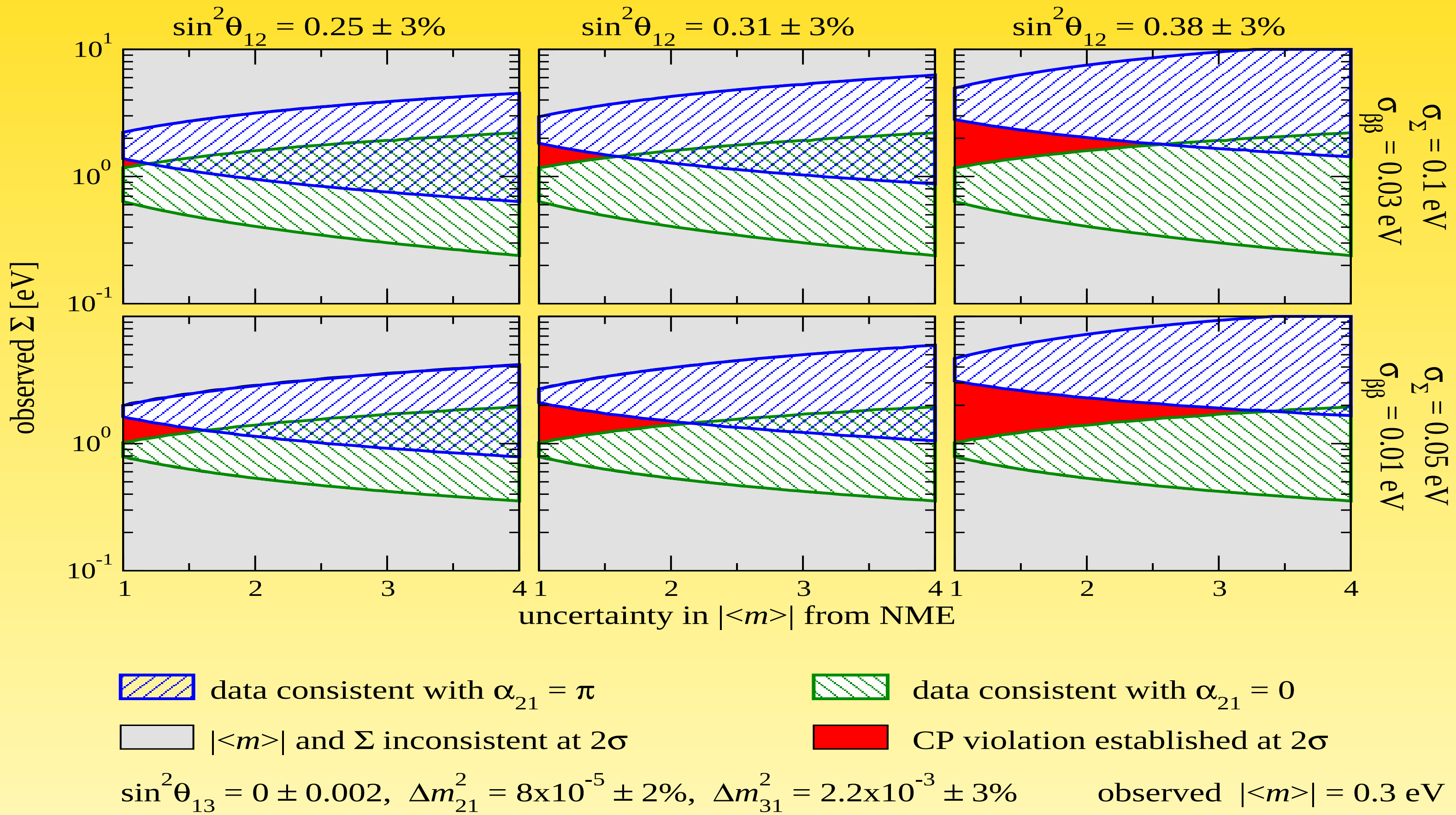
Majorana phases





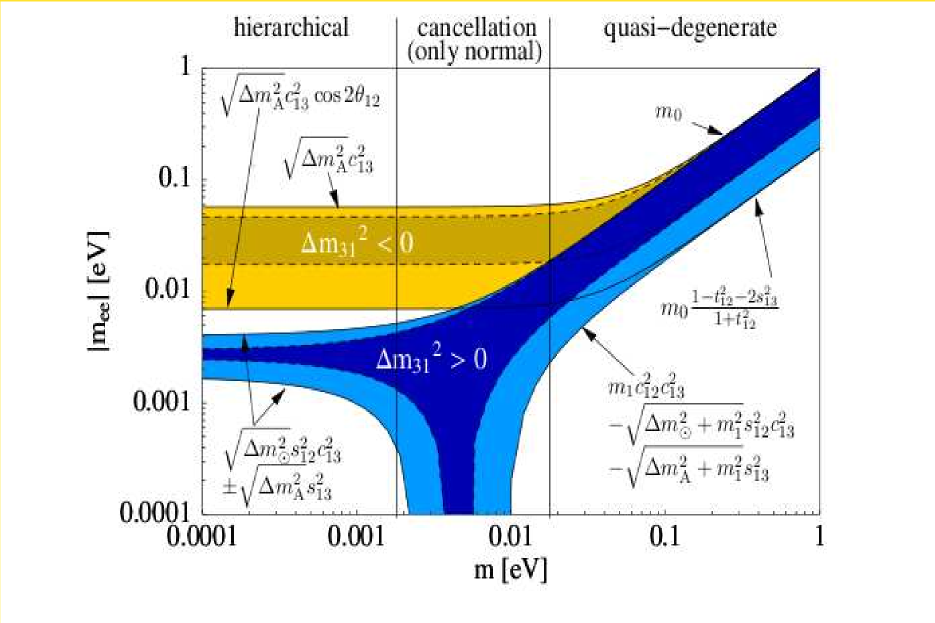
Majorana phases

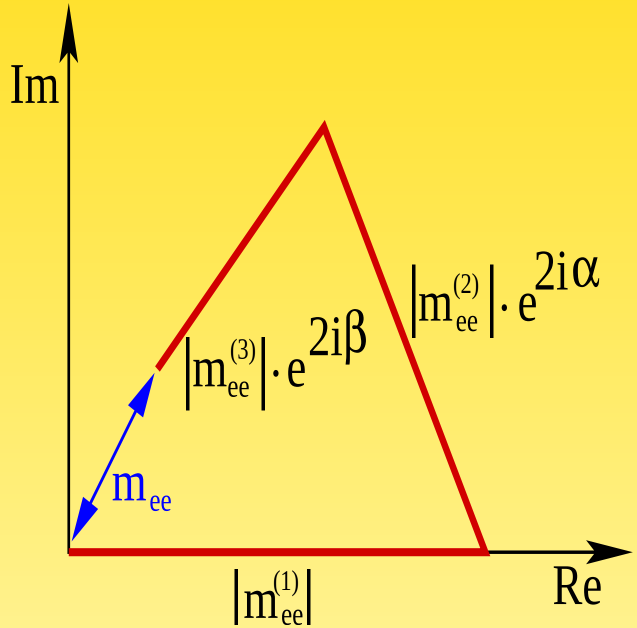
$$|m_{ee}| \propto \left|\cos^2\theta_{12} + e^{2i\alpha}\sin^2\theta_{12}\right| = \sqrt{1 - \sin^2 2\theta_{12}\,\sin^2\alpha}$$



Pascoli, Petcov, Schwetz, hep-ph/0505226

Vanishing $|m_{ee}|$





only for NH $\Rightarrow$ rule out possibility by ruling out NH

unnatural? texture zero!?!

Vanishing $|m_{ee}|$

does it stay zero?

- seesaw RG effects
- NLO see-saw terms:

$$m_\nu = m_D^2/M_R + \mathcal{O}(m_D^4/M_R^3)$$

- actually:

$$\mathcal{A} \propto \frac{U_{ei}^2\, m_i}{q^2 - m_i^2} \simeq \frac{|m_{ee}|}{q^2} + \mathcal{O}(m_i^3/q^4)$$

- Planck scale (Weinberg operator with Planck mass)

$$|m_{ee}| = \frac{v^2}{M_{\rm Pl}} \simeq 10^{-6}\ \text{eV}$$

Renormalization

$$m_\nu = I_{\alpha_\nu} \begin{pmatrix} (m_\nu^0)_{ee}\, I_e^2 & (m_\nu^0)_{e\mu}\, I_e\, I_\mu & (m_\nu^0)_{e\tau}\, I_e\, I_\tau \\ \cdot & (m_\nu^0)_{\mu\mu}\, I_\mu^2 & (m_\nu^0)_{\mu\tau}\, I_\mu\, I_\tau \\ \cdot & \cdot & (m_\nu^0)_{\tau\tau}\, I_\tau^2 \end{pmatrix}$$

where

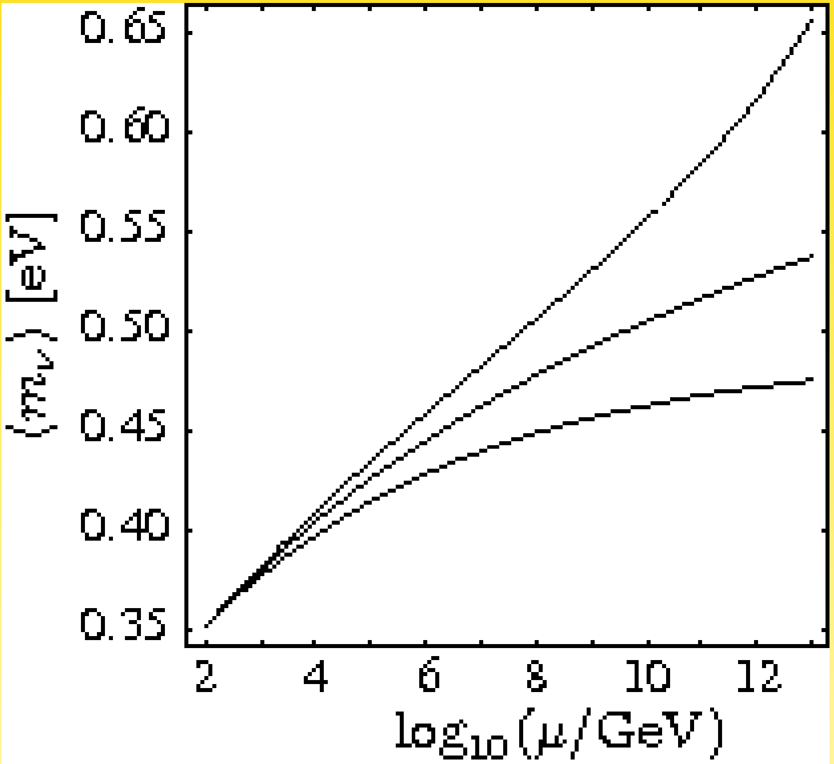
$$I_\alpha \simeq 1 + \frac{C}{16\pi^2}\, y_\alpha^2\, \ln\frac{\lambda}{\Lambda} \quad \text{and} \quad I_{\alpha_\nu} \simeq 1 + \frac{1}{16\pi^2}\alpha_\nu\, \ln\frac{\lambda}{\Lambda}$$

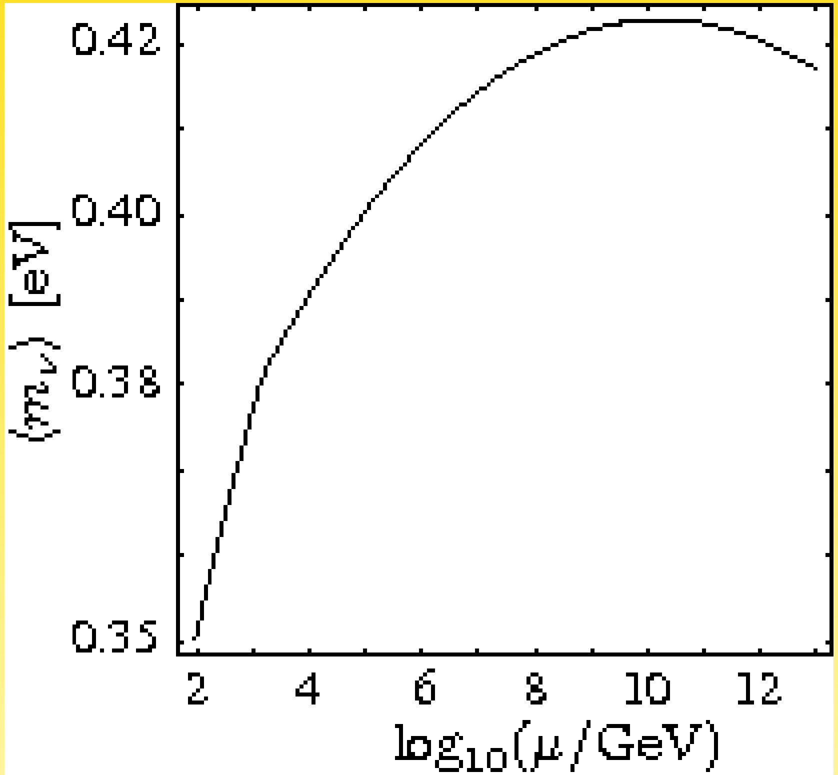
and

$$\alpha_\nu^{\rm SM} = -3g_2^2 + 2(y_\tau^2 + y_\mu^2 + y_e^2) + 6\left(y_t^2 + y_b^2 + y_c^2 + y_s^2 + y_d^2 + y_u^2\right) + \lambda_{\rm H}$$
$$\alpha_\nu^{\rm MSSM} = -\tfrac{6}{5}g_1^2 - 6g_2^2 + 6\left(y_t^2 + y_c^2 + y_u^2\right)$$

$\Rightarrow$ main effect: rescaling of $|m_{ee}|$, typically increases from low to high scale

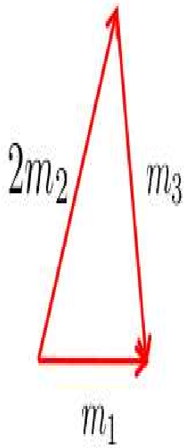
Renormalization



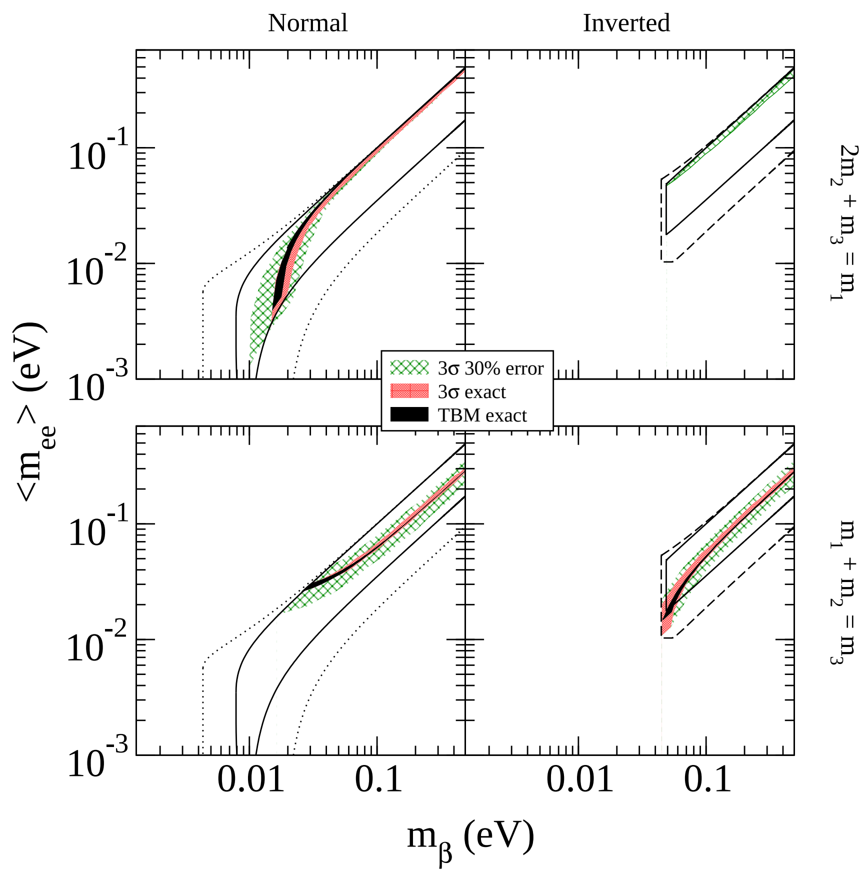


Antusch *et al.*, hep-ph/0305273

Flavor Symmetry Models: sum-rules



Sum-rule	Flavour symmetry
$2m_2 + m_3 = m_1$	$A_4, T', (S_4)$
$m_1 + m_2 = m_3$	$S_4, (A_4)$
$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$	A_4, T'
$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$	S_4



constraints on masses and Majorana phases

Barry, W.R., Nucl. Phys. **B842**

Predictions of $SO(10)$ theories

Yukawa structure of $SO(10)$ models depends on Higgs representations

10_H ($\leftrightarrow H$), $\overline{126}_H$ ($\leftrightarrow F$), 120_H ($\leftrightarrow G$)

Gives relation for mass matrices:

$$
\begin{aligned}
m_{\rm up} &\propto r(H + sF + it_u\, G) \\
m_{\rm down} &\propto H + F + iG \\
m_D &\propto r(H - 3sF + it_D\, G) \\
m_\ell &\propto H - 3F + it_l\, G \\
M_R &\propto r_R^{-1} F
\end{aligned}
$$

Numerical fit including RG, Higgs, θ_{13}

$10_H + \overline{126}_H$: 19 free parameters

$10_H + \overline{126}_H + 120_H$: 18 free parameters

20 (19) observables to be fitted

Predictions of $SO(10)$ theories

Model	Fit	$\lvert m_{ee}\rvert$ [meV]	m_0 [meV]	M_3 [GeV]	χ^2
$10_H + \overline{126}_H$	NH	0.49	2.40	3.6×10^{12}	23.0
$10_H + \overline{126}_H$ + SS	NH	0.44	6.83	1.1×10^{12}	3.29
$10_H + \overline{126}_H + 120_H$	NH	2.87	1.54	9.9×10^{14}	11.2
$10_H + \overline{126}_H + 120_H$ + SS	NH	0.78	3.17	4.2×10^{13}	6.9×10^{-6}
$10_H + \overline{126}_H + 120_H$	IH	35.52	30.2	1.1×10^{13}	13.3
$10_H + \overline{126}_H + 120_H$ + SS	IH	24.22	12.0	1.2×10^{13}	0.6

`Dueck, W.R., JHEP` **1309**

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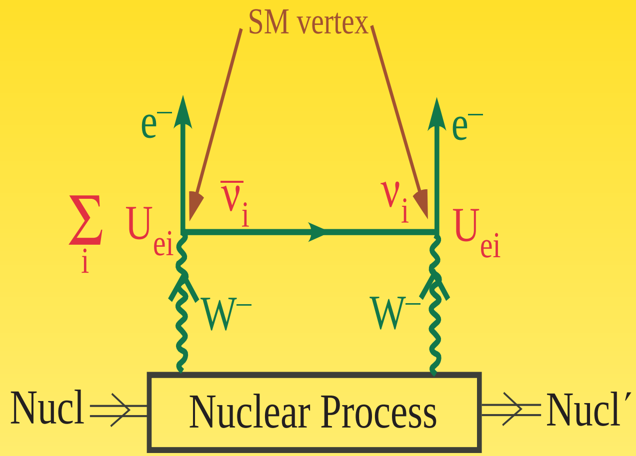
II3) Nuclear physics aspects

Nuclear physics



- 2 point-like Fermi vertices
- "long-range" neutrino exchange
- momentum exchange $q \simeq 1/r \simeq 0.1$ GeV
- wave functions of nucleons not exactly known, depend on nuclear model

typical model for NME: set of single particle states with a number of possible wave function configurations; obtained by solving Dirac equation in a mean background field; interaction known, treatment of fields differs

- Quasi-particle Random Phase Approximation (QRPA) **(many single particle states, few configurations)**
- Nuclear Shell Model (NSM) **(many configurations, few single particle states)**
- Interacting Boson Model (IBM) **(many single particle states, few configurations)**
- Generating Coordinate Method (GCM) **(many single particle states, few configurations)**
- projected Hartree-Fock-Bogoliubov model (pHFB)

tends to overestimate NMEs; tends to underestimate NMEs

gives uncertainty due to different approach plus uncertainty due to model details:

- short range correlations ($\leftrightarrow$ repulsive NN force): multiply two-body wave function with
  - Jastrow function
  - Unitary Correlation Operator method
  - Coupled Cluster method
  - (if heavy particles exchanged: NMEs suppressed)
- model parameters
  - correlated: e.g., g_A, g_{pp}, SRC
  - uncorrelated: e.g., model space of single particle states
  - some calculations include errors/ranges, some don't...

$$\mathcal{M}^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \left(\mathcal{M}^{0\nu}_{\rm GT} - \frac{g_V^2}{g_A^2}\mathcal{M}^{0\nu}_{\rm F}\right)$$

with Gamov-Teller (axial) and Fermi (vector) parts

$$\mathcal{M}^{0\nu}_{\rm GT} = \langle f | \sum_{lk} \sigma_l \, \sigma_k \, \tau_l^- \, \tau_k^- \, H(r_{lk}, E_a) | i \rangle$$

$$\mathcal{M}^{0\nu}_{\rm F} = \langle f | \sum_{lk} \tau_l^- \, \tau_k^- \, H(r_{lk}, E_a) | i \rangle$$

- $r_{lk} \simeq 1/p \simeq 1/(0.1 \text{ GeV})$ distance between the two decaying neutrons
- E_a average energy (closure approx.); $|i\rangle$ and $|f\rangle$ nucleon wave functions
- $J_\alpha = \sum \delta(\vec{x} - \vec{r}_n) \tau_+^n (g_{\alpha 0} \, J_n^0 + g_{\alpha k} \, J_n^k)$ with $J_n^0 = g_V$ and $J_n^k = g_A \, J_n^k \vec{\sigma}_n$
- 'neutrino potential' integrates over the virtual neutrino momenta

$$H(r, E_a) = \frac{2R}{\pi \, r} \int_0^\infty dq \frac{\sin qx}{q + E_a - (M_i + M_f)/2}$$

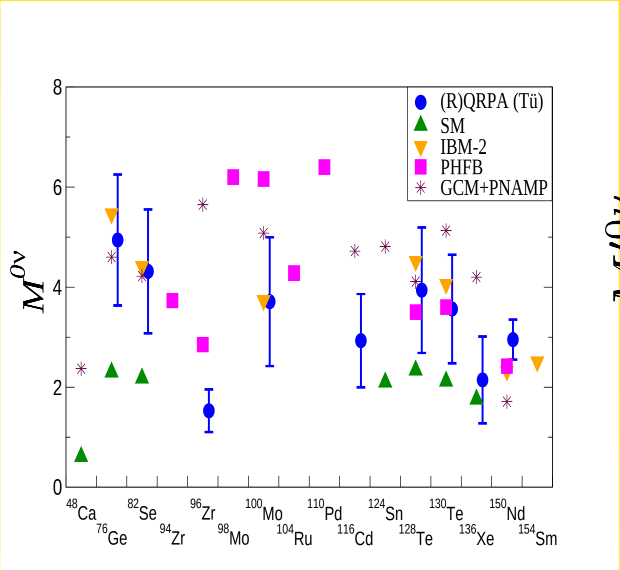
The $2\nu\beta\beta$ matrix elements can be written as

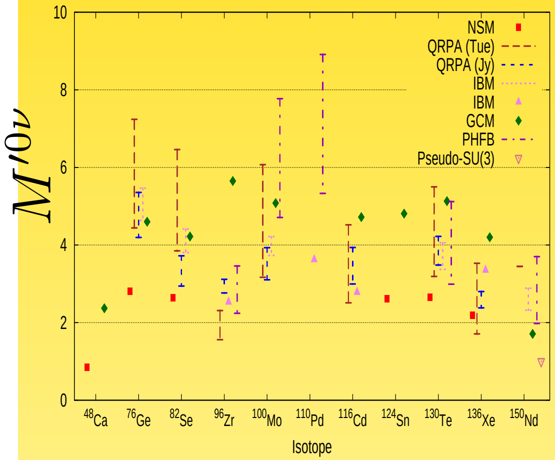
$$\mathcal{M}^{2\nu}_{\mathrm{GT}} = \sum_n \frac{\langle f|\sum_a \sigma_a \tau_a^- |n\rangle\langle n|\sum_b \sigma_b \tau_b^- |i\rangle}{E_n - (M_i - M_f)/2}$$

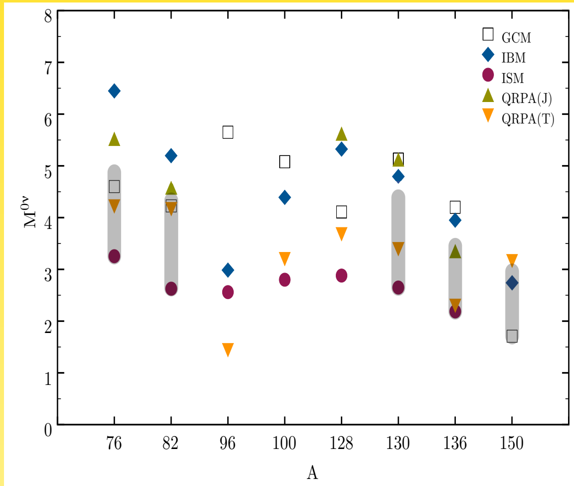
$$\mathcal{M}^{2\nu}_{\mathrm{F}} = \sum_n \frac{\langle f|\sum_a \tau_a^- |n\rangle\langle n|\sum_b \tau_b^- |i\rangle}{E_n - (M_i - M_f)/2}$$

- no direct connection to $0\nu\beta\beta\ldots$
- sum over 1^+ states (low momentum transfer); in $0\nu\beta\beta$ all multipolarities contribute
- adjust some parameters to reproduce $2\nu\beta\beta$-rates
- $2\nu\beta\beta$ observed in ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo (plus exc. state), ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd (plus exc. state) with half-lives from 10^{18} to 10^{24} yrs

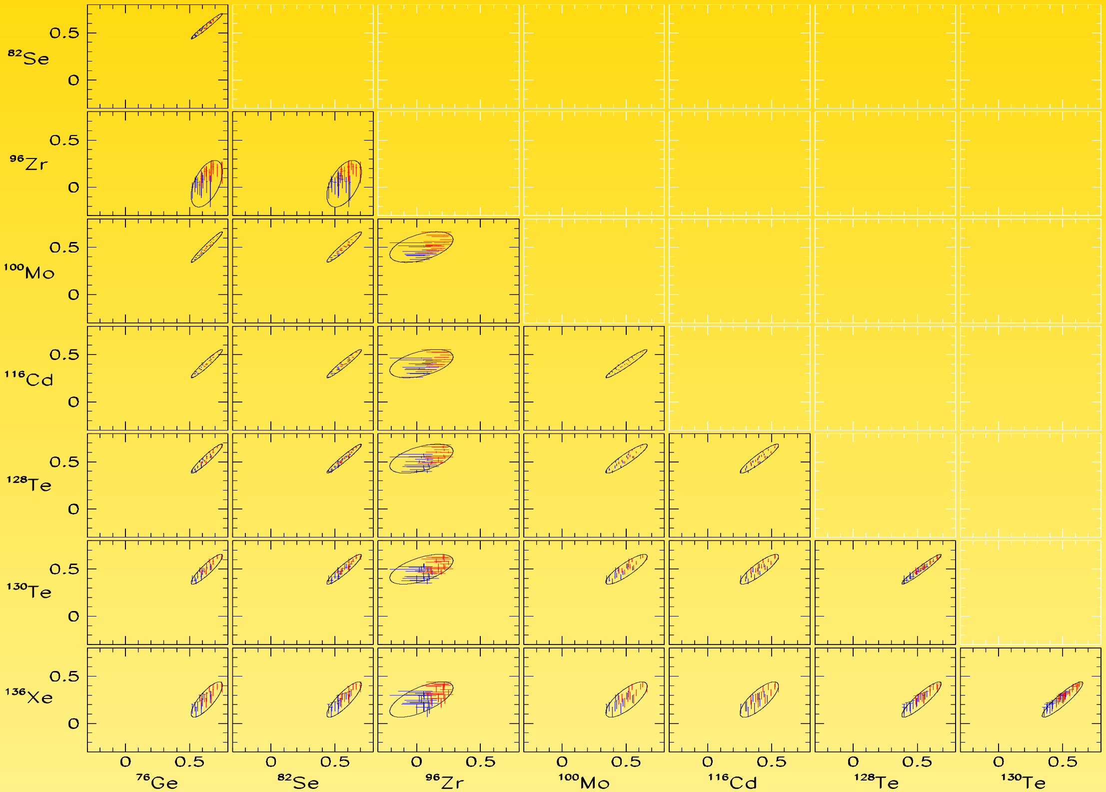
From life-time to particle physics: Nuclear Matrix Elements







to better estimate error range: correlations need to be understood



Faessler, Fogli *et al.*, PRD **79**

ellipse major axis: SRC (blue, red) and g_A

ellipse minor axis: g_{pp}

Quenching

$T^{0\nu}_{1/2} \propto g_A^{-4}$, where in $2\nu\beta\beta$-decay (Iachello)

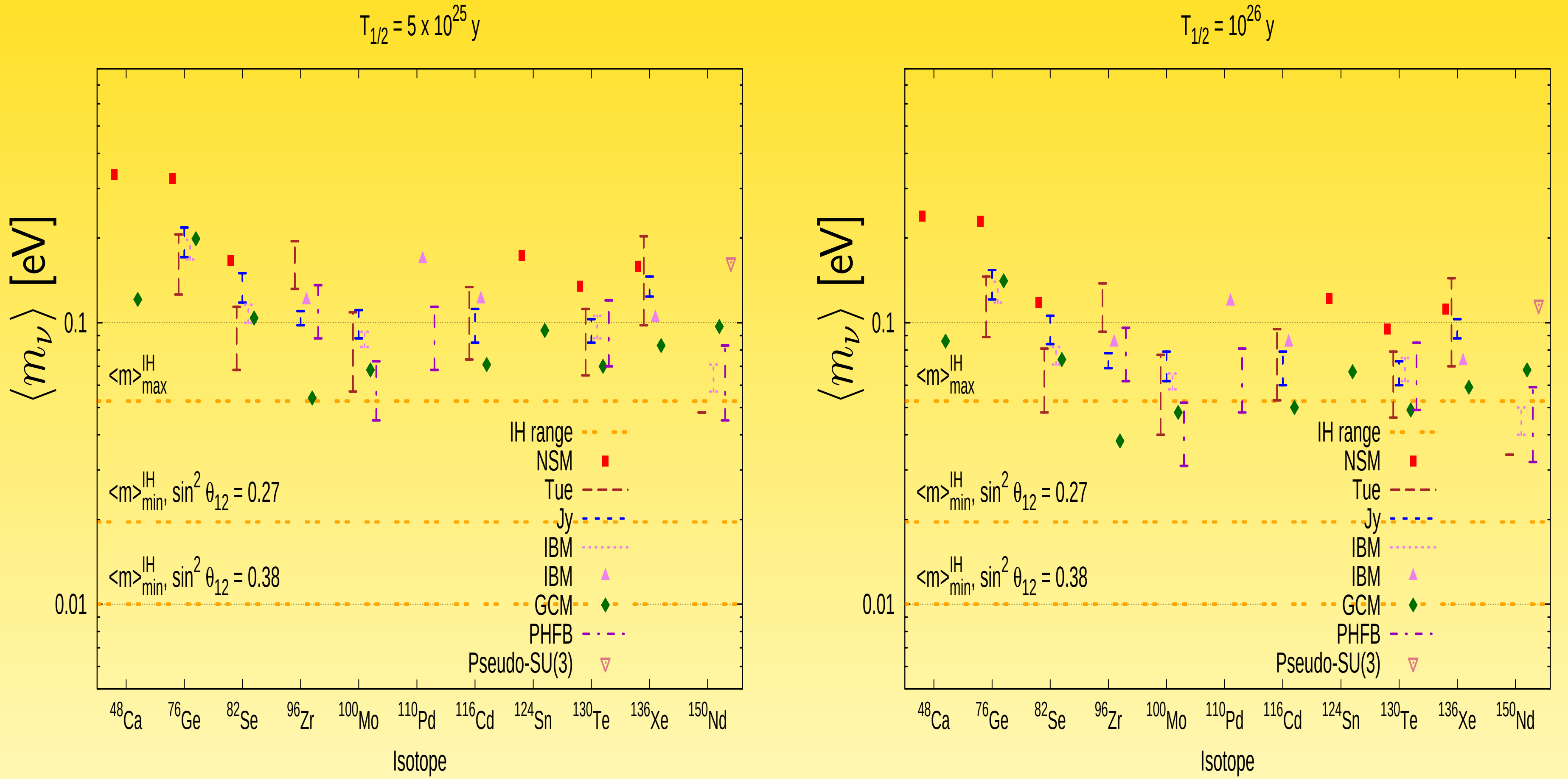
$$g_A^{\rm IBM} \simeq 1.27\, A^{-0.18} = \begin{cases} 0.58 & \text{Ge} \\ 0.53 & \text{Te} \\ 0.52 & \text{Xe} \end{cases}$$

would shift lifetimes an order of magnitude (in wrong direction...)

- model-space truncation (A-dep.)/omission of N^*, Δ (not A-dep.)
- energy scale (no quenching for muon capture)/many multipolarities in 0ν
- include 2 body currents (creation of 2p2h), weaker quenching in 0ν (Engel, Simkovic, Vogel, PRC**89**)

lots of nuclear theory work on the way, possibly no showstopper (20%?)

$0\nu\beta\beta$ and Neutrino physics



Testing NMEs

$$\frac{1}{T^{0\nu}_{1/2}(A,Z)} = G(Q,Z)\,|\mathcal{M}(A,Z)\,\eta|^2$$

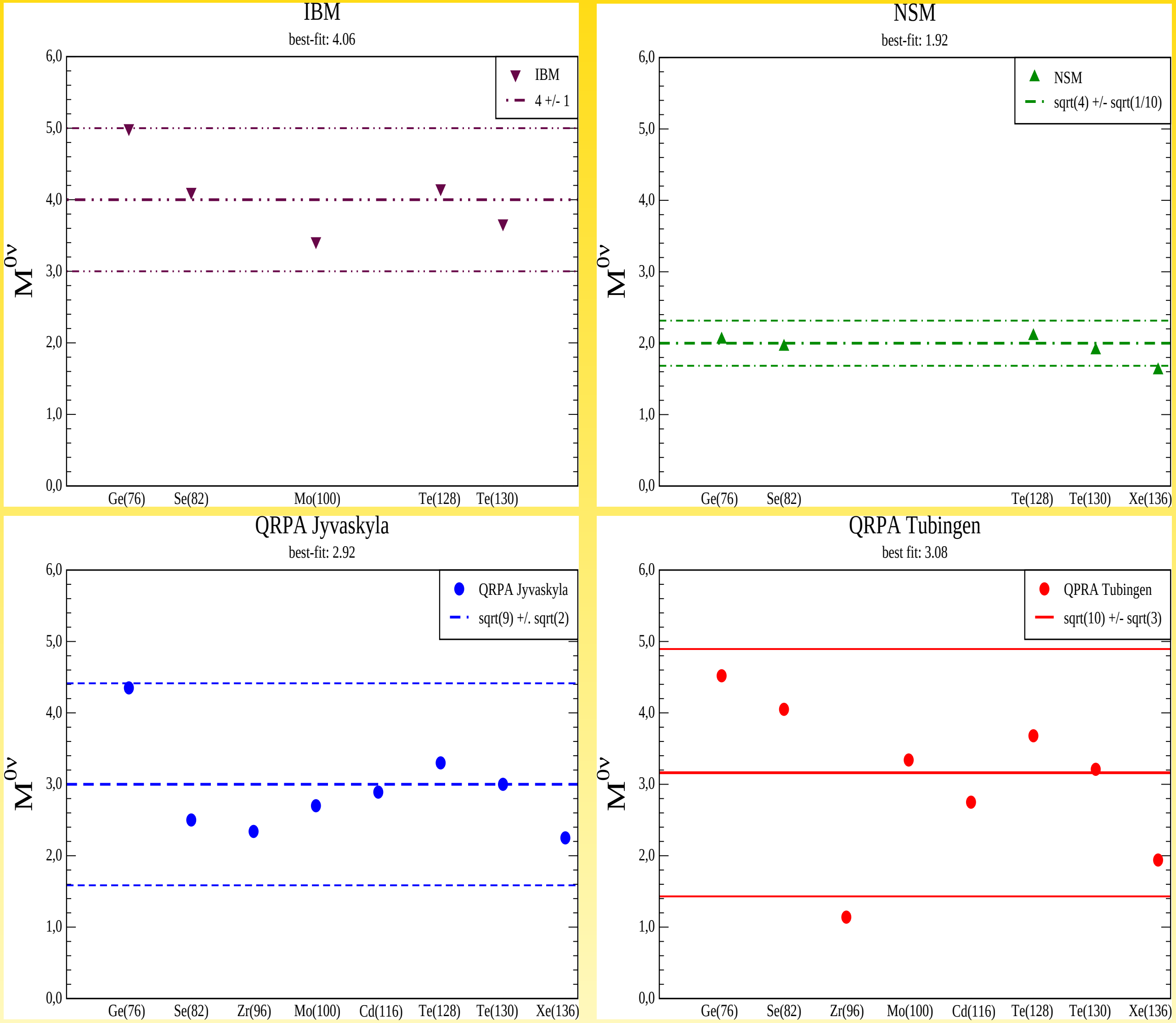
if you measured two isotopes:

$$\frac{T^{0\nu}_{1/2}(A_1,Z_1)}{T^{0\nu}_{1/2}(A_2,Z_2)} = \frac{G(Q_2,Z_2)\,|\mathcal{M}(A_2,Z_2)|^2}{G(Q_1,Z_1)\,|\mathcal{M}(A_1,Z_1)|^2}$$

systematic errors drop out, ratio sensitive to NME model

$\Rightarrow$ **second reason for multi-isotope determination**

NMEs are order one numbers :-)



Contents

II4) Exotic physics

- Dirac neutrinos
- Pseudo-Dirac neutrinos: for each mass state

$$m_i \begin{pmatrix} \epsilon & 1 \\ 1 & 0 \end{pmatrix} \to U = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 + \frac{\epsilon}{4} & -1 + \frac{\epsilon}{4} \\ 1 - \frac{\epsilon}{4} & 1 + \frac{\epsilon}{4} \end{pmatrix} \text{ and } m_i^{\pm} = m_i \left(\pm 1 + \frac{\epsilon}{2} \right)$$

and $|m_{ee}|^{(i)} = \epsilon\, m_i = \frac{1}{2}\, \delta m^2 / m_i$, with $\delta m^2 = (m_i^+)^2 - (m_i^-)^2$

- one neutrino could be (Pseudo-)Dirac, the other Majorana!

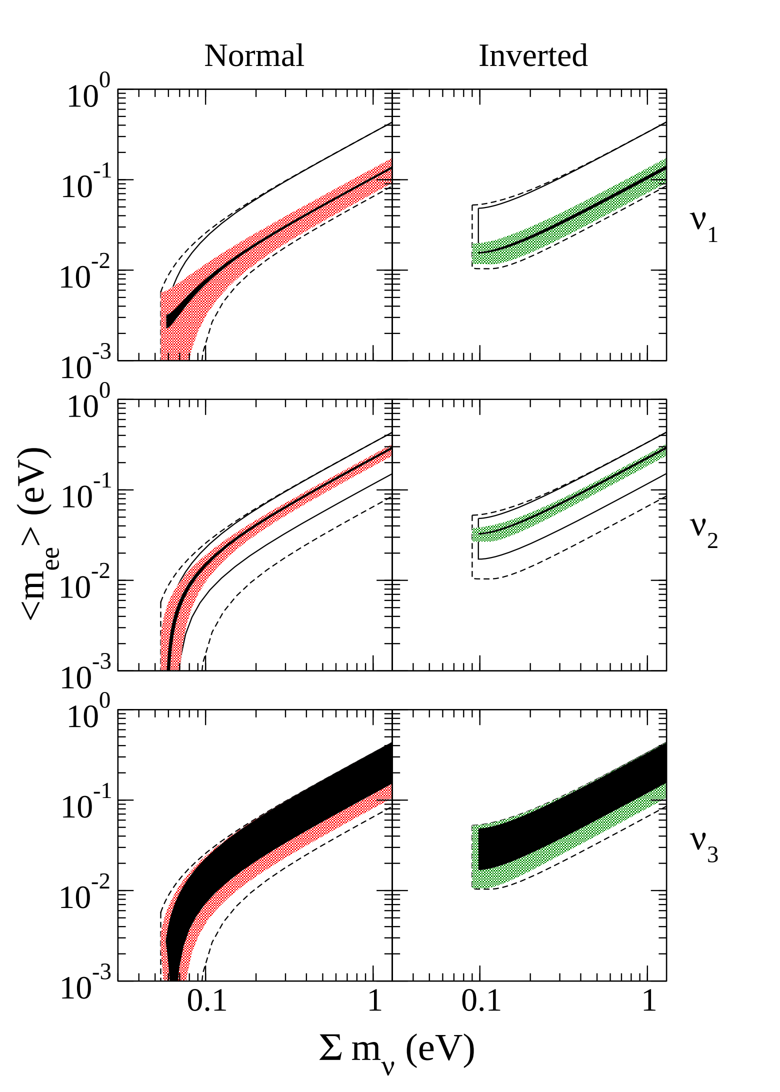
"Schizophrenic neutrinos"

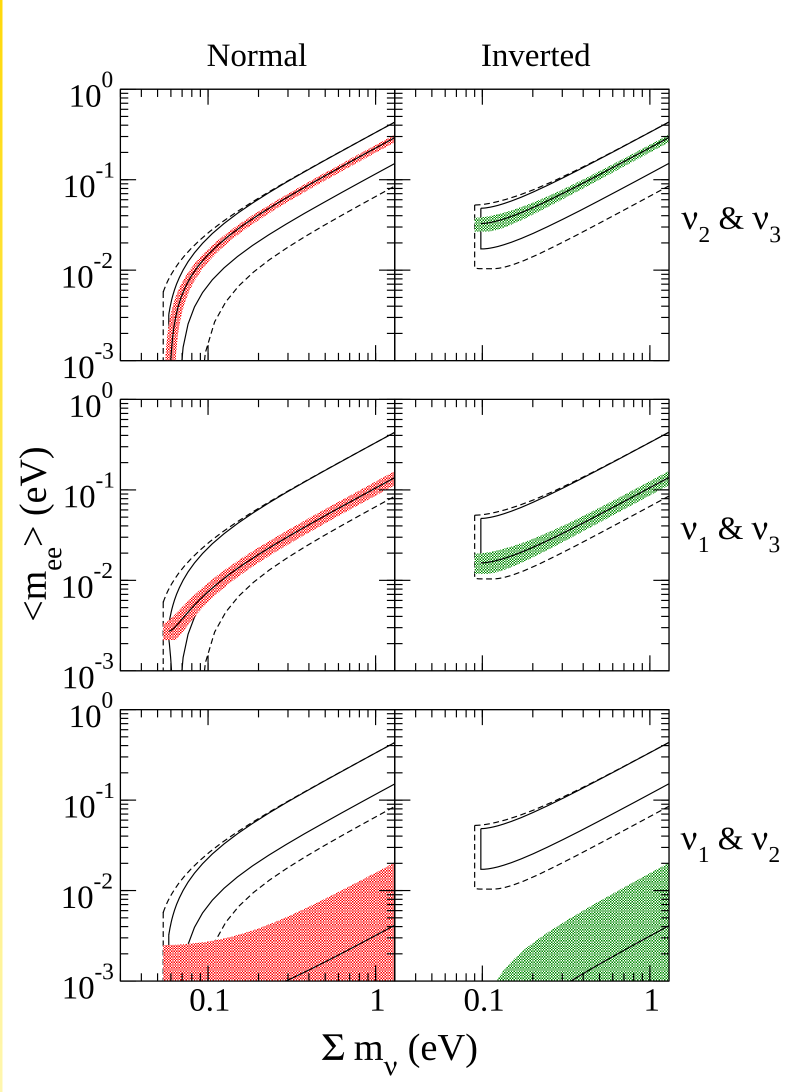
For instance: inverted hierarchy, m_2 Dirac:

$$|m_{ee}| = c_{12}^2\, c_{13}^2\, \sqrt{\Delta m_{\rm A}^2}$$

factor 2 larger than lower limit

Mohapatra *et al.*, 1008.1232





Barry, Mohapatra, W.R., 1012.1761

Lorentz invariance violation

Lorentz and CPT violating term for Majorana neutrinos (Kostelecky *et al.*)

$$\mathcal{L} \propto \nu_e^T \, g_{ee}^{\lambda\mu\nu} \, \sigma_{\lambda\mu} \, q_\nu \, \nu_e$$

gives contribution to propagator

$$S(q) = \frac{\not{q} + g_{ee}^{\lambda\mu\nu} \, \sigma_{\lambda\mu} \, q_\nu}{q^2}$$

and leads to $0\nu\beta\beta$ even for massless neutrinos

can identify

$$|m_{ee}| \leftrightarrow \frac{\frac{1}{3}\left(|\langle g_{ee}^{\lambda\mu 1}\rangle|^2 + |\langle g_{ee}^{\lambda\mu 2}\rangle|^2 + |\langle g_{ee}^{\lambda\mu 3}\rangle|^2\right)}{R}$$

and set limits of order 10^{-8} (Diaz, 1311.0930)

CPT Violation

MINOS, MiniBooNE, solar/KamLAND...

what about Dirac/Majorana??

Barenboim, Beacom, Borissov, Kayser, hep-ph/0203261

Define $\xi =$ CPT. A Majorana particle is defined as

$$\xi|\nu_i\rangle = e^{i\zeta_i}\,|\nu_i\rangle$$

Now we introduce CPT-violation

$$M_\nu = \begin{pmatrix} \mu+\Delta & y^* \\ y & \mu-\Delta \end{pmatrix} \text{ for basis } \nu, \overline{\nu}$$

y mixes ν and $\overline{\nu}$ and hence violates L

diagonalize M_ν to get eigenstates

$$|\nu_+\rangle = \cos\theta\,|\nu\rangle + e^{i\phi}\,\sin\theta\,|\overline{\nu}\rangle\ ,\quad m_+ = \mu + \sqrt{|y^2| + \Delta^2}$$
$$|\nu_-\rangle = -\sin\theta\,|\nu\rangle + e^{i\phi}\,\cos\theta\,|\overline{\nu}\rangle\ ,\quad m_- = \mu - \sqrt{|y^2| + \Delta^2}$$

mixing angle $\tan 2\theta = \frac{|y|}{\Delta}$

CPT properties are $\xi|\nu\rangle = e^{i\zeta}\,|\overline{\nu}\rangle$ and $\xi|\overline{\nu}\rangle = e^{i\zeta}\,|\nu\rangle$, hence

$$\xi|\nu_+\rangle = e^{i(\zeta-\phi)}\left(\sin\theta\,|\nu\rangle + e^{i\phi}\,\cos\theta\,|\overline{\nu}\rangle\right)\ ,$$
$$\xi|\nu_-\rangle = -e^{i(\zeta-\phi)}\left(-\cos\theta\,|\nu\rangle + e^{i\phi}\,\sin\theta\,|\overline{\nu}\rangle\right)$$

$\Rightarrow \nu_\pm$ are Majorana particles if and only if $\theta = \pi/4$

but $\theta = \pi/4$ means $\Delta = 0$ and thus CPT conservation:-)

if $\Delta \neq 0$: CPT is violated: neutrinos are no longer Majorana fermions; if $y \neq 0$ then L is violated and $0\nu\beta\beta$ can occur

observation of $0\nu\beta\beta$ implies non-zero y but not that neutrinos are CPT self-conjugate

Neutrinoless double beta decay?

mass eigenstates are for 3 generations

$$|\nu_i\rangle = U_{\alpha i}|\nu_\alpha\rangle + \overline{U}^*_{\alpha i}|\overline{\nu}_\alpha\rangle$$

amplitude for $0\nu\beta\beta$ is

$$\mathcal{A} \propto \sum m_i \, U_{ei} \, \overline{U}_{ei}$$

for 1-flavor case:

$$\mathcal{A} \propto m_+ \, U_{+\nu} \, U_{+\overline{\nu}} + m_- \, U_{-\nu} \, U_{-\overline{\nu}}$$
$$= m_+ \, \cos\theta \, e^{i\phi} \, \sin\theta + m_- \, (-\sin\theta) \, e^{i\phi} \, \cos\theta$$
$$\propto (m_+ - m_-) \, \cos\theta \, \sin\theta = y$$

as expected

CPT counter part of $0\nu\beta\beta$ gives same result

Light Sterile Neutrinos

$\leftrightarrow$ is there a **sterile** neutral lepton with $\Delta m^2 \sim$ eV^2 and mixing $\mathcal{O}(0.1)$?

- LSND ($\overline{\nu}_\mu \to \overline{\nu}_e$) 3.8σ
- MiniBooNE ($\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$) 3.8σ
- Gallium anomaly ($\nu_e \to \nu_e$) 2.9σ
- Reactor anomaly ($\overline{\nu}_e \to \overline{\nu}_e$) 2.8σ
- Cosmology and Astroparticle Physics ($N_{\rm eff}$, "Dark Radiation")

	$\Delta m^2_{41}[\mathrm{eV}^2]$	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	$\Delta m^2_{51}[\mathrm{eV}^2]$	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$
3+2/2+3	0.47	0.128	0.165	0.87	0.138	0.148
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163

or $\Delta m^2_{41} = 1.78$ eV^2 and $|U_{e4}|^2 = 0.151$

Kopp, Maltoni, Schwetz, 1103.4570

Sterile Neutrinos and $0\nu\beta\beta$

- recall: $|m_{ee}|^{\rm act}_{\rm NH}$ can vanish and $|m_{ee}|^{\rm act}_{\rm IH} \sim 0.03$ eV cannot vanish
- $|m_{ee}| = \Big| \underbrace{|U_{e1}|^2\, m_1 + |U_{e2}|^2\, m_2\, e^{2i\alpha} + |U_{e3}^2|\, m_3\, e^{2i\beta}}_{m_{ee}^{\rm act}} + \underbrace{|U_{e4}|^2\, m_4\, e^{2i\Phi_1}}_{m_{ee}^{\rm st}} \Big|$
- sterile contribution to $0\nu\beta\beta$ (assuming 1+3):

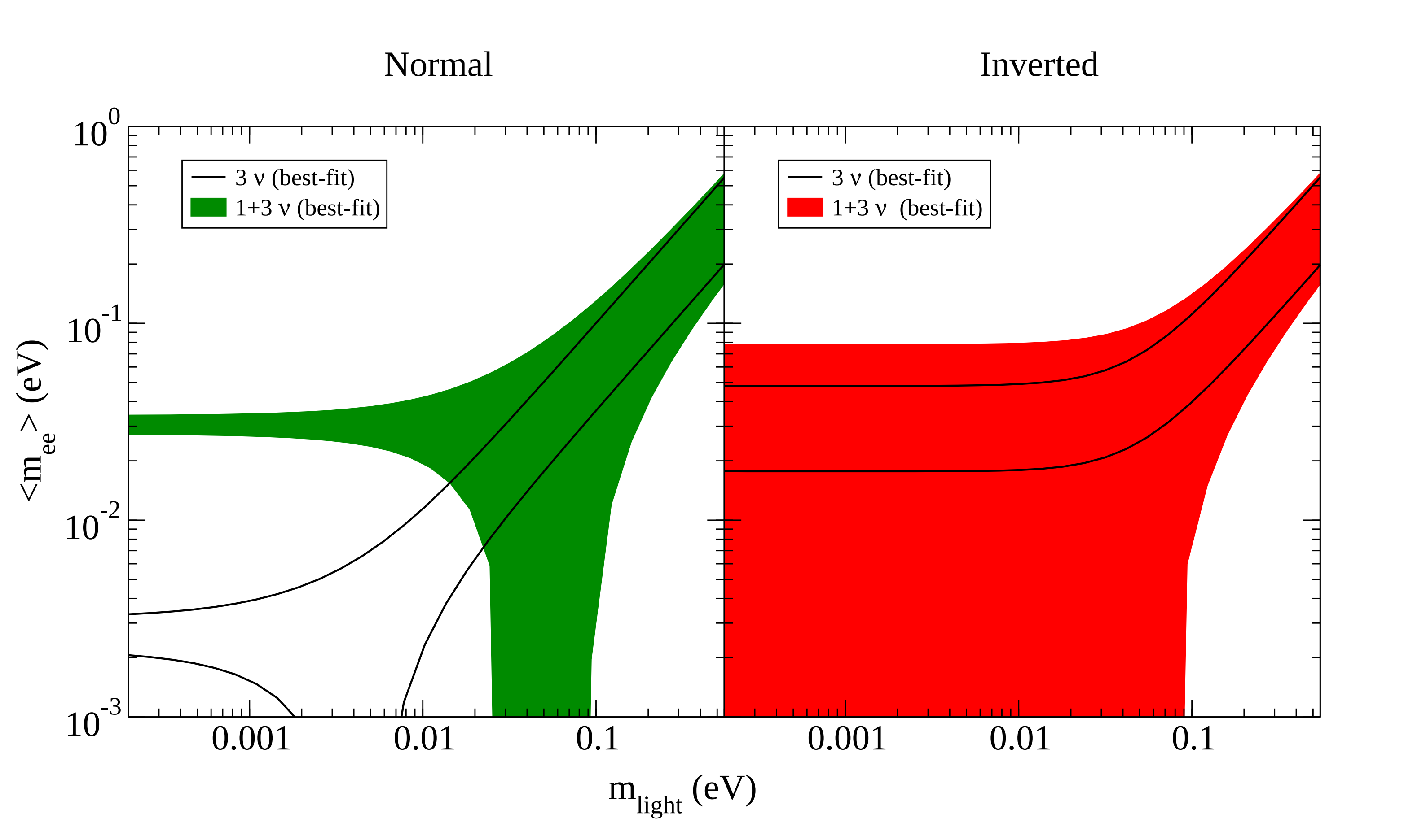
$$|m_{ee}|^{\rm st} \simeq \sqrt{\Delta m^2_{\rm st}}\, |U_{e4}|^2 \left\{ \begin{array}{l} \gg |m_{ee}|^{\rm act}_{\rm NH} \\ \simeq |m_{ee}|^{\rm act}_{\rm IH} \end{array} \right.$$

$\Rightarrow$ $|m_{ee}|_{\rm NH}$ cannot vanish and $|m_{ee}|_{\rm IH}$ can vanish!

$\Rightarrow$ usual phenomenology gets completely turned around!

Barry, W.R., Zhang, JHEP **1107**; Giunti *et al.*, PRD **87**; Girardi, Meroni, Petcov, JHEP **1311**; Giunti, Zavanin, 1505.00978

Usual plot gets completely turned around!



Do Dirac neutrinos mean there is no Lepton Number Violation?

model based on gauged $B - L$, broken by 4 units

$\Rightarrow$ Neutrinos are Dirac particles, $\Delta L = 2$ forbidden, but $\Delta L = 4$ allowed...

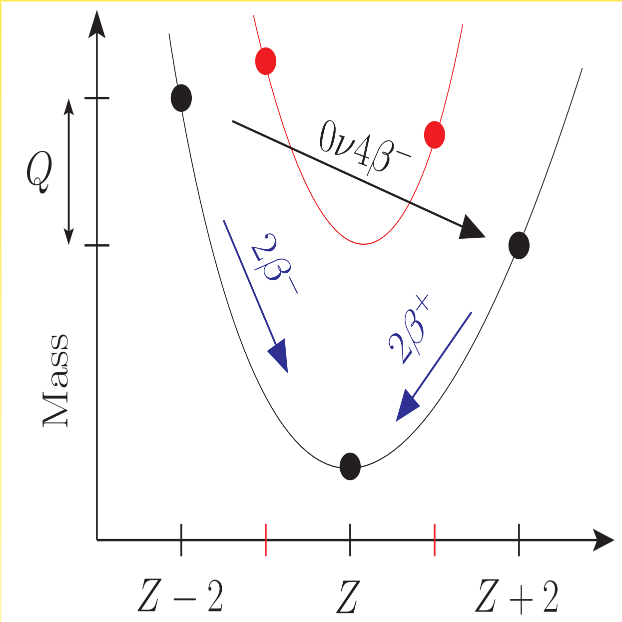
Heeck, W.R., EPL **103**

Do Dirac neutrinos mean there is no Lepton Number Violation?

model based on gauged $B - L$, broken by 4 units

$\Rightarrow$ Neutrinos are Dirac particles, $\Delta L = 2$ forbidden, but $\Delta L = 4$ allowed...

$\Rightarrow$ observable: neutrinoless quadruple beta decay $(A, Z) \to (A, Z + 4) + 4\, e^-$



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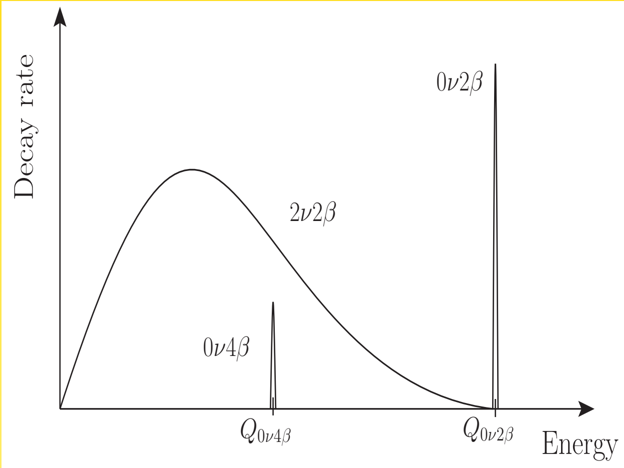
Candidates for neutrinoless quadruple beta decay



	$Q_{0\nu 4\beta}$	Other decays	NA
${}^{96}_{40}\text{Zr} \to {}^{96}_{44}\text{Ru}$	0.629	$\tau^{2\nu 2\beta}_{1/2} \simeq 2 \times 10^{19}$	2.8
${}^{136}_{54}\text{Xe} \to {}^{136}_{58}\text{Ce}$	0.044	$\tau^{2\nu 2\beta}_{1/2} \simeq 2 \times 10^{21}$	8.9
${}^{150}_{60}\text{Nd} \to {}^{150}_{64}\text{Gd}$	2.079	$\tau^{2\nu 2\beta}_{1/2} \simeq 7 \times 10^{18}$	5.6

Heeck, W.R., EPL **103**

With $0\nu\beta\beta$ one can

- test lepton number violation
- test Majorana nature of neutrinos
- probe neutrino mass scale
- extract Majorana phase
- constrain inverted ordering

conceptually, it would increase our believe in

- GUTs
- seesaw mechanism
- leptogenesis