XVII Frascati Spring School "Bruno Touschek," INFN-LNF 2014

Neutrinos:
Theory and Phenomenology

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Lecture I (1h 30')

Neutrino histo(ry)gram

The discovery of flavor oscillations has raised the level of interest in neutrino physics, at the level of ~1.4x10³ papers/year titled "…neutrino(s)…" on SPIRES

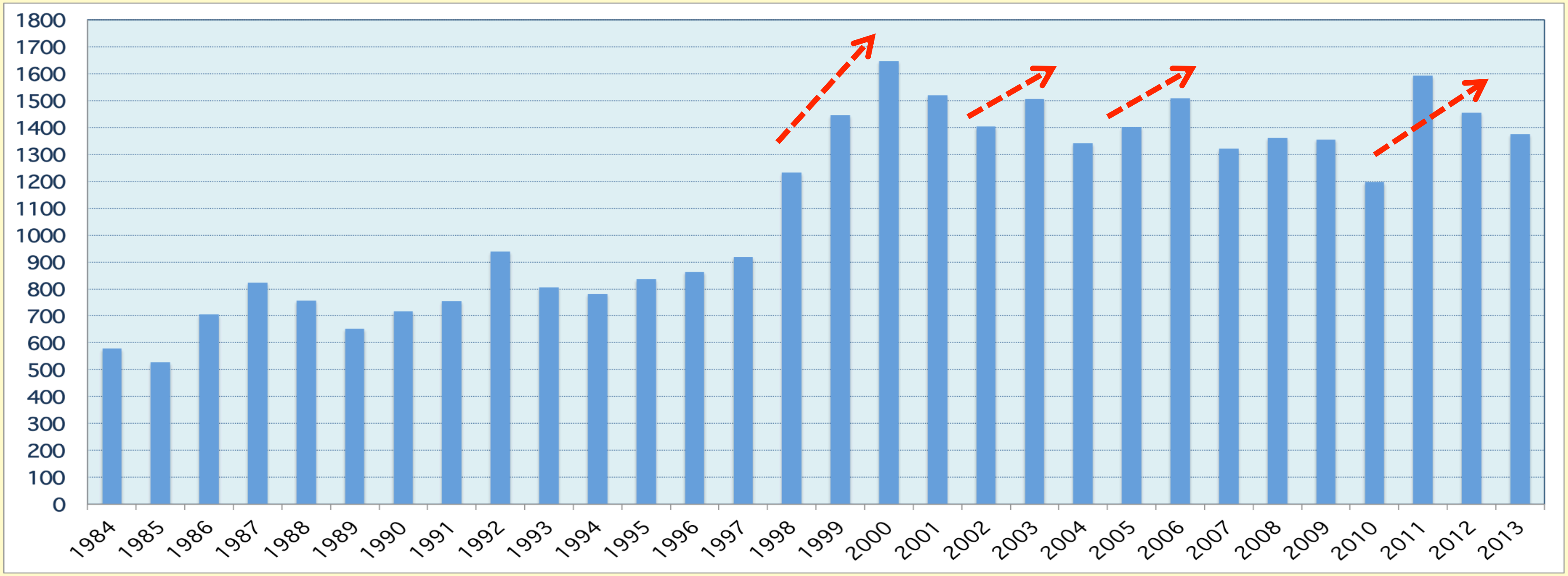
Recent peaks of interest:

Atmospheric ν oscillations, Limit from CHOOZ

Solar and react. ν oscillations, Nobel 2002 to Davis & Koshiba

Accelerator ν oscillations, Cosmo limits on abs.masses

1-3 ν oscill. at 2nd gen. react. + sterile nu anomalies



New fundamental ν parameters: $(\Delta m^2, \theta_{23})$ $(\delta m^2, \theta_{12})$ [Osc.patterns] (θ_{13})

A selection has to be made for 3h lectures...

Contents: only physics of **ν** masses and mixings (e.g., no cross sec's)
emphasis on interpretation of data (e.g., no mass models)

Format: phenomenology (yellowish bkgd slides, ~2/3 of total)
theory/exercises (handwritten notes, ~1/3 of total)

All exercises (even the simplest ones) are all worked out in detail, but will be just flashed in most cases ("skip"). If you wish to gain further understanding, work through (some of) the exercises by yourself.

The two lectures are supplemented by an appendix on statistics.

No bibliography herein: just browse Giunti & Laveder's www.nu.to.infn.it

General Outline

Today: **Neutrino oscillations in standard 3ν framework**

[Tomorrow: Absolute ν masses, open problems, perspectives]

Today's lecture – part 1:

* 3ν mass-mixing framework:
  - notation, conventions, simple calculations in vacuum

* Oscillation searches sensitive to Δm^2
  - various classes of experiments (production/detection/results/interpretation)

* Notes on neutrino CP violation in vacuum

[Then break → part 2]

Notation for neutrino masses

Physics facts :

- Three mass states $\nu_1\ \nu_2\ \nu_3$ with masses $m_1\ m_2\ m_3$
- For ultrarelativistic ν in vacuum : $E = \sqrt{m_i^2 + p^2} \simeq p + m_i^2/2p$
- Neutrino oscillations probe $\Delta E \propto \Delta m^2_{ij}$.
- 3 neutrinos $\rightarrow$ 2 independent Δm^2_{ij}, say, δm^2 and Δm^2
- Experimentally : very different values, $\delta m^2/\Delta m^2 \sim 1/30$

$$\boxed{\begin{array}{l} \delta m^2 \simeq 7.5 \times 10^{-5}\, eV^2 \\ \Delta m^2 \simeq 2.5 \times 10^{-3}\, eV^2 \end{array}}$$

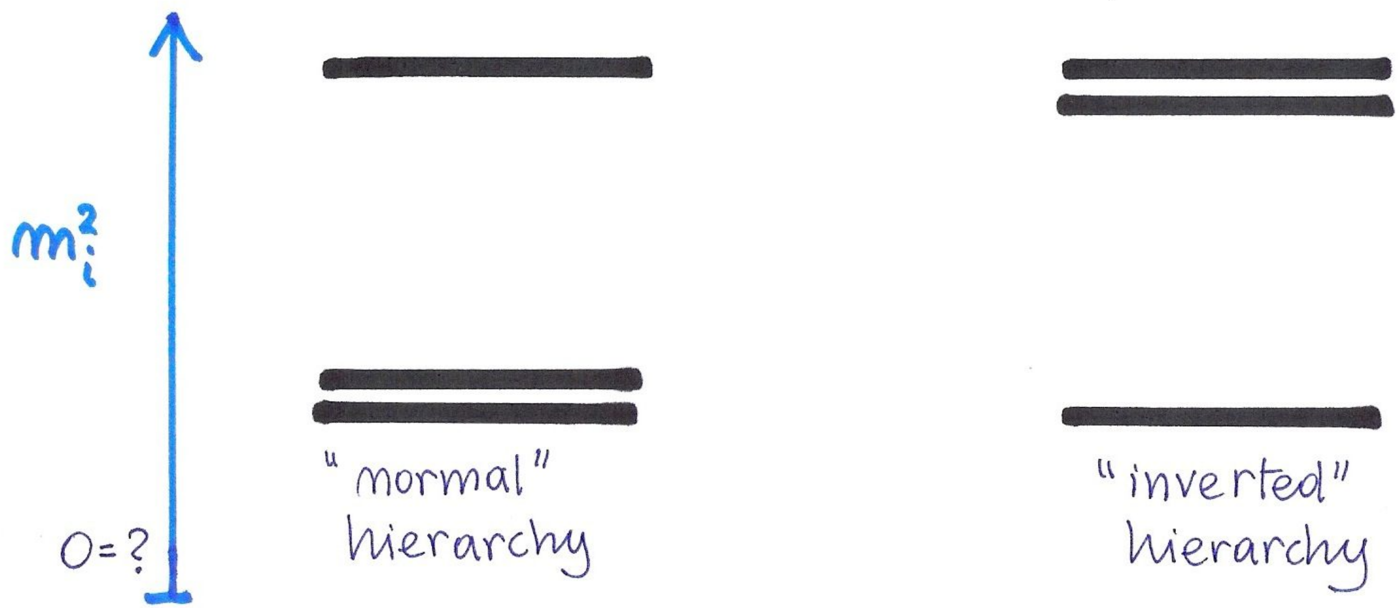
$\leftarrow$ "small" or "solar" splitting

$\leftarrow$ "large" or "atmospheric" splitting

Physics facts (cont'd):

- Very difficult to probe both splittings in the same experiment!

- Two possible arrangements ("hierarchies") for such splittings:



- Absolute ν mass scale unknown; lightest m_i could be zero

- However, upper limits exist: $m_i \lesssim \mathcal{O}(\text{eV})$

Conventions :

- In both hierarchies, there is a "doublet" of close mass states and a "lone" mass state. Universal convention:

  (ν_1, ν_2) is the doublet, with ν_1 being the lightest : $m_1 < m_2$

  ν_3 is the lone state, with $m_3 \gtrless m_{1,2}$

- Splittings : $\delta m^2 = m_2^2 - m_1^2 > 0$ (>0 by definition)

  $\Delta m^2 \simeq m_3^2 - m_{1,2}^2 \gtrless 0$ (has a physical sign)

- We use : $\Delta m^2 = \frac{1}{2}(\Delta m^2_{31} + \Delta m^2_{32})$ for the sake of precision

  (others prefer to use either $\Delta m^2 = \Delta m^2_{31}$ or $\Delta m^2 = \Delta m^2_{32}$)

Exercise: learn more about Δm^2 definition subtleties

- Very often, it is not declared if Δm^2 refers to $\Delta m^2_{31} = m^2_3 - m^2_1$, or to $\Delta m^2_{32} = m^2_3 - m^2_2$, or to some combination (sometimes, because $\delta m^2 = m^2_2 - m^2_1$ is assumed to be $\simeq 0$).

- However, the current 1σ error on Δm^2 is of the same order as δm^2! So, the ambiguity is approaching the $\sim 1\sigma$ level; need to be precise!

- We use the definition $\Delta m^2 = \frac{1}{2}(\Delta m^2_{31} + \Delta m^2_{32}) = m^2_3 - \frac{1}{2}(m^2_1 + m^2_2) \gtrless 0$ so that $(m^2_1, m^2_2, m^2_3) = \frac{m^2_1 + m^2_2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm \Delta m^2\right)$ where $+\Delta m^2 \to$ normal hierarchy, and $-\Delta m^2 =$ inverted hierarchy (+NH, -IH).

- Others prefer to use Δm^2_{31} or Δm^2_{32} as independent variables. However, this brings a disadvantage: Δm^2_{31} is the largest mass splitting in NH, and the next-to-largest in IH; viceversa for Δm^2_{32}; while our Δm^2 is always the average of the two in both NH and IH.

skip

Notation for neutrino mixing

Physics facts:

- Three flavor states $\nu_e\ \nu_\mu\ \nu_\tau$ mixed with mass states $\nu_1\ \nu_2\ \nu_3$:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad \nu_\alpha = U_{\alpha i}\, \nu_i$$

- If these are the only ν states in nature, the matrix U is unitary :
  $UU^\dagger = I$ (although $U \neq U^*$ in general)

- For antineutrinos, $U \to U^*$ (see also next exercise)

- As for quarks, the unitary mixing matrix U can be expressed in terms of four independent physical parameters :
  3 mixing angles + 1 CP-violating phase

Conventions:

- The Particle Data Group notation is currently "universal"

$$U = O_{23}\,\Gamma_\delta\,O_{13}\,\Gamma_\delta^\dagger\,O_{12}$$

← with $\Gamma_\delta =$ diag$(1, 1, e^{i\delta})$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

← with $c_{ij} = \cos\theta_{ij}$
$s_{ij} = \sin\theta_{ij}$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U is often called "Pontecorvo - Maki - Nakagawa - Sakata" (PMNS) matrix.

- Experimentally, we now know that:

$$U = (23) \qquad (13) \qquad (12)$$

$\uparrow$ $\sin^2\theta_{23} \sim 0.5$ $\sim$maximal

$\uparrow$ $\sin^2\theta_{13} \sim 0.02$ small ($\delta = ?$)

$\uparrow$ $\sin^2\theta_{12} \sim 0.3$ large

- The presence of two small parameters, $s^2_{13} \sim 0.02$ and $\delta m^2/\Delta m^2 \sim 1/30$, makes 3ν mixing approximately reducible to "effective 2ν mixing" in several cases of phenomenological interest.

- Goal of many current and future experiments is to find evidence of "genuine 3ν effects" beyond 2ν approximations.

Exercise: Learn more about U vs U^* subtleties

The PMNS matrix U actually connects quantum fields in the CC lagrangian, $\nu_{\alpha L} = \sum_{i=1}^{3} U_{\alpha i}\nu_{iL}$. But, for a field ψ, it is $\bar{\psi}$ (or ψ^+) which creates particles from vacuum $|0\rangle$. Therefore, in terms of created one-particle states (kets $|\nu\rangle$) one has that: $|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle$ (which is the PDG convention in terms of states). On the other hand, a generic $|\nu\rangle$ state can be decomposed as: $|\nu\rangle = \sum_i \nu^i|\nu_i\rangle$ or $|\nu\rangle = \sum_\alpha \nu^\alpha|\nu_\alpha\rangle$ where the components ν^i, ν^α (complex numbers) transform as: $\nu^\alpha = \sum_i U_{\alpha i}\nu^i$. Summarizing, for neutrinos:

$$\begin{cases} \nu_{\alpha L} = \sum_i U_{\alpha i}\nu_{iL} & \leftarrow \text{fields} \\ |\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle & \leftarrow \text{states} \\ \nu^\alpha = \sum_i U_{\alpha i}\nu^i & \leftarrow \text{components} \end{cases} \quad , \text{ where } \quad U_{\alpha i} = \langle\nu_\alpha|\nu_i\rangle$$

For antineutrinos, one should change $U \to U^*$ everywhere.

Note that neutrino "components" (rather than states or fields) are the numerical objects manipulated in computer calculations. They are often organized into a (column) vector; e.g.:

$\begin{pmatrix}\nu^e\\ \nu^\mu\\ \nu^\tau\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ is the vector of components of a pure $|\nu_e\rangle$ state in flavor basis.

skip

Neutrino flavor evolution

$m_i \ll E$ in almost all cases of phenomenological interest. Then:

- We can often set $\beta = v/c \simeq 1$, $x \simeq t$, $\partial_x \simeq \partial_t$
- Chirality flips (LH $\to$ RH) of $\mathcal{O}(m_i/E)$ can be ignored (more details tomorrow) $\to$ spinorial properties not relevant in flavor evolution
- Can thus adopt a simple description in terms of "scalar" states $|\nu\rangle$ governed by a Hamiltonian $\hat{H}$:

$$\boxed{i \frac{d}{dx} |\nu\rangle = \hat{H} |\nu\rangle}$$

with formal solution $|\nu(x)\rangle = \hat{S}(x,0)\,|\nu(0)\rangle$ where $\hat{S}$ is the evolution operator from 0 to x.

In vacuum :

- For a ν beam of momentum p traveling in vacuum, in the mass eigenstate basis the $\hat{H}$ matrix reads :

$$H_{mass} = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{pmatrix} \simeq p \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$ ← diagonal

- However, in flavor basis:

$$H_{flavor} = U\, H_{mass}\, U^{+}$$ ← non diagonal : flavor <u>not</u> conserved

- We shall work out several consequences of this simple hamiltonian, and then add corrections for propagation in matter. Main output : flavor oscillation probabilities,

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}|^2 \begin{cases} \alpha = \beta : \text{ survival probability} \\ \alpha \neq \beta : \text{ transition probability} \end{cases}$$

Exercise: Learn more about subtleties of flavor indices

- Fixing conventions when ordering indices is important because, e.g., (α,β) is inequivalent to (β,α) in many cases. Our conventions follow.
- Evolution equation: $\hat{H}|\nu\rangle = i\partial_x|\nu\rangle$
- Decomposition in mass basis:
  $$\hat{H} = \sum_{ij}|\nu_j\rangle\langle\nu_j|\hat{H}|\nu_i\rangle\langle\nu_i| = \sum_{ij} H_{ji}|\nu_j\rangle\langle\nu_i|$$
- Decomposition in flavor basis:
  $$\hat{H} = \sum_{\alpha\beta}|\nu_\beta\rangle\langle\nu_\beta|\hat{H}|\nu_\alpha\rangle\langle\nu_\alpha| = \sum_{\alpha\beta} H_{\beta\alpha}|\nu_\beta\rangle\langle\nu_\alpha|$$

  note (j,i) and (β,α) ordering!
- Relation among matrix elements:
  $$H_{ji} = \langle\nu_j|\hat{H}|\nu_i\rangle\,; \quad H_{\beta\alpha} = \langle\nu_\beta|\hat{H}|\nu_\alpha\rangle\,; \quad H_{\beta\alpha} = \sum_{ij} U_{\beta j} H_{ji} U^*_{\alpha i}$$
  In matrix form: $H_{flavor} = U\, H_{mass}\, U^+$
- Formal solution (evolution operator): $|\nu(x)\rangle = \hat{S}(x,0)|\nu(0)\rangle$
- Flavor oscillation probability: $P(\nu_\alpha\to\nu_\beta) = |S_{\beta\alpha}|^2$

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Exercise : 3ν oscillations in vacuum

Prove that

$$\boxed{P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\, J_{\alpha\beta}^{ij} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) - 2 \sum_{i<j} \mathrm{Im}\, J_{\alpha\beta}^{ij} \sin\left(\frac{\Delta m^2_{ij}\, x}{2E}\right)}$$

where

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$J_{\alpha\beta}^{ij} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$$

$$\frac{\Delta m^2_{ij}}{4E} = 1.267 \left(\frac{\Delta m^2_{ij}}{eV^2}\right)\left(\frac{x}{m}\right)\left(\frac{MeV}{E}\right)$$

(You find these formulae in the PDG 2009 ν review, with $i \leftrightarrow j$)

Solution –

- $H_{mass} = p\,\delta_{ij} + \frac{m_i^2}{2E}\delta_{ij} = p\,\mathbb{1} + \frac{1}{2E}\,\mathrm{diag}(m_1^2, m_2^2, m_3^2)$
- Any term proportional to $\mathbb{1}$ can be dropped in oscillation phenomena (it just shifts all energies by the same amount, and contributes only to an unobservable overall phase).
- Since H does not depend on x, the evolution operator is simply obtained by exponentiation: $\hat{S} = \exp(-i\hat{H}x)$.
- In mass basis: $S_{ji} = \langle \nu_j | \hat{S} | \nu_i \rangle = \delta_{ij}\, \cancel{e^{-ipx}}\, e^{-i\frac{m_i^2}{2E}x}$ and we drop the overall phase.
- In flavor basis: $S_{\beta\alpha} = \langle \nu_\beta | \hat{S} | \nu_\alpha \rangle = \sum_{ij} U_{\beta j} S_{ji} U_{\alpha i}^* = \sum_i U_{\alpha i}^* U_{\beta i}\, e^{-i\frac{m_i^2}{2E}x}$
- Flavor oscillation probability:

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) &= |S_{\beta\alpha}|^2 \\
&= \Big|\sum_i U_{\alpha i}^* U_{\beta i}\, e^{-i\frac{m_i^2}{2E}x}\Big|^2 \\
&= \sum_{ij} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\, e^{-i\frac{m_i^2}{2E}x}\, e^{+i\frac{m_j^2}{2E}x} \\
&= \sum_{ij} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \left(e^{i\frac{m_j^2 - m_i^2}{2E}x} - 1 + 1\right) \\
&= \sum_{ij} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \left(e^{i\frac{m_j^2 - m_i^2}{2E}x} - 1\right) + \sum_{ij} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \\
&= \Big(\sum_{i<j} + \sum_{i>j}\Big) U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \left(e^{i\frac{m_j^2 - m_i^2}{2E}} - 1\right) + \sum_i U_{\alpha i}^* U_{\beta i} \sum_j U_{\alpha j} U_{\beta j}^*
\end{aligned}
$$

→ cont'd

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$$= \sum_{i>j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \left(e^{i \frac{m^2_j - m^2_i}{2E} x} - 1 \right) + \sum_{i>j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \left(e^{-i \frac{m^2_j - m^2_i}{2E}} - 1 \right) + \delta_{\alpha\beta} \cdot \delta_{\alpha\beta}$$

$$= \sum_{i>j} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} + U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \left[\cos\left(\frac{m^2_j - m^2_i}{2E} x \right) - 1 \right]$$

$$+ \sum_{i>j} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} - U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \left[i \sin\left(\frac{m^2_j - m^2_i}{2E} x \right) \right] + \delta_{\alpha\beta}$$

$$= \delta_{\alpha\beta} - \sum_{i>j} 2 \,\mathrm{Re} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \left(\cos\left(\frac{m^2_j - m^2_i}{2E} x \right) - 1 \right) - \sum_{i>j} 2 \,\mathrm{Im} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \sin\left(\frac{m^2_j - m^2_i}{2E} x \right)$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \sin^2\left(\frac{m^2_j - m^2_i}{4E} x \right) - 2 \sum_{i>j} \mathrm{Im} \left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right) \sin\left(\frac{m^2_j - m^2_i}{2E} x \right)$$

$$\overset{i \leftrightarrow j}{=} \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\, J^{ij}_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{ij} x}{4E} \right) - 2 \sum_{i<j} \mathrm{Im}\, J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij} x}{2E} \right) .$$

$\hbar c = 197.327$ MeV $\cdot$ fm $\equiv 1$ in natural units $\rightarrow 1$ MeV $\cdot$ 1 m $= 5.0677 \times 10^{12}$

$$\frac{\Delta m^2_{ij}\, x}{4E} = \frac{1}{4} \left(\frac{\Delta m^2_{ij}}{eV^2} eV^2 \right) \left(\frac{x}{m} \cdot m \right) \left(\frac{MeV}{E} \cdot \frac{1}{MeV} \right)$$

$$= \frac{1}{4} \left(\frac{1\, eV^2 \cdot 1\, m}{1\, MeV} \right) \left(\frac{\Delta m^2_{ij}}{eV^2} \right) \left(\frac{x}{m} \right) \left(\frac{MeV}{E} \right)$$

$$= \frac{10^{-12}}{4} (MeV \cdot m) \left(\frac{\Delta m^2_{ij}}{eV^2} \right) \left(\frac{x}{m} \right) \left(\frac{MeV}{E} \right)$$

$$= 1.267 \left(\frac{\Delta m^2_{ij}}{eV^2} \right) \left(\frac{x}{m} \right) \left(\frac{MeV}{E} \right)$$

skip

Exercise: $3\nu \to 2\nu$ reduction for SBL reactor expts.

Short-baseline reactor experiments look for $\bar{\nu}_e$ oscillations at $x = L \sim O(1\,\mathrm{Km})$ and $E \sim$ few MeV. At these energies, CC reactions in the final state can produce e^+ but not μ^+ or τ^+; therefore, only $P(\bar{\nu}_e \to \bar{\nu}_e)$ is observable (disappearance) but not $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ or $P(\bar{\nu}_e \to \bar{\nu}_\tau)$ (appearance). Moreover, it is $\delta m^2 L/4E \ll 1$, while $\Delta m^2 L/4E \sim \mathcal{O}(1)$.

Prove that, in the limit $\delta m^2 \simeq 0$, effective 2ν oscillations occur:

$$\boxed{P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \underbrace{\sin^2 2\theta_{13}}_{\text{oscillation amplitude}} \; \underbrace{\sin^2\left(\frac{\Delta m^2 L}{4E}\right)}_{\text{oscillating factor}}}$$

Try to get an intuitive understanding of the dependence on θ_{13} only.

Solution – From the previous exercise (with $U \to U^*$), the only nonzero oscillating terms for $\alpha\beta = ee$ and $\delta m^2 = m_2^2 - m_1^2 \simeq 0$ are multiplied by: $J_{ee}^{13} = U_{e1}^* U_{e3} U_{e3}^* U_{e1} = |U_{e1}|^2 |U_{e3}|^2$ and $J_{ee}^{23} = |U_{e2}|^2 |U_{e3}|^2$. Then:

$\mathrm{Im}(J_{ee}^{13}) = 0 = \mathrm{Im}(J_{ee}^{23})$ and

$$
\begin{aligned}
P(\bar{\nu}_e \to \bar{\nu}_e) &= 1 - 4\left(|U_{e1}|^2|U_{e3}|^2 + |U_{e2}|^2|U_{e3}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&= 1 - 4|U_{e3}|^2\left(1 - |U_{e3}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \leftarrow |U_{e3}|^2 = s_{13}^2 \\
&= 1 - \sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)
\end{aligned}
$$

Intuitively: two of the three mixing rotations have ~no effect

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\cancel{(2\,3)}}_{\substack{\text{mixes}\\ \text{unobservable}\\ \text{flavors } (\nu_\mu, \nu_\tau)}} (1\,3)\, \underbrace{\cancel{(1\,2)}}_{\substack{\text{mixes}\\ \sim\text{degenerate}\\ \text{states } (\nu_1, \nu_2)}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \to \text{only } (1\,3) \text{ physical}
$$

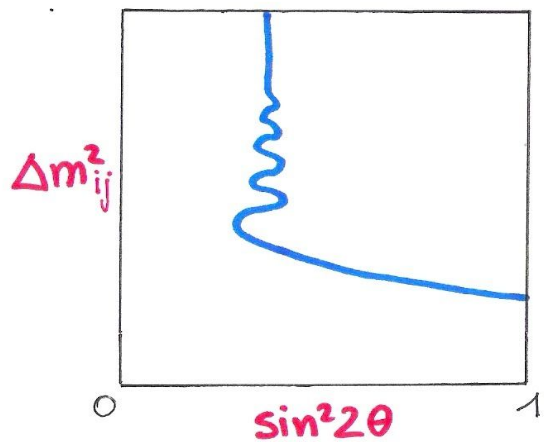
Note that, in this case, δ is unobservable, as well as sign$(\pm\Delta m^2)$, and $P(\bar{\nu}_e \to \bar{\nu}_e) = P(\nu_e \to \nu_e)$.

skip

Generic experimental constraints in 2ν approxim.

- Experiments measure some "averaged" $P_{\alpha\beta} \simeq \sin^2 2\theta \left\langle \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right)\right\rangle$

- Curve of iso-$P_{\alpha\beta}$:

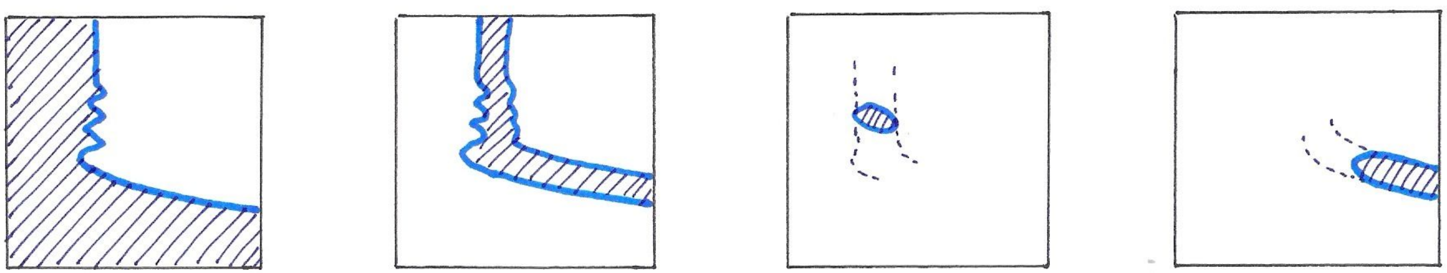


$\leftarrow \frac{\Delta m^2_{ij}\, x}{4E} \gg 1$, $\langle\cdots\rangle \sim \frac{1}{2}$, fast oscillations

$\leftarrow \frac{\Delta m^2_{ij}\, x}{4E} \sim \mathcal{O}(1)$

$\leftarrow \frac{\Delta m^2_{ij}\, x}{4E} \ll 1$, vanishing oscillations

- Possible expt. constraints:

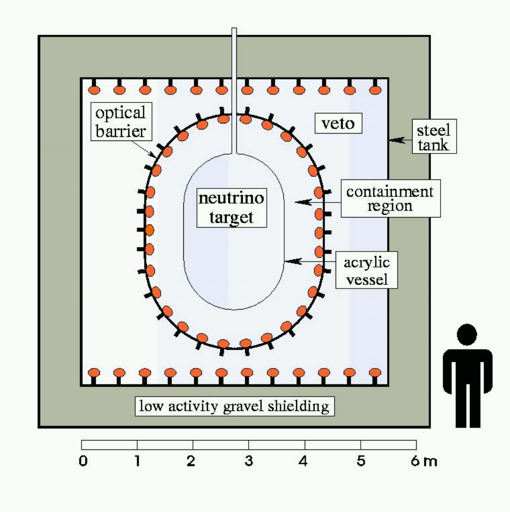


No signal — Signal — Precise signal at small mixing — Precise signal at large mixing

(need ≥ 2 expts or spectral data in 1 expt)

The short-baseline reactor experiment CHOOZ (1998)

L ~1 km →



Probably (one of) the most cited **negative** results ever!

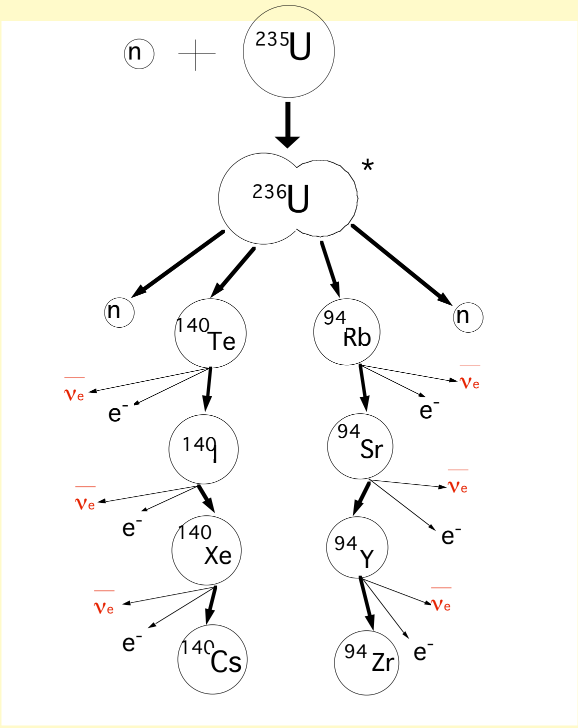
First data: Phys. Lett. B 466, 415 (1999) >1600 cites
Final data: Eur. Phys. J. C 27, 331 (2003) >1000 cites

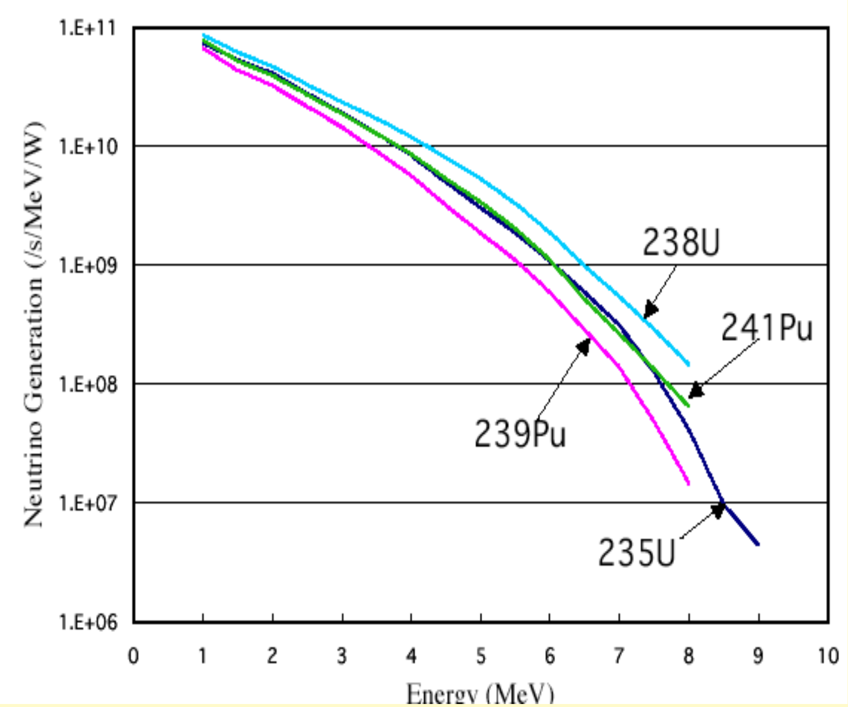
Production

Reactors: Intense sources of anti-ν_e (~6×10^{20}/s/reactor)

Typically, 6 neutron decays to reach stable matter from fission:

~200 MeV per fission / 6 decays: Typical available neutrino energy is E~ few MeV



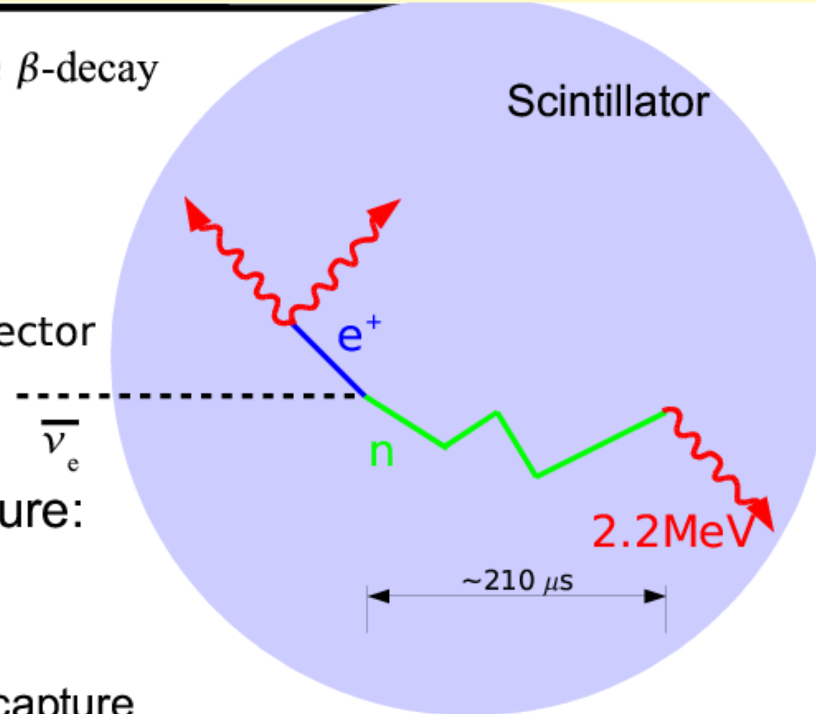


Detection

Reaction Process: inverse β-decay

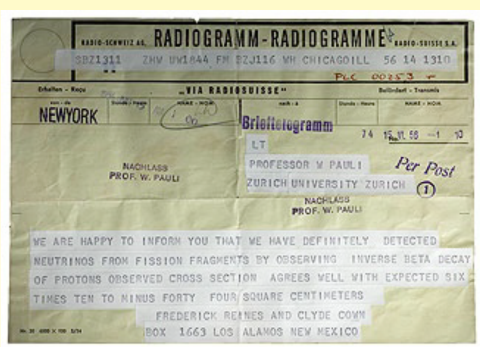
$$\overline{\nu}_e + p \rightarrow e^+ + n$$
$$n + p \rightarrow d + \gamma$$

Scintillator is target and detector



- Distinct two-step signature:
  - prompt event: positron
    $$E_\nu \approx E_{e^+} + 0.8\ MeV$$
  - delayed event: neutron capture after ~210μs
    - 2.2 MeV gamma

Delayed coincidence: good background rejection



RADIO-SCHWEIZ AG. RADIOGRAMM-RADIOGRAMME RADIO-SUISSE S.A.
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74 15.VI.56 -1 10
LT
NACHLASS PROF. W. PAULI
PROFESSOR W PAULI
Per Post
ZURICH UNIVERSITY ZURICH
NACHLASS PROF. W. PAULI

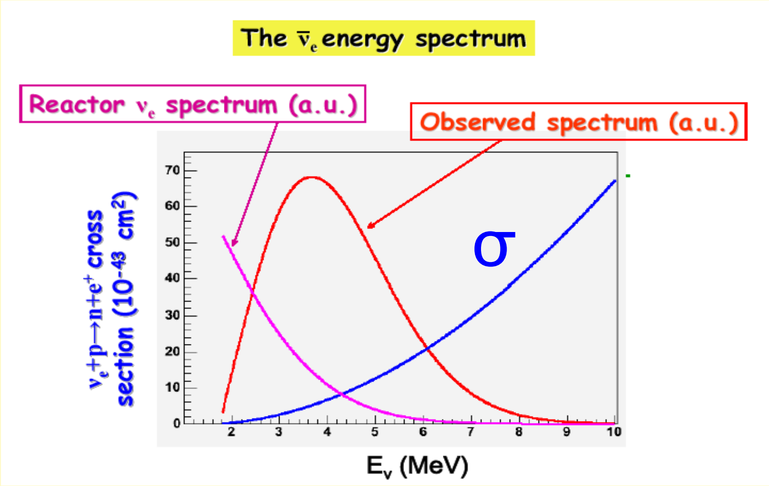
WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN
BOX 1663 LOS ALAMOS NEW MEXICO

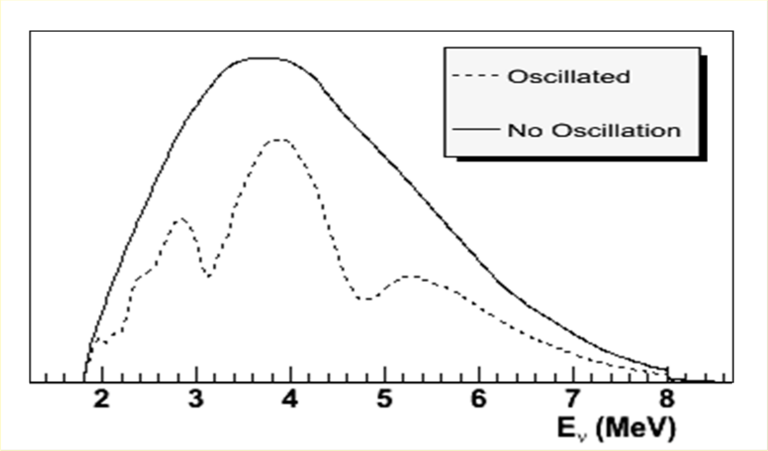
← This reaction allowed experimental neutrino discovery in 1956 (Reines & Cowan)

Results

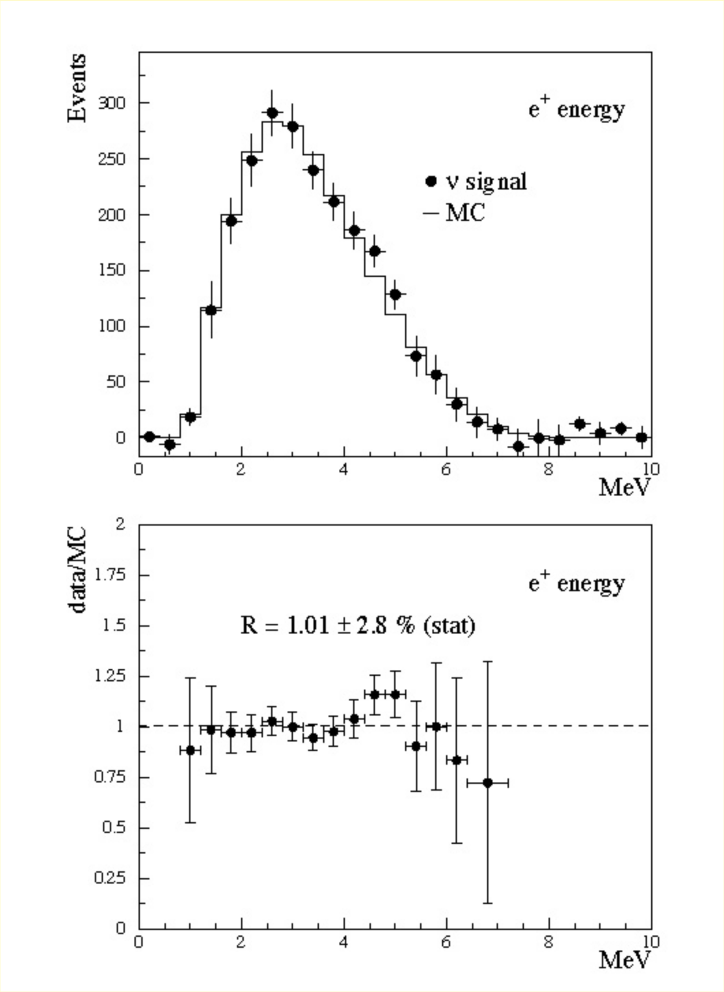
Expected spectrum (no oscill.):



With oscillations (qualitative):



CHOOZ: no oscillations
within few % error



Interpretation

One mass scale dominance:
$P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L/4E_\nu)$

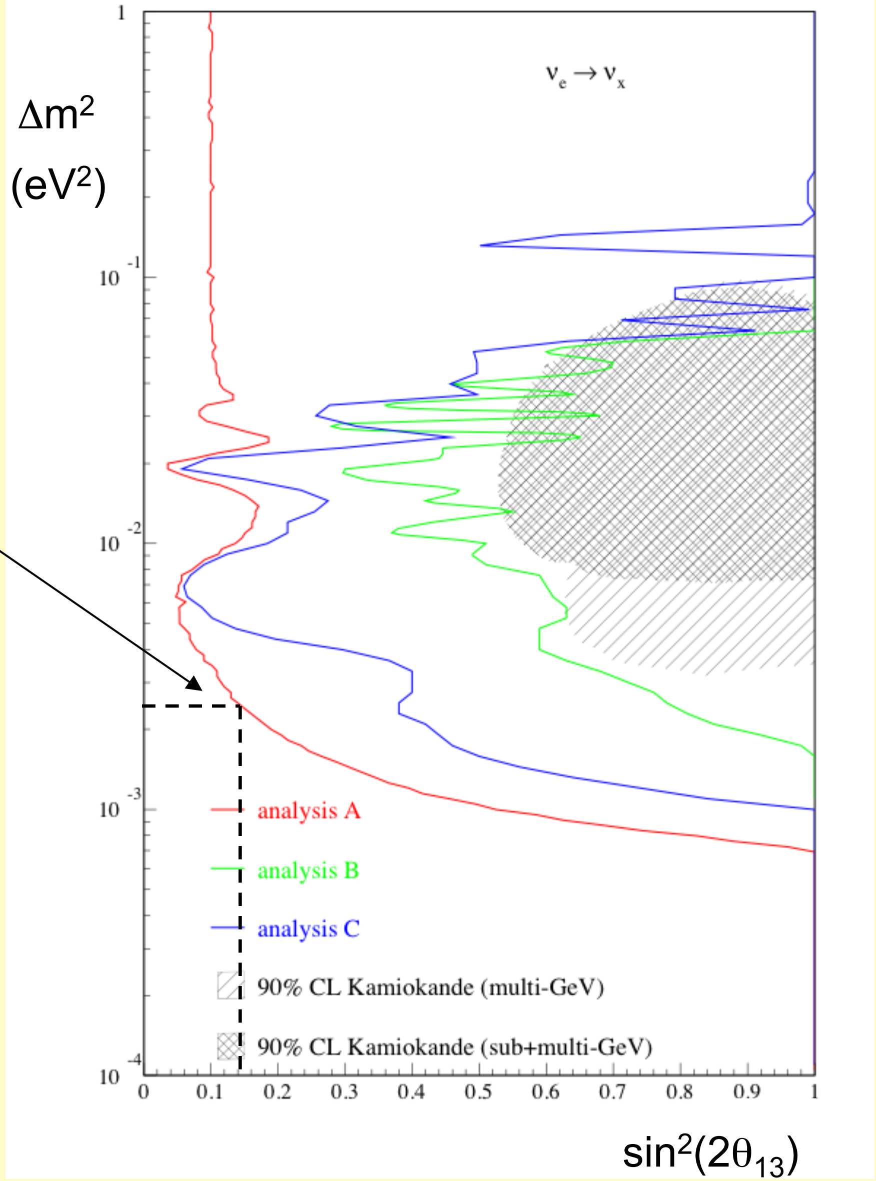
For any value of Δm^2 in the range allowed by atmospheric data (see next), get stringent upper bound on θ_{13}

$\sin^2 \theta_{13}$ < few %
(depending on Δm^2)

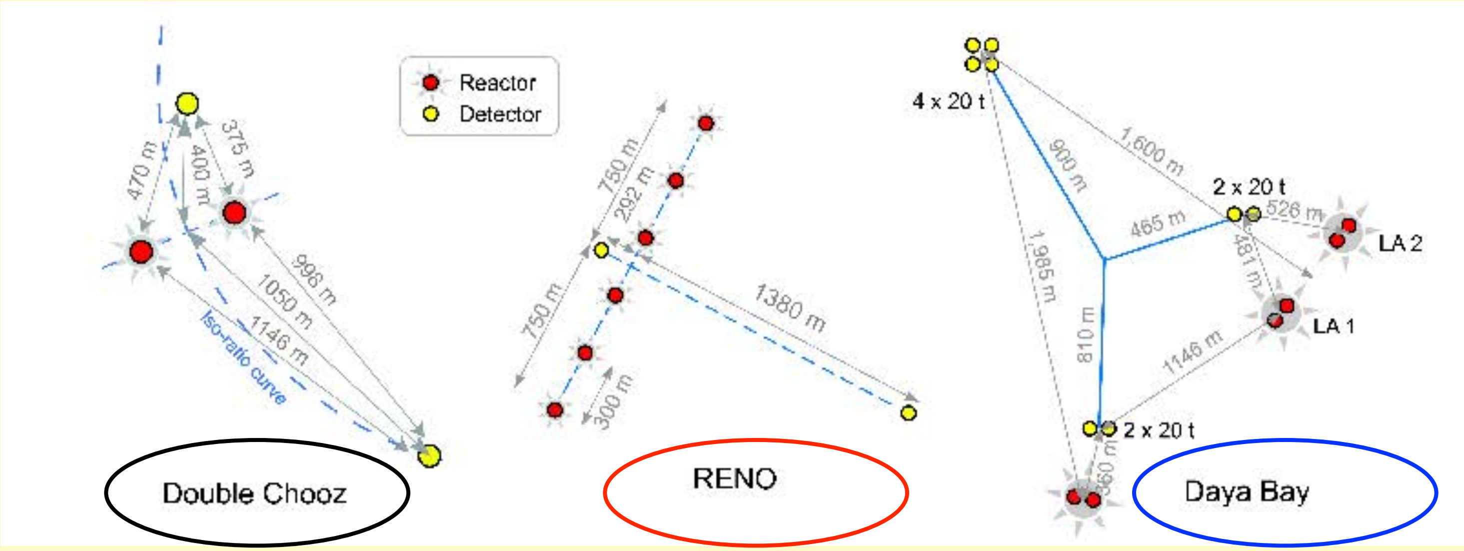
... Nobody could know at that time, but θ_{13} was just behind the corner (less than a factor of two in sensitivity!)

In any case, it was clear that, to reach higher θ_{13} sensitivity, need to use a second (close) detector to reduce systematics by far/near comparison →

CHOOZ exclusion plot



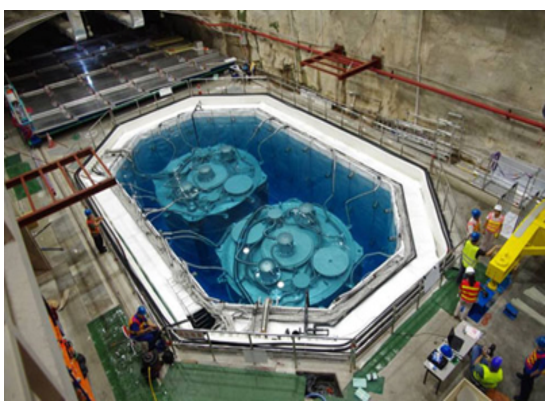
Reactor experiments with near & far detectors (ND & FD)



Running with FD;
ND this year!

Running with
ND & FD

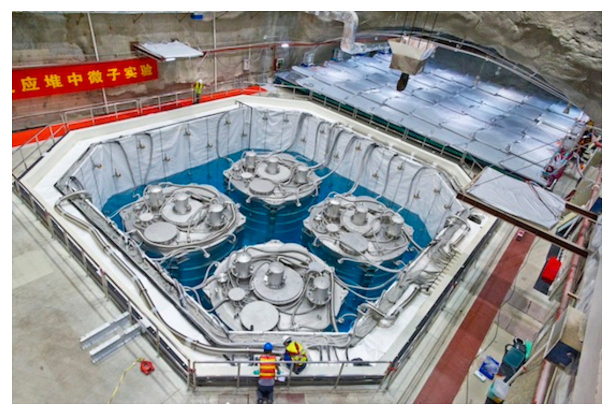
Running with
ND & FD



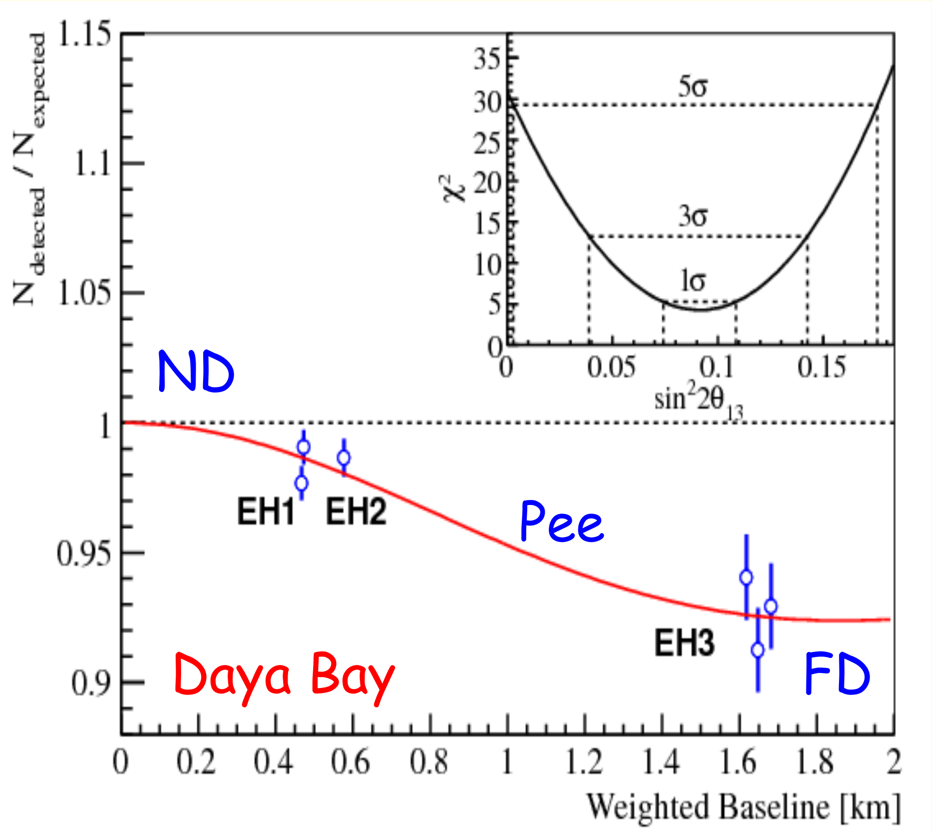
E.g, for
Daya Bay:

← ND

FD →



2012: discovery of θ_{13}>0 ! ($\sin^2\theta_{13}$~0.024 at ~fixed Δm^2)

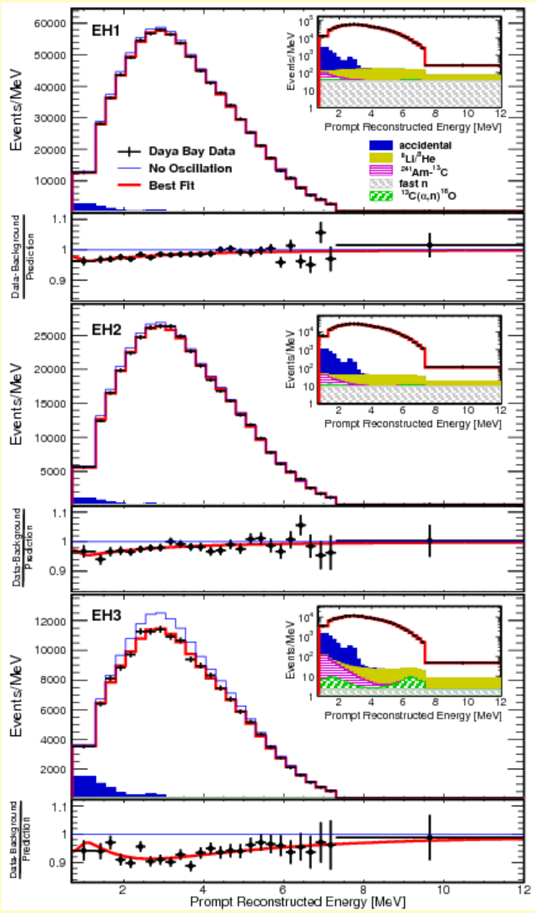


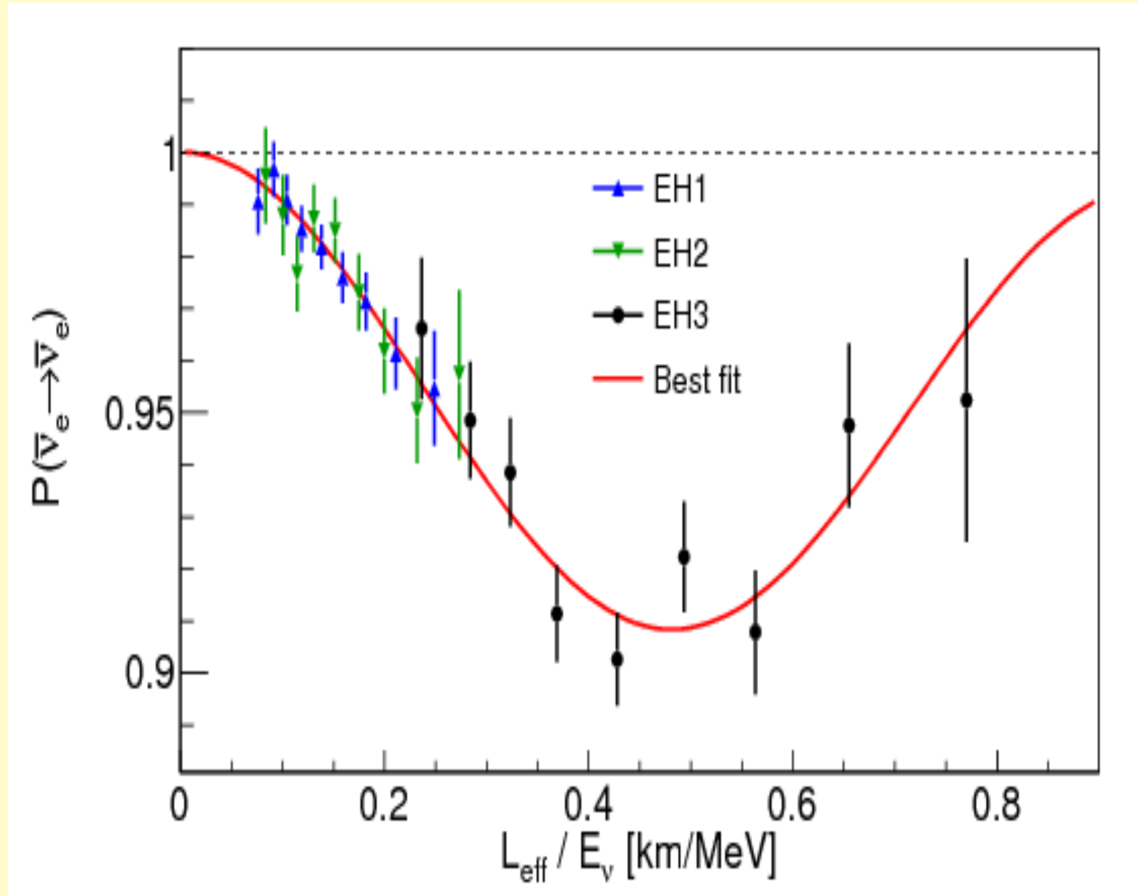
Daya Bay (& RENO): disappearance at FD w.r.t. ~unoscillated at ND

Double Chooz results (FD only) also consistent with Daya Bay & RENO.

Interestingly: approximate value of θ_{13} was previously hinted from other data.
Weaker signals were also coming from other expts (see later).

2013: more Daya Bay data → spectral analyses → ½ osc. cycle in L/E!





Let us now proceed with other expt's mainly sensitive to Δm^2 →

Exercise: One-dominant-mass-scale approximation (vacuum)

Prove that, in experiments mainly sensitive to Δm^2, i.e.:

$$\frac{\Delta m^2 x}{4E} \sim \mathcal{O}(1) \qquad \text{and} \qquad \frac{\delta m^2 x}{4E} \ll 1$$

the oscillation probabilities depend only on $|\Delta m^2|$ and on the mixing with ν_3 (elements $|U_{\alpha 3}|$, governed by θ_{23} and θ_{13}):

$$\boxed{\begin{aligned} P_{\alpha\alpha} &= P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \simeq 1 - 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\sin^2\left(\frac{\Delta m^2 x}{4E}\right) \\ P_{\alpha\beta} &= P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \simeq 4|U_{\alpha 3}|^2|U_{\beta 3}|^2\sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad \alpha \neq \beta \end{aligned}}$$

where $|U_{e3}|^2 = s_{13}^2$, $|U_{\mu 3}|^2 = c_{13}^2 s_{23}^2$, $|U_{\tau 3}|^2 = c_{13}^2 c_{23}^2$

→ no sensitivity to $(\delta m^2, \theta_{12})$ of course, but also:

→ no sensitivity to hierarchy or $\cancel{CP}$ phase δ

→ no difference $\nu/\bar{\nu}$

Solution

$$P_{\alpha\alpha} = 1 - 4\,\mathrm{Re}\left(J_{\alpha\alpha}^{13} + J_{\alpha\alpha}^{23}\right) \sin^2\left(\frac{\Delta m^2 x}{4E}\right) - 2\,\mathrm{Im}\left(J_{\alpha\alpha}^{13} + J_{\alpha\alpha}^{23}\right) \sin\left(\frac{\Delta m^2 x}{2E}\right)$$

$$J_{\alpha\alpha}^{13} + J_{\alpha\alpha}^{23} = |U_{\alpha 1}|^2 |U_{\alpha 3}|^2 + |U_{\alpha 2}|^2 |U_{\alpha 3}|^2 = |U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right), \text{ with imaginary part} = 0$$

$$P_{\alpha\alpha} = 1 - 4\,|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right) \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

$$P_{\alpha\beta} = -4\,\mathrm{Re}\left(J_{\alpha\beta}^{13} + J_{\alpha\beta}^{23}\right) \sin^2\left(\frac{\Delta m^2 x}{4E}\right) - 2\,\mathrm{Im}\left(J_{\alpha\beta}^{13} + J_{\alpha\beta}^{23}\right) \sin\left(\frac{\Delta m^2 x}{2E}\right), \quad \alpha \neq \beta$$

$$\begin{aligned} J_{\alpha\beta}^{13} + J_{\alpha\beta}^{23} &= U_{\alpha 1} U_{\beta 1}^* U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 2} U_{\beta 2}^* U_{\alpha 3}^* U_{\beta 3} \\ &= U_{\alpha 3}^* U_{\beta 3}^* \left(U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^*\right) \quad \leftarrow \text{unitarity of } U \\ &= U_{\alpha 3}^* U_{\beta 3} \left(-U_{\alpha 3} U_{\beta 3}^*\right) \\ &= -|U_{\alpha 3}|^* |U_{\beta 3}|^2, \quad \text{with imaginary part} = 0 \end{aligned}$$

$$P_{\alpha\beta} = 4\,|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m^2 x}{4E}\right), \quad \alpha \neq \beta$$

skip

Phenomenological note :

The one-dominant-mass-scale approximation can be applied, e.g., to :

- atmospheric ν experiments (ATM) Super-Kamiokande ...
- long-baseline accelerator expts. (LBL) K2K, MINOS, T2K, OPERA ...
- short-baseline reactor expts. (SBR) CHOOZ, Daya Bay, RENO, D-Chooz ...

OPERA (LBL) : $$P(\nu_\mu \to \nu_\tau) \simeq \cos^4\theta_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad (*)$$

ATM + LBL : $$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4\, c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad (*)$$

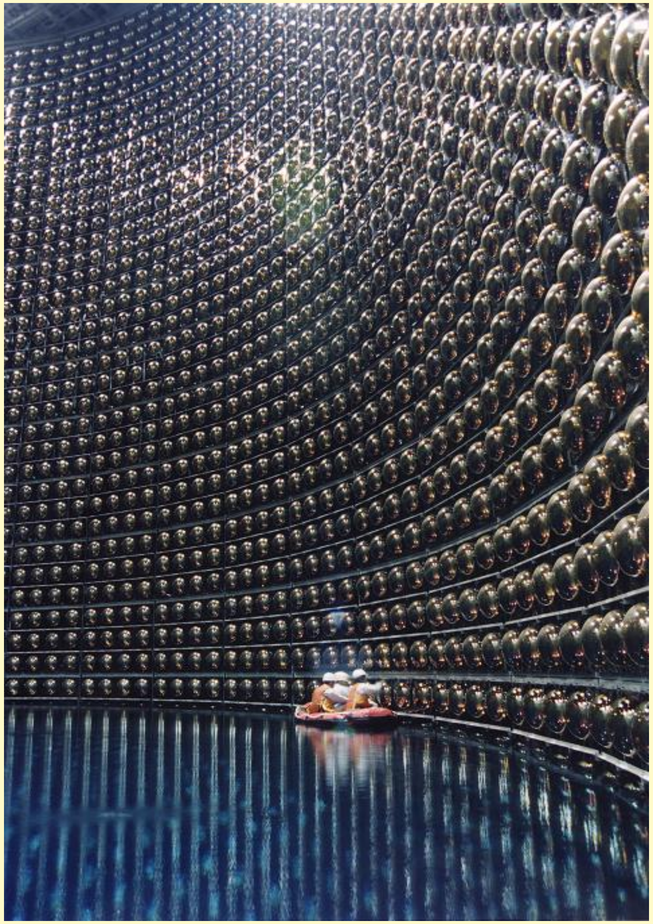
ATM + LBL : $$P(\nu_\mu \to \nu_e) \simeq s_{23}^2 \cdot \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad (**)$$

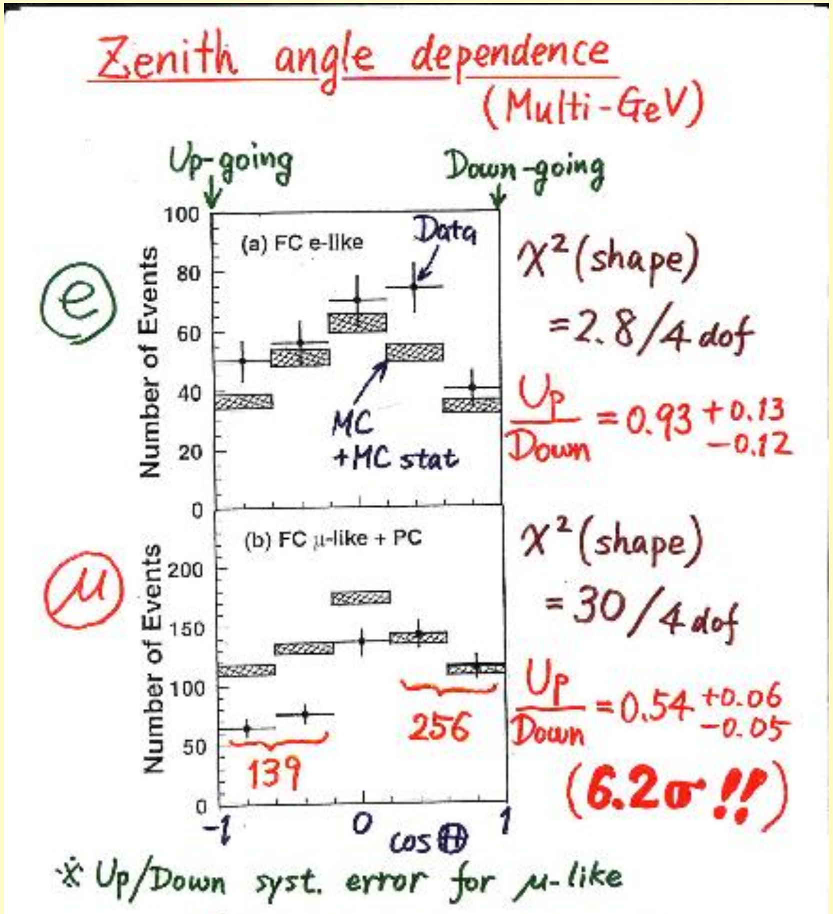
SBR : $$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad (**)$$

(*) : reduces to 2ν form for $\theta_{13} \to 0$ (pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations)

(**) : vanishes for $\theta_{13} \to 0$

Atmospheric neutrinos:
The 1998 Super-Kamiokande breakthrough

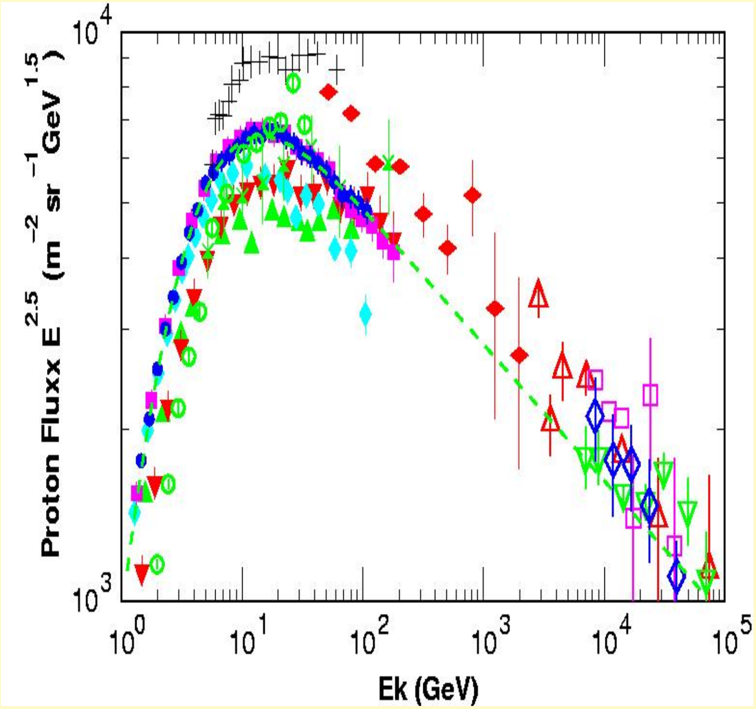


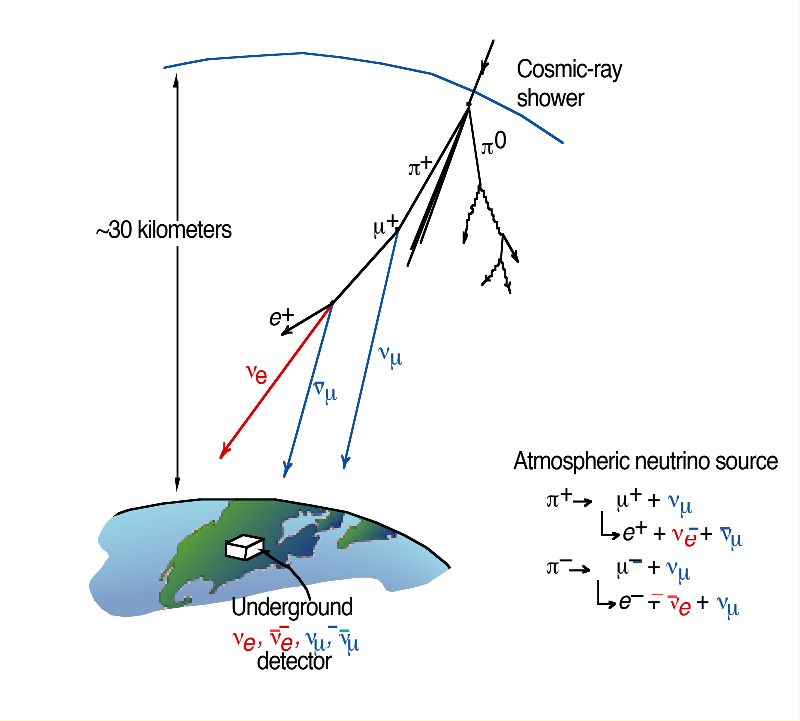


(T. Kajita at Neutrino' 98, Takayama)

Production

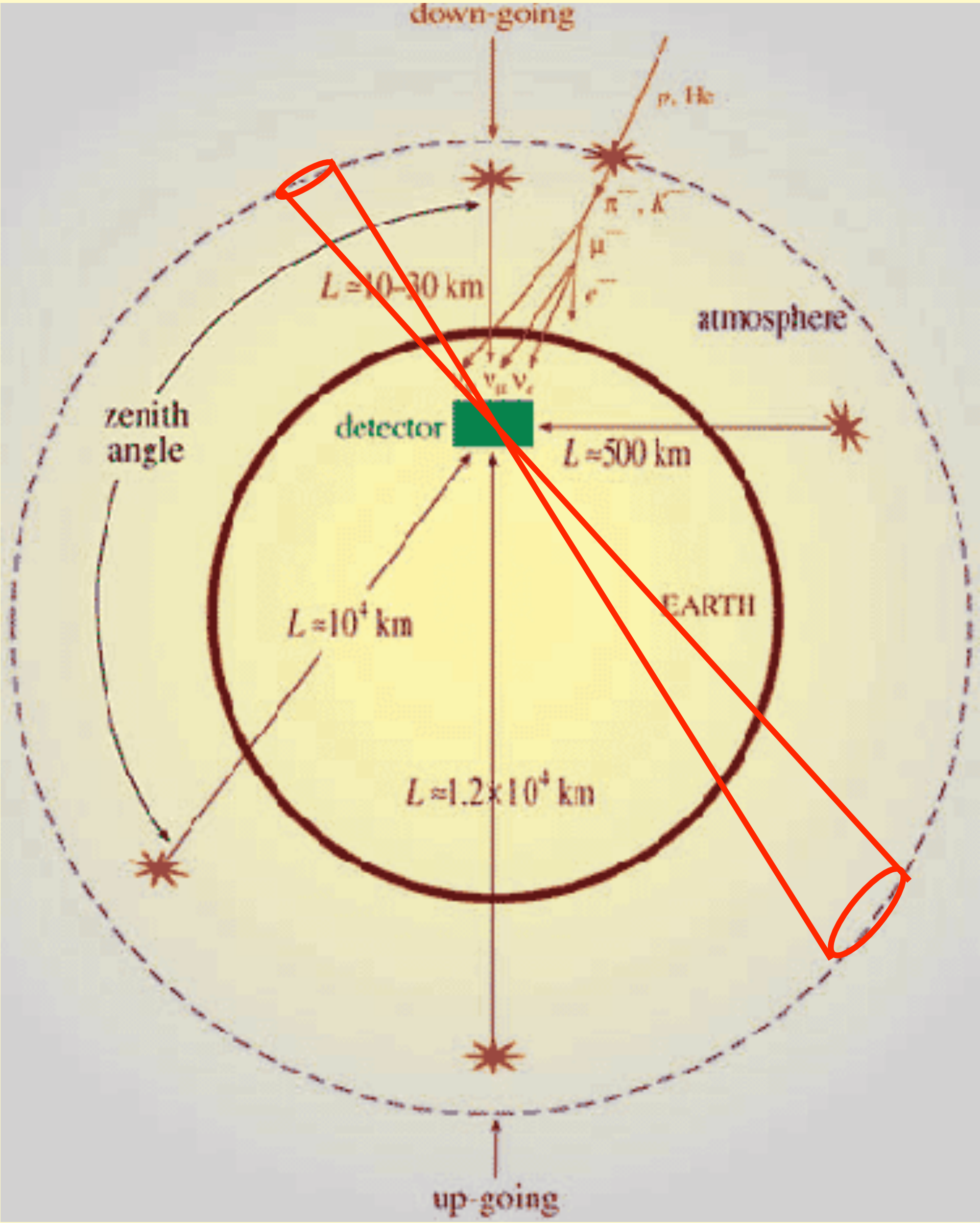
Cosmic rays hitting the atmosphere can generate secondary (anti)neutrinos with electron and muon flavor via meson decays.





Primary flux affected by large normalization uncertainties…

… but (anti)neutrino **flavor ratio (μ/e ~ 2)** robust within few %



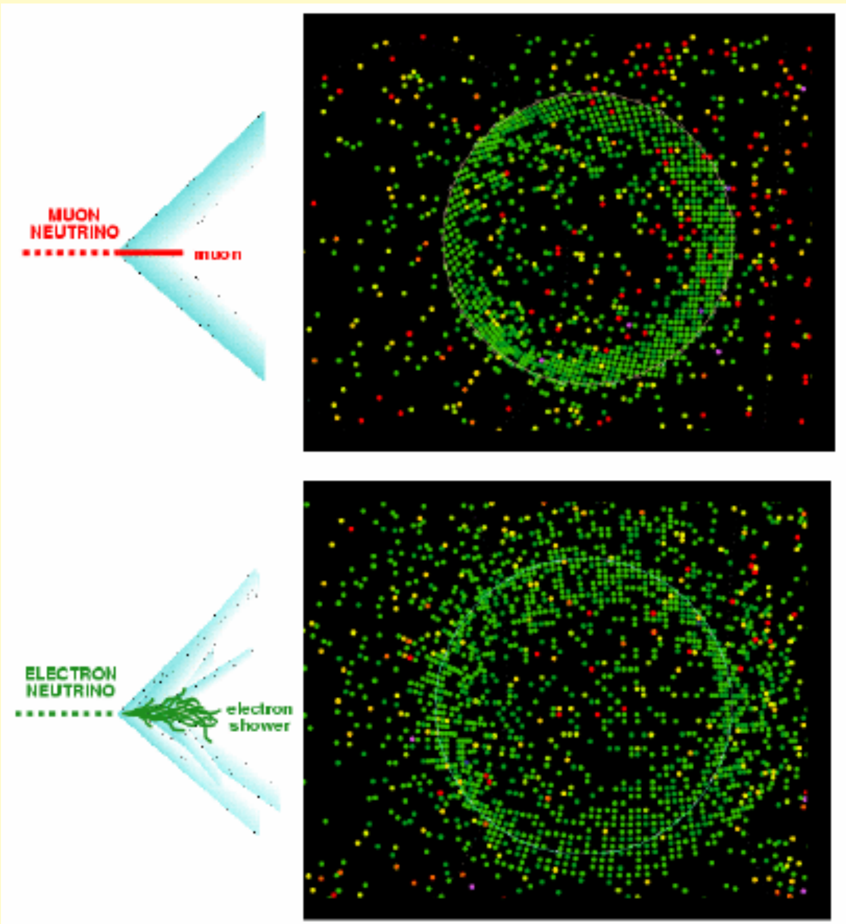
Moreover: same ν flux from opposite solid angles (up-down symmetry)

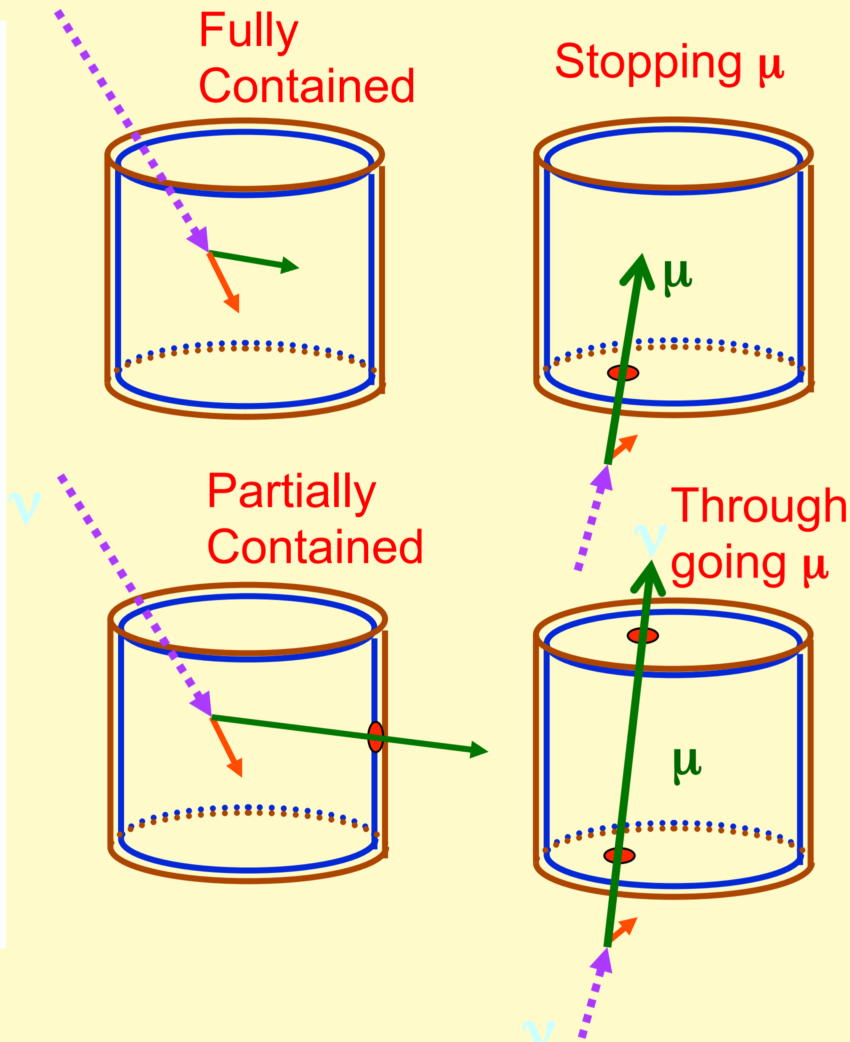
[Flux dilution (~1/r²) is compensated by larger production surface (~r²)]

Should be reflected in symmetry of event zenith spectra, if energy & angle can be reconstructed well enough

Detection in SK

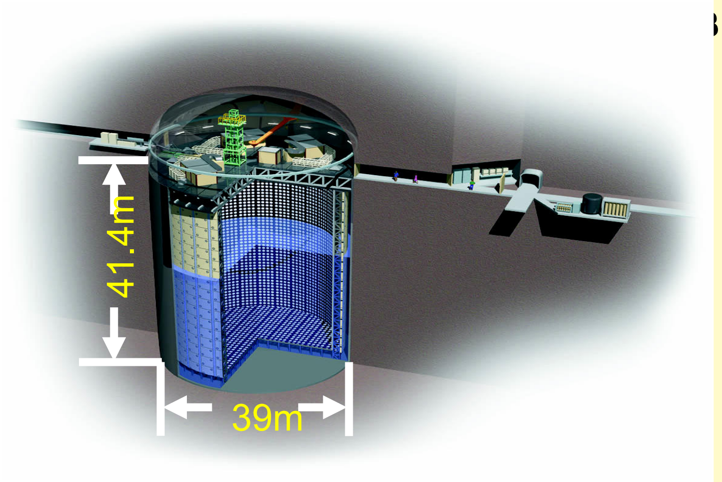
Parent neutrinos detected via CC interactions in the target (water).
Final-state μ and e distinguished by ≠ Cherenkov ring sharpness.
(But: no charge discrimination, no τ event reconstruction). Topologies:



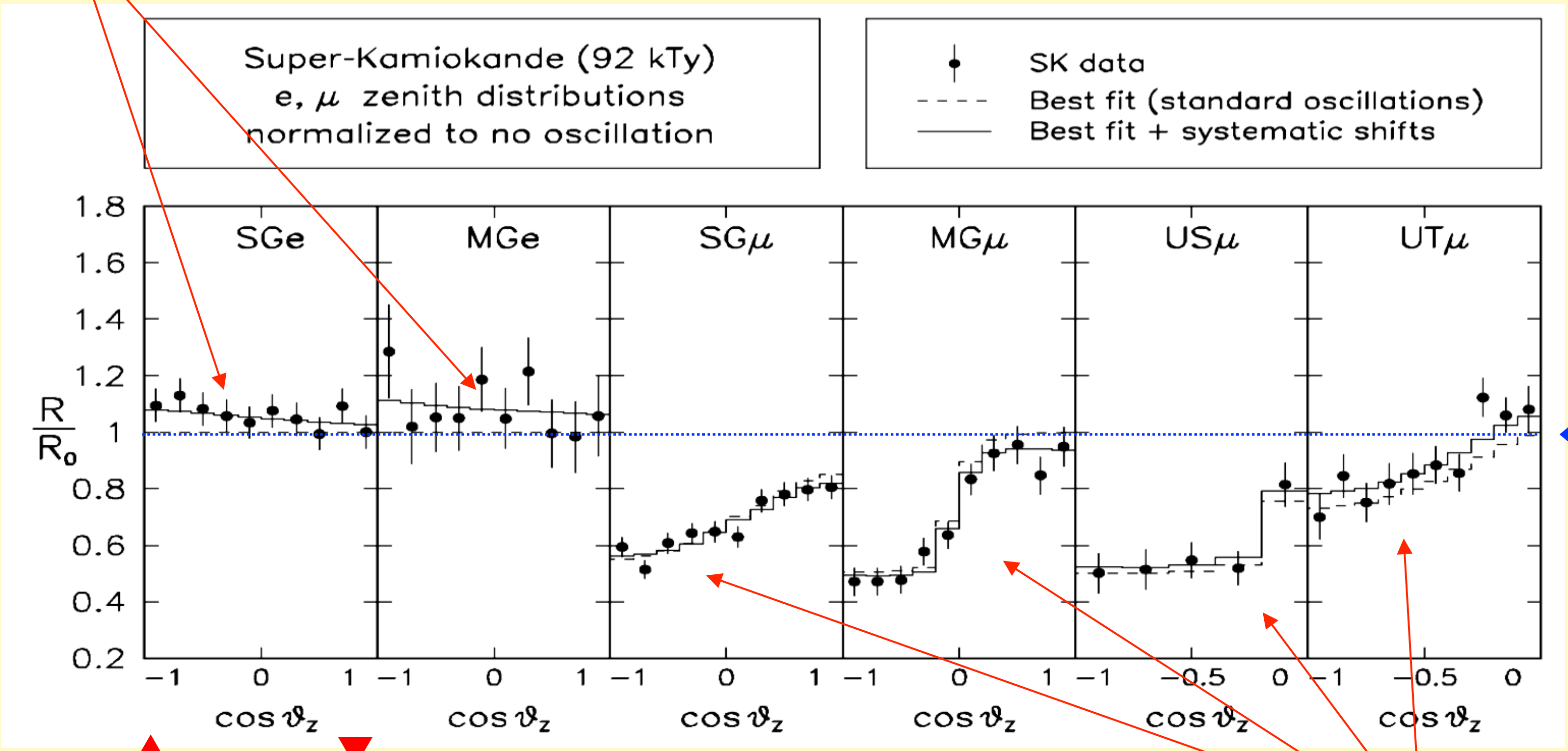


RESULTS SK zenith distributions

SGe	Sub-GeV electrons
MGe	Multi-GeV electrons
SGμ	Sub-GeV muons
MGμ	Multi-GeV muons
USμ	Upward Stopping muons
UTμ	Upward Through-going muons



electrons ~OK



up down

muon deficit from below

Observations over several decades in L/E:

ν_e induced events: ~ as expected

ν_μ induced events: disappearance from below

Interpretation in terms of oscillations:

Channel $\nu_\mu \rightarrow \nu_e$? No (or subdominant) ← CHOOZ OK!

Channel $\nu_\mu \rightarrow \nu_\tau$? Yes (dominant)

One-mass-scale approximation (for θ_{13}~0):

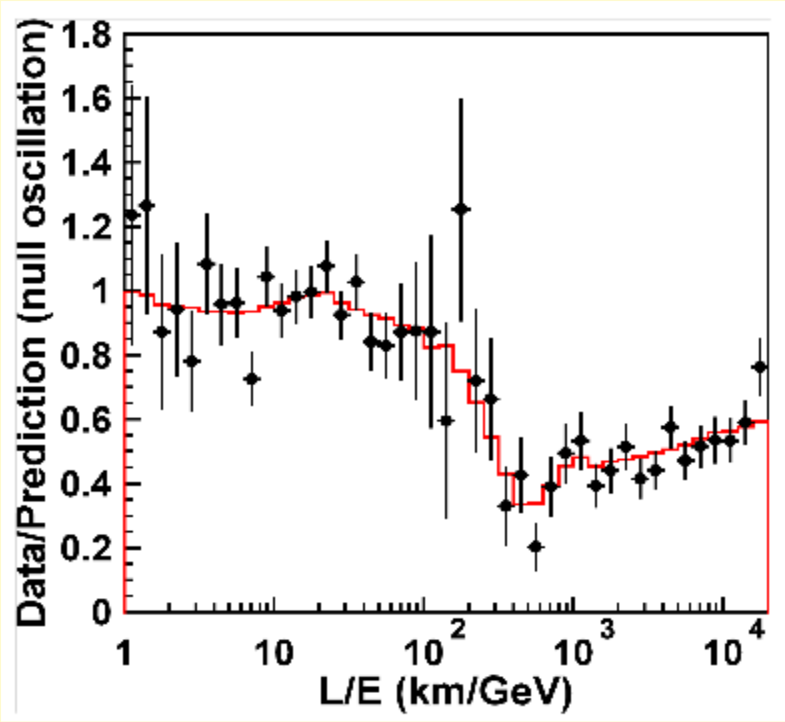
$$P_{\mu\tau} = \sin^2(2\theta_{23}) \sin^2(\Delta m^2 L/4E_\nu)$$

[In this channel, oscillations are ~vacuum-like, despite the presence of Earth matter]

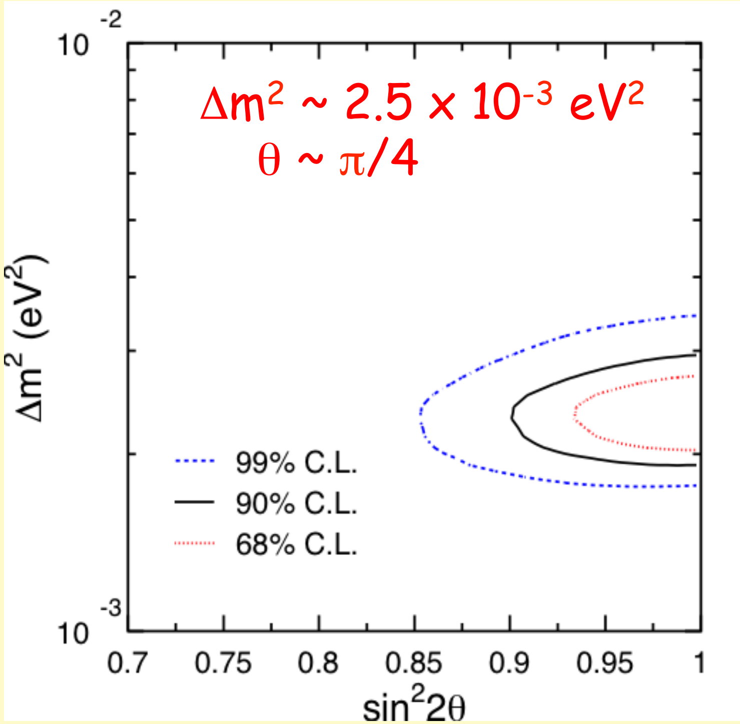
Results consistent with other atmos. expts. using different techniques (MACRO, Soudan2) but with lower statistics

Dedicated L/E analysis in SK "sees" half-period of oscillations

1st oscillation dip still visible despite large L & E smearing



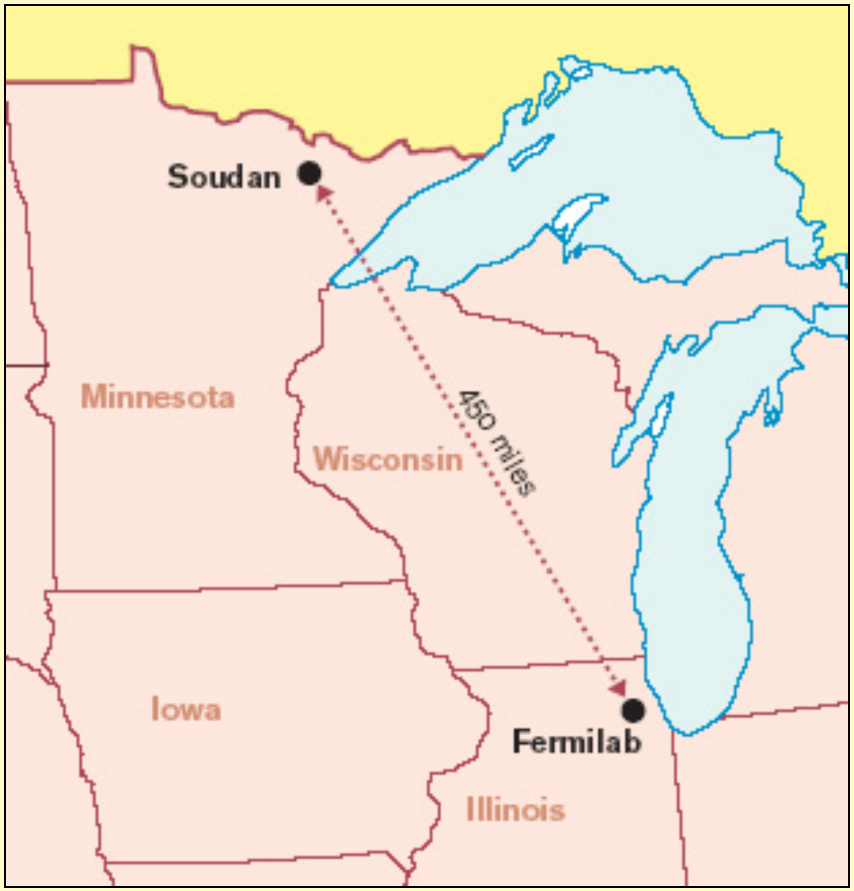
Strong constraints on the parameters (Δm^2, θ)

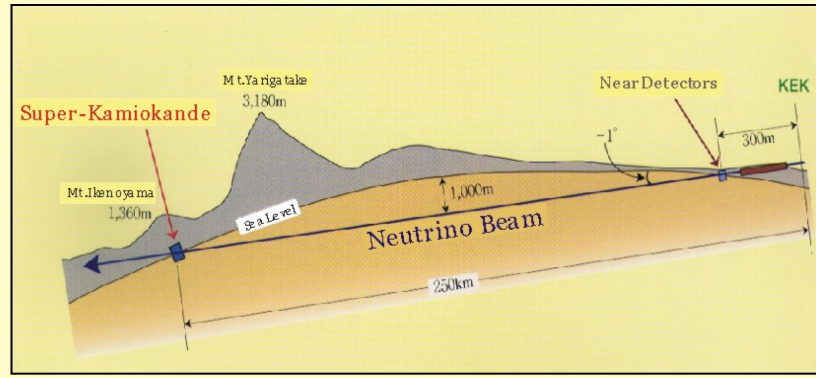


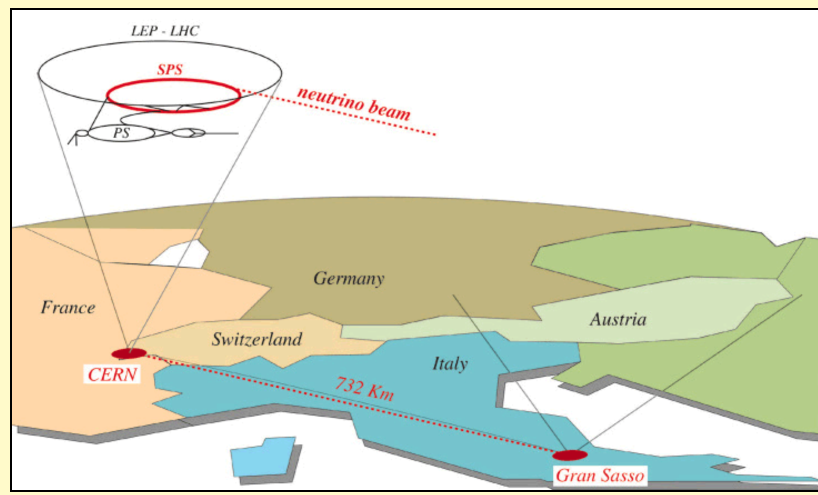
[Latest SK data analyses more refined: include many bins and syst. in order to "squeeze" subleading effects beyond dominant L/E]

Long-baseline neutrino experiments (K2K, MINOS, OPERA, T2K)

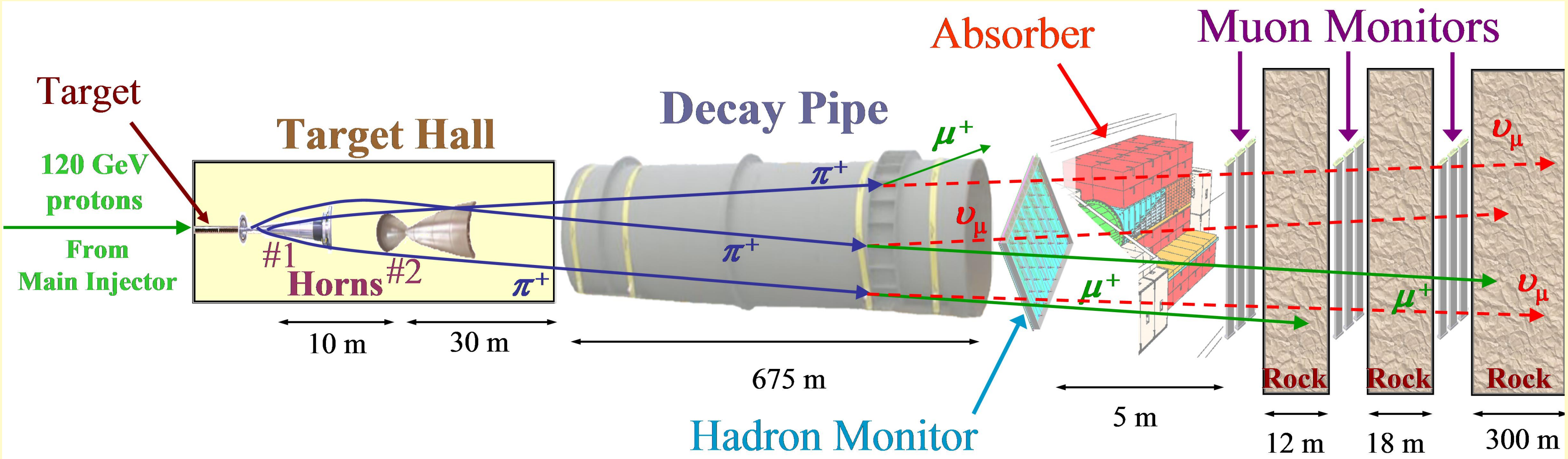
"Reproducing atmospheric ν_μ physics" in controlled conditions





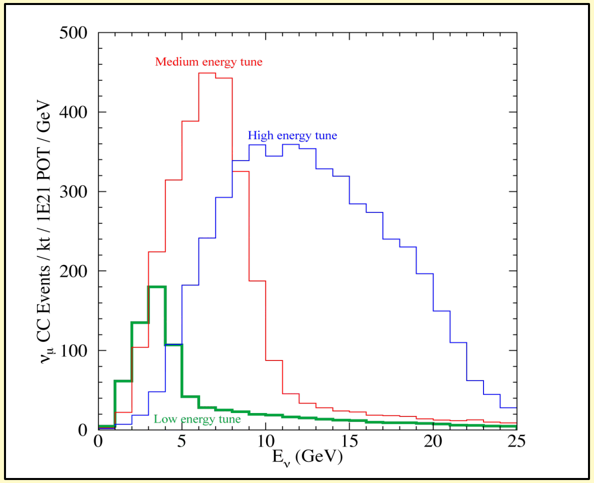


Production (e.g., MINOS)



π decay: ν energy is only function of $\nu\pi$ angle and π energy

Spectra:

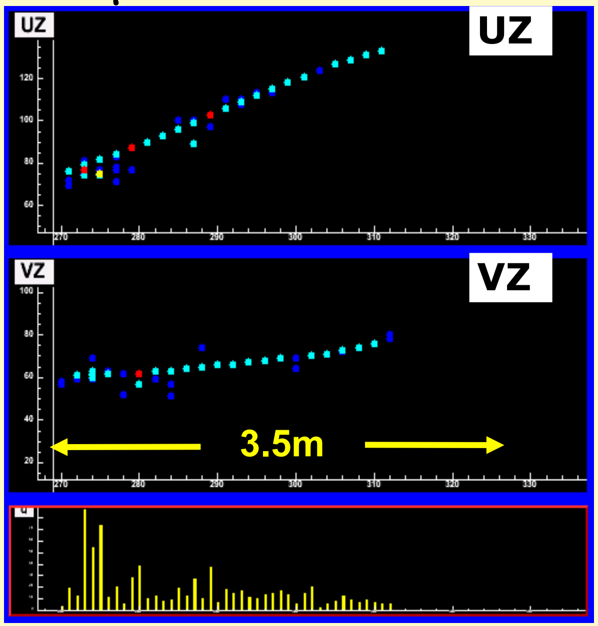


(Far) Detection

K2K, T2K: Cherenkov technique in SK

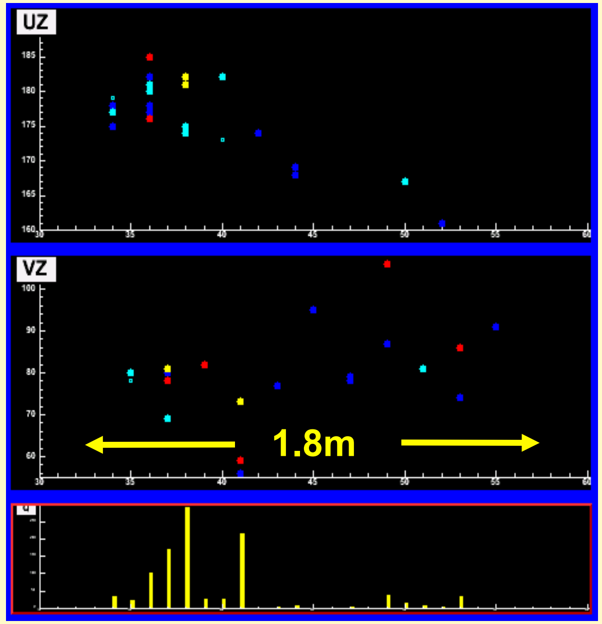
MINOS: Steel/Scintillator detector (+ magnetic field)

ν_μ CC Event



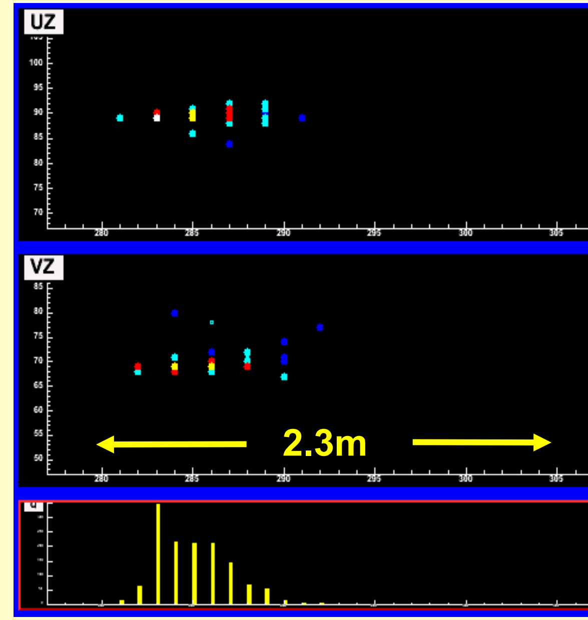
- Long muon track + hadronic activity at vertex

NC Event



- Short showering event, often diffuse

ν_e CC Event

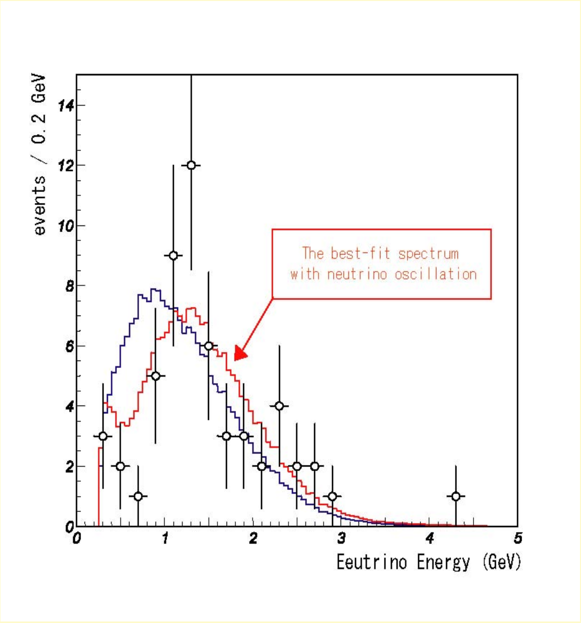


- Short event with typical EM shower profile

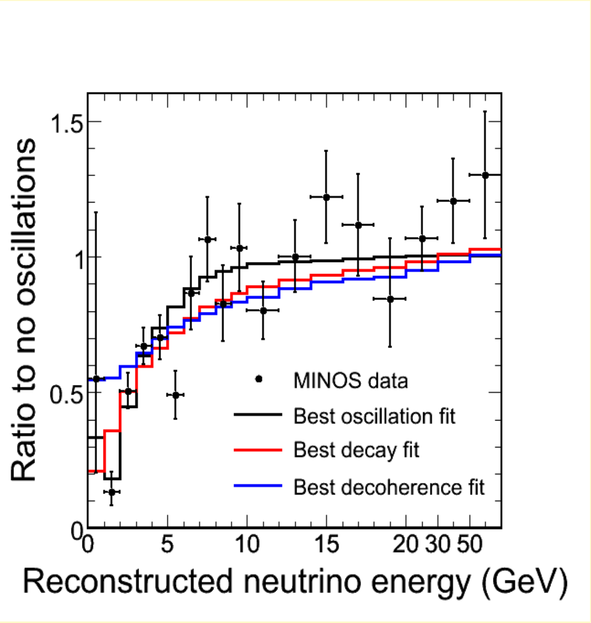
K2K, MINOS, T2K supplemented by near detectors to measure disappear. $P_{\mu\mu}$

Results in muon neutrino disappearance mode, $P_{\mu\mu}$

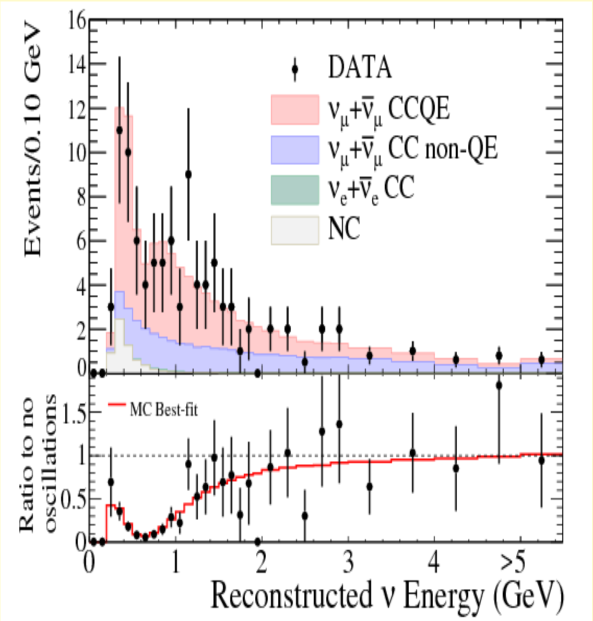
K2K



MINOS



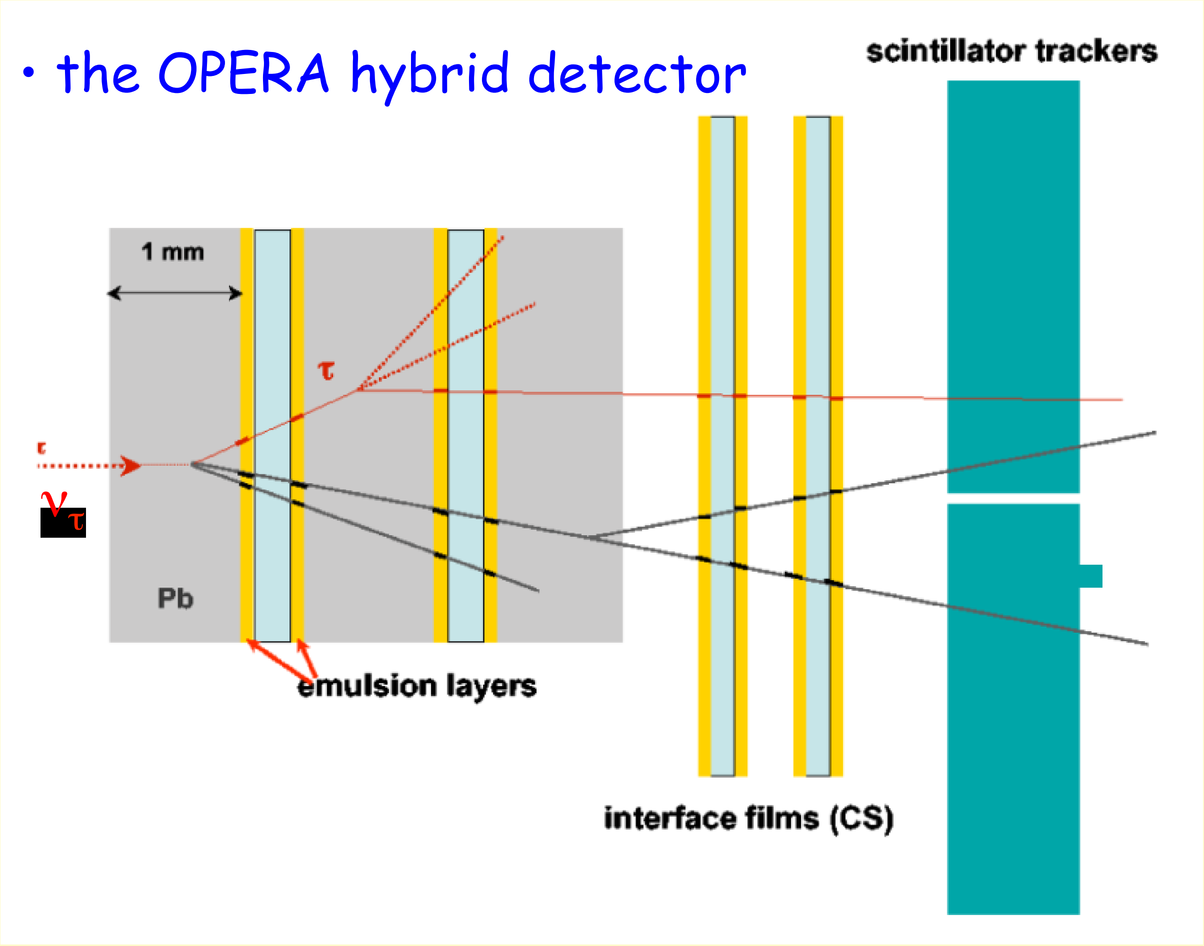
T2K



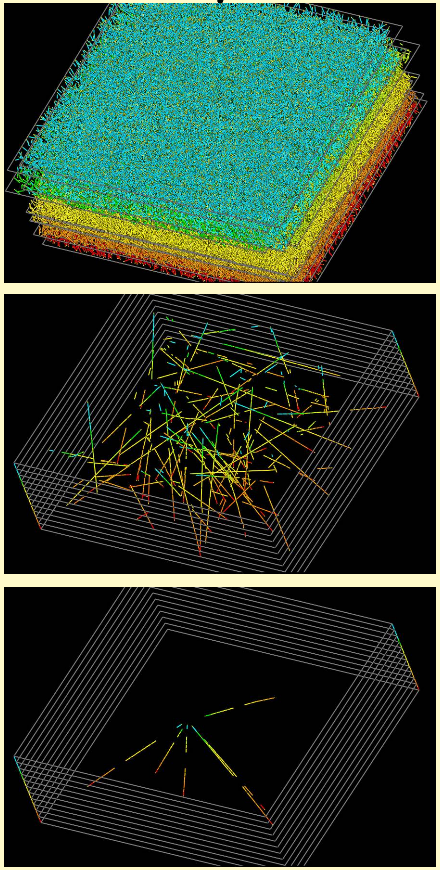
1st oscillation dip observed in energy spectrum (equivalent to L/E spectrum since L is fixed).

[Exotic explanations without dip (decay, decoherence) excluded]

Testing dominant oscillations via τ appearance: **OPERA**



Finding needles in a haystack…

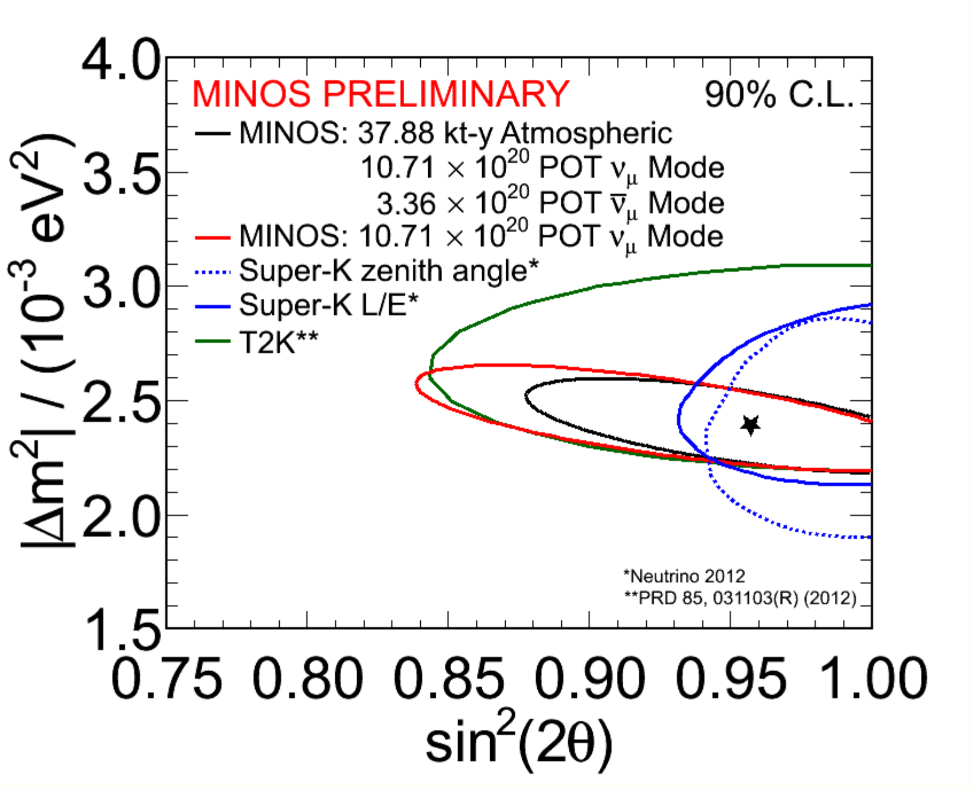


Four "τ needles" found! (consistent with expected signal)

Interpretation

Once more... dominant $P_{\mu\tau} = \sin^2(2\theta_{23})\,\sin^2(\Delta m^2 L/4E_\nu)$

Osc. parameters consistent among atm/LBL experiments...
... with recent, possible hints of nonmaximal θ_{23} mixing in MINOS

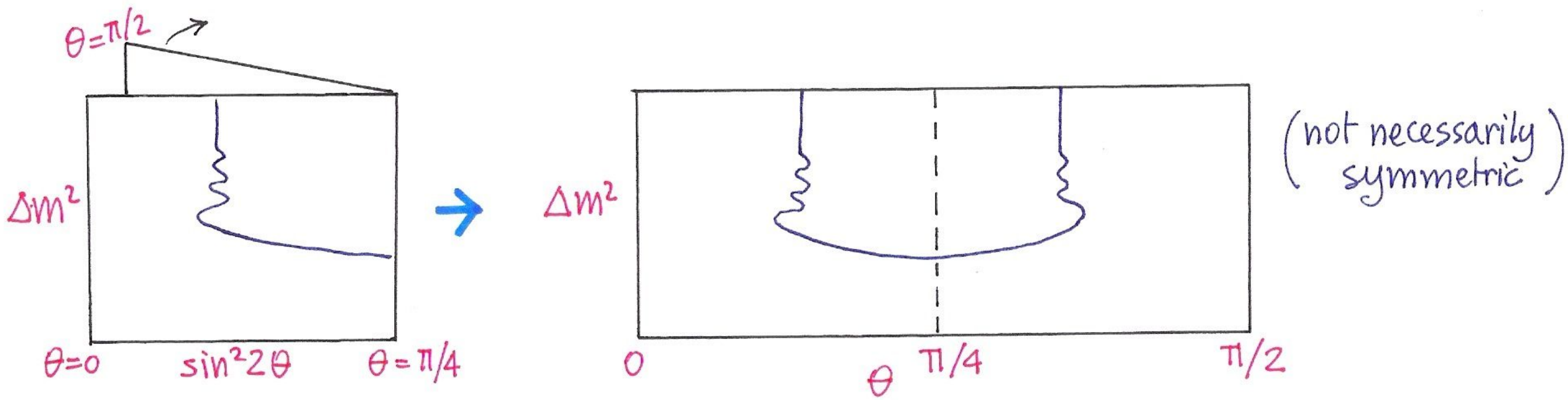


The format of such a "2ν" plot is, however, obsolete...

(Δm^2, θ_{23}) parameters...

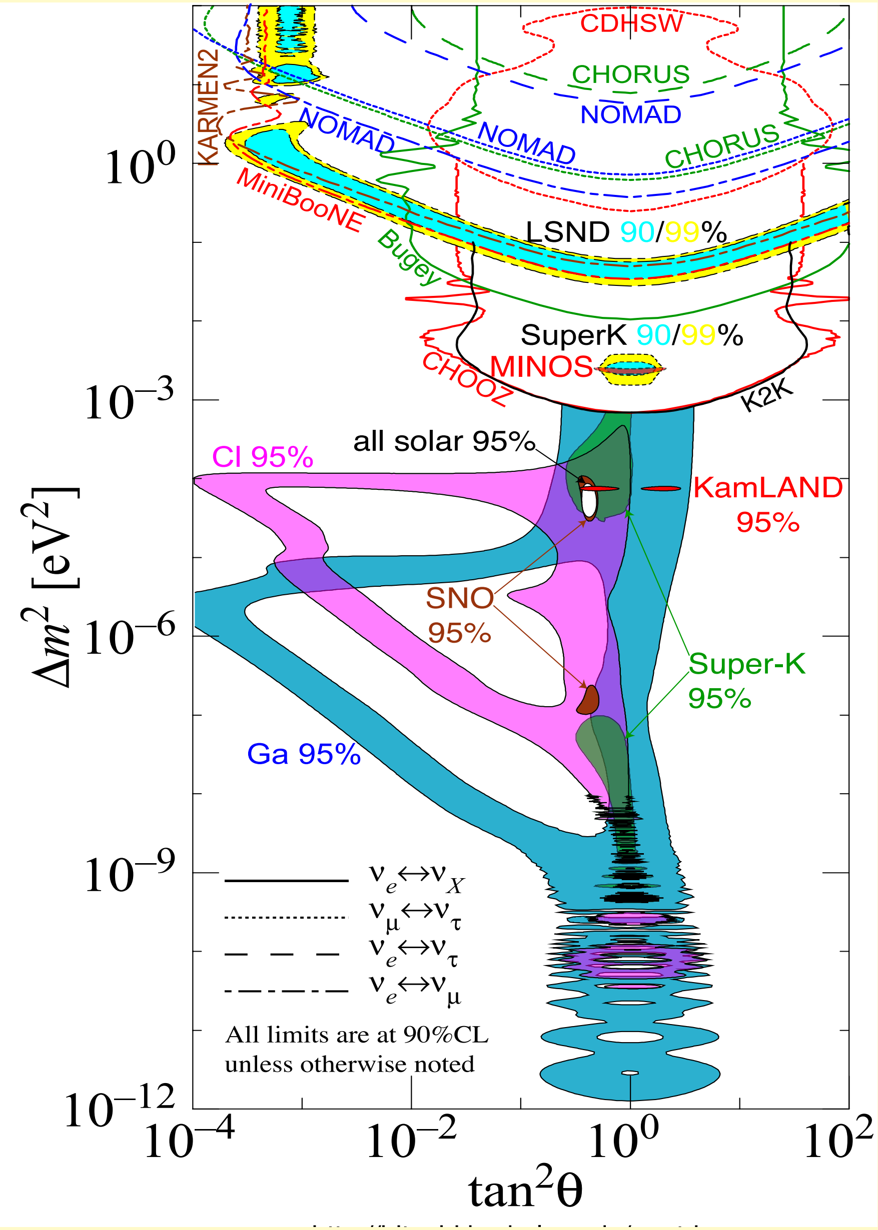
... are mainly determined by ATM+LBL expts. via $P(\nu_\mu \to \nu_\mu)$.

- $P_{\mu\mu}$ is octant symmetric (i.e., invariant for $\theta_{23} \to \frac{\pi}{2} - \theta_{23}$) only in the limit $\delta m^2 \to 0$ <u>and</u> $\theta_{13} \to 0$: $P_{\mu\mu} \cong 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$
- For $\theta_{13} \neq 0$ it is no longer octant-symmetric :
  $$P_{\mu\mu} \cong 1 - 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$
- Further effects ($\delta m^2 \neq 0$, matter) also contribute to asymmetry
  → need to unfold 2nd octant in general :



- Typical abscissa : either $\mathbf{\sin^2\theta}$ (linear scale) or $\mathbf{\tan^2\theta}$ (log scale)

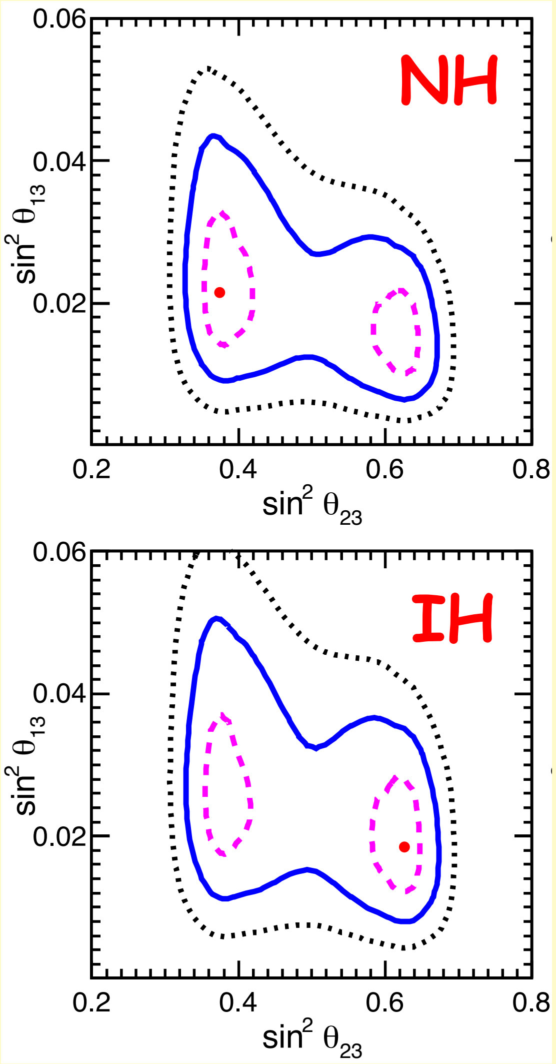
E.g., PDG 2012:



CDHSW
CHORUS
NOMAD
KARMEN2
NOMAD
NOMAD
CHORUS
MiniBooNE
LSND 90/99%
Bugey
SuperK 90/99%
MINOS
CHOOZ
K2K
all solar 95%
Cl 95%
KamLAND 95%
SNO 95%
Super-K 95%
Ga 95%
$\nu_e \leftrightarrow \nu_X$
$\nu_\mu \leftrightarrow \nu_\tau$
$\nu_e \leftrightarrow \nu_\tau$
$\nu_e \leftrightarrow \nu_\mu$
All limits are at 90%CL
unless otherwise noted
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
Δm^2 [eV^2]
10^{-4}
10^{-2}
10^{0}
10^{2}
$\tan^2\theta$

E.g., LBL appearance: $P_{\mu e} = \sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$ + corrections $\rightarrow$

θ_{23} octant asymmetry, anticorrel. of θ_{23} and θ_{13}: the lower θ_{23}, the higher θ_{13}



Bari group analysis of LBL results, 2012:

Latest LBL disappearance data from T2K and MINOS favored nonmaximal θ_{23}

From LBL appearance+disappear. data, two quasi-degenerate θ_{23} solutions emerged, in anticorrelation with θ_{13}
The two solutions merged above ~1σ.

No clear preference for one octant or for one hierarchy yet.

[2014 update at the end of this lecture]

Note:

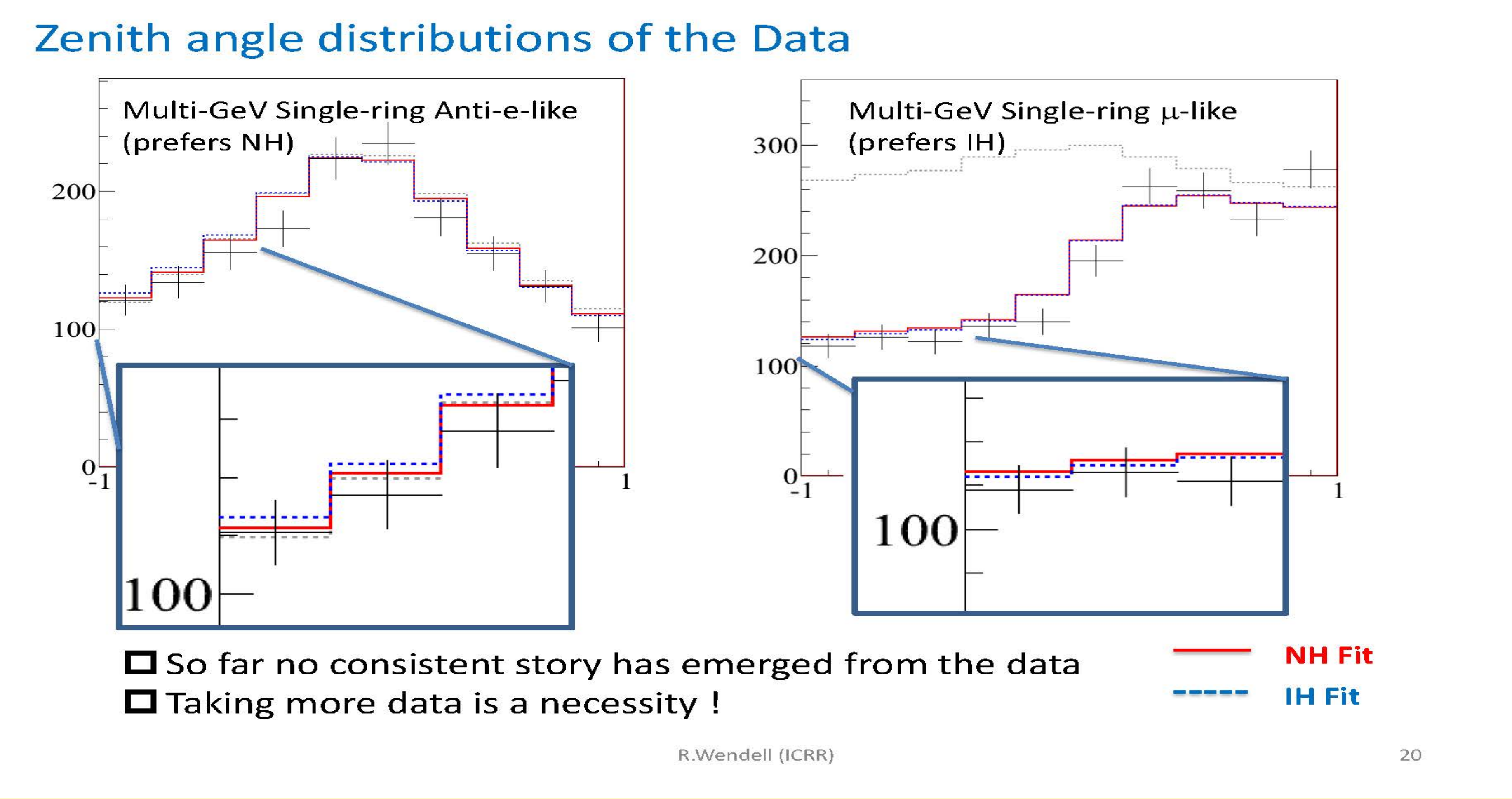
The previous experiments (LBL + ATM + SBR) allow to set constraints on $|\Delta m^2|$ and on the 3rd-column elements of the PMNS matrix (in absolute value):

$$|U| = \begin{pmatrix} \cdot & \cdot & |U_{e3}| \\ \cdot & \cdot & |U_{\mu 3}| \\ \cdot & \cdot & |U_{\tau 3}| \end{pmatrix} \leftarrow \text{functions of } \theta_{23}, \theta_{13}$$

Next frontier: subleading effects related to sign(Δm^2), δ, θ_{12}, δm^2, matter, ...

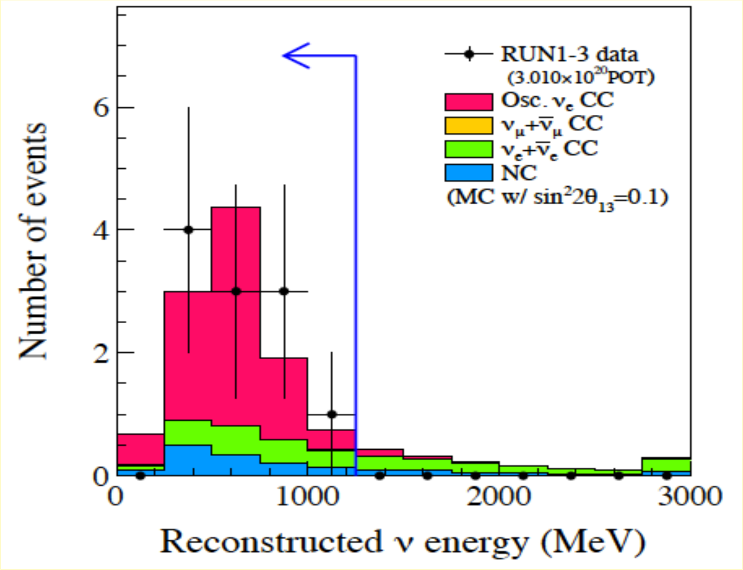
Even if not observed, all these effects "are there" and must be accounted for in state-of-the-art analyses; unfortunately, it is difficult to see (and then disentangle) them within current uncertainties.

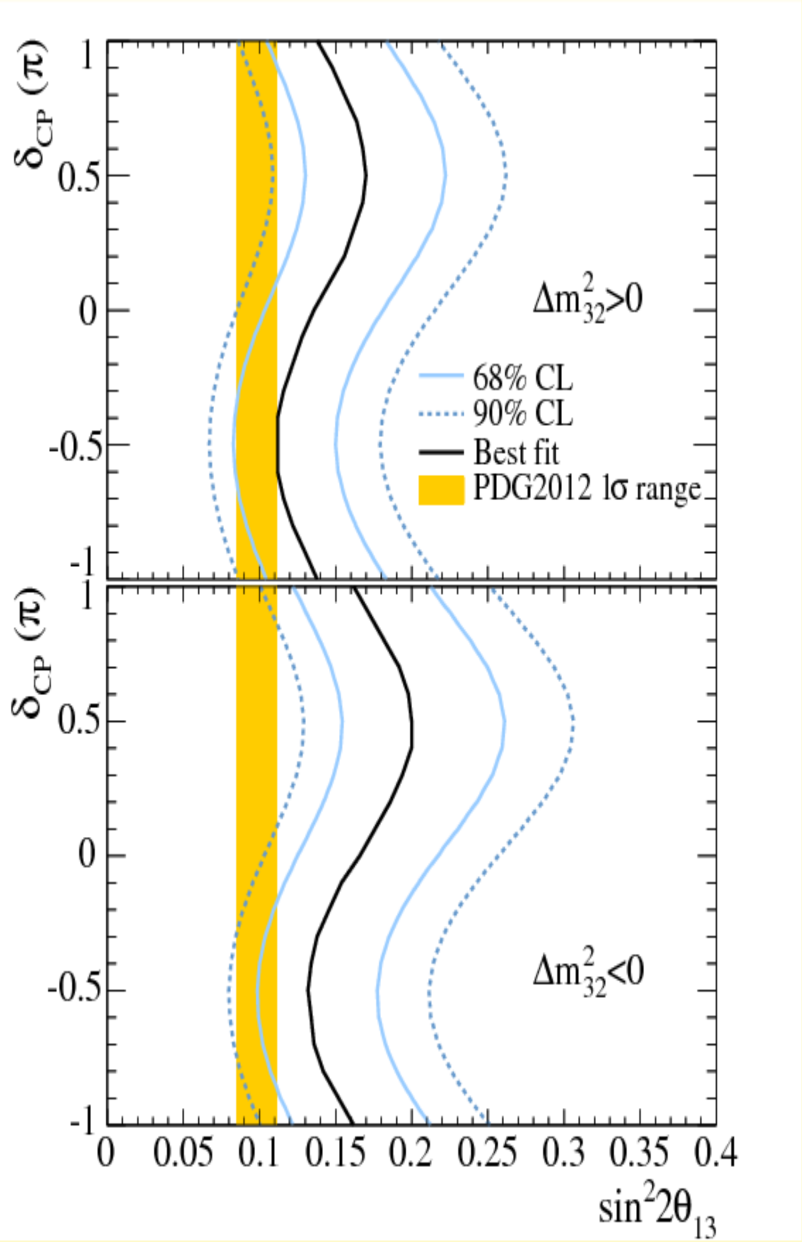
Example: size of hierarchy effects in atmospheric neutrinos at Super-K:



This is even more important in the light of ν_e appearance signals in T2K (and MINOS), consistent with the same θ_{13} as reactors (up to subleading CPV, and hierarchy osc. terms).

T2K 2012/2013
e-like appearance:





Exercise : CP(T) properties of $P_{\alpha\beta}$ in vacuum

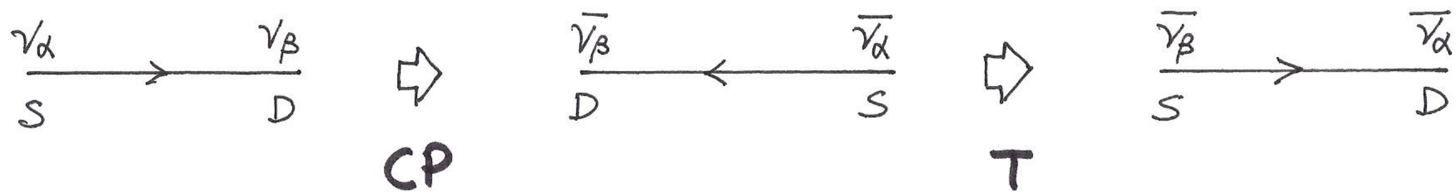
One of the next frontiers is to investigate $\cancel{CP}$ in the ν sector. Prove that the general form of $P(\nu_\alpha \to \nu_\beta)$ is naturally split in a CP-conserving and a CP-violating part, $P = P_{CP} + P_{\cancel{CP}}$:

$$\boxed{\begin{aligned} P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} &- 4 \sum_{i<j} \mathrm{Re}\, J_{\alpha\beta}^{ij} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) \quad \leftarrow P_{CP} \\ &- 2 \sum_{i<j} \mathrm{Im}\, J_{\alpha\beta}^{ij} \sin\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) \quad \leftarrow P_{\cancel{CP}} \end{aligned}}$$

where CP invariance would imply $U = U^*$ and $P(\nu) = P(\bar{\nu})$.

Prove also that CPT invariance holds, and implies $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$.

Action of CP and T transformations on $\nu_\alpha \to \nu_\beta$ process from source (S) to detector (D):



CP invariance: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ $\Leftarrow (\nu \leftrightarrow \bar{\nu})$

T invariance: $\begin{cases} P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha) \\ P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) \end{cases}$ $\Leftarrow (\alpha \leftrightarrow \beta)$

CPT invariance: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$ $\Leftarrow (\nu \leftrightarrow \bar{\nu}) \oplus (\alpha \leftrightarrow \beta)$

For 3ν in vacuum: in the general form of $P_{\alpha\beta}$, it is easy to check that either $(\alpha \leftrightarrow \beta)$ or $(\nu \leftrightarrow \bar{\nu})$ exchange amount to $(U \leftrightarrow U^*)$, only affecting the $P_{\cancel{CP}}$ part. Therefore, CP invariance requires $U = U^*$, while CPT invariance holds in any case.

skip

Exercise: Conditions to observe $\cancel{CP}$ in vacuum

Consider the general form $P = P_{CP} + P_{\cancel{CP}}$. Prove that, in order to have $P_{\cancel{CP}} \neq 0$, the following conditions must be satisfied:

- $\delta \neq 0$ or π ← U must be complex
- $\alpha \neq \beta$ ← need appearance experiments
- $\theta_{ij} \neq 0$ ← all mixing angles $\neq 0$
- $\Delta m^2_{ij} \neq 0$ ← sensitivity to both δm^2 and Δm^2

→ The smallness of θ_{13} and of $\delta m^2 \ll \Delta m^2$ make it difficult to test CP violation in the neutrino sector!

(Learn about Jarlskog invariant in this exercise)

- For $\delta = 0, \pi$: $U = U^* \rightarrow P_{\cancel{CP}} = 0$
- For $\alpha = \beta$: $\mathrm{Im}(J_{\alpha\alpha}^{ij}) = 0 \rightarrow P_{\cancel{CP}} = 0$
- For $\alpha \neq \beta$: It turns out that all $\mathrm{Im}(J_{\alpha\beta}^{ij})$ are equal to each other up to a sign.

Define $J = \mathrm{Im}(J_{e\mu}^{12})$. Then : $\mathrm{Im}(J_{\alpha\beta}^{ij}) = \pm J$ for $\alpha \neq \beta$ and $i \neq j$, where :

$$\begin{cases} +J \text{ for } (\alpha,\beta) = (e\mu), (\mu\tau), (\tau e) & \leftarrow \text{flavor cyclic} \\ +J \text{ for } (ij) = (12), (23), (31) & \leftarrow \text{generation cyclic} \\ -J \text{ otherwise} \end{cases}$$

(J is called Jarlskog invariant)

E.g. : $\mathrm{Im}(J_{e\tau}^{12}) = \mathrm{Im}(U_{e1} U_{\tau 1}^* U_{e2}^* U_{\tau 2})$

$= \mathrm{Im}(U_{e1} U_{e2}^* (-U_{e1}^* U_{e2} - U_{\mu 1}^* U_{\mu 2}))$ ← U unitary

$= \mathrm{Im}(-U_{e1} U_{e2}^* U_{\mu 1}^* U_{\mu 2}) = -\mathrm{Im}(J_{e\mu}^{12}) = -J$ etc...

Note : $\mathrm{Im}(J_{e\tau}^{21}) = +J = (-1)(-1)J$ since both $(e\tau)$ and (21) are anticyclic.

Cyclic rules can be embedded in compact form via completely antisymmetric tensors ϵ : $\mathrm{Im}(J_{\alpha\beta}^{ij}) = J \cdot \sum_\gamma \epsilon_{\alpha\beta\gamma} \sum_k \epsilon_{ijk}$

Now, using the PDG convention for $U = U(\theta_{ij}, \delta)$, it is :

$J = \mathrm{Im}(J_{e\mu}^{12}) = \mathrm{Im}(U_{e1} U_{\mu 1}^* U_{e2}^* U_{\mu 2}) = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$

which vanishes for any $\theta_{ij} = 0$ and/or for $\delta = 0, \pi$ $(\sin\delta = 0)$.

skip

- $P_{\not{CP}}$, previously derived in "sum" form, can also be cast in "product" form,

$$P_{\not{CP}}(\nu_\alpha \to \nu_\beta) = \pm 8J \sin\left(\frac{\Delta m^2_{12} x}{4E}\right) \sin\left(\frac{\Delta m^2_{23} x}{4E}\right) \sin\left(\frac{\Delta m^2_{31} x}{4E}\right)$$

which vanishes for any $\Delta m^2_{ij} \cong 0$ (i.e., for oscillation phase $\ll 1$).

The proof makes use of the following trigonometric identity:

If $x+y+z=0$, then $\sin 2x + \sin 2y + \sin 2z = -4 \sin x \sin y \sin z$.

In our case, the identity is applied to $\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0$. Indeed:

$$\begin{aligned}
P_{\not{CP}}(\nu_\alpha \to \nu_\beta) &= -2 \sum_{i<j} \mathrm{Im}\, J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij}}{2E} x\right) \\
&= -2 \left[\mathrm{Im}\, J^{12}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{12} x}{2E}\right) + \mathrm{Im}\, J^{23}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{23} x}{2E}\right) + \mathrm{Im}\, J^{13}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{13} x}{2E}\right)\right] \\
&\doteq -2\, \mathrm{Im}\, J^{12}_{\alpha\beta} \left[\sin\left(\frac{\Delta m^2_{12} x}{2E}\right) + \sin\left(\frac{\Delta m^2_{23} x}{2E}\right) - \sin\left(\frac{\Delta m^2_{13} x}{2E}\right)\right] \\
&= -2\, \mathrm{Im}\, J^{12}_{\alpha\beta} \left[\sin\left(\frac{\Delta m^2_{12} x}{2E}\right) + \sin\left(\frac{\Delta m^2_{23} x}{2E}\right) + \sin\left(\frac{\Delta m^2_{31} x}{2E}\right)\right] \\
&= +8\, \mathrm{Im}\, J^{12}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{12} x}{4E}\right) \sin\left(\frac{\Delta m^2_{23} x}{4E}\right) \sin\left(\frac{\Delta m^2_{31} x}{4E}\right) \\
&= +8\, \mathrm{Im}\, J^{12}_{\alpha\beta} \prod_{(ij)}^{\text{cyclic}} \sin\left(\frac{\Delta m^2_{ij} x}{4E}\right)
\end{aligned}$$

In particular, $P_{\not{CP}} \to 0$ for $\delta m^2 \to 0$.

skip

So far we have mainly discussed Δm^2-driven oscillations

Next: 2nd part of lecture I

* Oscillation searches sensitive to δm^2:
  - probabilities for $\Delta m^2/\delta m^2 \rightarrow \infty$
  - matter effects (theory)
  - experimental results

* Global 3ν analysis of all oscillation data

Exercise: Expts. sensitive to δm^2 in the limit $\Delta m^2 \to \infty$

Previously we have considered experiments with sensitivity to Δm^2 in the limit $\delta m^2 \to 0$. At the other end of the spectrum, there are expts. with leading sensitivity to δm^2, for which one can take $\Delta m^2 \to \infty$:

$$\frac{\delta m^2 x}{4E} \sim \mathcal{O}(1) \qquad \text{and} \qquad \frac{\Delta m^2 x}{4E} \gg 1$$

This is the case, for instance, of long-baseline reactor experiments (KamLAND) with large x and relatively low E. At low $E \sim$ few MeV, the main observable is the disappearance probability P_{ee}. Prove that:

$$\boxed{P_{ee} \simeq \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\delta m^2 x}{4E}\right)\right] + \sin^4\theta_{13}}$$

(which does not depend on hierarchy, $\nu/\bar{\nu}$, $\cancel{CP}$)

For P_{ee} (disappearance) it is $P_{\cancel{CP}} = 0$.

For $\frac{\Delta m^2 x}{4E}$, on average it is: $\sin^2\left(\frac{\Delta m^2_{31} x}{4E}\right) \simeq \frac{1}{2} \simeq \sin^2\left(\frac{\Delta m^2_{32} x}{4E}\right)$

Then:

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - 4\,\mathrm{Re}\,(J^{12}_{ee}) \sin^2\left(\frac{\delta m^2 x}{4E}\right) - 2\,\mathrm{Re}\,(J^{13}_{ee} + J^{23}_{ee}) \\
&= 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\left(\frac{\delta m^2 x}{4E}\right) - 2|U_{e3}|^2\left(|U_{e1}|^2 + |U_{e2}|^2\right) \\
&= 1 - 4c^4_{13}\, s^2_{12} c^2_{12} \sin^2\left(\frac{\delta m^2 x}{4E}\right) - 2 s^2_{13} c^2_{13} \\
&= c^4_{13} + s^4_{13} - 4 c^4_{13} s^2_{12} c^2_{12} \sin^2\left(\frac{\delta m^2 x}{4E}\right) \\
&= c^4_{13}\, P^{2\nu} + s^4_{13} \quad \text{where} \quad P^{2\nu} = 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\delta m^2 x}{4E}\right)
\end{aligned}
$$

Namely, the 3ν probability (for $\theta_{13} \neq 0$) is related to the 2ν probability (at $\theta_{13} = 0$) by the relation:

$$\boxed{P^{3\nu}_{ee} = \cos^4\theta_{13}\, P^{2\nu}_{ee} + \sin^4\theta_{13}}$$

skip

Important note: The Δm^2-averaged form for P_{ee},

$$P_{ee}^{3\nu} = c_{13}^4 \, P_{ee}^{2\nu}(\delta m^2; \theta_{12}) + s_{13}^4$$

holds not only for KamLAND, but also for <u>solar</u> neutrinos (proof omitted) where, however, $P_{ee}^{2\nu}$ takes a very different form due to <u>matter</u> effects in the Sun.

Therefore, via $P_{ee}^{3\nu}$, solar + KamLAND experiments allow to set constraints on δm^2 and on the 1st-row elements of the PMNS matrix (in absolute value):

$$|U| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot \end{pmatrix} \leftarrow \text{functions of } \theta_{12}, \theta_{13}$$

Recap of leading sensitivity:

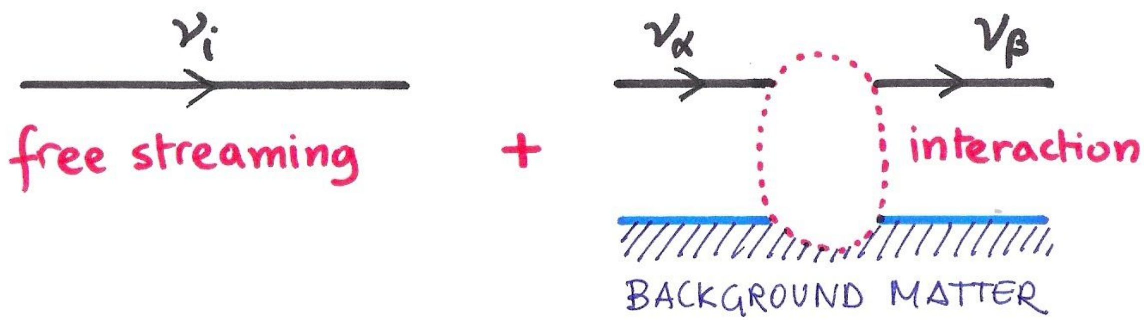
Solar + KamLAND	→	θ_{12}	θ_{13}	δm^2
ATM + LBL accel.	→	θ_{23}	θ_{13}	$\|\Delta m^2\|$
SBL reactors	→		θ_{13}	$\|\Delta m^2\|$

skip

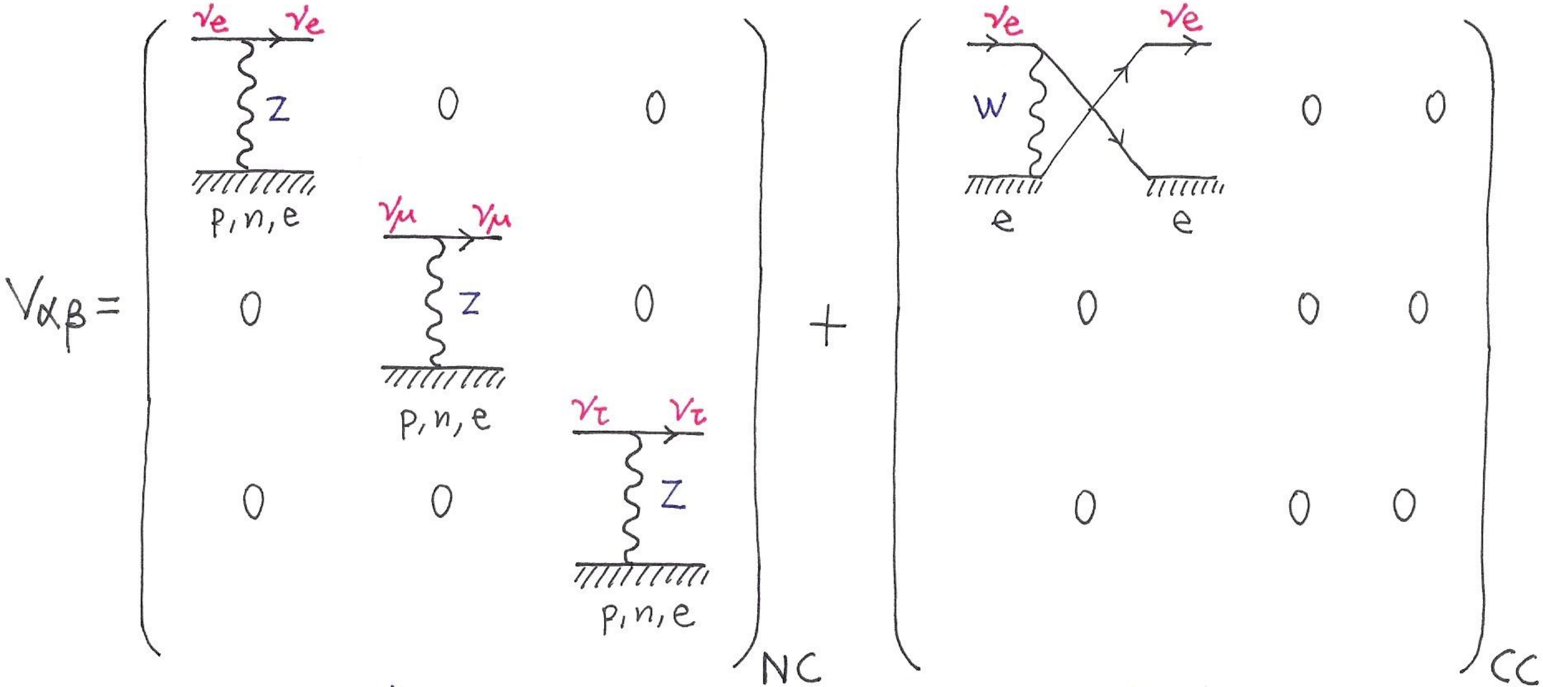
Hamiltonian for ν oscillations in matter (MSW)

It was first realized by Wolfenstein, and later elaborated by Mykheev and Smirnov, that neutrinos traveling in matter receive a contribution to coherent forward scattering, in the form of a tiny interaction energy $V_{\alpha\beta}$:

$$H_{\text{flavor}} = \underbrace{\frac{1}{2E} U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^{+}}_{\text{VACUUM (KINEMATICS)}} + \underbrace{\begin{pmatrix} V_{ee} & V_{e\mu} & V_{e\tau} \\ V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\ V_{\tau e} & V_{\tau\mu} & V_{\tau\tau} \end{pmatrix}}_{\text{MATTER (DYNAMICS)}}$$

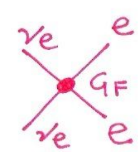


Within the Standard Model, and in ordinary matter:



↑ proportional to unity and unobservable

↑ observable in ν_e oscillations

Relevant term is the $\nu_e - \nu_{\mu,\tau}$ energy difference: $V_{CC} \simeq$ 

(No analogous for μ, τ, which are absent in ordinary matter)

- It turns out that the V_{CC}^{ee} interaction energy is (see next page):

$$V = \sqrt{2}\, G_F N_e$$

where N_e = electron number density, and $V \to -V$ for $\nu \to \bar{\nu}$

- Then, the Hamiltonian of ν propagation in matter reads:

$$\boxed{H_{flavor} = \frac{1}{2E} U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^+ + \frac{1}{2E} \begin{pmatrix} A & & \\ & 0 & \\ & & 0 \end{pmatrix}}$$

where $\quad A = 2\sqrt{2}\, G_F N_e E$

- The relative size of matter/vacuum terms is given by $A/\Delta m_{ij}^2$. Roughly speaking, one may expect sizable effects for $A/\Delta m_{ij}^2 \sim \mathcal{O}(1)$.

- The dependence $A = A(x)$ makes the evolution nontrivial in many cases.

Exercise: Sketchy proof of $V = \sqrt{2}\, G_F N_e$.

From the CC Hamiltonian of the Standard Model,

$$\mathcal{H}_{cc} = \frac{G_F}{\sqrt{2}}\, \bar{e}\gamma^\mu(1-\gamma_5)\nu_e \cdot \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \qquad \longleftarrow \qquad J_{cc} \otimes J_{cc}$$

$$\stackrel{(\text{FIERZ})}{=} \frac{G_F}{\sqrt{2}}\, \bar{e}\gamma^\mu(1-\gamma_5) e \cdot \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e \qquad \longleftarrow \qquad J_e \otimes J_\nu$$

From the neutrino viewpoint, the e^- is ~nonrelativistic and ~unpolarized ($\langle \vec{\sigma} \rangle = 0$) in ordinary matter. It is then useful to use the Dirac representation, where only the upper components of e^- survive:
$e \simeq \begin{bmatrix} \xi \\ 0 \end{bmatrix}$, with ξ = Pauli spinor.

Then: $\bar{e}\gamma^\mu(1-\gamma_5)e \simeq (\underbrace{\xi^+\xi}_{\text{density } N_e},\ \underbrace{\xi^+\vec{\sigma}\,\xi}_{\text{polarization} \sim 0}) \simeq N_e\, \delta_{\mu 0}$

and: $\mathcal{H}_{cc} \simeq \frac{G_F}{\sqrt{2}} N_e\, \bar{\nu}_e \gamma_0 (1-\gamma_5) \nu_e = \underbrace{[\sqrt{2}\, G_F N_e]}_{\text{coupling}} \cdot \bar{\nu}_{e_L} \gamma_0\, \nu_{e_L}$

which adds to the ν_e energy in the evolution equation.

skip

Exercise : Prove that $\dfrac{A}{\Delta m^2_{ij}} = 1.526\times10^{-7}\left(\dfrac{N_e}{mol/cm^3}\right)\left(\dfrac{E}{MeV}\right)\left(\dfrac{eV^2}{\Delta m^2_{ij}}\right)$

$1\ mol = 6.022\times10^{23}$ particles

$$1\ \frac{mol}{cm^3} = \frac{6.022\times10^{23}}{10^{-6}\,m^3}\left(\frac{MeV^3}{MeV^3}\right) = 6.022\times10^{29}\ \frac{1}{(m\cdot MeV)^3}\ MeV^3$$

$$= \frac{6.022\times10^{29}}{(5.0677\times10^{12})^3}\ MeV^3 = 4.627\times10^{-9}\ MeV^3 \qquad \text{(in natural units)}$$

$$G_F = 1.16637\times10^{-5}\ GeV^{-2} = 1.16637\times10^{-11}\ MeV^{-2}$$

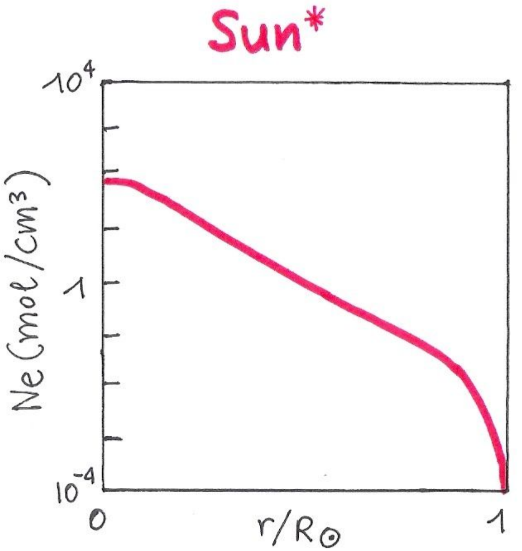
$$\frac{2\sqrt{2}G_F N_e E}{\Delta m^2_{ij}} = 2\sqrt{2}\left(1.1664\times10^{-11}\ MeV^{-2}\right)\left(\frac{N_e}{mol/cm^3}\ mol/cm^3\right)\left(\frac{E}{MeV}\ MeV\right)\left(\frac{eV^2}{\Delta m^2_{ij}}\cdot\frac{1}{eV^2}\right)$$

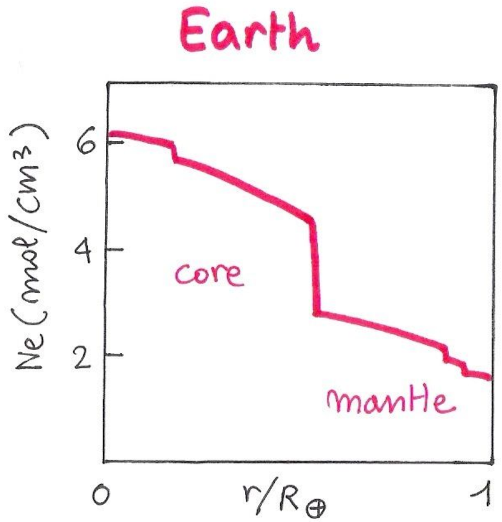
$$= 3.299\times10^{-11}\ \frac{MeV^{-2}\,MeV}{eV^2}\ \frac{mol}{cm^3}\left(\frac{N_e}{mol/cm^3}\right)\left(\frac{E}{MeV}\right)\left(\frac{eV^2}{\Delta m^2_{ij}}\right)$$

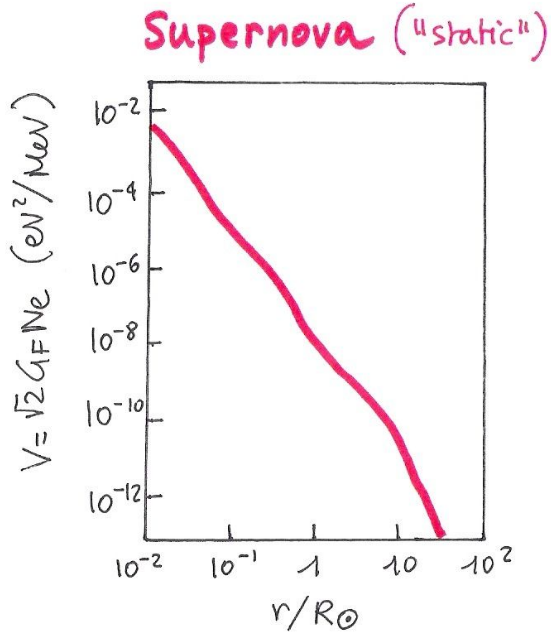
$$3.299\times10^{-11}\ \frac{MeV^{-2}\,MeV}{eV^2}\ \frac{mol}{cm^3} = 3.299\times10^{-11}\ \frac{10^{12}}{MeV^3}\times 4.627\times10^{-9}\ MeV^3 = 1.526\times10^{-7}$$

skip

Examples of matter density profiles:







~exponential ~step-like ~power law

There is a huge literature about (semi)analytical solutions of flavor evolution equations for (approximations of) these and other profiles, in 2ν, 3ν or $N\nu$ cases. Analytical understanding is useful, because numerical solutions are prone to artifacts.

* For solar ν_e at $r \sim 0$: $\frac{A}{\delta m^2} \gtrsim 1$ for $E \gtrsim$ few MeV $\rightarrow$ expect large matter effects!

Exercise : 2ν oscillations in matter at constant density

Prove that, in the 2ν limit ($\theta_{13}=0$), the ν_e survival probability reads:

$$\boxed{P_{ee}^{2\nu}(\mathrm{mat}) = 1 - \sin^2 2\tilde{\theta}_{12} \sin^2\left(\frac{\delta\tilde{m}^2 x}{4E}\right)} \quad \text{for } N_e = \text{const}$$

i.e., it has the same vacuum-like structure, but with the replacements:

$$\boxed{\sin 2\tilde{\theta}_{12} = \frac{\sin 2\theta_{12}}{\sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}} \,, \quad \delta\tilde{m}^2 = \delta m^2 \frac{\sin 2\theta_{12}}{\sin 2\tilde{\theta}_{12}} \quad \left(\begin{array}{c} A = \pm 2\sqrt{2}\, G_F N_e \\ + : \nu \\ - : \bar{\nu} \end{array}\right)}$$

Solution- In the 2ν limit, the relevant oscillation parameters are $(\delta m^2, \theta_{12})$ and the hamiltonian is:

$$\tilde{H} = U \begin{pmatrix} \frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E} \end{pmatrix} U^T + \begin{pmatrix} V & \\ & 0 \end{pmatrix} = \frac{1}{2E}\left[\begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}\begin{pmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{pmatrix} + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}\right]$$

where $U = U^* = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}$. It is convenient to put $\tilde{H}$ in traceless form:

$\rightarrow$ cont'd

$$\tilde{H} = \frac{1}{4E}\begin{bmatrix} A - \cos 2\theta_{12}\,\delta m^2 & \sin 2\theta_{12}\,\delta m^2 \\ \sin 2\theta_{12}\,\delta m^2 & -A + \cos 2\theta_{12}\,\delta m^2 \end{bmatrix}$$

which has eigenvalues $\pm \frac{\delta \tilde{m}^2}{4E}$, with $\delta\tilde{m}^2 = \delta m^2 \sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}$.

Diagonalizing rotation: $\tilde{H} = \begin{pmatrix} \cos\tilde{\theta}_{12} & \sin\tilde{\theta}_{12} \\ -\sin\tilde{\theta}_{12} & \cos\tilde{\theta}_{12} \end{pmatrix}\begin{pmatrix} -\frac{\delta\tilde{m}^2}{4E} & 0 \\ 0 & +\frac{\delta\tilde{m}^2}{4E} \end{pmatrix}\begin{pmatrix} \cos\tilde{\theta}_{12} & -\sin\tilde{\theta}_{12} \\ \sin\tilde{\theta}_{12} & \cos\tilde{\theta}_{12} \end{pmatrix} = \tilde{U}(\cdot\,.)\,\tilde{U}^T$

where $\sin 2\tilde{\theta}_{12} = \dfrac{\sin 2\theta_{12}}{\sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}}$, $\cos 2\tilde{\theta}_{12} = \dfrac{\cos 2\theta_{12} - A/\delta m^2}{\sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}}$

so that $\delta\tilde{m}^2 = \delta m^2 \sin 2\theta_{12} / \sin 2\tilde{\theta}_{12}$

The evolution operator in matter is obtained by exponentiation of $\tilde{H}$:

$$\tilde{S} = e^{-i\tilde{H}x} = \tilde{U}\begin{pmatrix} e^{i\frac{\delta\tilde{m}^2 x}{4E}} & \\ & e^{-i\frac{\delta\tilde{m}^2 x}{4E}} \end{pmatrix}\tilde{U}^T = \cos\left(\frac{\delta\tilde{m}^2}{4E}x\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\sin\left(\frac{\delta\tilde{m}^2 x}{4E}\right)\begin{pmatrix} -\cos 2\tilde{\theta}_{12} & \sin 2\tilde{\theta}_{12} \\ \sin 2\tilde{\theta}_{12} & \cos 2\tilde{\theta}_{12} \end{pmatrix}$$

Squaring the diagonal element of $\tilde{S}$ one gets the survival probability:

$$P_{ee} = |\tilde{S}_{diag}| = 1 - \sin^2 2\tilde{\theta}_{12}\,\sin^2\left(\frac{\delta\tilde{m}^2 x}{4E}\right).$$

(Squaring the off-diagonal element one would get the complementary transition probability).

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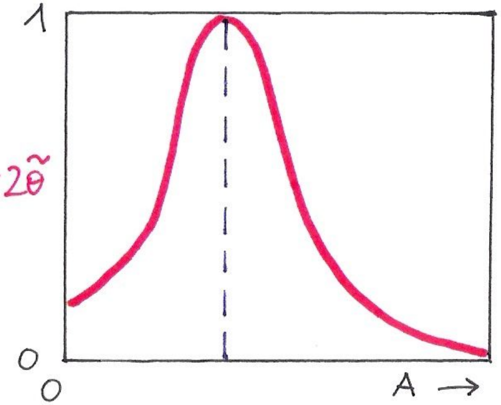
Comments: ($\theta_{12} \equiv \theta$ for simplicity)

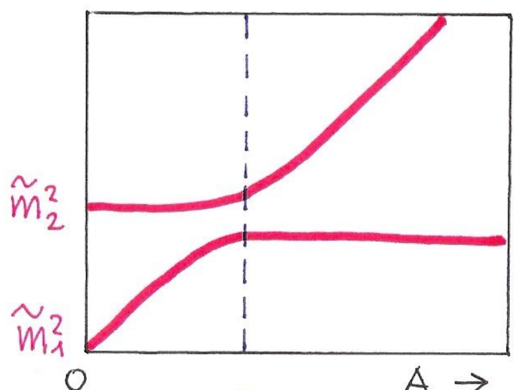


Mykheev-Smirnov-Wolfenstein (MSW) resonance:

For $A/\delta m^2 > 0$, the effective parameters have a resonant behavior around:

$$\frac{A}{\delta m^2} \simeq \cos 2\theta$$

(only for ν: no resonance for $\bar{\nu}$, since $A<0$ for $\bar{\nu}$)



$$\frac{A}{\delta m^2} \simeq \cos 2\theta$$

($A = 2\sqrt{2}\, G_F N_e E$)

Limiting cases:

$A/\delta m^2 \ll 1$: $(\delta\tilde{m}^2, \tilde{\theta}) \simeq (\delta m^2, \theta)$ ← vacuum-like

$A/\delta m^2 \simeq \cos 2\theta$: $(\delta\tilde{m}^2, \tilde{\theta}) \simeq (\delta m^2 \sin 2\theta, \pi/4)$ ← reson.

$A/\delta m^2 \gg 1$: $(\delta\tilde{m}^2, \tilde{\theta}) \simeq (A, \pi/2)$ ← matter dominance

Confirms expectations of large matter effects for $A/\delta m^2 \sim \mathcal{O}(1)$.

skip

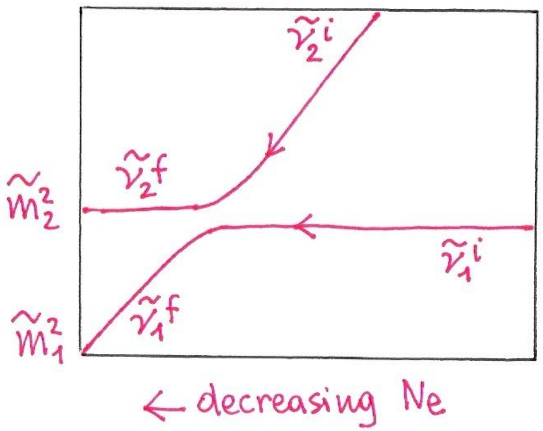
Exercise: 2ν osc. in matter with slowly varying density

If $N_e(x)$ changes slowly from $x = x_i$ (with $\tilde{\theta} = \tilde{\theta}_i$) to $x = x_f$ (with $\tilde{\theta} = \tilde{\theta}_f$) while oscillations are fast, then the averaged P_{ee} probability takes the form:

$$\boxed{P_{ee}^{2\nu} \simeq \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f}$$

← "adiabatic" approximation

Solution. For a quasi-constant hamiltonian, one can solve the evolution equation "one x" at a time, and then patch the solutions from x_i to x_f. This means that, given the initial state $|\nu_e^i\rangle = \cos\tilde{\theta}_i |\tilde{\nu}_1^i\rangle + \sin\tilde{\theta}_i |\tilde{\nu}_2^i\rangle$, the effective mass eigenstates $|\tilde{\nu}_1^i\rangle$ and $|\tilde{\nu}_2^i\rangle$ at x_i slowly transform into $|\tilde{\nu}_1^f\rangle$ and $|\tilde{\nu}_2^f\rangle$ at x_f, respectively:



$$\Leftarrow \quad |\langle\tilde{\nu}_1^f|\tilde{\nu}_1^i\rangle| = 1 = |\langle\tilde{\nu}_2^f|\tilde{\nu}_2^i\rangle| \quad ; \quad \underbrace{|\langle\tilde{\nu}_{2,1}^f|\tilde{\nu}_{1,2}^i\rangle| = 0}_{\text{no "level crossing"}}$$

Then:

$$P_{ee}^{2\nu} = |\langle\tilde{\nu}_e^f|\tilde{\nu}_e^i\rangle|^2 = |\cos\tilde{\theta}_i \cos\tilde{\theta}_f \langle\tilde{\nu}_1^f|\tilde{\nu}_1^i\rangle + \sin\tilde{\theta}_i \sin\tilde{\theta}_f \langle\tilde{\nu}_2^f|\tilde{\nu}_2^i\rangle|^2$$

Averaging out interference terms after many oscillations:

$$P_{ee}^{2\nu} \simeq \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f \,|\langle\tilde{\nu}_1^f|\tilde{\nu}_1^i\rangle|^2 + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f \,|\langle\tilde{\nu}_2^f|\tilde{\nu}_2^i\rangle|^2$$

$$= \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f \,.$$

Application to solar ν.

It turns out that, for the $(\delta m^2, \theta_{12})$ values chosen by nature, the adiabatic approximation can be applied to solar ν_e. In this case, $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at the exit from the Sun), while $\tilde{\theta}_{12}(x_i)$ must be evaluated at the production point x_i. Limiting cases:

$E \lesssim$ few MeV (vacuum dominance): $A/\delta m^2 \lesssim 1$ and $\tilde{\theta}_{12}(x_i) \simeq \theta_{12}$

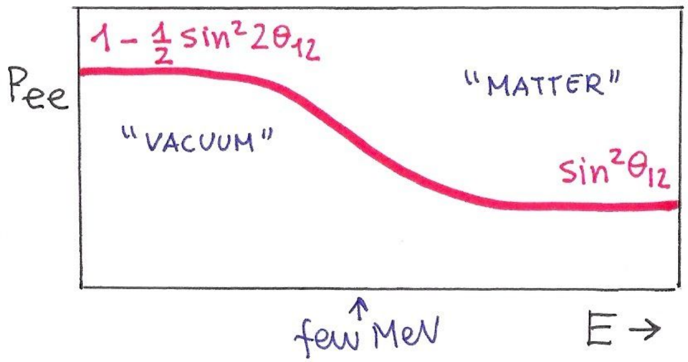
$P_{ee} \simeq c_{12}^4 + s_{12}^4 = 1 - \frac{1}{2}\sin^2 2\theta_{12}$

This is the averaged vacuum probability, octant symmetric.

$E \gtrsim$ few MeV (matter dominance): $A/\delta m^2 \gtrsim 1$ and $\tilde{\theta}_{12}(x_i) \sim \pi/2$

$P_{ee} \simeq \sin^2\theta_{12}$

This is the matter-dominated probability, octant-asymmetric

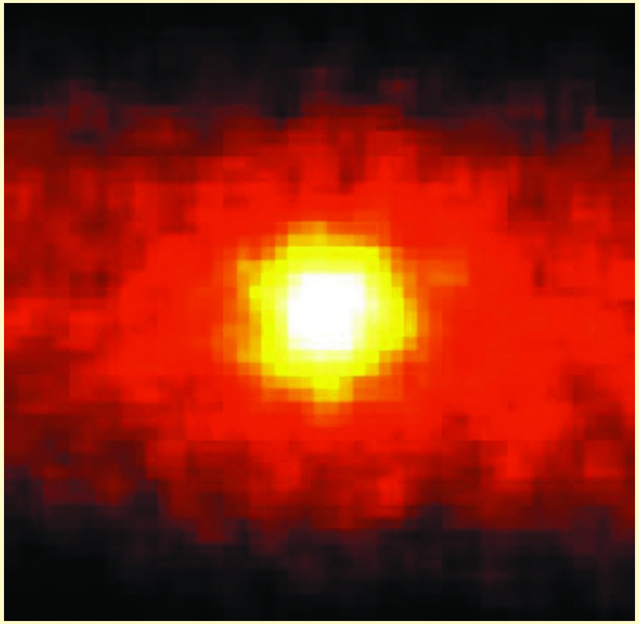


The Pee transition from "low" to "high" E is a signature of matter effects in the Sun.

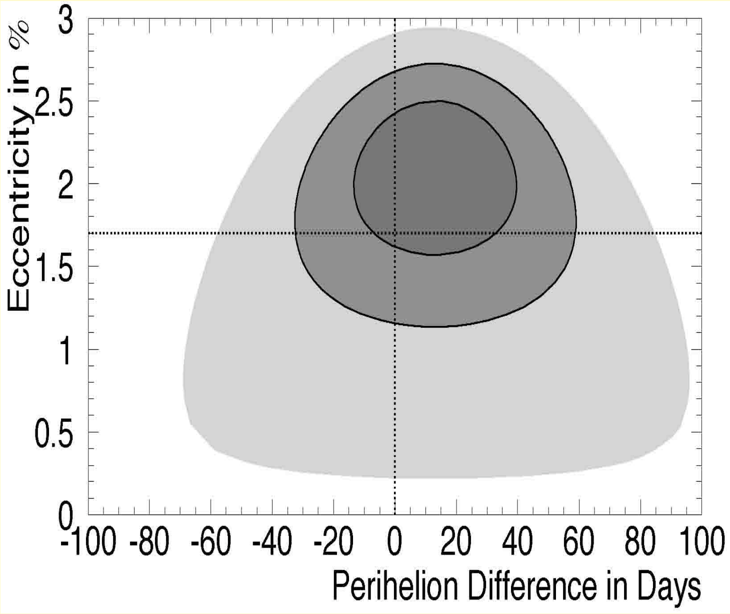
Thanks to matter effects we can determine the octant of the mixing angle θ_{12}.

Experiments sensitive to the "small" δm^2:

Solar neutrinos



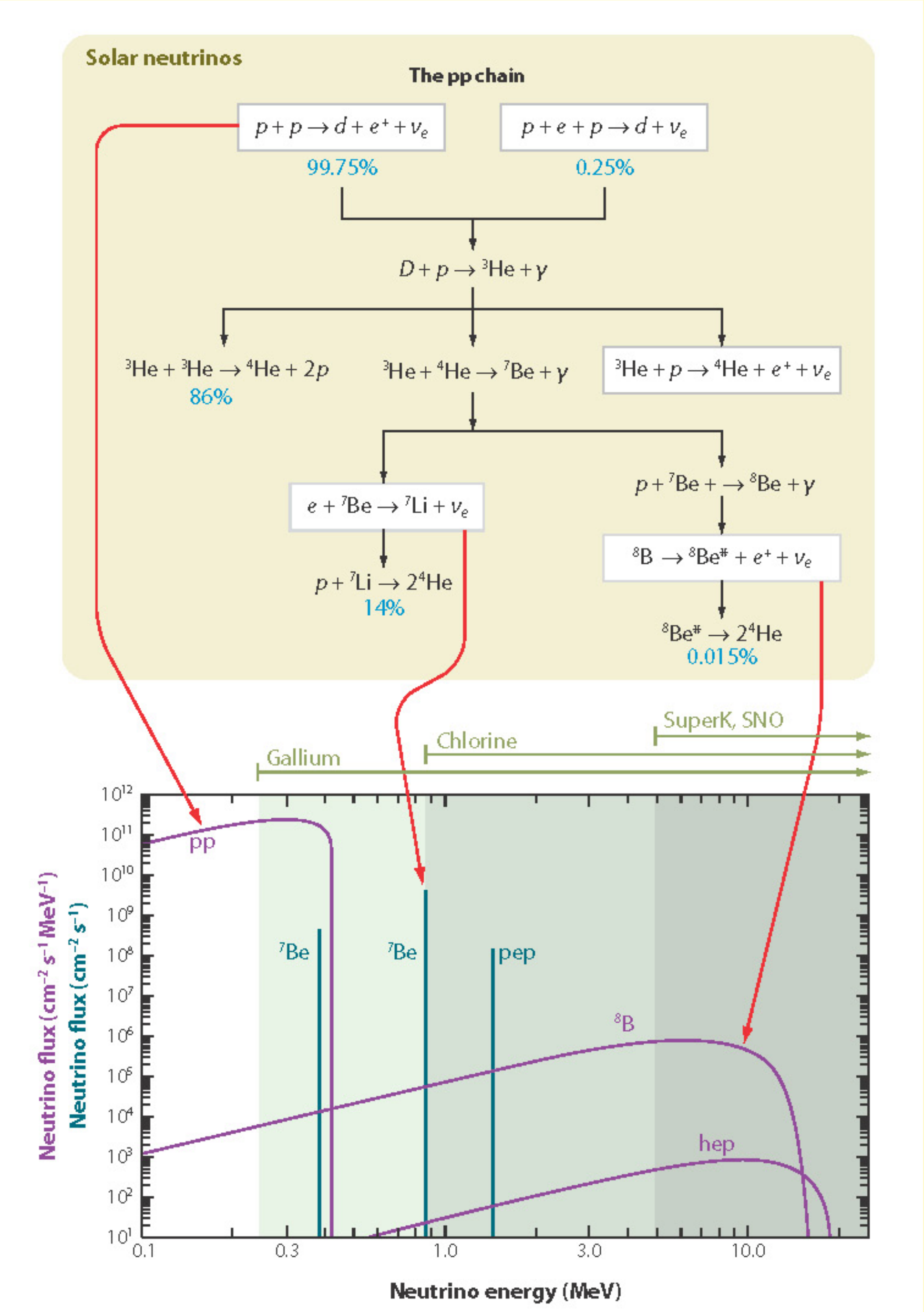
The Sun seen with neutrinos (SK)

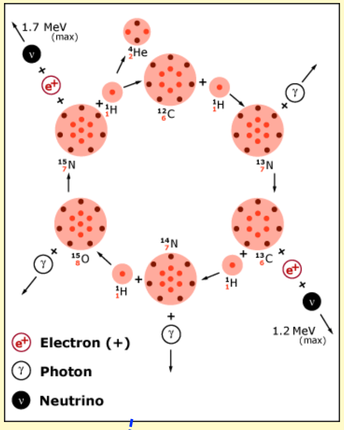


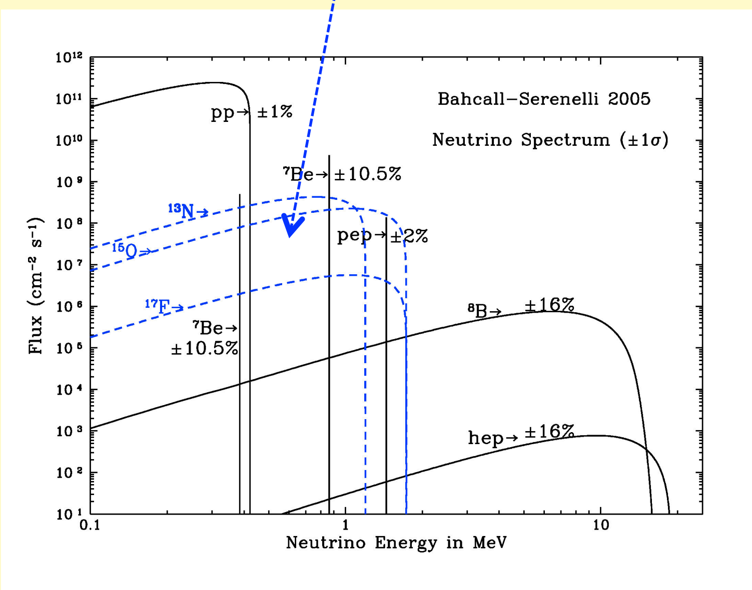
Earth orbit from solar ν (SK)

Production

pp (+CNO) cycle







Detection

Radiochemical: count the decays of unstable final-state nuclei. (low energy threshold, but energy and time info lost/integrated)

$$^{37}Cl + \nu_e \rightarrow {}^{37}Ar + e \quad (CC)$$

Homestake

$$^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^- \quad (CC)$$

GALLEX/GNO, SAGE

Elastic scattering: events detected in real time with either "high" threshold (Č, directional) or "low" threshold (Scintillators)

$$\nu_x + e^- \rightarrow \nu_x + e^- \quad (NC, CC)$$

SK, SNO, Borexino

Interactions on Deuterium: *CC* events detected in real time; NC events separated statistically + using neutron counters.

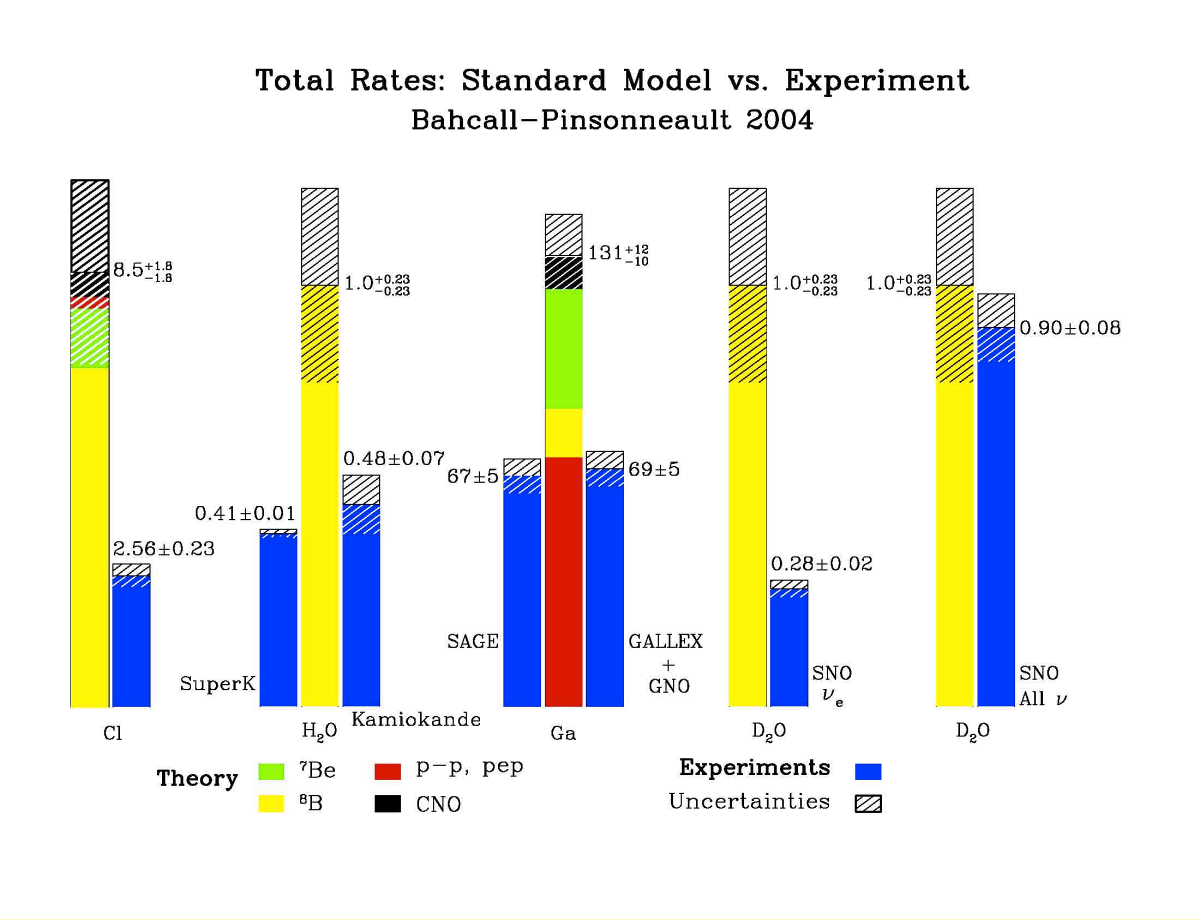
$$\nu_e + d \rightarrow p + p + e^- \quad (CC)$$

$$\nu_x + d \rightarrow p + n + \nu_x \quad (NC)$$

SNO (Sudbury Neutrino Observatory)

Results

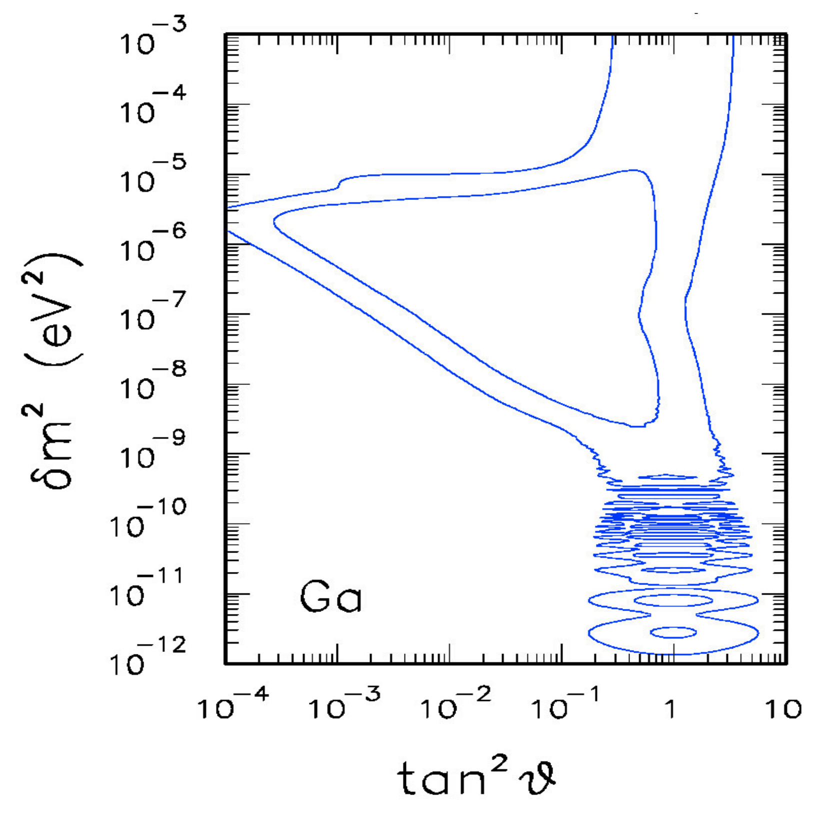
All CC-sensitive results indicated a ν_e deficit…



…as compared to solar model expectations

Interpretation

In the "past millennium": Oscillations? Maybe, but…
- large uncertainties in the parameter space or solar model
- no unmistakable evidence for flavor transitions ("smoking gun")



E.g., in Gallium expts:

"matter" (MSW) solutions

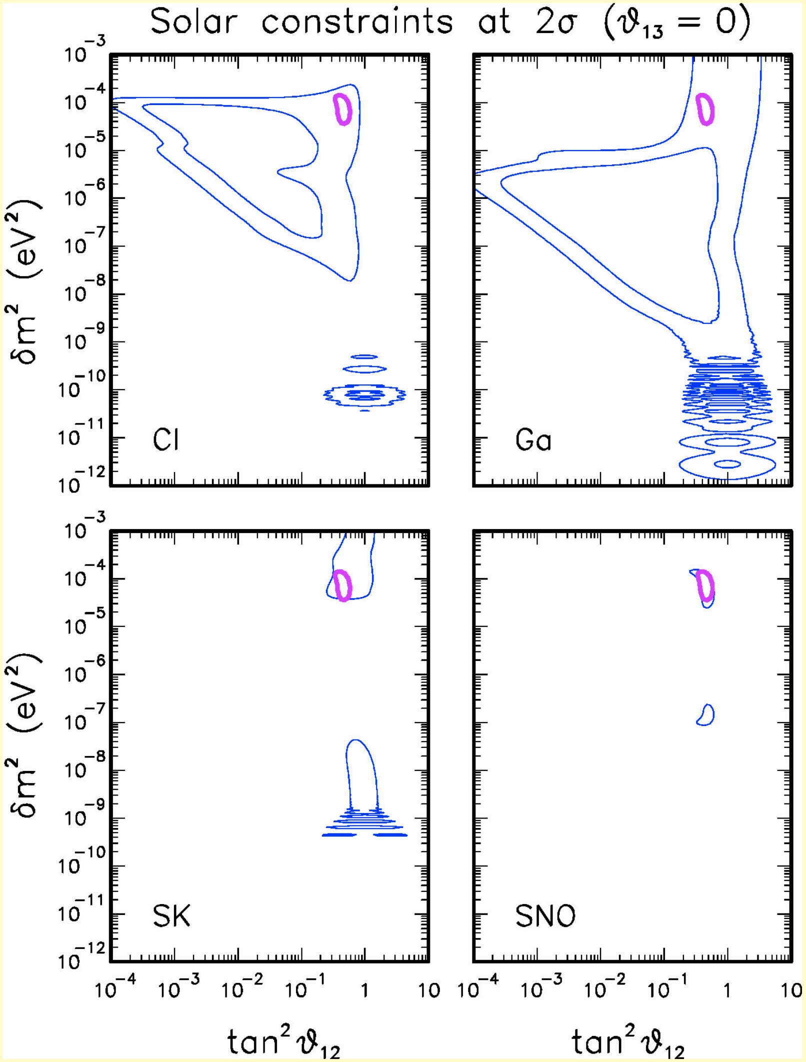
"vacuum" solutions

"small" mixing

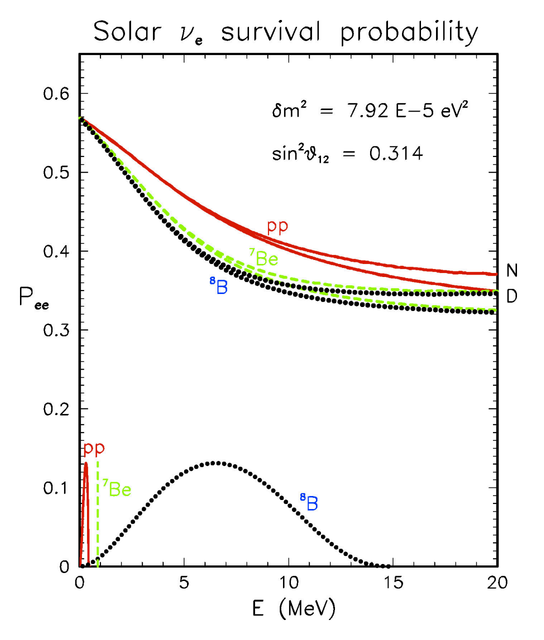
"large" mixing

+ many "exotic" or non-oscillatory solutions…

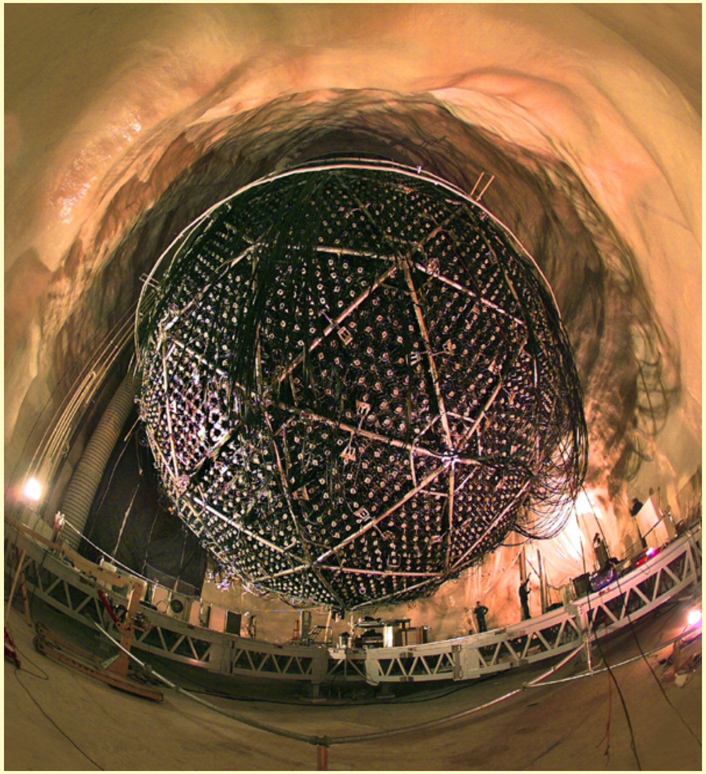
But, in 2002 ("annus mirabilis"), one global solution was finally singled out by combination of all solar data ("large mixing angle" or LMA).



For LMA parameters, evolution is adiabatic in solar matter.



In the Earth: small day/night (D/N) effects, not yet seen.



crucial role played by

Sudbury Neutrino Observatory:

The breakthrough: in deuterium one can separate CC events (induced by ν_e only) from NC events (induced by ν_e, ν_μ, ν_τ), and double check via Elastic Scattering events (due to both NC and CC)

$$
\begin{aligned}
\mathrm{CC}: &\quad \nu_e + d \to p + p + e \\
\mathrm{NC}: &\quad \nu_{e,\mu,\tau} + d \to p + n + \nu_{e,\mu,\tau} \\
\mathrm{ES}: &\quad \nu_{e,\mu,\tau} + e \to e + \nu_{e,\mu,\tau}
\end{aligned}
$$

$$\frac{\mathrm{CC}}{\mathrm{NC}} \sim \frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})}$$

thus:

$$\frac{\mathrm{CC}}{\mathrm{NC}} < 1 \;\Rightarrow\; \phi(\nu_{\mu,\tau}) > 0 \;\Rightarrow\; \nu_e \to \nu_{\mu,\tau}$$

CC/NC ~ 1/3 < 1

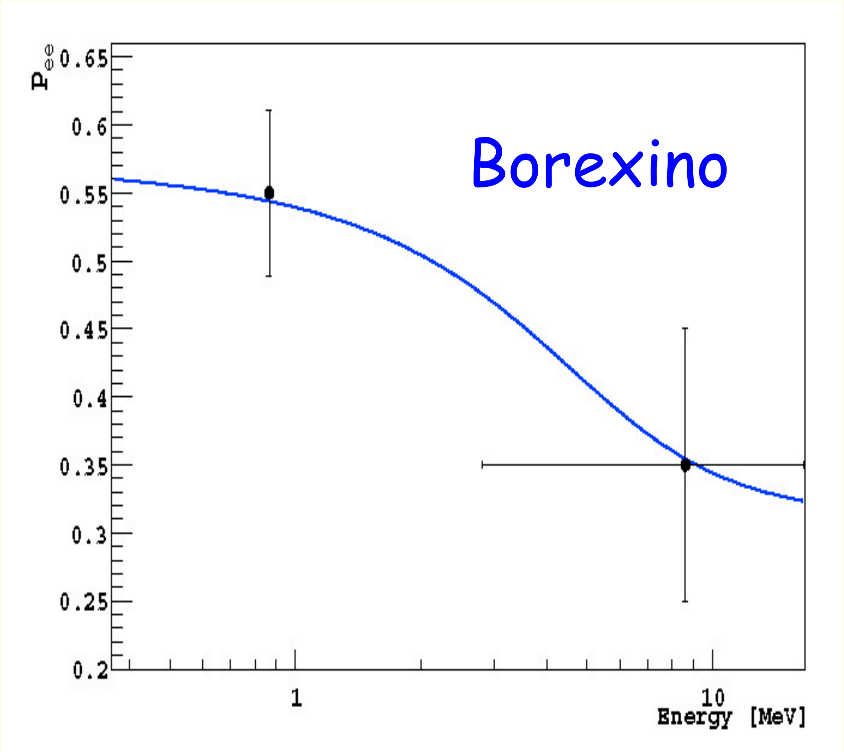
"Smoking gun" proof of flavor change. Solar model OK! Also:

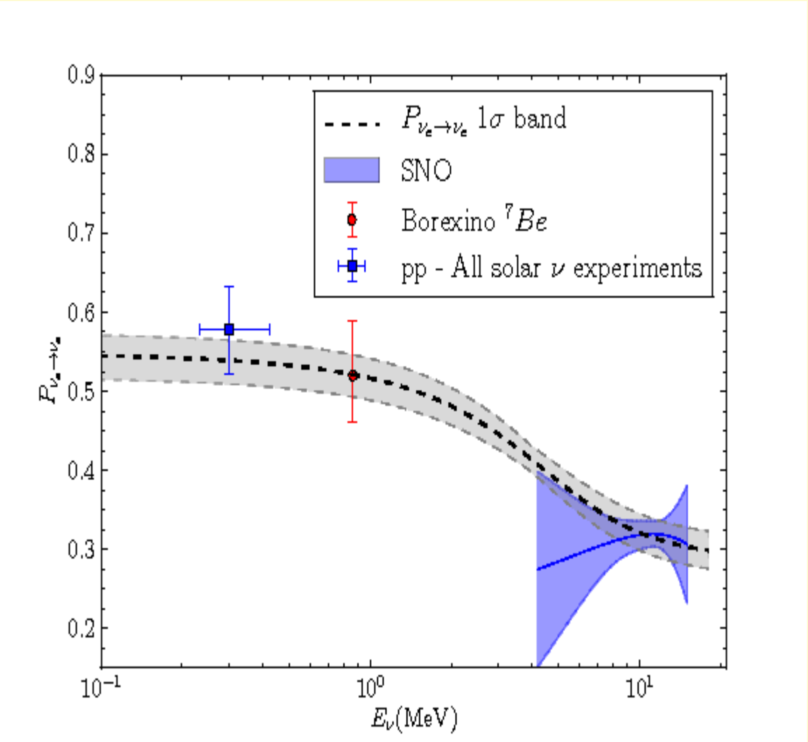
CC/NC ~ Pee ~ $\sin^2\theta_{12}$ (LMA) ~1/3 < ½

Evidence of: mixing in first octant + matter effects

Recent, direct confirmation of adiabatic Pee pattern at LMA in a single solar ν experiment: BOREXINO at Gran Sasso

Overall picture including final SNO data [Spectral rise at low energy not yet directly observed – anomaly?]





Also in 2002… KamLAND: 1000 ton mineral oil detector, "surrounded" by nuclear reactors producing anti-ν_e. Characteristics:

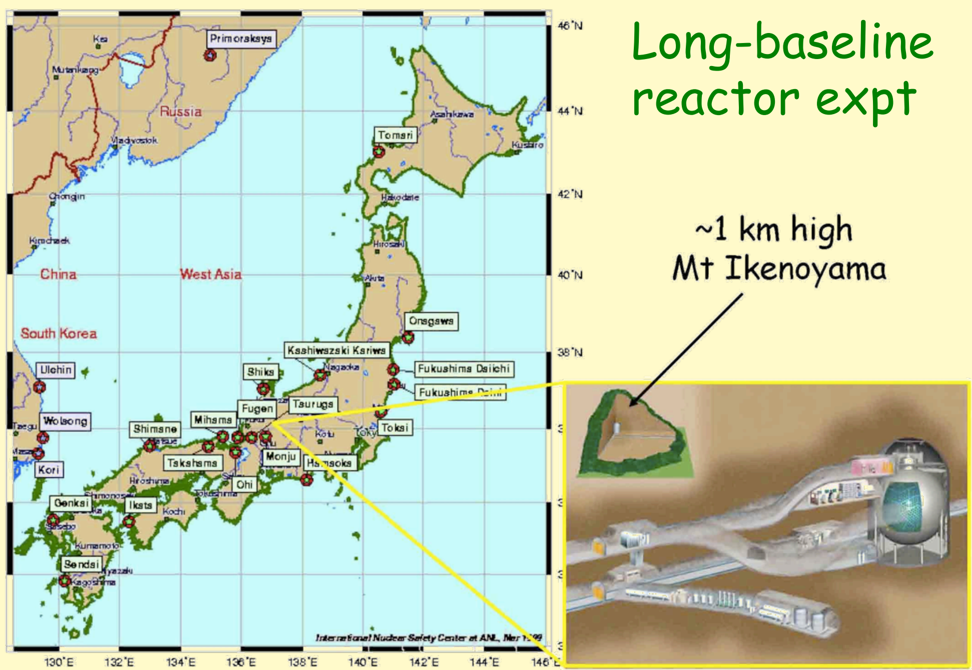
$A/\delta m^2 \ll 1$ in Earth crust
(vacuum approxim. OK)
L~100-200 km
E_ν~ few MeV

⇒

With previous $(\delta m^2,\theta)$ parameters it is $(\delta m^2 L/4E)$~O(1) and reactor neutrinos should oscillate with large amplitude (large θ)

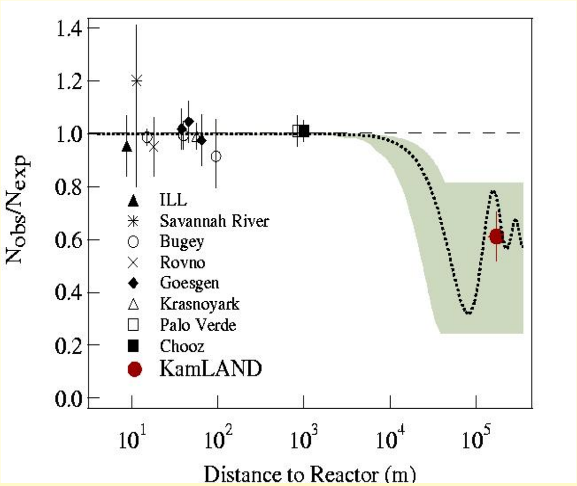
Long-baseline reactor expt

~1 km high
Mt Ikenoyama

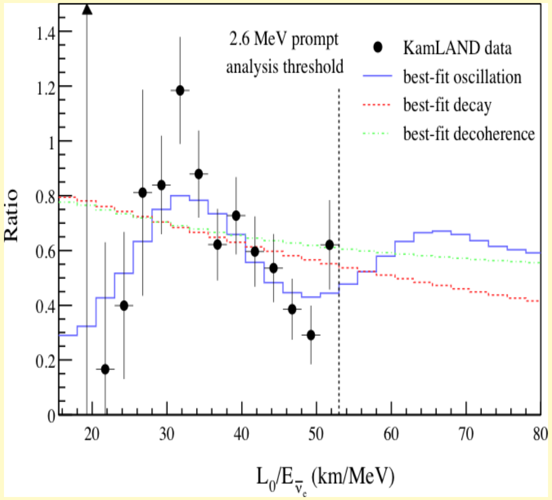


KamLAND results

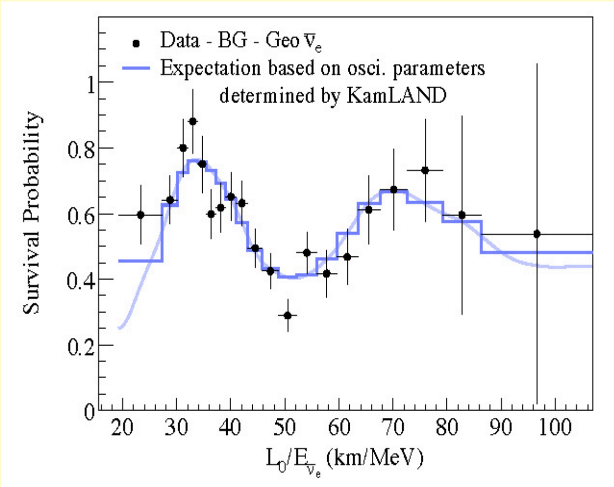
2002: electron flavor disappearance observed



2004: half-period of oscillation observed



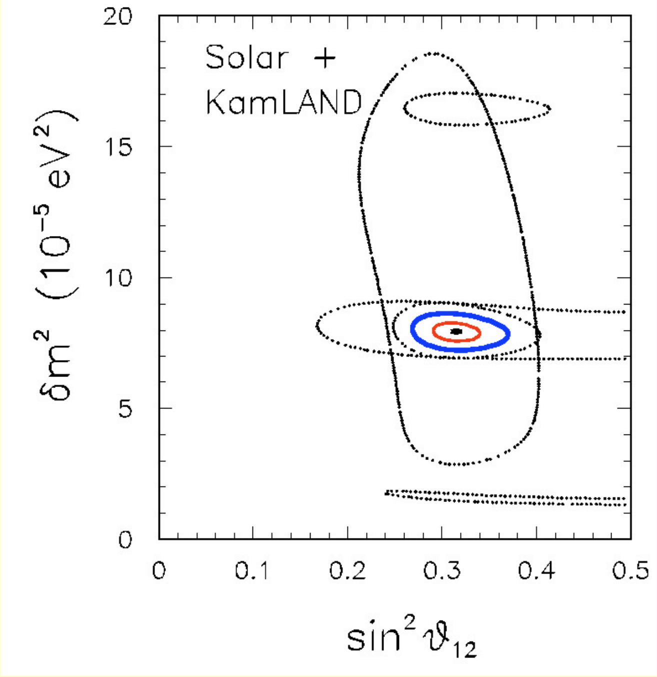
2007: one period of oscillation observed



Direct observation of δm^2 oscillations!

Interpretation in terms of 2ν oscillations

(δm^2, θ_{12}) - complementarity of solar/reactor neutrinos



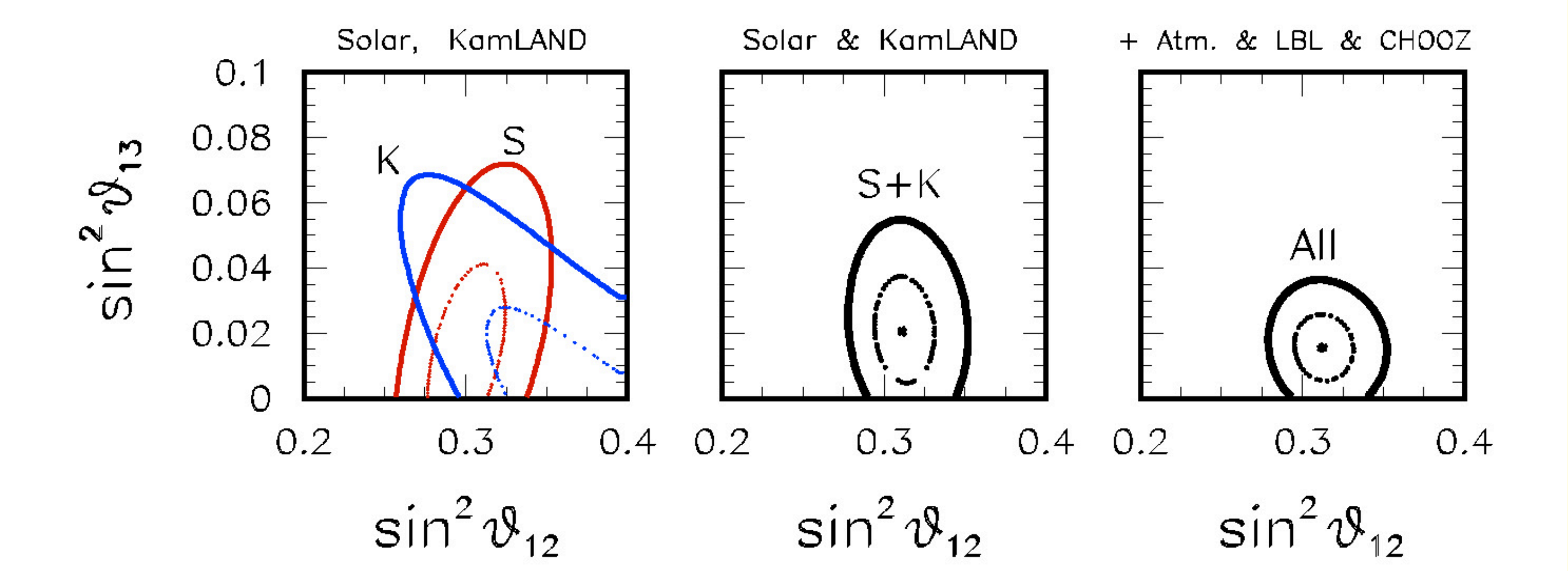
KamLAND

Solar

More refined (3ν) interpretation

Go beyond dominant 3ν oscillations. Include subleading θ_{13} effects in solar+KamLAND (as well as other data).

Interesting hints for $\theta_{13} > 0$ emerged as early as 2008…



Solar, high energy (LMA MSW):

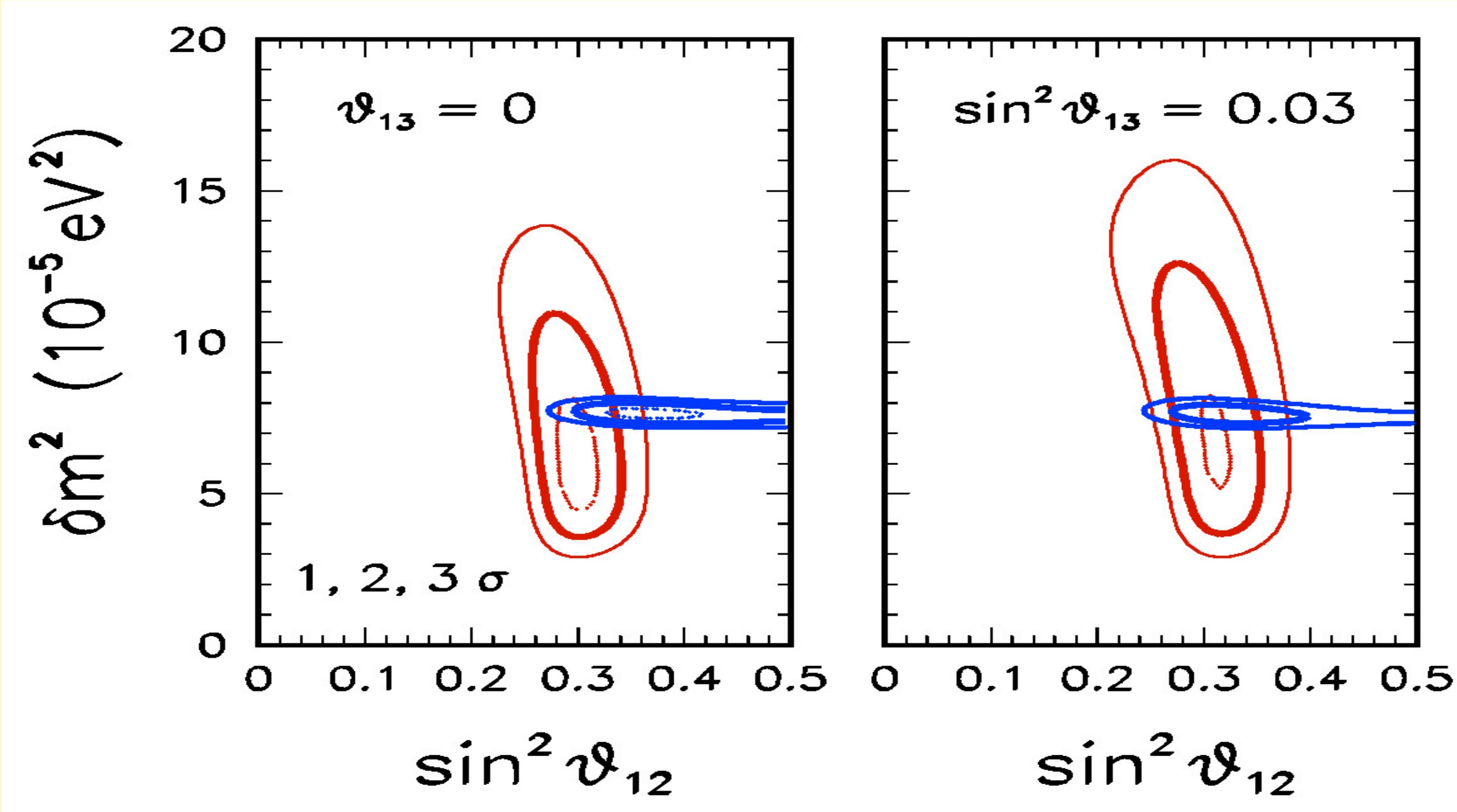
$$P_{ee} \simeq (1 - 2s^2_{13})(+s^2_{12})$$

− +

Reactor (~vacuum): KamLAND

$$P_{ee} \simeq (1 - 2s^2_{13})(1 - 4s^2_{12}c^2_{12}\sin^2(\delta m^2 L/4E))$$

− −



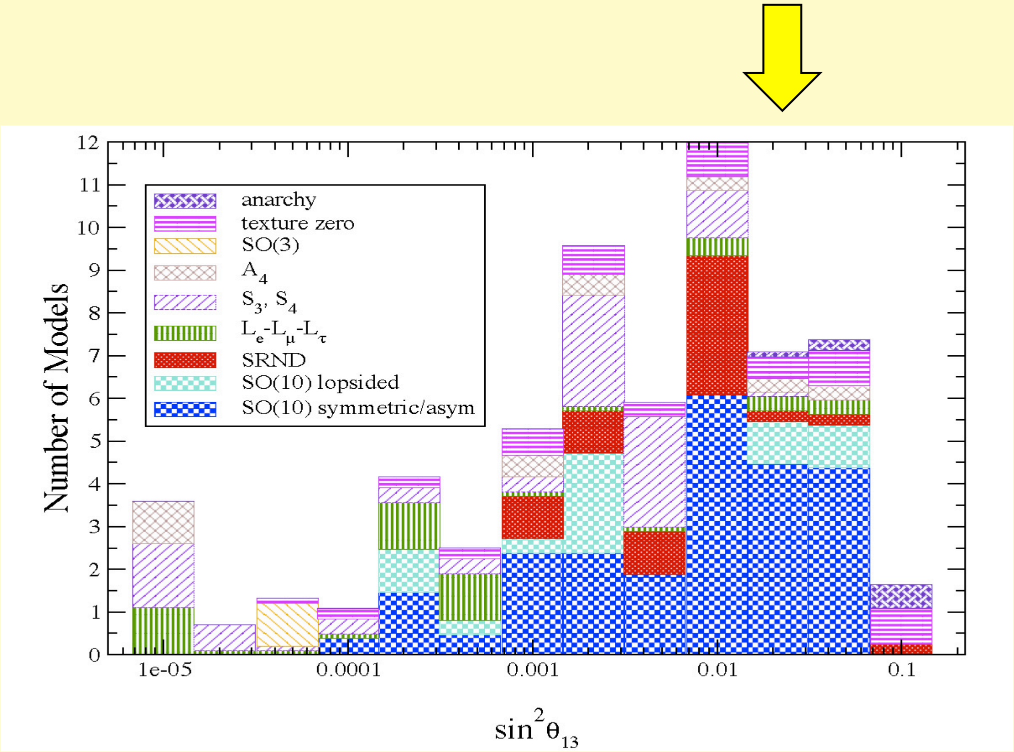
Slight "tension" on θ_{12} could be reduced for $\theta_{13}>0$

Also consistent with MINOS+T2K appearance, 2009-2011. This was an important test of 3ν oscillation consistency, confirmed by the discovery of $\theta_{13}>0$ at reactors in 2012.

PDG 2012: $\sin^2\theta_{13} \approx 0.024 \pm 0.003$

θ_{13} determination essential for further progress in terrestrial oscillation searches (CP violat., matter effects, mass hierarchy)

Also: very important to restrict theoretical models for ν masses



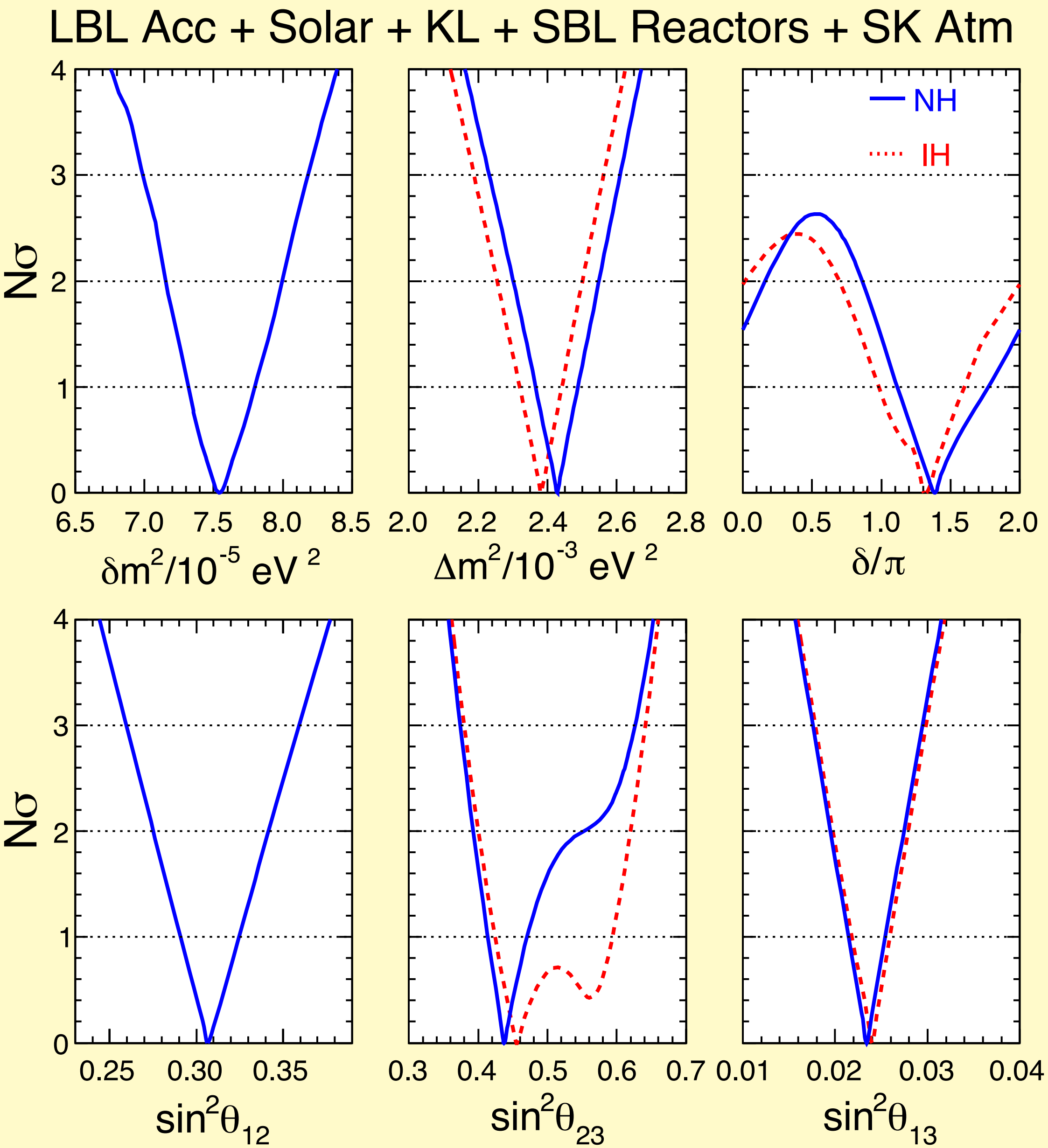
E.g.: CH Albright, 2008, "distribution" of published predictions

Global analysis of world ν oscillation data within the 3ν framework

Extracting oscillation parameters and their covariances from all the available data (solar, atmosph., accelerator and reactor), circa 2013-2014 - arXiv:1312.2878v2.

Full 3ν probabilities included, no approximation.

Individual oscillation parameters from ALL data



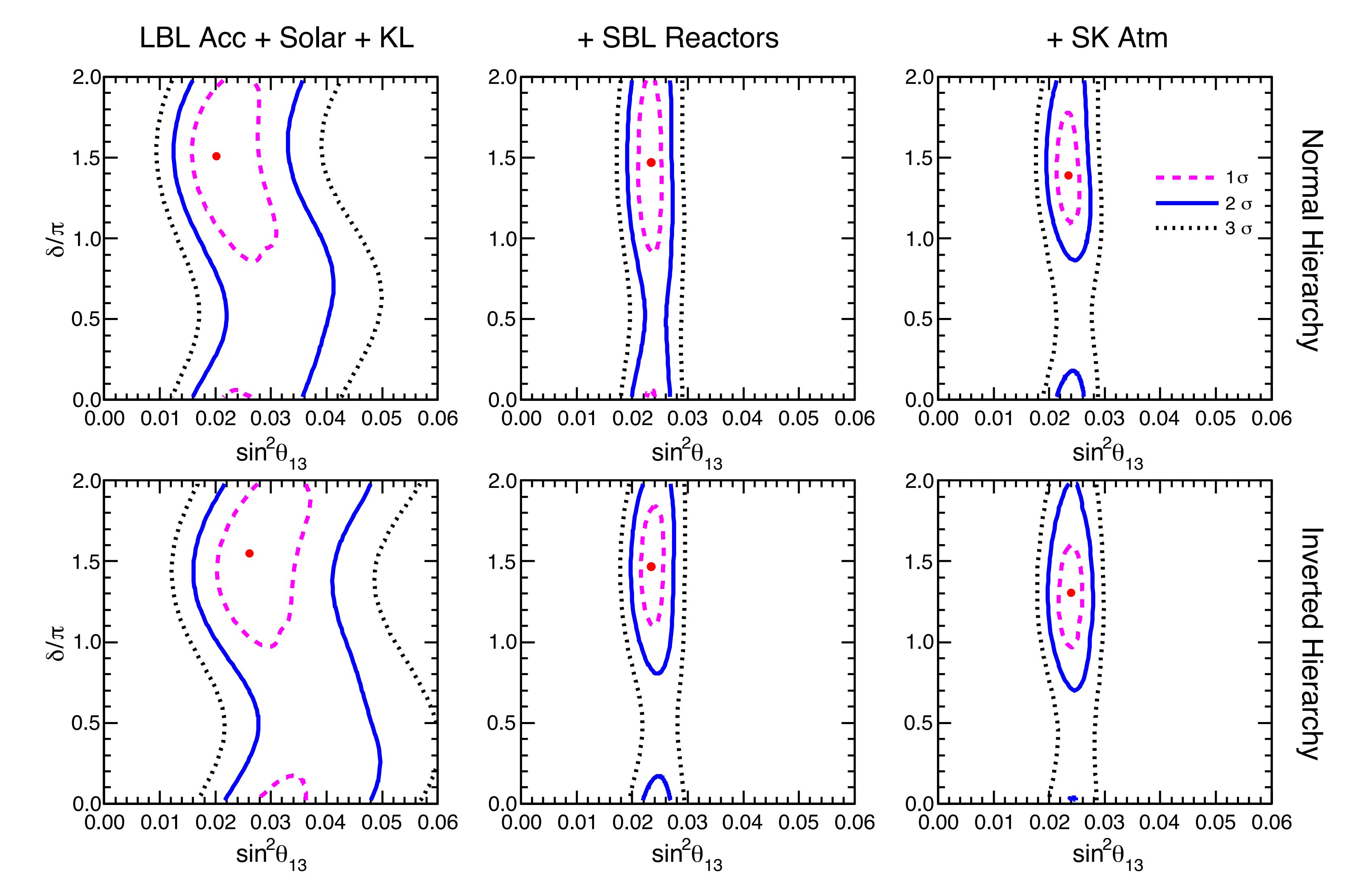
Four parameters have upper and lower bounds at > 4σ. Previous indications for $\theta_{13} > 0$ have now become precise measurements!

Weak hints of $\theta_{23} < \pi/4$, 2nd octant also allowed; more precise data needed.

Hints of $\delta_{CP} \sim 3/2\,\pi$ emerging from all data ?
Is there (maximal) CPV in neutrinos ?

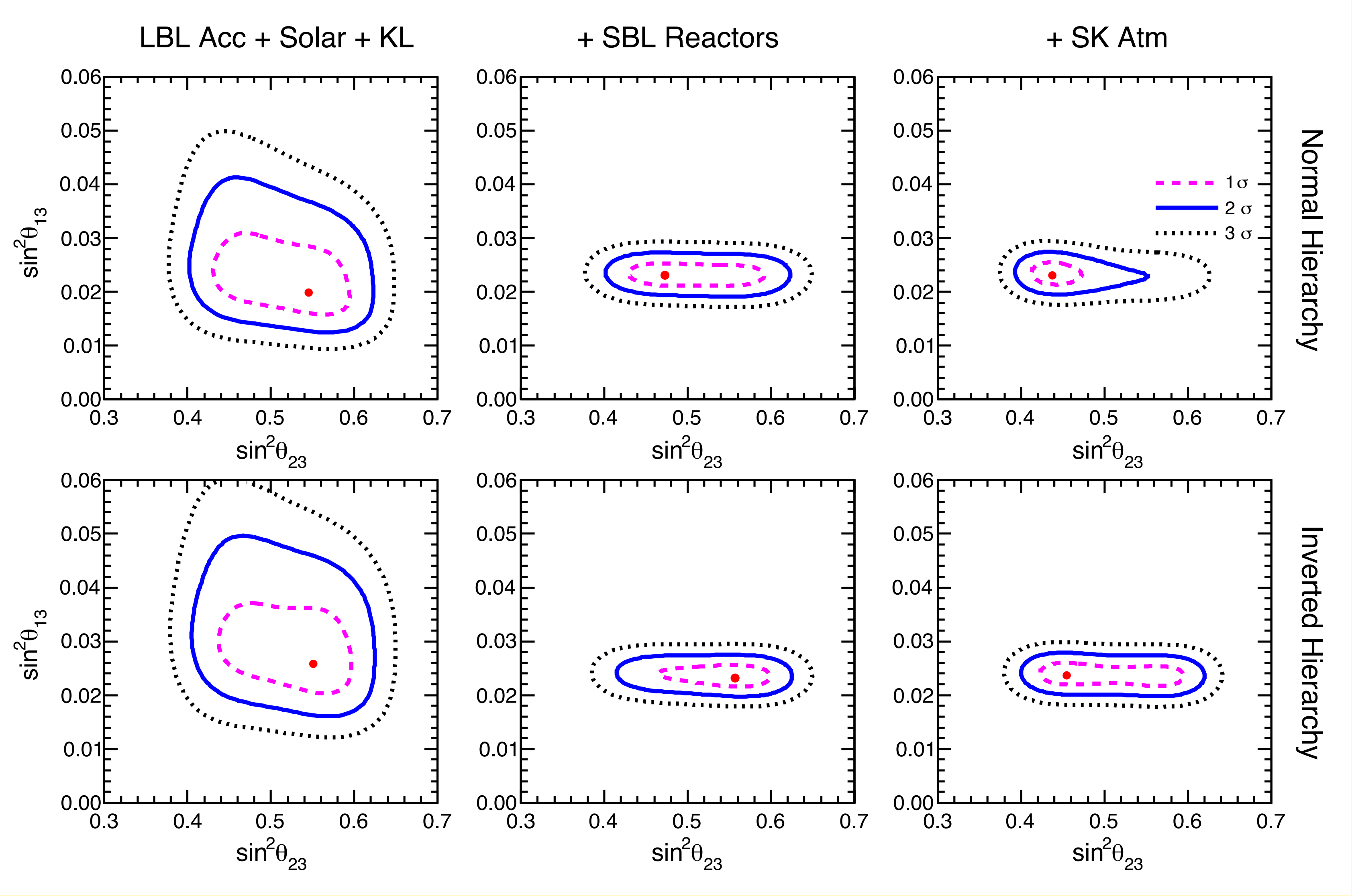
(So far, no clue about NH ⟷ IH)

More on CPV constraints from latest data



Further understanding from future accelerator & reactor data (+SK atm.)

More on octant asymmetry from latest data



Further understanding from future T2K and MINOS+NOvA data (+SK atm.)

Numerical 1σ, 2σ, 3σ ranges:

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. See also Fig. 3 for a graphical representation of the results. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The overall χ^2 difference between IH and NH is insignificant ($\Delta\chi^2_{\mathrm{I-N}} = -0.3$).

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 $\oplus$ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 $\oplus$ 0.70 – 2.00	—

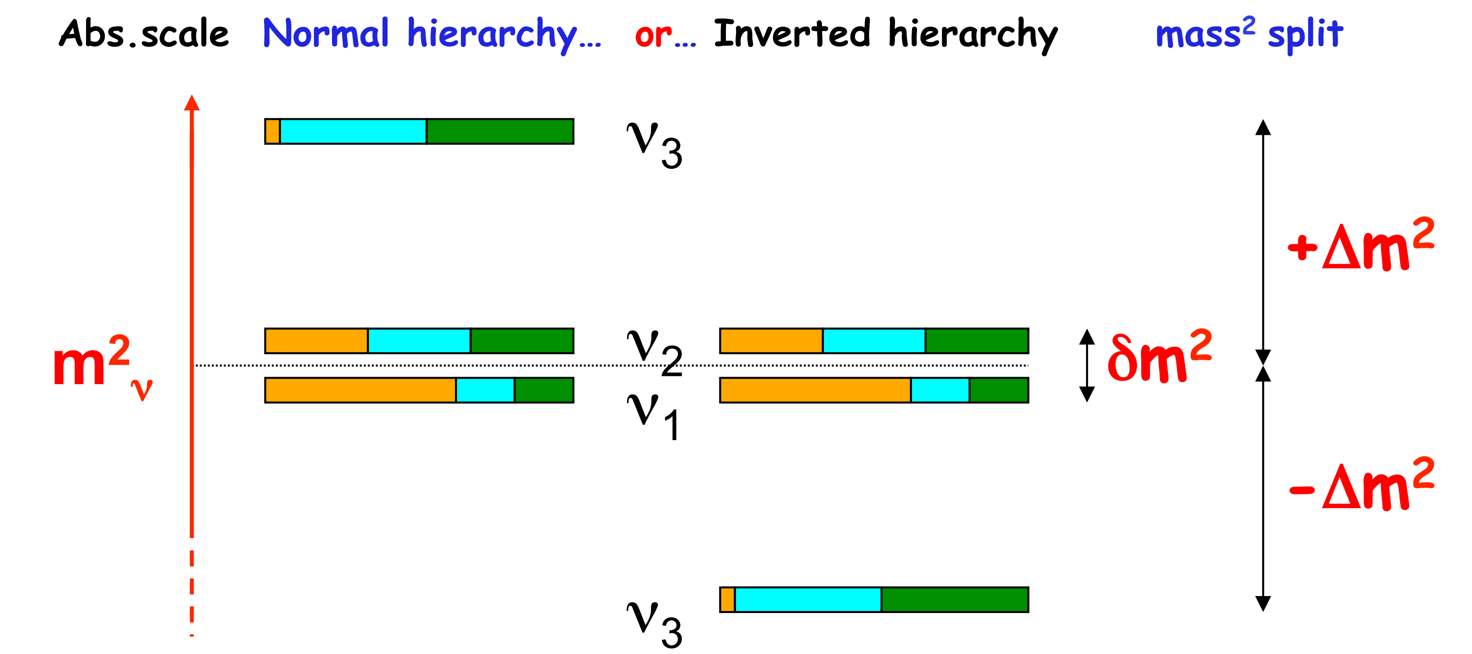
Fractional 1σ accuracy [defined as 1/6 of $\pm 3\sigma$ range]

δm^2	Δm^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
2.6%	2.6%	5.4%	8.5%	9.6%

Hierarchy differences at $\sim 1\sigma$ or below for various data combinations

With 1 digit accuracy: 3ν summary in just one slide!

Flavors = e μ τ



Knowns:

- $\delta m^2 \sim 8 \times 10^{-5}$ eV^2
- $\Delta m^2 \sim 2 \times 10^{-3}$ eV^2
- $\sin^2\theta_{12} \sim 0.3$
- $\sin^2\theta_{23} \sim 0.5$
- $\sin^2\theta_{13} \sim 0.02$

Unkowns:

- δ (CP)
- sign(Δm^2)
- octant($\sin^2\theta_{23}$)
- absolute mass scale
- Dirac/Majorana nature

Epilogue of 1^{st} Lecture:

Nomen [est] Omen

"Name [is] Destiny"

Neutrino - What's in a name?

The root of the name [neutrino] ... is a [kwa]stion

Language	Word tree	*...Some branches*	Meaning
Physics (Fermi 1934)	NEUTR-INO		Little neutral one
Italian	NEUTRO		Neutral
Latin	NE-UTER		Not either; neutral
Latin	UTER		Either
Greek		*OUDETEROS*	*Neutral*
Old High German		*HWEDAR*	*Which of two; whether*
Phonetic change/loss	[K]UOTER[US]		Which of the two?
Ionic Greek	KOTEROS		Which of the two?
Sanskrit	KATARAS		Which of the two?
Latin		*QUANTUS*	*How much?*
Sanskrit		*KATAMAS*	*Which out of many?*
Sanskrit		*KATHA*	*How?*
Sanskrit		*KAS*	*Who?*
Indo-European root	KA *or* KWA		Interrogative base

If "name is destiny," then … neutrino's destiny is to raise questions!

We shall address some of the open questions tomorrow.

Thank you for your attention today!