



# QCD at the LHC

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# Claims and Aims

LHC data is there!!!!

There has been a number of key theoretical results recently in the quest of achieving the best possible **predictions** and **description** of events at the LHC.

Perturbative  QCD applications to LHC physics in conjunction with Monte Carlo developments are **VERY** active lines of theoretical research in particle phenomenology.

In fact, **new dimensions** have been added to  
Theory  $\Leftrightarrow$  Experiment interactions



# Claims and Aims

Cindy's Recipes / About Cindy / Informing You / Home

## McCain Family Recipe

### Passion Fruit Mousse

You'll need:

- 1 1/4 cups passion fruit puree
- 1 1/4 cups orange juice
- 3/4 cup sugar
- Scant 1 tablespoon gelatin dissolved in 2 tablespoons water
- 3 cups heavy cream, whipped
- 2 finger bananas
- Coarse sugar
- 1 kiwi, peeled, cut in 1/2 and sliced

In a saucepan, heat the passion fruit puree, orange juice, and sugar until dissolved.

Add the gelatin to the hot juice and stir to melt and combine. Strain the liquid into a bowl and place it over an ice bath. Stir it constantly with a rubber spatula and when it just starts to set, fold in the whipped cream. Pour this into soup plates or dessert bowls and chill. If storing them overnight, cover them with plastic wrap.

To serve the mousse, remove the bowls from the refrigerator. Peel and slice the bananas in half lengthwise and dip the flat side in coarse sugar and caramelize them under a broiler or with a blowtorch. Place them on the mousse fanning them then tuck in a few half slices of kiwi.



### Passion Fruit Mousse

Recipe courtesy Gale Gand  
Show: [Sweet Dreams](#)  
Episode: [Fabulous Fruit](#)



You can pour the mousse into individual ~~dessert~~ cups or eggcups for a cute presentation. Finger bananas are half size and more yellow but starchier than regular bananas. Passion fruit grows in Bermuda and tropical climates like Asia and New Zealand. My mom brought me home some passion flower perfume from a visit to Bermuda in 1964 when she went to see my aunt Greta who lived there for a bit. The flower almost looks like an exotic insect of purple and yellow.

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My aim is not to swamp you with technical details on **how**,

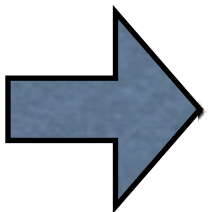


# Claims and Aims

- **perspective:** the big picture
- **physics issues:** QCD from high- to low- $Q^2$ , Parton showers, Angular ordering, jet algos
- **recent progress:** NLO computations, merging Monte Carlo with FO.
- **key applications at the LHC:** Drell-Yan, Top, Higgs, Jets, BSM,...



# Claims and (your) Aims



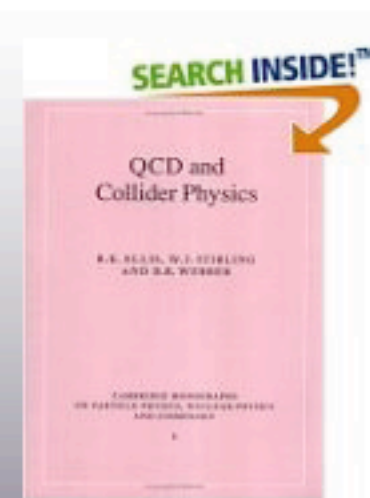


# Minimal references and write-ups

Ellis, Stirling, Webber: The pink book

Subtitle:

“All you want to know about perturbative QCD  
and never dared to ask...”



\*Very useful recent talks/lectures by (just google the names):  
Gavin Salam, Stefano Frixione, Michelangelo Mangano.



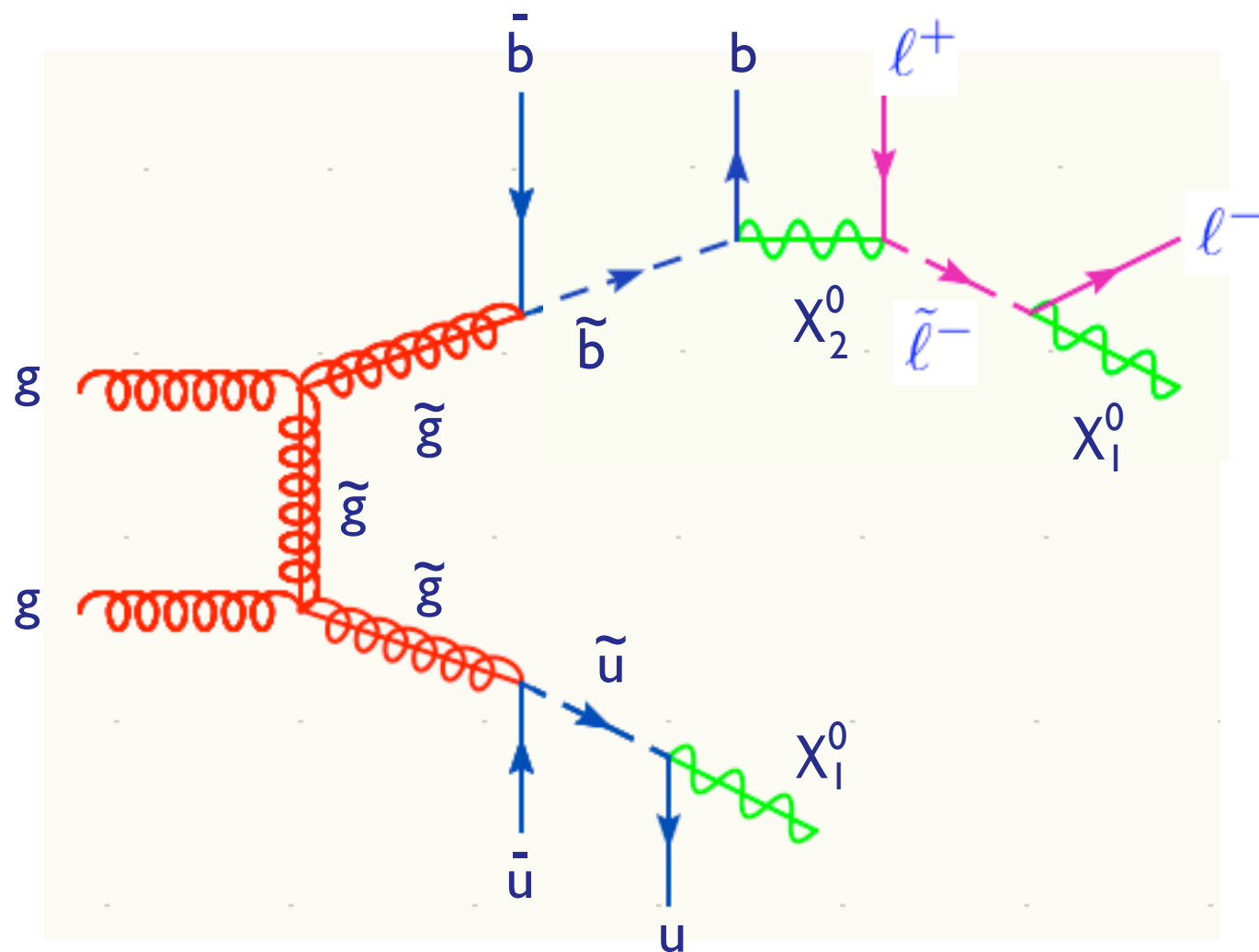
# Discoveries at hadron colliders





# A new challenge

Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing  $E_T$ ...

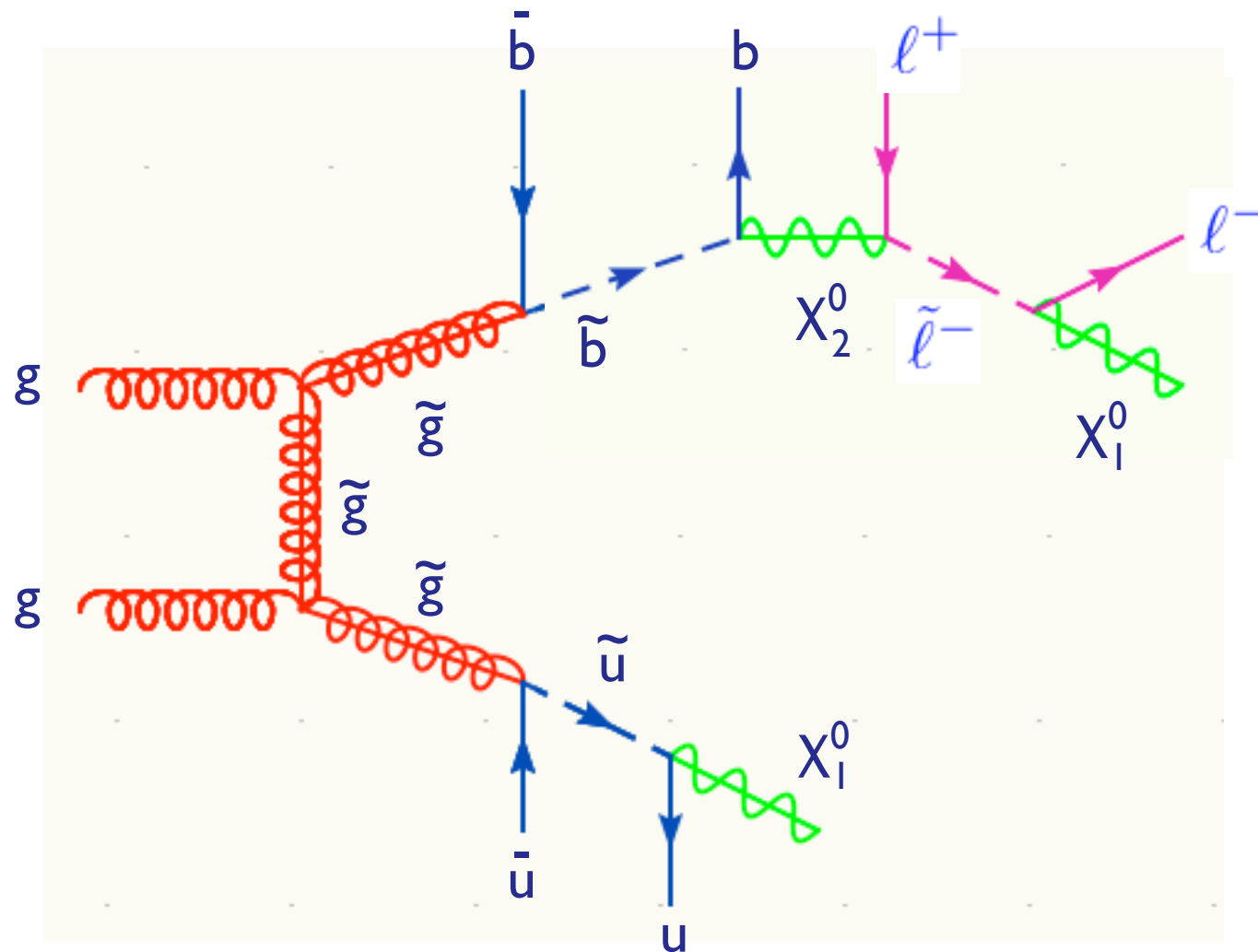




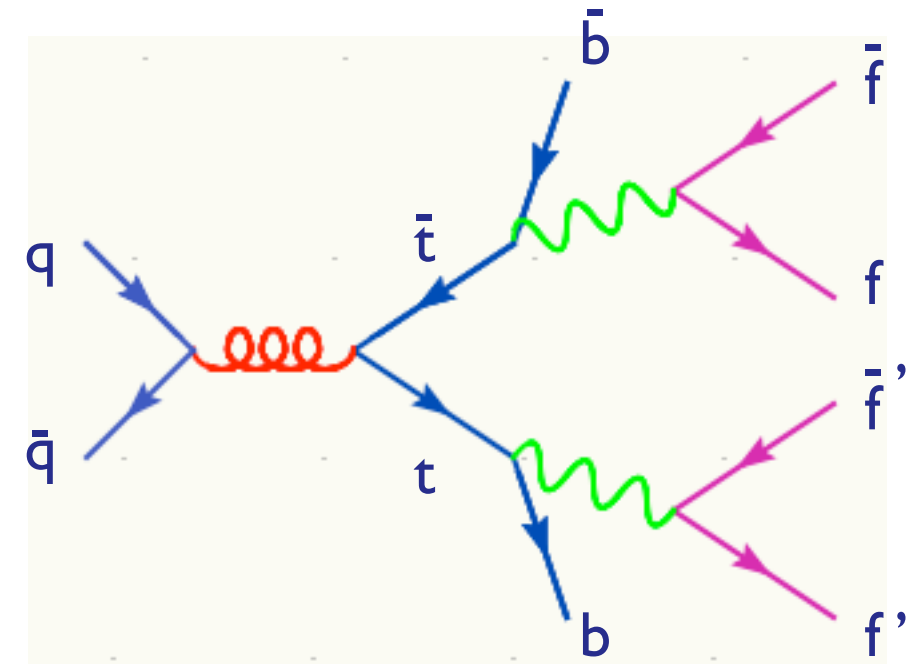


# A new challenge

Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing  $E_T$ ... We have already a very good example of a similar discovery!



VS

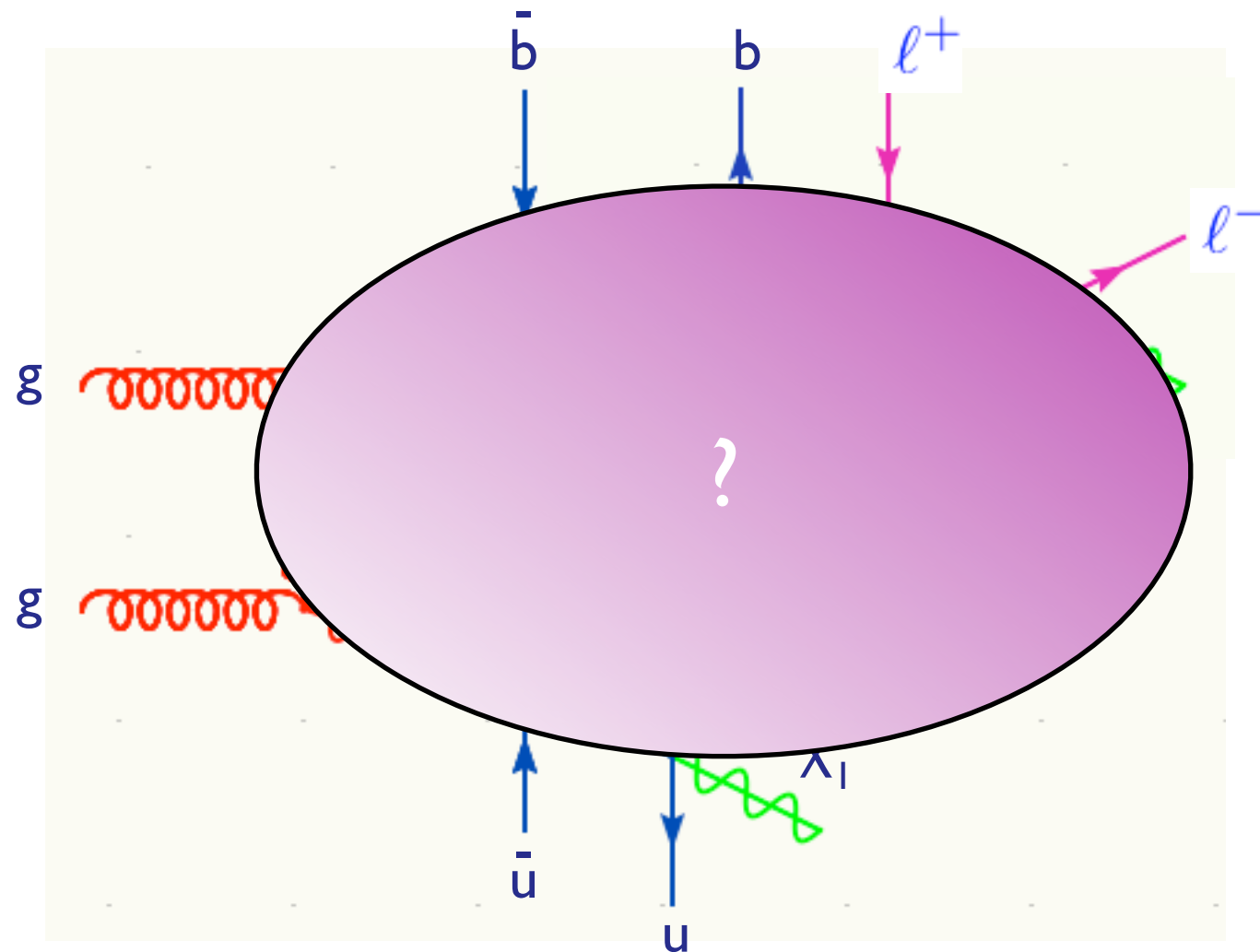


Follow the same approach of CDF in 1995 to establish first evidence of an excess wrt to SM-top and then consistency with SM top production [ $m_t=174$ ,  $t \rightarrow b \nu$ ,  $\sigma(tt)$ ], works for the SM Higgs, but in general beware that...

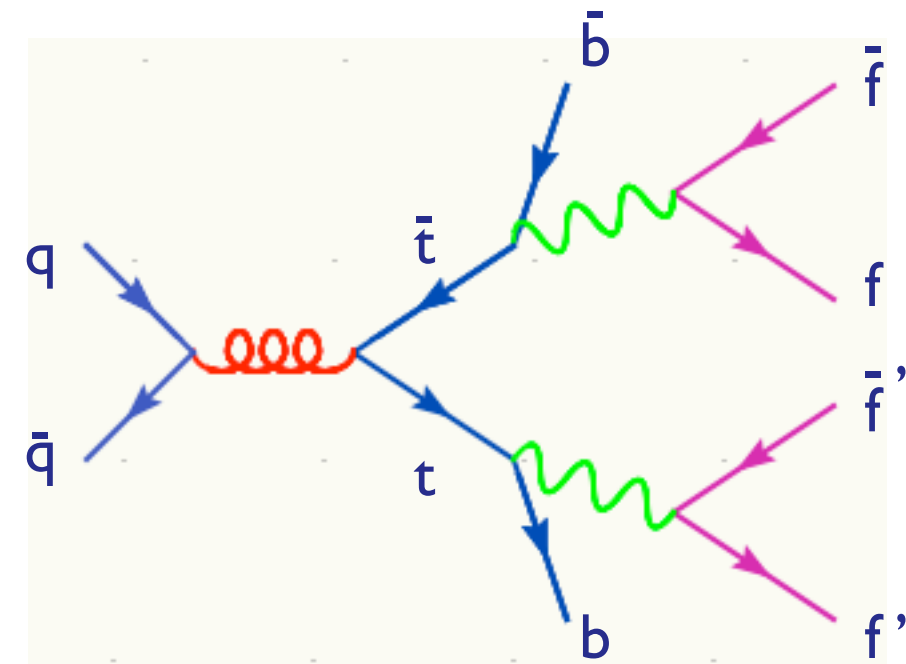


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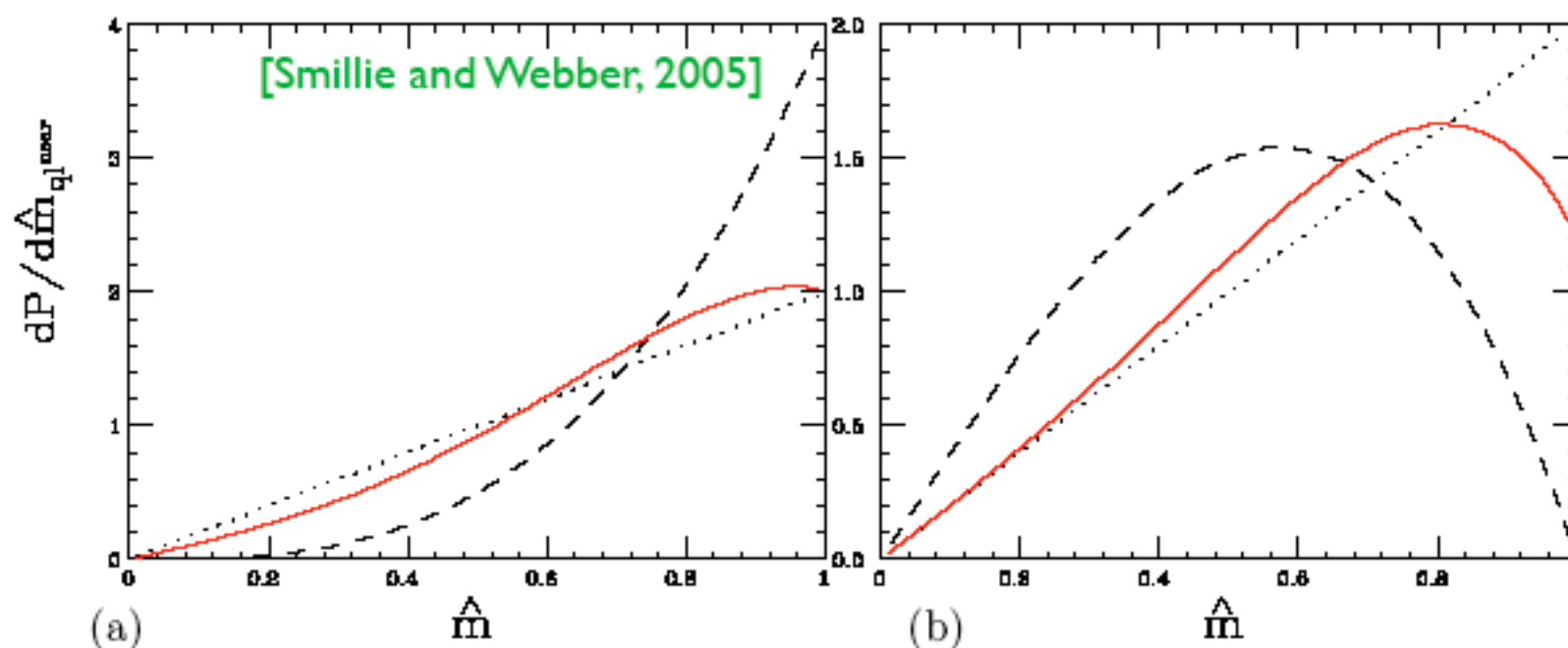
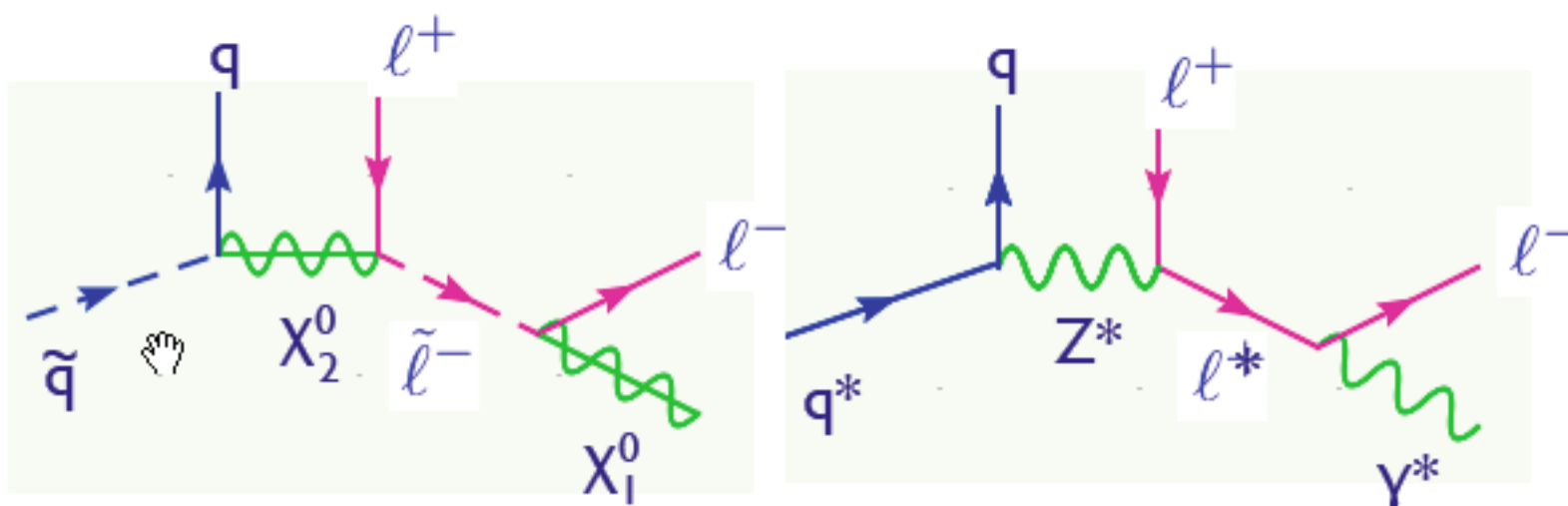


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# Example: SUSY vs UED

Information on the mass of the intermediate states can be obtained through the study of kinematical edges. The shape of the edges can give information on the spin of the intermediate states. Compare for instance SUSY and UED:



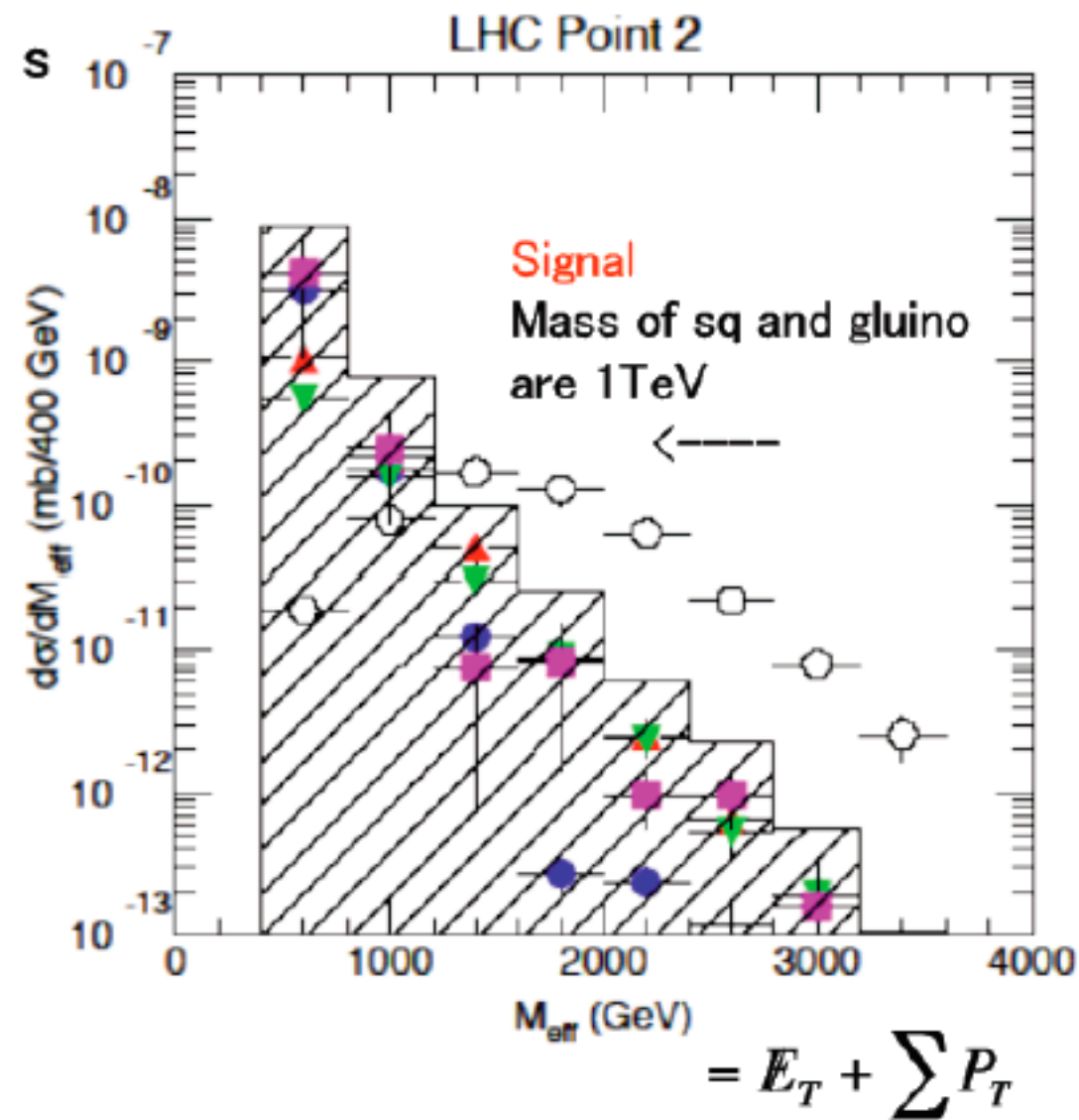
Beware that most of the MC's make some of or all the following simplifications:

1. production and decay are factorized.
2. Spin is ignored.
3. Chains proceed only through  $1 \rightarrow 2$  decays.
4. The narrow width approximation is employed.
5. Non-resonant diagrams are ignored.

Flexible and powerful ME tools are needed to check and in case go beyond the above approximations!



# Example: early discovery SuperSymmetry at the LHC



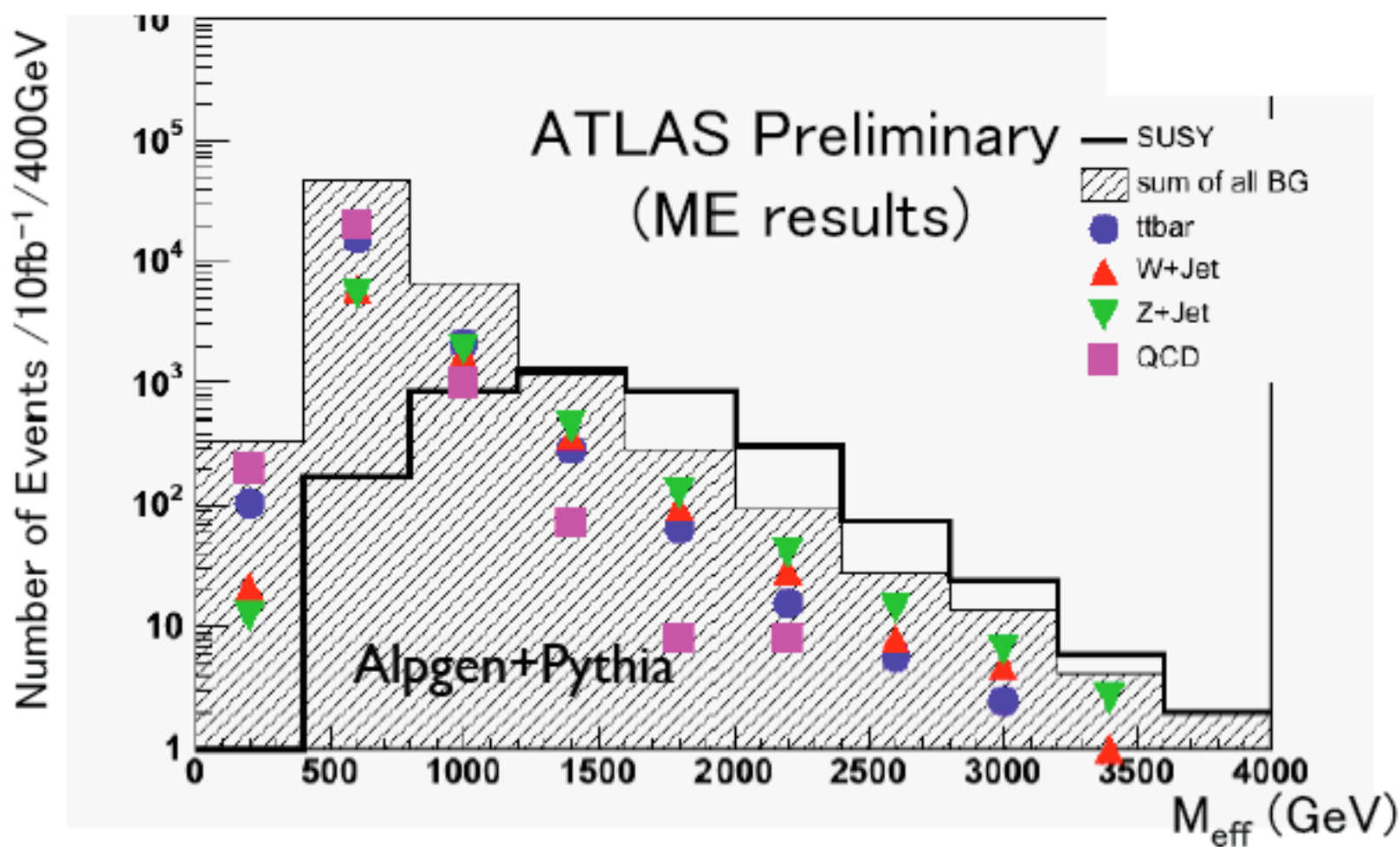
“Old MC”

**Background:**  $t \bar{t} + \text{jets}, (Z, W) + \text{jets}, \text{jets}$ . Very difficult to estimate theoretically: many parton calculation ( $2 \rightarrow 8$  gluons = 10 millions Feynman diagrams diagrams!!). Now MC's for this are available...





# Example: early discovery SuperSymmetry at the LHC

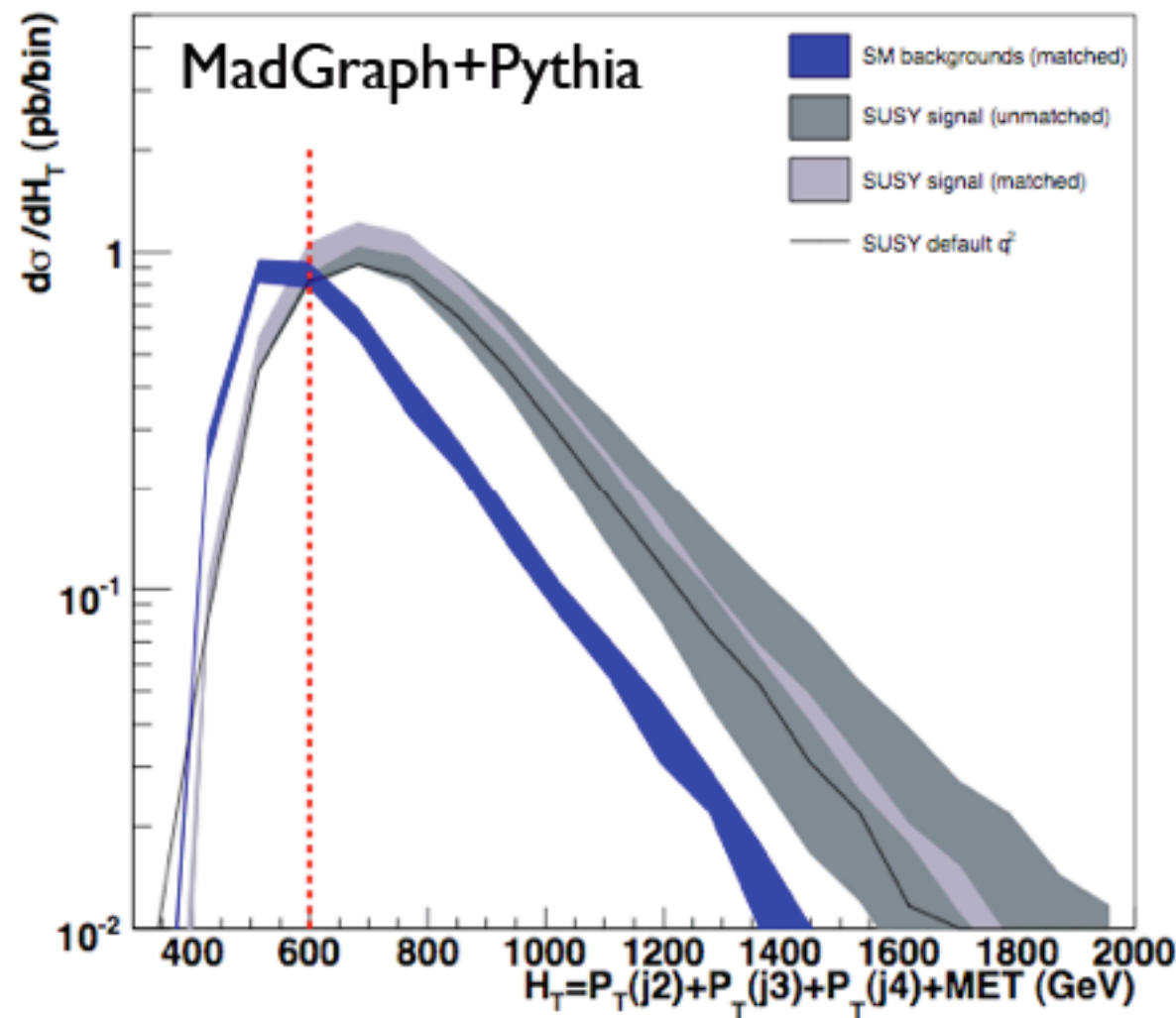


“New MC  
for the BKG”

**Background:**  $t\bar{t}$  + jets, (Z,W) + jets, jets. Very difficult to estimate theoretically: many parton calculation ( $2 \rightarrow 8$  gluons = 10 millions Feynman diagrams diagrams!!). Now MC's for this are available...



# Example: early discovery SuperSymmetry at the LHC



“New MC  
for Signal & BKG”

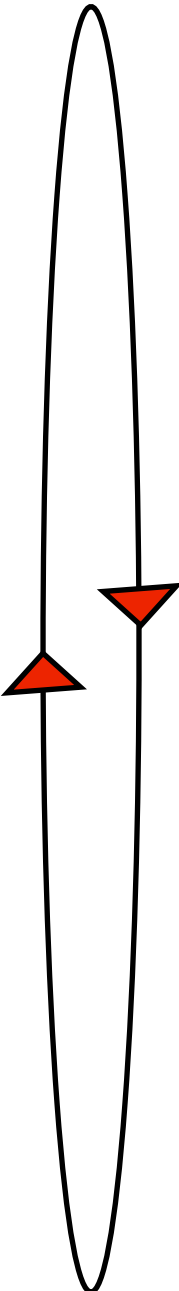
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**Texte:** signal matched ME+PS. Predictability improved. Same theoretical status as the background.



# The path towards discoveries

$$\text{LHC physics} = \text{QCD} + \epsilon$$



1. Rediscover the known SM at the LHC (top's, W's, Z's) + jets.

New regime for QCD. Exclusive **description** for rich and energetic final states with flexible MC to be validated and tuned to control samples. Shapes for multi-jet final states and normalization for key process important. Accurate **predictions** (NLO, NNLO) needed only for standard candle cross sections.

2. Identify excess(es) over SM

Importance of a good theoretical description depends on the nature of the physics discovered: from none (resonances) to fundamental (inclusive SUSY).

3. Identify the nature of BSM:  
from coarse information to  
measurements of mass spectrum,  
quantum numbers, couplings.

Not fully worked out strategy. Several approaches proposed (MARMOSSET, VISTA,...). Only in the final phase accurate QCD predictions and MC tools for SM as well as for the BSM signals will be needed.





# Bottom-line

No QCD  $\Rightarrow$  No Party



# A simple plan

- Intro: the LHC challenge
- Precision QCD: from LO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

today



# Master QCD formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

I. Parton Distribution functions (from exp, but evolution from th).



# Progress in the PDF

PDF measured at HERA and fixed-target experiments.  $x$  dependence from data.  
 $Q^2$  dependence from DGLAP evolution.

## Recently:

NNLO calculation of the 3-loop splitting kernels (“the hardest calculation in QCD”)

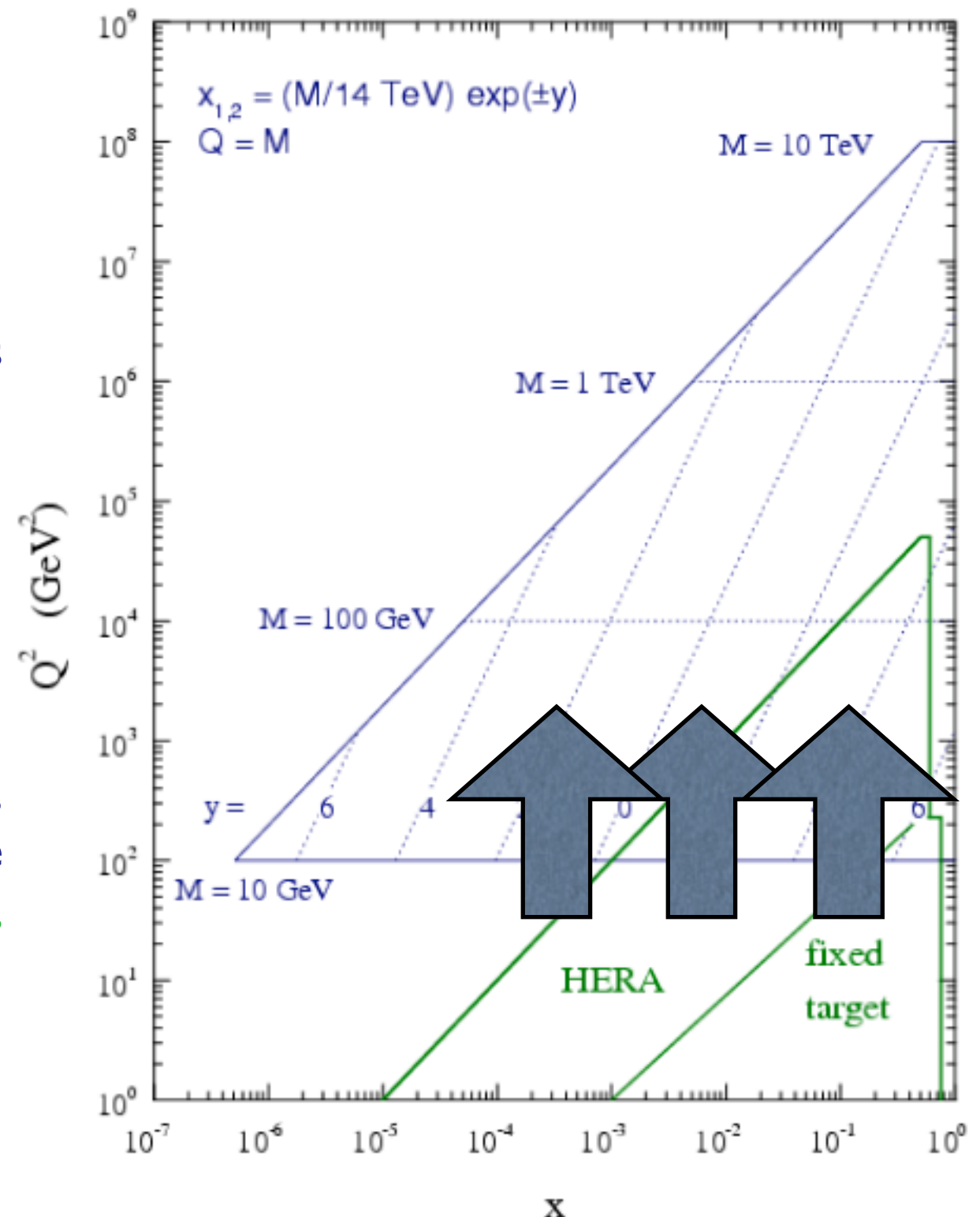
[Moch, Vermaseren, Vogt. 2004]

Together with short distance NNLO calculation first sets of NNLO PDF sets. [MRST and Alekhin, 2004]

PDF's with errors: Various “traditional methods”, [CTEQ and MRST, 2003]. Also new approaches, the functional space [Giele, Keller, Kosower.2001] and the Neural Network (NNPDF) approach [Del Debbio, Forte, La Torre, Piccione, Rojo. 2002,2005].

## Issues:

1. small- $x$  effects
2. Heavy flavors pdf





# Master QCD formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).
2. Short distance coefficients as an expansion in  $\alpha_S$  (from th).

$$\hat{\sigma}_{ab \rightarrow X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order



# LO : the technical challenges

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses ( $gg \rightarrow ggg$ ,  $qg \rightarrow qgg$ ....) in

$$\sigma(pp \rightarrow 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \rightarrow k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\}, \{h\}, \{c\}) = \sum_i D_i$$

difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space ( $D \sim 3n$ )

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

very hard



# Leading Order

\* Many available algorithms for automatic generation of tree-level matrix element, some of which in a public tools:

- **Feynman diagrams** (with tricks to reduce factorial growth) :  
CompHEP/CalcHEP, AMEGIC++, MadGraph
- **off-shell recursive relations**: Berends-Giele, ALPHA/ALPGEN, HELAC, COMIX
- **on-shell recursive relations** (twistor inspired) : CSW, BCFW

\* Automatic/modular integration over phase space and event generation:

- HELAC/PHEGAS, MadEvent, SHERPA, ALPGEN

\* Merging with PS : HELAC (MLM), SHERPA (CKKW), ALPGEN (MLM), MadEvent (CKKW, KTMLM)

[Duhr, Hoeche, FM]

| Final State | BG      |        | BCF      |           | CSW      |          |
|-------------|---------|--------|----------|-----------|----------|----------|
|             | CO      | CD     | CO       | CD        | CO       | CD       |
| 2g          | 0.24    | 0.28   | 0.28     | 0.33      | 0.31     | 0.26     |
| 3g          | 0.45    | 0.48   | 0.42     | 0.51      | 0.57     | 0.55     |
| 4g          | 1.20    | 1.04   | 0.84     | 1.32      | 1.63     | 1.75     |
| 5g          | 3.78    | 2.69   | 2.59     | 7.26      | 5.95     | 5.96     |
| 6g          | 14.20   | 7.19   | 11.90    | 59.10     | 27.80    | 30.60    |
| 7g          | 58.50   | 23.70  | 73.60    | 646.00    | 146.00   | 195.00   |
| 8g          | 276.00  | 82.10  | 597.00   | 8690.00   | 919.00   | 1890.00  |
| 9g          | 1450.00 | 270.00 | 5900.00  | 127000.00 | 6310.00  | 29700.00 |
| 10g         | 7960.00 | 864.00 | 64000.00 |           | 48900.00 |          |

The “good and old” BG provide the fastest approach. Need to work also for complex momenta





## Leading Order

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- Matrix element calculators provide our first estimation of rates for **inclusive** final states.
- Extra radiation **is** included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.
- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.
- Any tree-level calculation for a final state F can be promoted to the exclusive F + X through a shower. More on this tomorrow...

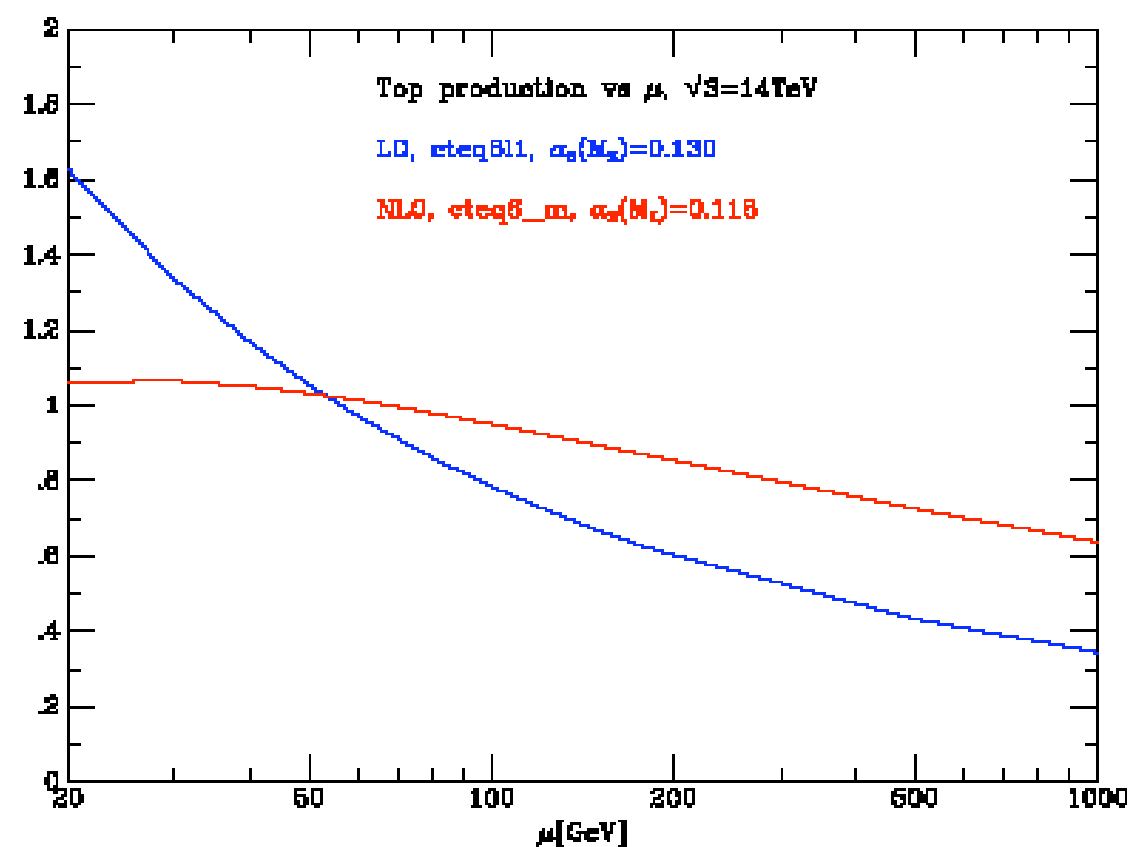
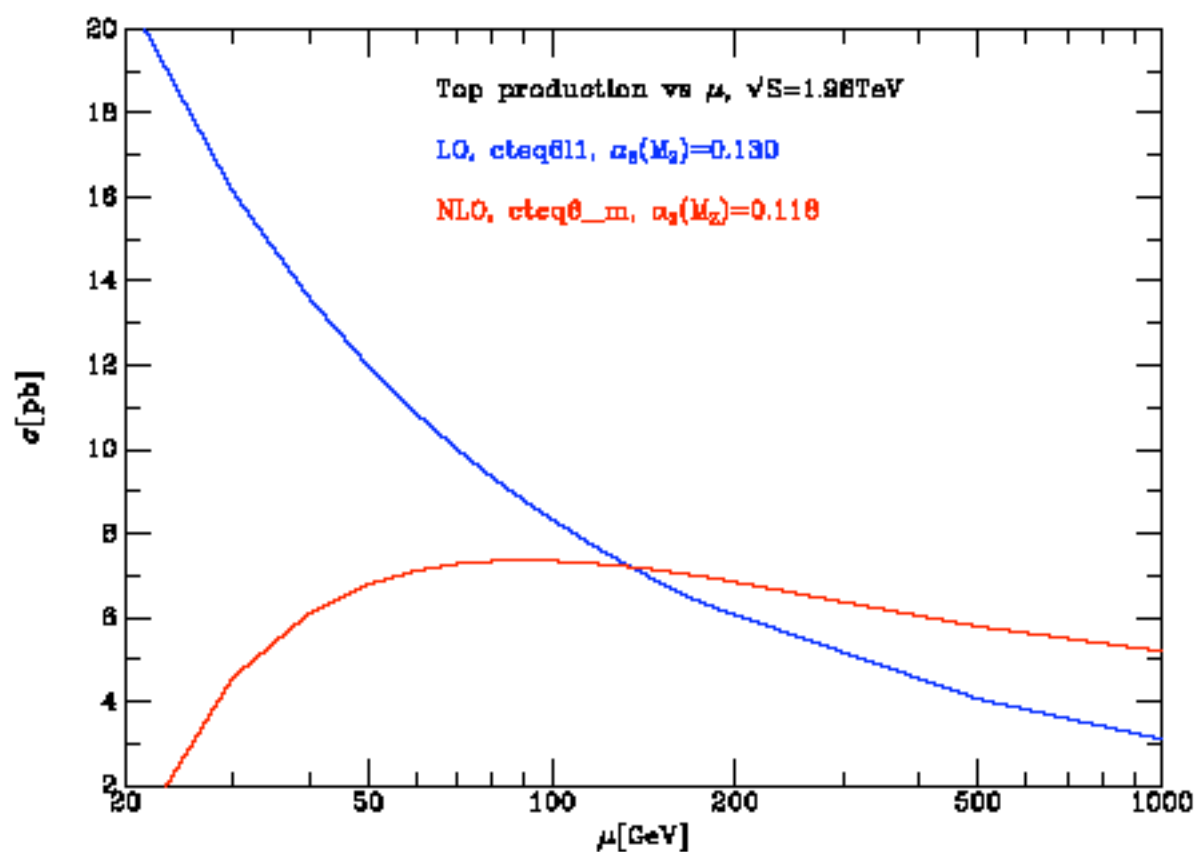
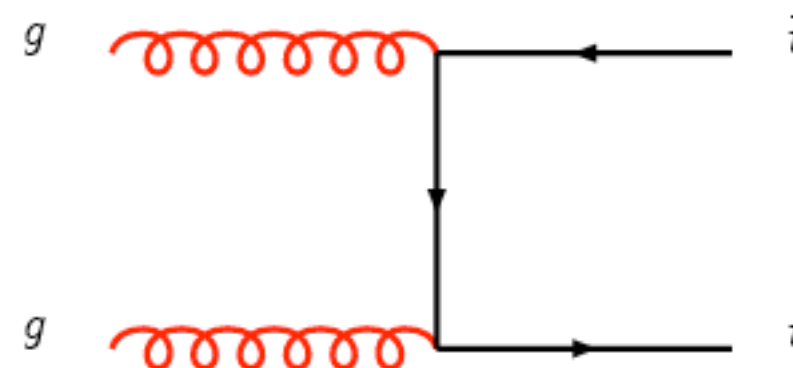
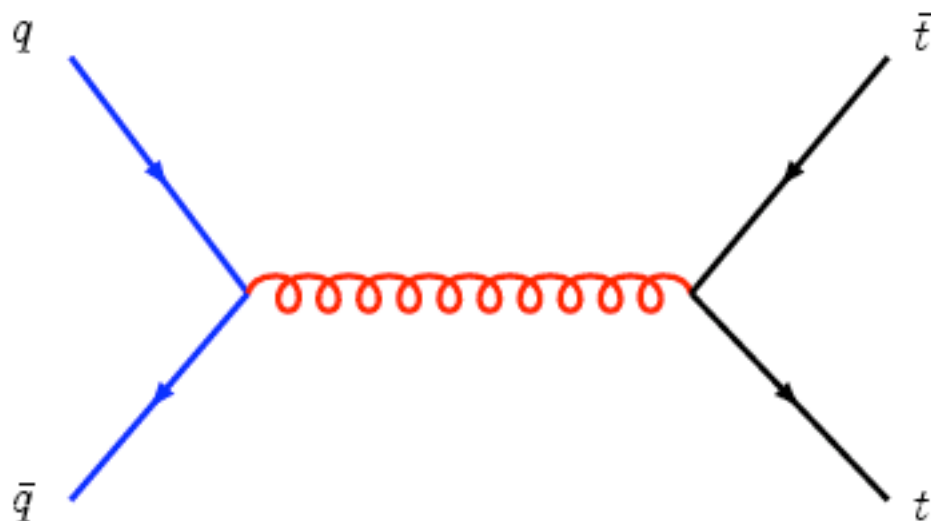


# NLO calculations

- NLO calculations are needed to perform measurements where the knowledge of total and differential rates is essential. This is true not only for the signal but also for the backgrounds.
- Standard NLO programs do not produce unweighted events and therefore are not suitable for direct experimental analysis.
- In fact, it can be highly non-trivial to establish an accurate connection between what is computed at the partonic level and what is measured (hadronic quantities).
- Comparison with data can be done once detector and hadronization effects have been deconvoluted.
- Be aware that there are many possibly dangerous (mal)practices in the exp community (K-factor, reweighting of distributions,...)
- Suggestion: always consult with the authors of the code in case of doubts...



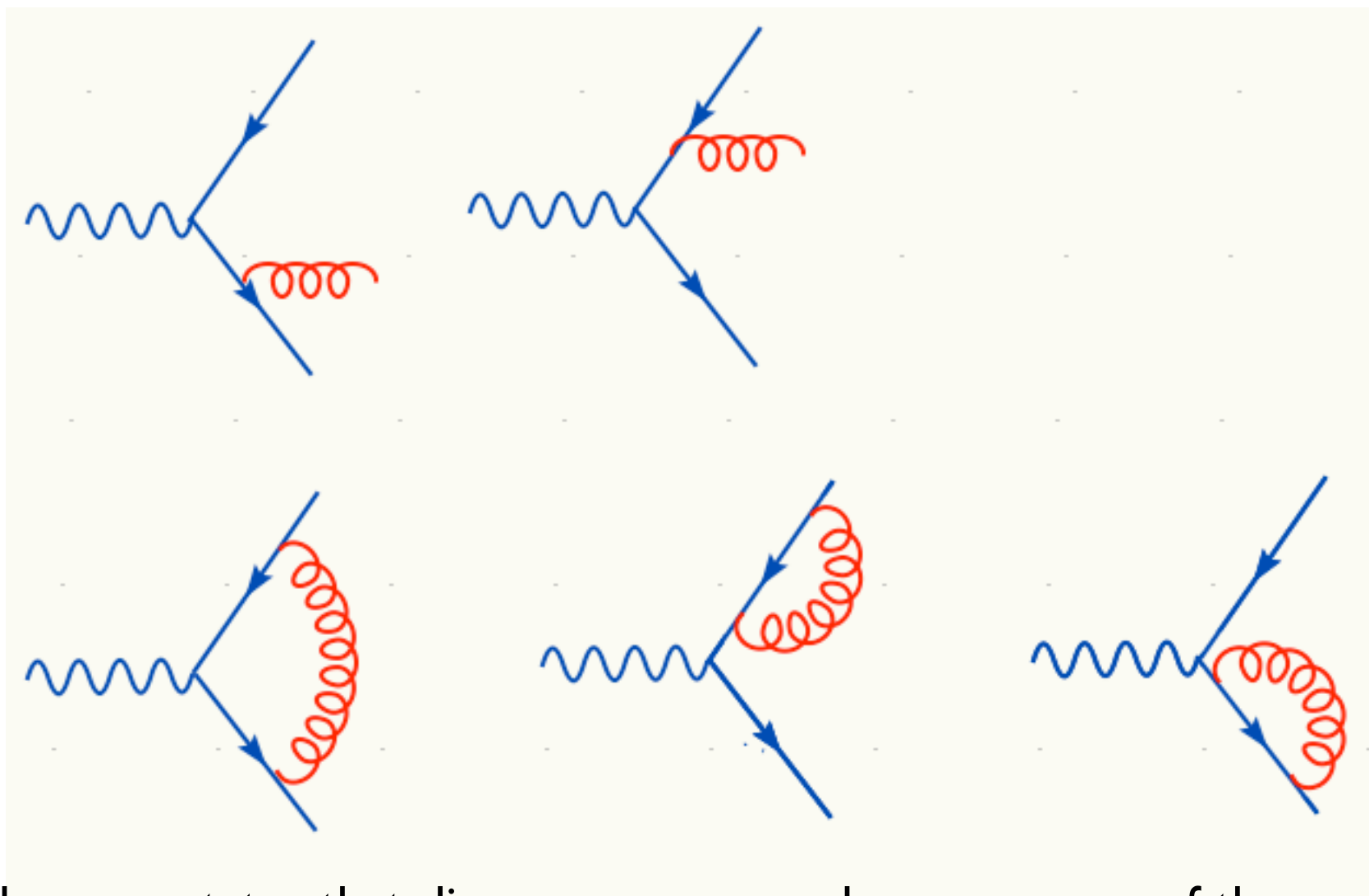
# Tevatron vs LHC



Inclusion of higher order corrections leads to a stabilization of the prediction.  
At the LHC scale dependence is more difficult to estimate.



# The elements of NLO calculation



Real

Virtual

The KLN theorem states that divergences appear because some of the final state are physically degenerate but we treated them as different. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual).

$$\sigma^{\text{NLO}} = \int_R |M_{\text{real}}|^2 d\Phi_3 + \int_V 2\text{Re} (M_0 M_{\text{virt}}^*) d\Phi_2 = \text{finite!}$$

$$\int \frac{d^d k}{(2\pi)^d} \cdots$$



# Infrared divergences

Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.

When distances become comparable to the hadron size of  $\sim 1$  Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

We have seen that in total cross sections such divergences cancel. But what about for other quantities?

Well obviously the only possibility is to try to use the pQCD calculations for quantities that are not sensitive to the to the long-distance physics.

Can we formulate a criterium that is valid in general?

**YES! It is called INFRARED SAFETY**



# Infrared-safe quantities

**DEFINITION:** quantities are that are insensitive to soft and collinear branching.

For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel between real and virtual or are simply removed by kinematic factors.

Such quantities are determined primarily by hard, short-distance physics. Long-distance effects give power corrections, suppressed by the inverse power of a large momentum scale (which must be present in the first place to justify the use of PT).

**EXAMPLES:** total rates & cross sections, jet distributions, shape variables...

NLO codes calculate IR safe quantities  
and return histograms (calculators)



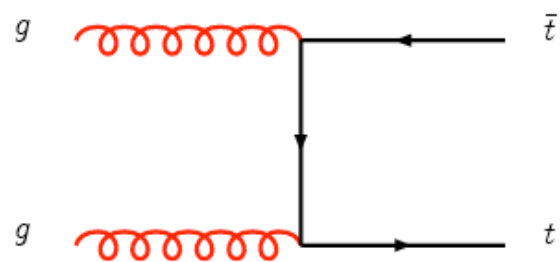
# Something to remember well

Calling a code “a NLO code” is an abuse of language and can be confusing.

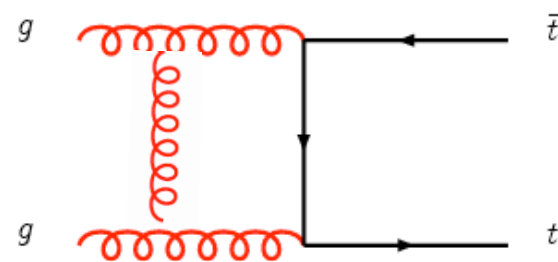
A NLO calculation always refers to **an IR-safe observable**.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

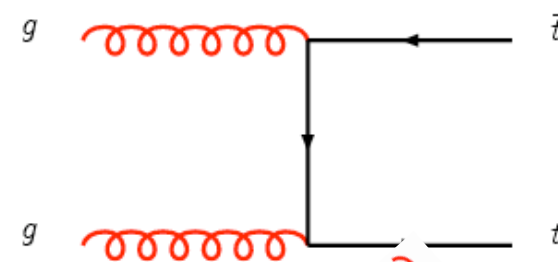
Example: Suppose we use the NLO code for  $pp \rightarrow t\bar{t}$



LO



Virt



Real

- ☞ Total cross section,  $\sigma(t\bar{t})$  ..... ✓
- ☞  $P_T$  of one top quark ..... ✓
- ☞  $P_T$  of the  $t\bar{t}$  pair ..... ✗
- ☞  $P_T$  of the jet ..... ✗
- ☞  $t\bar{t}$  invariant mass,  $m(t\bar{t})$  ..... ✓
- ☞  $\Delta\Phi(t\bar{t})$  ..... ✗





# Anatomy of $pp \rightarrow \text{Higgs}$ at NLO

- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC



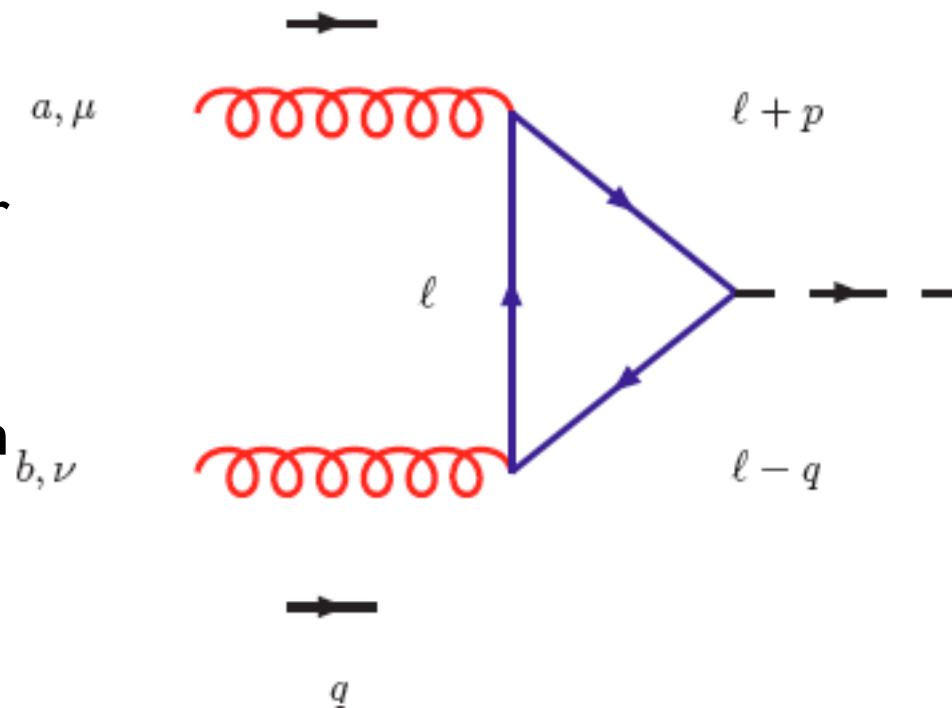
# $pp \rightarrow H$ at LO $p$

This is a “simple”  $2 \rightarrow 1$  process.

However, at variance with  $pp \rightarrow W$ , the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!

Let's do the calculation!



$$i\mathcal{A} = -(-ig_s)^2 \text{Tr}(t^a t^b) \left( \frac{-im_t}{v} \right) \int \frac{d^d \ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\text{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q)$$

where

$$\text{Den} = (\ell^2 - m_t^2)[(\ell + p)^2 - m_t^2][(\ell - q)^2 - m_t^2]$$

We combine the denominators into one by using  $\frac{1}{ABC} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[Ax + By + C(1-x-y)]^3}$

$$\frac{1}{\text{Den}} = 2 \int dx dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}.$$

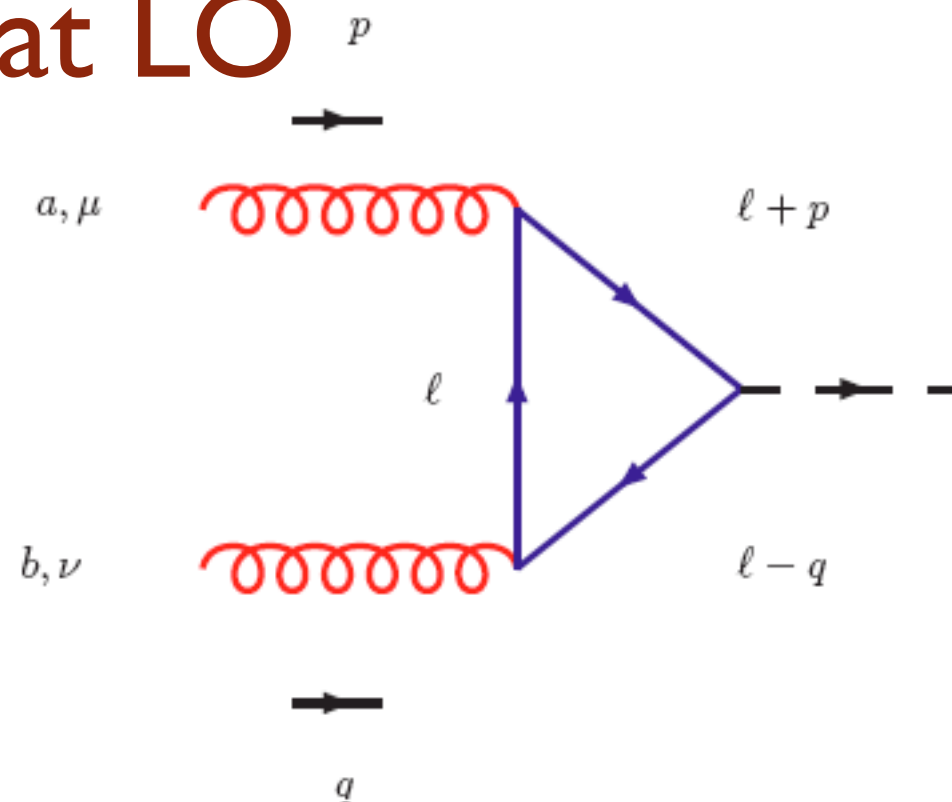


# $pp \rightarrow H$ at LO

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - C)^3} = \frac{i}{32\pi^2} (4\pi)^\epsilon \frac{\Gamma(1 + \epsilon)}{\epsilon} (2 - \epsilon) C^{-\epsilon}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - C)^3} = -\frac{i}{32\pi^2} (4\pi)^\epsilon \Gamma(1 + \epsilon) C^{-1-\epsilon}.$$

where  $d=4-2\epsilon$ . By substituting we arrive at a very simple final result!!



$$\mathcal{A}(gg \rightarrow H) = -\frac{\alpha_S m_t^2}{\pi v} \delta^{ab} \left( g^{\mu\nu} \frac{M_H^2}{2} - p^\nu q^\mu \right) \int dx dy \left( \frac{1 - 4xy}{m_t^2 - m_H^2 xy} \right) \epsilon_\mu(p) \epsilon_\nu(q).$$

## Comments:

- \* The final dependence of the result is  $m_t^2$ : one from the Yukawa coupling, one from the spin flip.
- \* The tensor structure could have been guessed by gauge invariance.
- \* The integral depends on  $m_t$  and  $m_h$ .



# LO cross section

$$\sigma(pp \rightarrow H) = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 g(x_1, \mu_f) g(x_2, \mu_f) \hat{\sigma}(gg \rightarrow H)$$

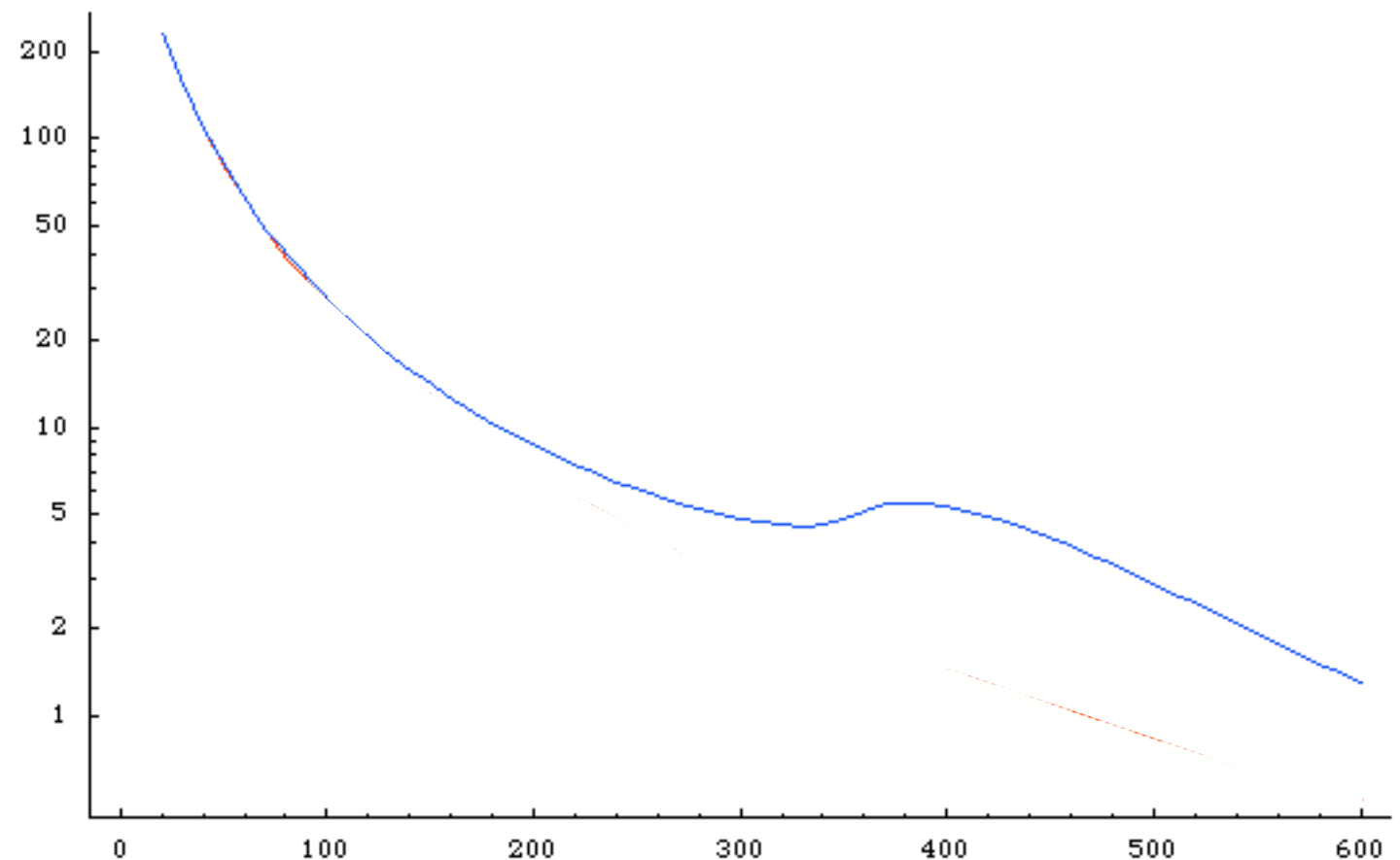
$$x_1 \equiv \sqrt{\tau} e^y \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \quad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \left| I\left(\frac{M_H^2}{m^2}\right) \right|^2 \tau_0 \int_{\log \sqrt{\tau_0}}^{-\log \sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^y) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

$I(x)$  has both a real and imaginary part, which develops at  $m_h=2m_t$ .

This causes a bump in the cross section.





# $pp \rightarrow H @ \text{NLO}$

At NLO we have to include an extra parton (virtual or real).

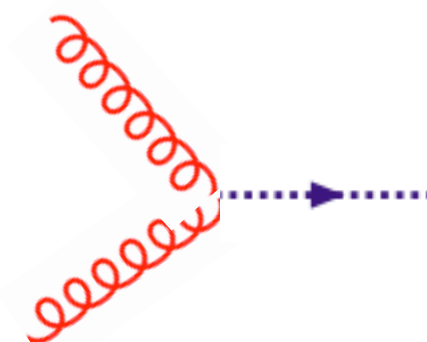
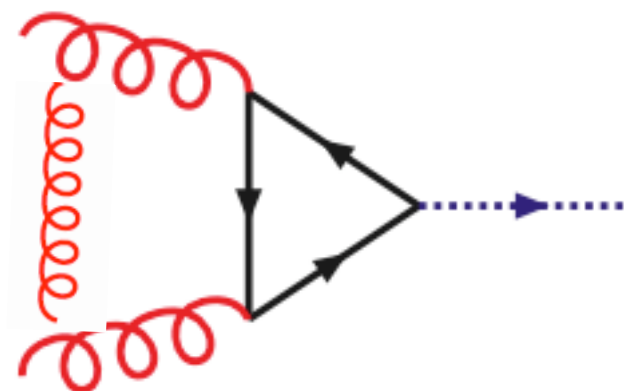
The virtuals will become a two-loop calculation!!

Can we avoid that?

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \rightarrow H) = -\frac{\alpha_S m_t^2}{\pi v} \delta^{ab} \left( g^{\mu\nu} \frac{M_H^2}{2} - p^\nu q^\mu \right) \int dx dy \left( \frac{1 - 4xy}{m_t^2 - m_H^2 xy} \right) \epsilon_\mu(p) \epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} -\frac{\alpha_S}{3\pi v} \delta^{ab} \left( g^{\mu\nu} \frac{M_H^2}{2} - p^\nu q^\mu \right) \epsilon_\mu(p) \epsilon_\nu(q).$$



This looks like a local vertex,  $ggH$ .

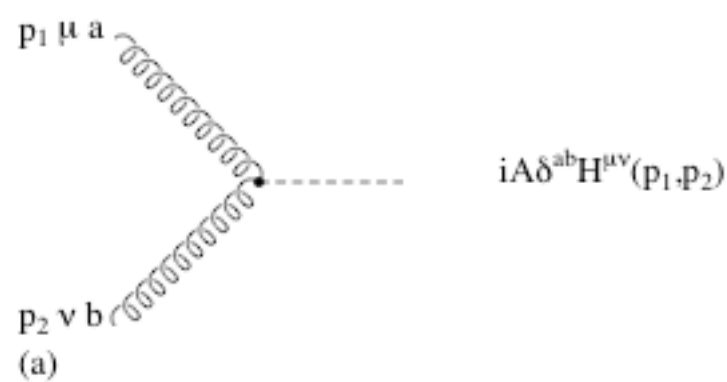
The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).



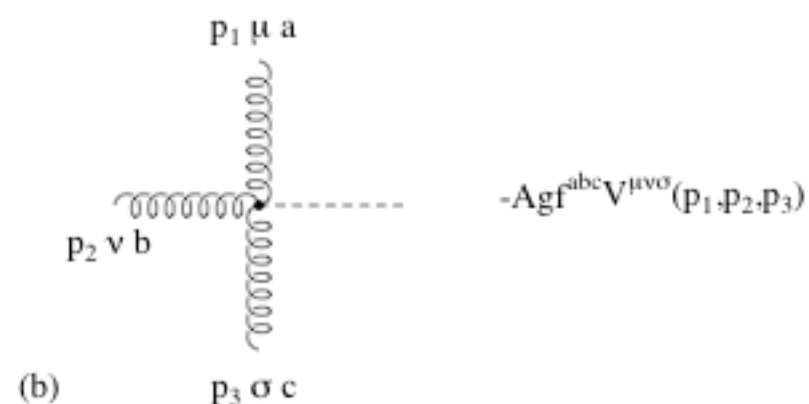
# Higgs effective field theory

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \left( 1 - \frac{\alpha_S}{3\pi} \frac{H}{v} \right) G^{\mu\nu} G_{\mu\nu}$$

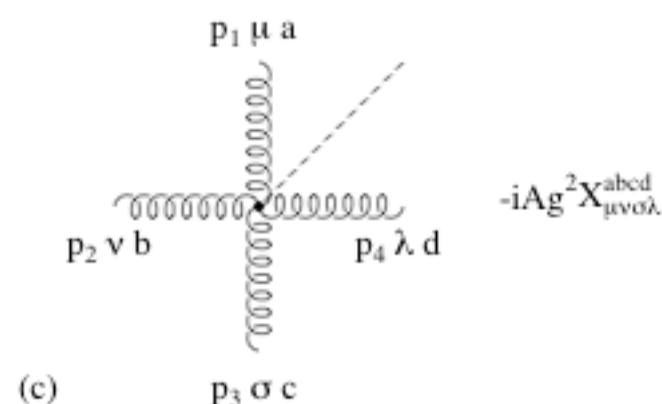
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu.$$



$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu},$$



$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} = & f_{abe} f_{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f_{ace} f_{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f_{ade} f_{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}). \end{aligned}$$

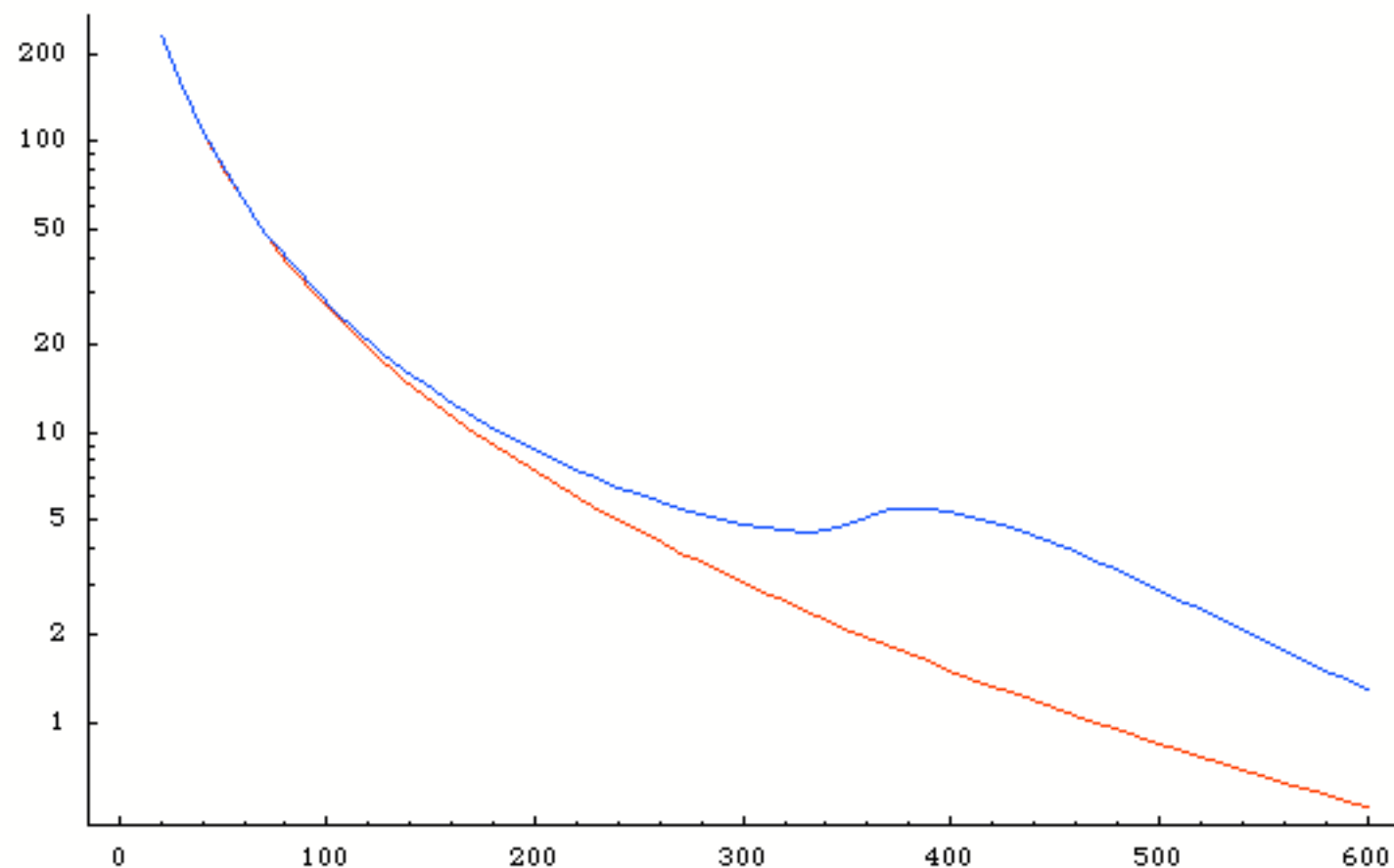


# LO cross section: full vs HEFT

$$\sigma(pp \rightarrow H) = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 g(x_1, \mu_f) g(x_2, \mu_f) \hat{\sigma}(gg \rightarrow H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit  $m \rightarrow \infty$ .

For light Higgs is better than 10%.



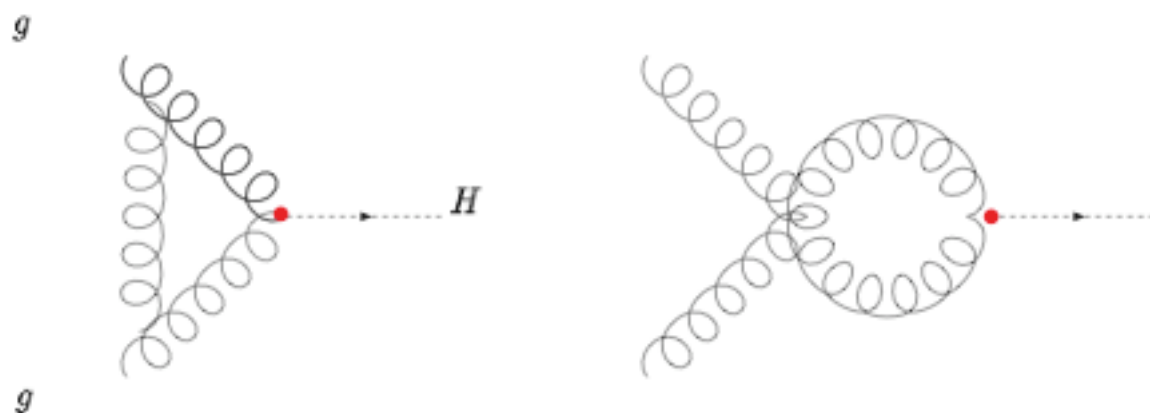
So, if we are interested in a light Higgs we use the HEFT and simplify our life.  
If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can do it!!





# Virtual contributions



Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

Then the integration over the tensor decomposition into scalar integrals and loop integration has to be performed.

One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

$$\mathcal{L}_{\text{eff}}^{\text{NLO}} = \left(1 + \frac{11}{4} \frac{\alpha_S}{\pi}\right) \frac{\alpha_S}{3\pi} \frac{H}{v} G^{\mu\nu} G_{\mu\nu}$$

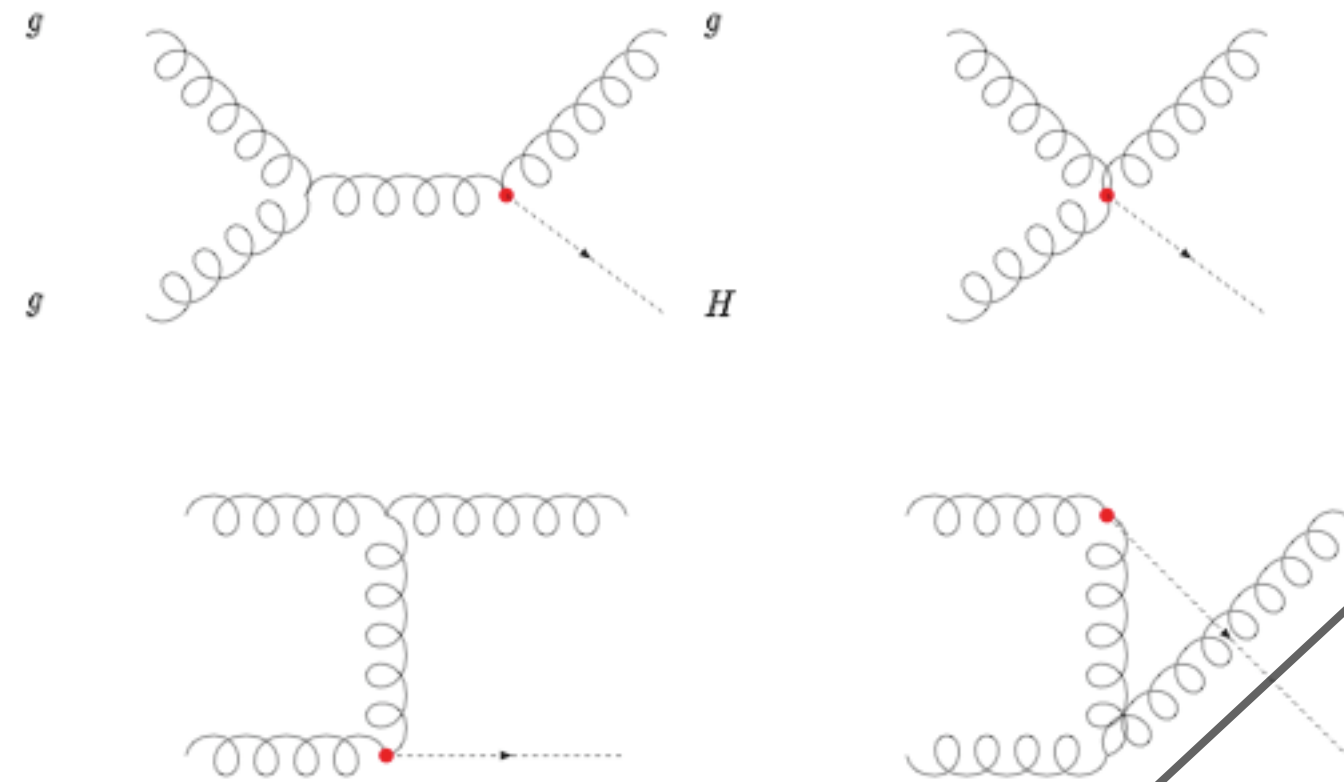
The result is:

$$\sigma_{\text{virt}} = \sigma_0 \delta(1-z) \left[ 1 + \frac{\alpha_S}{2\pi} C_A \left( \frac{\mu^2}{m_H^2} \right)^\epsilon c_\Gamma \left( -\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2 \right) \right],$$

$$\sigma_{\text{Born}} = \frac{\alpha_S^2}{\pi} \frac{m_H^2}{576 v^2 s} (1 + \epsilon + \epsilon^2) \mu^{2\epsilon} \delta(1-z) \equiv \sigma_0 \delta(1-z) \quad z = m_H^2/s$$



# Real contributions



This is the last piece: the result at the end must be finite!

$2/\epsilon$  cancels with the virtual contribution ✓

This is the renormalization of the coupling!!

$$\sigma_{\text{c.t.}}^{\text{UV}} = 2 \sigma_{\text{Born}} \frac{\alpha_S}{2\pi} \left[ - \left( \frac{\mu^2}{\mu_{\text{UV}}^2} \right)^\epsilon c_\Gamma \frac{b_0}{\epsilon} \right] \checkmark$$

$$\sigma_{\text{real}} = \sigma_0 \frac{\alpha_S}{2\pi} C_A \left( \frac{\mu^2}{m_H^2} \right)^\epsilon c_\Gamma \left[ \left( \frac{2}{\epsilon^2} + \frac{2 b_0}{\epsilon C_A} - \frac{\pi^2}{3} \right) \delta(1-z) - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3} \frac{(1-z)^3}{z} - 4 \frac{(1-z)^2(1+z^2) + z^2}{z(1-z)} \log z + 4 \frac{1+z^4 + (1-z)^4}{z} \left( \frac{\log(1-z)}{1-z} \right)_+ \right]$$

This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma_{\text{c.t.}}^{\text{coll.}} = 2 \sigma_0 \frac{\alpha_S}{2\pi} \left[ \left( \frac{\mu^2}{\mu_F^2} \right)^\epsilon \frac{c_\Gamma}{\epsilon} P_{gg}(z) \right] \checkmark$$



# Final results = we made it!!

$$\sigma(pp \rightarrow H) = \sum_{ij} \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij) [\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels:  $q \bar{q}$ ,  $q g$ , and  $g g$ .

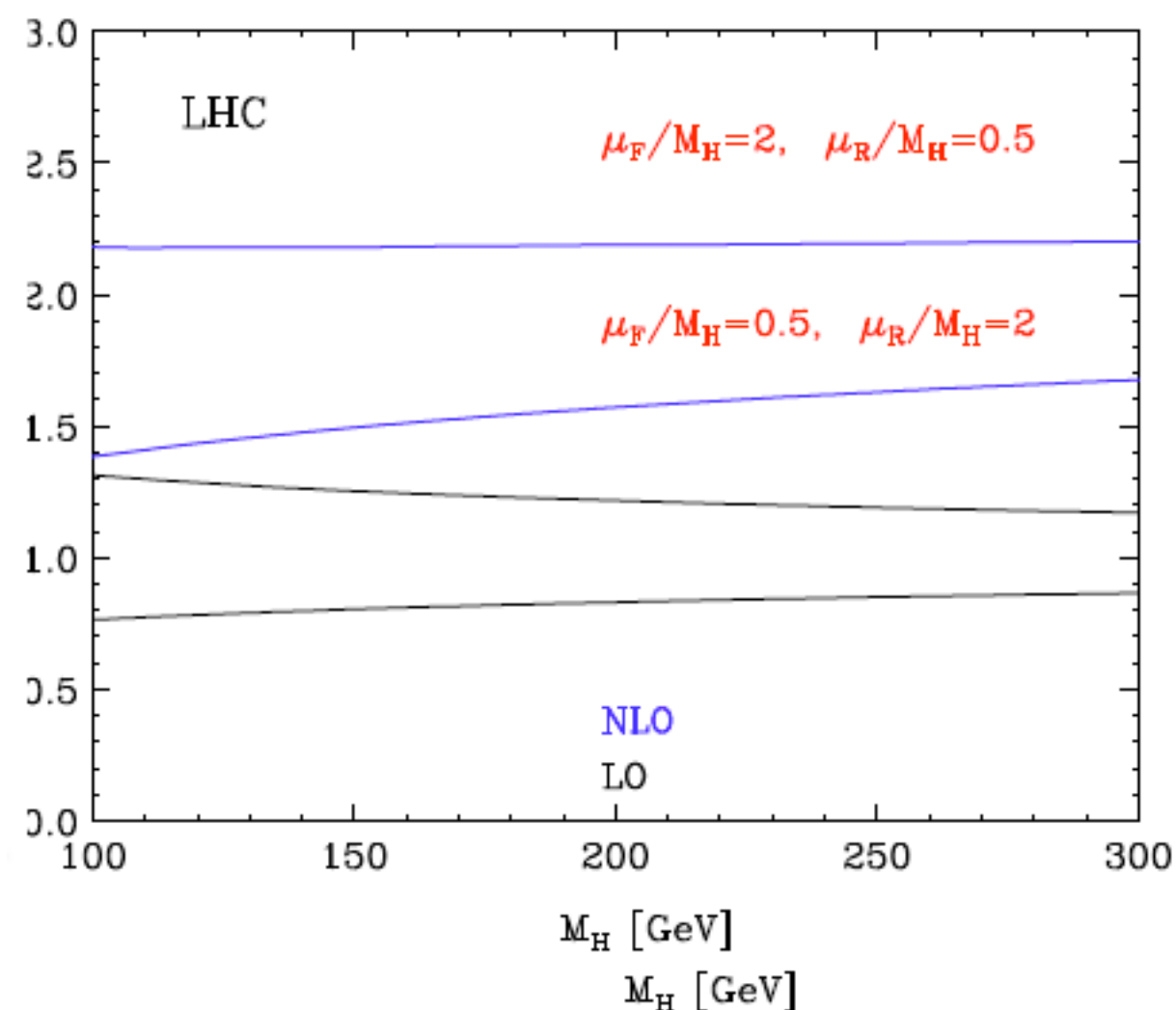
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is  $\sim 2$  and scale dependence not really very much improved.

Is perturbation theory valid?  
NNLO is mandatory...





# General algorithm for calculations of observables at NLO

As we discussed, the form of the soft and collinear terms are UNIVERSAL, i.e., they don't depend on the short distance coefficients, but only on the color and spin of the partons participating soft or collinear limit.

Therefore it is conceivable to have an algorithm that can handle any process, once the real and virtual contributions are computed.

There are several such algorithms available, but the conceptually simplest is the Subtraction Method [Catani & Seymour ; Catani, Dittmaier, Seymour, Trocsanyi]

$$\begin{aligned}\sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V\end{aligned}$$



# General algorithm for calculations of observables at NLO

One can use the universality to construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

which only depend on the partons involved in the divergent regions,  $d\sigma^B$  denotes the appropriate colour and spin projection of the Born-level cross section and the counter terms are independent on the process under considerations.

These counter terms cancel all non-integrable singularities in  $d\sigma^R$ , so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

where the space integration in the first term can be performed numerically in four dimensions and the integral of the counter terms can be done once for all.



# An (incomplete) list of NLO codes

- NLOJET++ [Nagy]  $pp \rightarrow (2,3) \text{ jets}$
- AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer]  $pp \rightarrow (W, Z) + (W, Z, \gamma)$
- DIPHOX/EPHOX [Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen]  $pp \rightarrow \gamma + 1 \text{ jet}, pp \rightarrow \gamma\gamma, \gamma^* p \rightarrow \gamma + 1 \text{ jet}$
- MCFM [Campbell, Ellis]  $pp \rightarrow (W, Z) + (0,1,2) \text{ jets}, pp \rightarrow (W, Z) + b\bar{b}, \dots$
- heavy-quark production [Mangano, Nason, Ridolfi]  $pp \rightarrow Q\bar{Q}$
- single-top production [Harris, Laenen, Phaf, Sullivan, Weinzierl]  $pp \rightarrow Q\bar{q}$
- associated Higgs production with  $t\bar{t}$  [Dawson, Jackson, Orr, Reina, Wackerroth, Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas]  $pp \rightarrow H Q\bar{Q}$
- VBFNLO [Figy, Zeppenfeld, C.O.]  $pp \rightarrow (W, Z, H, WW, ZZ, WZ) + 2 \text{ jets}$ , QCD corrections to electroweak production, when typical vector-boson fusion cuts are applied
- di-photon production [del Duca, Maltoni, Nagy, Trocsanyi]  $pp \rightarrow \gamma\gamma + 1 \text{ jet}$

For a **more complete list**, and the corresponding web pages, see:

<http://www.cedar.ac.uk/hepcode>





# Example:MCFM

Downloadable general purpose NLO code (Campbell & Ellis)

$$p\bar{p} \rightarrow W^\pm / Z$$

$$p\bar{p} \rightarrow W^\pm + Z$$

$$p\bar{p} \rightarrow W^\pm + \gamma$$

$$p\bar{p} \rightarrow W^\pm + g^* (\rightarrow b\bar{b})$$

$$p\bar{p} \rightarrow W^\pm / Z + 1 \text{ jet}$$

$$p\bar{p}(gg) \rightarrow H$$

$$p\bar{p}(VV) \rightarrow H + 2 \text{ jets}$$

$$p\bar{p} \rightarrow H + b$$

$$p\bar{p} \rightarrow W^+ + W^-$$

$$p\bar{p} \rightarrow Z + Z$$

$$p\bar{p} \rightarrow W^\pm / Z + H$$

$$p\bar{p} \rightarrow Z b\bar{b}$$

$$p\bar{p} \rightarrow W^\pm / Z + 2 \text{ jets}$$

$$p\bar{p}(gg) \rightarrow H + 1 \text{ jet}$$

$$p\bar{p} \rightarrow t + q$$

$$p\bar{p} \rightarrow Z + b$$

- ➡ Plus all single-top channels,  $Wc$ ,  $WQJ$ ,  $ZQJ$ ,...
- ➡ Extendable/sizeable library of processes,  
relevant for signal and background studies, including spin correlations.
- ➡ Cross sections and distributions at NLO are provided
- ➡ Easy and flexible choice of parameters/cuts (input card).





# Next-to-leading order : Loops



Any one-loop amplitude can be written as (PV decomposition):

$$\mathcal{M} = \sum_i a_i(D) \text{Boxes}_i + \sum_i b_i(D) \text{Triangles}_i + \sum_i c_i(D) \text{Bubbles}_i + \sum_i d_i(D) \text{Tadpoles}_i$$

\* All the scalar loop integrals are known and now easily available [Ellis, Zanderighi]

\* Open issue is to compute the D-dimensional coefficient in the expansion:  
large number of terms forbid a direct evaluation with symbolic algebra. In addition  
normally large gauge cancellation, inverse Gram determinants, spurious phase-space  
singularities lead to **numerical instabilities**.

Sometimes it is better to calculate

$$\mathcal{M} = \sum_i a_i(4) \text{Boxes}_i + \sum_i b_i(4) \text{Triangles}_i + \sum_i c_i(4) \text{Bubbles}_i + \sum_i d_i(4) \text{Tadpoles}_i + R$$

Where **R** is a rational function



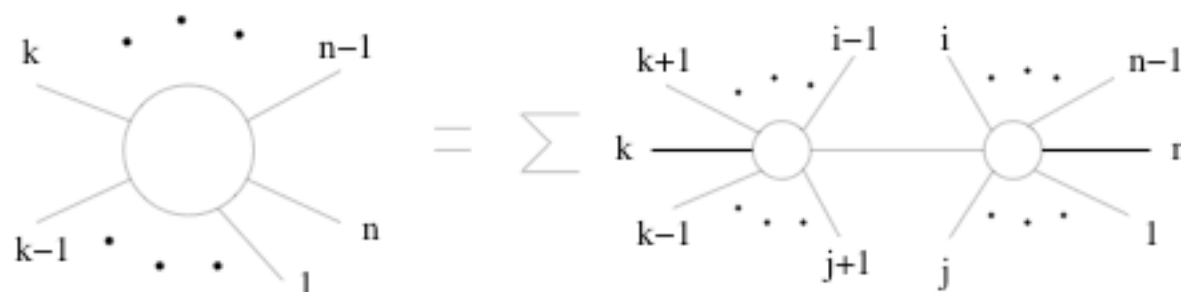
# Progress in loops

Several new developments coming from the idea

A scattering amplitude is an analytic function of the external momenta and (most) its structure can be reconstructed from the poles and the branch cuts.

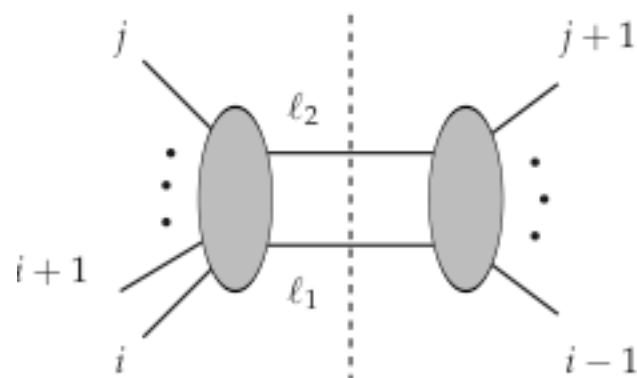
LOOPS can be calculated from tree-level amplitudes

✓ **POLES** : lower number of external lines. Cauchy residue theorem



[Cachazo, Svrcek, Witten]  
[Witten]  
[Britto, Cachazo, Feng]

✓ **BRANCH CUTS** : lower number of loops



$$\text{Disc} = \int d^4\Phi A^{\text{tree}}(\ell_1, i, \dots, j, \ell_2) A^{\text{tree}}(-\ell_2, j+1, \dots, i-1, -\ell_1)$$

$$d^4\Phi = d^4\ell_1 d^4\ell_2 \delta^{(4)}(\ell_1 + \ell_2 - P_{ij}) \delta^{(+)}(\ell_1^2) \delta^{(+)}(\ell_2^2)$$

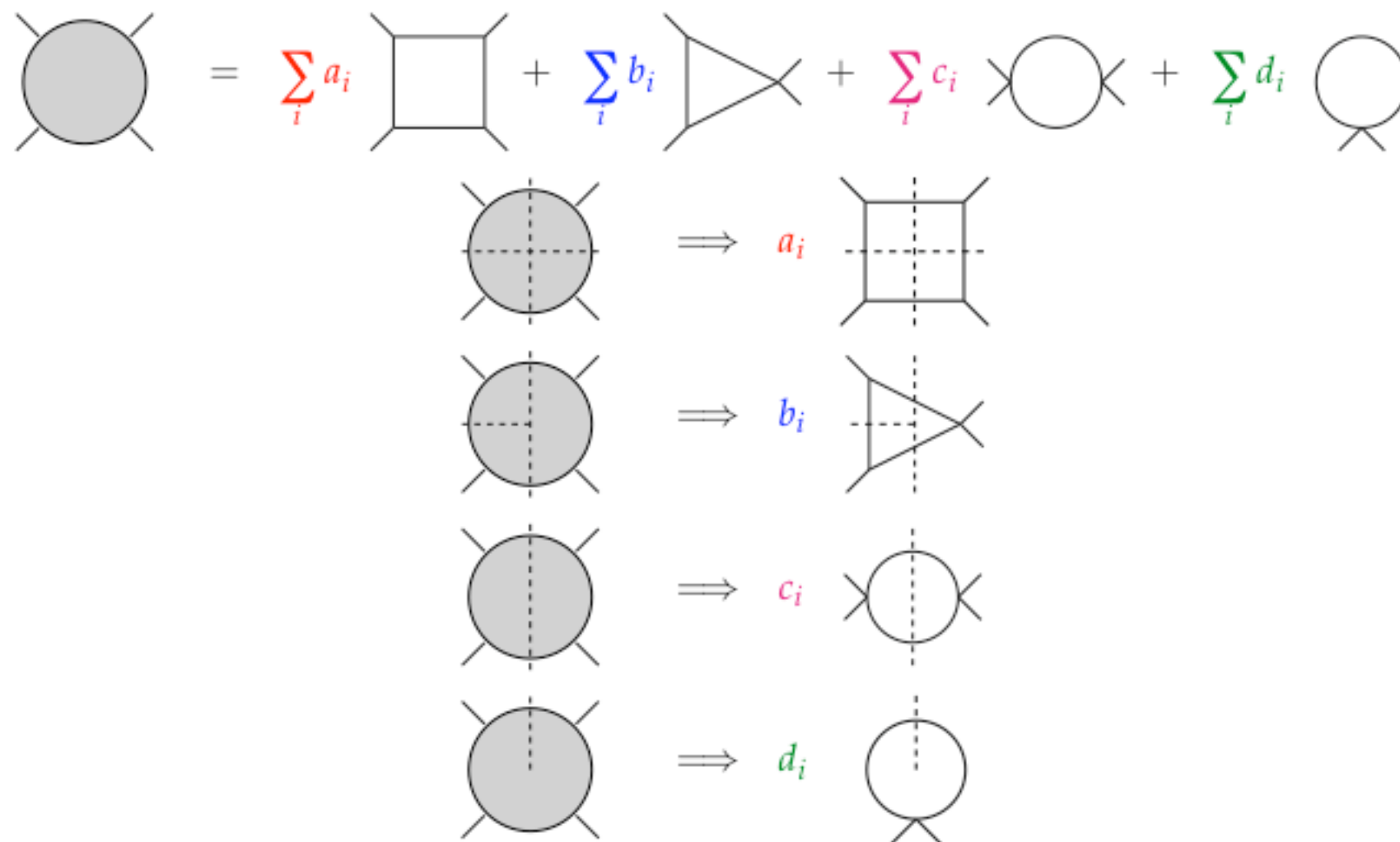
$$\delta^{(+)}(p^2) = \delta(p^2) \theta(p_0) \quad \text{on-shell condition}$$

[Vermaseren, van Neerven]  
[Bern, Dixon, Dunbar, Kosower]  
[Britto, Cachazo, Feng]



# Generalized unitarity

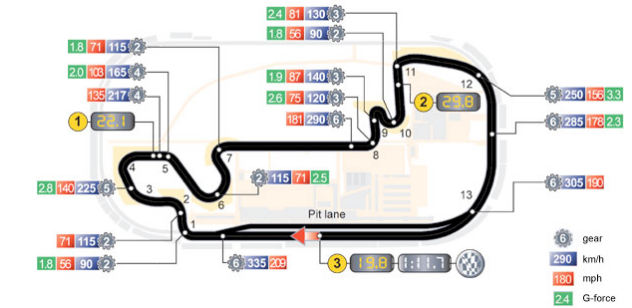
[Bern, Dixon, Kosower]  
[Britto, Cachazo, Feng]  
[Anastasiou, Kunszt, Mastrolia]



Three and four particle cuts are non zero due to the continuation of momenta into complex values!



# The loop race



Impressive developments in the last year(s) : automatic and multipurpose method to 1-loop calculation in sight

Unitarity-based methods

Bern, Dixon, Dunbar and Kosower  
Britto, Cachazo, Feng and Witten  
Bern, Dixon and Kosower  
Anastasiou, Britto, Feng, Kunszt, Mastrolia  
Anastasiou, Kunszt, Forde  
Ossola, Papadopoulos, Pittau [CutTools]

....

On-shell recurrence relations

Ellis, Giele, Kunszt, Melnikov  
Moretti, Piccinini, Polosa  
Catani, Gleisberg, Krauss, Rodrigo, Winter  
Berger, Bern, Dixon, Febres Cordero, Forde,  
Ita, Kosower, Maitre [BlackHAT]  
Giele, Zanderighi [Rocket]

....

Improved tensor reduction

Binoth, Guillet, Pilon, Heinrich and Schubert  
Denner and Dittmaier  
Xiao, Yang, and Zhu

...

New papers and proposals on daily basis....



# The future of NLO

- VERY active field of research, with a lot of progress achieved in the last one or two years. New approaches for numerical evaluation of the scalar integrals and also of the tensor decomposition proposed. Several results achieved (Ex. ttjj at NLO by the HELAC-NLO coll.).
- Several new general algorithms to interface NLO calculations with parton showers have been also proposed and tools available (POWHEG BOX).
- Full automatization of NLO calculations interfaced with showers ( $\sim$  Pythia@NLO) imminent.



# What about NNLO?

- At present only  $2 \rightarrow 1$  calculations available, all of them (parton) exclusive final state.
- From loop integrals to phase space integrals...all of them are an art!
- General algorithms and checked only in  $e^+e^- \rightarrow 3j$  at NNLO.

Let's consider two physics cases:

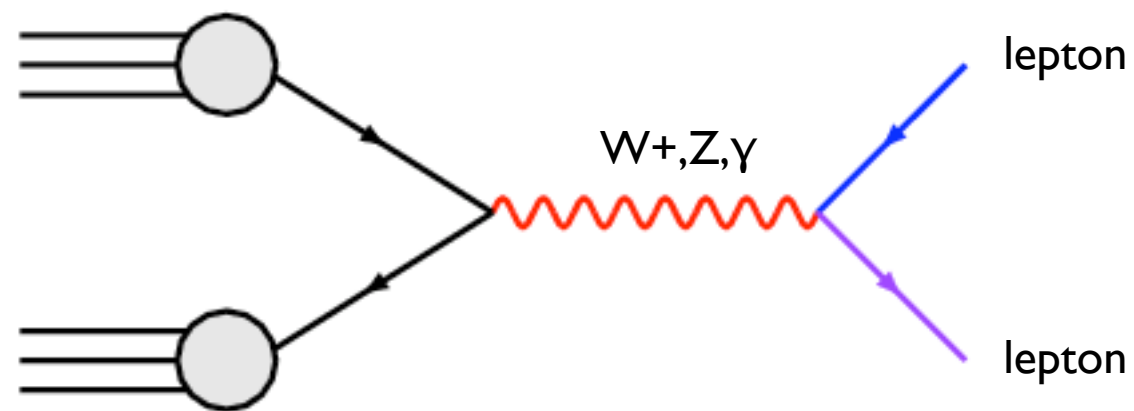
a. Drell-Yan

b. Higgs





# Drell-Yan

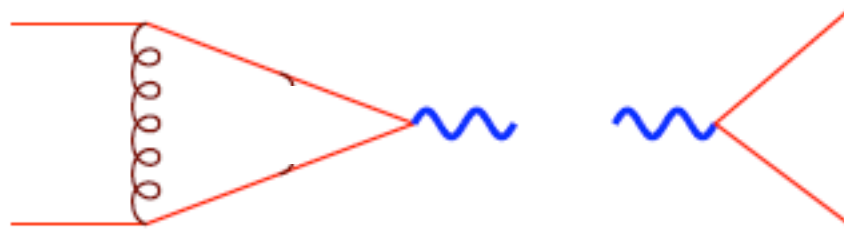


- Clean final state ( no hadrons from the hard process).
- Nice test of QCD and EW interactions. The cross sections are known up to NNLO (QCD) and at NLO (EW).
- Measure  $m_W$  to be used in the EW fits together with the top mass to guess the Higgs mass.
- Constraint the PDF
- Channel to search for new heavy gauge bosons or new kind of interactions



# Elements of $pp \rightarrow W$ NLO calculation

## Virtual

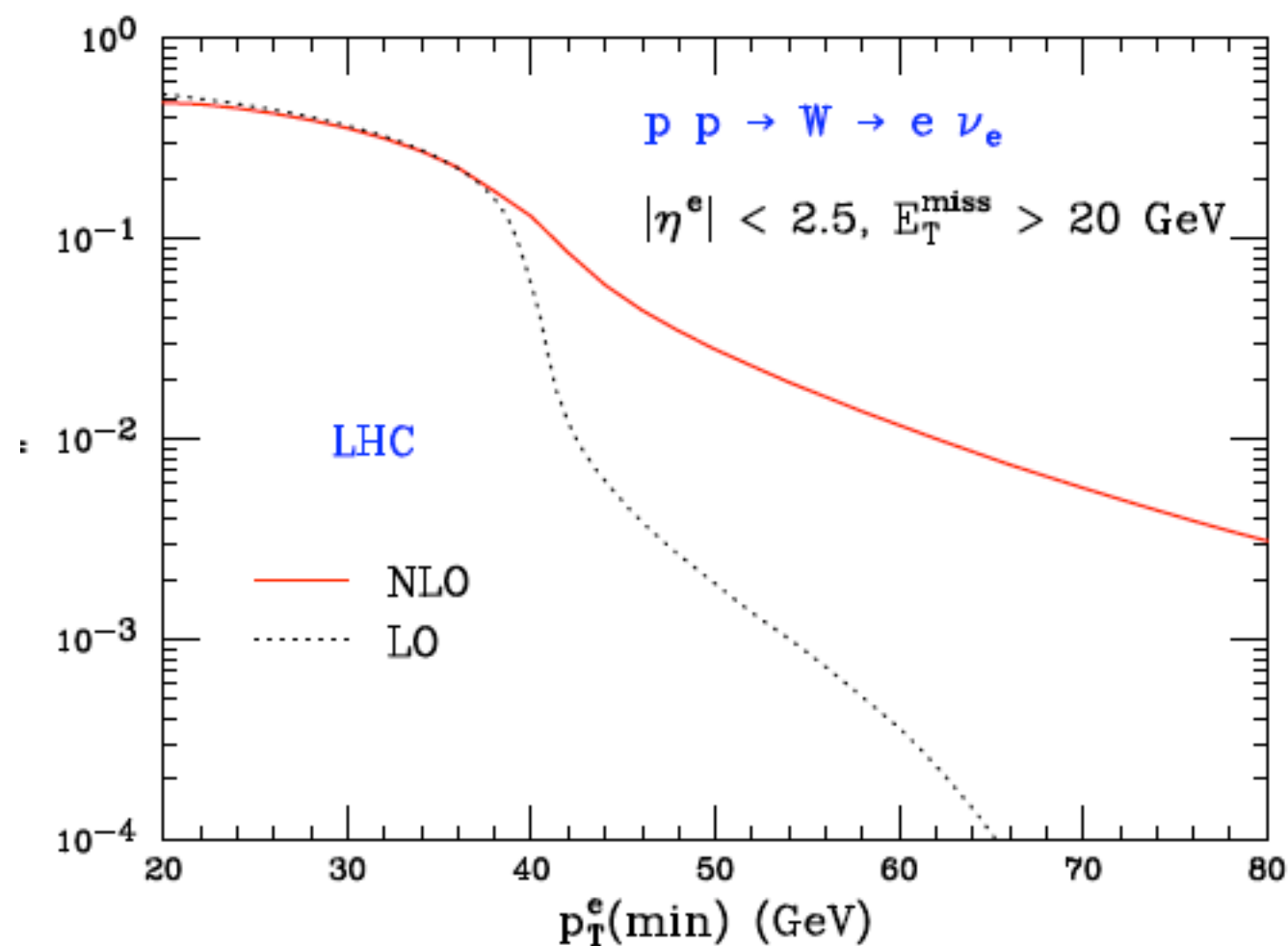


## Real





# Drell-Yan @ NLO



$$\checkmark A_W = \frac{1}{\sigma^{(tot)}} \int_{p_T^e(\text{min})}^{\sqrt{s}/2} dp_T^e \frac{d\sigma}{dp_T^e} (\text{cuts})$$

$$\checkmark K(x) = \frac{d\sigma_{\text{NLO}}/dx}{d\sigma_{\text{LO}}/dx}$$

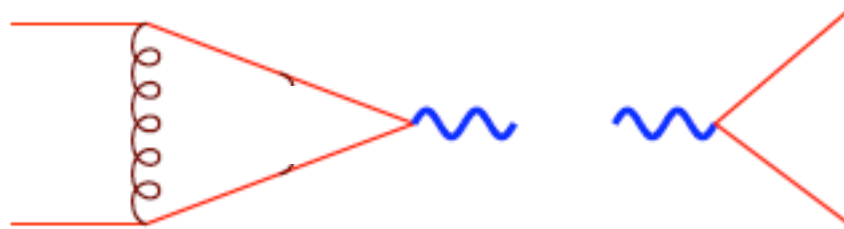
K factors **STRONGLY** phase-space dependent.

Lepton **spin correlations** have to be taken account correctly!



# Elements of $pp \rightarrow W$ NLO calculation

## Virtual



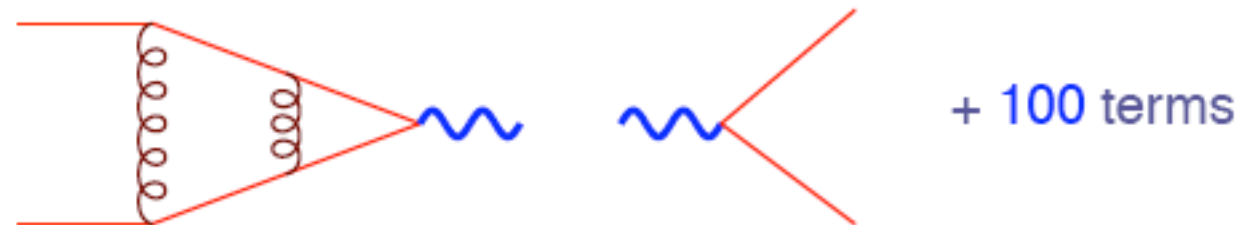
## Real



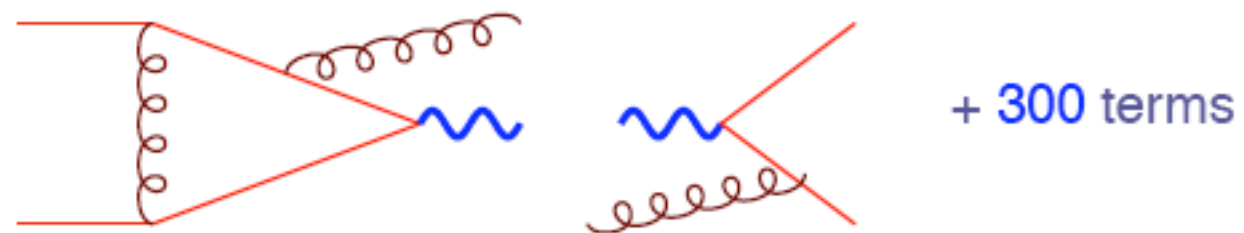


# Elements of $pp \rightarrow W$ NNLO calculation

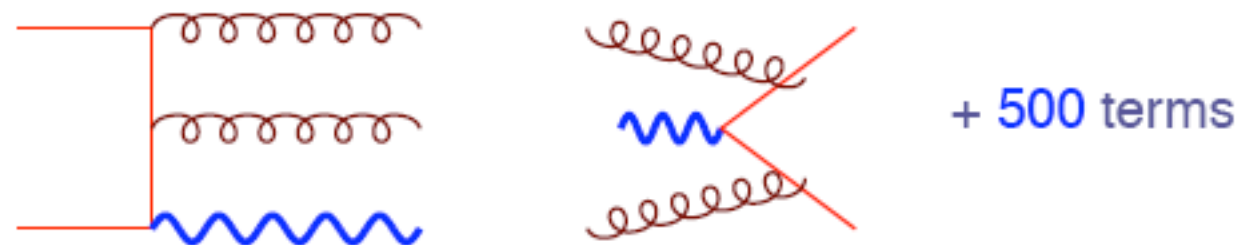
## Virtual-Virtual



## Real-Virtual



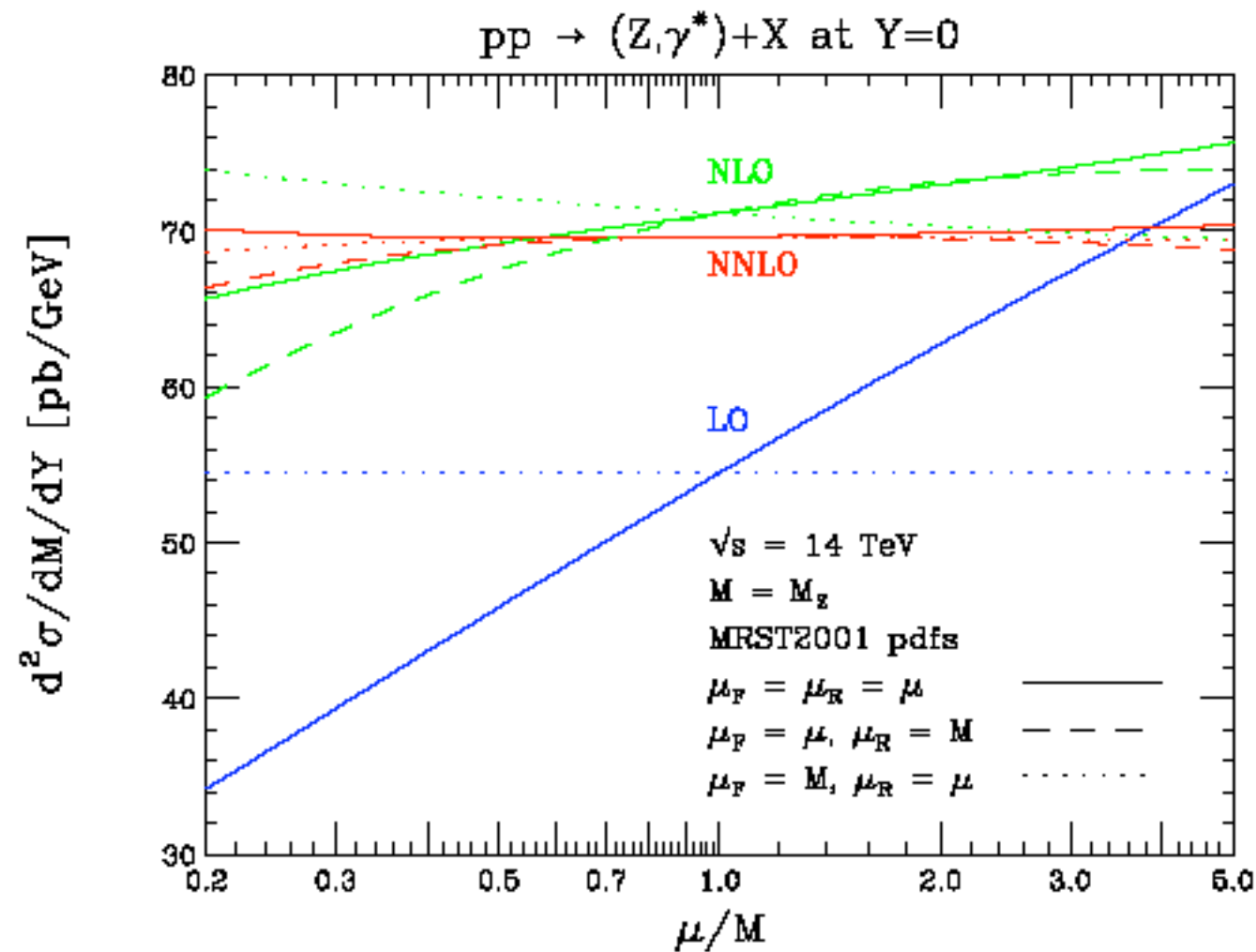
## Real-Real



⇒ Need clever algorithms to handle!



# The NNLO result



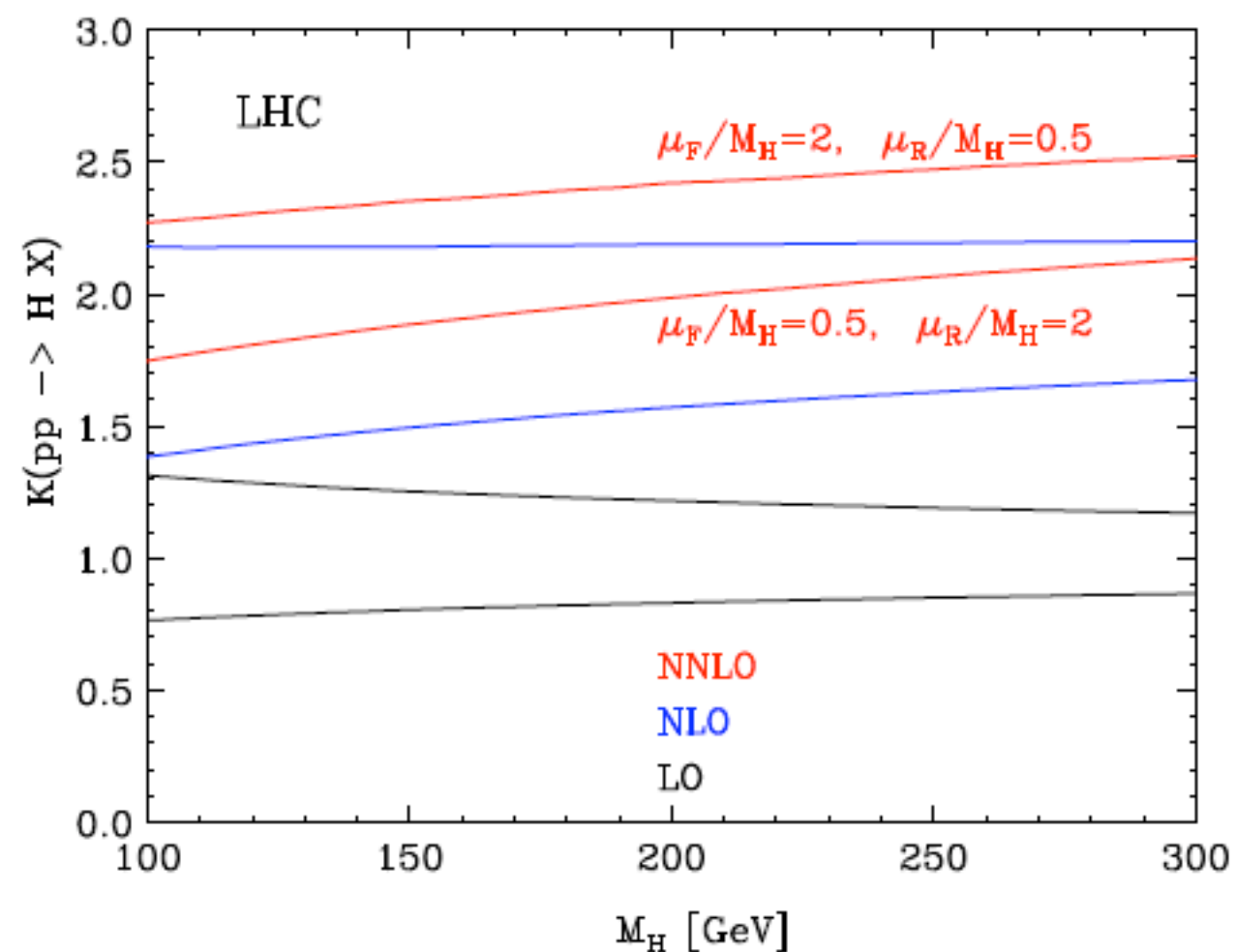
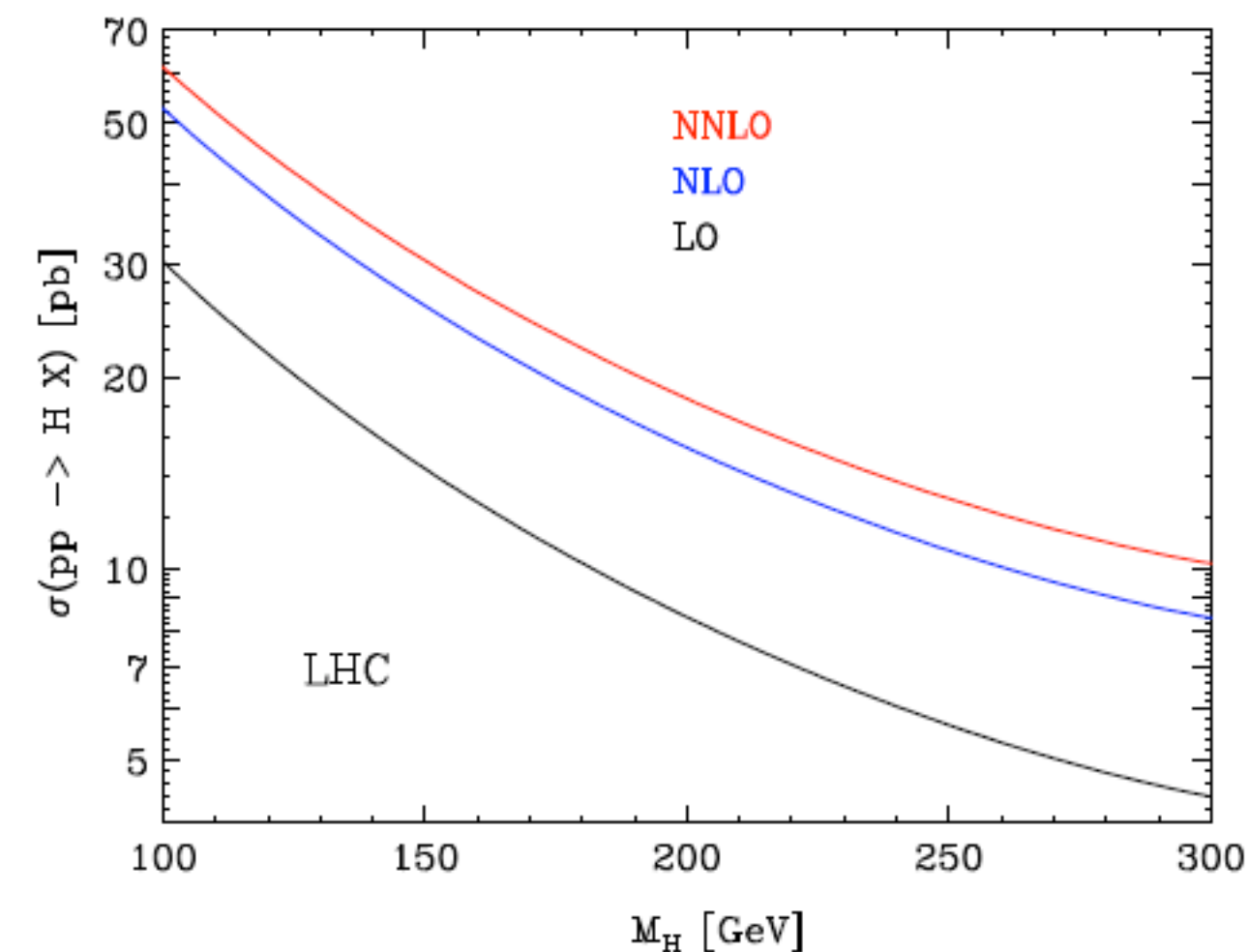
- Precision predictions at NNLO
- Also miss qualitative effects at lower orders
  - Few initial channels open; sensitivity to pdfs underestimated
  - Few jets in final state
  - Jets modeled by too few partons
  - Incorrect kinematics, e.g., no  $p_T$

[Anastasiou, Dixon, Melnikov, Petriello. 2004]





# $pp \rightarrow H$ at NNLO



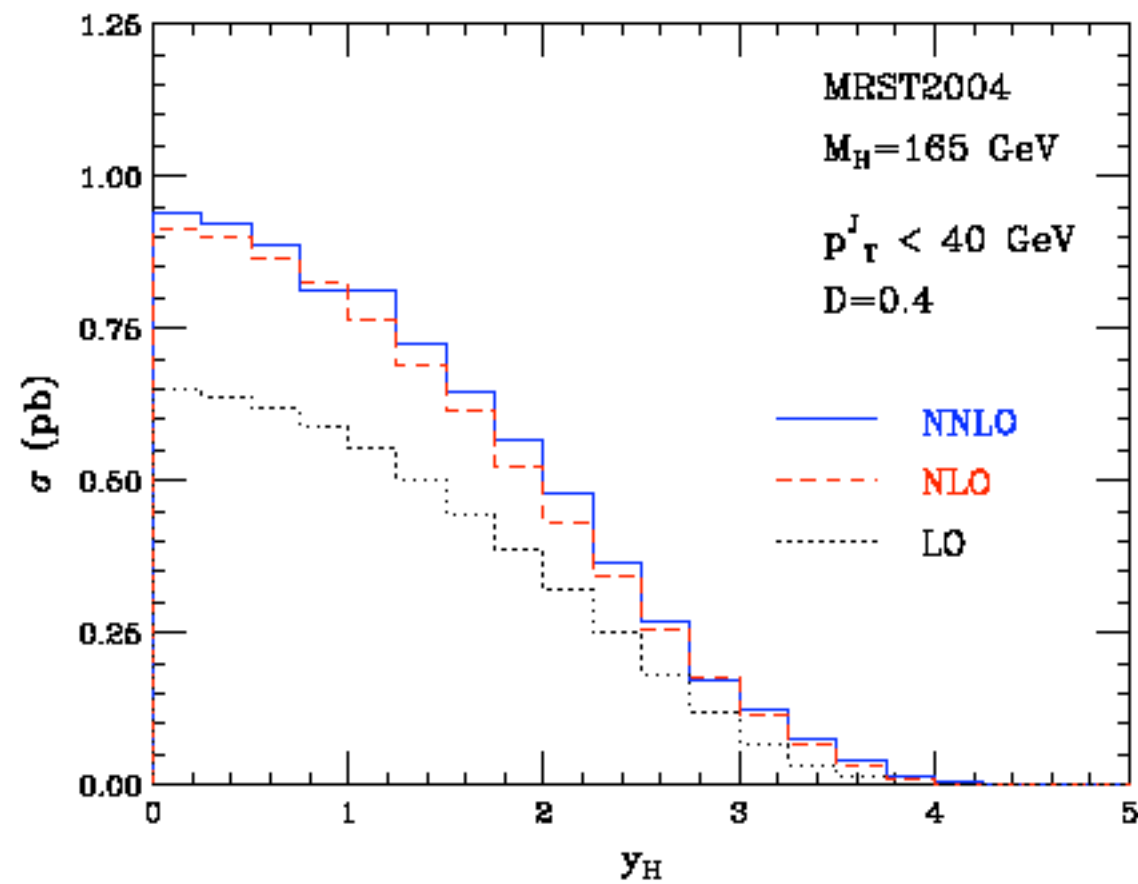
Is the series well behaved?  $\Rightarrow$  YES NNLO 15%

The current TH QCD uncertainty on the total cross section is about 10%.

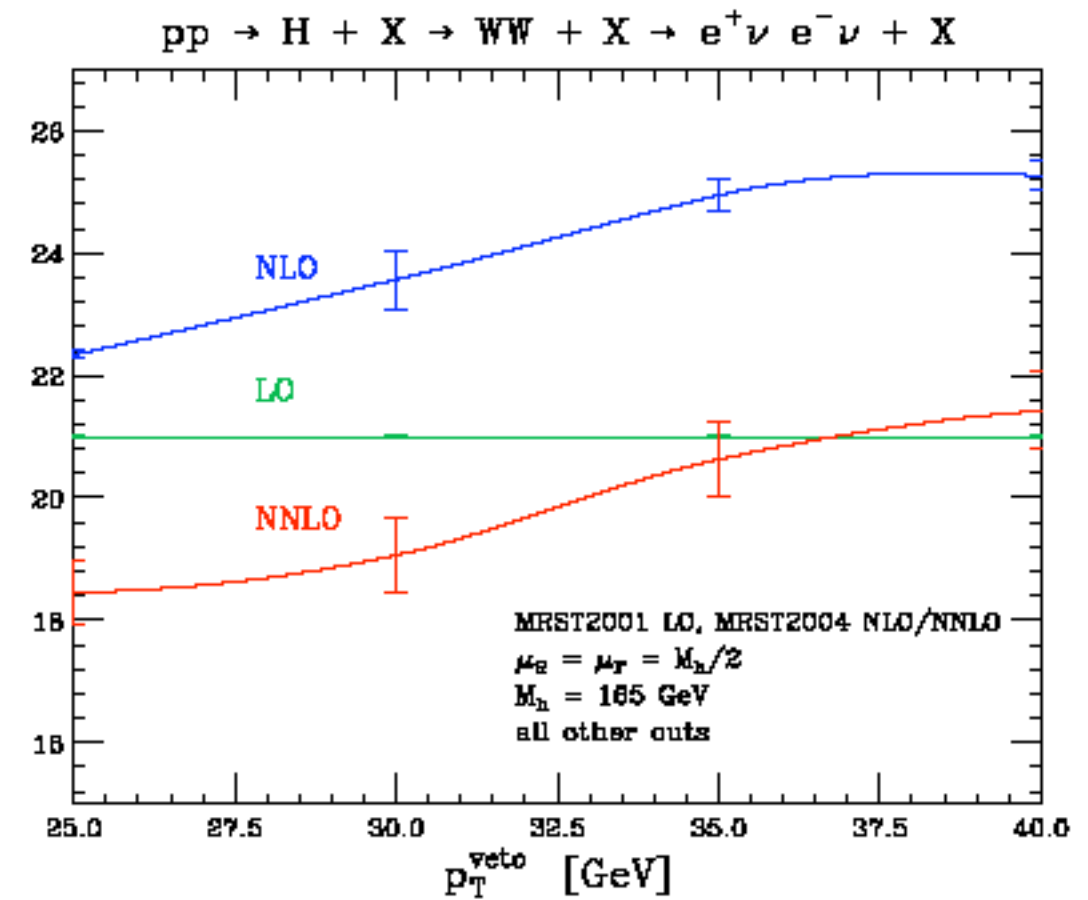
What about our predictions for limited areas of the phase space?



# $pp \rightarrow H$ at NNLO



[Catani, grazzini, 2007]



[Anastasiou, Melnikov, Petriello. 2005]  
[Anastasiou, Dissertori, Stockli. 2007]

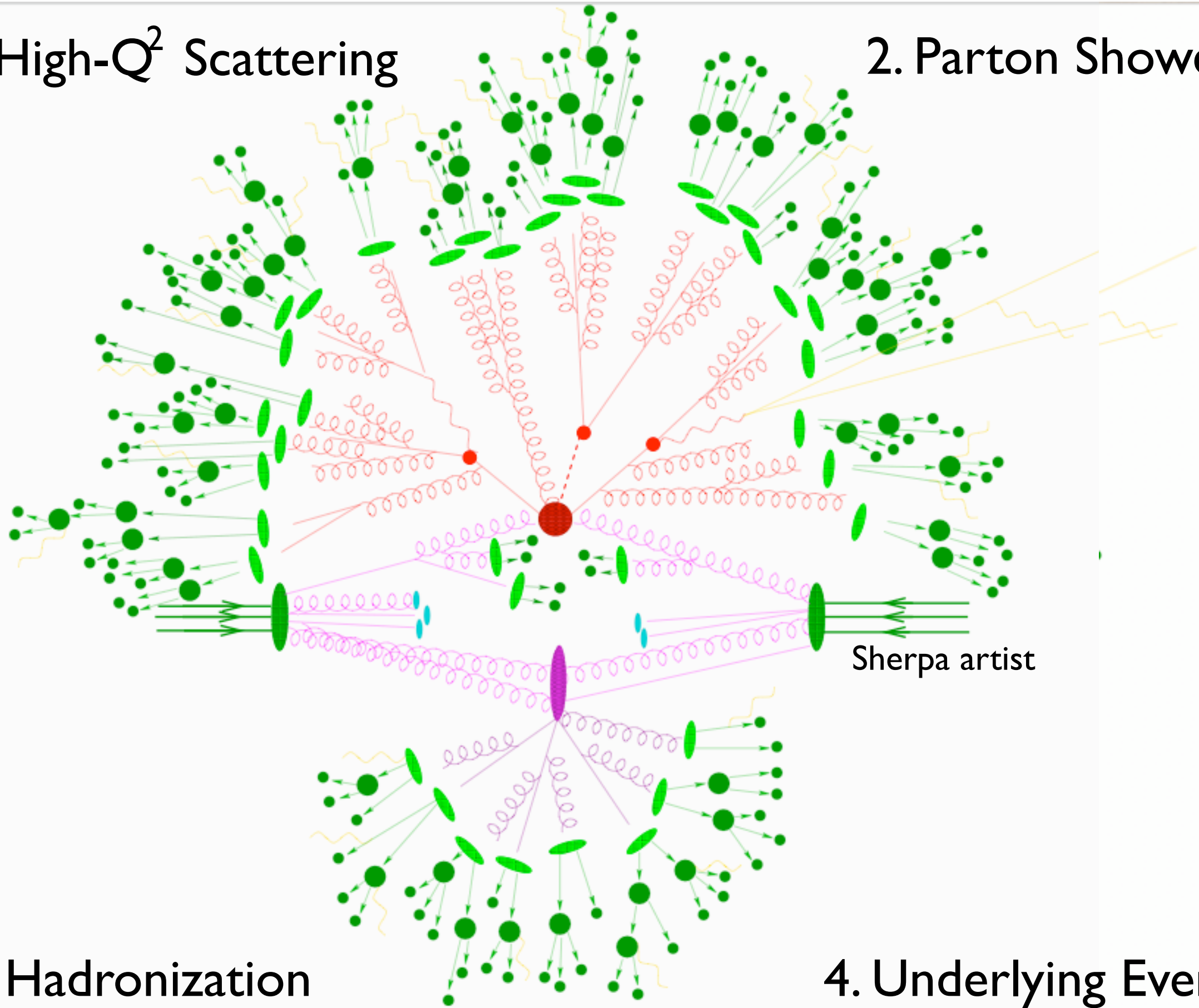


# A simple plan

- Intro: the LHC challenge
- Precision QCD: from LO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

1. High- $Q^2$  Scattering

2. Parton Shower



3. Hadronization

4. Underlying Event

# I. High- $Q^2$ Scattering

## 2. Parton Shower

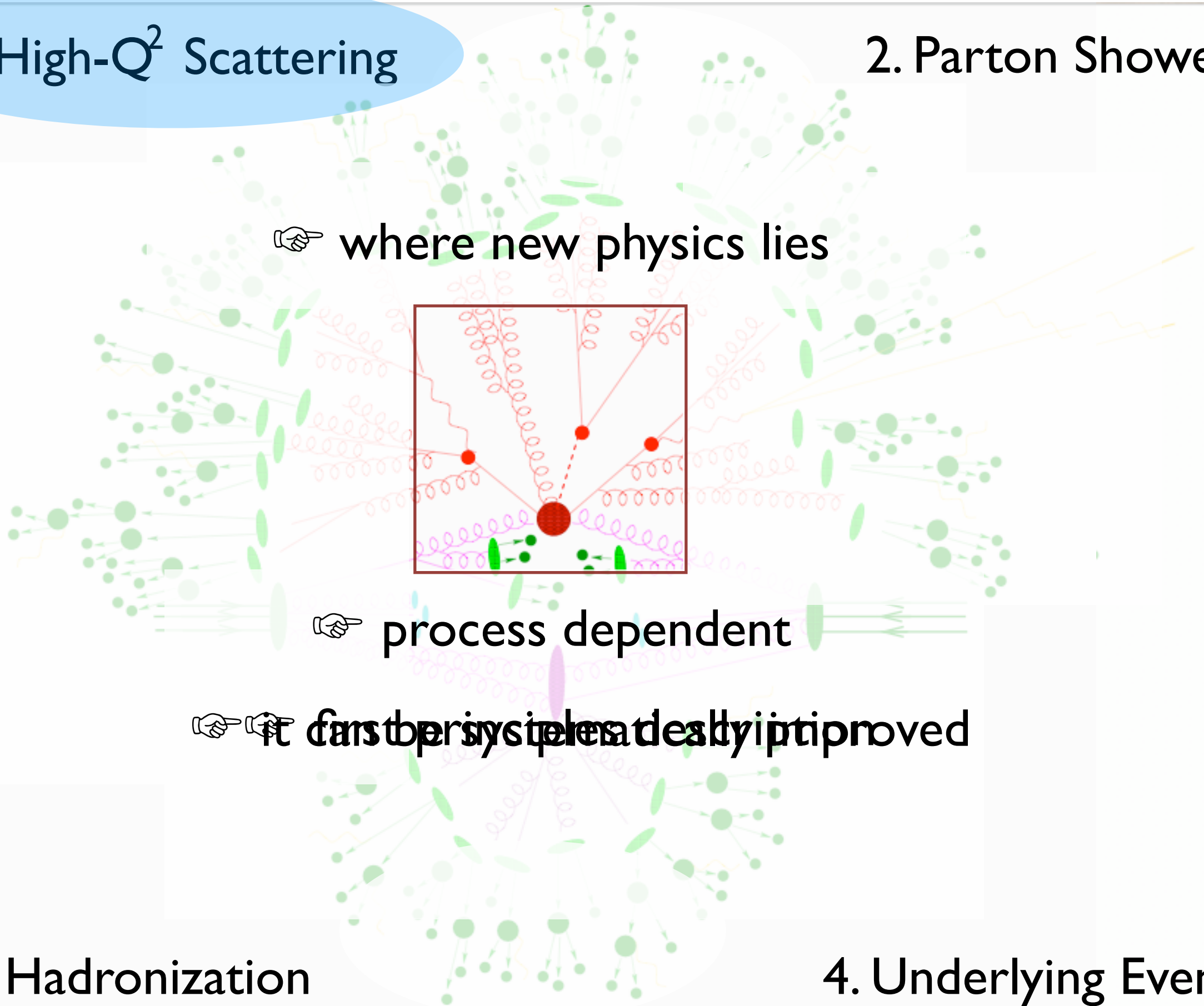
☞ where new physics lies

☞ process dependent

☞ it can be systematically improved

## 3. Hadronization

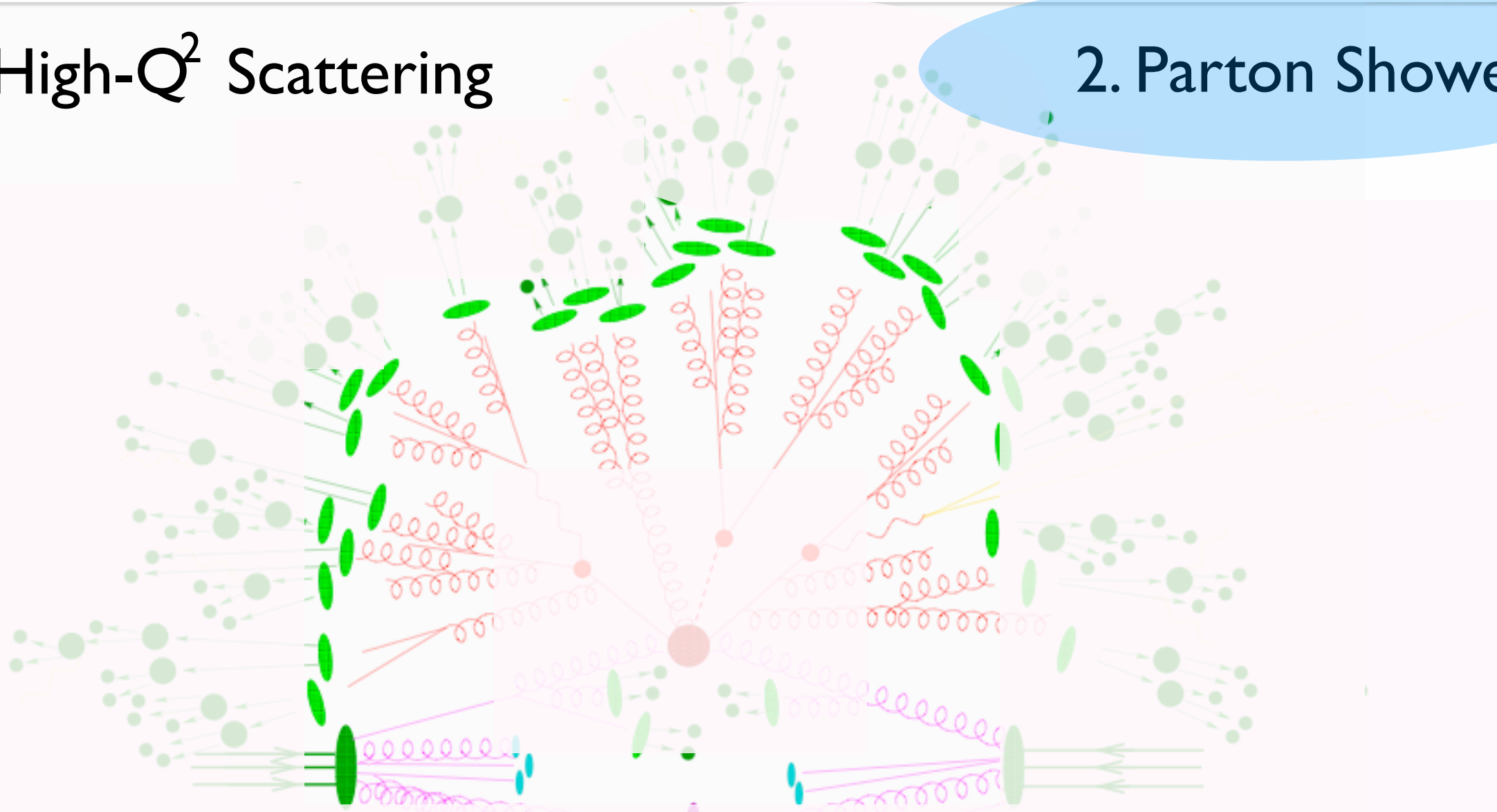
## 4. Underlying Event





# I. High- $Q^2$ Scattering

## 2. Parton Shower



☞ QCD - "known physics"

☞ universal/ process independent

☞ first principles description

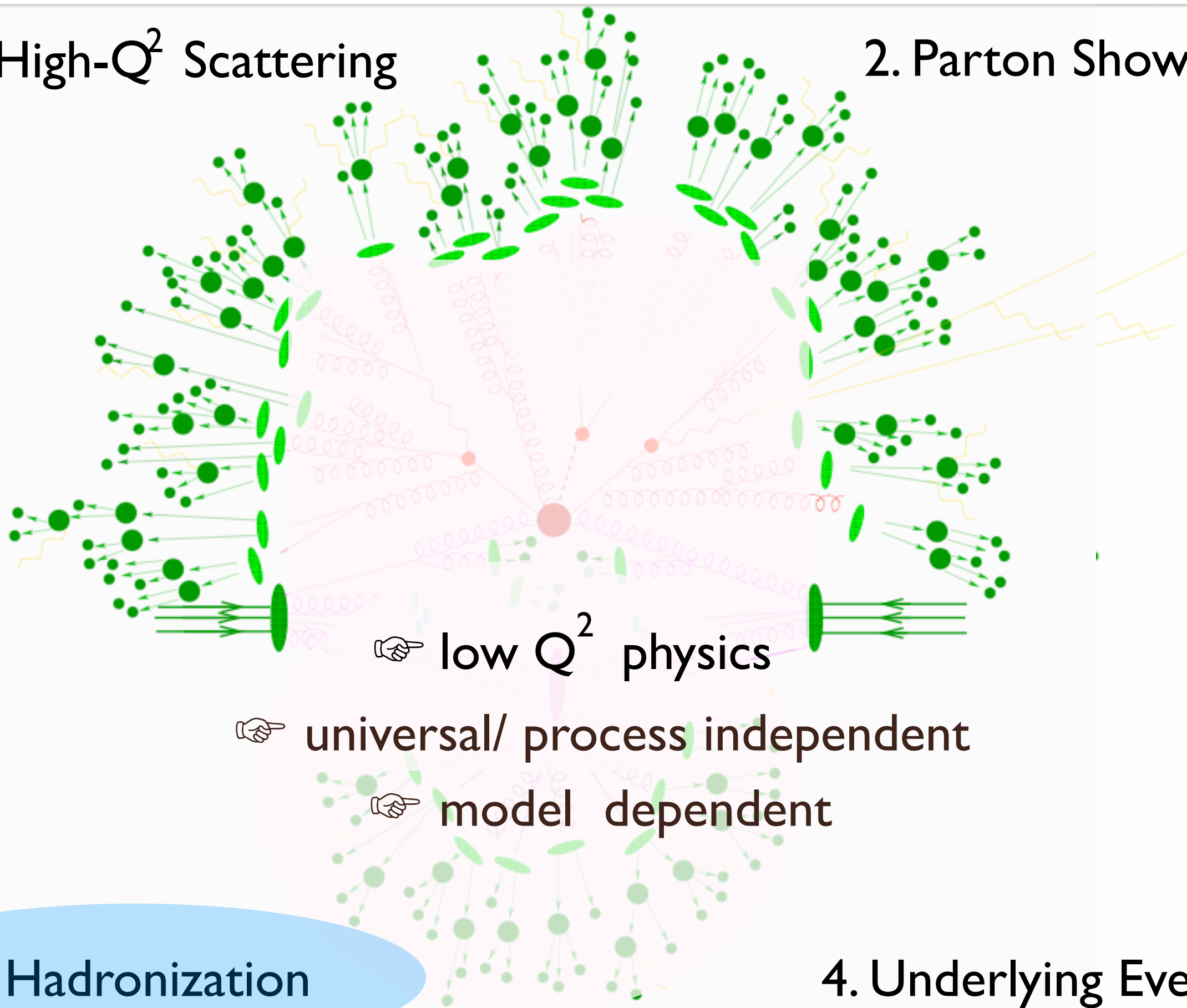
## 3. Hadronization

## 4. Underlying Event



1. High- $Q^2$  Scattering

2. Parton Shower



low  $Q^2$  physics

universal/ process independent

model dependent

3. Hadronization

4. Underlying Event

1. High- $Q^2$  Scattering

2. Parton Shower

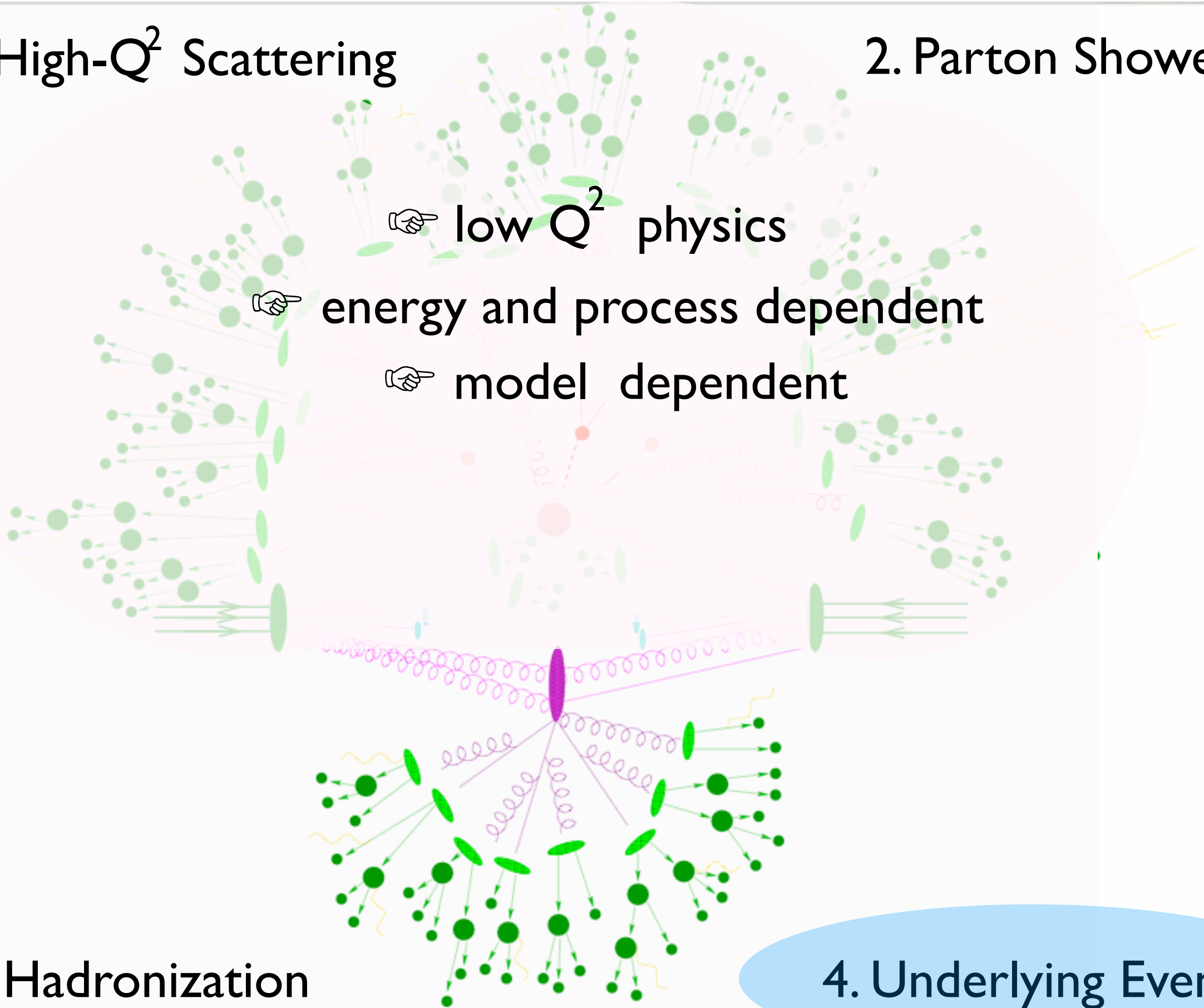
low  $Q^2$  physics

energy and process dependent

model dependent

3. Hadronization

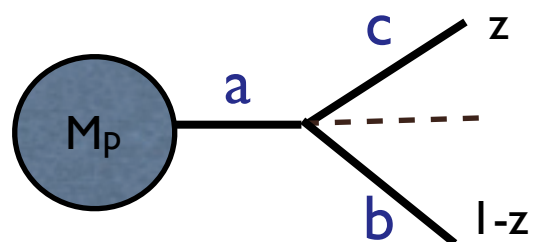
4. Underlying Event





# Parton branching

ME involving  $q \rightarrow q g$  ( or  $g \rightarrow gg$ ) are strongly enhanced when they are close in the phase space:



$$\frac{1}{(p_q + p_g)^2} \simeq \frac{1}{2E_q E_g (1 - \cos \theta)}$$

$$z = E_b / E_a, t = k_a^2$$

$$\theta = \theta_b + \theta_c$$

$$= \frac{\theta_b}{1 - z} = \frac{\theta_c}{z}$$

$$= \frac{1}{E_a} \sqrt{\frac{t}{z(1 - z)}}$$

$$d\sigma_{N+1} = d\sigma_N \frac{dt}{t} \frac{d\phi}{2\pi} dz \frac{\alpha_s}{2\pi} |K_{ba}(z)|^2$$

$$d\bar{\sigma}_{N+1} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{\alpha_s}{2\pi} P_{ba}(z)$$

The cross section factorizes. The splitting can be iterated



# Parton branching

It is easy to iterate the branching process:

$$a(t) \longrightarrow b(z) + c, \quad b(t') \longrightarrow d(z') + e$$
$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{dt'}{t'} dz' \left( \frac{\alpha_s}{2\pi} \right)^2 P_{ba}(z) P_{db}(z')$$

This is a generalized Markov process (in the continuum), where the probability of the system to change (discontinuously) to another state, depends only on present state and not how it got there.,

$$\tau_1 < \dots < \tau_n \implies$$
$$P\left(x(\tau_n) < x_n | x(\tau_{n-1}), \dots, x(\tau_1)\right) = P(x(\tau_n) < x_n | x(\tau_{n-1}))$$



# Parton branching

Following a given line in a branching tree, it is clear that contributions coming from the strongly-ordered region will be leading

$$Q^2 \gg t_1 \gg t_2 \gg \dots t_N \gg Q_0^2$$

$$\sigma_N \propto \sigma_0 \alpha_s^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_s^N}{N!} \left( \log \frac{Q^2}{Q_0^2} \right)^N$$

Denote by

$$\Phi_a[E, Q^2]$$

the ensemble of parton cascades initiated by a parton  $a$  of energy  $E$  and emerging from a hard process with scale  $Q^2$  (Generating functional). Also, define

$$\Delta(Q_1^2, Q_2^2)$$

as the probability that a **does not branch** for virtualities  $Q_1^2 > t > Q_2^2$





# Evolution equation and Sudakov

With this, it easy to write a formula that takes into account all the branches associated to a parton a:

$$\begin{aligned}\Phi_a[E, Q^2] &= \Delta_a(Q^2, Q_0^2) \Phi_a[E, Q_0^2] \\ &+ \int_{Q_0^2}^{Q^2} \frac{dt}{t} \Delta_a(Q^2, t) \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z) \Phi_b[zE, t] \Phi_c[(1-z)E, t]\end{aligned}$$

Simple interpretation. First term describes the evolution to  $Q_0$ , where no branching has occurred. The second term is the contribution coming from evolving with no branching up to a given  $t$  and then branching there.

Now conservation of probability imposes that:

$$1 = \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dt}{t} \Delta_a(Q^2, t) \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z)$$

Which can be solved to give an explicit expression for  $\Delta$ .





# Evolution equation and Sudakov

$$\Delta_a(Q^2, Q_0^2) = \exp \left( - \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z) \right)$$

Proof: derive the conservation of probability equation

$$0 = \frac{d\Delta_a}{dQ_0^2}(Q^2, Q_0^2) - \frac{\mathcal{P}_a}{Q_0^2} \Delta_a(Q^2, Q_0^2), \quad \mathcal{P}_a = \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z)$$

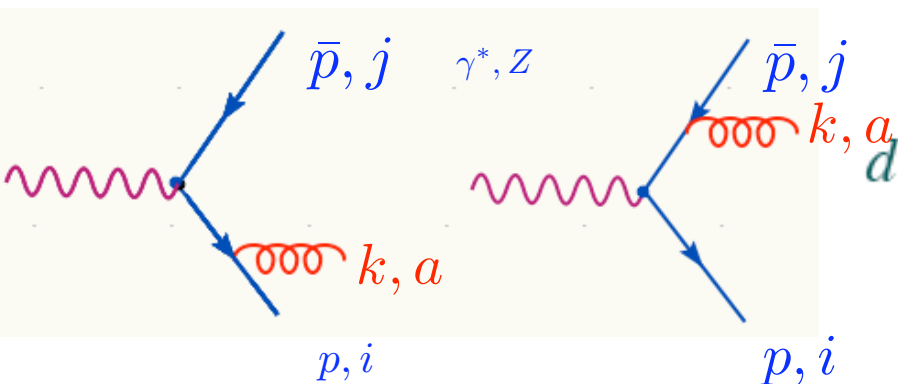
and impose the initial condition

$$\Delta_a(Q^2, Q^2) = 1$$

Note that  $\Delta_a(Q^2, t) = \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(t, Q_0^2)}$  and therefore sometimes the second argument is not used.



# Angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \epsilon_\mu \epsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} d\cos \theta$$

You can easily prove that:

$$\frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} = \frac{1}{2} \left[ \frac{\cos \theta_{jk} - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} + \frac{1}{1 - \cos \theta_{ik}} \right] + \frac{1}{2} [i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

where

$$W_{(i)} \rightarrow \text{finite if } k \parallel j \text{ (} \cos \theta_{jk} \rightarrow 1 \text{)}$$

$$W_{(j)} \rightarrow \text{finite if } k \parallel i \text{ (} \cos \theta_{ik} \rightarrow 1 \text{)}$$

The probabilistic interpretation of  $W_{(i)}$  and  $W_{(j)}$  is a priori spoiled by their non-positivity. However, you can prove **[EXERCISE]** that after azimuthal averaging:

$$| \text{diagram} |^2 = | \text{diagram} |^2 \Theta(\varphi - \varphi_1) + | \text{diagram} |^2 \Theta(\varphi - \varphi_2)$$

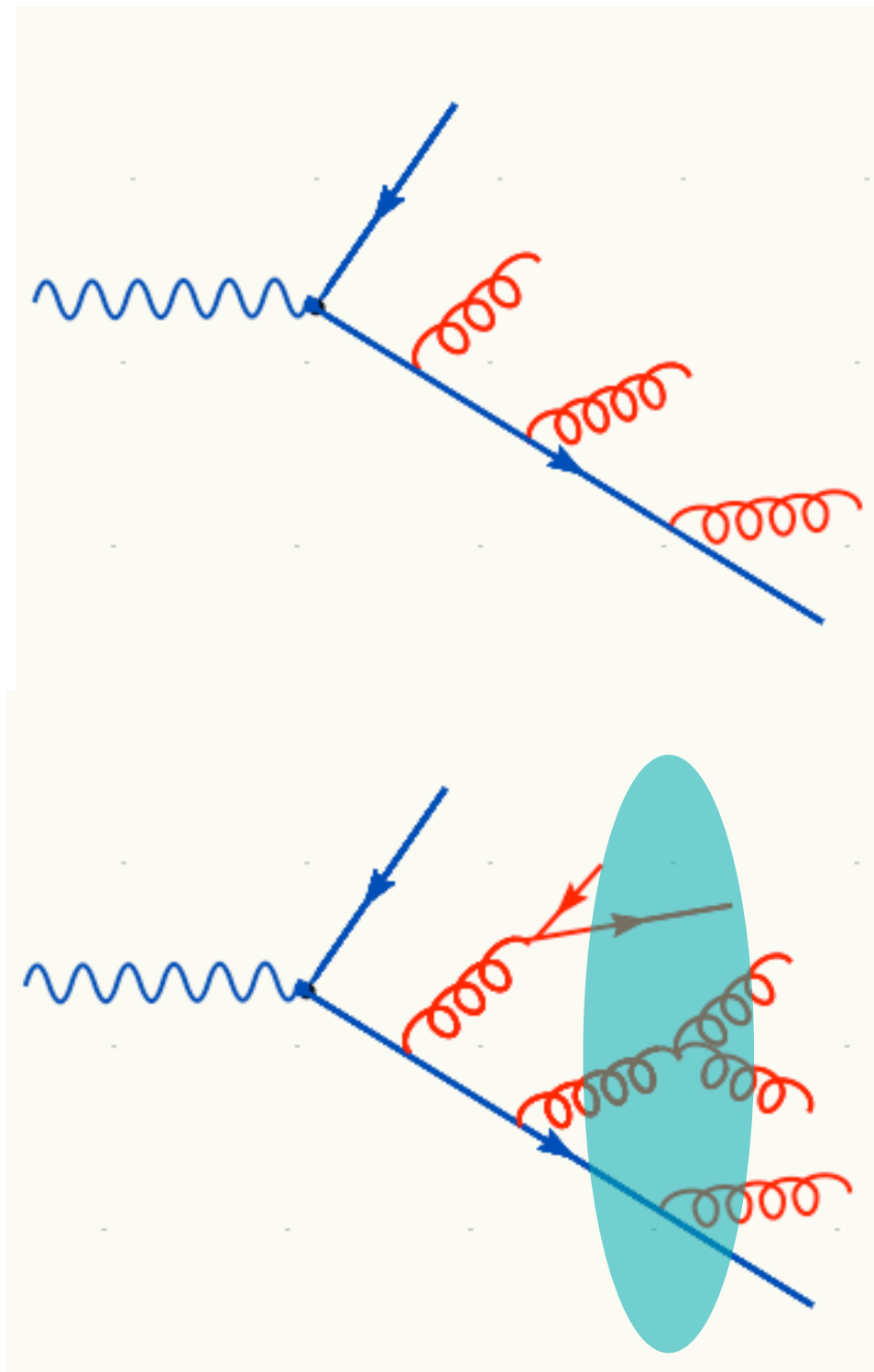
$$\int \frac{d\phi}{2\pi} W_{(i)} = \frac{1}{1 - \cos \theta_{ik}} \text{ if } \theta_{ik} < \theta_{ij}, \quad 0 \text{ otherwise}$$

$$\int \frac{d\phi}{2\pi} W_{(j)} = \frac{1}{1 - \cos \theta_{jk}} \text{ if } \theta_{jk} < \theta_{ij}, \quad 0 \text{ otherwise}$$

Further branchings will obey angular ordering relative to the new angles. As a result emission angles get smaller and smaller, squeezing the jet.



# Angular ordering



The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.

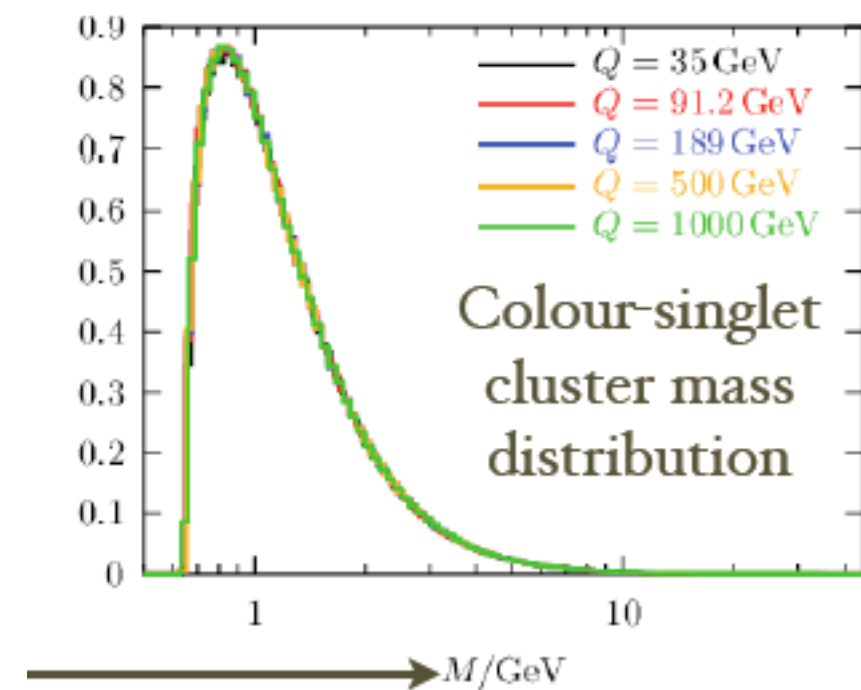
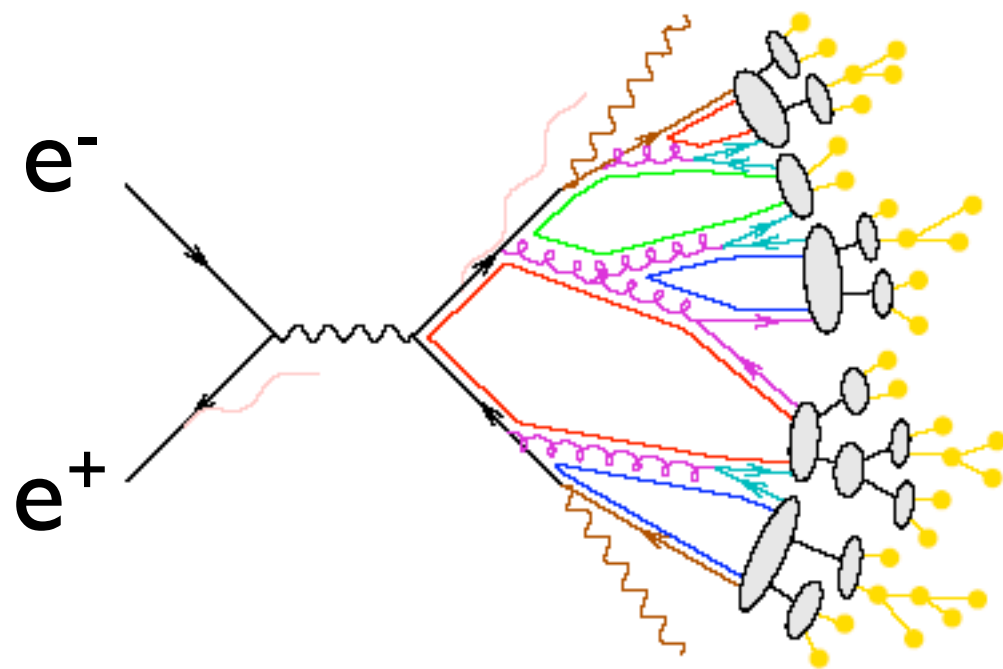
In fact one can generalize the treatment before to a generic parton of color charge  $Q_k$  splitting into two partons  $i$  and  $j$ ,  $Q_k = Q_i + Q_j$ . The result is that inside the cones  $i$  and  $j$  emit as independent charges, and outside their angular-order cones the emission is coherent and can be treated as if it was directly from color charge  $Q_k$ .

This has an effect on the multiplicity of hadrons in jets (INTRAjet radiation), since the radiation is more suppressed with respect to the total phase space available, which one would get from an incoherent radiation. Color ordering enforces coherence and leads to the proper evolution with energy of particle multiplicities.



# Monte Carlo approach to PS

The structure of the perturbative evolution, including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





# A simple plan

- Intro: the LHC challenge
- Precision QCD: from LO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS



# How we (used to) make predictions?

## First way:

- For low multiplicity include higher order terms in our fixed-order calculations (LO→NLO→NNLO...)

$$\Rightarrow \hat{\sigma}_{ab \rightarrow X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

- For high multiplicity use the tree-level results



## Comments:

1. The theoretical errors systematically decrease.
2. Pure theoretical point of view.
3. A lot of new techniques and universal algorithms are developed.
4. Final description only in terms of partons and calculation of IR safe observables  $\Rightarrow$  not directly useful for simulations





# How we (used to) make predictions?

## Second way:

- Describe final states with high multiplicities starting from  $2 \rightarrow 1$  or  $2 \rightarrow 2$  procs, using parton showers, and then an hadronization model.



## Comments:

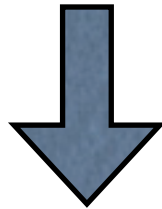
1. Fully exclusive final state description for detector simulations
2. Normalization is very uncertain
3. Very crude kinematic distributions for multi-parton final states
4. Improvements are only at the model level.



# ME vs PS

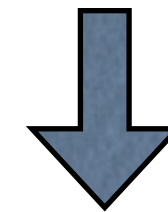
[Mangano]  
[Catani, Krauss, Kuhn, Webber]  
[Frixione, Nason, Webber]

ME



1. parton-level description
2. fixed order calculation
3. quantum interference exact
4. valid when partons are hard and well separated
5. needed for multi-jet description

Shower MC



1. hadron-level description
2. resums large logs
3. quantum interference through angular ordering
4. valid when partons are collinear and/or soft
5. needed for realistic studies

Approaches are complementary: merge them!

Difficulty: avoid double counting



# How to improve our predictions?

New trend:

TH & EXP

Match fixed-order calculations and parton showers to obtain the most accurate predictions in a detector simulation friendly way!

Two directions:

1. Get fully exclusive description of many parton events correct at LO (LL) in all the phase space.

ME+PS

2. Get fully exclusive description of events correct at NLO in the normalization and distributions.

NLO<sub>w</sub>PS

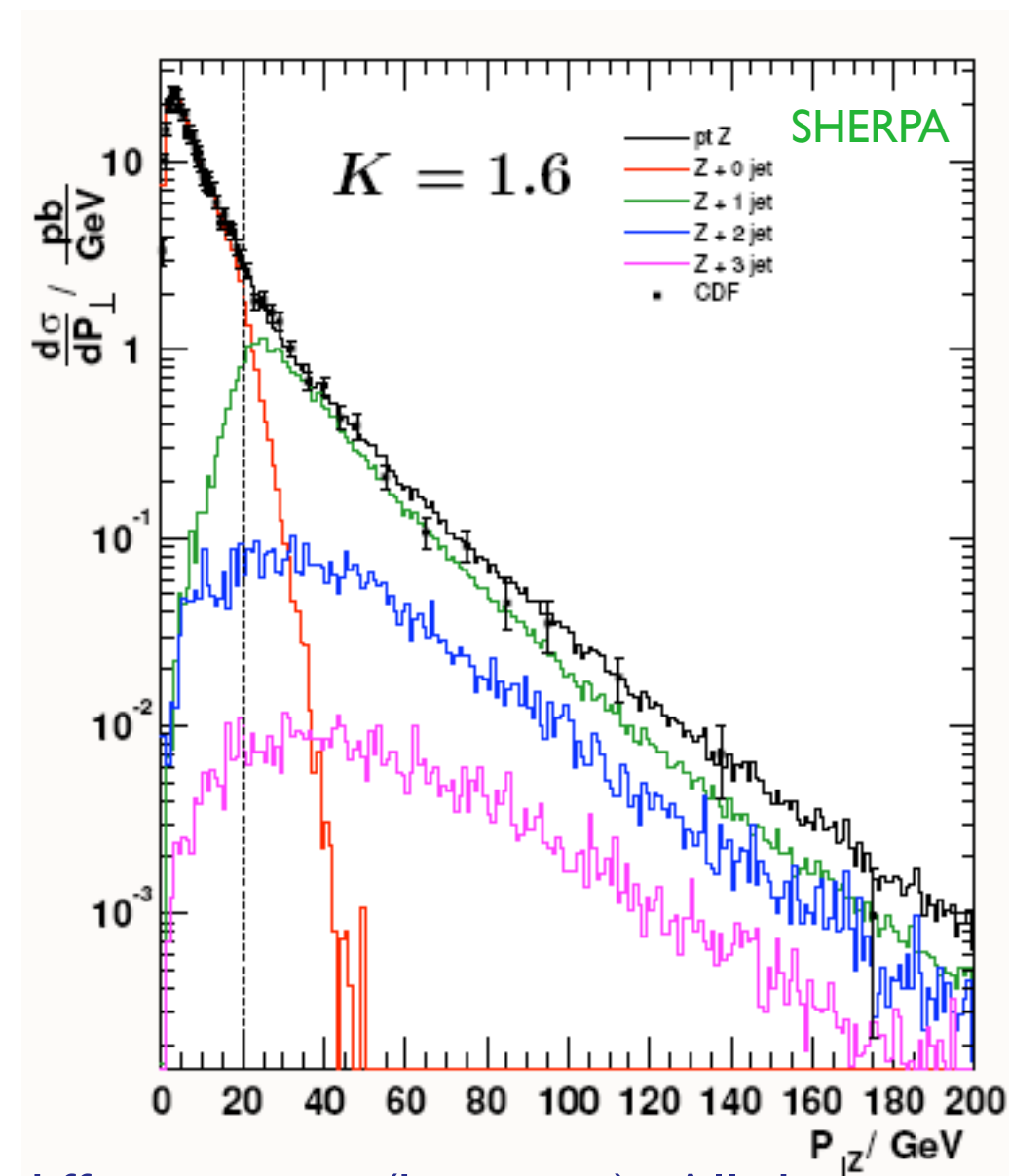
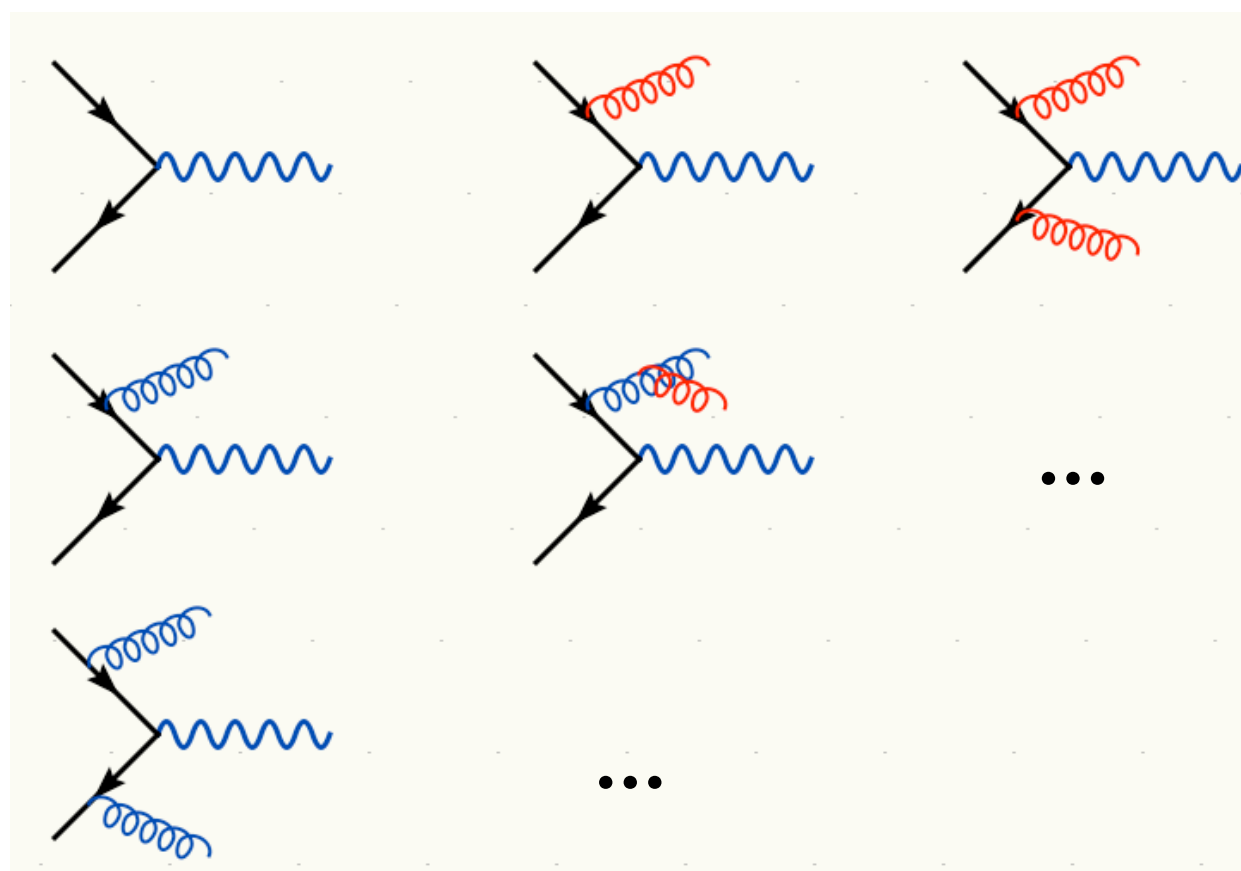


# Merging fixed order with PS

[Mangano]  
[Catani, Krauss, Kuhn, Webber]

PS →

ME

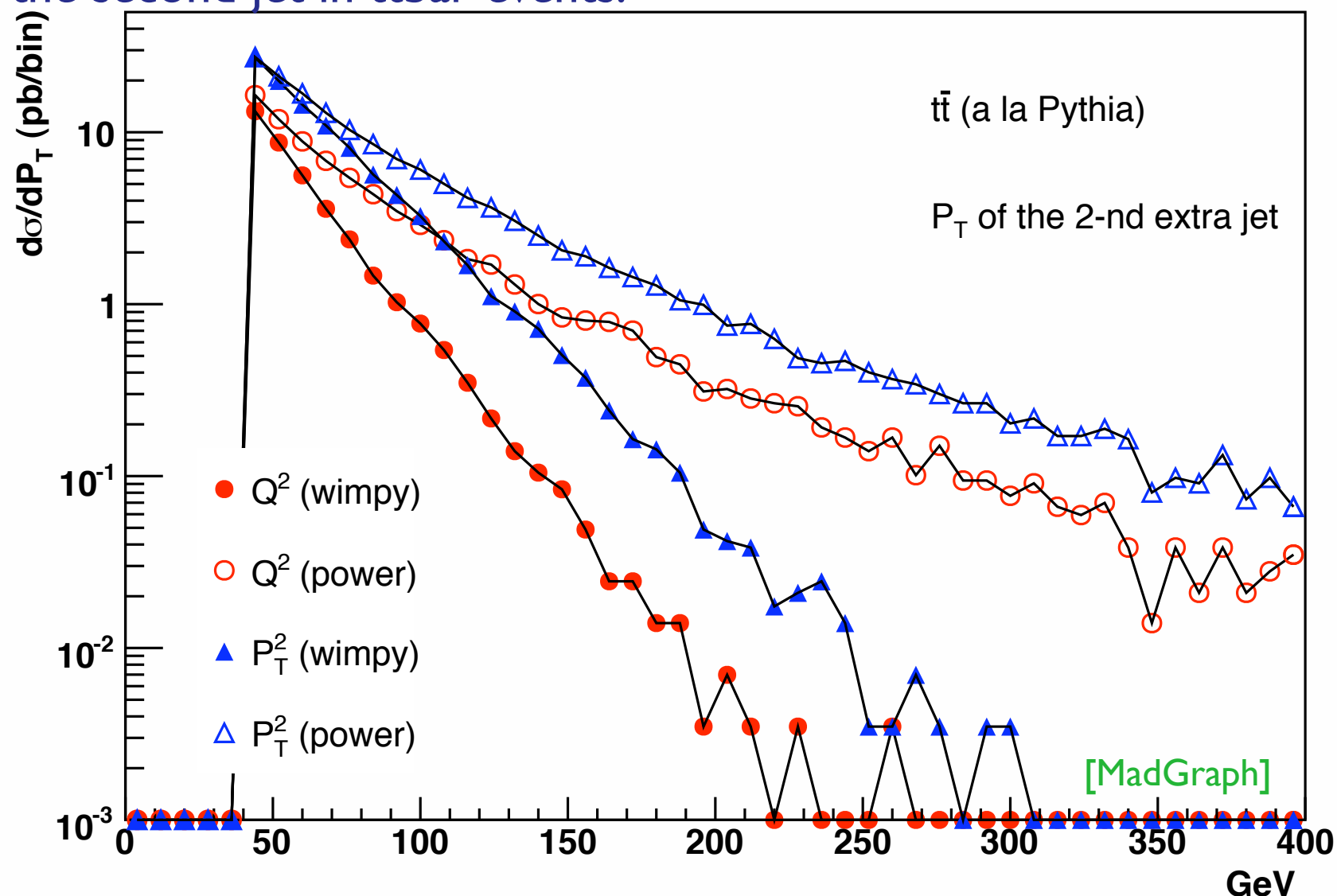


Double counting of configurations that can be obtained in different ways (histories). All the matching algorithms (CKKW, MLM,...) apply criteria to select only one possibility based on the hardness of the partons. As the result events are exclusive and can be added together into an inclusive sample. Distributions are accurate but overall normalization still “arbitrary”.



# PS alone vs matched samples

A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in  $t\bar{t}$  events:



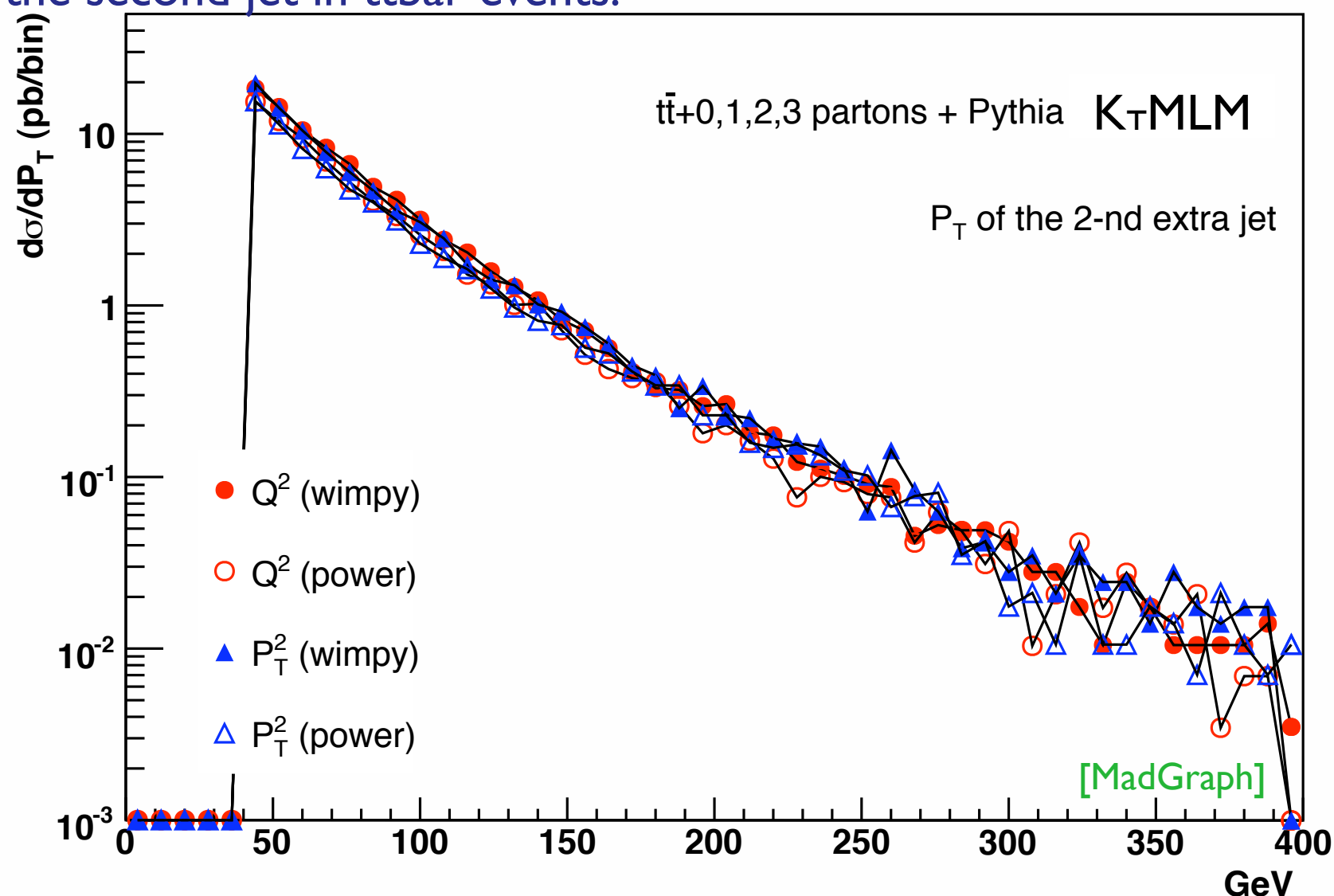
Changing some choices/parameters leads to huge differences  $\Rightarrow$  self diagnosis. Trying to tune the log terms to make up for it is not a good idea  $\Rightarrow$  mess up other regions/shapes, process dependence.





# PS alone vs matched samples

A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in  $t\bar{t}$  events:

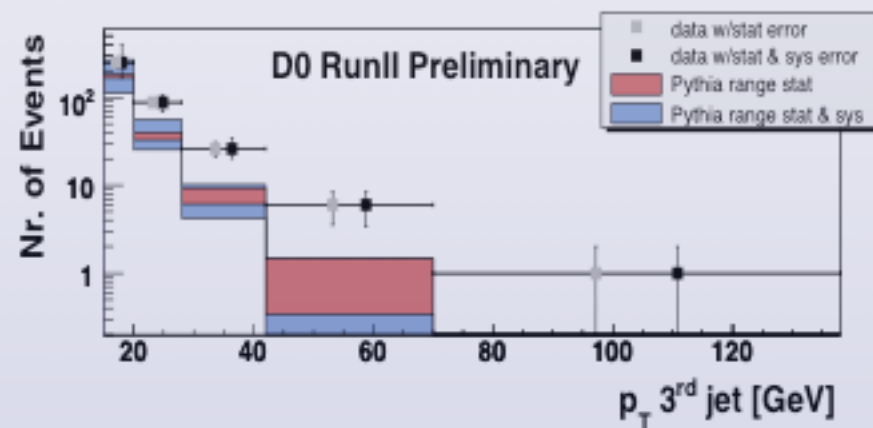
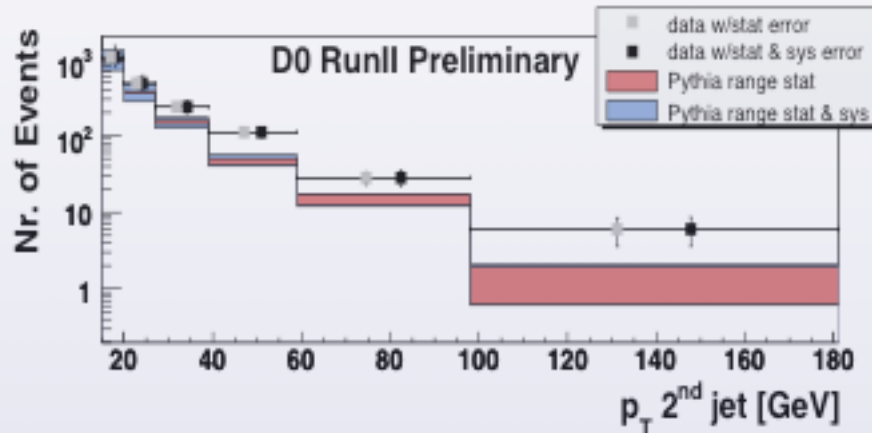
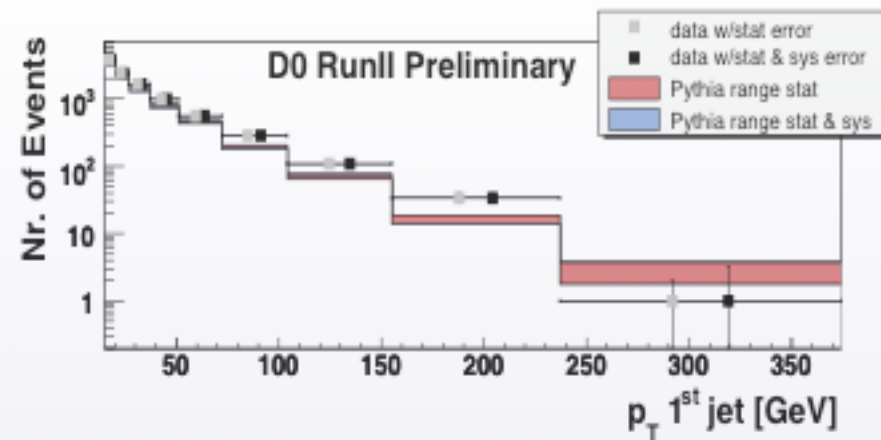


In a matched sample these differences are irrelevant since the behaviour at high pt is dominated by the matrix element. LO+LL is more reliable. (Matching uncertainties not shown.)

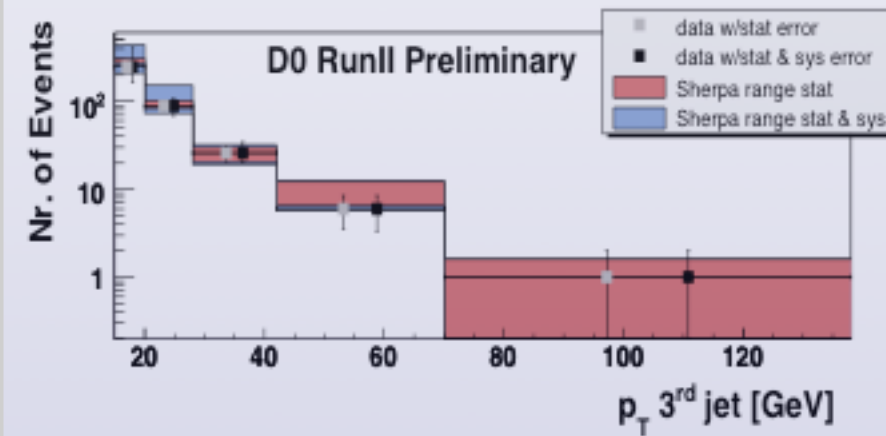
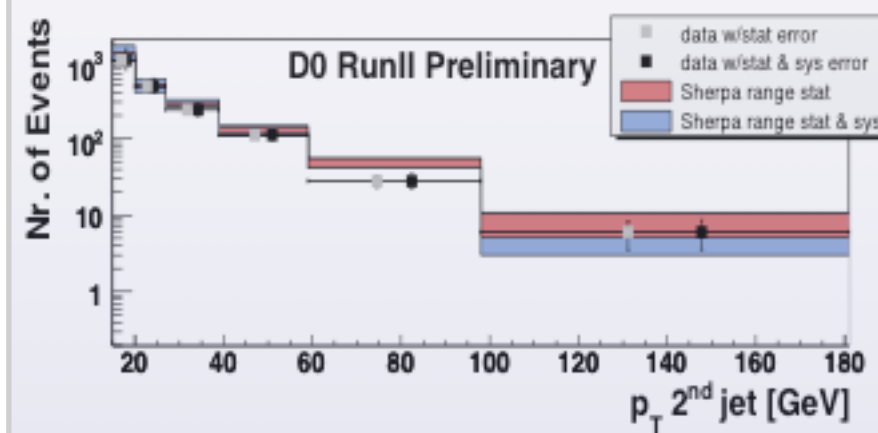
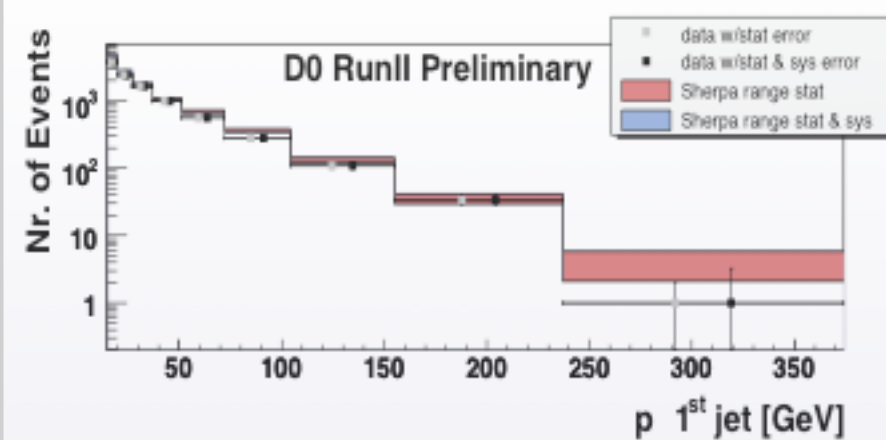




# PS alone vs matched samples : Z+jets at D0



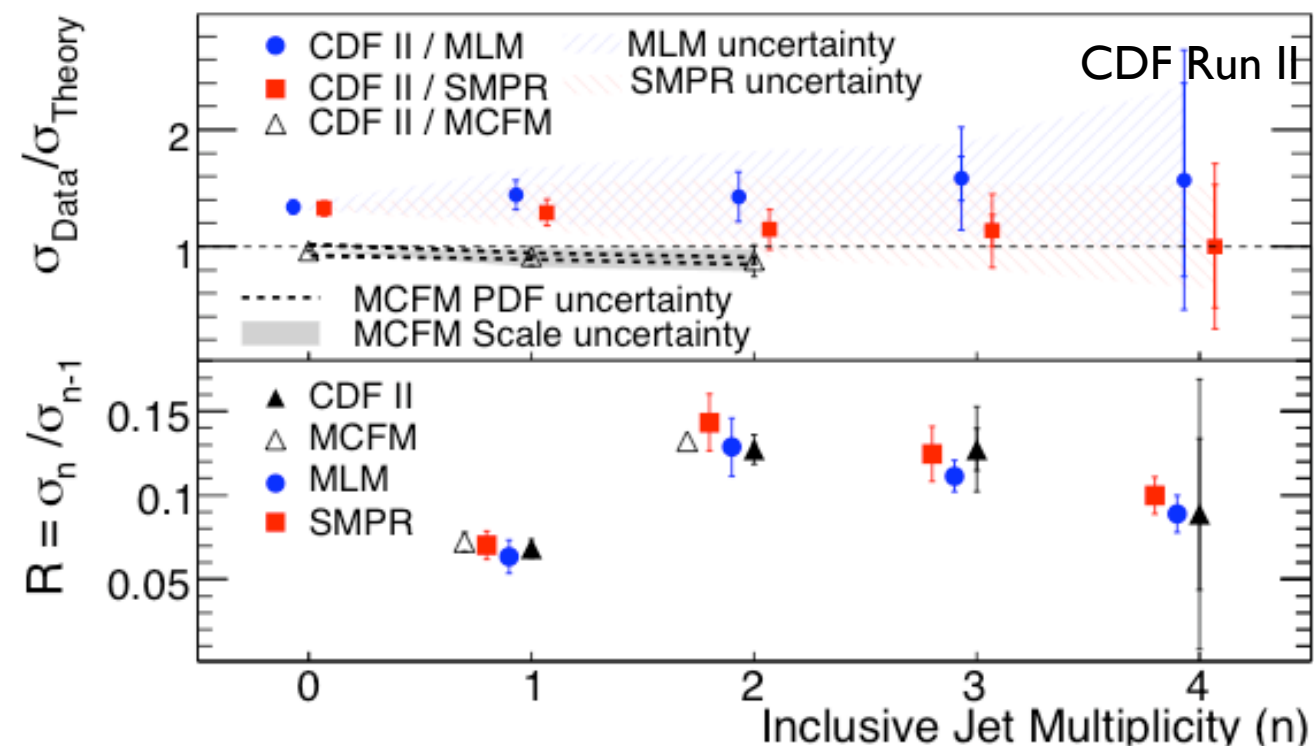
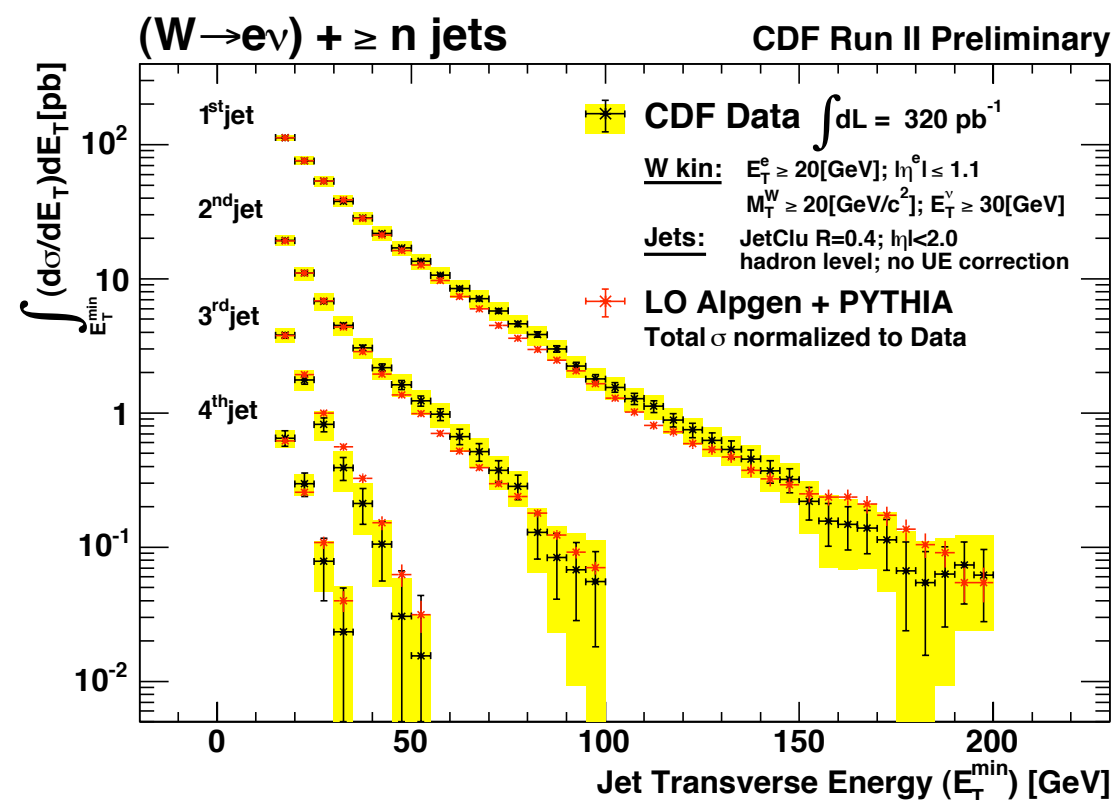
**PS alone** (Pythia)



**ME+PS** (SHERPA)



# W+jets at CDF



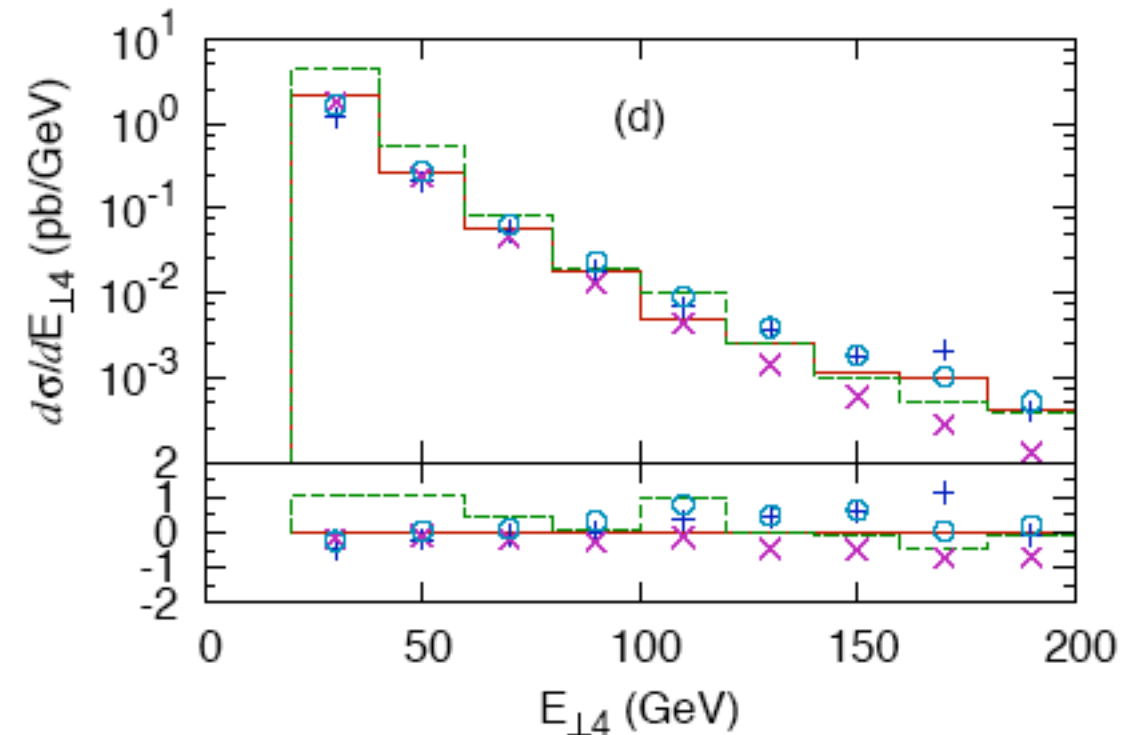
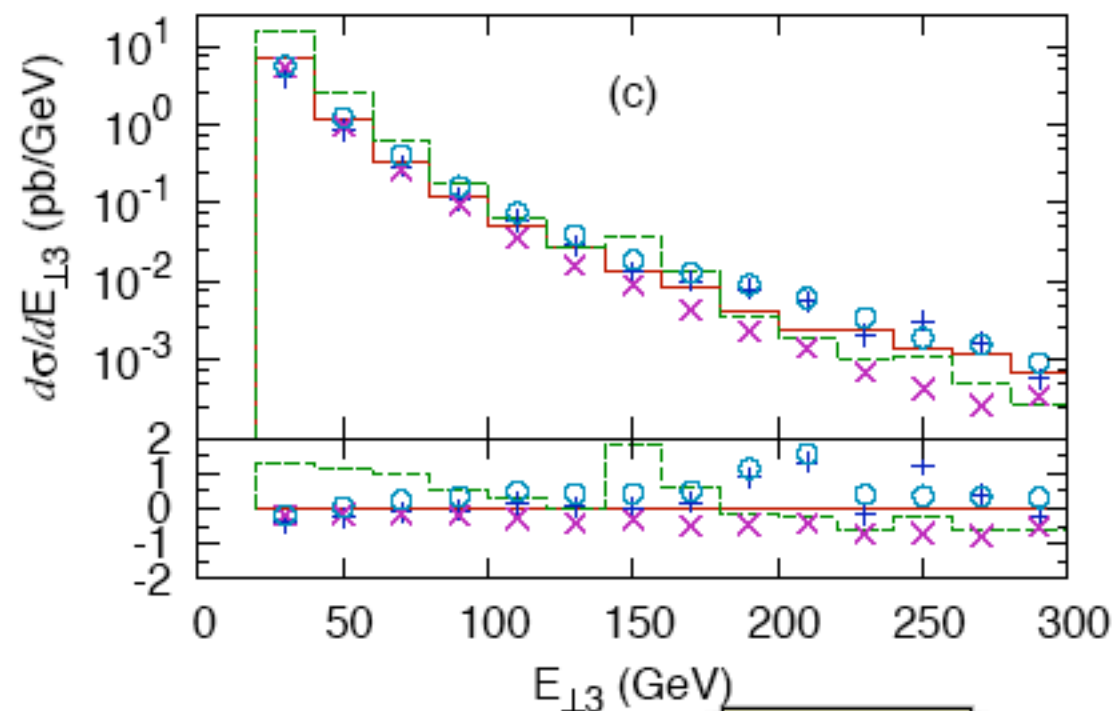
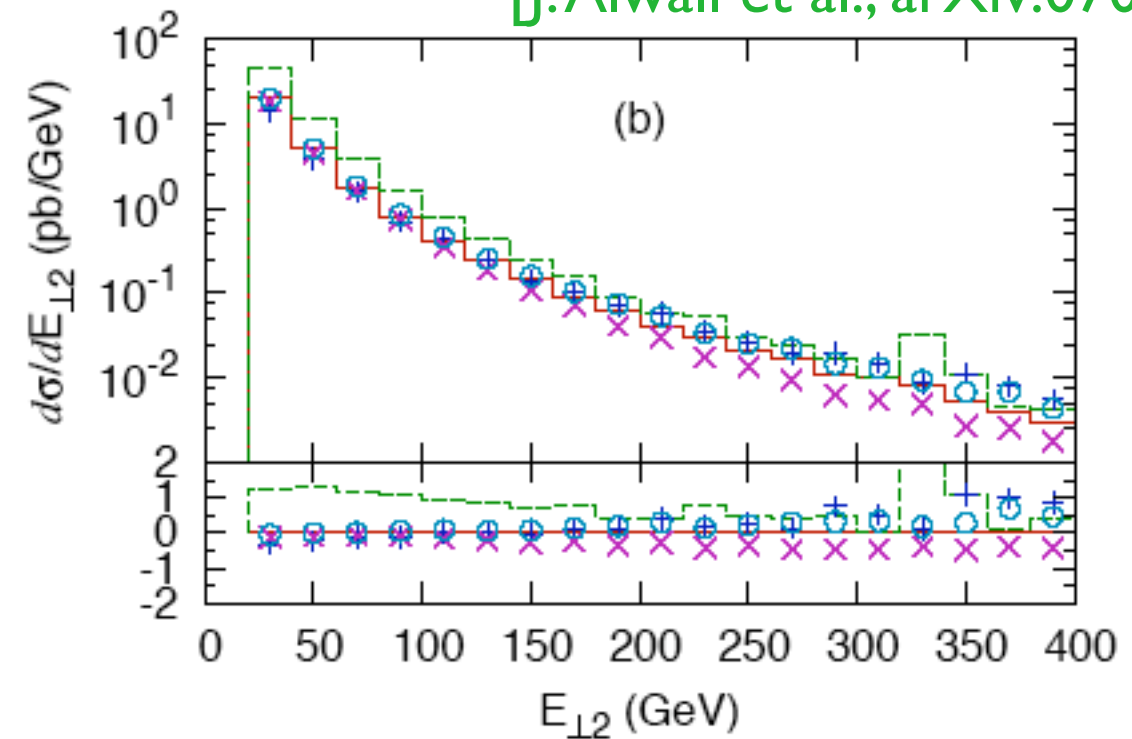
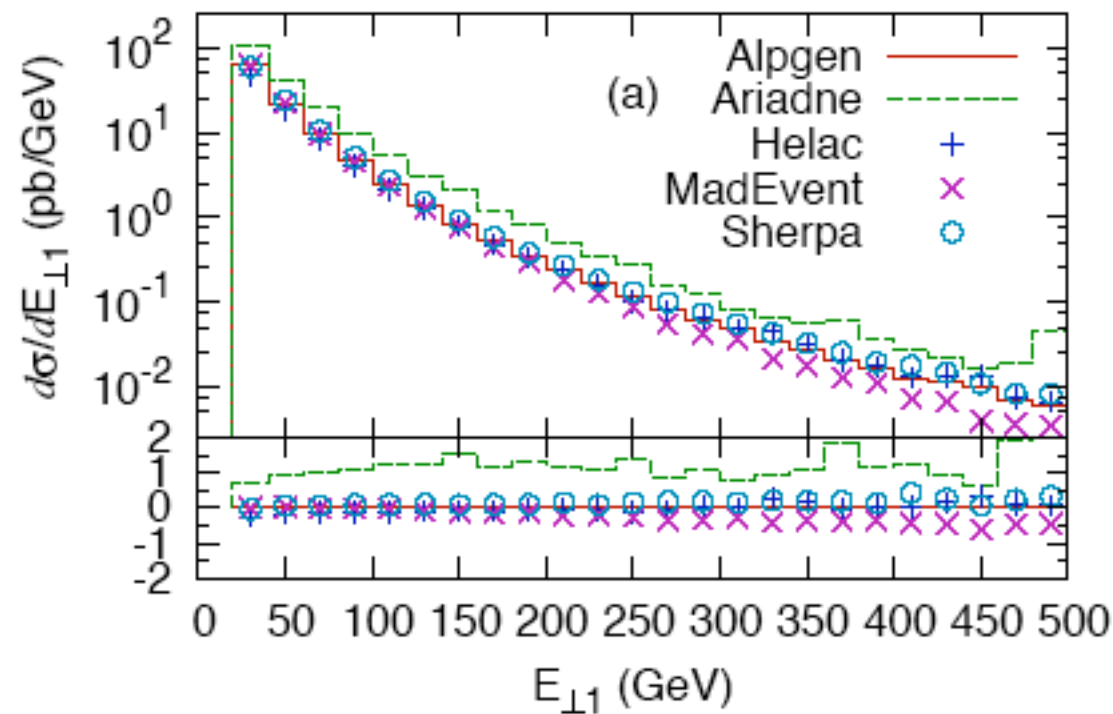
- \* Very good agreement in shapes (left) and in relative normalization (right).
- \* NLO rates in outstanding agreement with data.
- \* Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertainties. Differences might arise in more exclusive quantities.



# $W^+$ jets: first comparison

$W^\pm$  + jets comparison plots: Jet  $E_T$  for LHC

[J. Alwall et al., arXiv:0706.2569]





# NLOwPS

Problem of double counting becomes even more severe at NLO

- \* Real emission from NLO and PS has to be counted once
- \* Virtual contributions in the NLO and Sudakov should not overlap

Current available (and working) solutions:

**MC@NLO** [Frixione, Webber, 2003; Frixione, Nason, Webber, 2003]

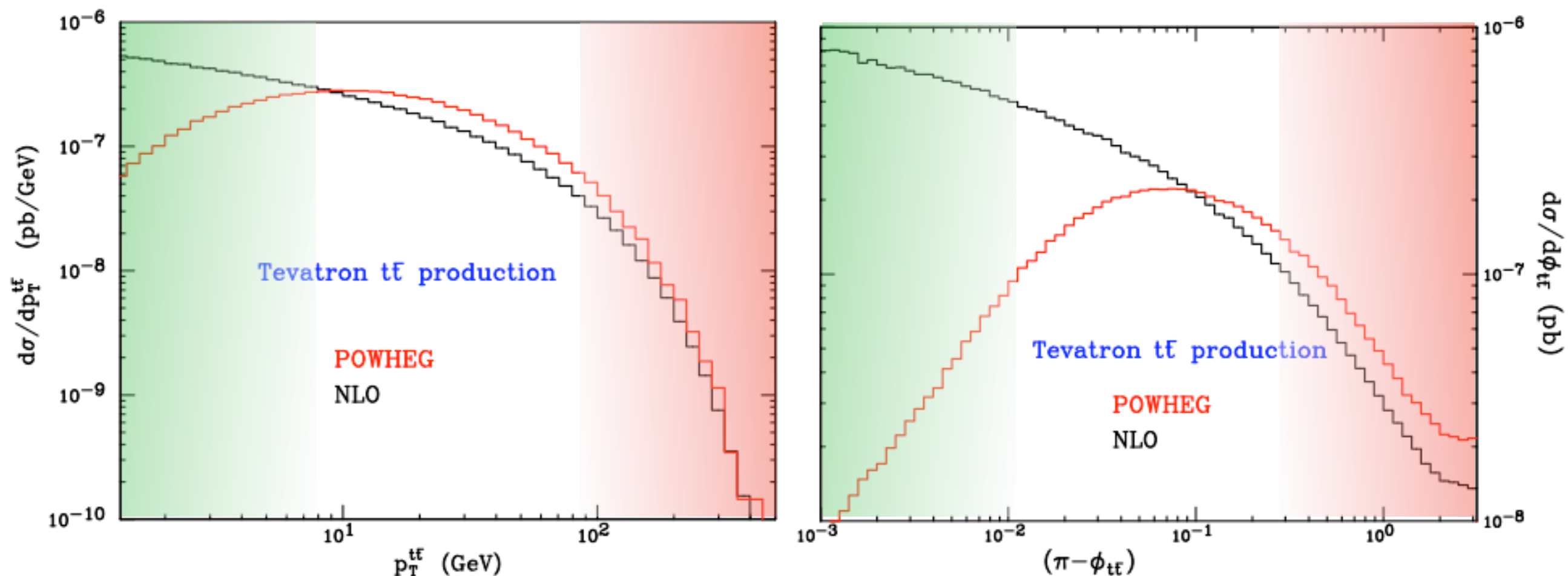
- Matches NLO to HERWIG angular-ordered PS.
- “Some” work to interface an NLO calculation to HERWIG.  
Uses only FKS subtraction scheme.
- Some events have negative weights.
- Sizable library of procs now.

**POWHEG** [Nason 2004; Frixione, Nason, Oleari, 2007]

- Is independent from the PS. It can be interfaced to PYTHIA or HERWIG.
- Can use existing NLO results.
- Generates only positive unit weights.
- For top only ttbar (with spin correlations) is available so far.



# $t\bar{t}$ : NLOwPS vs NLO



- \* Soft/Collinear resummation of the  $p_T(t\bar{t}) \rightarrow 0$  region.
- \* At high  $p_T(t\bar{t})$  it approaches the  $t\bar{t}$ +parton (tree-level) result.
- \* When  $\Phi(t\bar{t}) \rightarrow 0$  ( $\Phi(t\bar{t}) \rightarrow \pi$ ) the emitted radiation is hard (soft).
- \* Normalization is FIXED and non trivial!!





# NLOwPS

“Best” tools when NLO calculation is available (i.e. low jet multiplicity).

- \* Main points:

- \* NLOwPS provide a consistent way to include K-factors into MC's
- \* Scale dependence is meaningful
- \* Allows a correct estimates of the PDF errors.
- \* Non-trivial dynamics beyond LO included for the first time.

N.B.: The above is true for observables which are at NLO to start with!!!

- \* Current limitations:

- \* Considerable manual work for the implementation of a new process.
- \* Only SM.
- \* Only available for low multiplicity.





# Status

$pp \rightarrow n$  particles

accuracy  
[loops]

III

2

II

1

I

0

Two-loop:

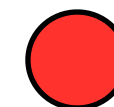
- . Limited number of  $2 \rightarrow 1$  processes
- . No general algorithm for divs cancellation
- . Completely manual
- . No matching known

One-loop:

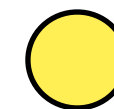
- . Large number of processes known up to  $2 \rightarrow 3$
- . General algorithms for divergences cancellation
- . Not automatic yet (loop calculation)
- . Matching with the PS available for several processes (MC@NLO)

Tree-level:

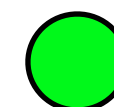
- . Any process  $2 \rightarrow n$  available
- . Many algorithms
- . Completely automatized
- . Matching with the PS at NLL



fully inclusive



parton-level



fully exclusive

1

2

3

4

5

6

7

8

9

10

complexity [n]



# Status: SUSY

$pp \rightarrow n$  particles

accuracy  
[loops]

III

2

II

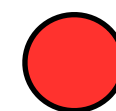
1

I

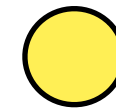
0

NLO:

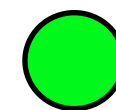
- .  $2 \rightarrow 1$  (SM) and  $2 \rightarrow 2$
- . Fully inclusive ("K factors only")
- . Completely manual



fully inclusive



parton-level



fully exclusive



+ SM

Tree-level:

- . Any process  $2 \rightarrow 2k$  susy + i sm
  - . Feynman-diagram based
  - . Completely automatized
  - . Double counting
  - . Merging ME&PS
- NEW!

1

2

3

4

5

6

7

8

9

10

complexity [n]



## Conclusions

- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements.
- A new generation of tools and techniques has been is available. Among the most useful is the matching between fixed-order and parton-shower both at tree-level and at NLO.
- Fully efficient and flexible BSM simulation chain being completed. Same level of sophistication as SM processes attained.
- Shift in paradigm: useful TH predictions in the form of tools that can be used by EXP's. Communication and collaboration between THs & EXPs easier  $\Rightarrow$  emergence of an integrated LHC community.