The Cosmological Dark Puzzles

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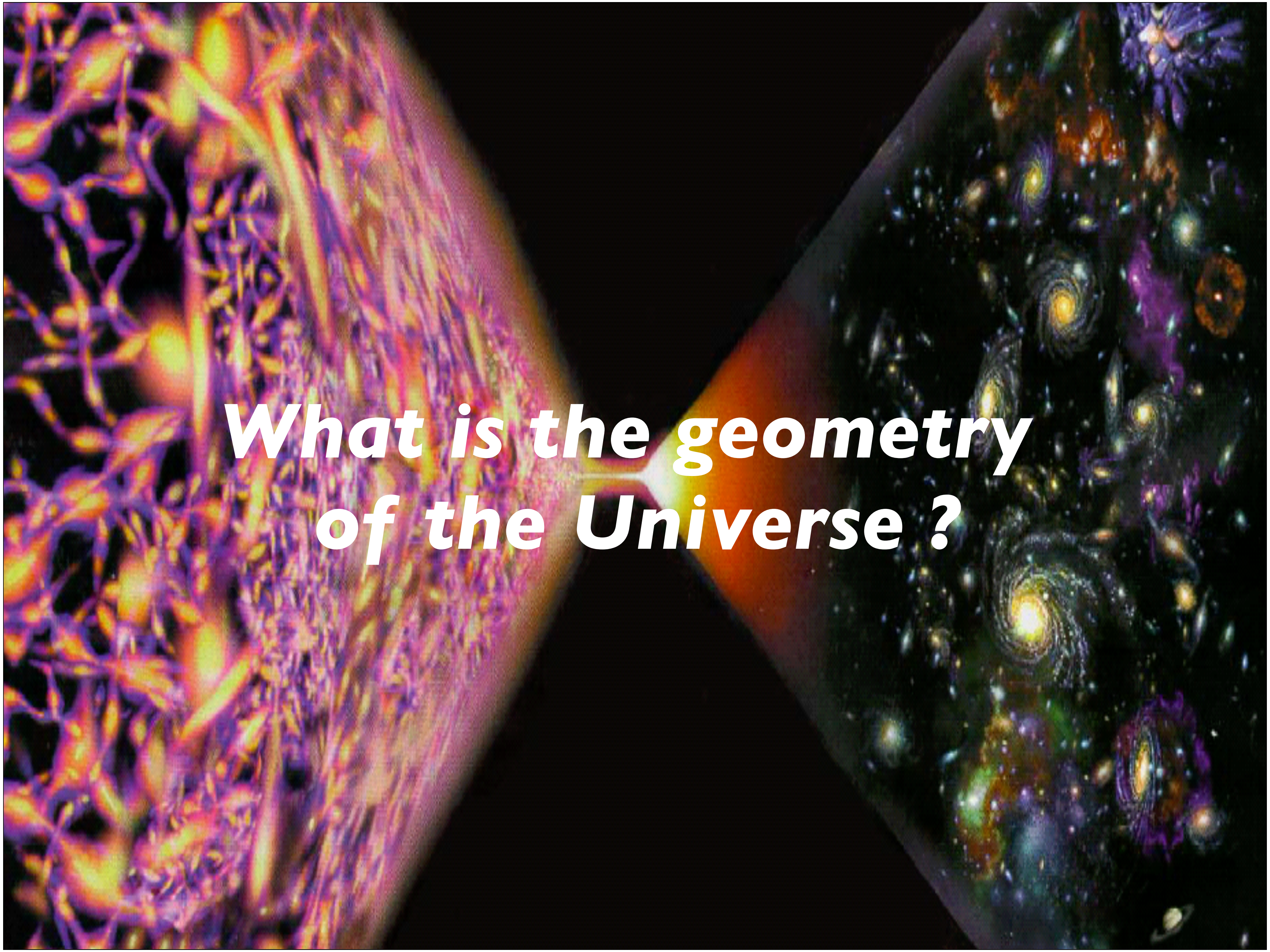
CERN

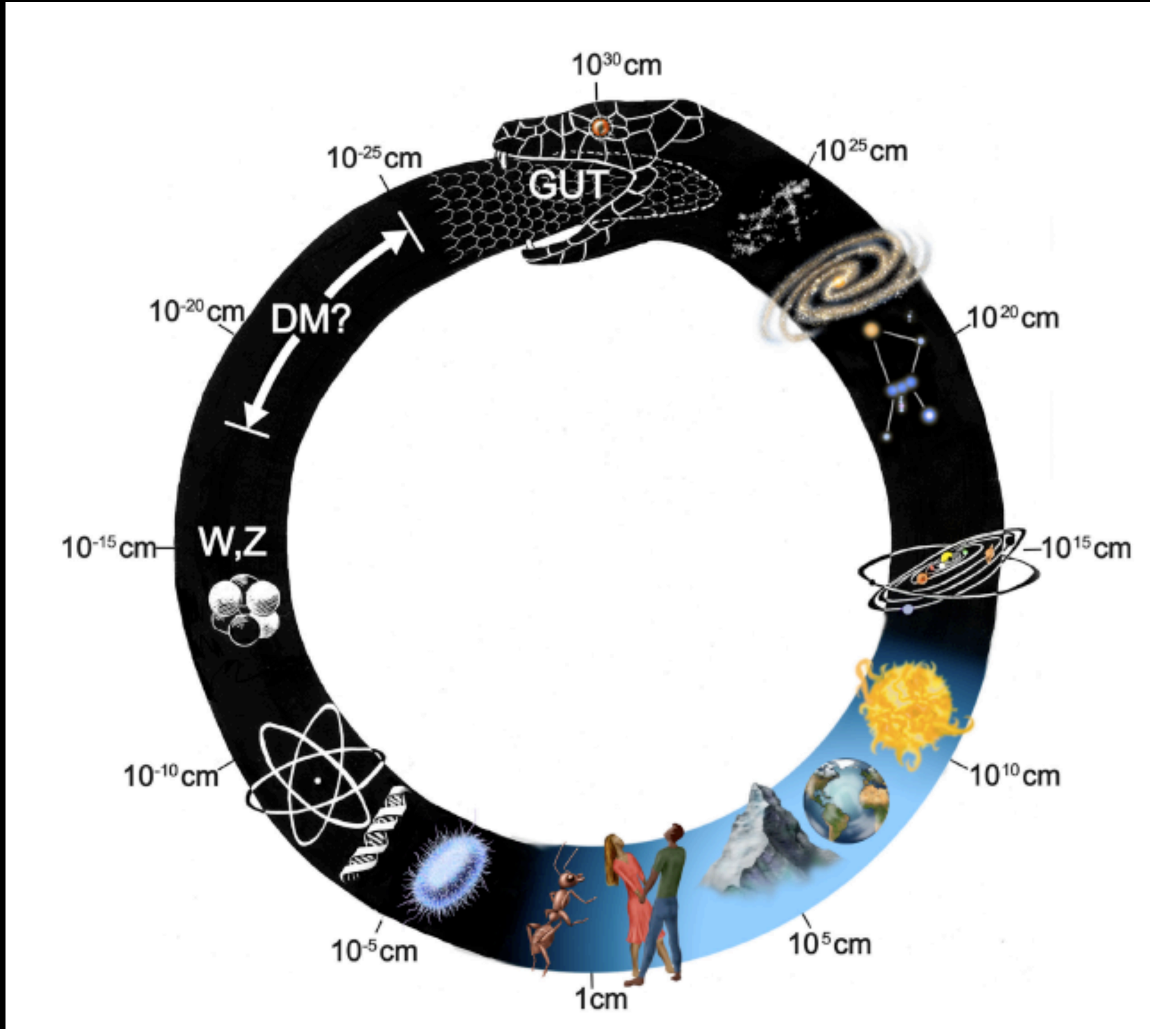
XIV Frascati Spring School " Bruno Touschek"


Where is our Universe
coming from ?

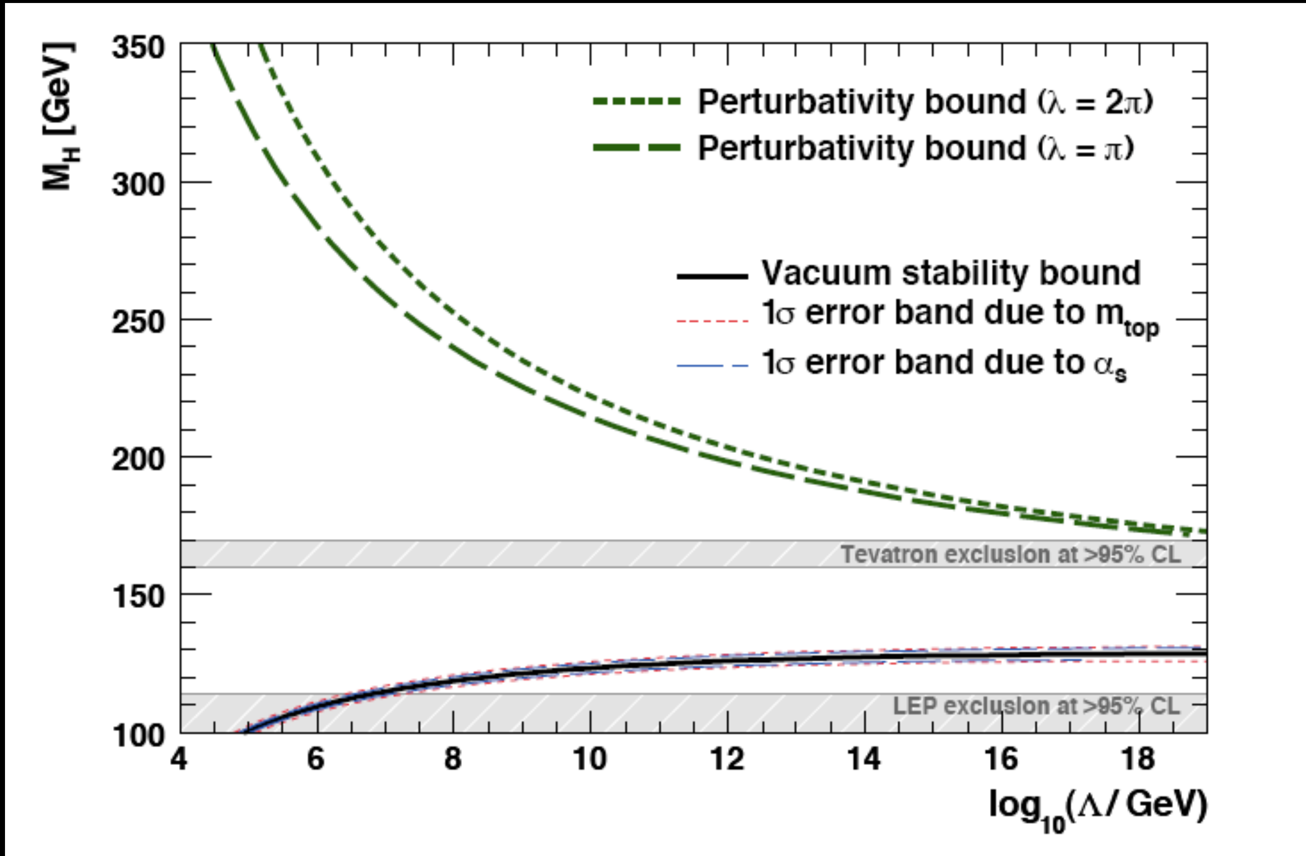

What is the geometry
of the Universe ?


What is the fate
of the Universe ?


What is our Universe made of?



10^30 cm
10^-25 cm
10^25 cm
GUT
10^-20 cm
DM?
10^20 cm
10^-15 cm
W,Z
10^15 cm
10^-10 cm
10^10 cm
10^-5 cm
10^5 cm
1cm



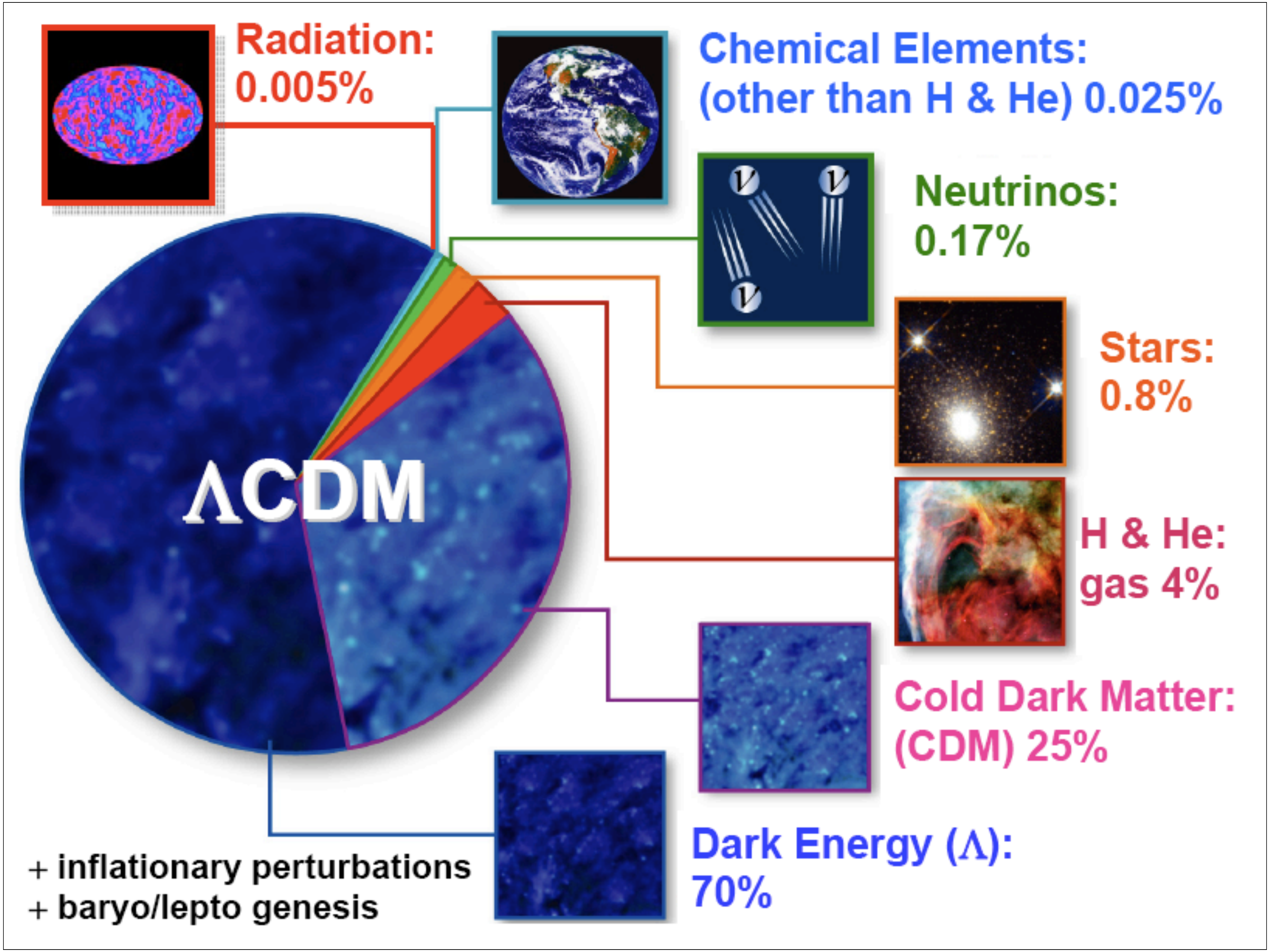
J. Ellis, J. Espinosa, G. Giudice, A. Hoecker and A.R., to appear

First collisions at CERN

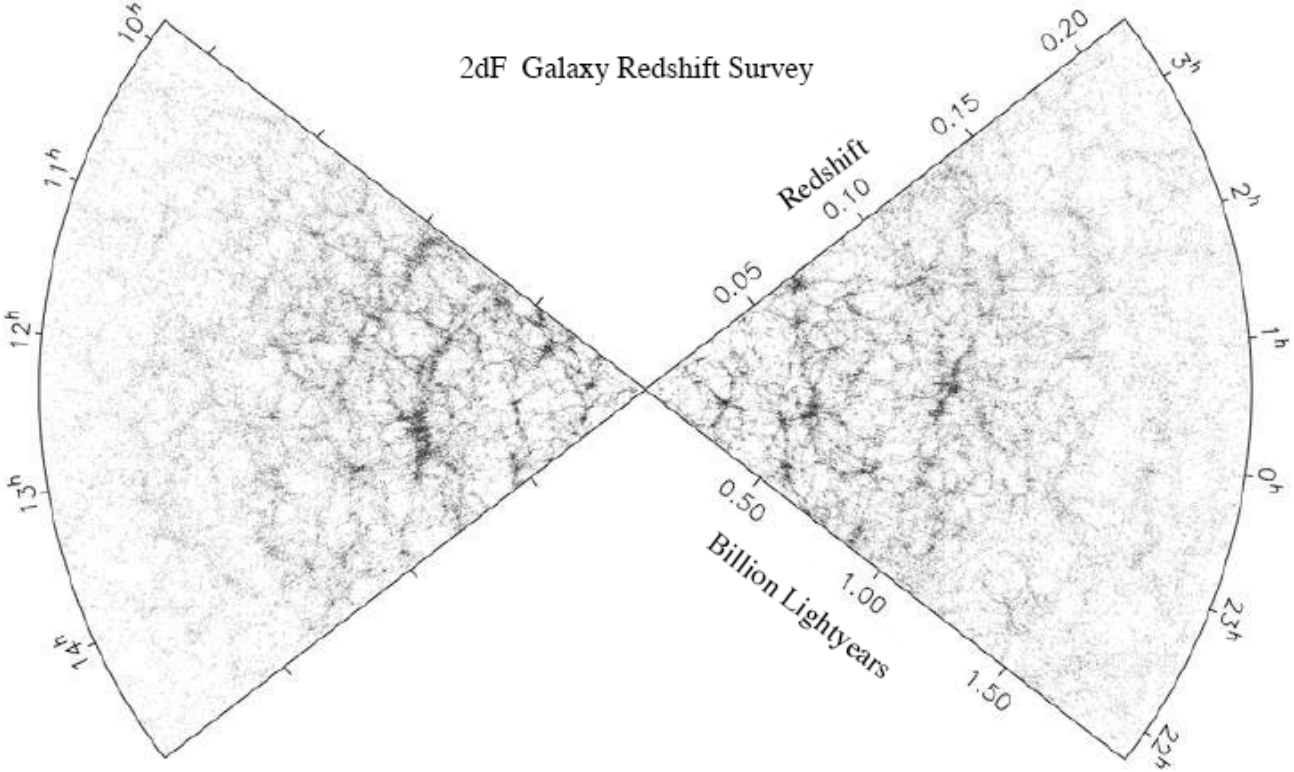
Plan of the lectures

- Introduction to the Standard Big-Bang cosmology
- The Dark Energy puzzle
- The DM puzzle

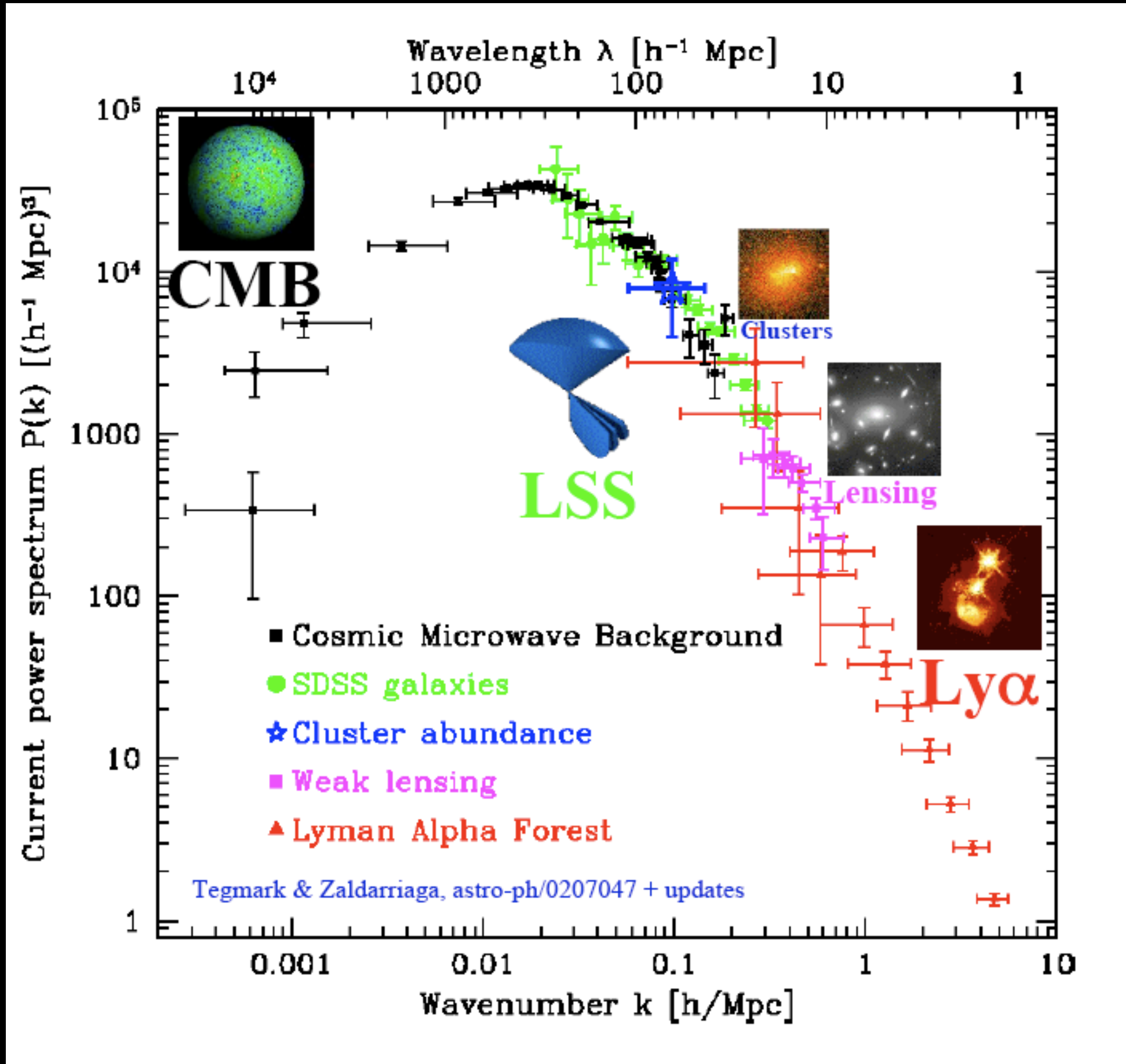
Lecture one: Introduction to Standard Big Bang Cosmology and the Dark Energy Puzzle


Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
Stars:
0.8%
ΛCDM
H & He:
gas 4%
Cold Dark Matter:
(CDM) 25%
Dark Energy (Λ):
70%
+ inflationary perturbations
+ baryo/lepto genesis

The Universe has structure

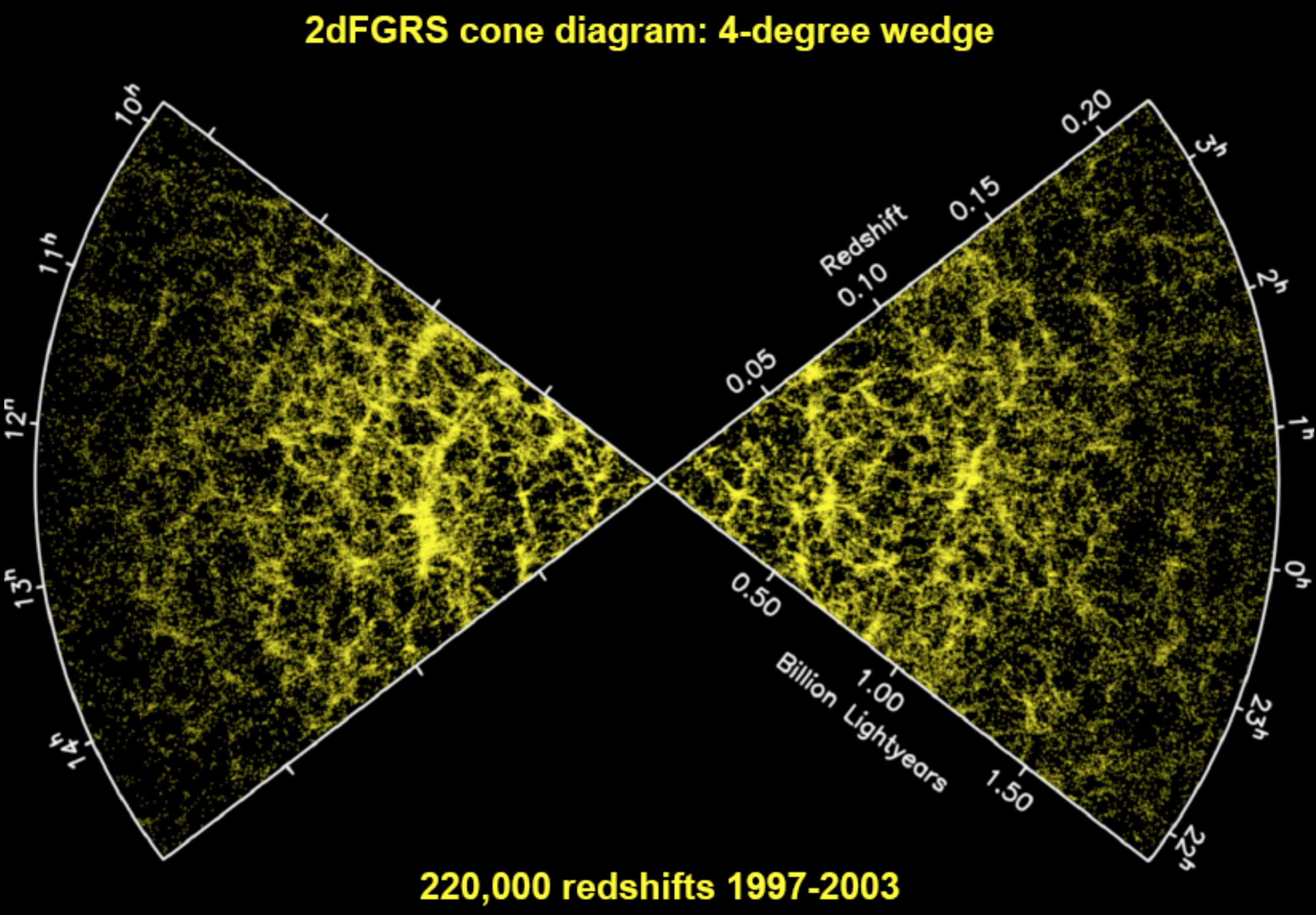


221414 galaxies with redshifts from 10° strips in APM survey

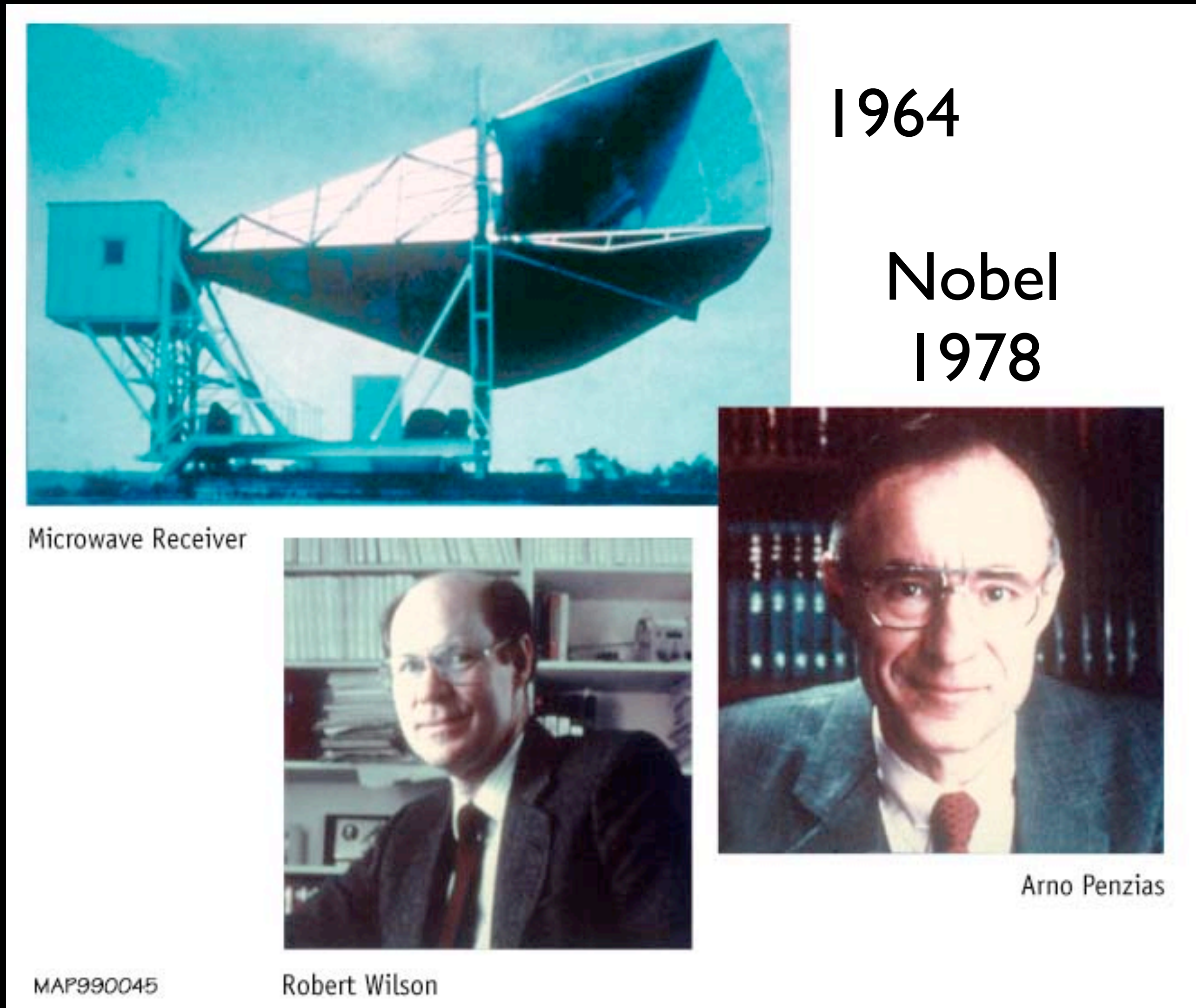


Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
CMB
LSS
Clusters
Lensing
Lyα
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
Tegmark & Zaldarriaga, astro-ph/0207047 + updates
Current power spectrum P(k) [(h⁻¹ Mpc)³]
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]

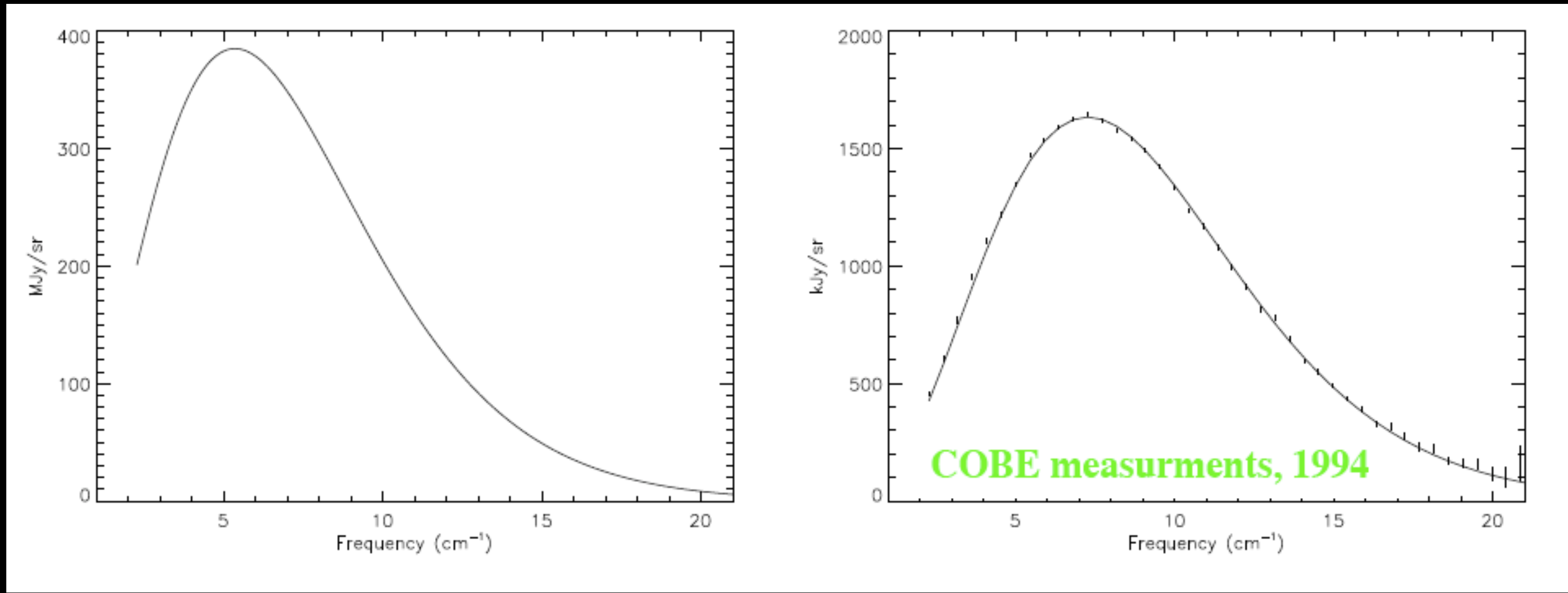
The Universe is homogeneous and isotropic on sufficiently large scales



Cosmic Microwave Background

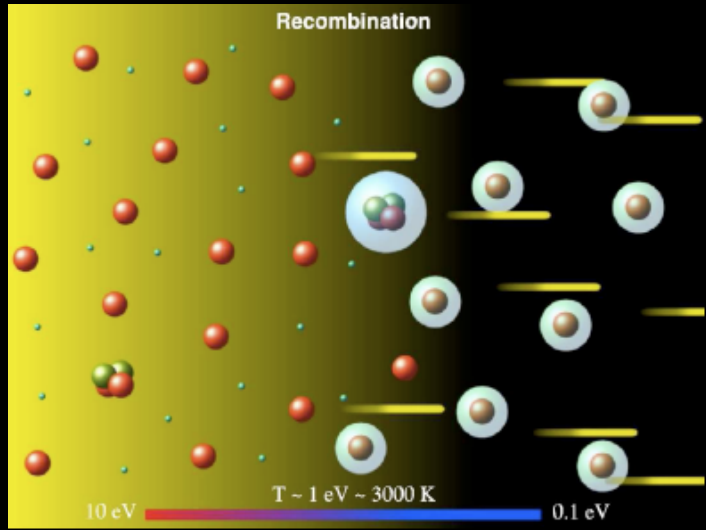


The Cosmic Microwave Background Radiation



- 2.725 K above absolute zero
- mm-cm wavelength
- 410.4 photons per cubic cm
- Perfect black-body spectrum
- Nobel prize 1978: Penzias & Wilson
- Nobel Prize 2006: Mather & Smoot

Hydrogen Recombination & Last Scattering Surface

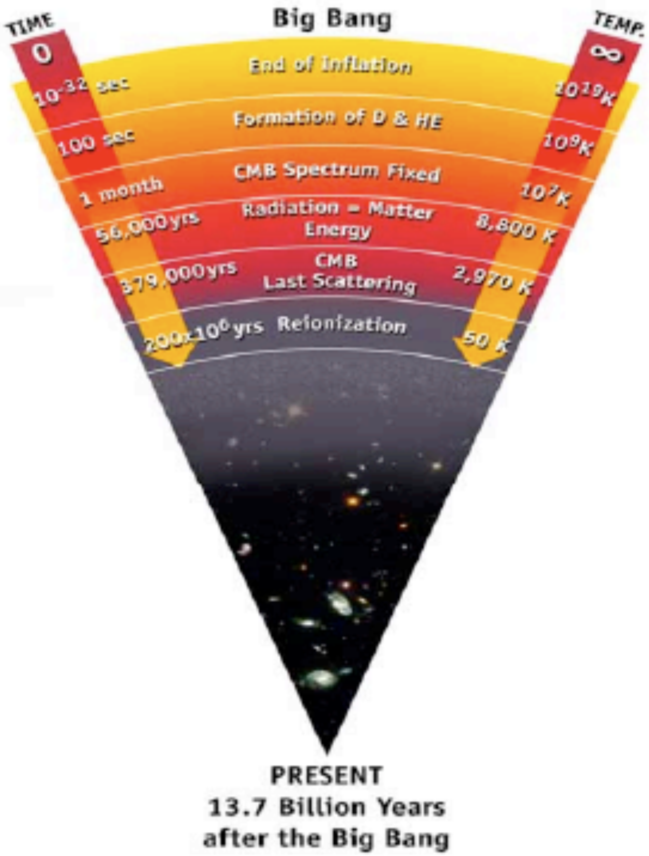


Matter is ionized at temperatures higher than the hydrogen ionization energy of 13.6 eV

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-E_{\rm ion}/T}$$

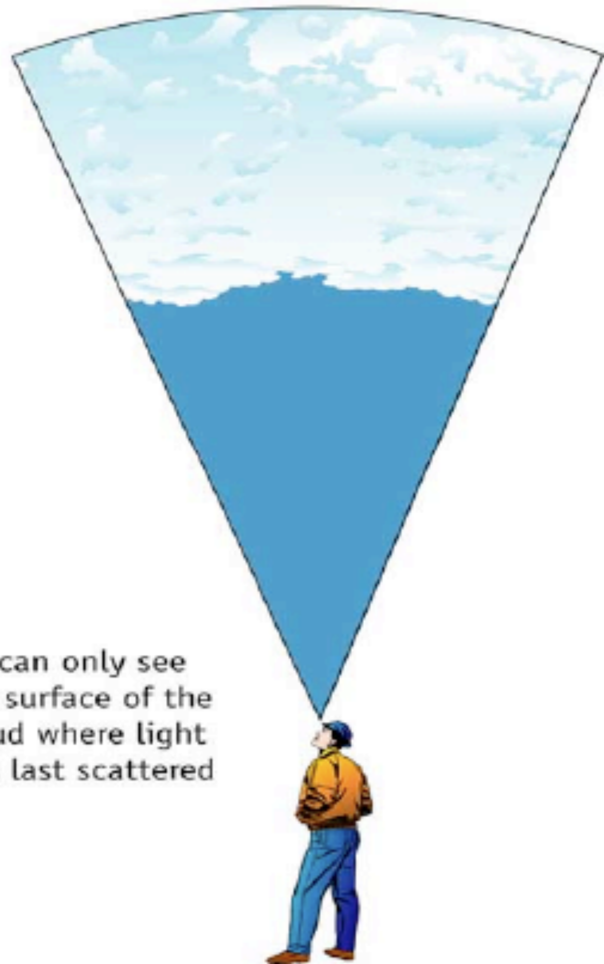
The Universe becomes transparent to photons when

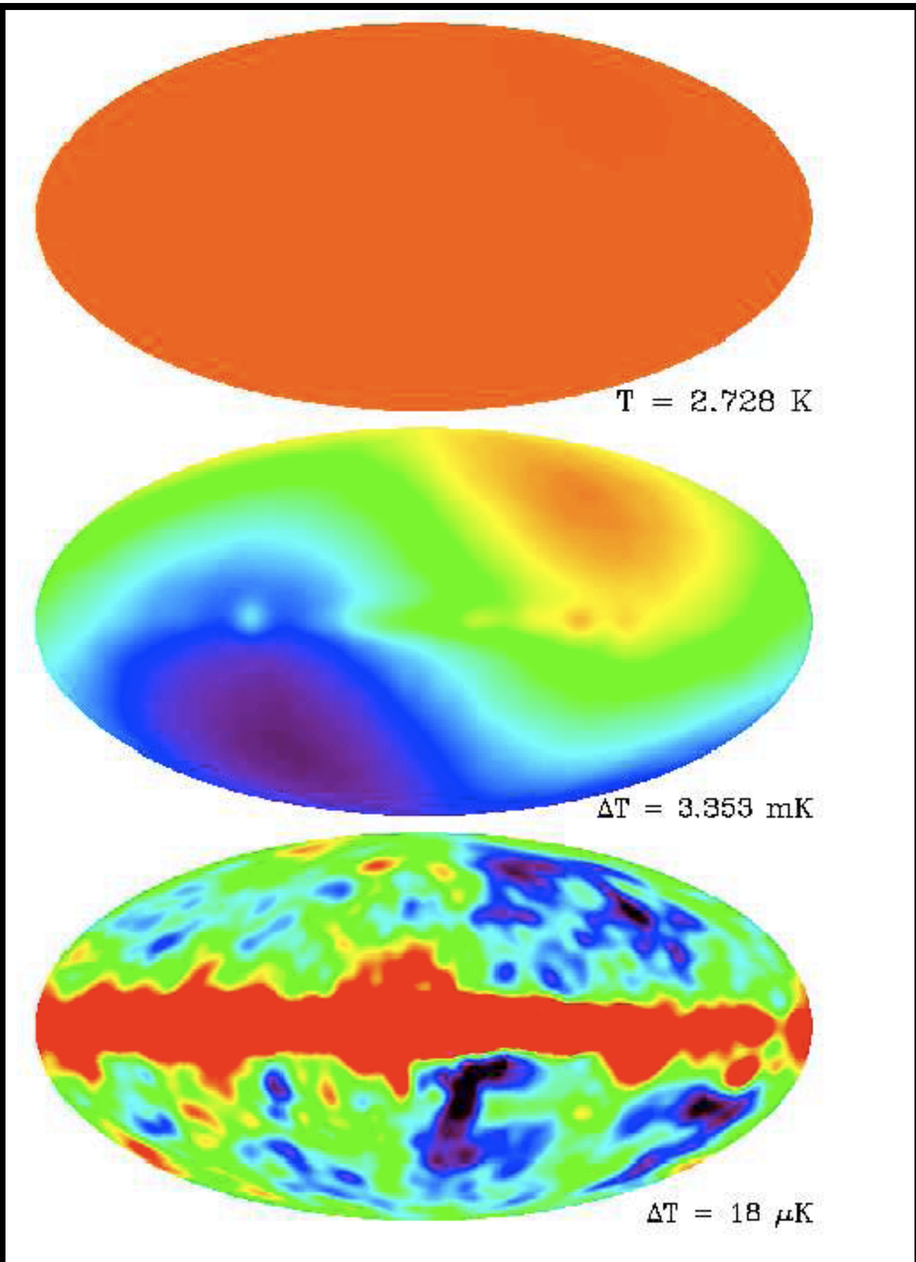
$$(\sigma_{e\gamma} n_e)^{-1} \sim t,\ \sigma_{e\gamma} = 8\pi\alpha^2/3m_e^2,\ T_{\rm LS} \simeq 0.26\ {\rm eV}$$



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

We can only see the surface of the cloud where light was last scattered



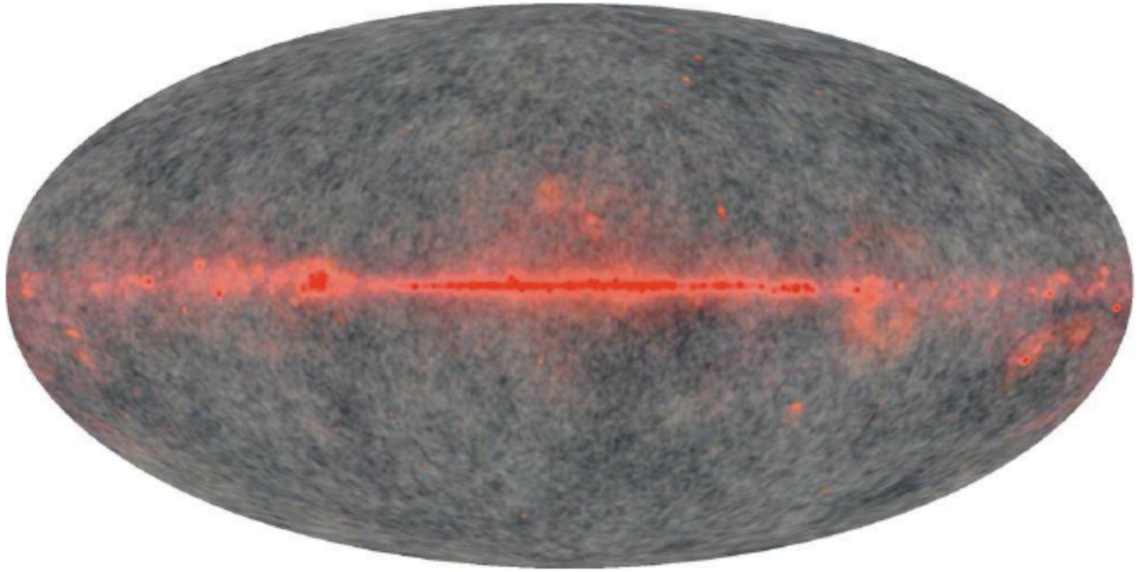


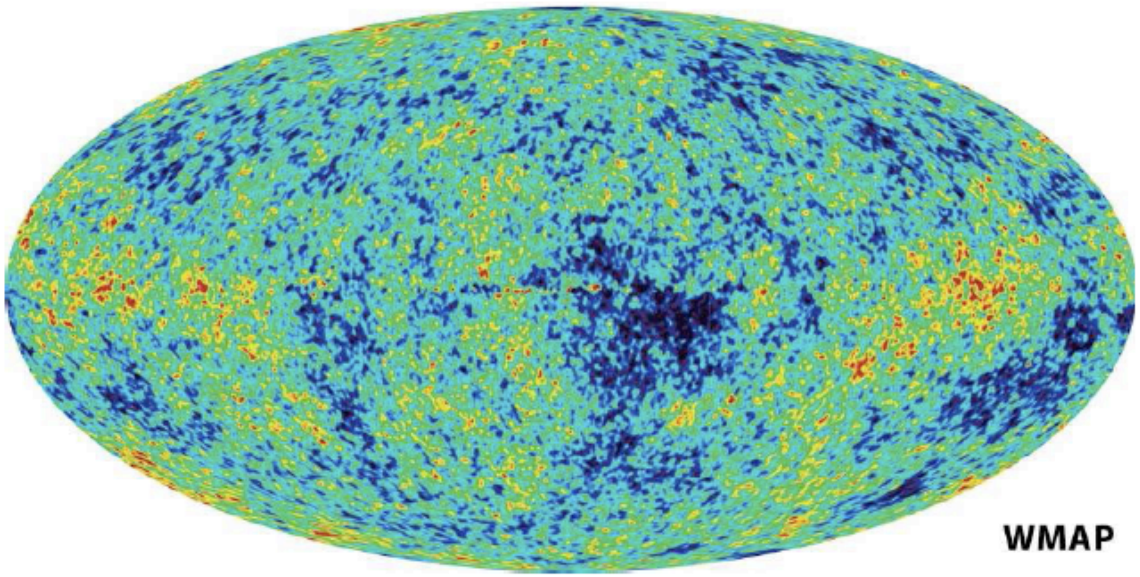
Background

Dipole

Milky Way and fluctuations

After substruction of monopole and dipole components

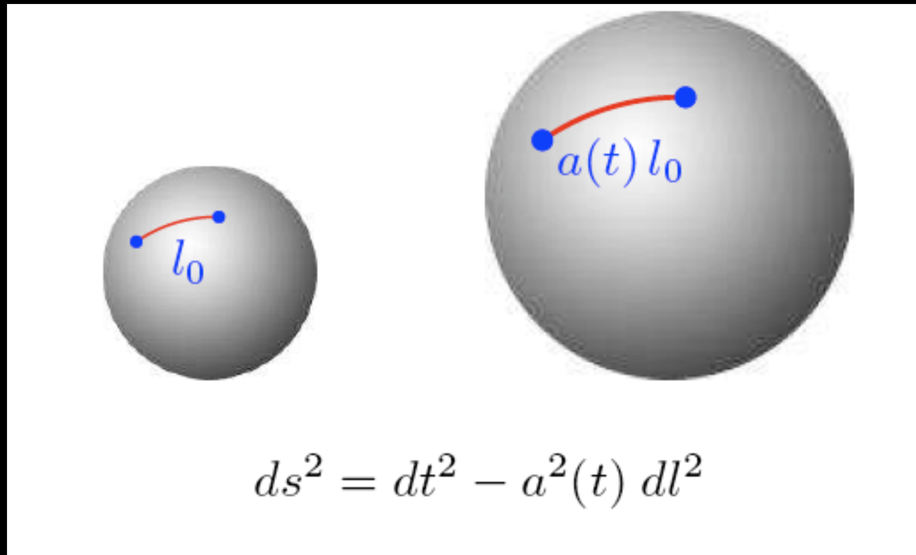




The Cosmological Principle:

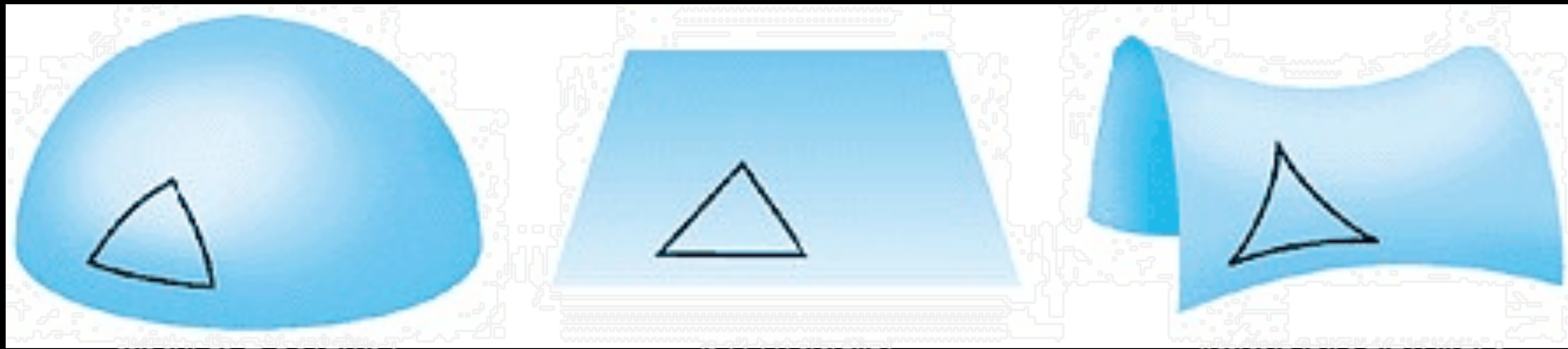
The Universe is
homogeneous and isotropic
(ON LARGE SCALES)

The Universe is homogeneous and isotropic: Friedmann-Robertson-Walker metric



$$\boxed{dl^2 = \frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)}$$

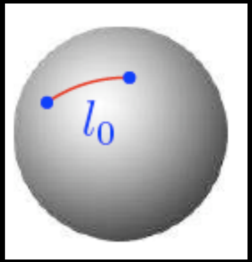
The geometry of space



$k = 1$ | $k = 0$ | $k = -1$

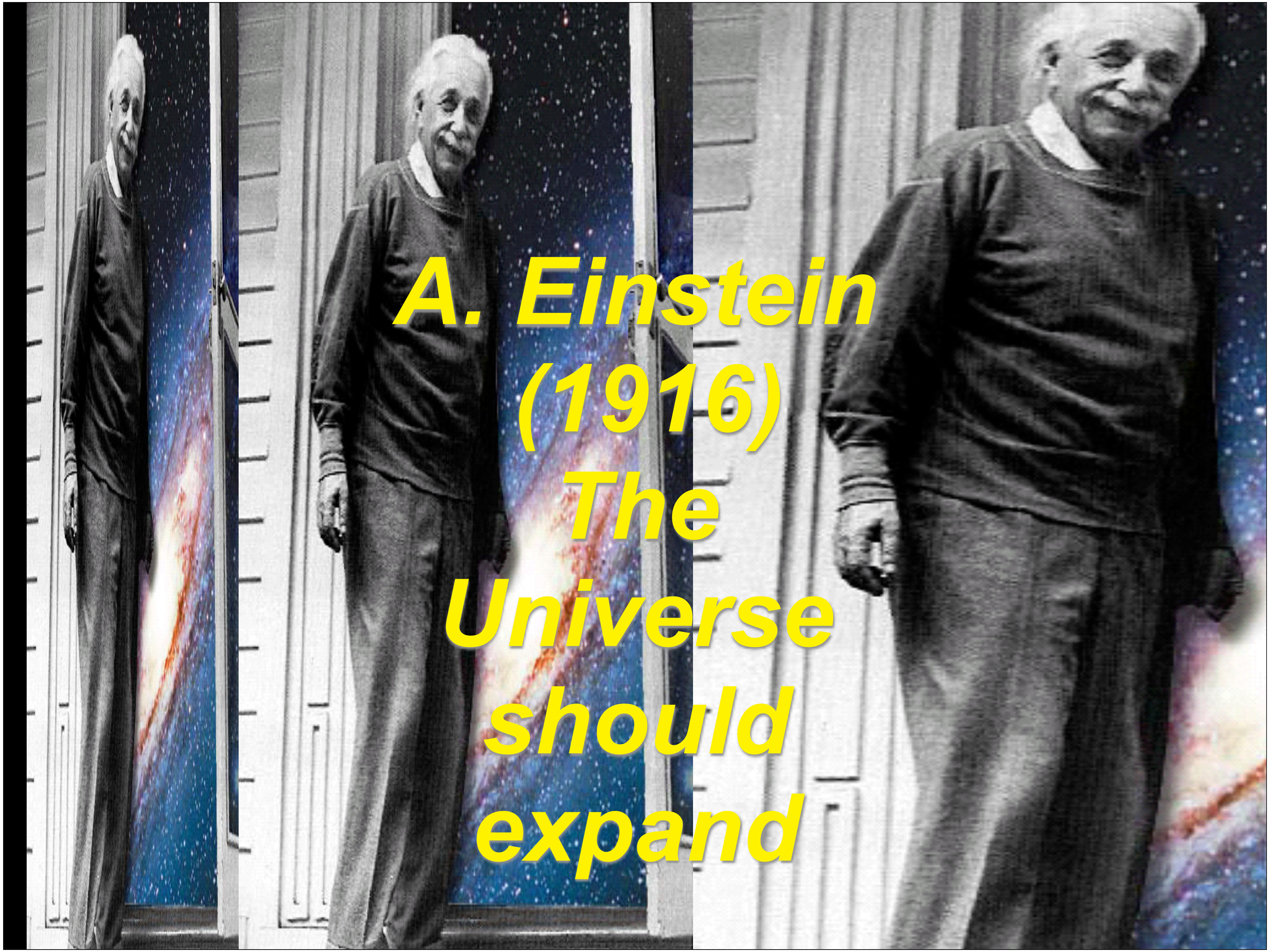
Sphere | Plane | Hyperboloid

Example: geometry of a sphere

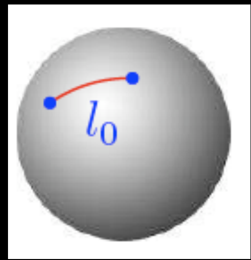


$$x^2 + y^2 + z^2 = 1 \Rightarrow z^2 = 1 - x^2 - y^2$$

$$x = r\cos\theta,\ y = r\sin\theta \Rightarrow\ dl^2 = \frac{dr^2}{1 - r^2} + r^2 d\theta^2$$


A. Einstein
(1916)
The
Universe
should
expand

Implication: Hubble's Law (1929)

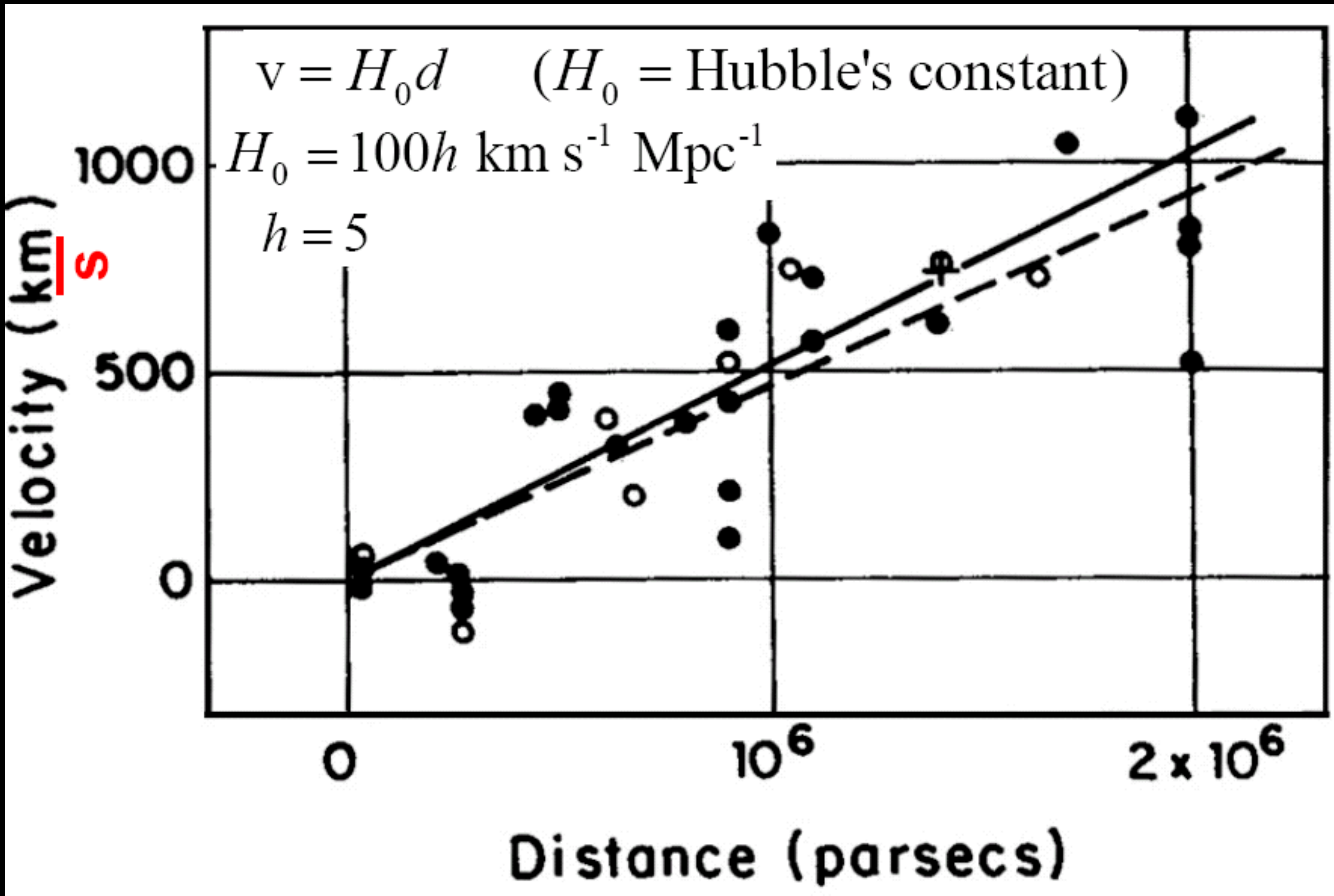


$$\vec{x} = a(t)\vec{l}_0 \Rightarrow \vec{v} = \frac{d\vec{x}}{dt} = H(t)\vec{x}$$

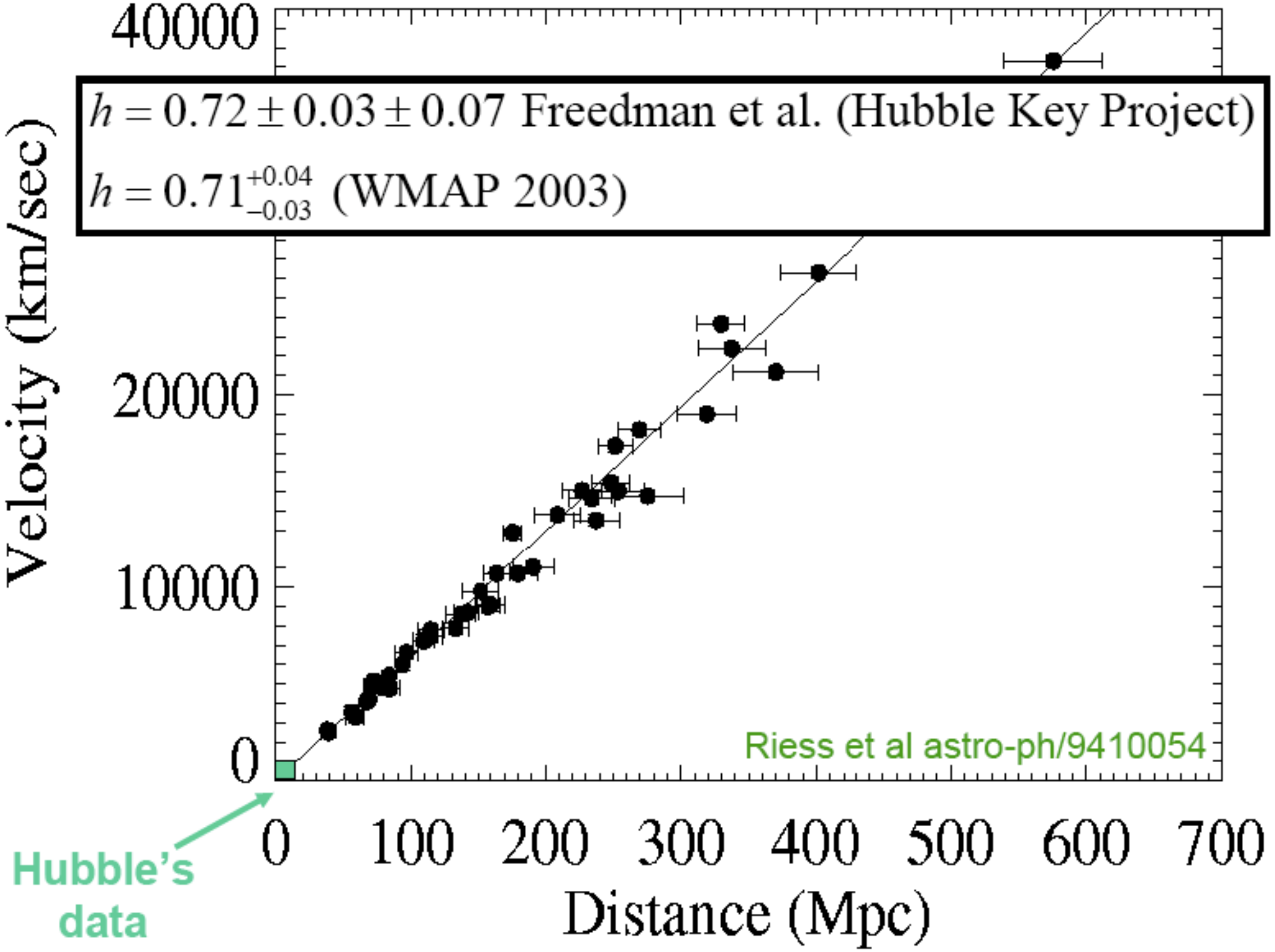
Recession velocities are proportional to distance

$$\boxed{H(t) \equiv \frac{\dot{a}}{a}}$$

Hubble's law

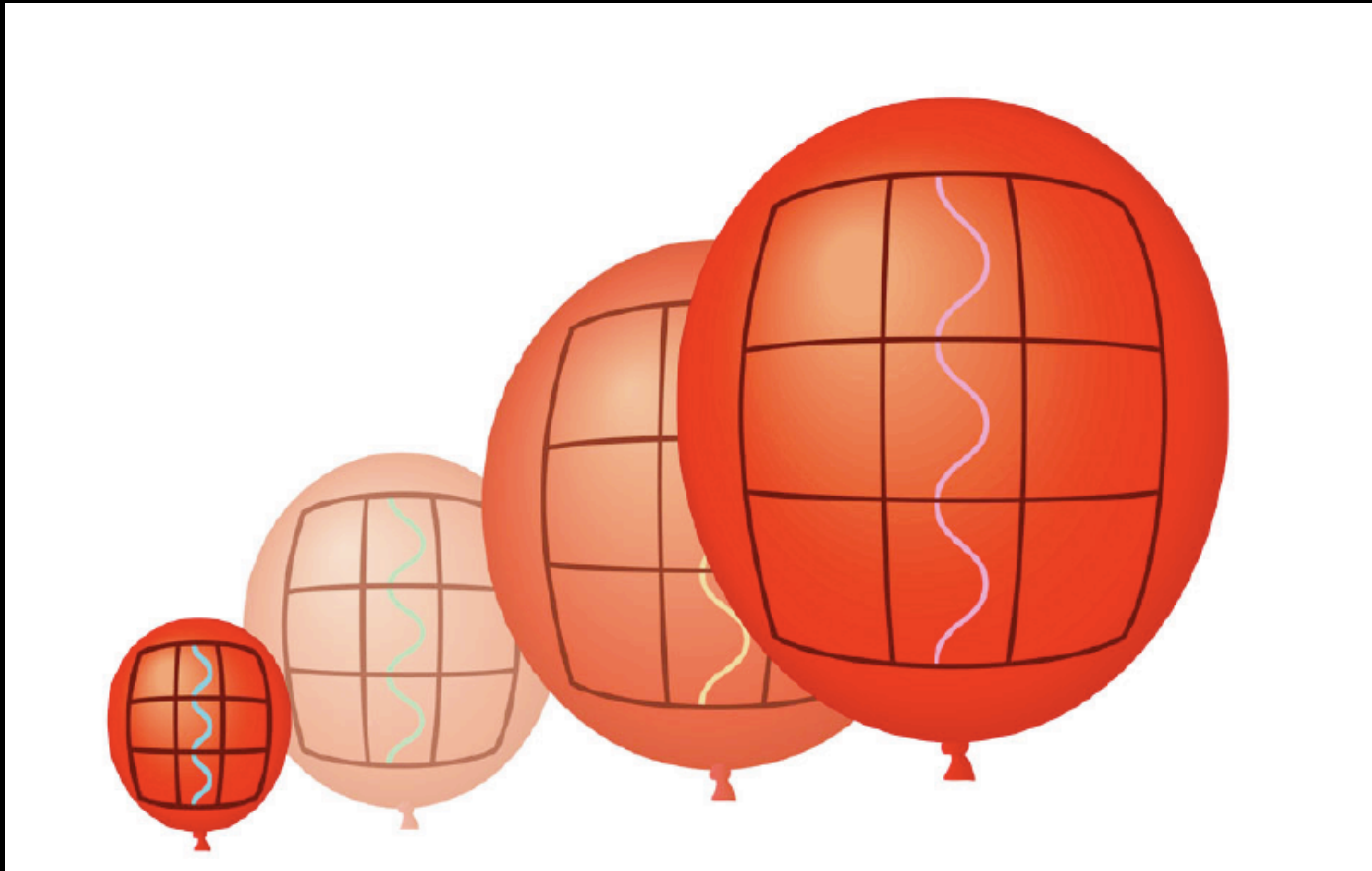


$v = H_0 d \quad (H_0 = \text{Hubble's constant})$
$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$
$h = 5$
Velocity (km/s)
1000
500
0
0
10^6
2×10^6
Distance (parsecs)



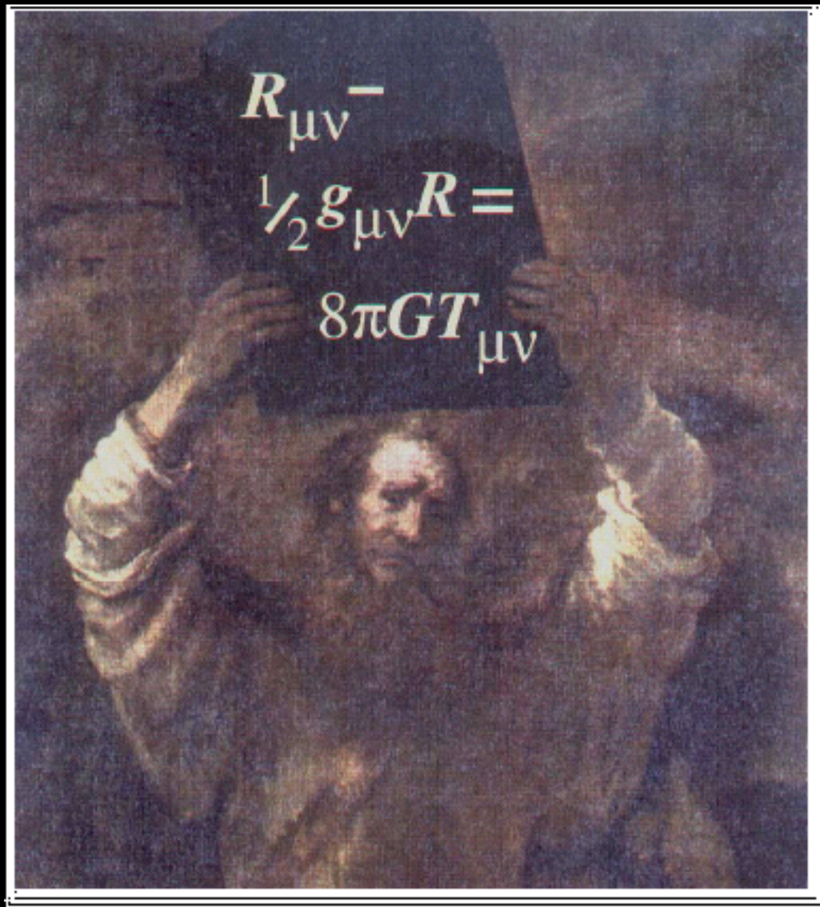
40000
20000
10000
0
Velocity (km/sec)
$h = 0.72 \pm 0.03 \pm 0.07$ Freedman et al. (Hubble Key Project)
$h = 0.71^{+0.04}_{-0.03}$ (WMAP 2003)
Riess et al astro-ph/9410054
Hubble's data
0 100 200 300 400 500 600 700
Distance (Mpc)

It is space which is expanding



How is the Universe expanding?

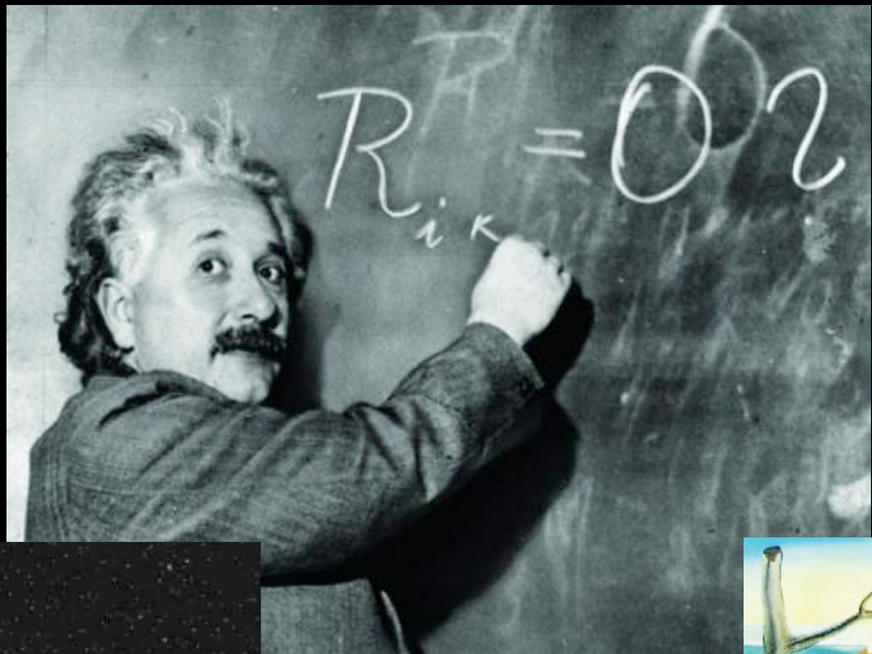
The scale factor in the Friedman-Robertson-Walker metric satisfies Einstein equations

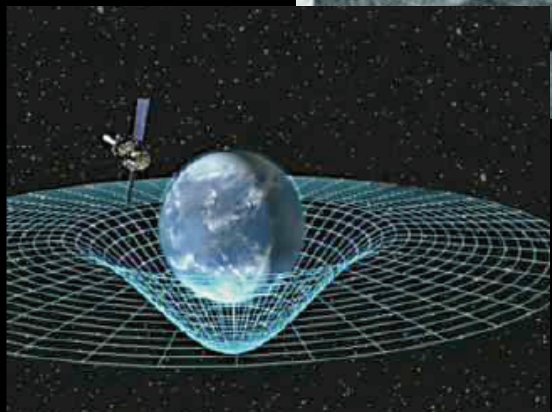


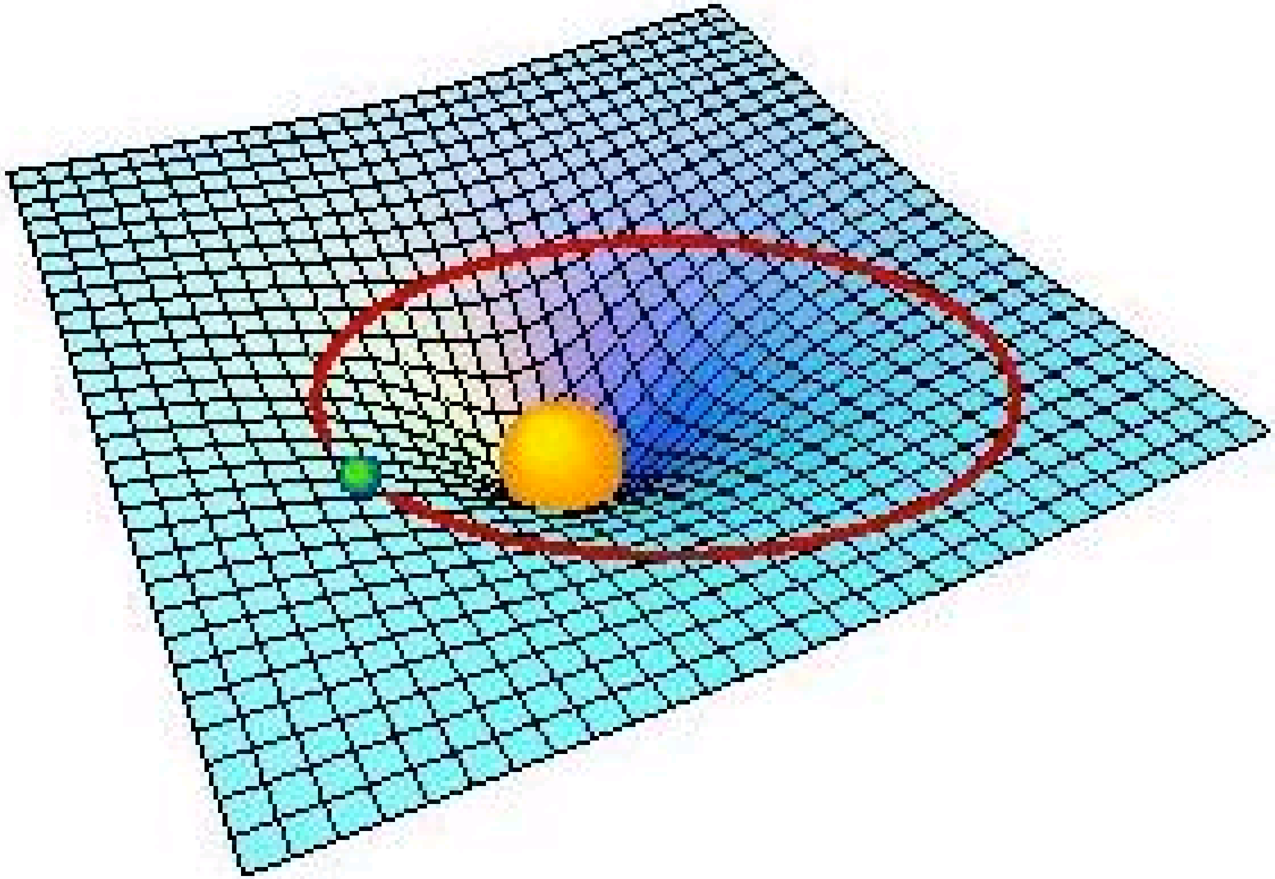
Space-time geometry = energy

Space and time are linked (1905)

Space and time are dynamical (1915)







The Cosmological Principle imposes that the energy momentum tensor is of the form

$$T^{\mu}{}_{\nu} = \mathrm{Diag}\,(\rho, -P, -P, -P)$$

$\rho =$ Energy density $\quad P =$ Pressure

Einstein equations take the form

$$\boxed{\begin{aligned} H^2 &= \frac{8\pi G_N}{3}\,\rho - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3}\,(\rho + 3P) \end{aligned}}$$

Energy momentum conservation takes the form

$$\dot{\rho} + 3H(\rho + P) = 0$$

Physics behind:

Take a test particle of unit mass immersed in a pressureless fluid of given energy density

$$r = ar_0, \;\; M = \frac{4\pi}{3}\rho r^3$$

$$\boxed{\frac{1}{2}\dot{r}^2 - \frac{G_N M}{r} = -\frac{kr_0^2}{2}}$$

Energy conservation of a test particle:
the value of the binding energy tells if the Universe will recollapse or expand for ever

The Golden Rule of the expansion

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

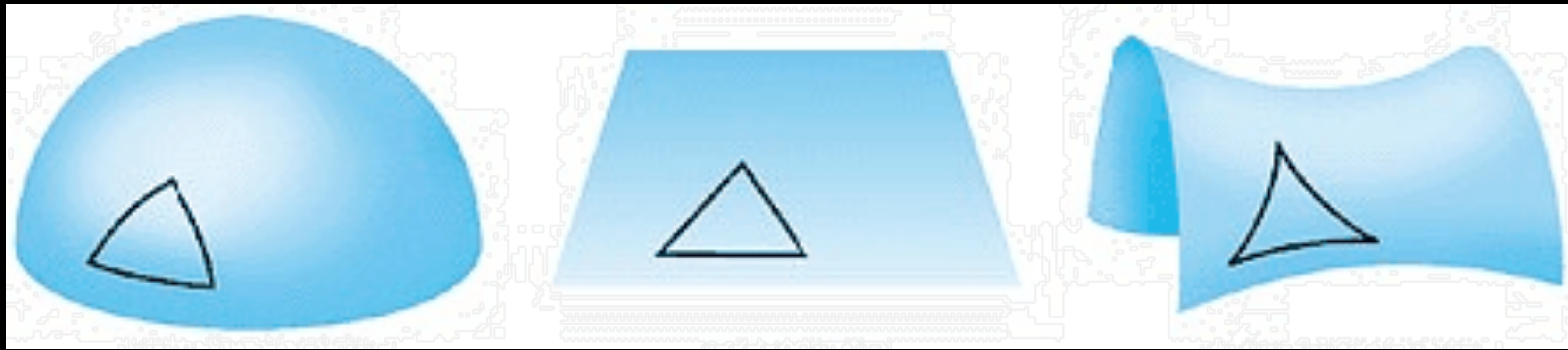
is equivalent to

$$\Omega - 1 = \frac{k}{a^2 H^2}$$

$$\Omega = \frac{\rho}{\rho_c}, \ \rho_c = \frac{3H^2}{8\pi G_N}$$

$$\text{Today } \rho_c \simeq 10^4 \, \text{eV cm}^{-3}$$

The Geometry of space



$\Omega > 1$ $\Omega = 1$ $\Omega < 1$

A measurement of the total energy density of the Universe implies a measurement of the geometry of space

Various types of fluids:

$$\boxed{\text{Suppose } P = w\rho \quad \Rightarrow \rho \propto a^{-3(1+w)}}$$

Relativistic $\quad w = 1/3 \Rightarrow \rho_R \propto a^{-4} = a^{-3} \times a^{-1}$

Nonrelativistic $\quad w \simeq 0 \Rightarrow \rho_{NR} \propto a^{-3}$

Cosmological constant $\quad w \simeq -1 \Rightarrow \rho \propto a^0$

Curvature term $\quad w = -1/3 \Rightarrow \rho \propto a^{-2}$

Time evolution

$$\boxed{\begin{aligned} a &\propto t^{\frac{2}{3}(1+w)} \\ H &= \frac{2}{3}(1+w)\frac{1}{t} \end{aligned}}$$

RD

$$a \propto t^{\frac{1}{2}}$$

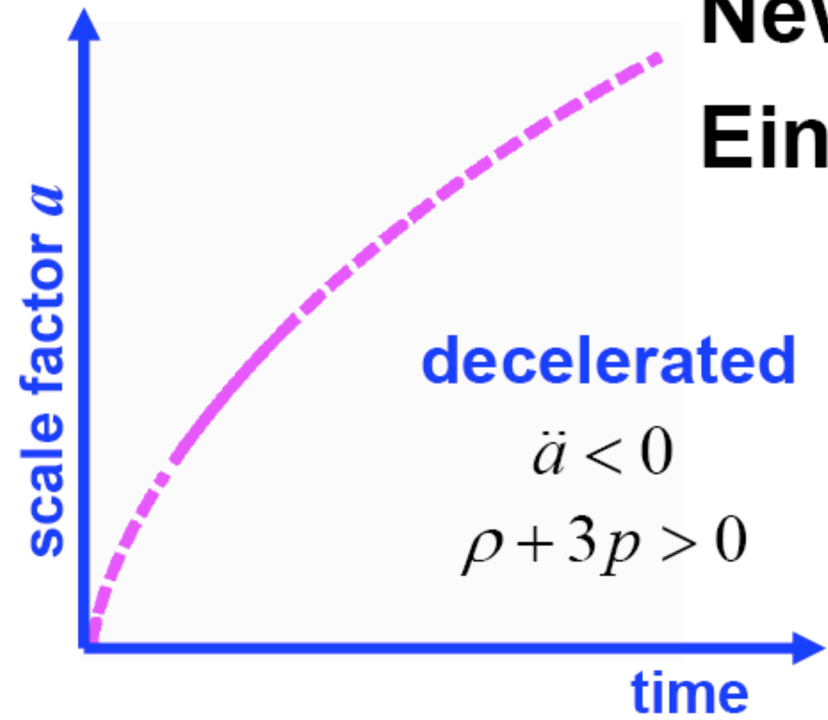
MD

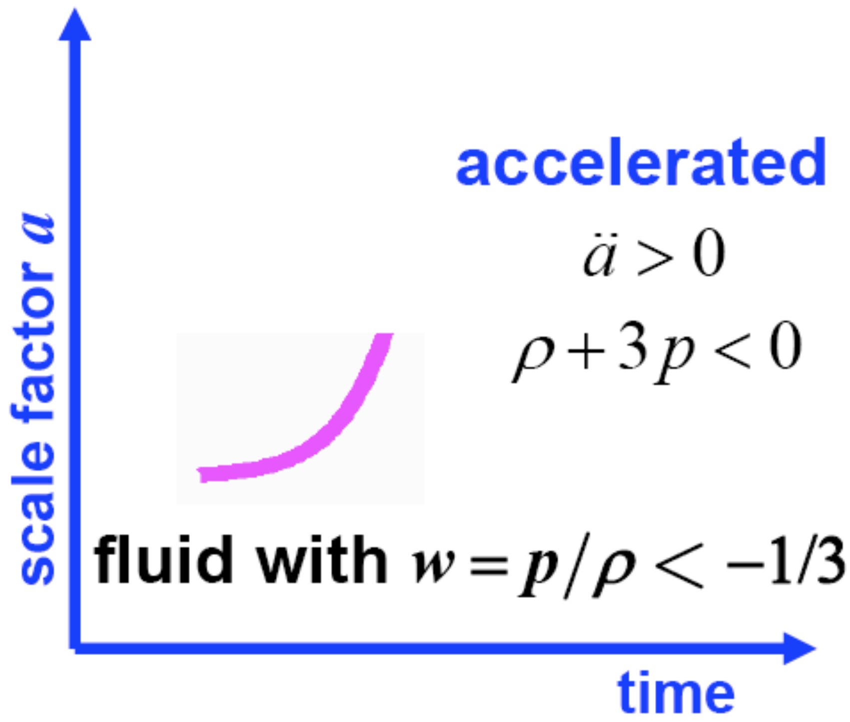
$$a \propto t^{\frac{2}{3}}$$

The expansion is decelerated

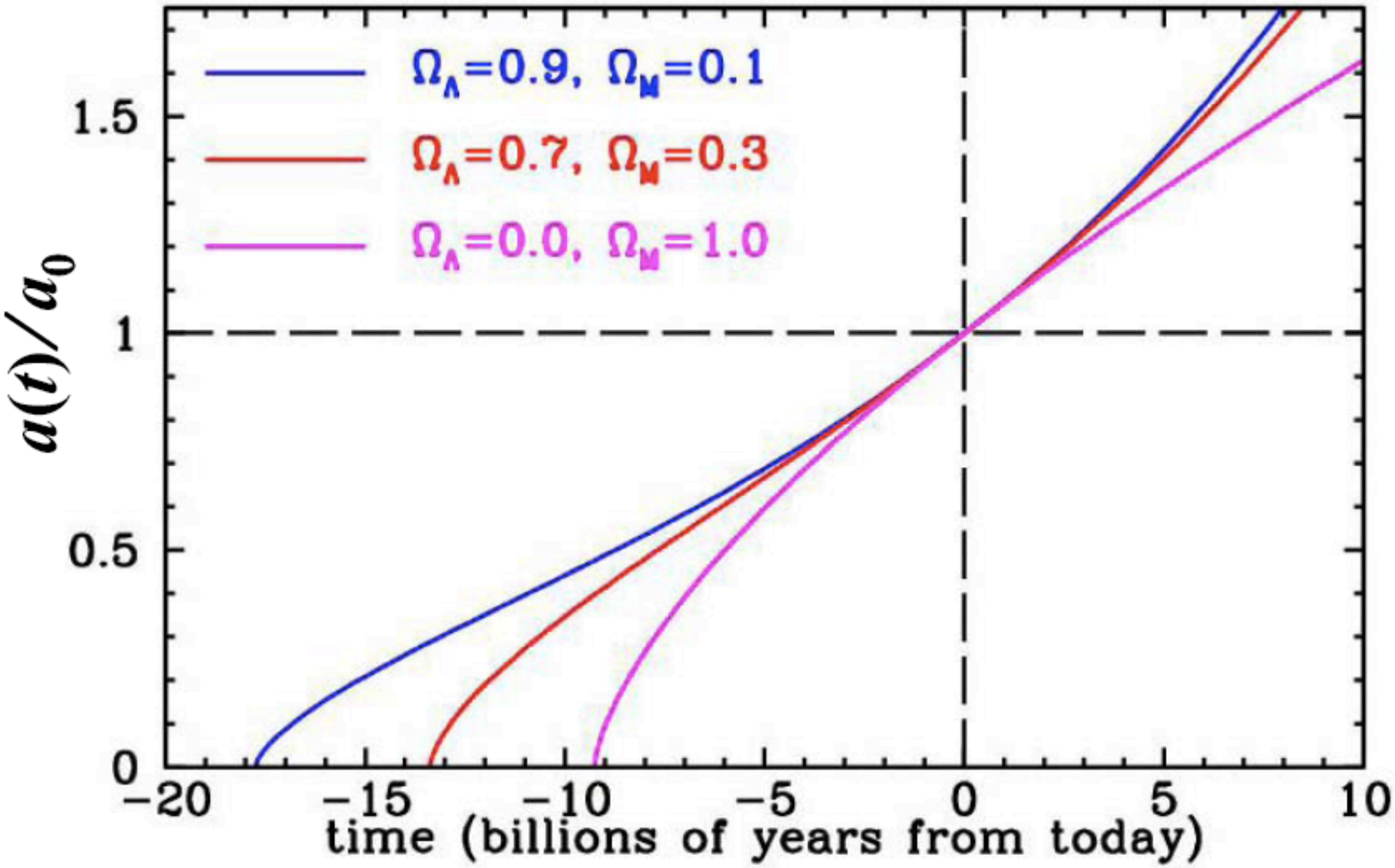
Newton
Einstein

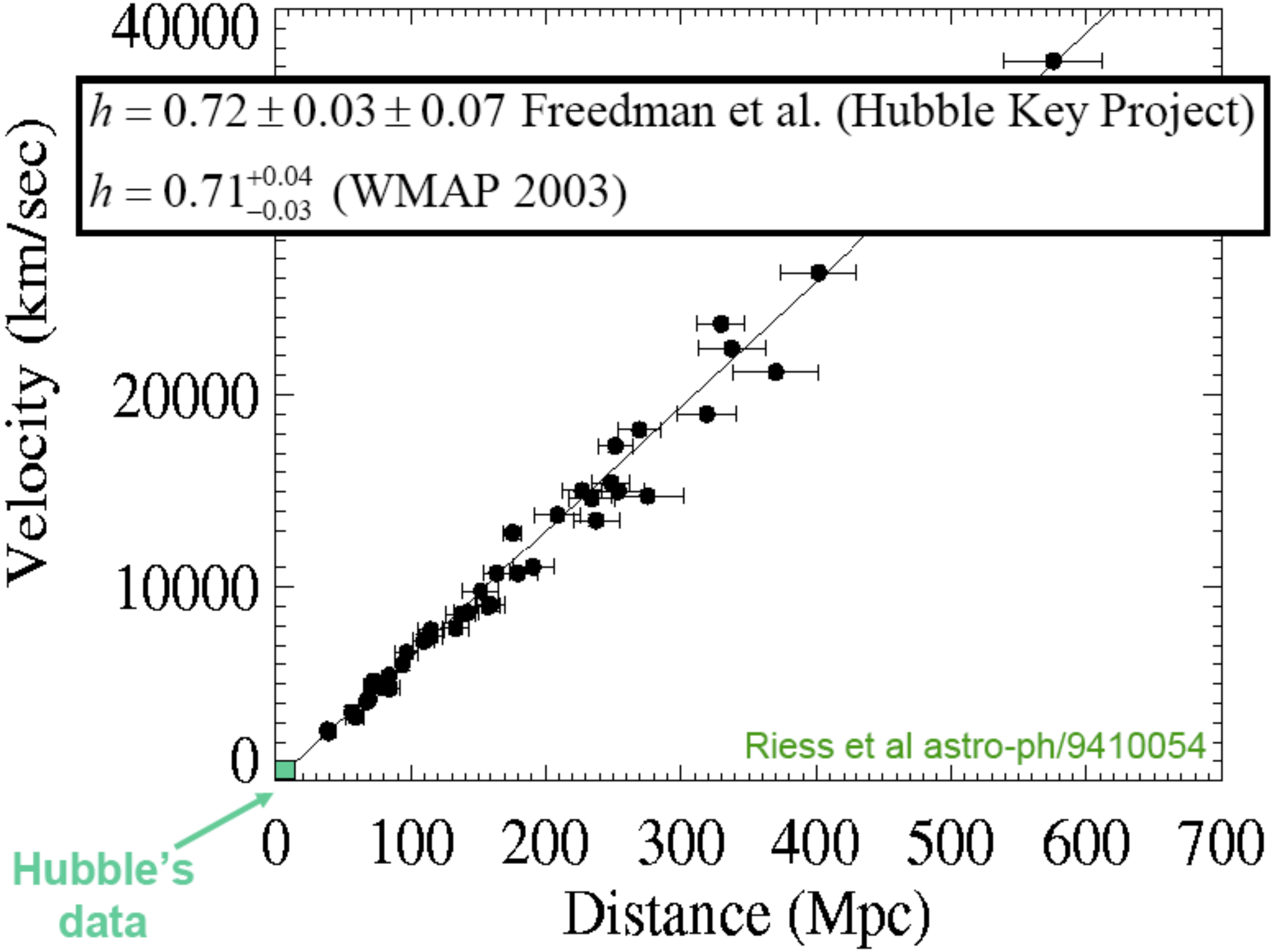
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)$$





$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_{M0}\left(\frac{a_0}{a}\right)^3 + \rho_\Lambda\right]$$

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$h = 0.71^{+0.04}_{-0.03}$ (WMAP 2003)
Velocity (km/sec)
40000
20000
10000
0
Riess et al astro-ph/9410054
0
100
200
300
400
500
600
700
Distance (Mpc)
Hubble's data

Distance-Redshift Relation

$$F = \frac{L}{4\pi d_L^2}$$ defines luminosity distance, know L, measure F

$4\pi d_L^2$ area of ${}^2S$ centered on source at time of detection

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \Rightarrow \text{area} = 4\pi a_0^2 r^2$$

$$1 + z = \frac{a_0}{a}$$

Energy redshifted: $(1+z)$

Time interval redshifted: $(1+z)$

Flux redshifted: $(1+z)^2$

$$\boxed{d_L^2 = a_0^2 r^2 (1+z)^2}$$

Distance-Redshift Relation

Light travels on geodesics

$$ds^2 = 0 \Rightarrow \int \frac{dr}{\sqrt{1-kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2}$$

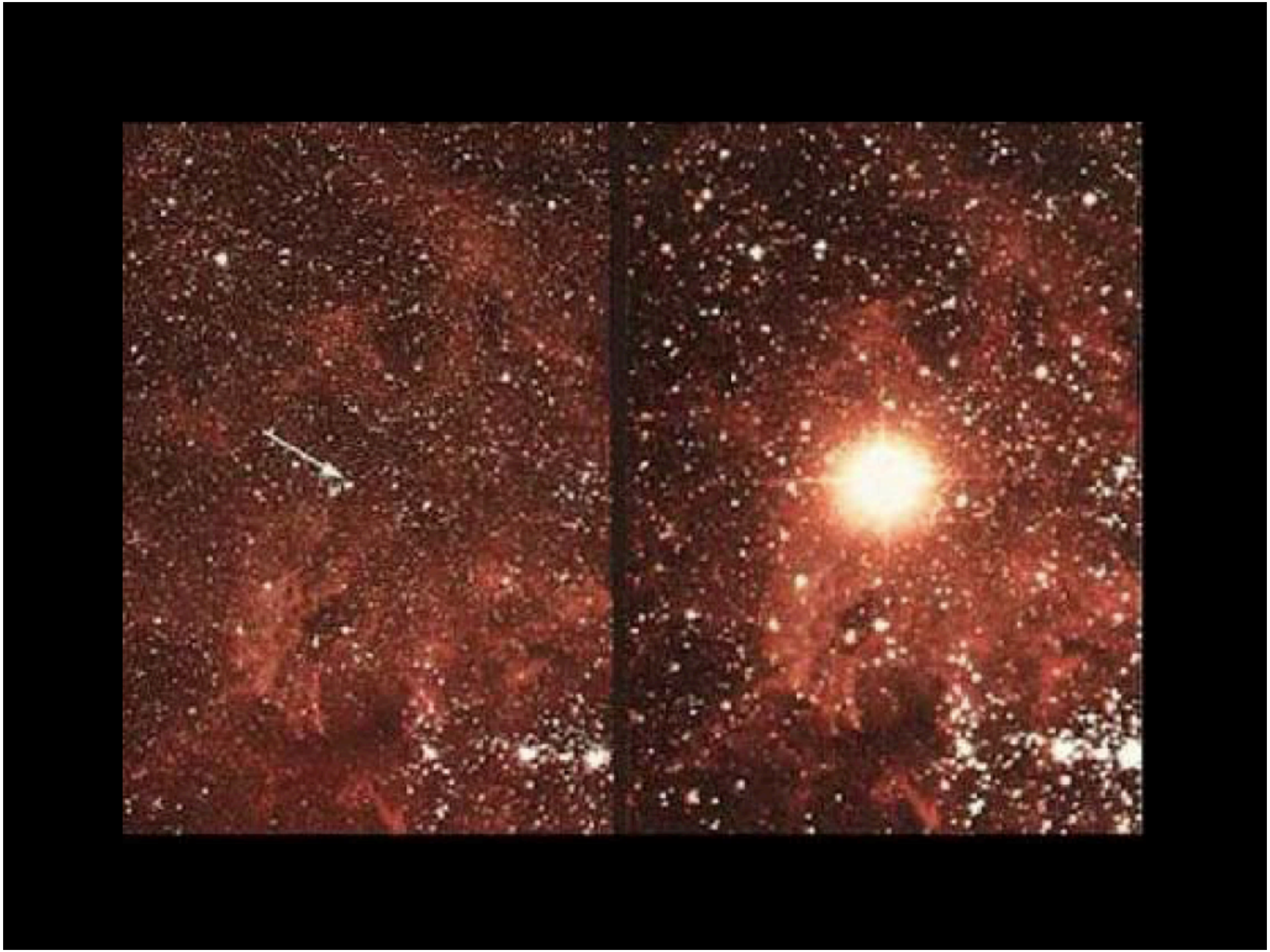
$$\boxed{\int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a^{-1}(t_0)H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_0)(1+z')^2 + \Omega_M(1+z')^3 + \Omega_w(1+z')^{3(1+w)} + \ldots}}}$$

<u>Program:</u>

- measure d_L (via $d_L{}^2 = L/4\pi F$) and z
- input a model cosmology (Ω_i) and calculate $a_0\, r$
- compare to data
- need bright "standard candle"

Distance-Redshift Relation

$$
\begin{aligned}
d_L(z) &= \frac{1}{H_0}\left[z + (1-q_0)\frac{z^2}{2} + \left(-j_0 + 3q_0^2 - 1 - \frac{k}{a_0^2 H_0^2}\right)\frac{z^3}{6} + \mathcal{O}(z^4)\right] \\
q &\equiv -(\ddot{a}/a)/H^2, \text{ jerk } j \equiv (\dddot{a}/a)/H^3
\end{aligned}
$$



Monastic Chronicles re: Supernova 1006:

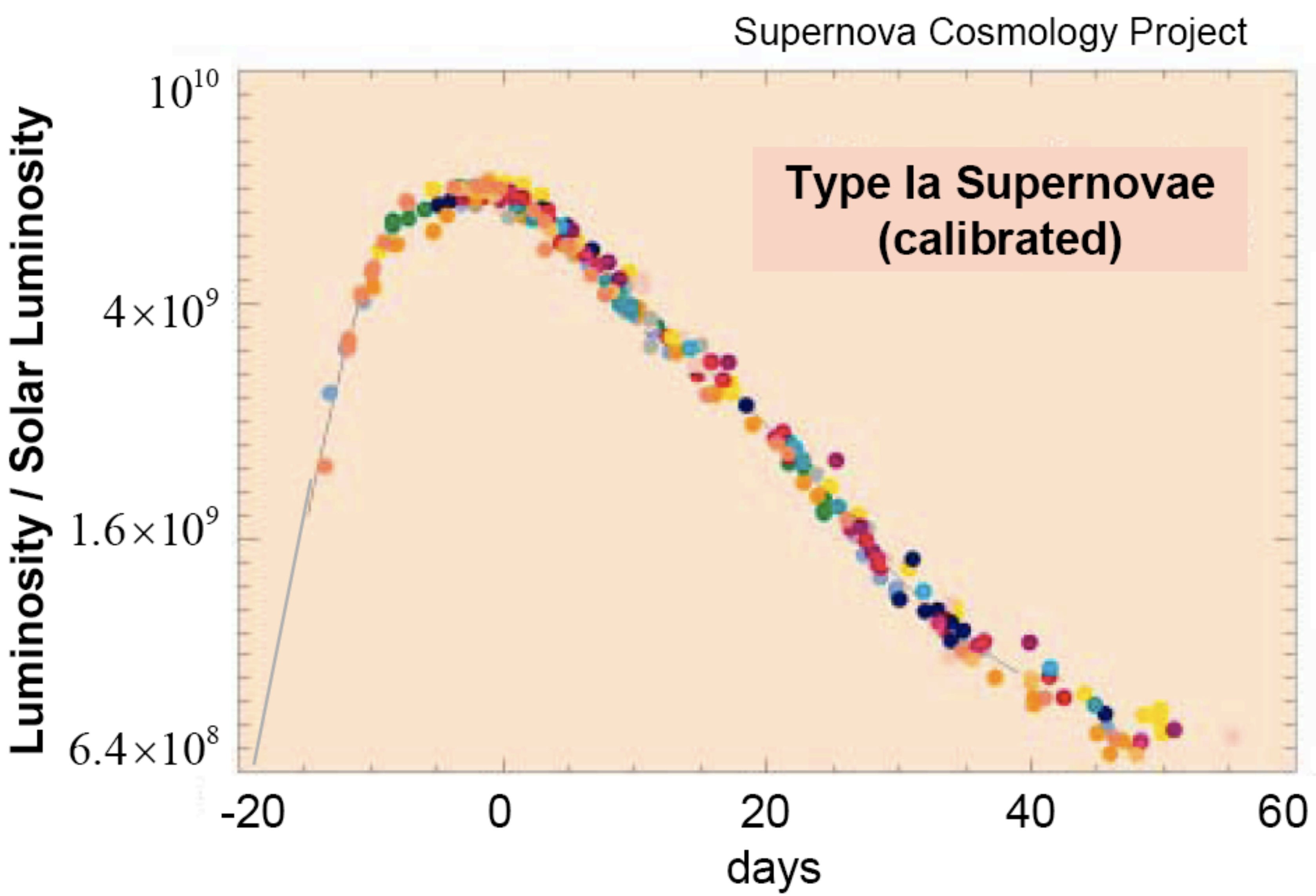
"in 1006 there was a very great famine and a comet appeared for a long time …."

"at the same time a comet, which always announces human shame, appeared in the southern regions, which was followed by a great pestilence…"

"three years after the king was raised to the throne, a comet with a horrible appearance was seen in the southern part of the sky, emitting flames this way and that…"

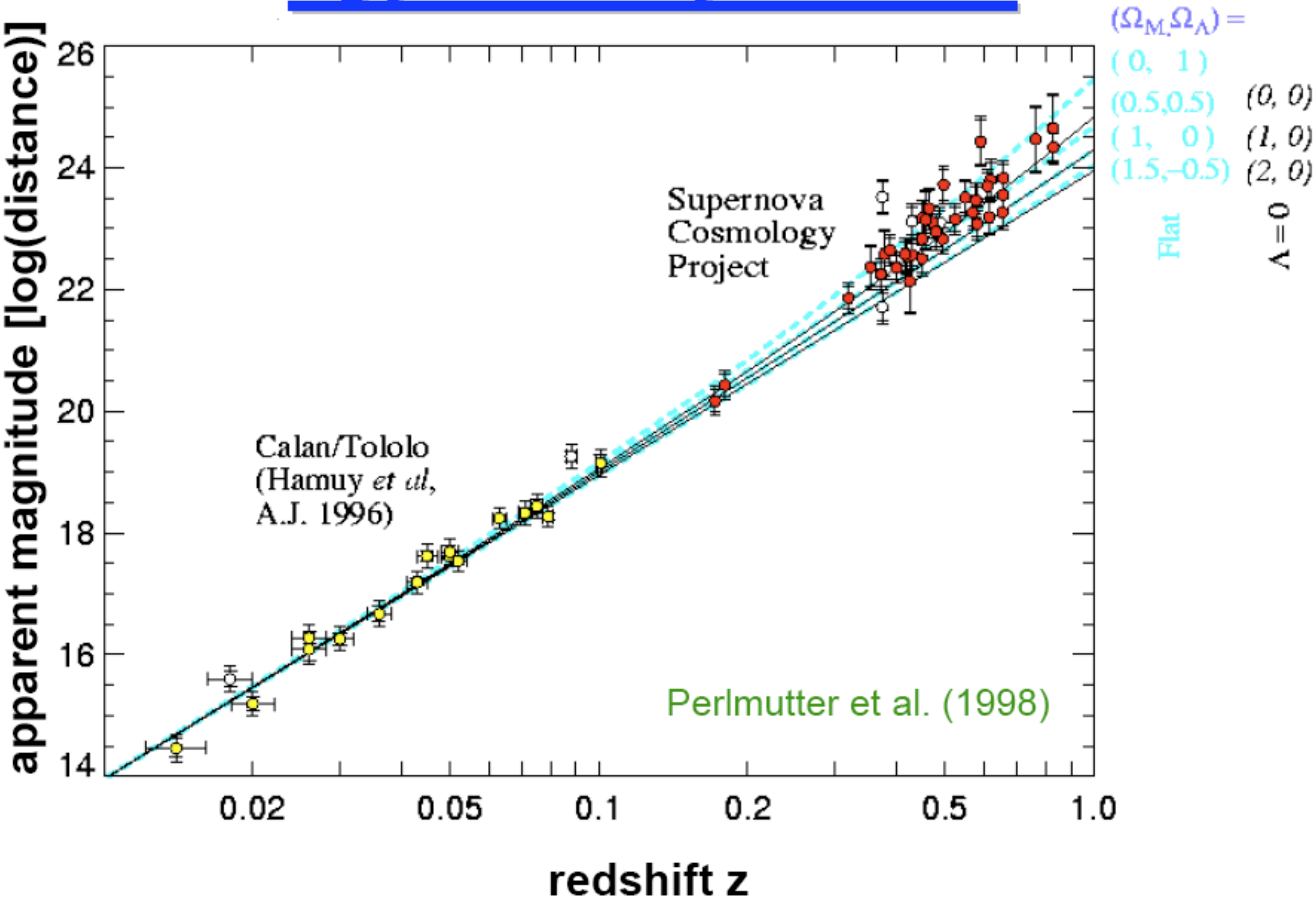
Georg Busch (German painter) in 1572:

"It is a sign that we will be visited by all sorts of calamities such as inclement weather, pestilence, and Frenchmen."



Supernova Cosmology Project
Type Ia Supernovae
(calibrated)
Luminosity / Solar Luminosity
10^{10}
4×10^{9}
1.6×10^{9}
6.4×10^{8}
-20
0
20
40
60
days

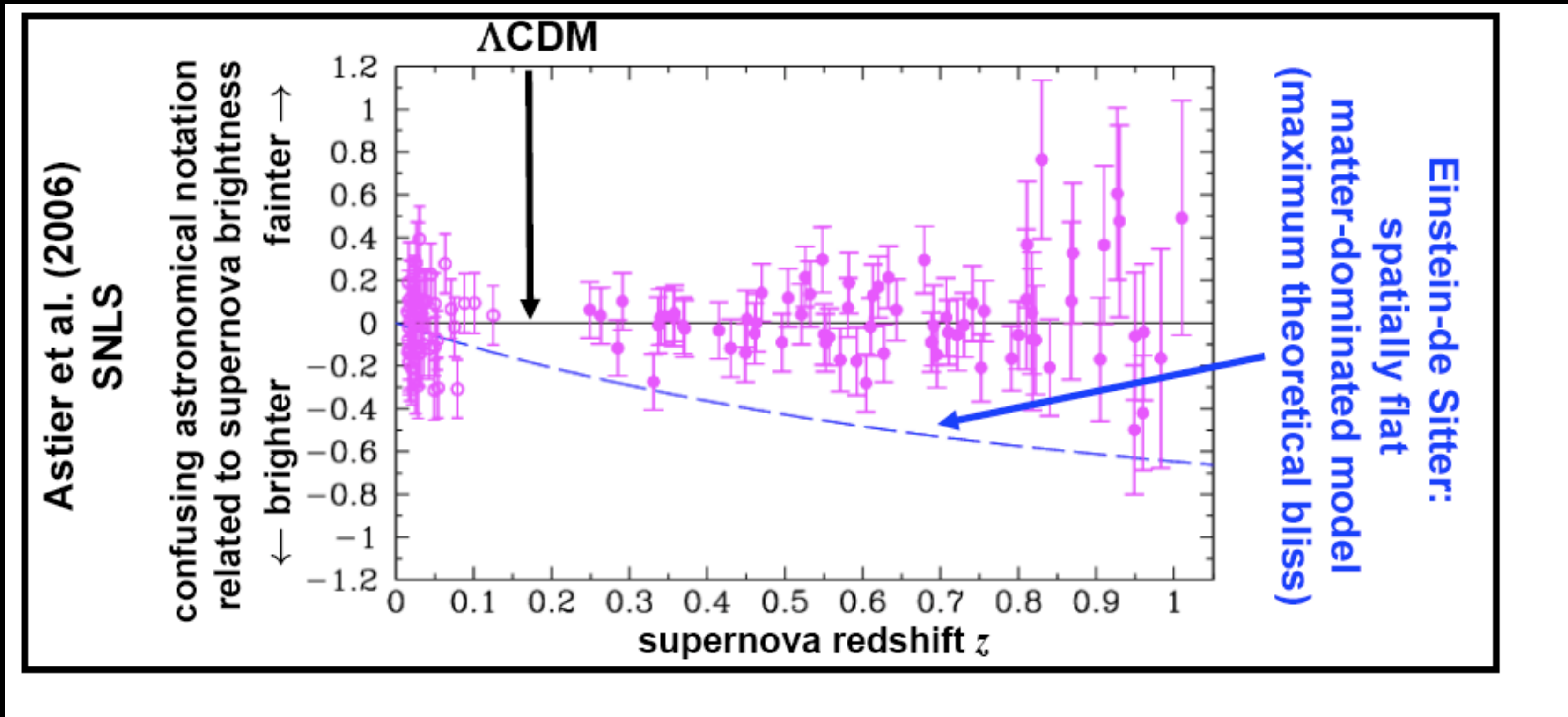
Type Ia supernova

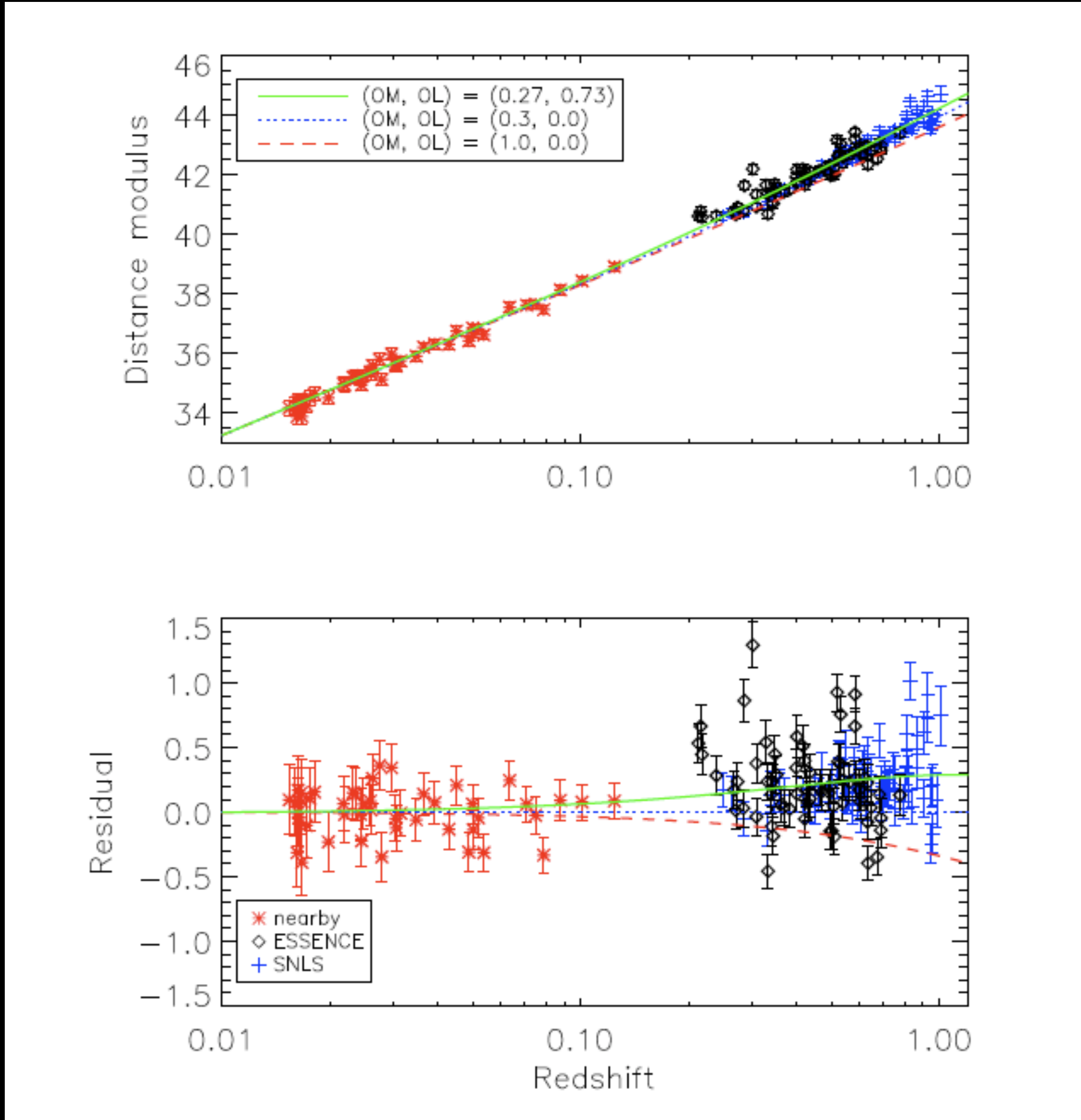


Evidence for acceleration

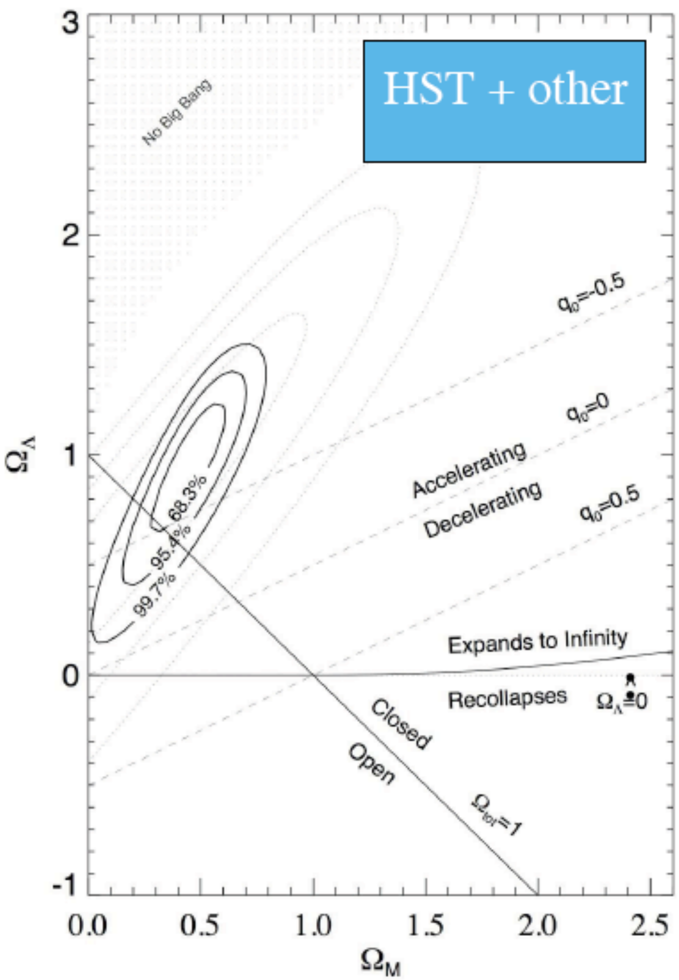
$$d_L(z) = \frac{1}{H_0}\left[z + (1-q_0)\frac{z^2}{2} + \left(-j_0 + 3q_0^2 - 1 - \frac{k}{a_0^2 H_0^2}\right)\frac{z^3}{6} + \mathcal{O}(z^4)\right.$$

$$q \equiv -(\ddot{a}/a)/H^2,\ \text{jerk}\, j \equiv (\dddot{a}/a)/H^3$$





(OM, OL) = (0.27, 0.73)
(OM, OL) = (0.3, 0.0)
(OM, OL) = (1.0, 0.0)
Distance modulus
46
44
42
40
38
36
34
0.01
0.10
1.00
Residual
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
nearby
ESSENCE
SNLS
Redshift

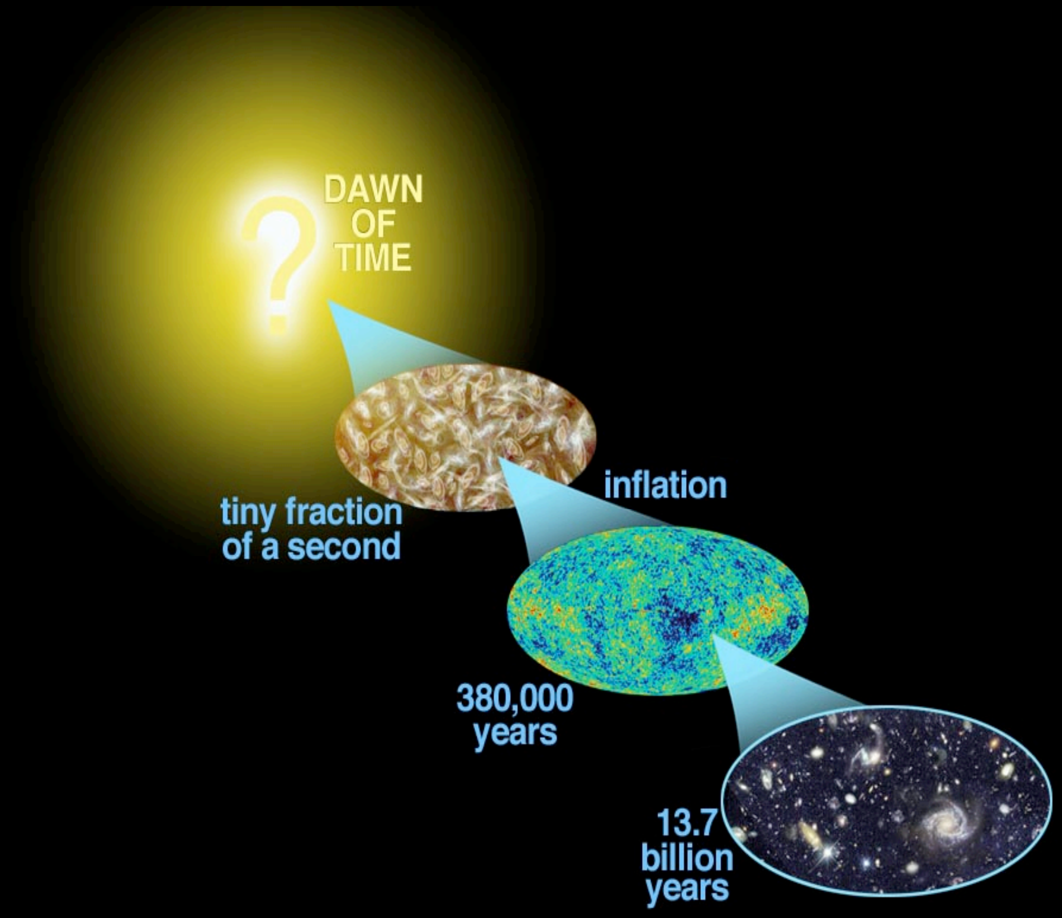


Riess et al. 2004

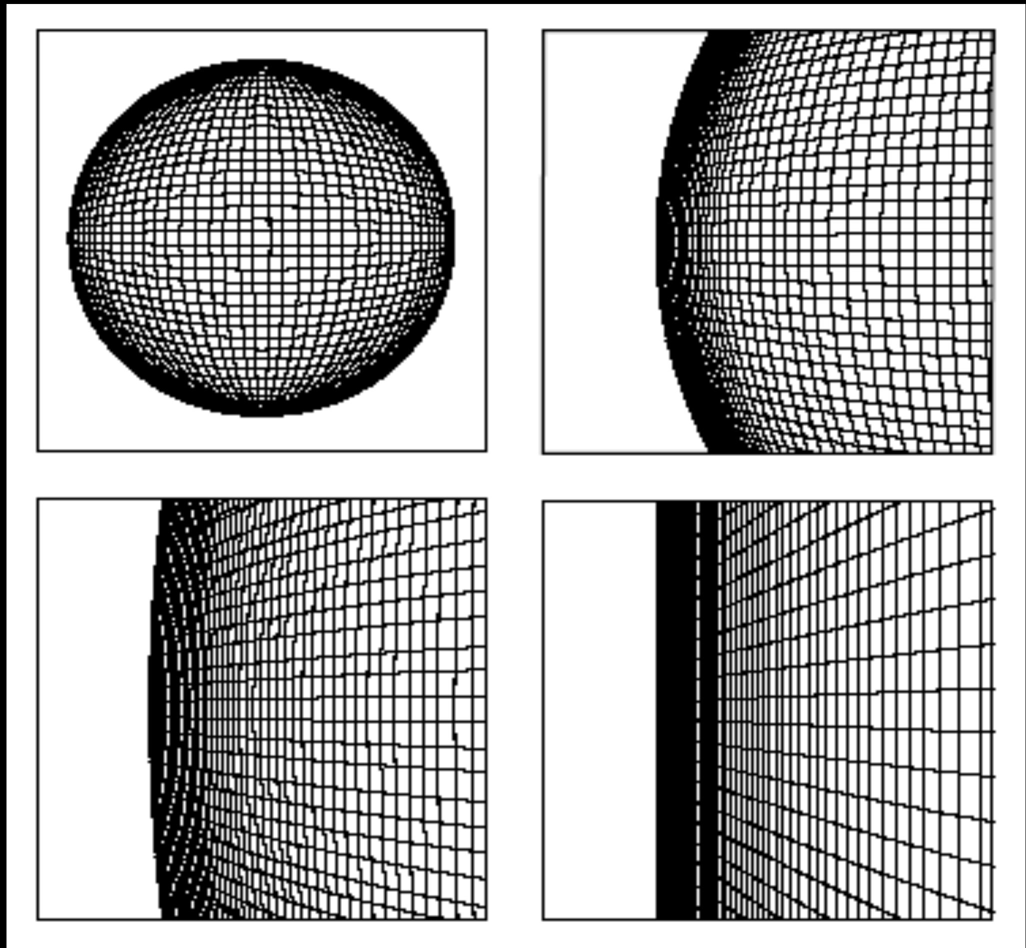
Theoretical prejudice

$$\Omega_0 = 1$$

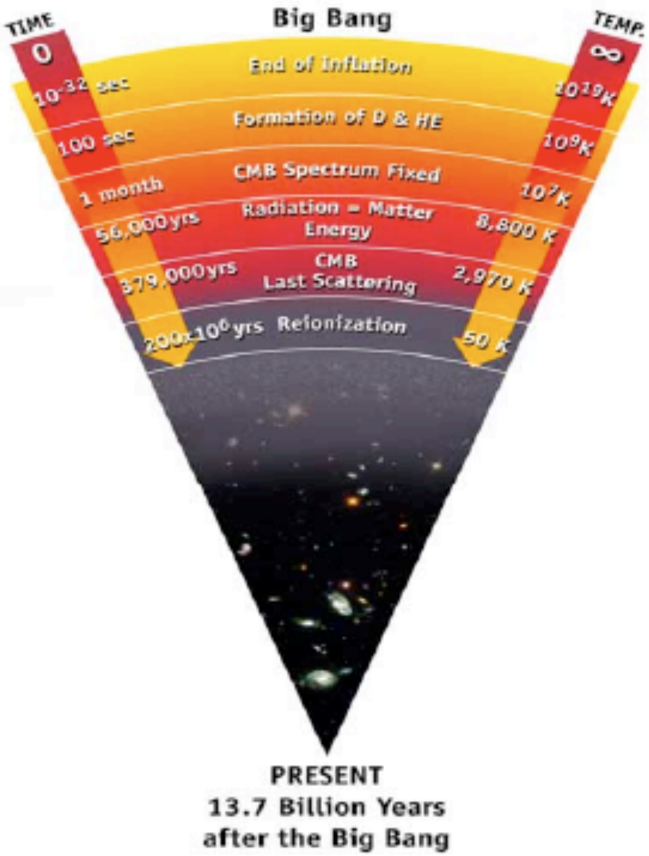
The Inflationary Cosmology



Inflation does NOT change the global structure of space, but LOCALLY it makes it flat



$$\text{IF } N \gg 60 \Rightarrow \Omega_0 = 1 + \mathcal{O}\left(e^{60-N}\right)$$



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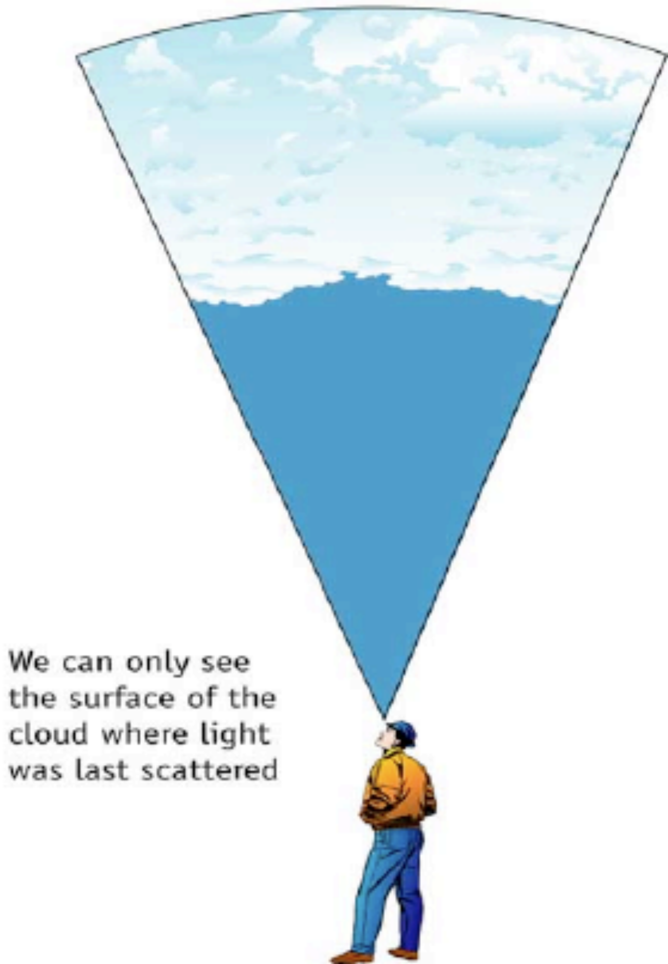
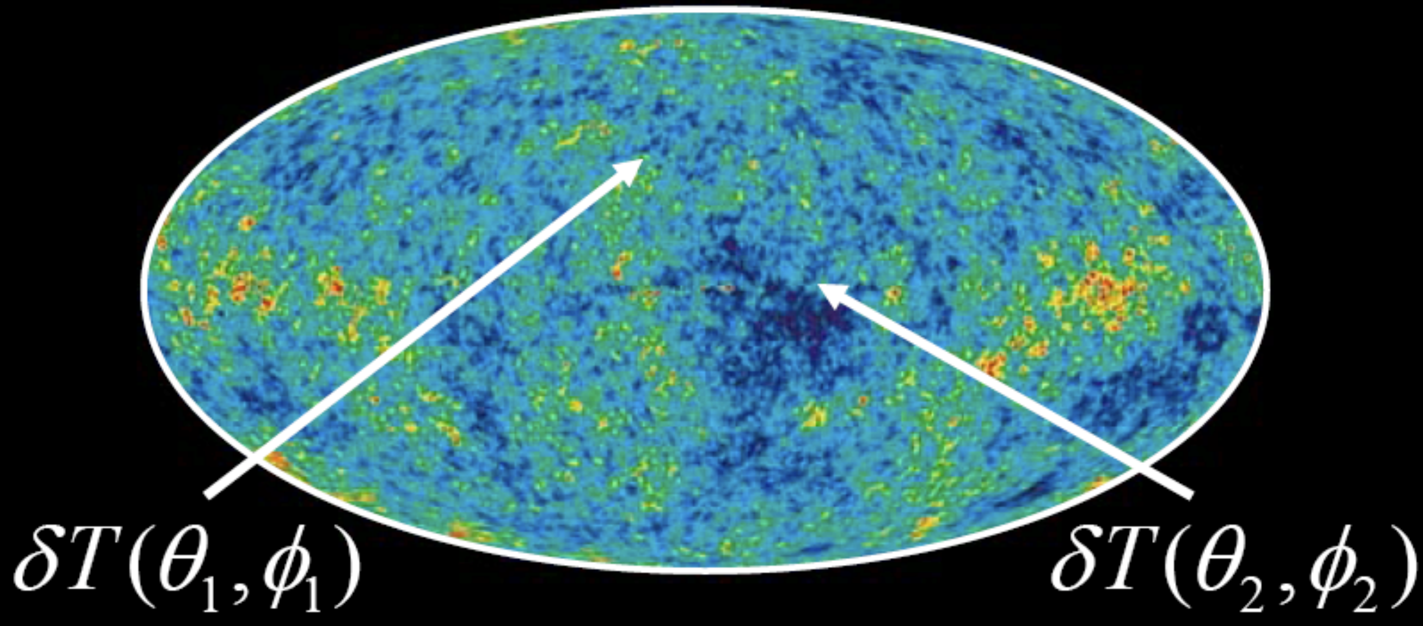


CMB anisotropy



$$\frac{\Delta T}{T}(x_0, \tau_0, \mathbf{n}) = \sum_{\ell m} a_{\ell m}(x_0) Y_{\ell m}(\mathbf{n})$$

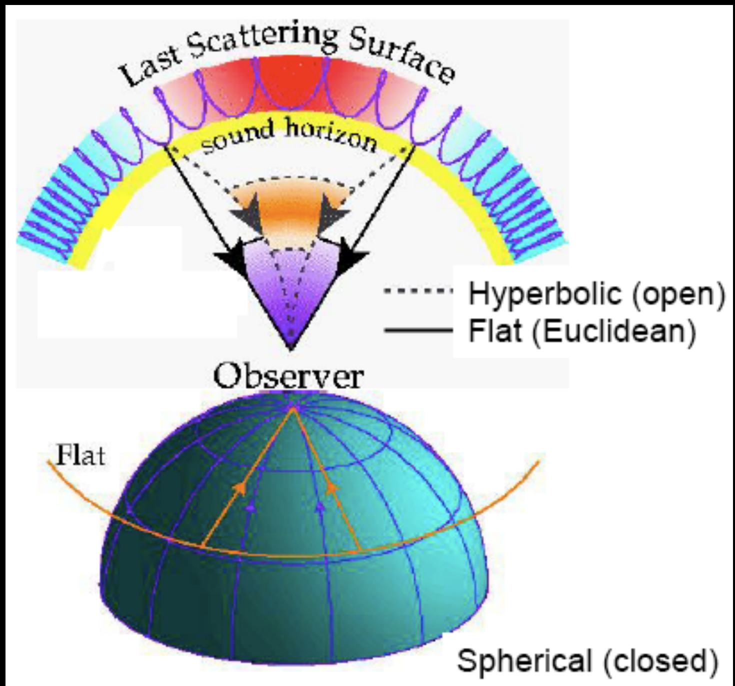
$$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

$$\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle = \sum_{\ell} \frac{(2\ell + 1)}{4\pi} C_\ell P_\ell(\mathbf{n} \cdot \mathbf{n}')$$

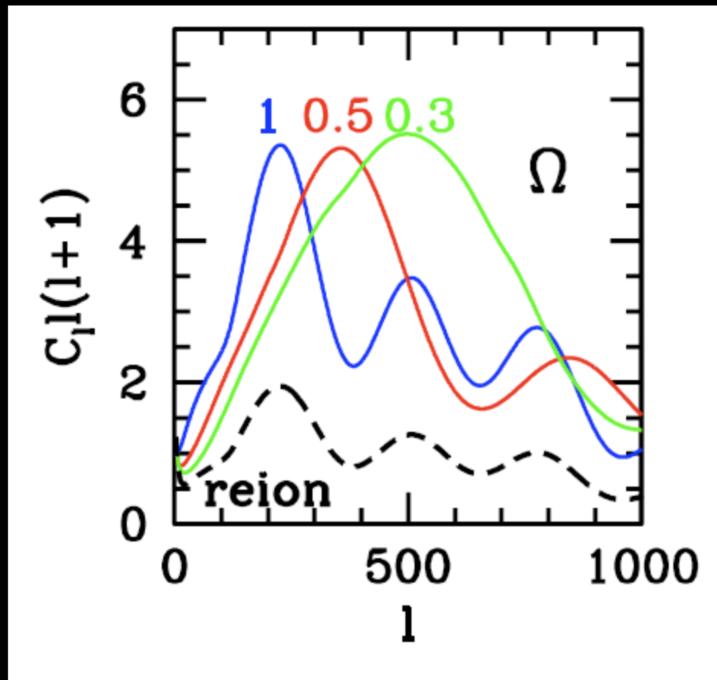
(ensemble averages)

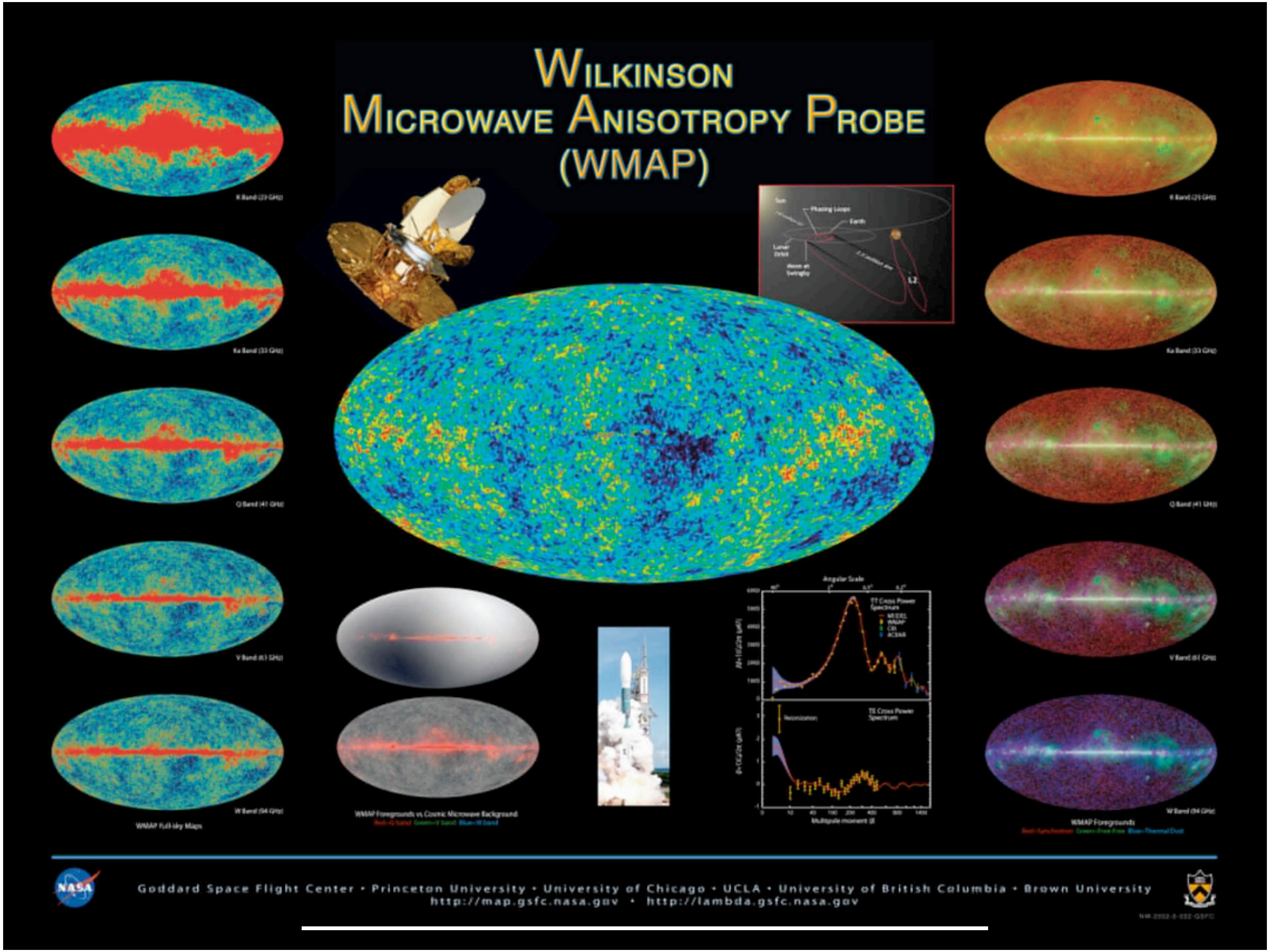
Position of the first peak

Modes caught in the extrema of their oscillation at recombination will have enhanced fluctuations, yielding a fundemental scale or frequency related to the Universe sound horizon

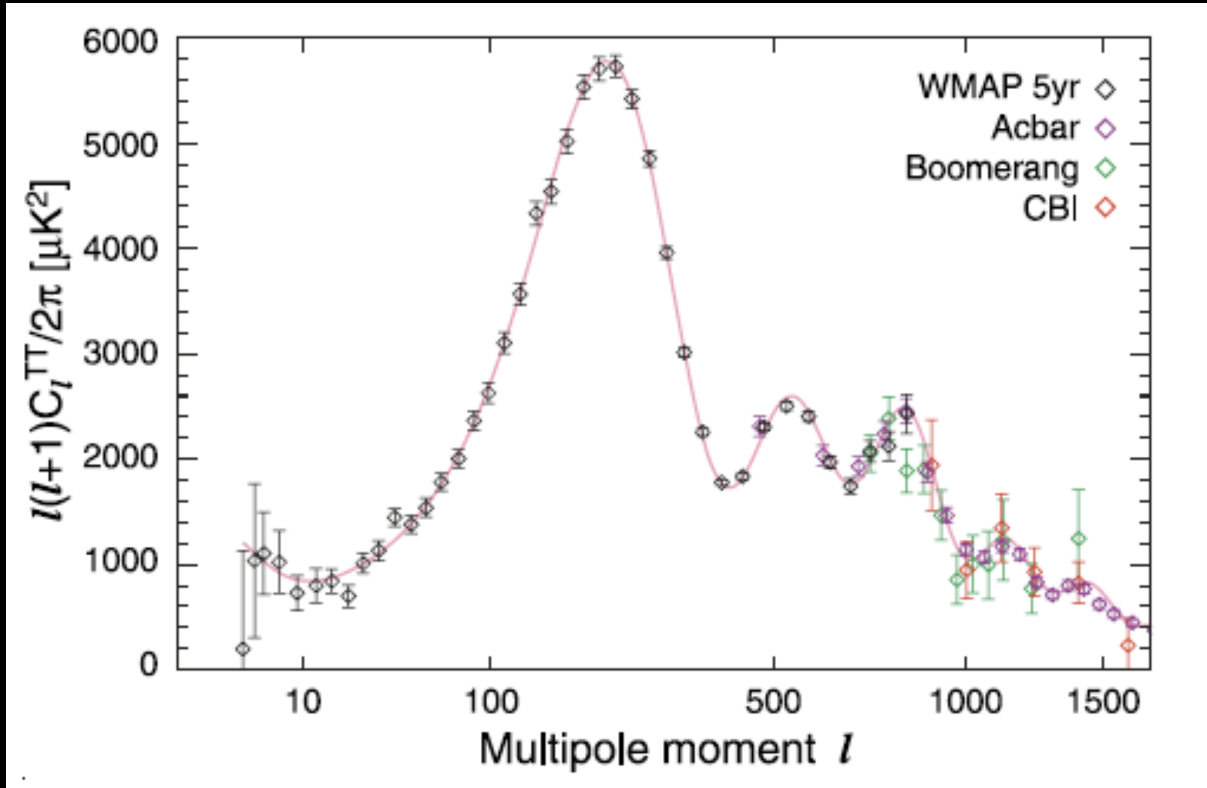


$$\ell_{\text{first peak}} \simeq \frac{220}{\sqrt{\Omega_{\text{tot}}}}$$





Wilkinson
Microwave Anisotropy Probe
(WMAP)
Angular Scale
TT Cross Power Spectrum
TE Cross Power Spectrum
Reionization
Multipole moment (l)
WMAP Full-sky Maps
WMAP Foregrounds vs Cosmic Microwave Background
WMAP Foregrounds
Goddard Space Flight Center • Princeton University • University of Chicago • UCLA • University of British Columbia • Brown University
http://map.gsfc.nasa.gov • http://lambda.gsfc.nasa.gov

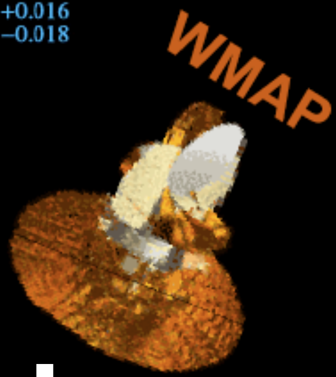


6000
5000
4000
3000
2000
1000
0
10
100
500
1000
1500
WMAP 5yr
Acbar
Boomerang
CBI
$l(l+1)C_l^{TT}/2\pi$ [μK²]
Multipole moment l

Precision Cosmology

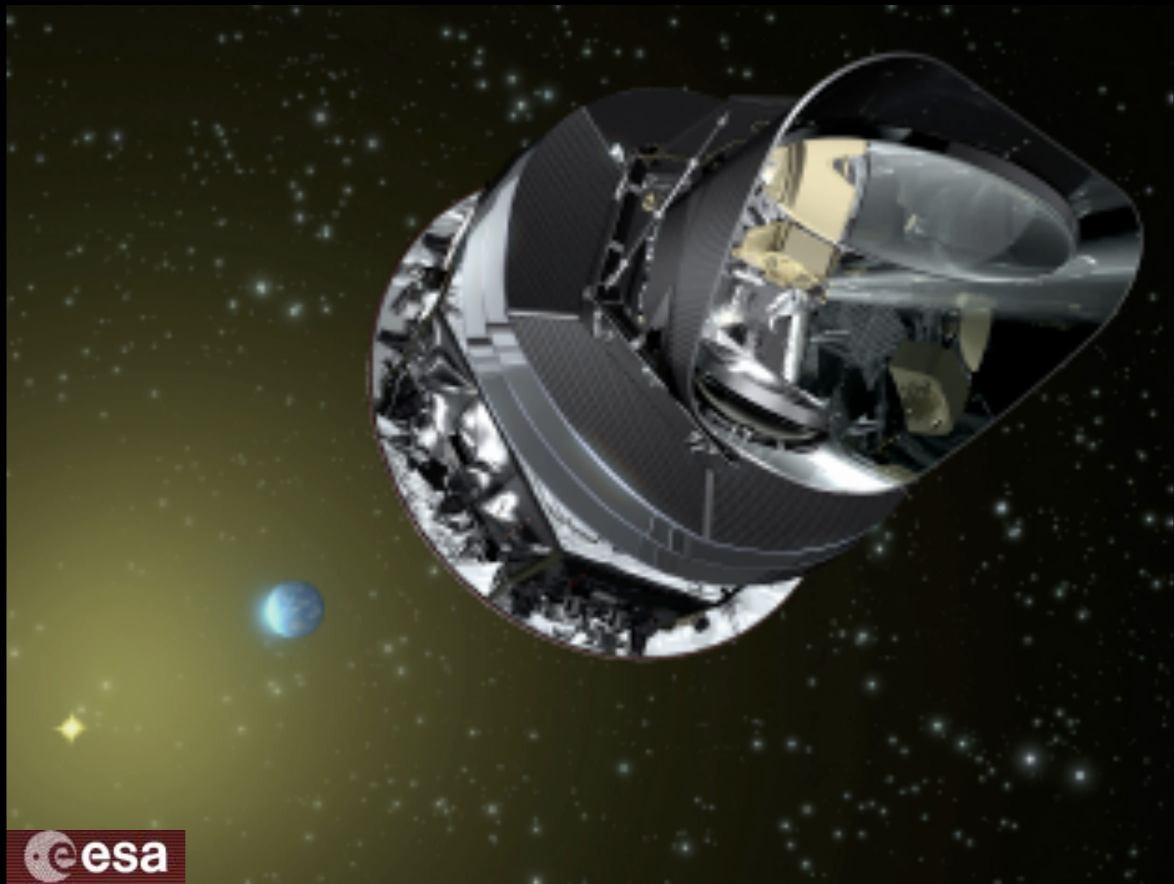
$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$
$w < -0.78$ (95% CL)
$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$
$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$
$\Omega_b = 0.044^{+0.004}_{-0.004}$
$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}
$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$
$\Omega_m = 0.27^{+0.04}_{-0.04}$
$\Omega_\nu h^2 < 0.0076$ (95% CL)
$m_\nu < 0.23$ eV (95% CL)
$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K
$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}
$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$
$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$
$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc
$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$
$A = 0.833^{+0.086}_{-0.083}$

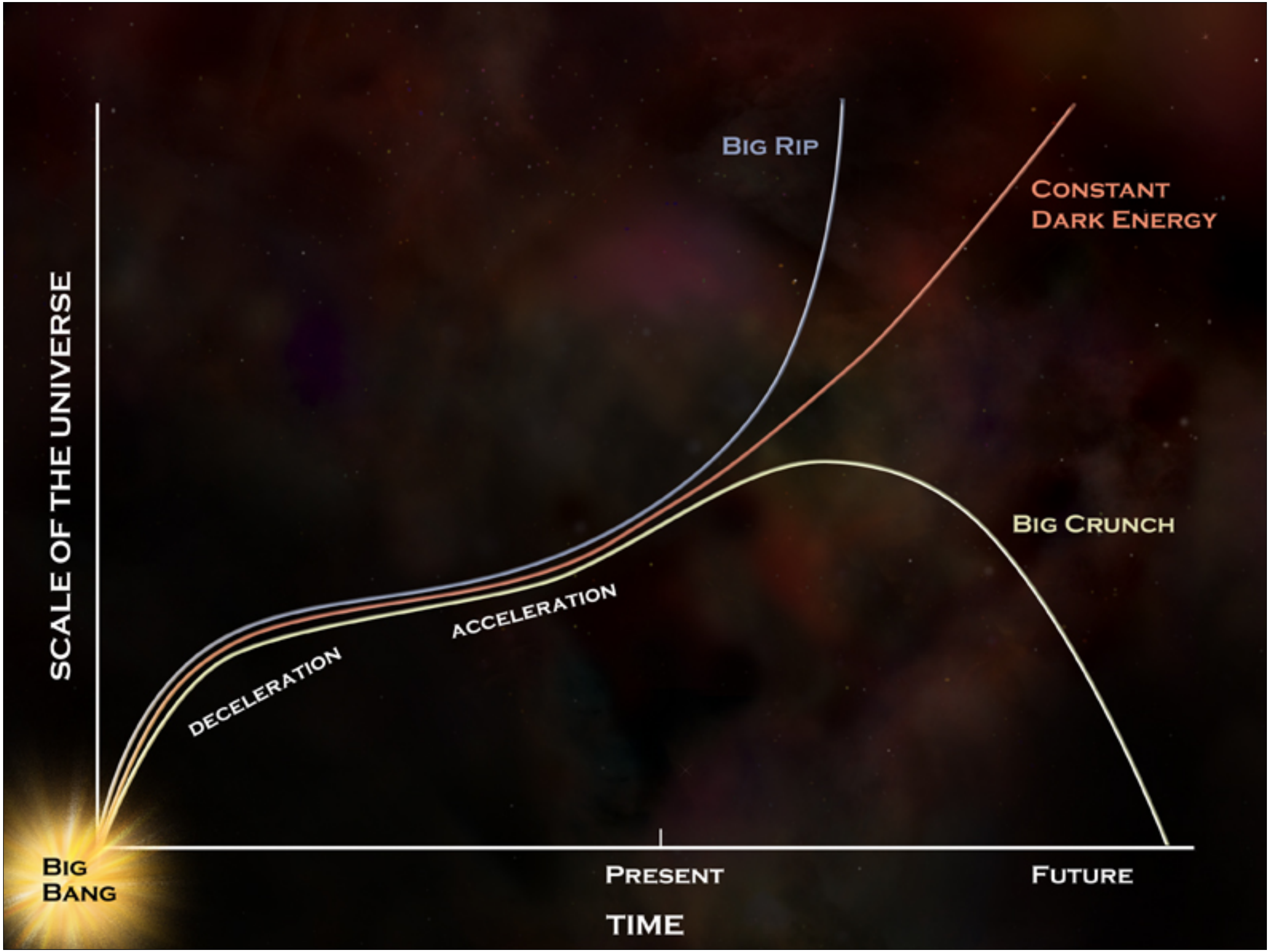
$n_s = 0.93^{+0.03}_{-0.03}$
$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$
$r < 0.71$ (95% CL)
$z_{dec} = 1089^{+1}_{-1}$
$\Delta z_{dec} = 195^{+2}_{-2}$
$h = 0.71^{+0.04}_{-0.03}$
$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr
$t_{dec} = 379^{+8}_{-7}$ kyr
$t_r = 180^{+220}_{-80}$ Myr (95% CL)
$\Delta t_{dec} = 118^{+3}_{-2}$ kyr
$z_{eq} = 3233^{+194}_{-210}$
$\tau = 0.17^{+0.04}_{-0.04}$
$z_r = 20^{+10}_{-9}$ (95% CL)
$\theta_A = 0.°598^{+0.002}_{-0.002}$
$d_A = 14.0^{+0.2}_{-0.3}$ Gpc
$l_A = 301^{+1}_{-1}$
$r_s = 147^{+2}_{-2}$ Mpc



INFLATION

Planck




Big Rip
Constant
Dark Energy
Big Crunch
Scale of the Universe
Deceleration
Acceleration
Big
Bang
Present
Future
Time

Taking sides:

$$\boxed{G_{00}(\mathrm{FRW}) = 8\pi\, G\, T_{00}}$$

1) Modify the RHS of Einstein equations
   a) Cosmological constant
   b) Not constant (scalar field)

2) Modify the LHS of Einstein equations
   a) Beyond Einstein (mod. of gravity)
   b) Just Einstein (BR of inhomog.)

The dark side of the Universe

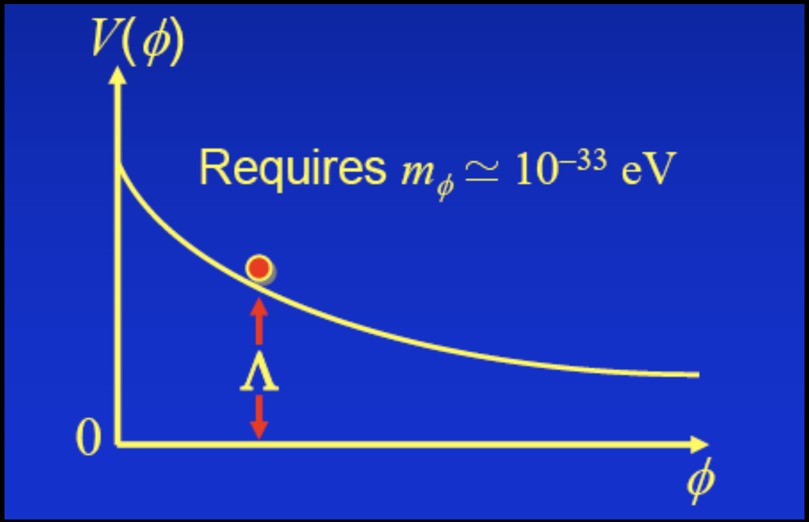
70% of the energy density of the Universe
is in the form of dark energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3P)$$

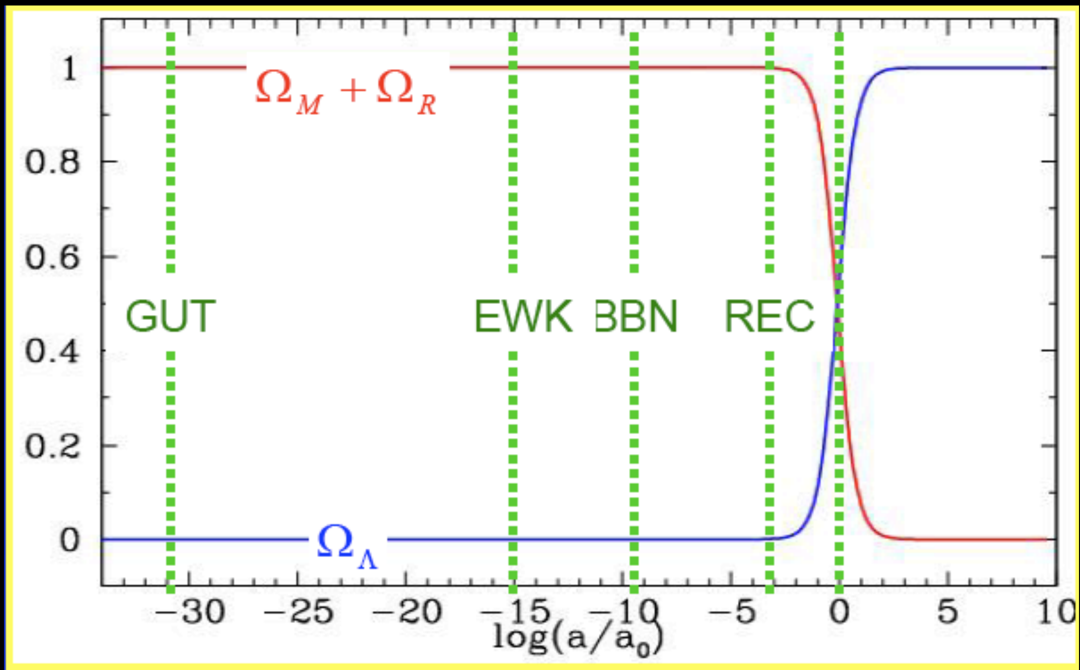
$$\ddot{a} > 0 \Leftrightarrow w \equiv P/\rho < -1/3$$

Modify the RHS: CC/Quintessence

- Many possible contributions?
- Why then is the total so small?
- Perhaps some unknown dynamics sets the total CC to zero, but we are not there yet



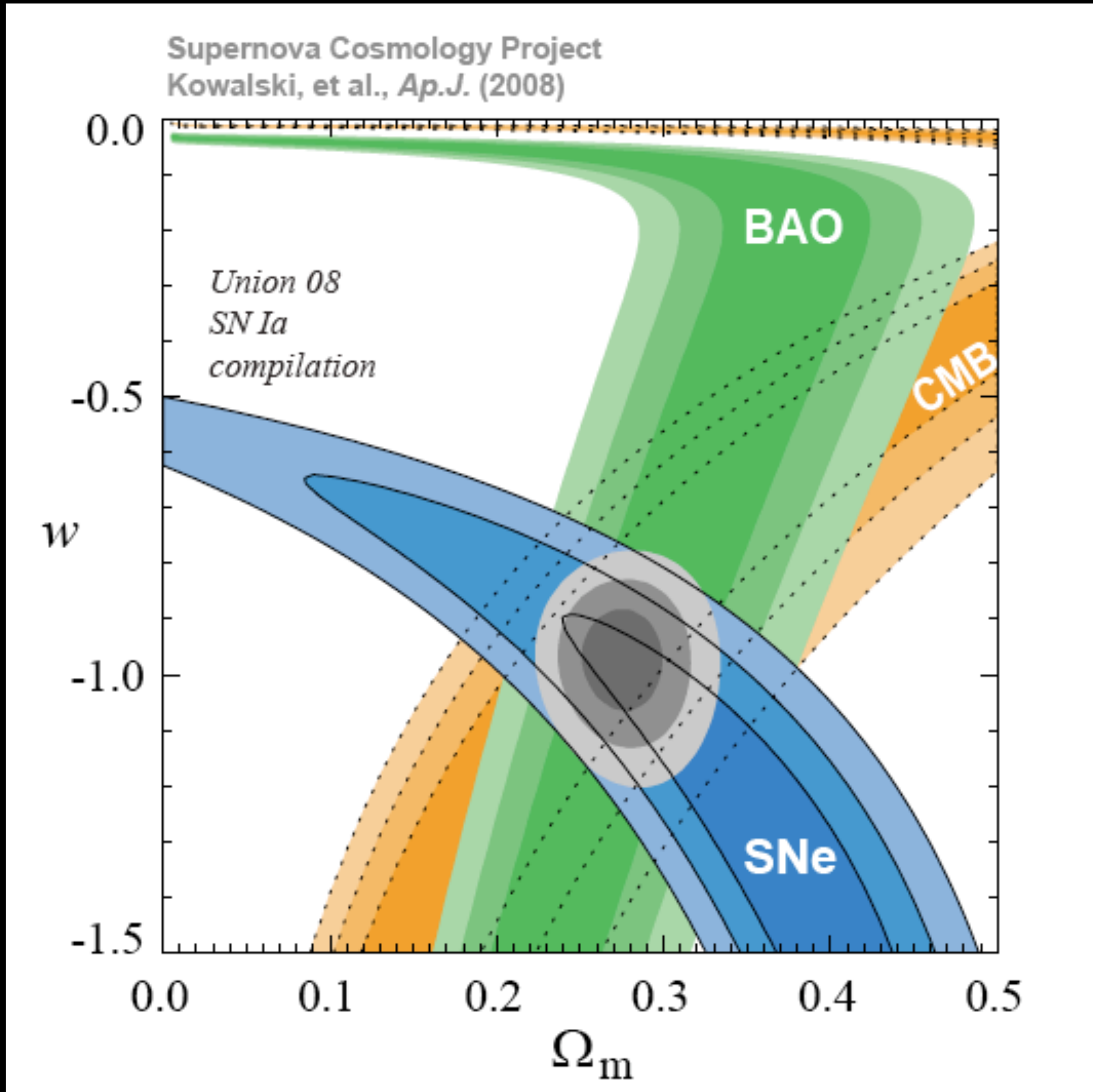
Why now?



$\Omega_M + \Omega_R$
Ω_Λ
GUT
EWK
BBN
REC
1
0.8
0.6
0.4
0.2
0
−30
−25
−20
−15
−10
−5
0
5
10
log(a/a$_0$)

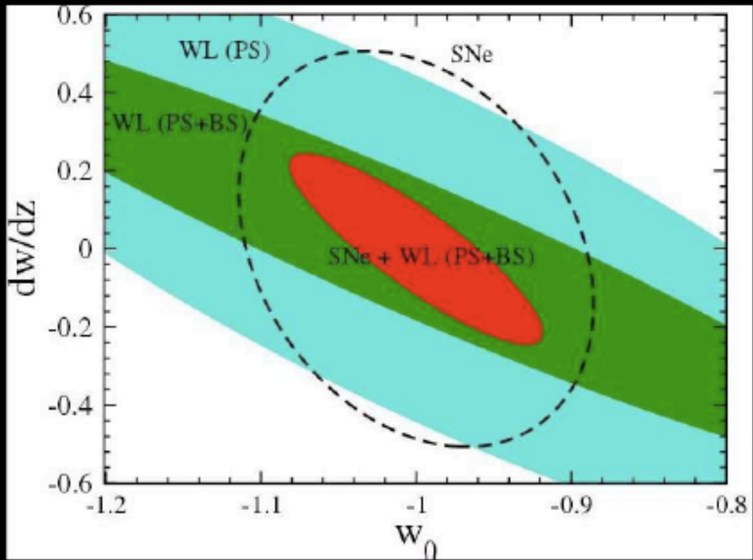
How do we know DE exists?

- Assume FRW model of cosmology: $H^2 = 8\pi G\rho/3 - k/a^2$
- Assume energy and pressure content: $\rho = \rho_M + \rho_\gamma + \rho_\Lambda + \cdots$
- Input cosmological parameters
- Compute observables: $d_L(z),\ d_A(z),\ H(z)$
- Model cosmology fits with ρ_Λ , but not without ρ_Λ
- All evidence for DE is INDIRECT : the observed Hubble rate is not the one predicted through all the previous steps


Supernova Cosmology Project
Kowalski, et al., Ap.J. (2008)
Union 08
SN Ia
compilation
BAO
CMB
SNe
w
0.0
-0.5
-1.0
-1.5
0.0
0.1
0.2
0.3
0.4
0.5
Ω_m

The dark energy is probably vacuum energy.
Requires dramatic fine-tuning, but every alternative requires even more. Observational signature: constant energy density ($w = -1$, and $w' = 0$).

If it is vacuum energy, cosmological observations won't tell us anything; we'll have to understand fundamental physics (extra dimensions, susy), probably through accelerator experiments.



But knowing whether it is vacuum is of paramount importance!

Modify the LHS: non-standard gravity

$$F_g = G_N \frac{m_1 m_2}{r^2} \text{ per } r < r_c$$

$$F_g = G_N \frac{m_1 m_2}{r^3} \text{ per } r > r_c$$

Degravitation

$$\boxed{G_{\rm N}^{-1}\left(L^2\Box\right)G_{\mu\nu} = 8\,\pi\,T_{\mu\nu}}$$

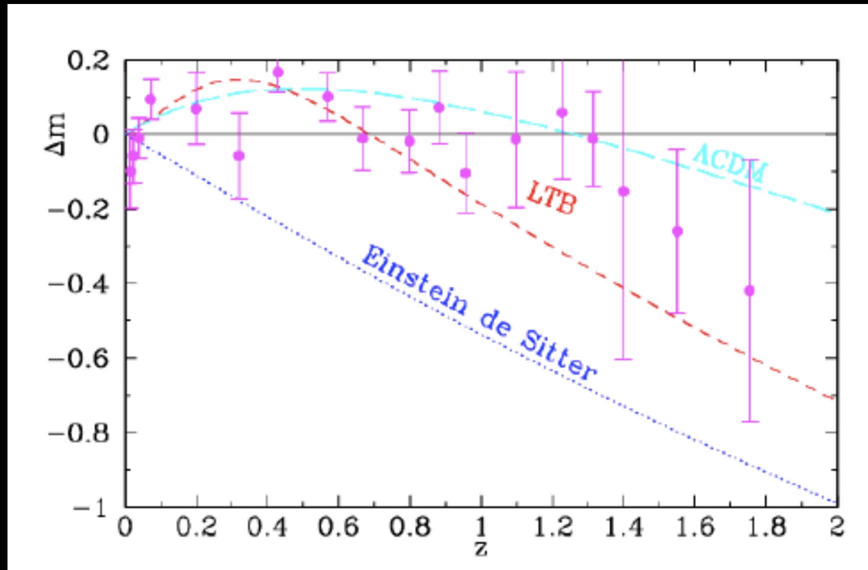
All these class of theories predict the presence of extra longitudinal degrees of freedom of the gravitaton which becomes strongly coupled at distances

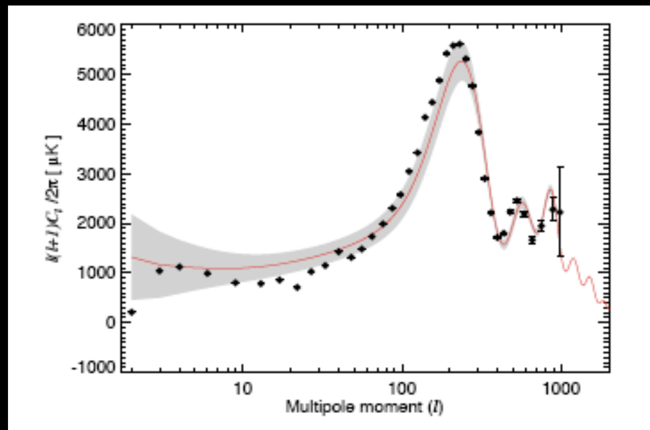
$$\begin{aligned} r_\star &= \left(L^{4(1-\alpha)}r_g\right)^{\frac{1}{1+4(1-\alpha)}}, \\ r_g &= 2\,G_{\rm N}\,M \end{aligned}$$

$$\delta \sim \left(\frac{r}{L}\right)^{2(1-\alpha)}\sqrt{\frac{r}{r_g}}$$

Lemaitre-Tolman-Bondi Models

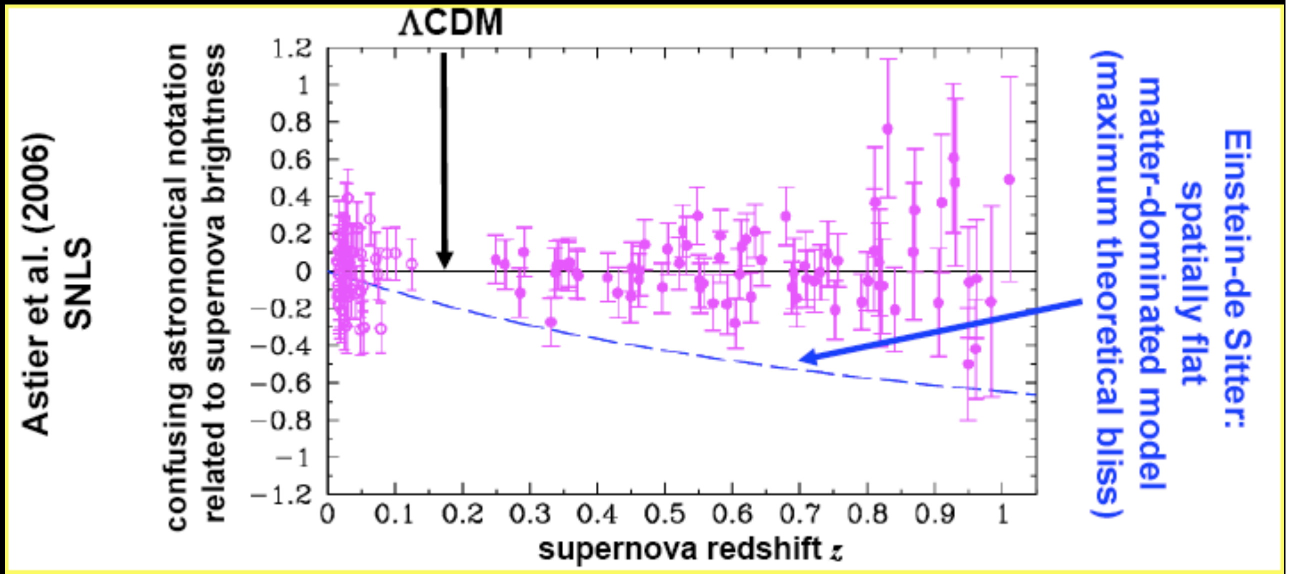
- Spherical model
- Overall Einstein-de Sitter
- Inner underdense Gpc region
- Calculate $d_L(z)$
- Compare to data
- Fit with $\Lambda = 0$



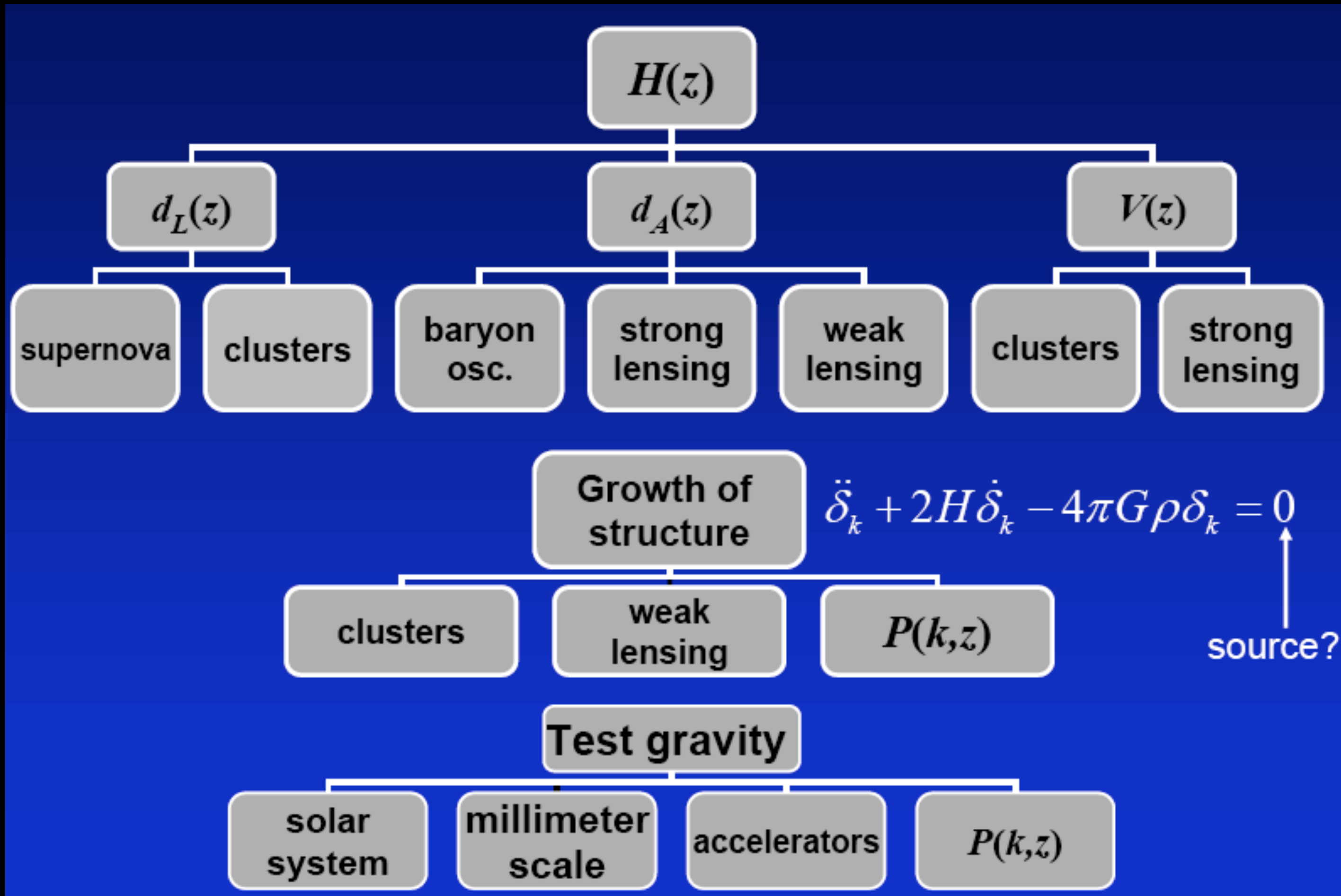


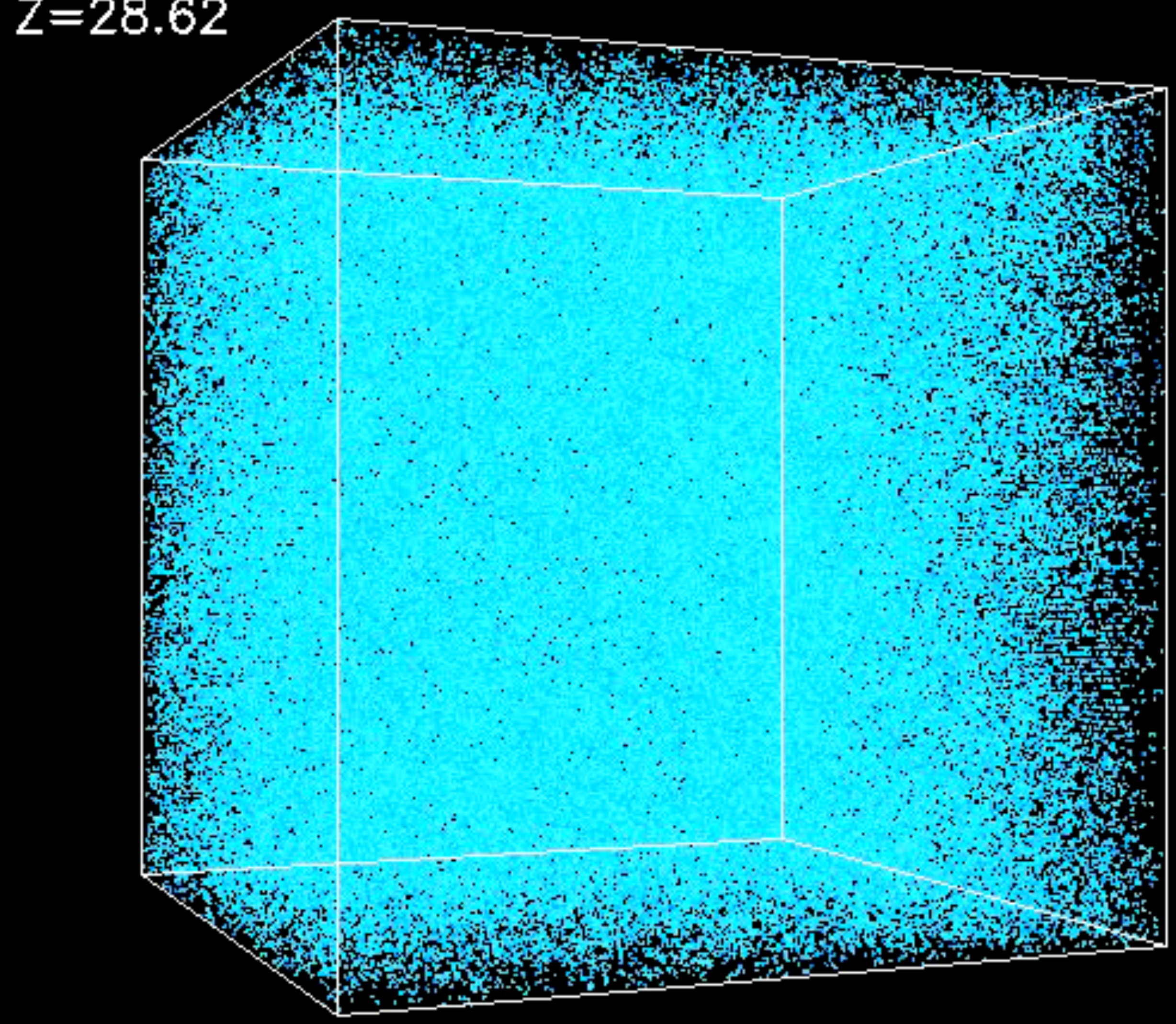
Techniques to descriminate

- Hubble Diagram (SNe)
- Baryon acoustic oscillations
- Weak lensing
- Galaxy clusters
- Age of the Universe
- Structure formation



Observational strategy





The Millenium Simulation Project:

http://www.mpa-garching.mpg.de/galform/virgo/millennium/

The structure in the Universe

Perturbing around the average energy density
we may define the density contrast

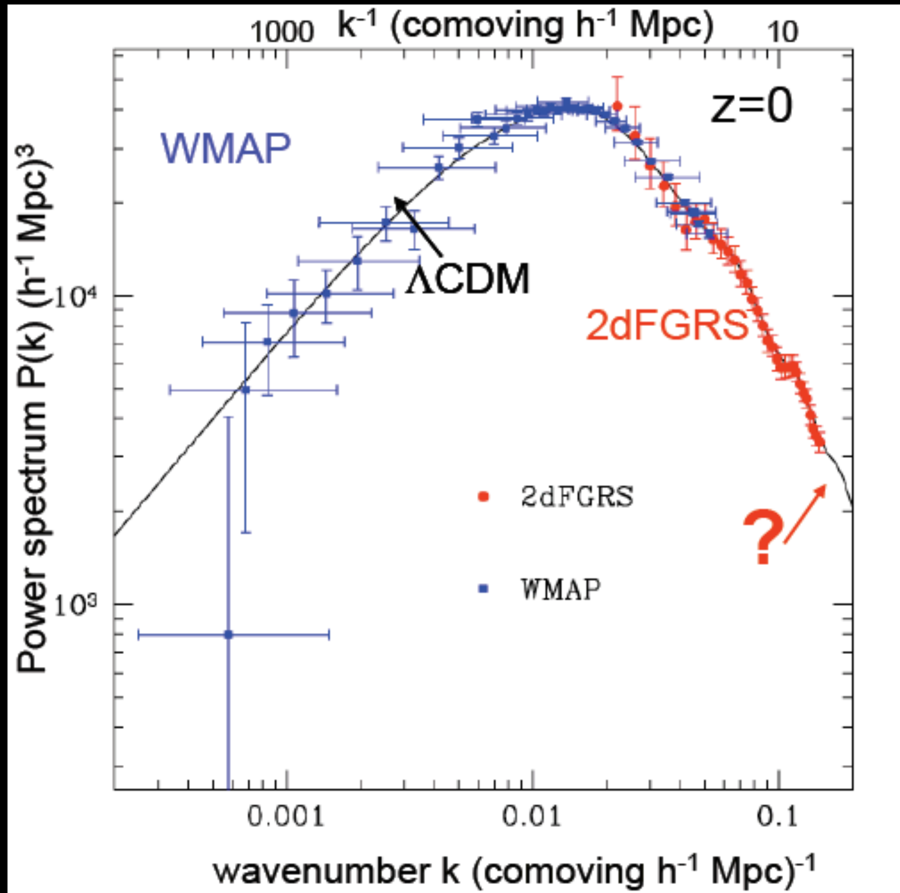
$$\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t)-\bar{\rho}}{\bar{\rho}} = \int \frac{d^3k}{(2\pi)^3}\,\delta_{\mathbf{k}}(t)\,e^{-i\,\mathbf{k}\cdot\mathbf{x}}$$

The power spectrum is defined by

$$\langle \delta_{\mathbf{k}}\,\delta_{\mathbf{k}'}\rangle = (2\pi)^3\,P_\delta(k)\,\delta(\mathbf{k}-\mathbf{k}')$$

$$\boxed{\Delta_\delta(k) = \frac{k^3\,P_\delta(k),}{2\pi^2}\,,\;\; P_\delta = A\,k^n\,T(k)}$$

$$n \simeq 1,\; T(k) = \text{transfer function}$$



$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} = 4\pi G\,\overline{\rho}\,\delta_{\mathbf{k}} \Rightarrow \delta_{\mathbf{k}} \propto a$$

$$\delta_{\mathbf{k}} = -\frac{2}{3}\frac{k^2}{\Omega_m H^2}\,\Phi_{\mathbf{k}}$$

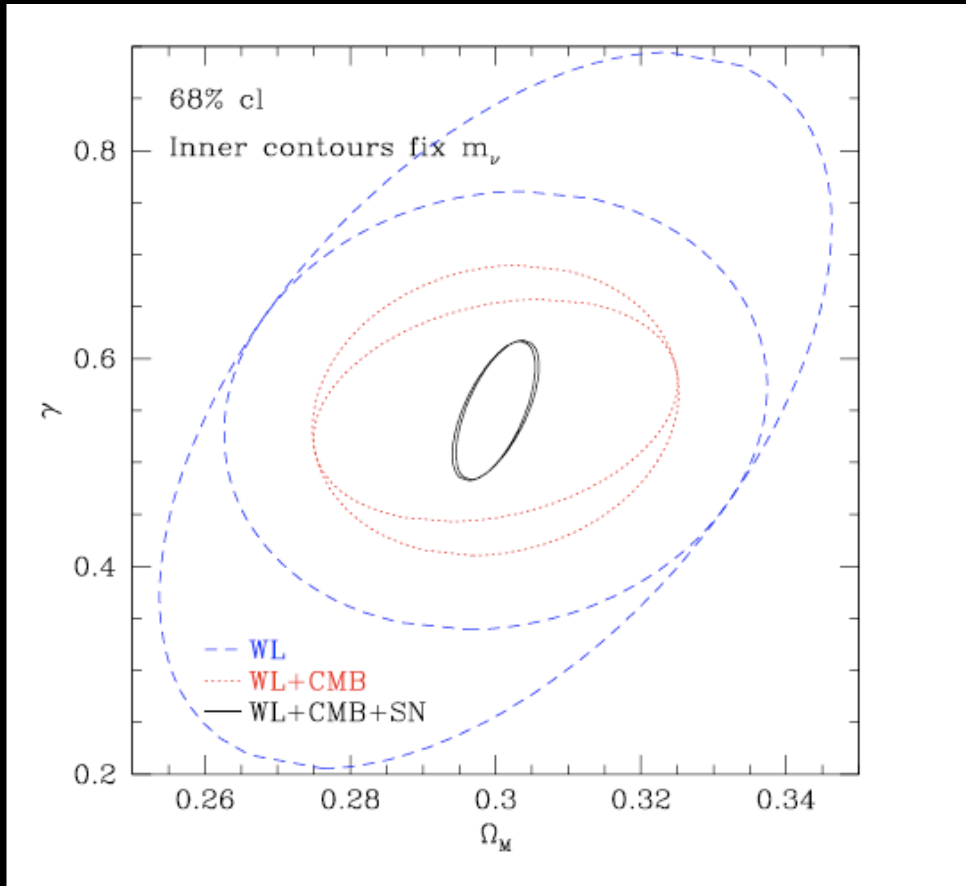
Cosmological Perturbations are sensitive to energy contentand to modified gravity

$$\boxed{\ddot{\delta}_m + 2H\dot{\delta}_m = \frac{3}{2}H^2\delta_m, \;\; \delta_m = \delta\rho_m/\rho_m}$$

$$\delta_m(a) = D(a) = \text{growth function}, \; D(a) = a \text{ in MD}$$

Perturbations can be probed at different epochs:
1) CMB, z ~ 1100
2) 21 cm, z ~ 10-20
3) Ly-alpha forest, z ~ 2-4
4) Weak lensing, z ~ 0.3-2
5) Galaxy clustering, z ~ 0-2

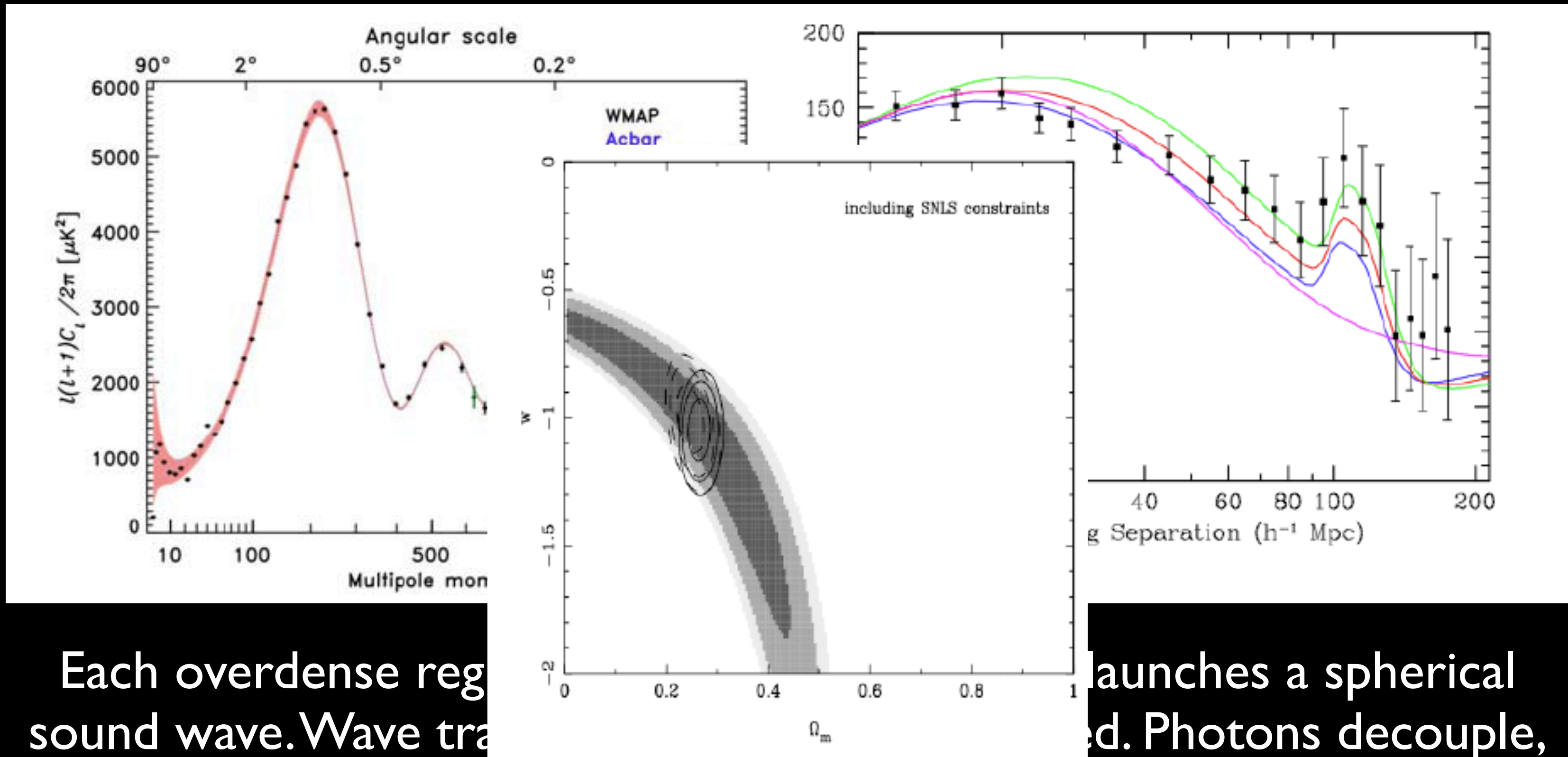
$$w(a) = w_0 + (1-a)w_a$$



$$g(a) \equiv \delta_m/a = e^{\int_0^a d\ln a' \left[\Omega_M^\gamma(a') - 1\right]}$$

D. Huterer, E. Linder, 2005

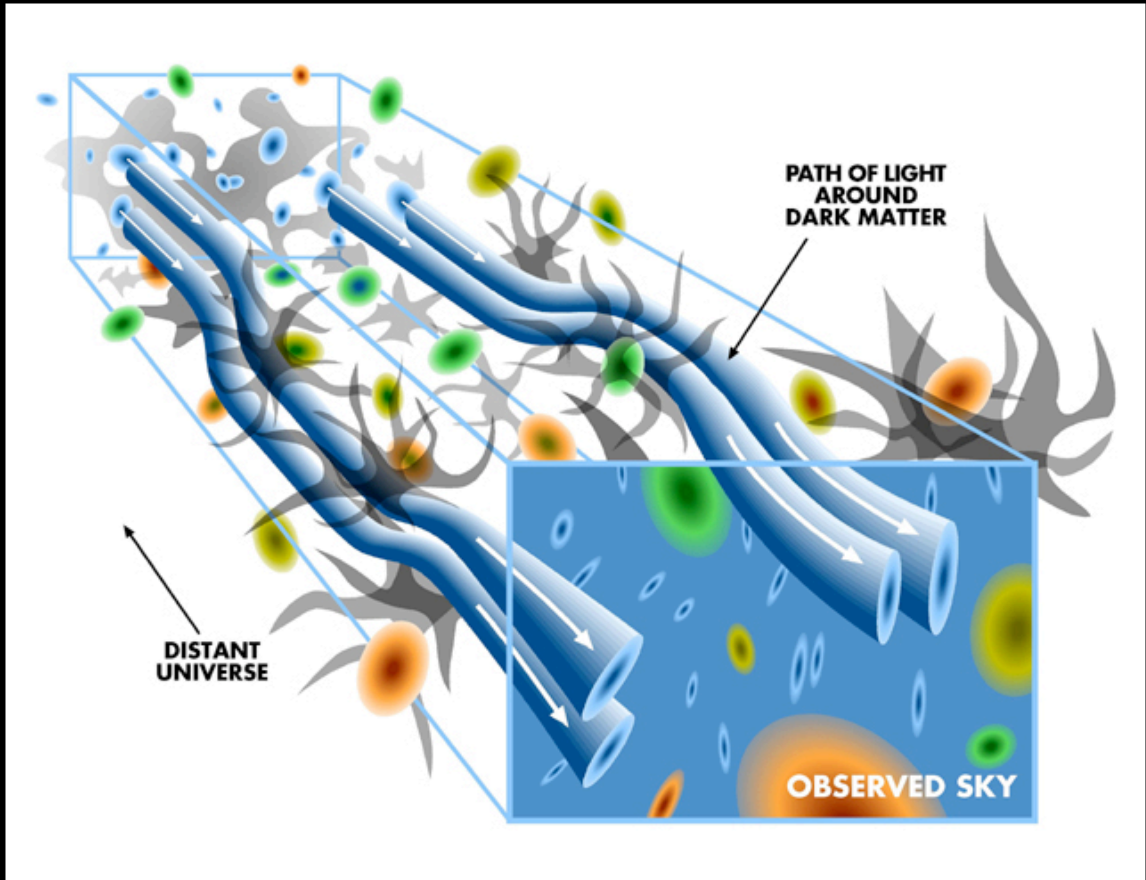
Acoustic Baryonic Oscillations



Each overdense reg launches a spherical sound wave. Wave tra d. Photons decouple, travel to us and are observable as CMB acoustic peaks. For matter, sound speed plummets, wave stalls, total distance travelled 150 Mpc imprinted on power spectrum.

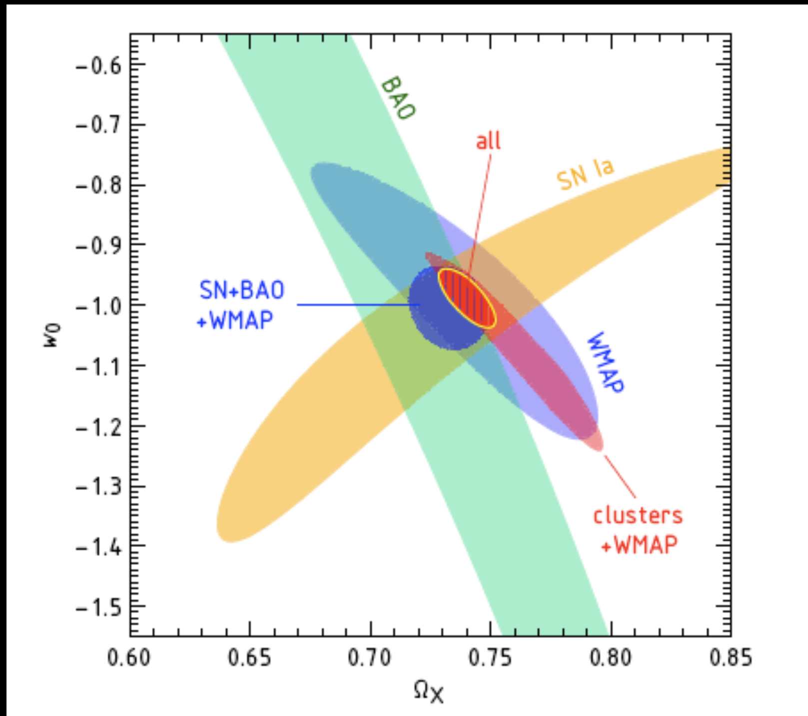
DE enters in the determination of the angular distance

Weak Lensing



Why the halo mass function is so relevant ?

$dn/dV\,dz$ is exponentially sensitive to the Dark Energy through the growth function



X-ray cluster cosmology white paper, arXiv: 0903.5320

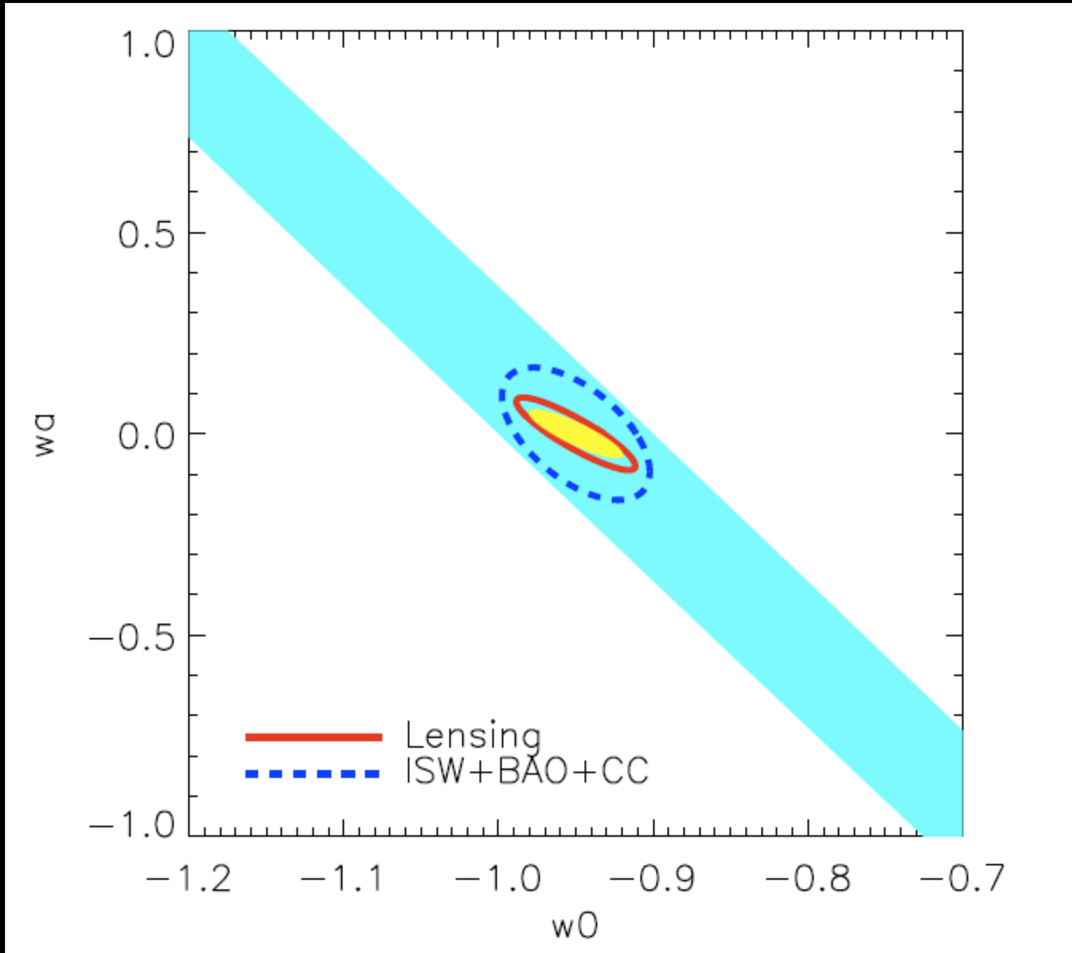
Main current/future BAO surveys

Name	Telescope	N(z) / 10^6	Dates	Status
SDSS/2dFGRS	SDSS/AAT	0.8	Now	Done
WiggleZ	AAT(AAOmega)	0.4	2007-2011	Running
FastSound	Subaru(FMOS)	0.6	2009-2012	Proposal
BOSS	SDSS	1.5	2009-2013	Proposal
HETDEX	HET(VIRUS)	1	2010-2013	Part funded
WFMOS	Subaru	>2	2013-2016	Part funded
ADEPT	Space	>100	2012+	JDEM
SKA	SKA	>100	2020+	Long term

Most data will come at $z \sim 1$ (U-band bottleneck for LBGs)

Σ WiggleZ/FastSound/BOSS = 2m by ~2012 (~7% on w)

$$w(a) = w_0 + (1 - a)w_a$$



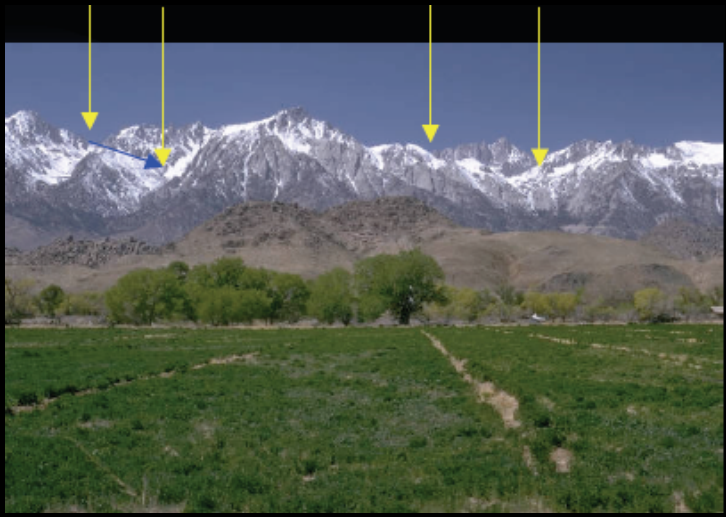
$$\sigma(w_0) \sim 0.05$$

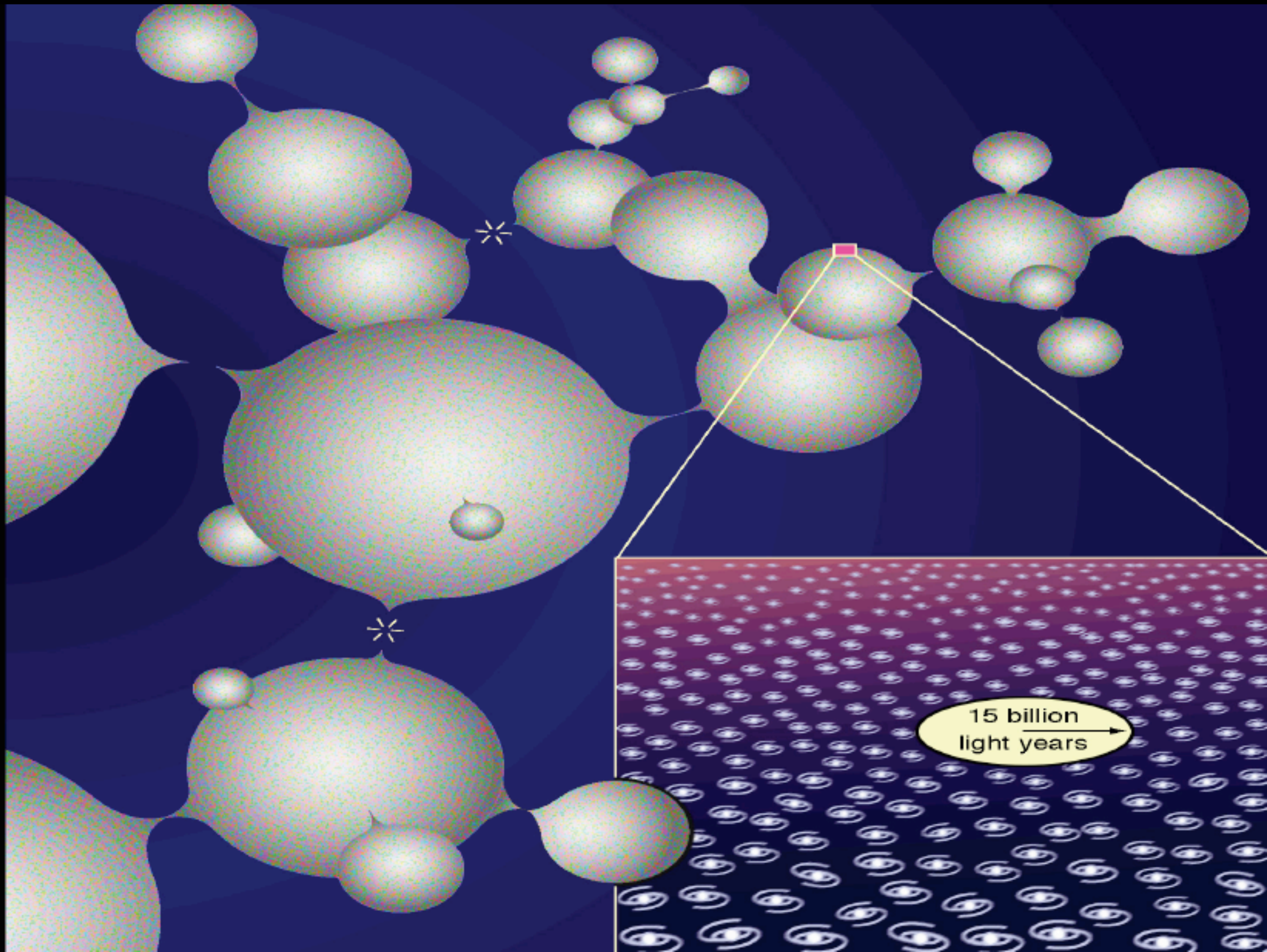
Anthropic/Landscape

- Many sources of vacuum energy
- String Theory has many vacua $> 10^{500}$
- Some of them correspond to a cancellation leading to the observed small cosmological constant

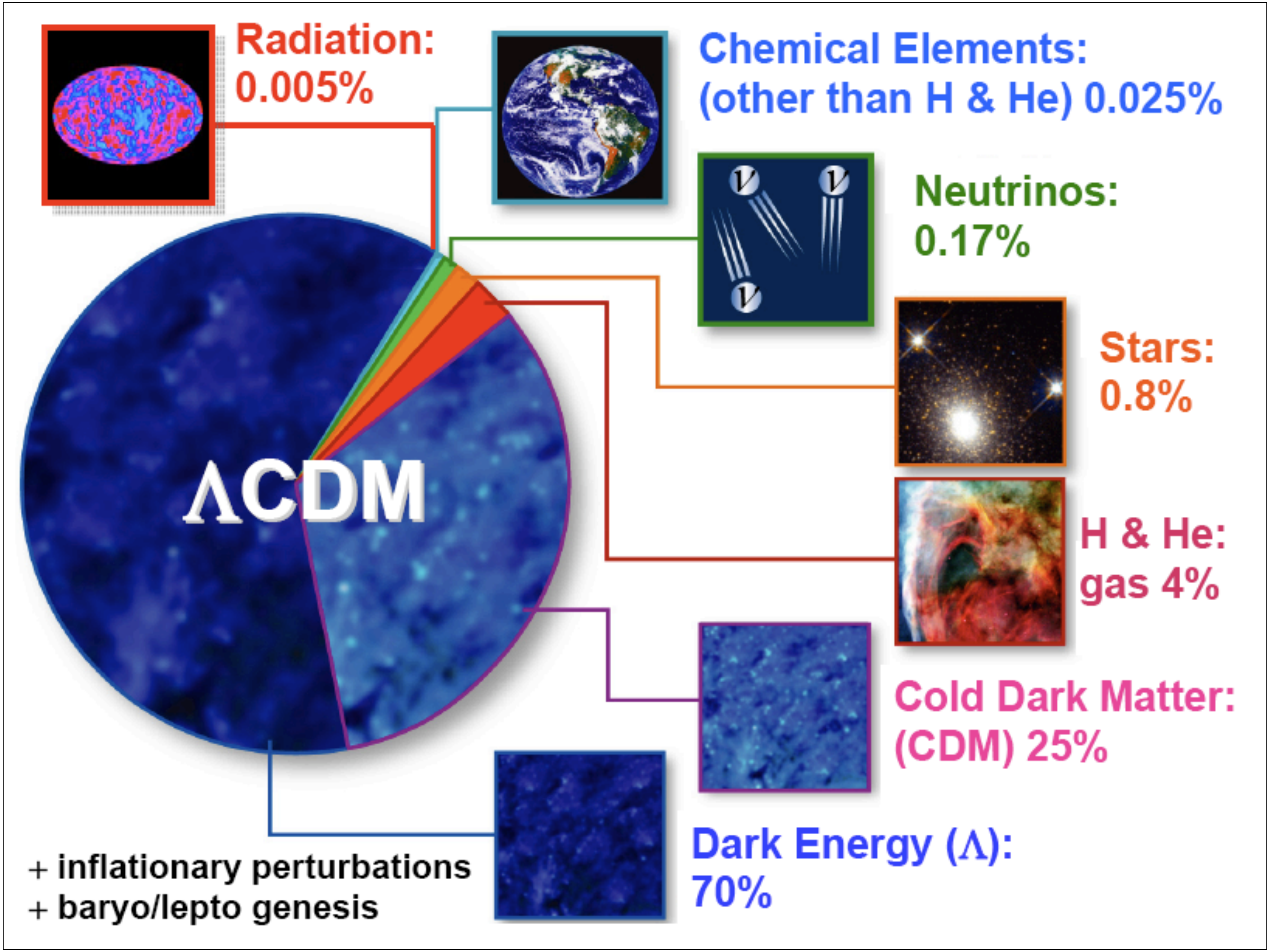
Galaxies require
(Weinberg)
$$\Lambda < 10^{-118}\, M_{\rm Pl}^4$$

- Although the are exponentially uncommon, they are preferred because...
- More common values of the CC results in an inhospitable Universe

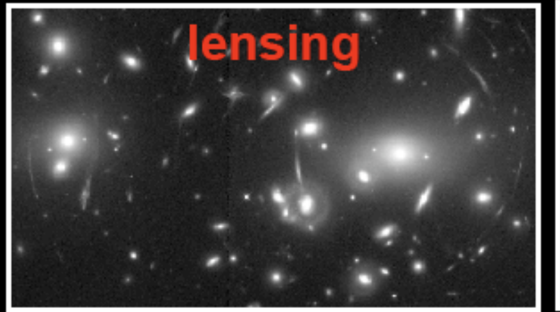


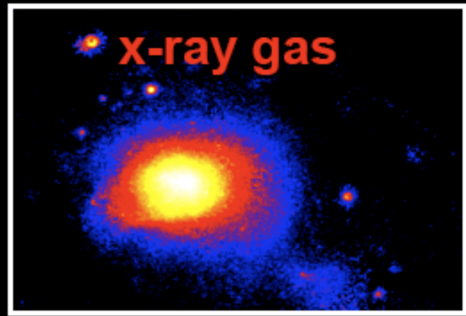

15 billion
light years

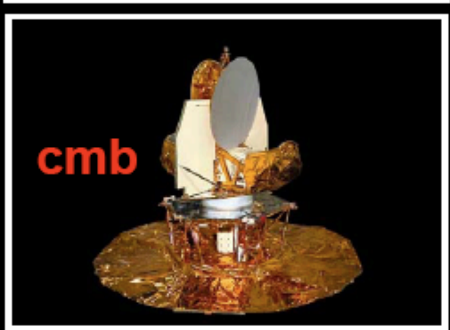
Lecture two:
the Dark Matter Puzzle

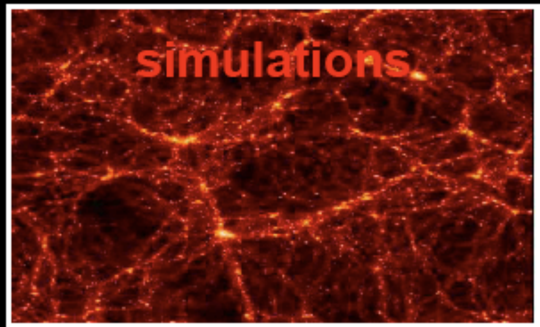

Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
Stars:
0.8%
H & He:
gas 4%
Cold Dark Matter:
(CDM) 25%
Dark Energy (Λ):
70%
ΛCDM
+ inflationary perturbations
+ baryo/lepto genesis

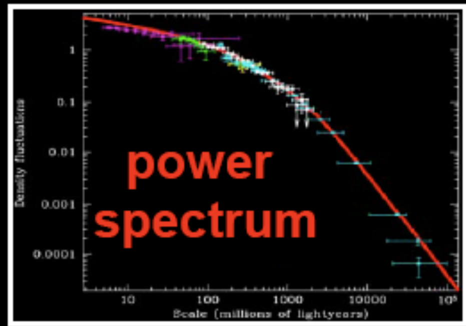
$\Omega_M \sim 0.3$





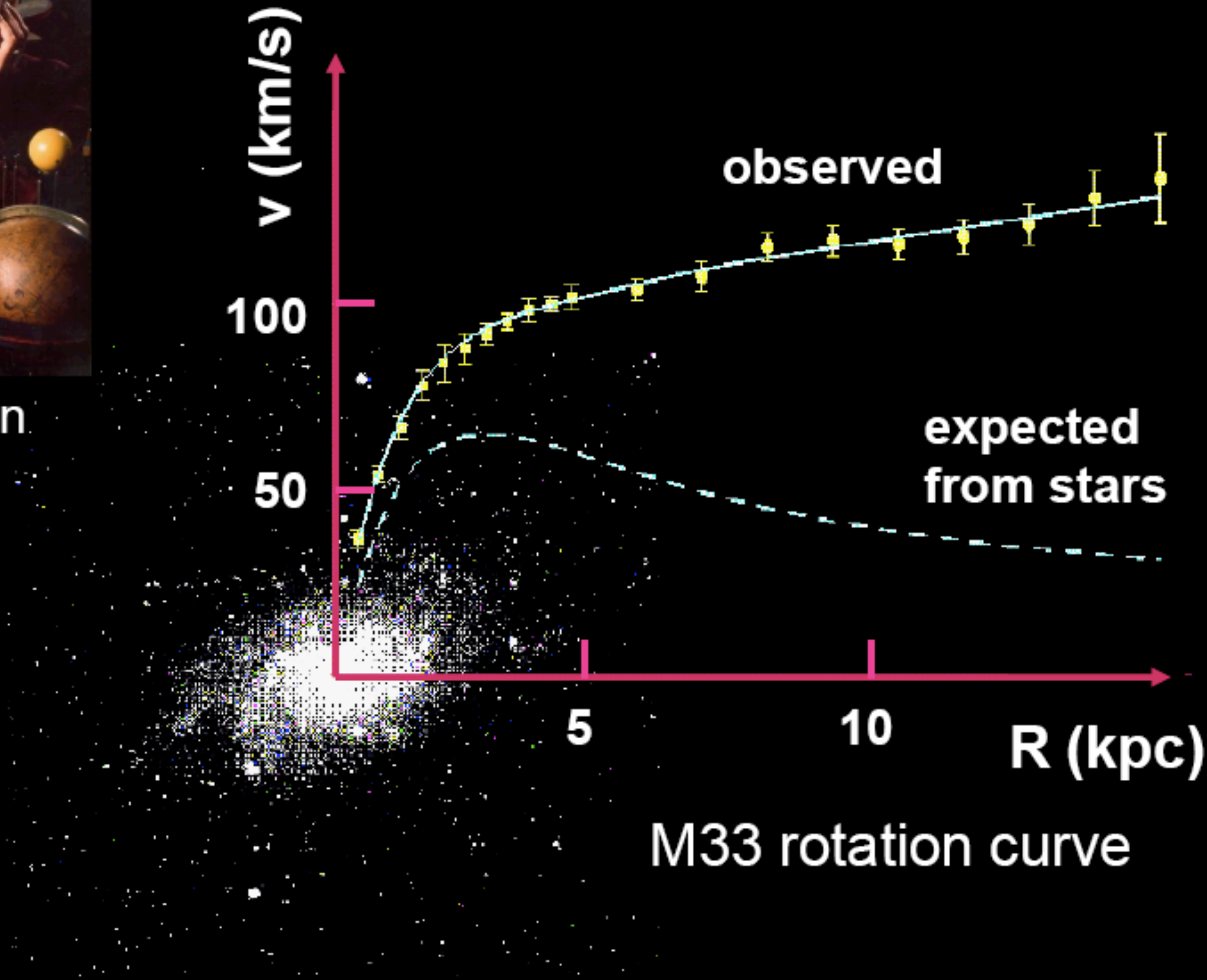




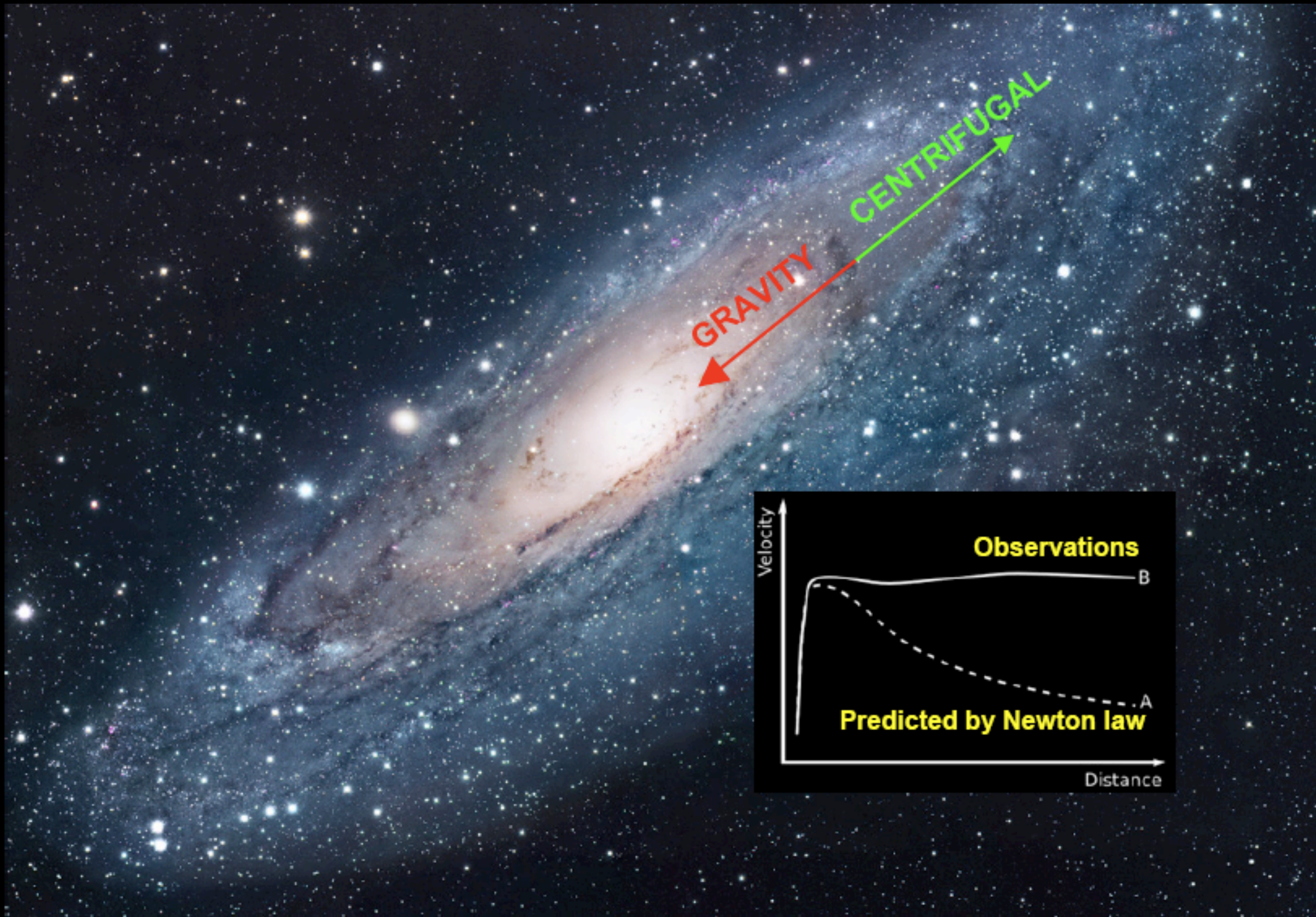


The Dark Universe

Vera Rubin

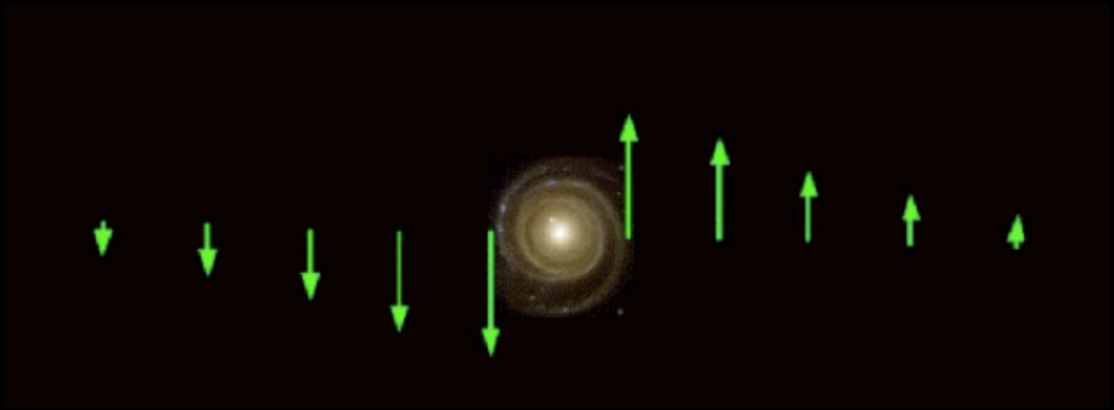


M33 rotation curve

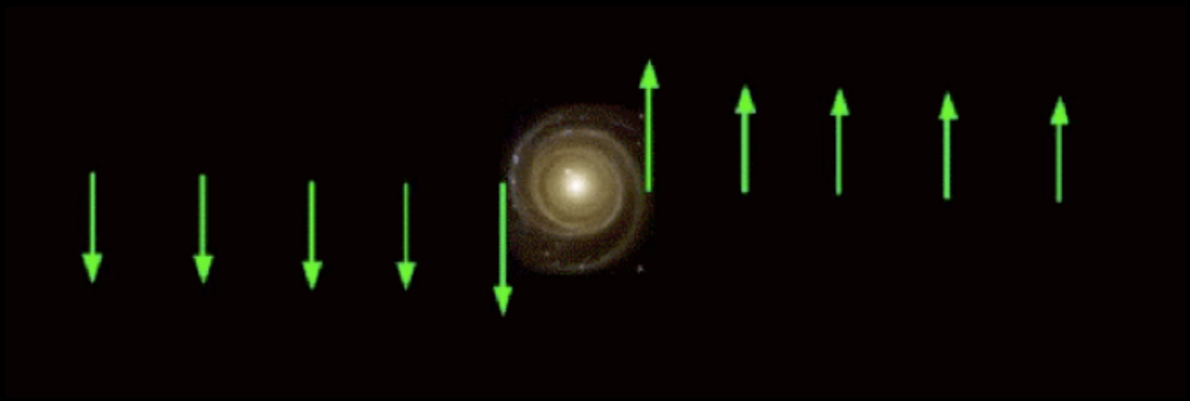


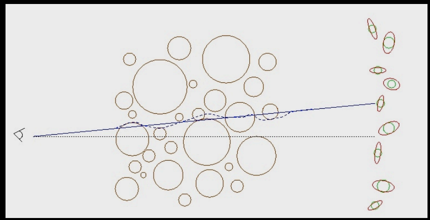
The Andromeda Galaxy (M31)

What we should see

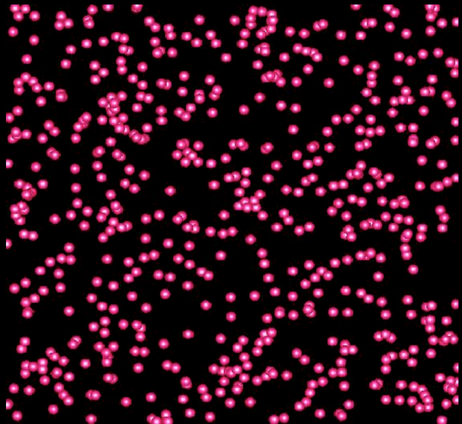


What we do see

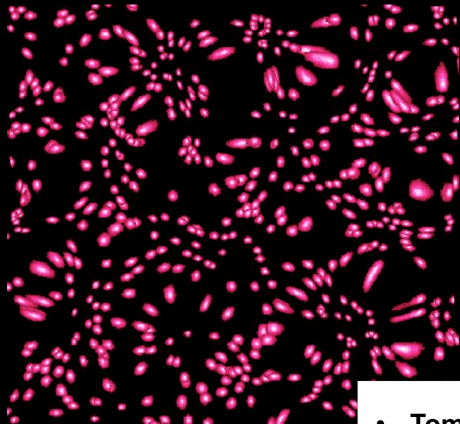




Distortion of background images by foreground matter

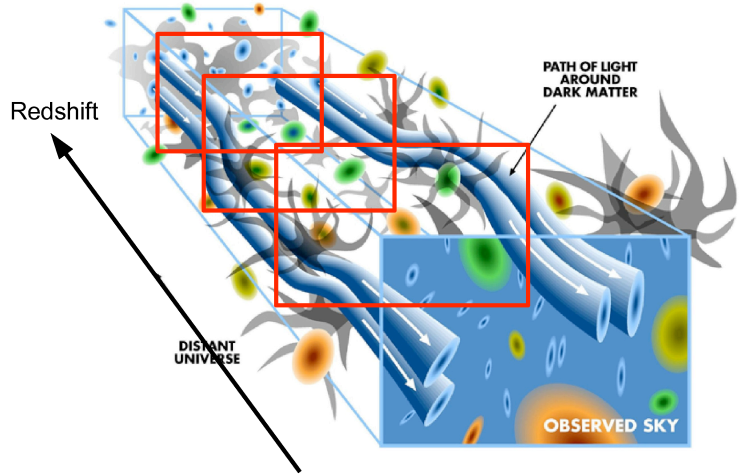


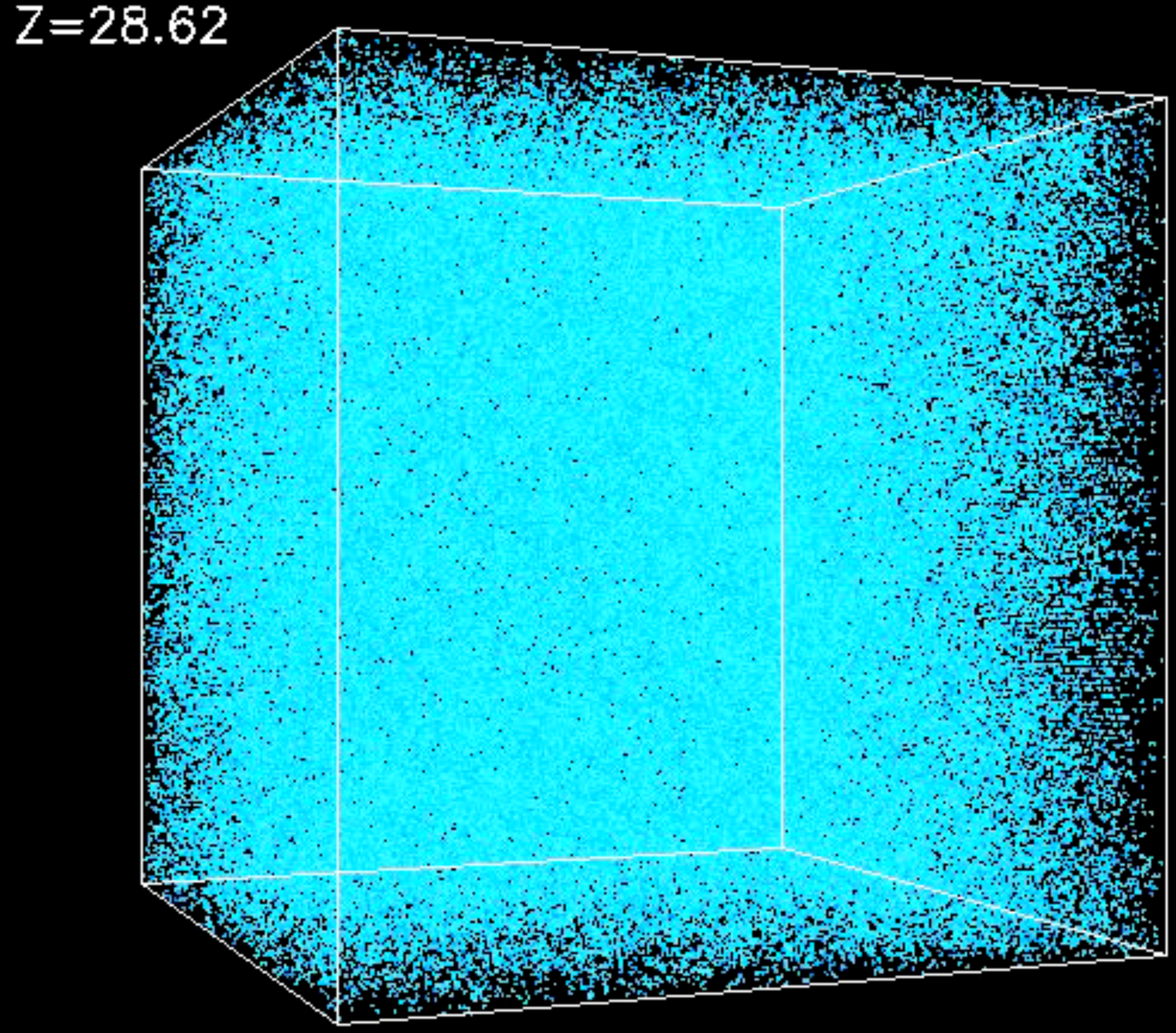
Unlensed



Lensed

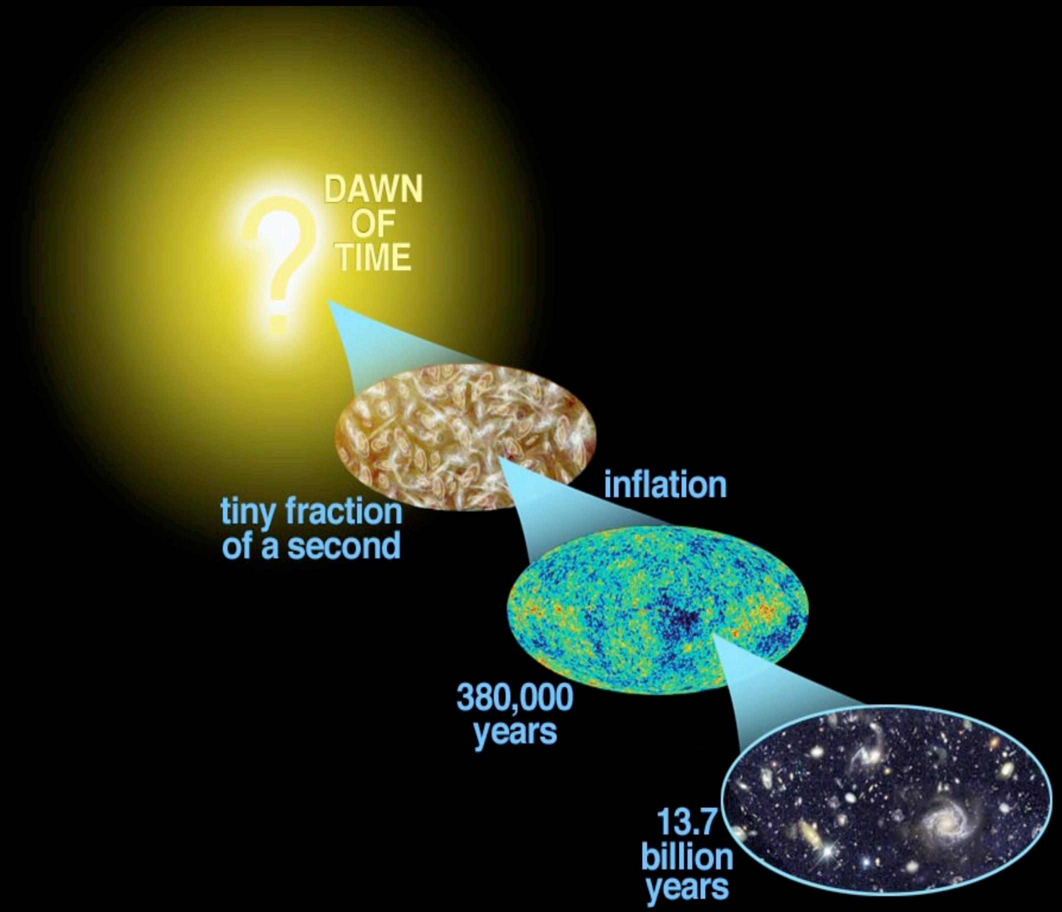
- **Tomography** = bin galaxies by redshift





The Millenium Simulation Project:
http://www.mpa-garching.mpg.de/galform/virgo/millennium/

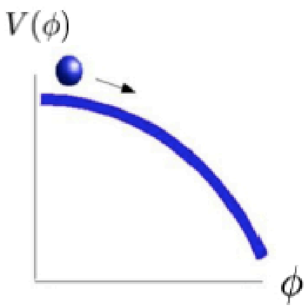
The Inflationary Cosmology



How to get Inflation

Inflation

For a review,see
D.H. Lyth and A.R.,
Phys. Rept. 314
(1999) 1



Potential $\rightarrow$ Vacuum Energy $\rightarrow$ Inflation

Friedmann equation:

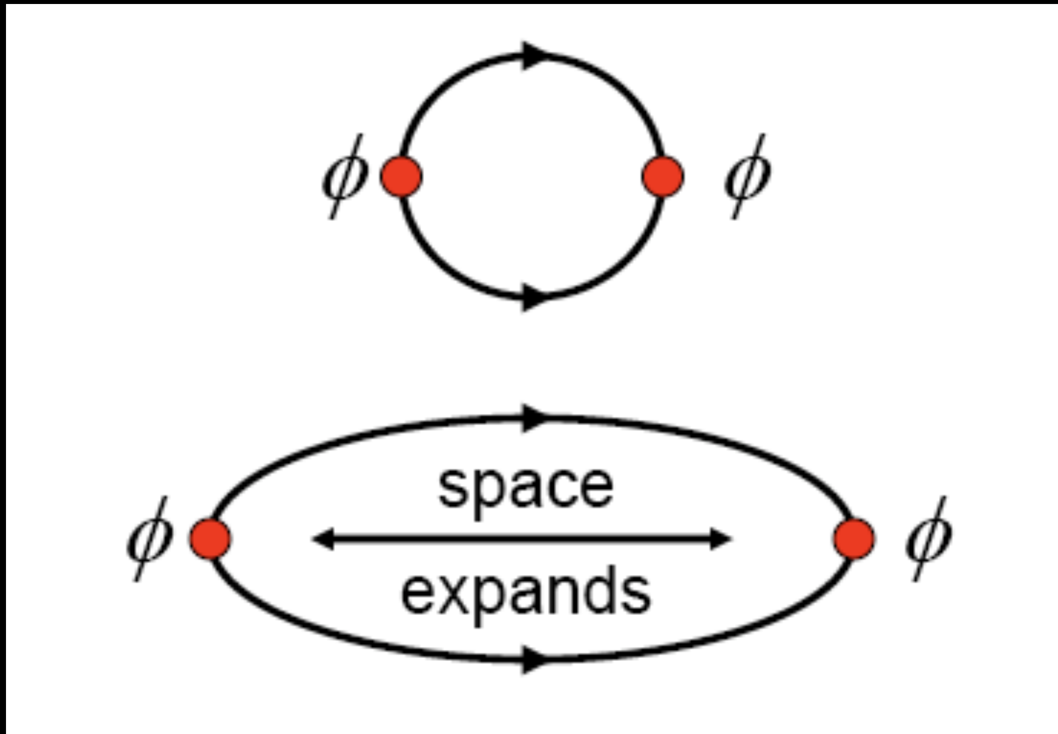
slow roll

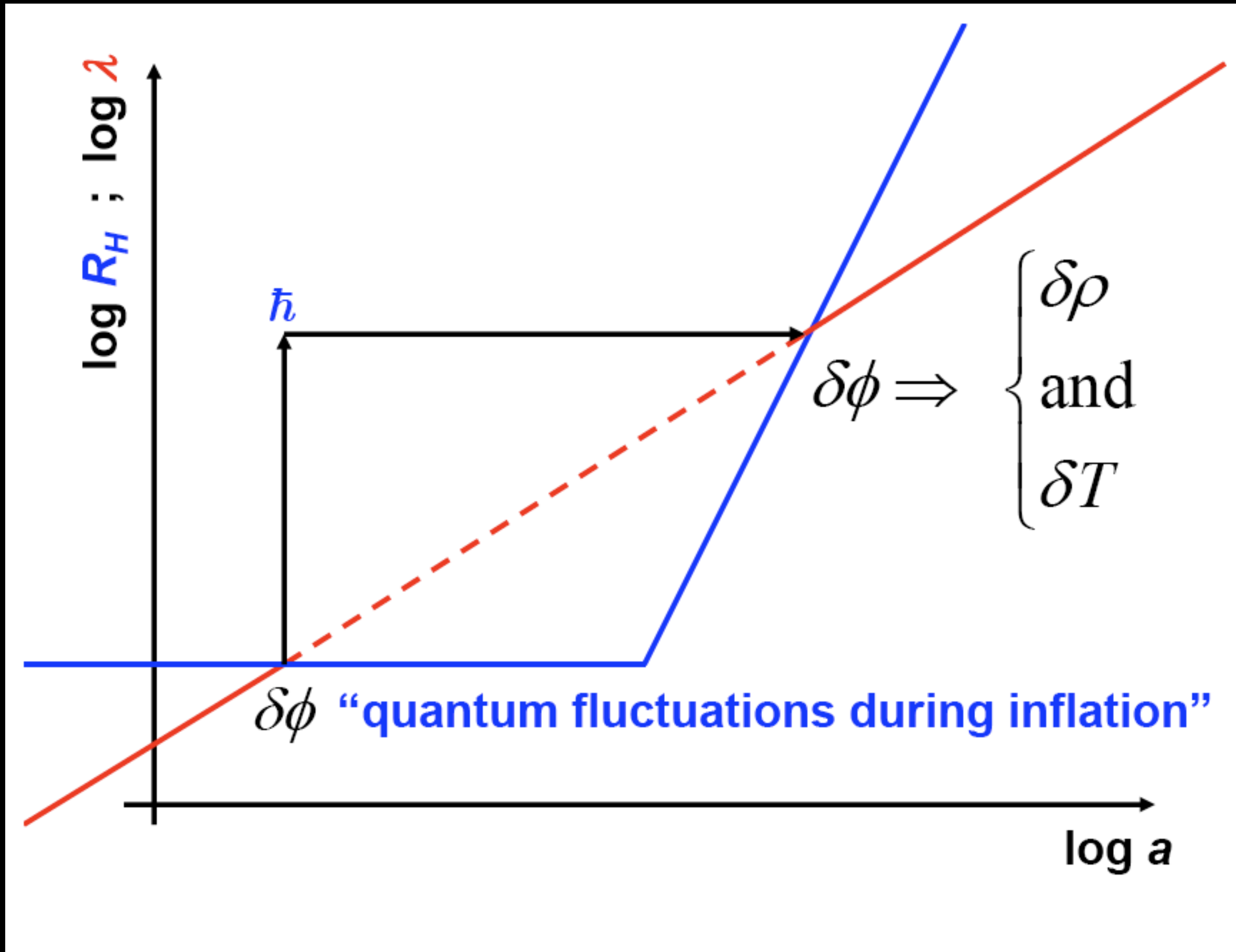
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\left[\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right] \simeq \mathrm{const.}$$

Scalar field equation of motion:

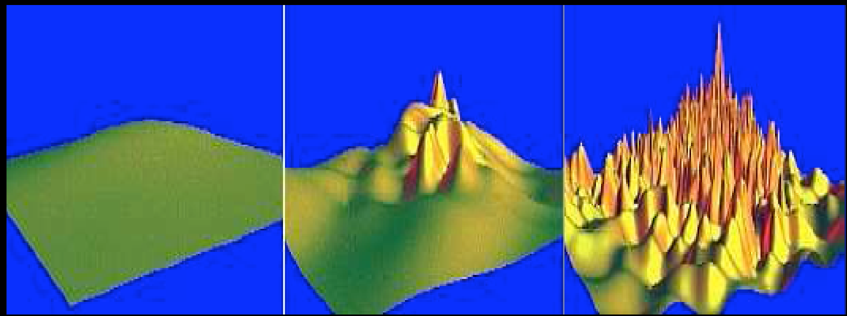
$$\cancel{\ddot{\phi}} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} + V'(\phi) = 0 \qquad a(t) \propto e^{\int H dt} \equiv e^N$$

Particle production in an expanding Universe





log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a



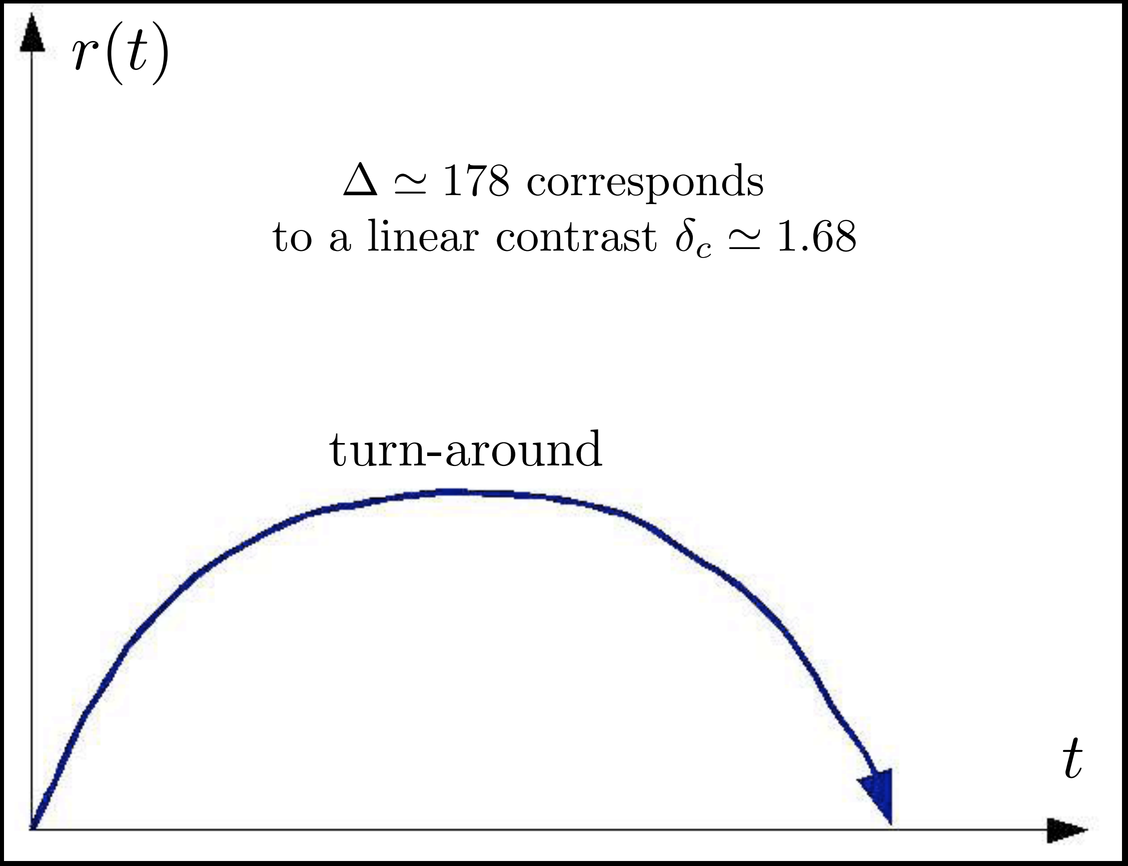
From Quantum Fluctuations to the Large Scale Structure

The cornerstones of structure formation

- Initial seeds provided by primordial inflation: spectrum of perturbations nearly flat and nearly gaussian
- Density perturbations grow because of the gravitational instability. They grow like the scale factor at the linear level
- In the CDM scenario, the first objects to collapse and form dark matter haloes are of low mass
- Merger trees: a halo that exists at a given time will have been constructed by the merging of smaller fragments over time
- When haloes merge, their cores survive as distinct subhaloes for some time. In group/cluster scale haloes, these will mark the locations of the galaxies

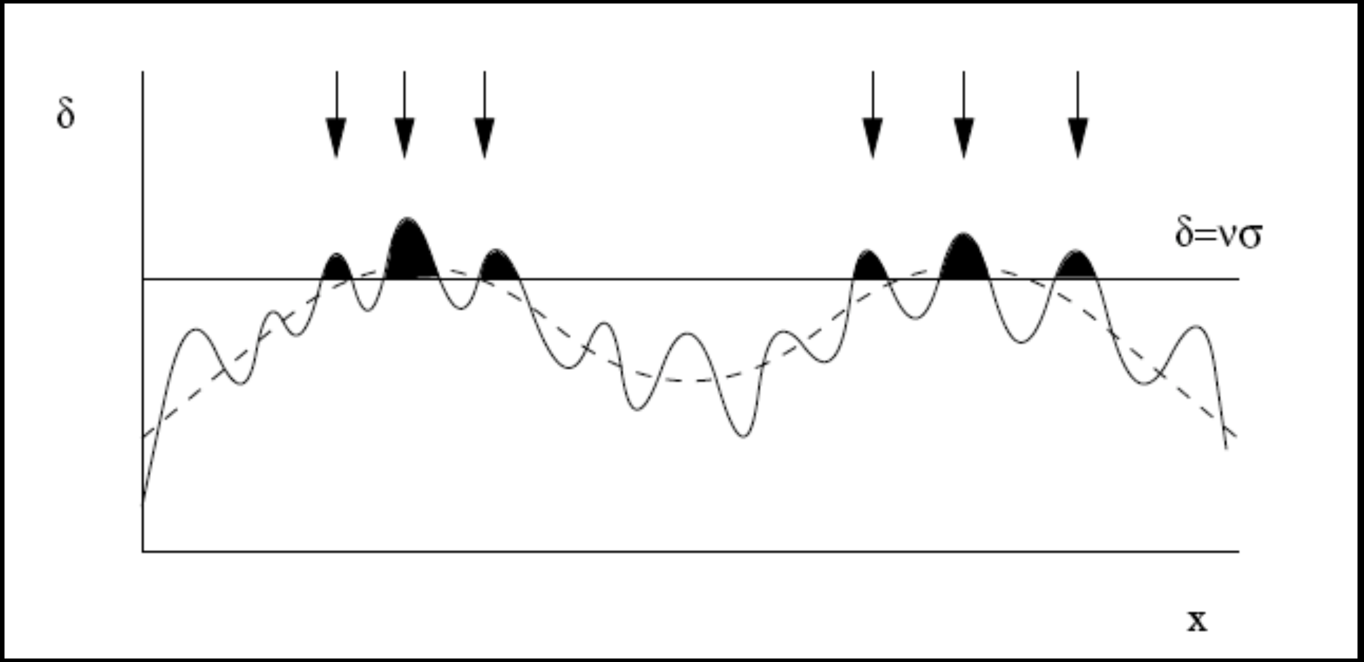
The spherical collapse model

According to Birkhoff's theorem, a spherical density overdense perturbation departs from the background evolution and behaves in exactly the same way as part of of a closed Universe



Why the halo mass function is so relevant

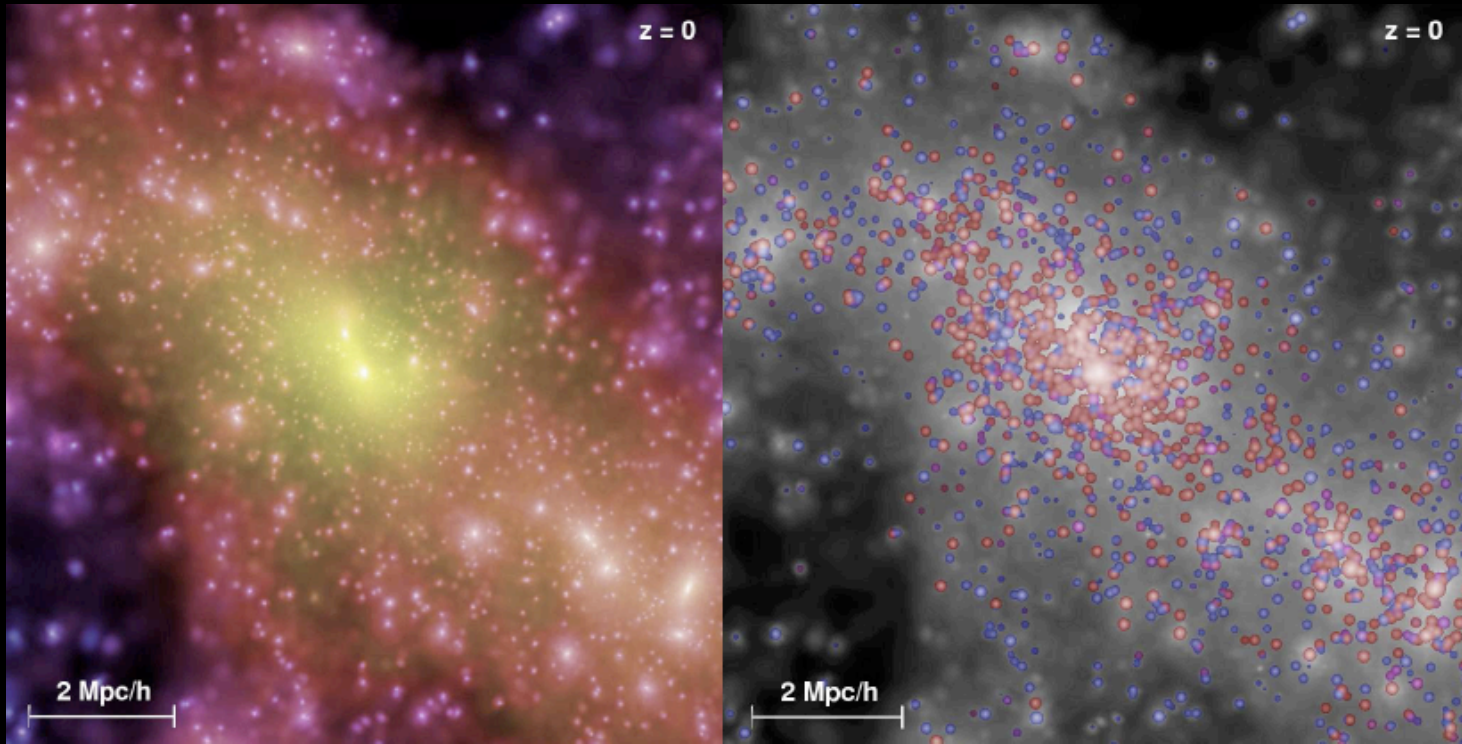
The high-peak bias model, based on the knowledge of the halo mass function, correctly predicts that high-mass haloes are positivey biased:



$$\left(\frac{\delta\rho}{\rho}\right)_{\rm galaxies} = b\left(\frac{\delta\rho}{\rho}\right)_{\rm mass}$$

Dark Matter halo: $10^{14}\, M_{\odot}$

Dark Matter | Galaxies



The Millenium Simulation Project:

Informations on the DM halo mass function from

- Optical detection of their member galaxies
- X-ray emission from hot electrons confined by the gravitational potential wells
- SZ effect whereby hot electrons up-scatter the CMB photons leaving an apparent deficit of low-frequency CMB flux in their direction
- Weak lensing (clusters selected as peaks in a smoothed two-dimensional shear map)
- Systematic, not statistical uncertainties, provide the limiting factor in cosmological measurements: none of these technique measure mass directly, but some proxy quantity as galaxy counts, X-ray flux and/or temperature or the SZ decrement (e.g. X-ray selection requires the intra-cluster gas to be heated to a detectable level, bias effects; weak lensing techniques may miss a fraction of the real mass)

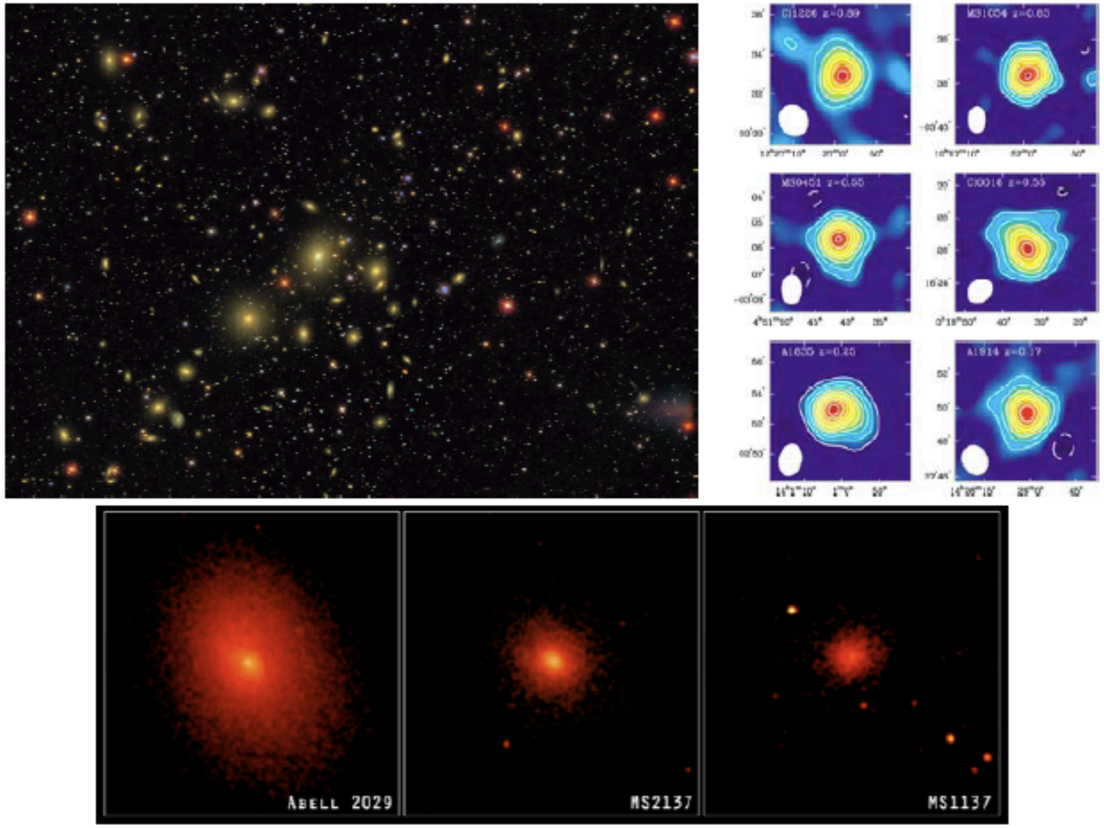
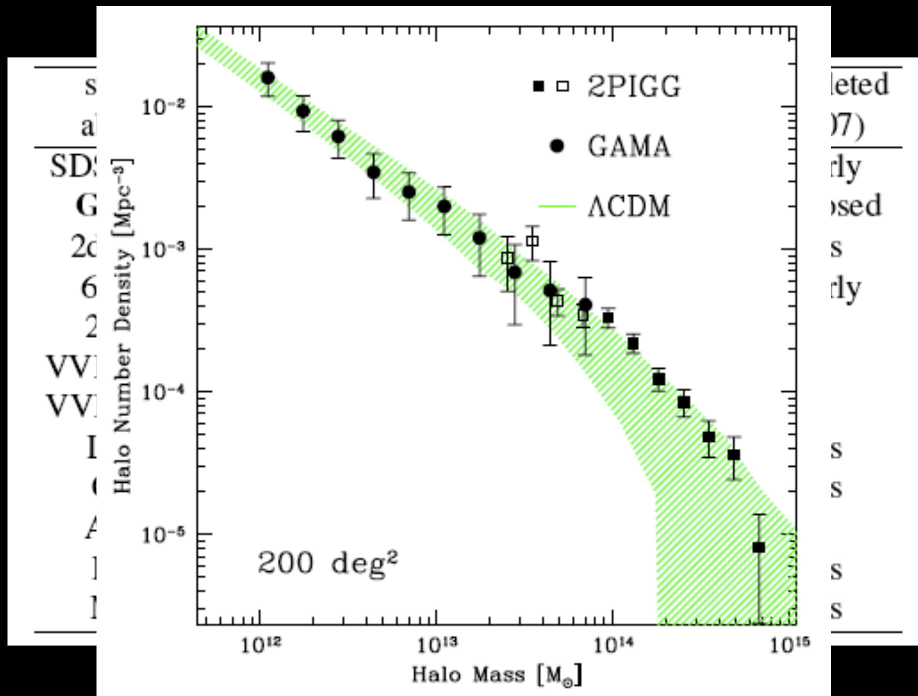


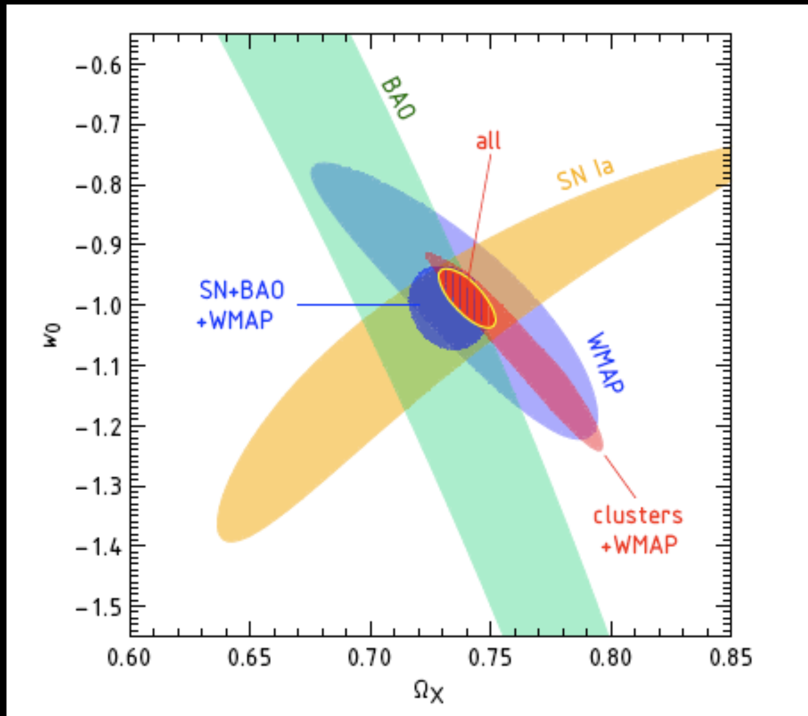
Fig. VI-5: Galaxy clusters as viewed in three different spectral regimes: top left, an optical view showing the concentration of yellowish member galaxies (SDSS); top right, Sunyaev, Zel'dovich flux decrements at 30 GHz (Carlstrom, et al. 2001); bottom, x-ray emission (Chandra Science Center). These images are not at a common scale.

- The DM halo mass function will be accurately tested by planned large-scale galaxy surveys, both ground (e.g. Large Synoptic Survay Telescope, Galaxy And Mass Assembly, volume comparable to horizon size) and satellite (e.g. the ESA EUCLID) based (optical, weak lensing, X-ray emission, SZ effect are complementary)



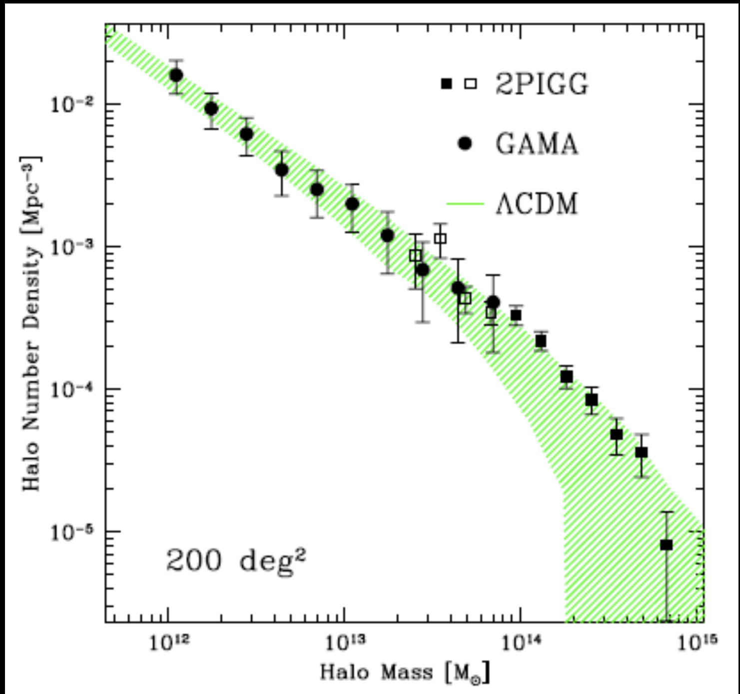
Why the halo mass function is so relevant ?

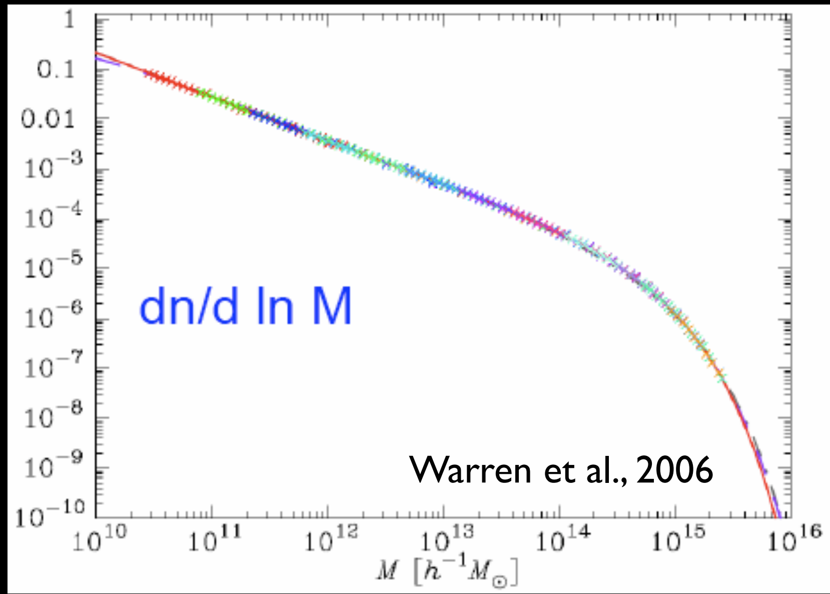
$dn/dV\,dz$ is exponentially sensitive to the Dark Energy through the growth function



X-ray cluster cosmology white paper, arXiv: 0903.5320

How dark matter mass is distributed





At present the knowledge of the halo mass function comes mainly from N-body simulations

The structure in the Universe

Perturbing around the average energy density
we may define the density contrast

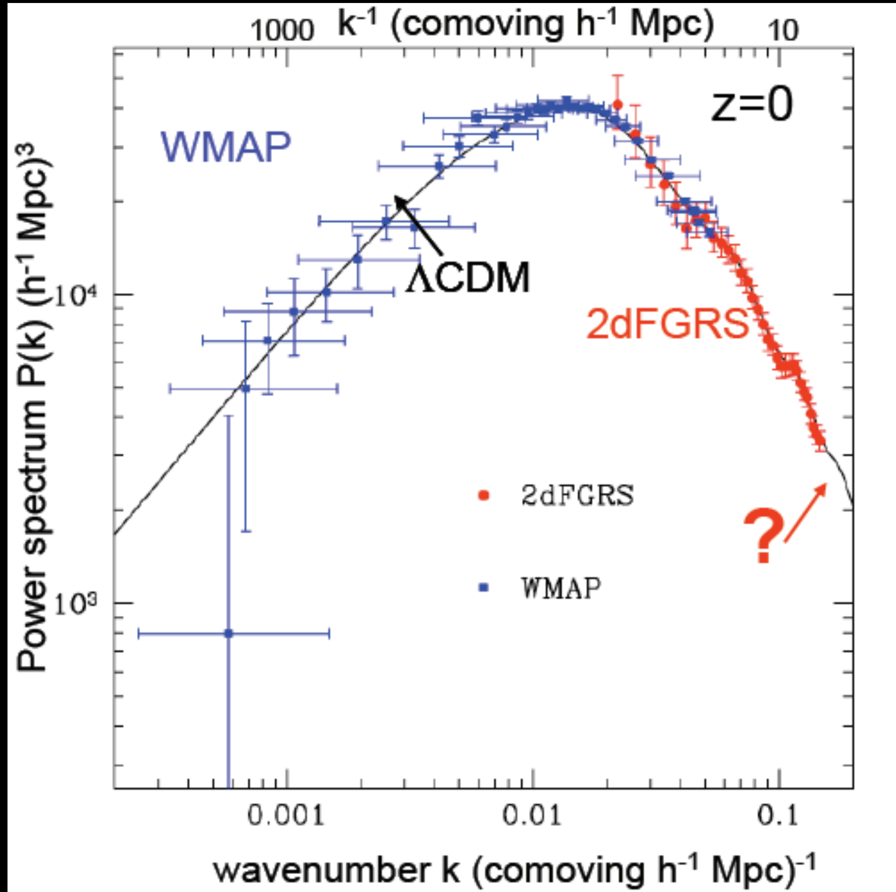
$$\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t)-\overline{\rho}}{\overline{\rho}} = \int \frac{d^3k}{(2\pi)^3}\, \delta_{\mathbf{k}}(t)\, e^{-i\,\mathbf{k}\cdot\mathbf{x}}$$

The power spectrum is defined by

$$\langle \delta_{\mathbf{k}}\, \delta_{\mathbf{k}'} \rangle = (2\pi)^3\, P_\delta(k)\, \delta(\mathbf{k}-\mathbf{k}')$$

$$\boxed{\Delta_\delta(k) = \frac{k^3\, P_\delta(k),}{2\pi^2},\ \ P_\delta = A\, k^n\, T(k)}$$

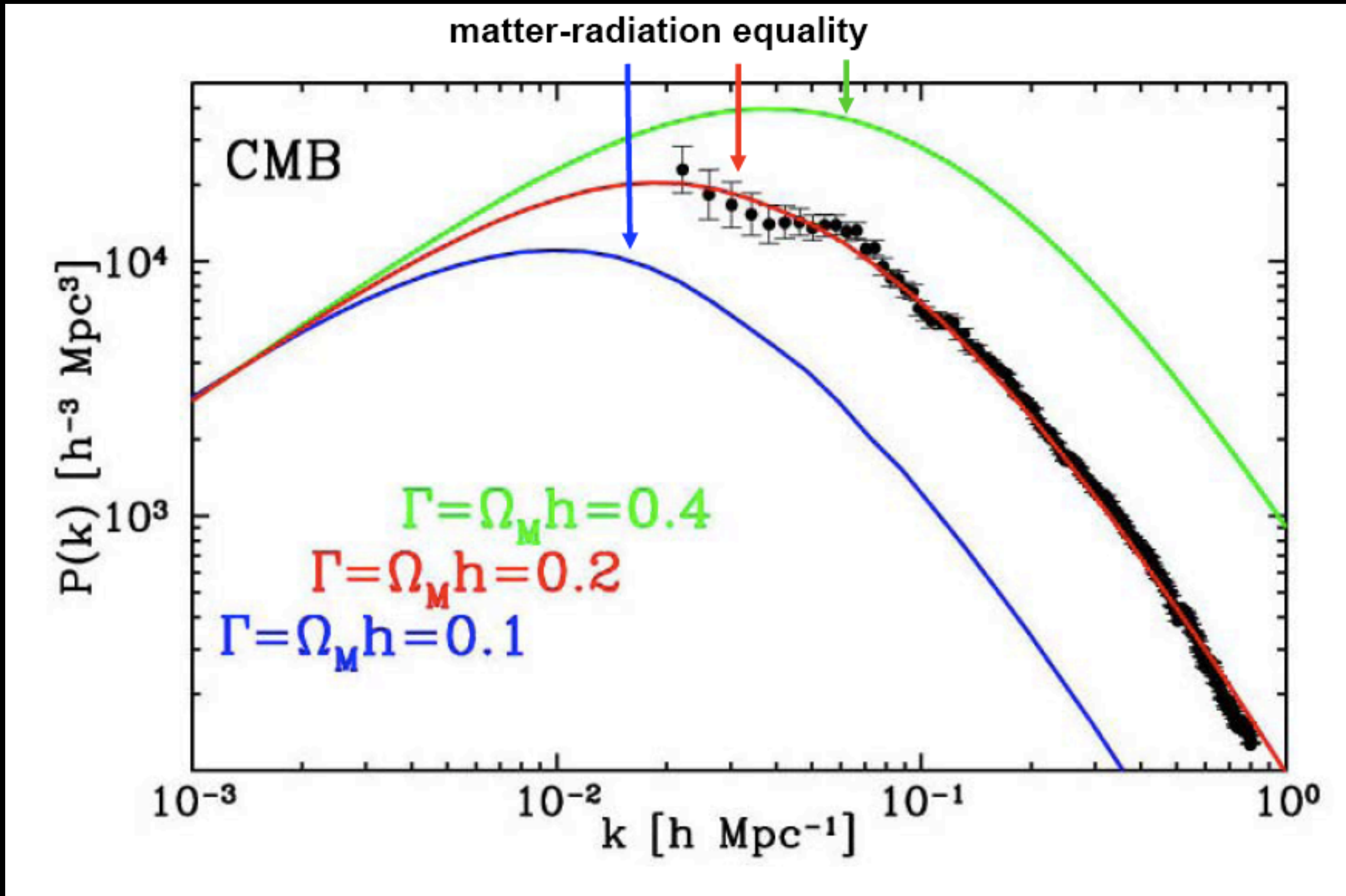
$$n \simeq 1,\ T(k) = \text{transfer function}$$

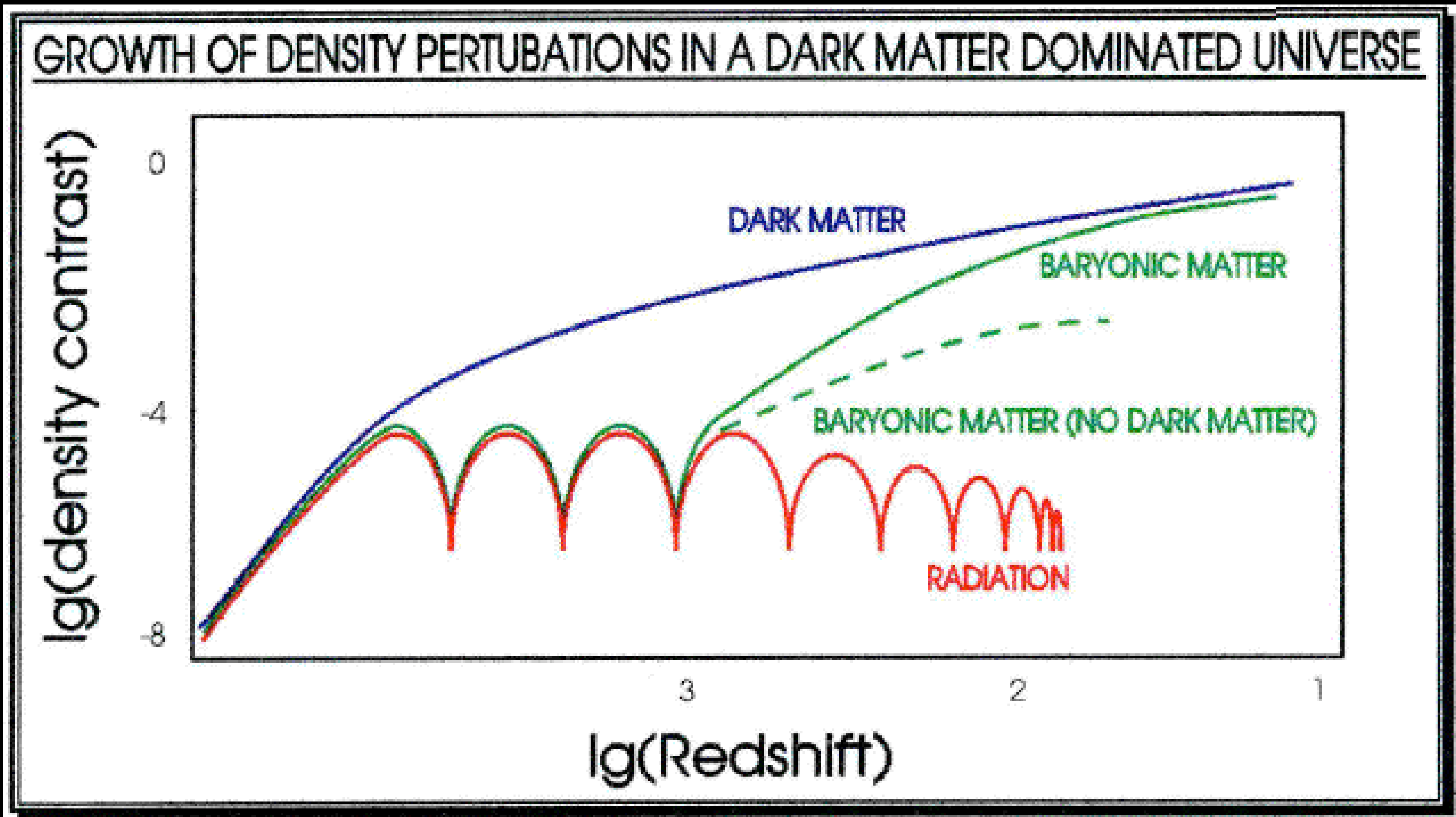


$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} = 4\pi G\,\overline{\rho}\,\delta_{\mathbf{k}} \Rightarrow \delta_{\mathbf{k}} \propto a$$

$$\delta_{\mathbf{k}} = -\frac{2}{3}\frac{k^2}{\Omega_m H^2}\,\Phi_{\mathbf{k}}$$

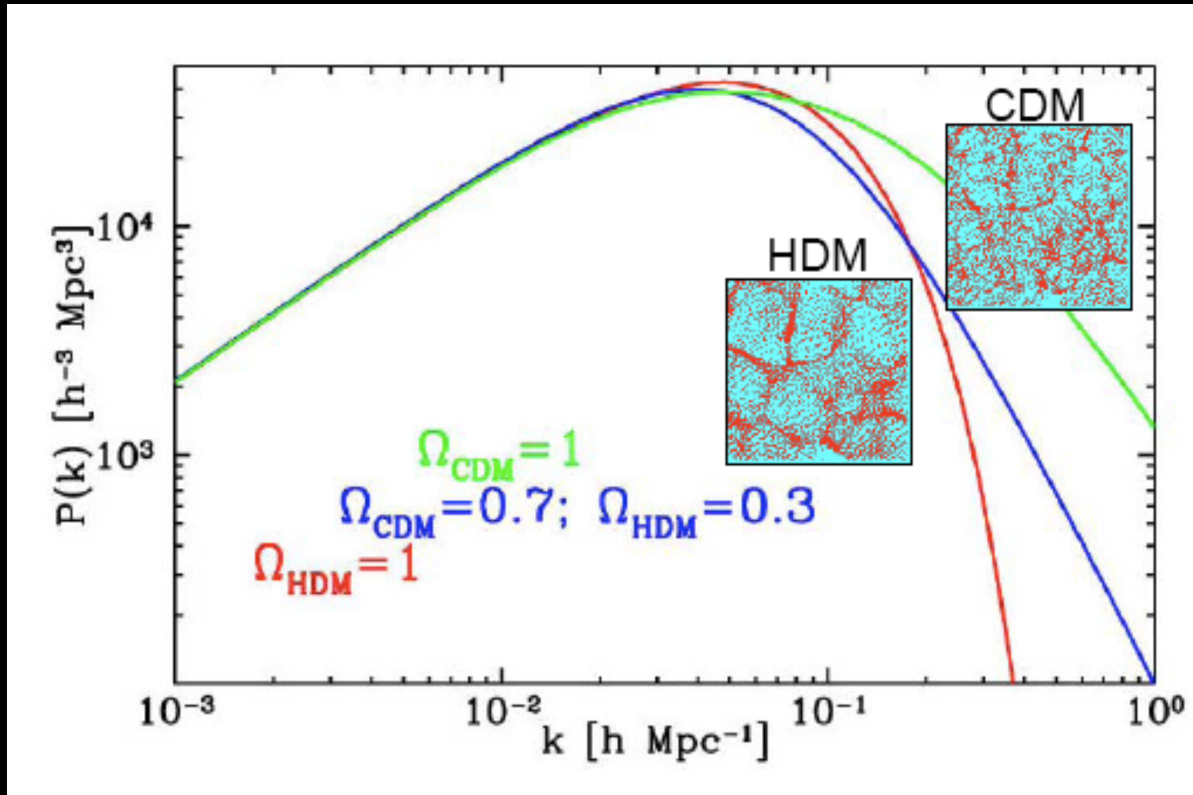
Power spectrum for CDM




GROWTH OF DENSITY PERTUBATIONS IN A DARK MATTER DOMINATED UNIVERSE
lg(density contrast)
0
-4
-8
DARK MATTER
BARYONIC MATTER
BARYONIC MATTER (NO DARK MATTER)
RADIATION
3
2
1
lg(Redshift)

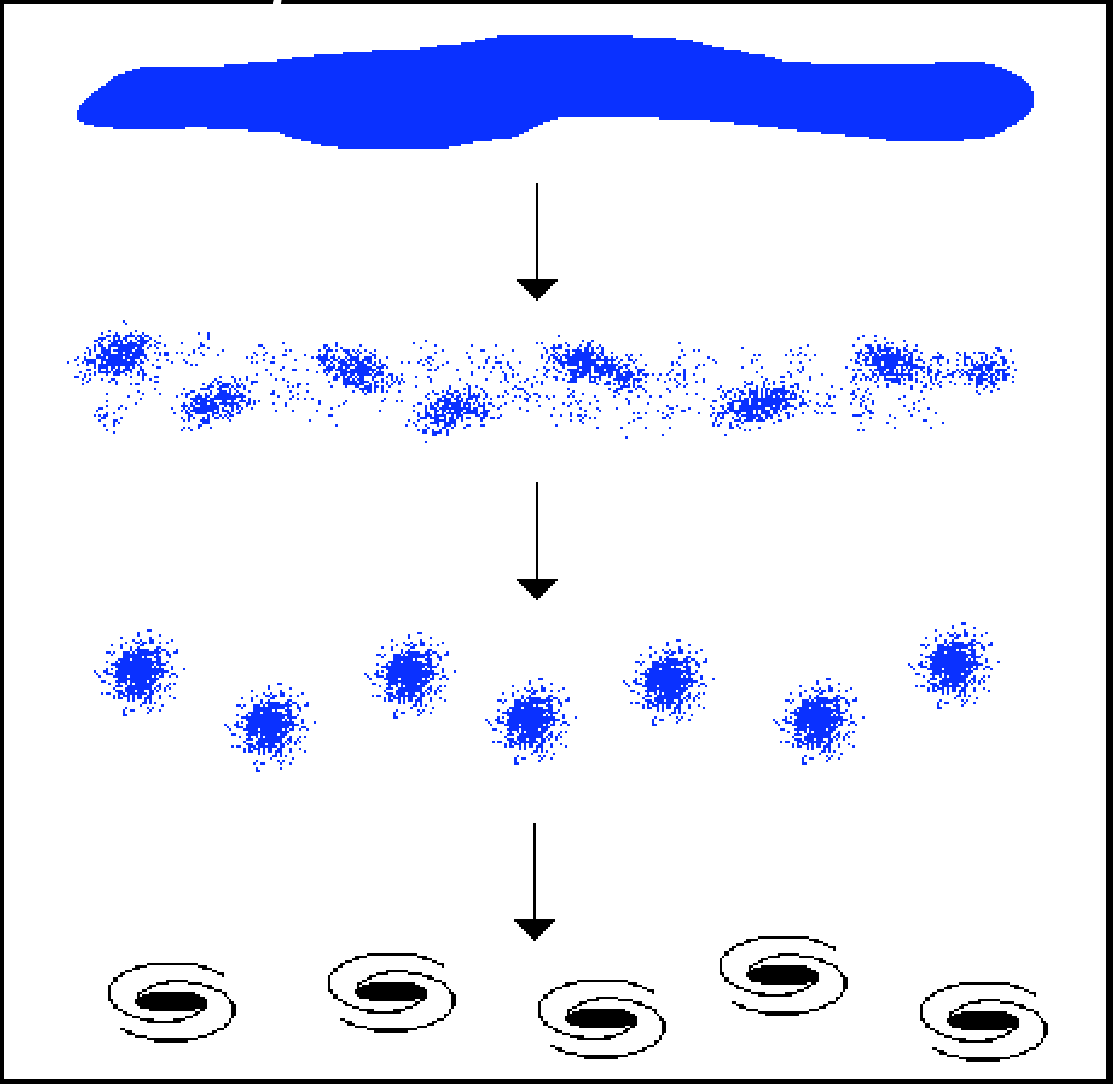
Particle Dark Matter

DM is not in the form of light massive neutrinos

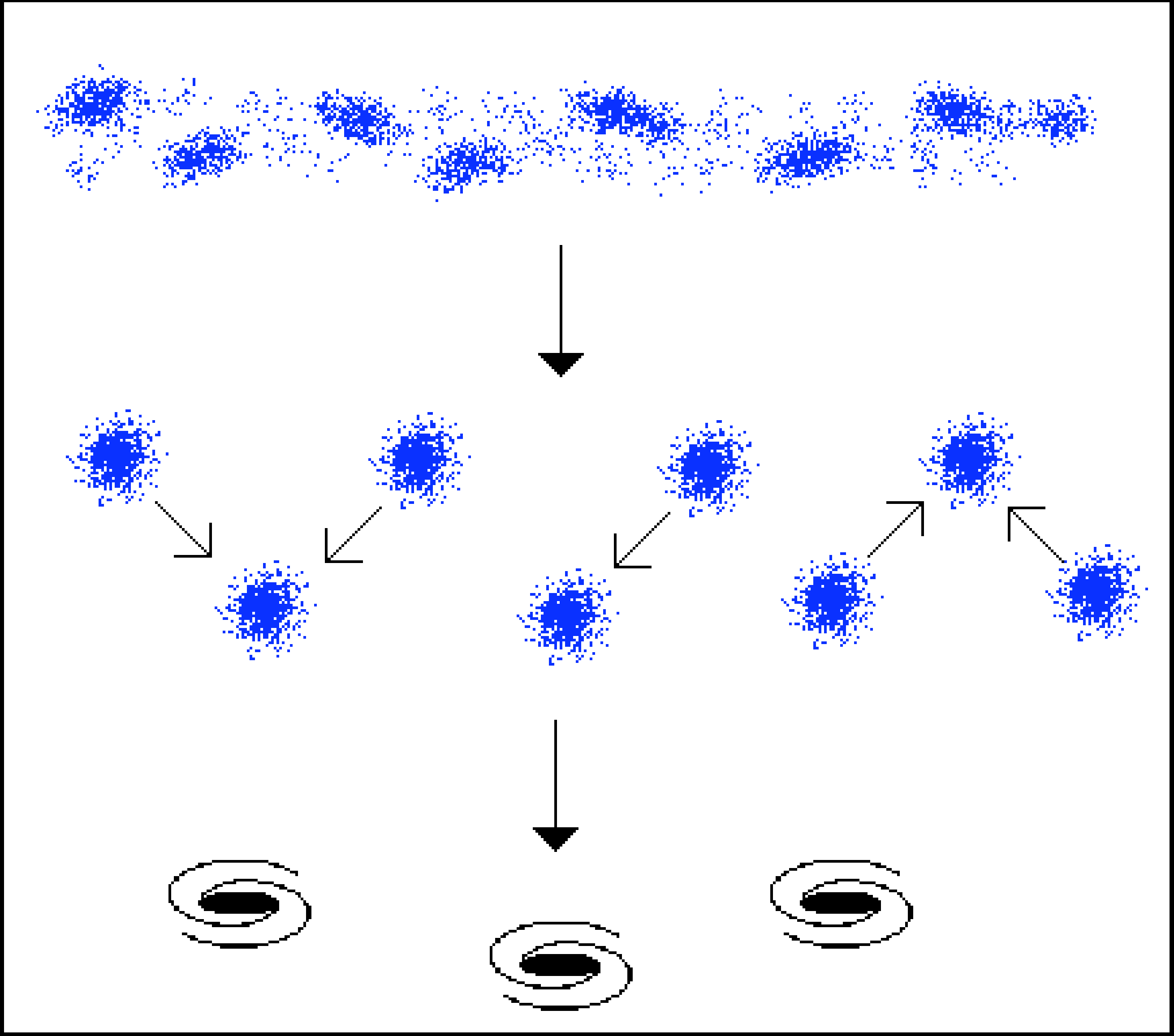


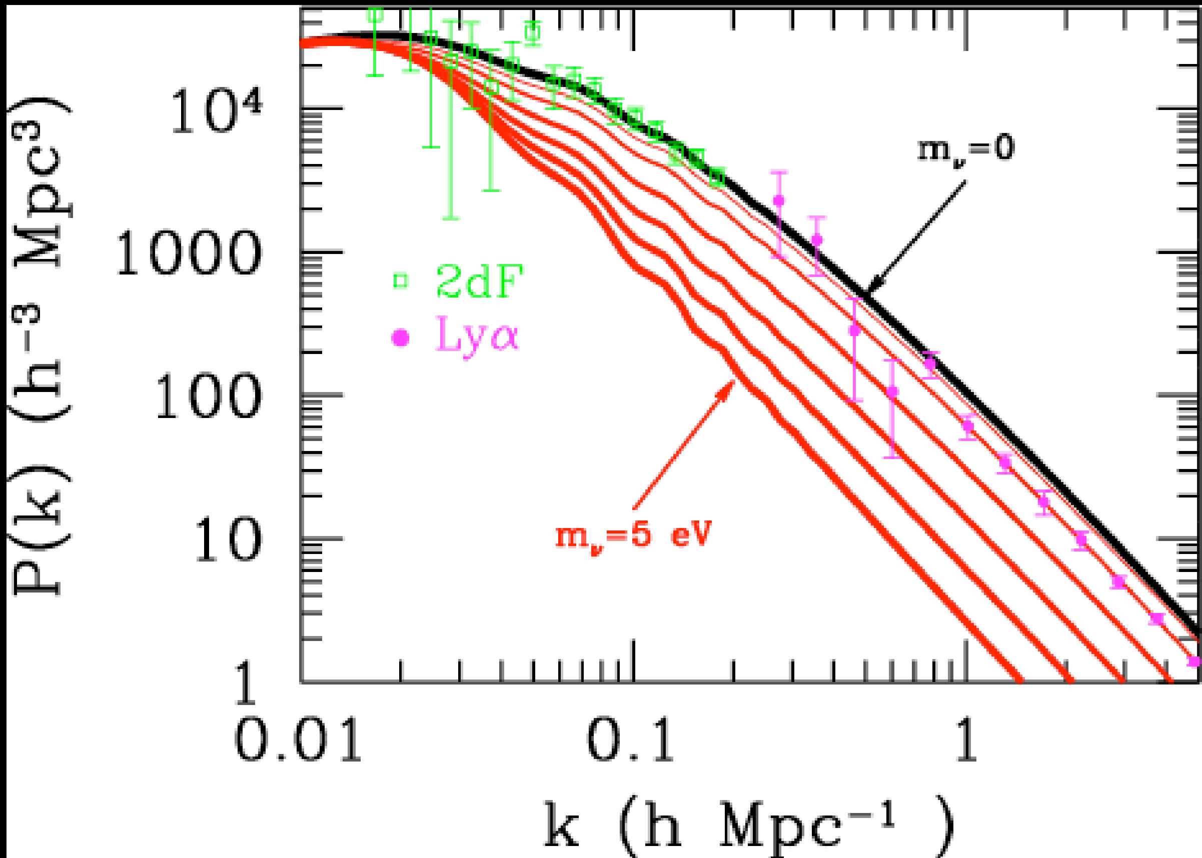
$$\lambda_\nu \sim 40\,\mathrm{Mpc}\,(30\,\mathrm{eV}/m_\nu)$$

HDM - Top-Down Pancake Scenario

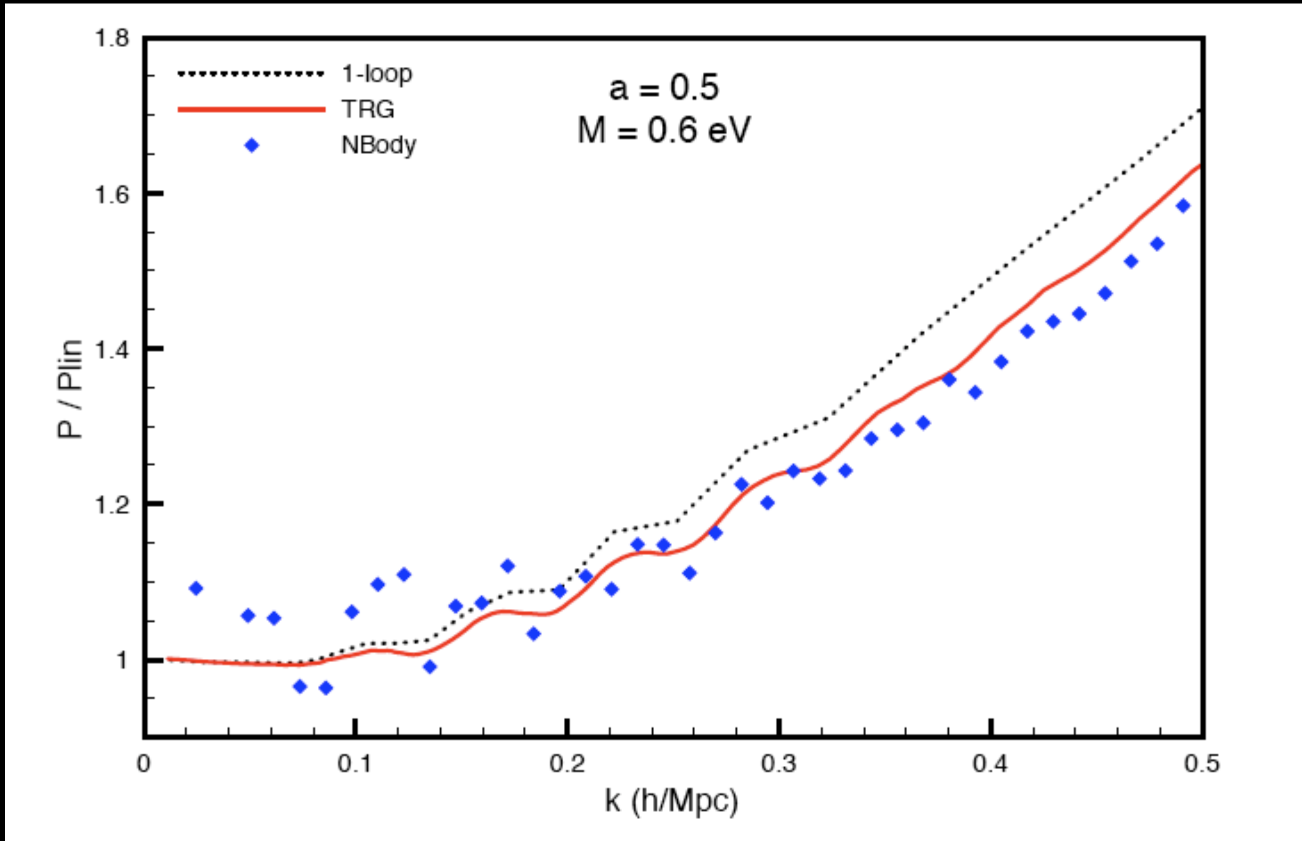


CDM paradigm





P(k) (h^-3 Mpc^3)
10^4
1000
100
10
1
0.01
0.1
1
k (h Mpc^-1)
2dF
Lyα
m_ν=0
m_ν=5 eV

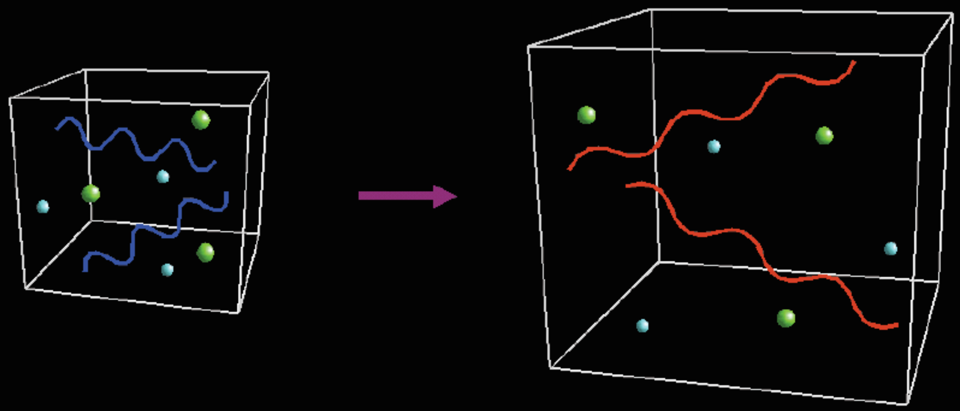


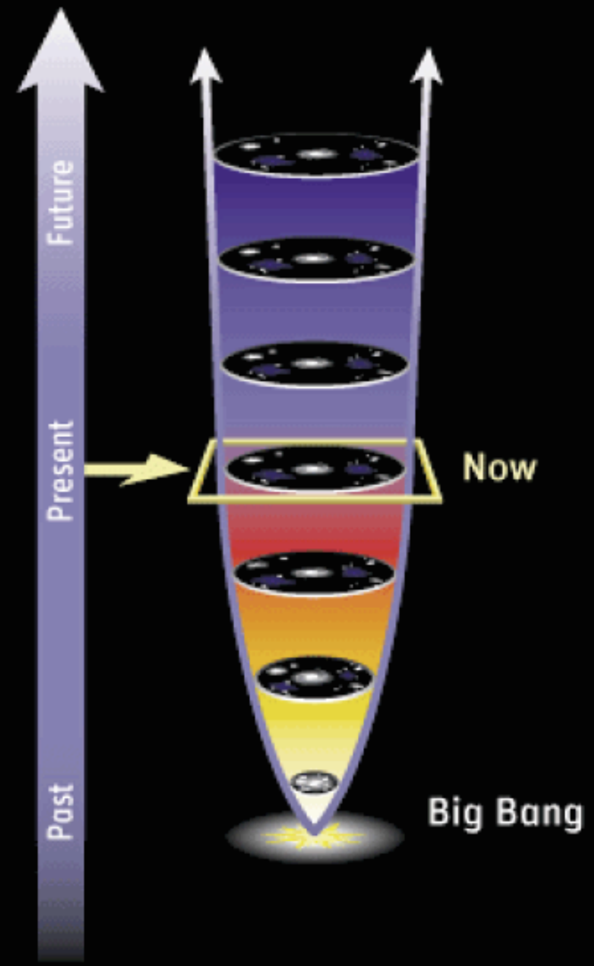
J. Lesgourgues, M. Pietroni and A.R., 2009

Known facts

- Must be cold: by the time of matter-radiation equality and until now, DM must be nonrelativistic and clump together by gravitational attraction
- must be neutral
- WIMP paradigm

The past





Future
Present
Past
Now
Big Bang

At high temperatures, the Universe is expected to be Radiation Dominated

IF equilibrium holds, then

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \;\; (T \gg m)$$

$$g_* = \sum_{\text{bosons}} g_{\mathrm{b}} + \frac{7}{8} \sum_{\text{fermions}} g_{\mathrm{f}}$$

$$8\pi G_N = \frac{1}{M_p^2}$$

$$\boxed{H \simeq 1.66\, g_*^{1/2} \frac{T^2}{M_p}, \;\; M_p \simeq 1.2 \times 10^{19}\,\mathrm{GeV}}$$

$$\frac{t}{\mathrm{sec}} \sim \left(\frac{\mathrm{MeV}}{T}\right)^2$$

$$\boxed{t_{\mathrm{LHC}} \sim 10^{-14}\,\mathrm{sec}}$$

Entropy Density

$$\boxed{s = \frac{\rho_R + P_R}{T} = \frac{4}{3}\frac{\rho_R}{T} = \frac{2\pi^2}{45} g_* T^3}$$

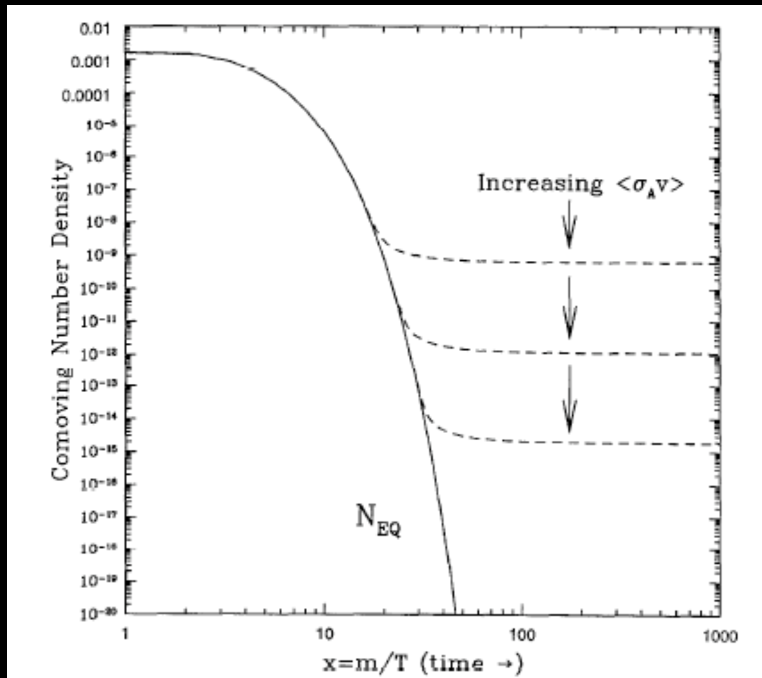
If expansion is adiabatic:

$$S \equiv s \times V = \text{constant} \Rightarrow g_* (Ta)^3 = \text{constant}$$

$$\boxed{T \propto \frac{1}{g_*^{1/3} a}}$$

Thermal relic

- Thermal equilibrium when T>m
- Once T<m, no more WIMPs created
- If stable, the only way to loose them is by annihilation
- Universe expands: at some point comoving number of WIMPs freeze out



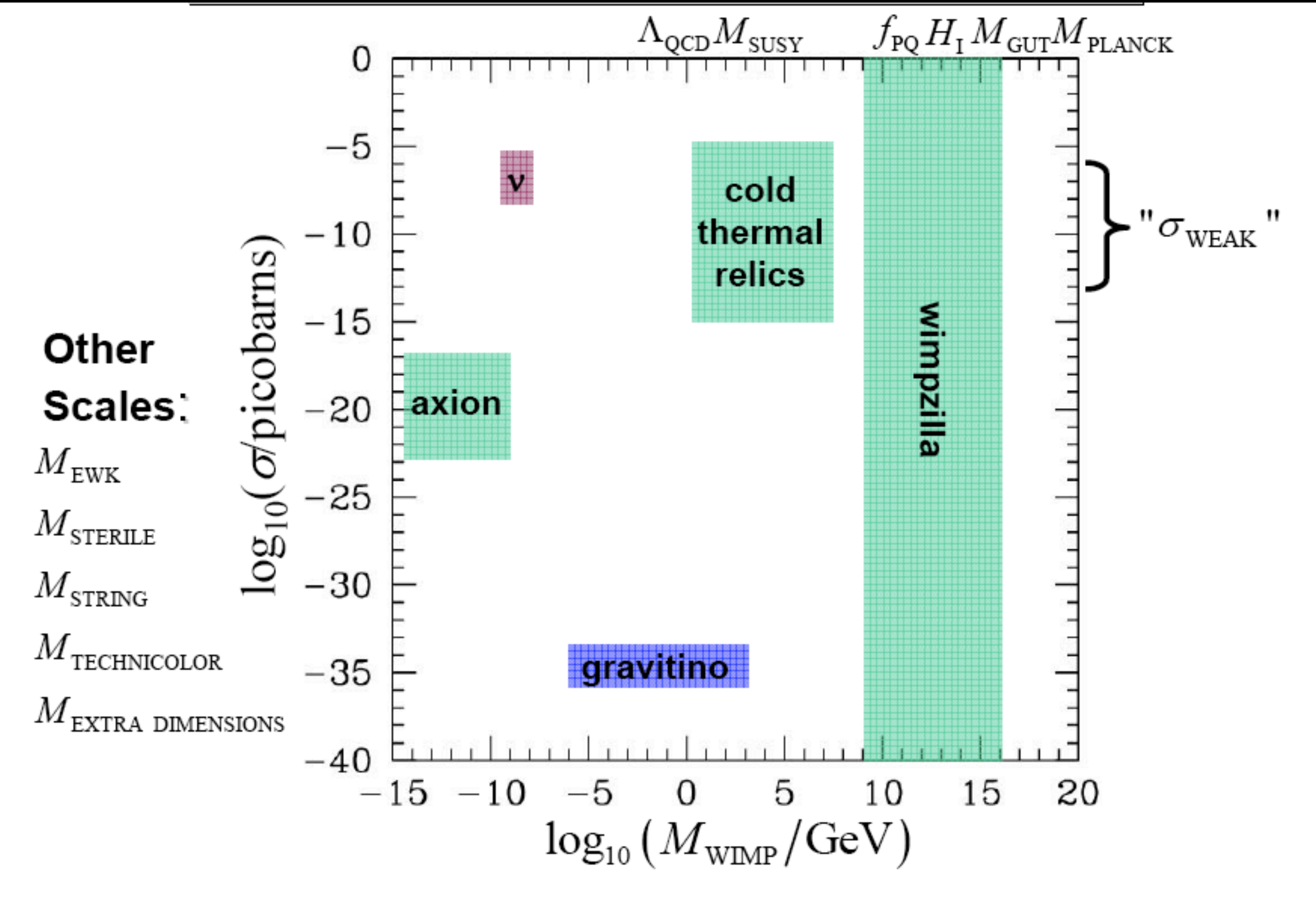
Order of magnitude estimate

$$\begin{aligned}
H &\sim g_*^{1/2}\frac{T^2}{M_p} \\
\Gamma_{\rm ann} &\simeq \langle\sigma_{\rm ann}v\rangle n \\
H(T_f) &\simeq \Gamma_{\rm ann} \\
n &\simeq g_*^{1/2}\frac{T_f^2}{M_p\langle\sigma_{\rm ann}v\rangle} \\
Y &= \frac{n}{s} \simeq g_*^{-1/2}\frac{1}{M_P T_f\langle\sigma_{\rm ann}v\rangle}
\end{aligned}$$

$$\boxed{\Omega\sim(0.2-0.3)\Rightarrow\langle\sigma_{\rm ann}v\rangle\simeq 10^{-9}\,{\rm GeV}^{-2}\simeq\frac{\pi\alpha^2}{m^2}\ \text{for } m\simeq 300\ {\rm GeV}}$$

Coincidence or evidence for new physics at TeV scale?
Impossible to overestimate the discovery of DM at LHC

Particle DM candidates



Split Supersymmetry

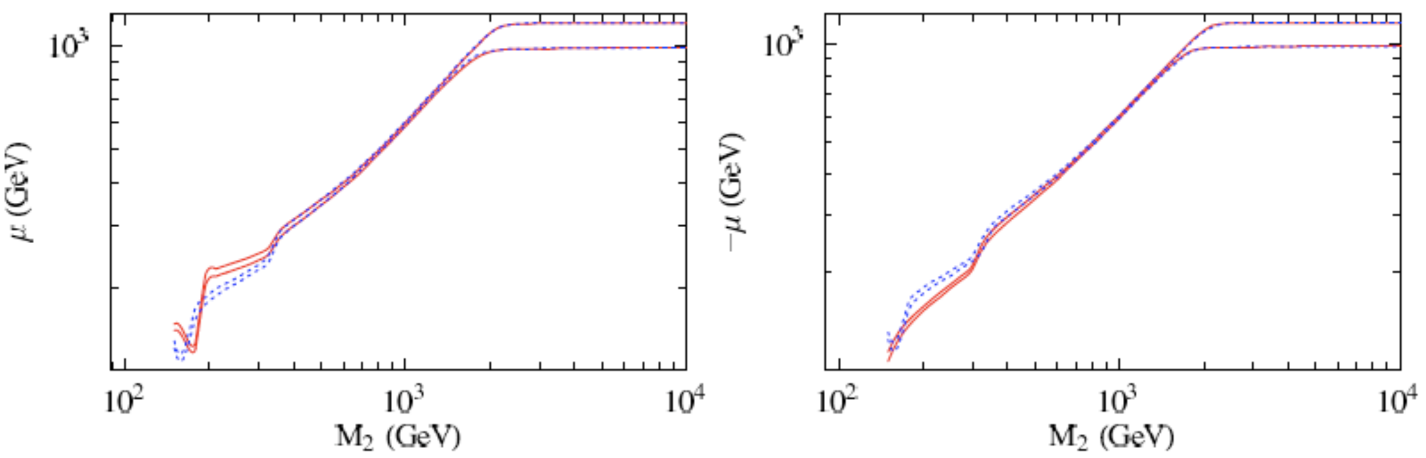
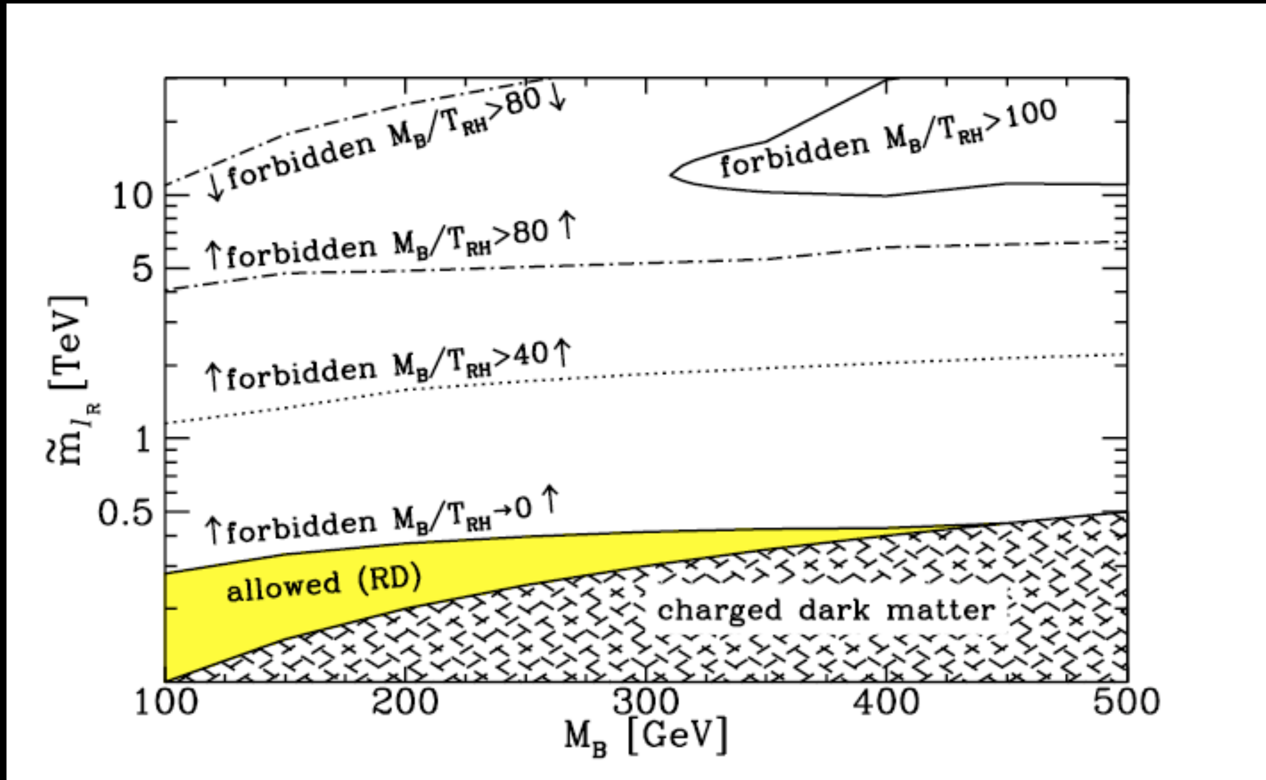


Figure 11: The bands give the values of μ, as functions of M_2, consistent with the dark matter constraint $0.094 < \Omega_{DM}h^2 < 0.129$, with gaugino mass unification The solid lines correspond to $\tan\beta = 4$ and the dashed lines to $\tan\beta = 20$. The left frame is for positive values of μ and the right frame for negative values of μ.

Lots of uncertainties

- Thermal history of the Universe above nucleosynthesis unknown
- DM may be non-thermal
- Hubble rate at freeze-out may depend on DE

Reheating temperature below the freeze-out temperature



DM affected related to the baryon asymmetry?

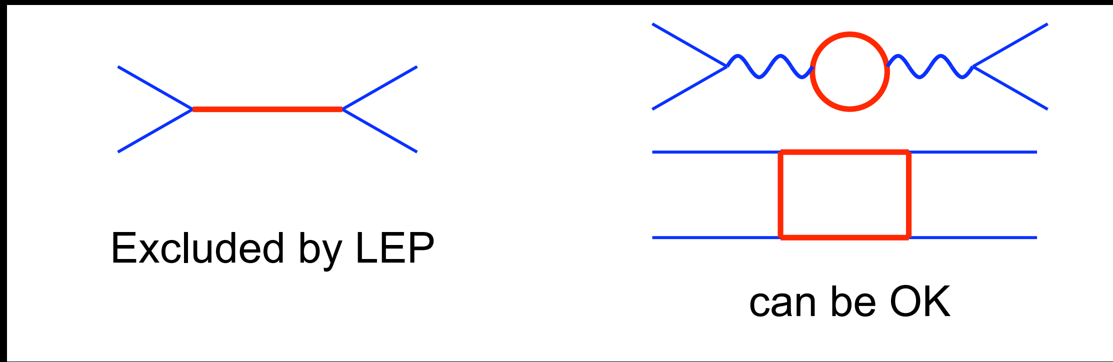
$$\Omega_{\rm DM} \simeq 0.6 \left(\frac{Y_\infty}{10^{-11}}\right)\left(\frac{m_X}{100\,{\rm GeV}}\right)$$

If DM carries an asymmetry

$$\Omega_X \sim \left(\frac{(n_X - n_{\overline{X}})/s}{10^{-11}}\right)\left(\frac{m_X}{30\,{\rm GeV}}\right)$$

Is there an influence of the baryon asymmetry on the DM abundance?

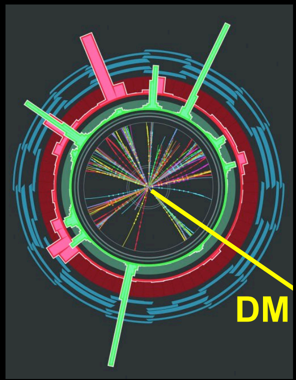
Most viable new theories at EW scale have a DM candidate



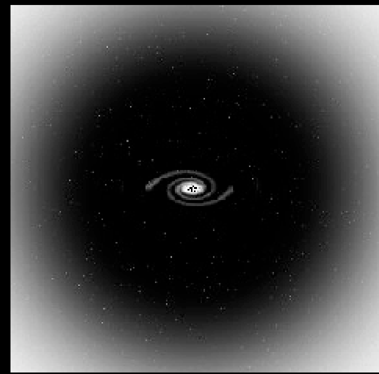
Discrete symmetry (R,T,KK) makes the lightest particle stable

Supersymmetry, Little Higgs, extra-dimensions have a DM candidate

How can LHC establish that a new discovery is the DM of the Universe?

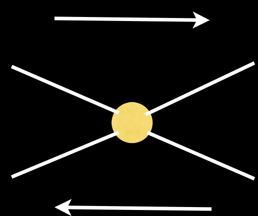


1) Excess of missing energy



2) Reconstructing DM abundance

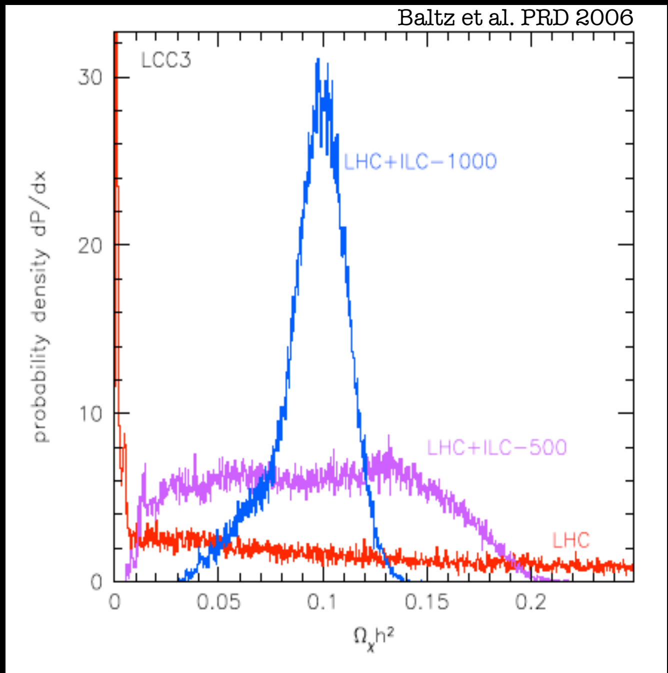
Early Universe

DM  matter $\Rightarrow \Omega_{\rm DM}$

LHC

Warning:
LHC will not provide all the answers
&
will explore DM candidates in a limited energy range

- Even if SUSY particles are discovered, it will be challenging to reconstruct the abundance



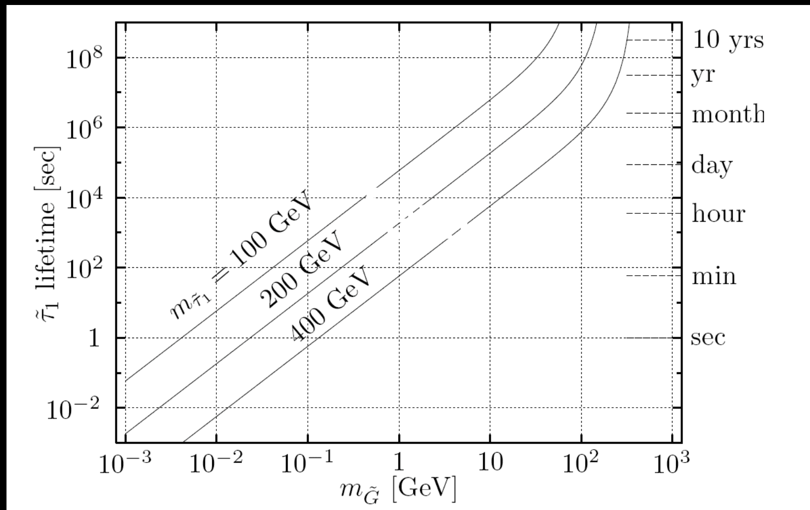
	Ωh^2	LHC
LCC1	0.192	7.2%
LCC2	0.109	82.%
LCC3	0.101	167%
LCC4	0.114	405%

- Collider experiments do not distinguish between stable $(\tau > 10^{17}\,\mathrm{sec})$ and long-lived $(\tau > 10^{-7}\,\mathrm{sec})$ particle

$$\Omega_{\mathrm{DM}} = \frac{m_{\mathrm{DM}}}{m_{\mathrm{dec}}}\Omega_{\mathrm{dec}}$$

Long-lived charged particle at LHC

$$\widetilde{\tau} \to \tau + \widetilde{G}$$

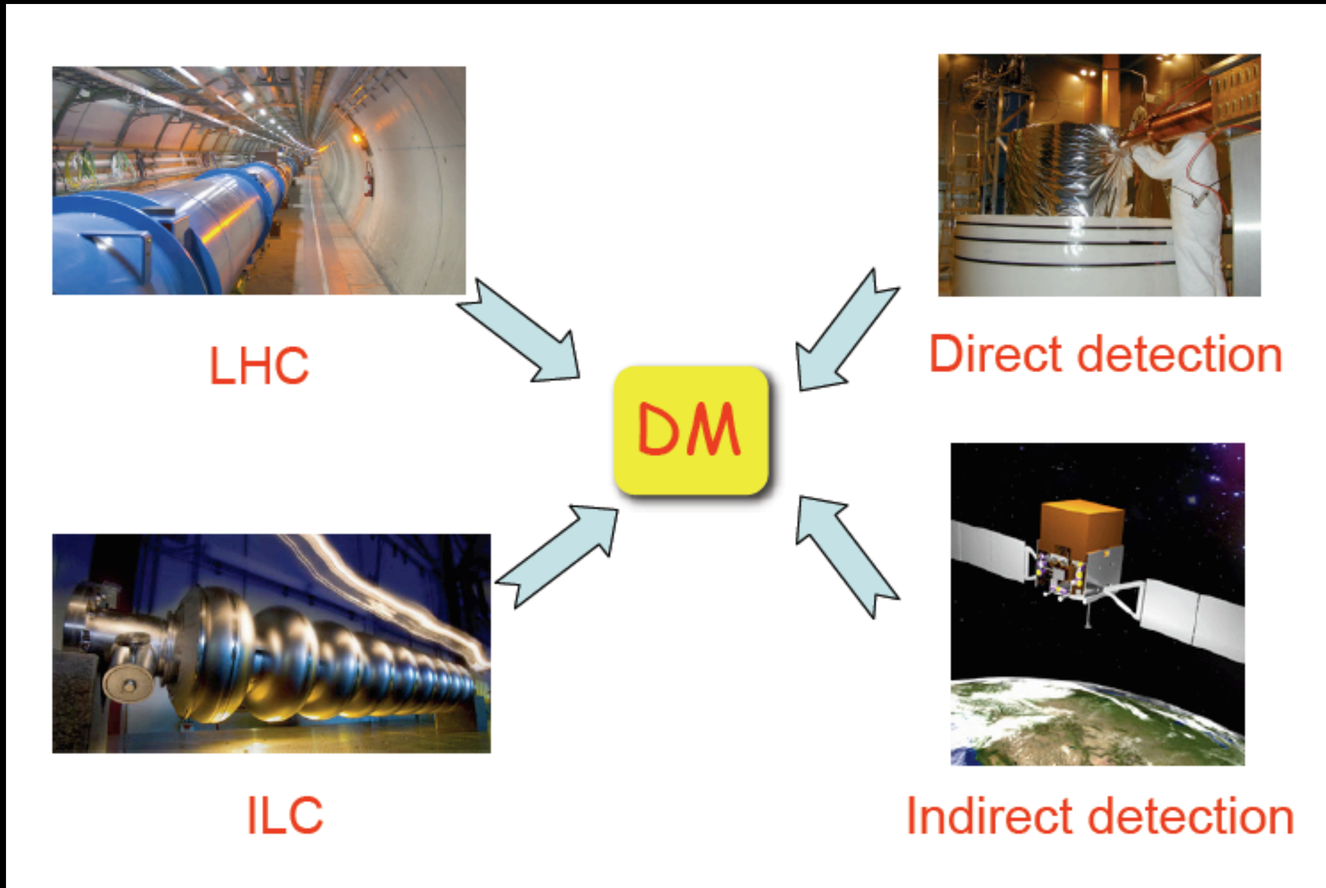


Gravitino: distinctive ToF and energy loss signatures

Measure position and time of stopped stau; time and energy of tau
Reconstruct SUSY scale and gravitational coupling

Hamaguchi, Kuno, Nakaya and Nojiri; Feng and Smith; Ellis, Raklev and Oye, ...

Combining collider with DM searches



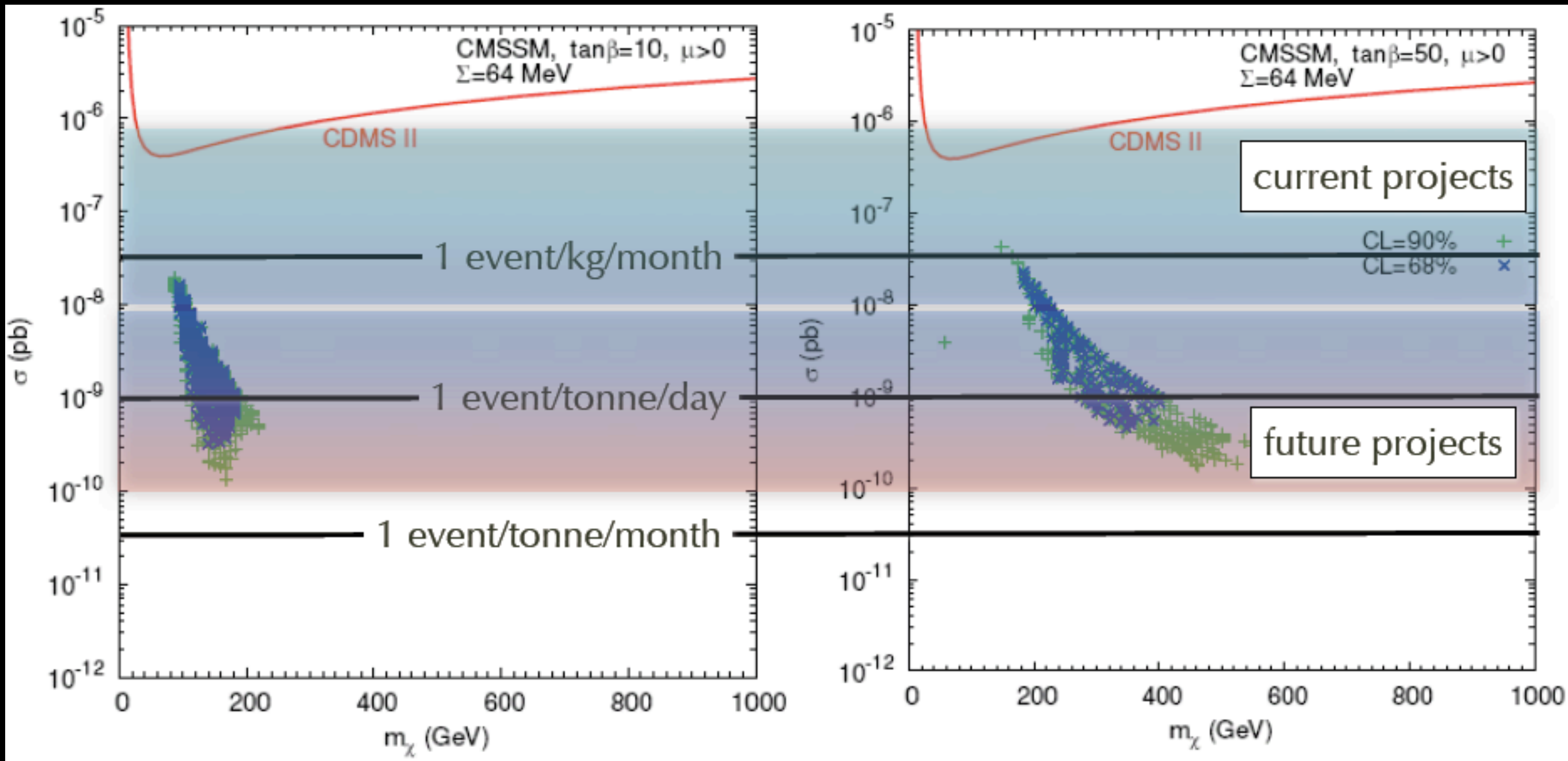
Collider: precise test of DM mass and particle physics parameters, but no information on stability, halo density, etc.

Only detection by different methods may conclusively identify the nature of DM

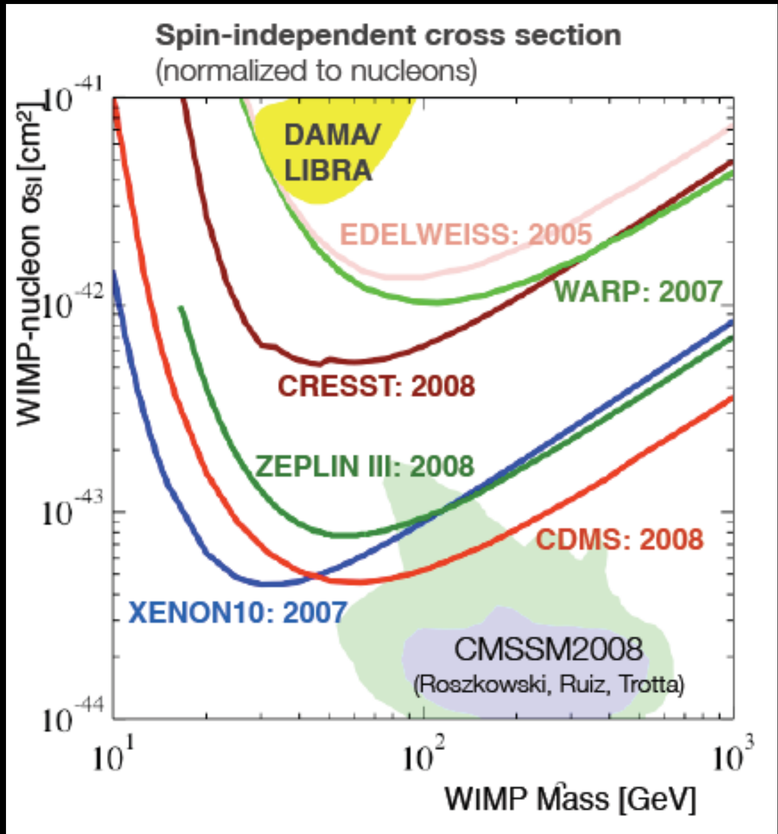
Finding DM: Direct Method

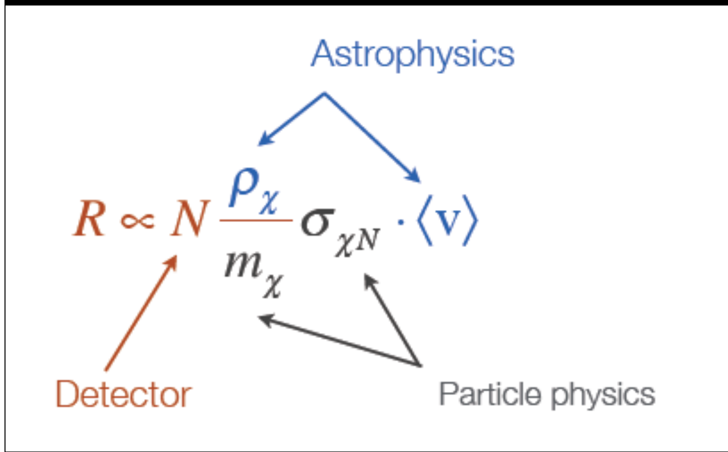
What are the challenges?

- Event rate is small, energy deposited is small, need large scale detectors
- Low (KeV) thresholds for nuclea recoils
- Low background, especially neutrons
- Good background rejection

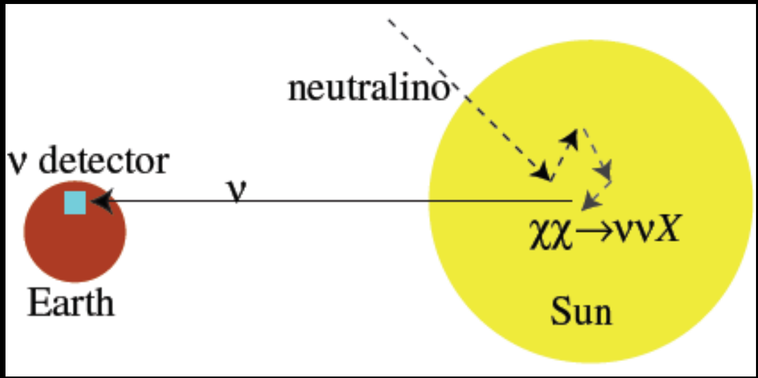


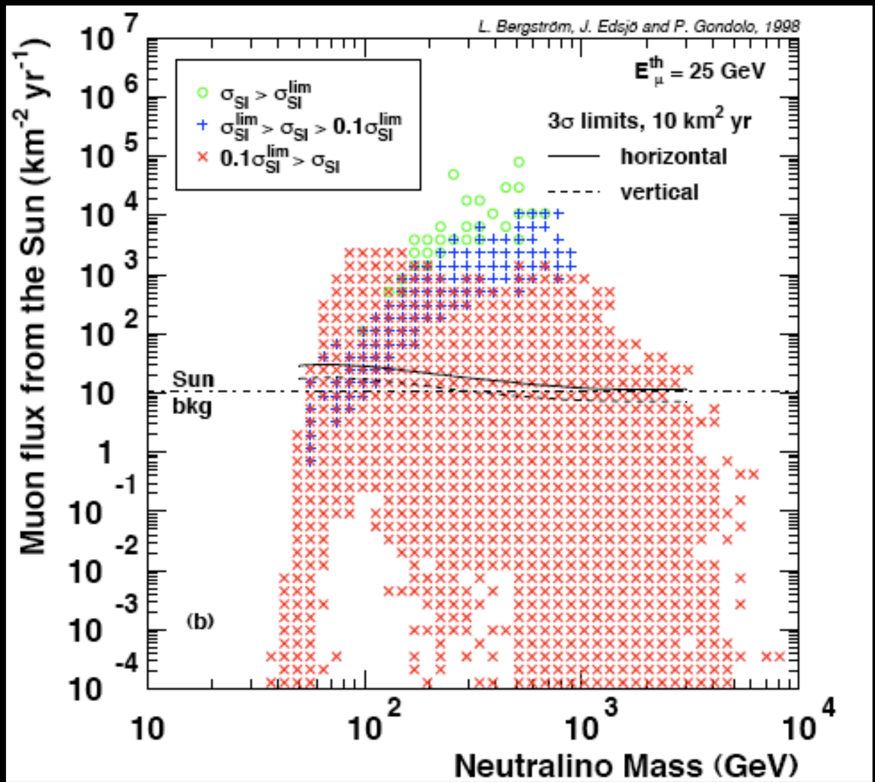
CMSSM, tanβ=10, μ>0
Σ=64 MeV
CDMS II
1 event/kg/month
1 event/tonne/day
1 event/tonne/month
σ (pb)
m_χ (GeV)
CMSSM, tanβ=50, μ>0
Σ=64 MeV
CDMS II
current projects
CL=90%
CL=68%
future projects
σ (pb)
m_χ (GeV)

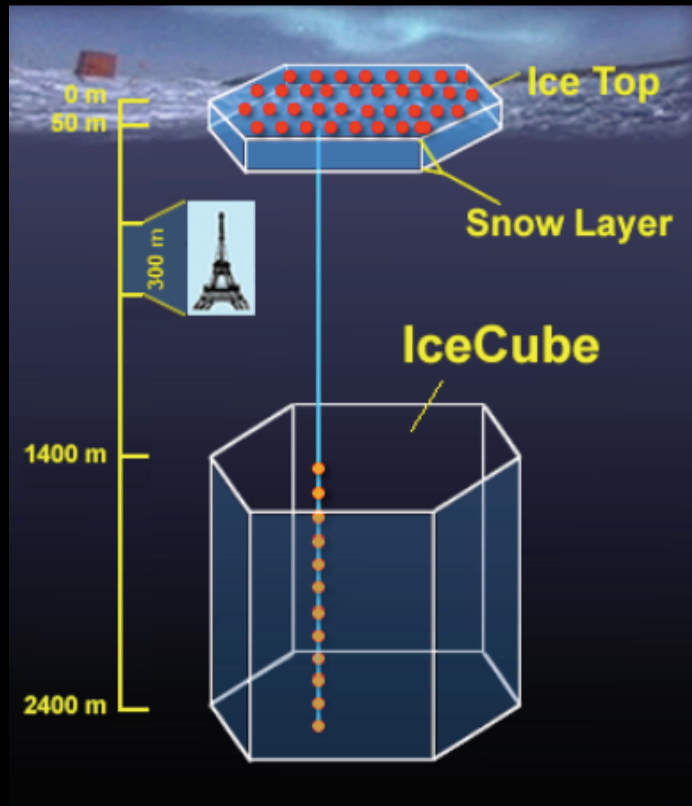

Spin-independent cross section
(normalized to nucleons)
WIMP-nucleon σ_{SI} [cm^2]
WIMP Mass [GeV]
10^{-41}
10^{-42}
10^{-43}
10^{-44}
10^{1}
10^{2}
10^{3}
DAMA/ LIBRA
EDELWEISS: 2005
WARP: 2007
CRESST: 2008
ZEPLIN III: 2008
CDMS: 2008
XENON10: 2007
CMSSM2008
(Roszkowski, Ruiz, Trotta)


Astrophysics
$R \propto N \frac{\rho_\chi}{m_\chi} \sigma_{\chi N} \cdot \langle v \rangle$
Detector
Particle physics

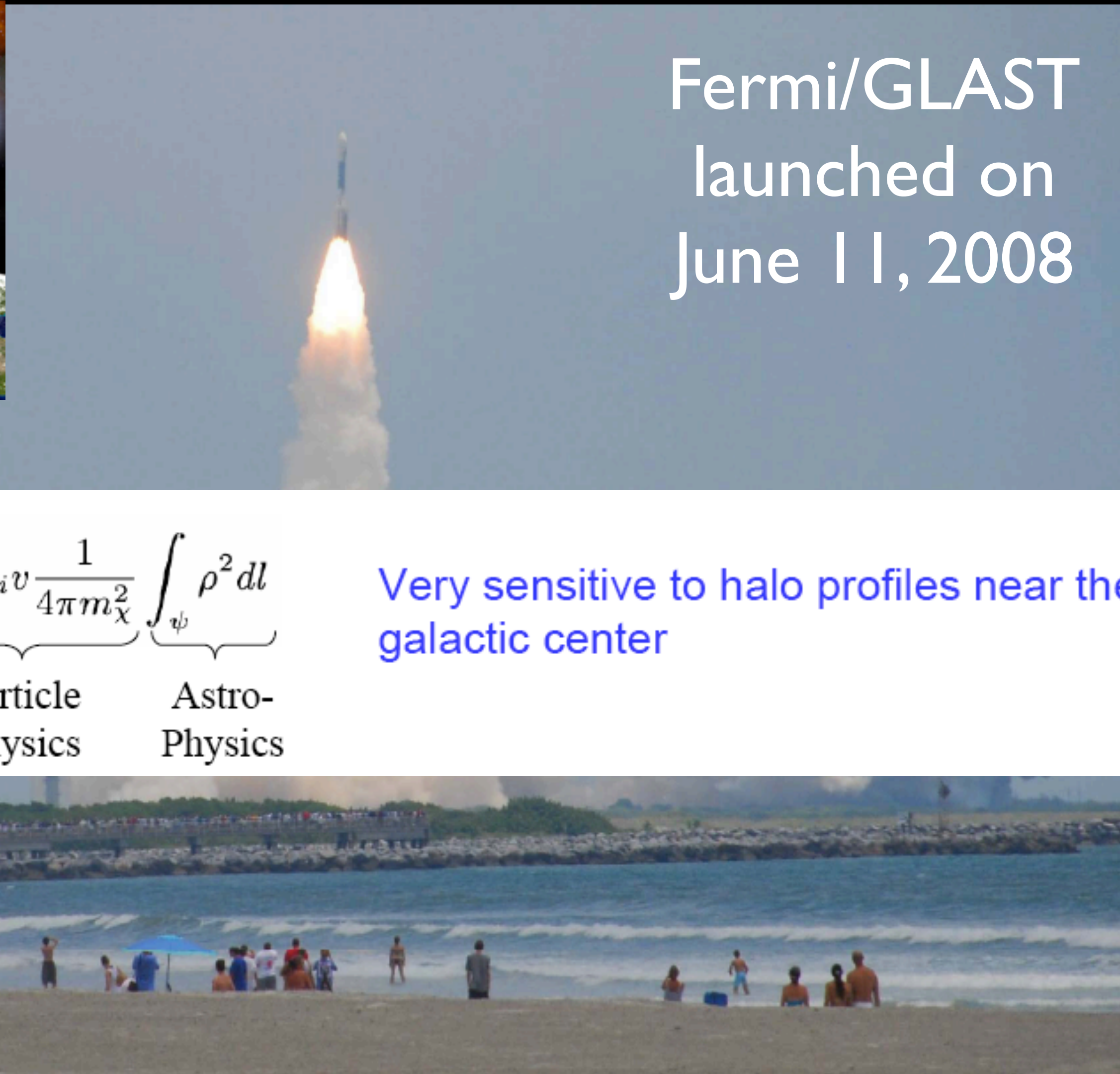
Finding DM: Indirect method







Look for signals from the sky



$$\frac{d\Phi_\gamma}{d\Omega dE} = \sum_i \underbrace{\frac{dN^i_\gamma}{dE}\sigma_i v \frac{1}{4\pi m_\chi^2}}_{\text{Particle Physics}} \underbrace{\int_\psi \rho^2 dl}_{\text{Astro-Physics}}$$

Very sensitive to halo profiles near the galactic center

ICC University of Durham

Cold dark matter searches - 3 key questions -

Q1: What is the mean density structure of a CDM halo?

Q2: How smooth are the dark matter mass and velocity distributions at the solar position?

Q3: Does the halo formation process leave "observable" imprints?

C. Frenk, ENTAap DM meeting, CERN, Feb. 2009

Galactic halos (without the galaxies)

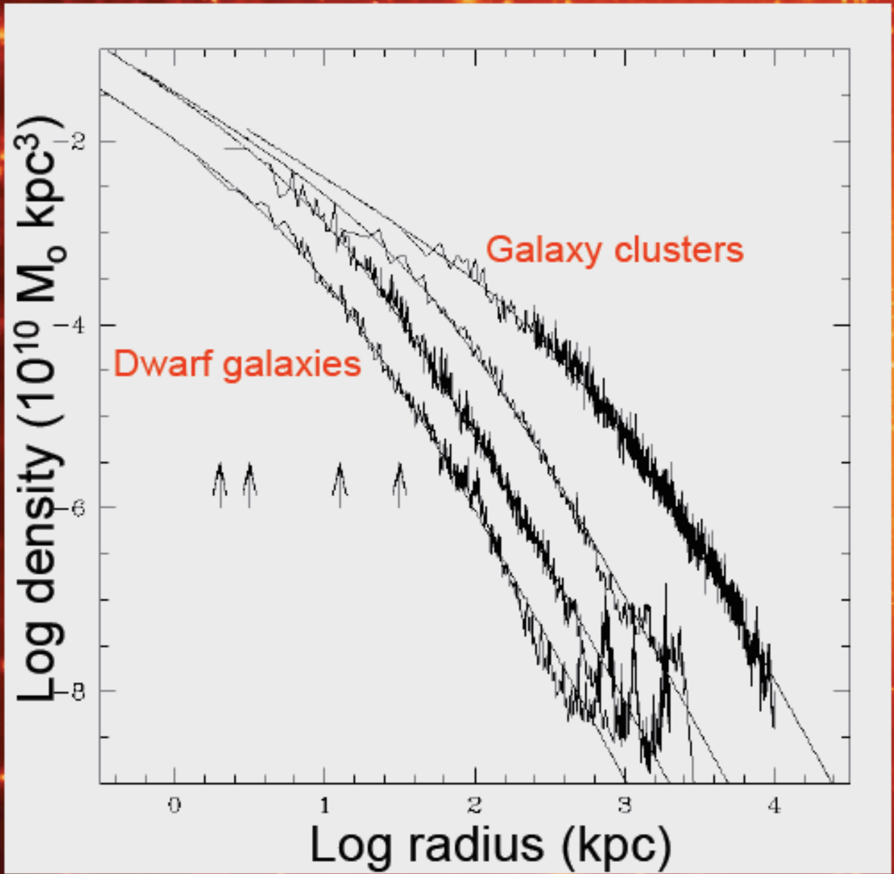
University of Durham

- Halos are extended
  - Extend to ~10 times the 'visible' radius of galaxies and contain ~10 times the mass in the visible regions
- Halos are triaxial ellipsoids (not spherical)
  - more prolate than oblate
  - axial ratios greater than 2 are common
- Halos have nearly universal "cuspy" density profiles
  - $d\ln\rho / d\ln r = \gamma$ with $\gamma < -2.5$ at large r
    $\gamma > -1.2$ at small r
- Halos are clumpy
  - Substantial numbers of self-bound subhalos containing ~10% of the halo's mass

Institute for Computational Cosmology

C. Frenk, ENTAap DM meeting, CERN, Feb. 2009

The Density Profile of Cold Dark Matter Halos



Halo density profiles are independent of halo mass & cosmological parameters

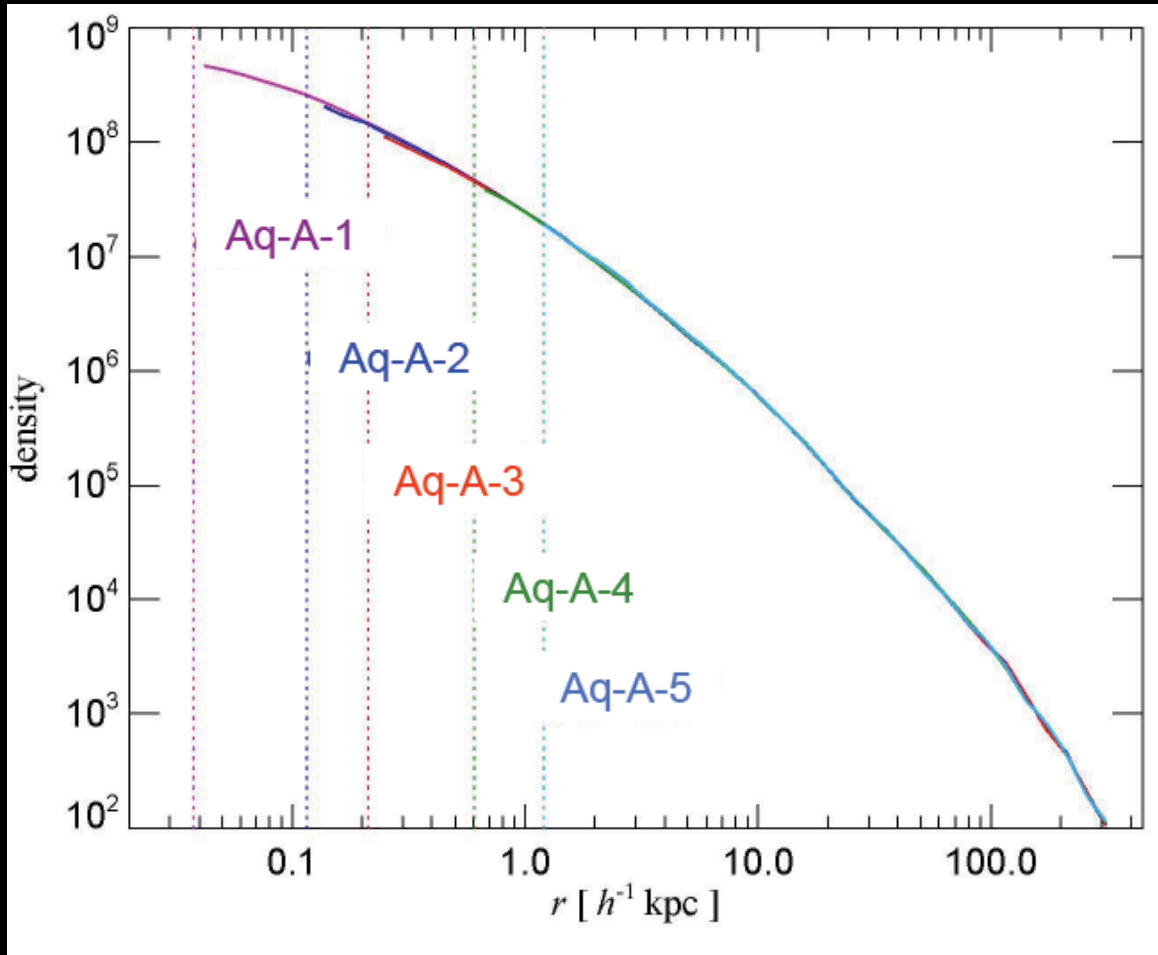
There is no obvious density plateau or `core' near the centre.

(Navarro, Frenk & White '97)

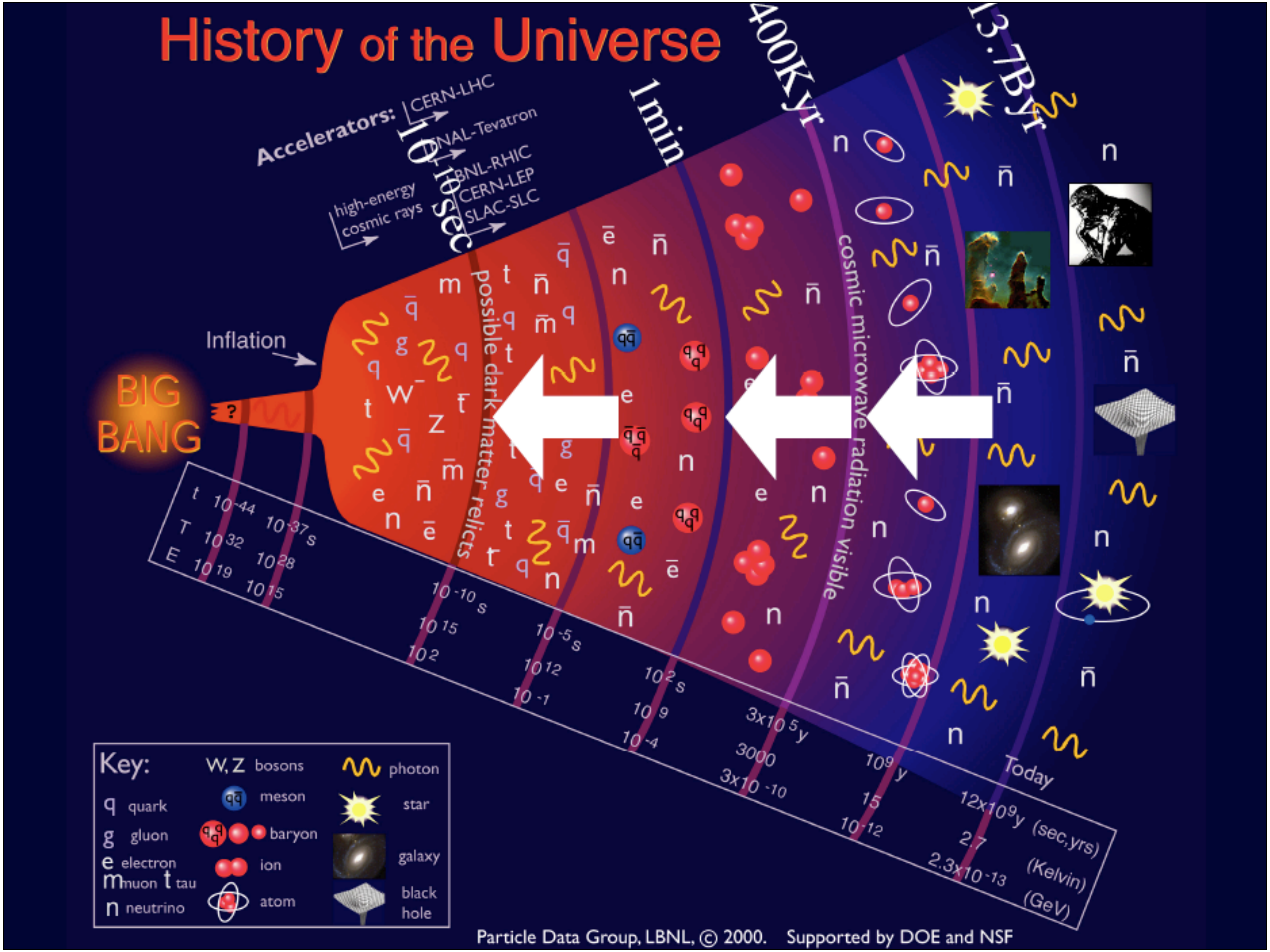
$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

More massive halos and halos that form earlier have higher densities (bigger δ)

C. Frenk, ENTAap DM meeting, CERN, Feb. 2009



10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
density
Aq-A-1
Aq-A-2
Aq-A-3
Aq-A-4
Aq-A-5
0.1
1.0
10.0
100.0
r [h^{-1} kpc]

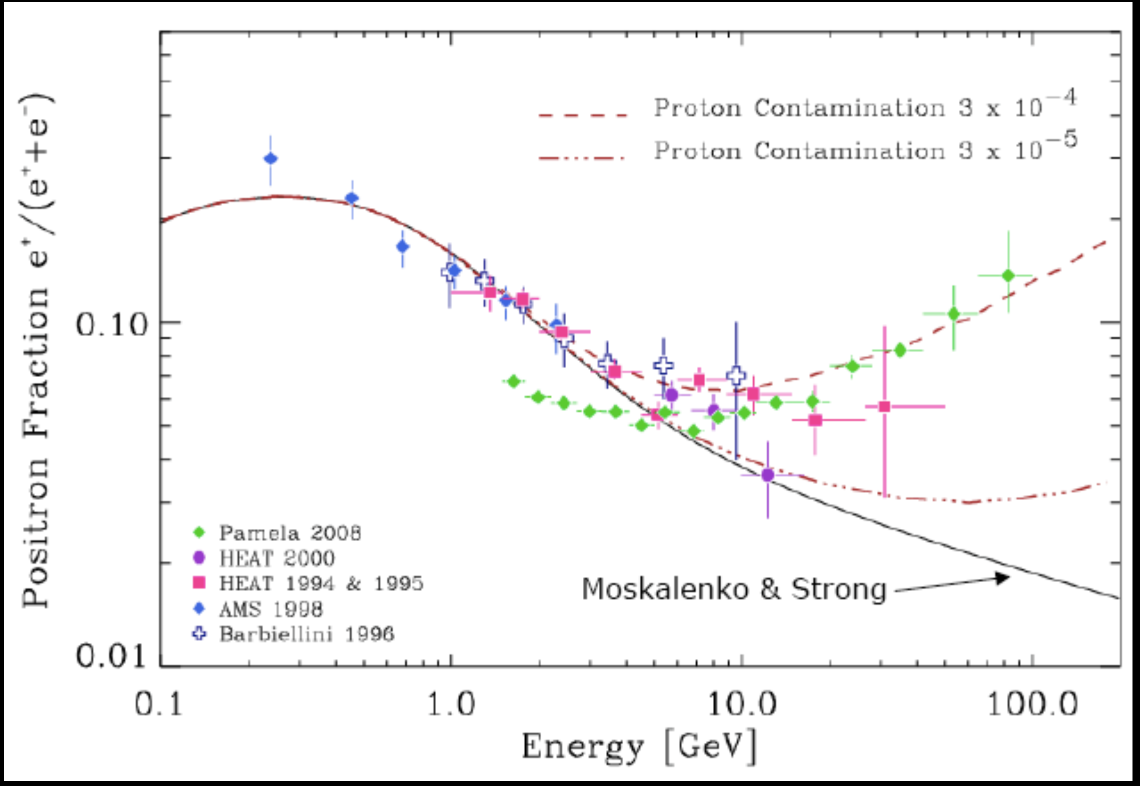

History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
10^{-10} sec
1 min
400Kyr
13.7Byr
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10^{-44} 10^{-37} s
T 10^{32} 10^{28}
E 10^{19} 10^{15}
10^{-10} s
10^{15}
10^{2}
10^{-5} s
10^{12}
10^{-1}
10^{2} s
10^{9}
10^{-4}
$3x10^{5}$ y
3000
$3x10^{-10}$
10^{9} y
15
10^{-12}
Today
$12x10^{9}$ y (sec,yrs)
2.7 (Kelvin)
$2.3x10^{-13}$ (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Despite the Dark Puzzles, the future is brighter

Positrons / electrons from Secondary Production (p-ISM $\rightarrow \pi^+ \rightarrow \mu^+ \rightarrow e^+$)

Flux of CR protons in the energy range 1 – 50 GeV exceeds that of positrons by a factor of ~ 5×10^4

Pamela



Galactic and local pulsars

D. Hooper, P. Blasi & P. D. Serpico, **arXiv:0810.1527** S. Profumo, **arXiv:0812.4457**

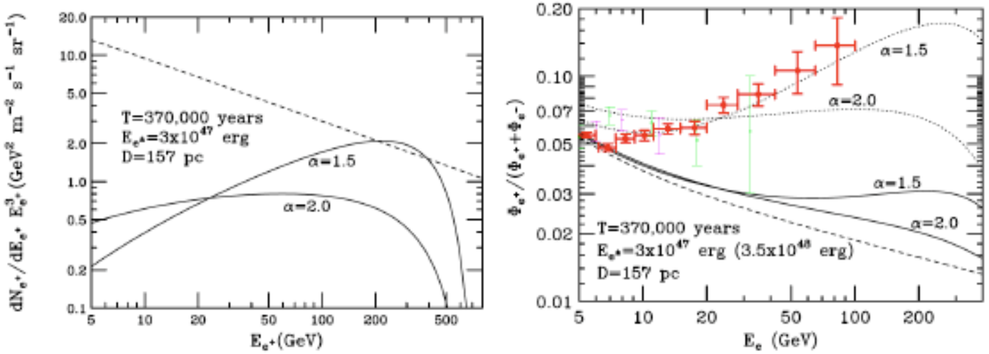


FIG. 2: The spectrum of positrons (left) and ratio of positrons to electrons plus positrons (right) from the pulsar Geminga, with the dashed lines as in Fig. 1. In the right frames, the measurements of HEAT [3] (light green and magenta) and measurements of PAMELA [2] (dark red) are also shown. Here we have used an injected spectrum such that $dN_e/dE_e \propto E^{-\alpha}\exp(-E_e/600\,\mathrm{GeV})$, with $\alpha = 1.5$ and 2.2. The solid lines correspond to an energy in pairs given by 3.5×10^{47} erg, while the dotted lines require an output of 3×10^{48} erg.

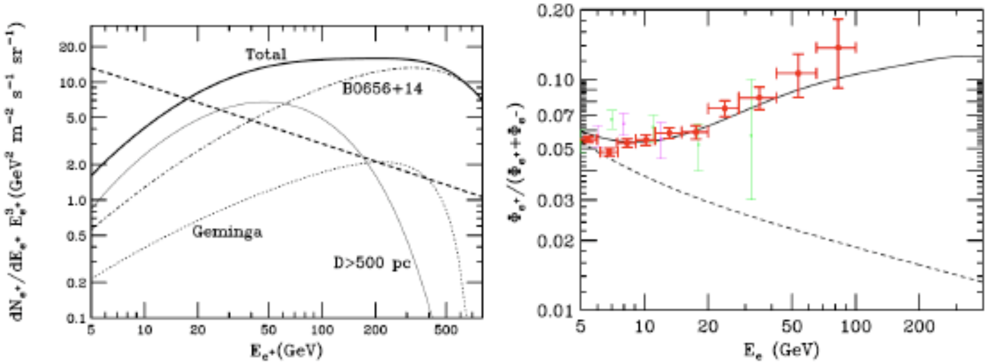
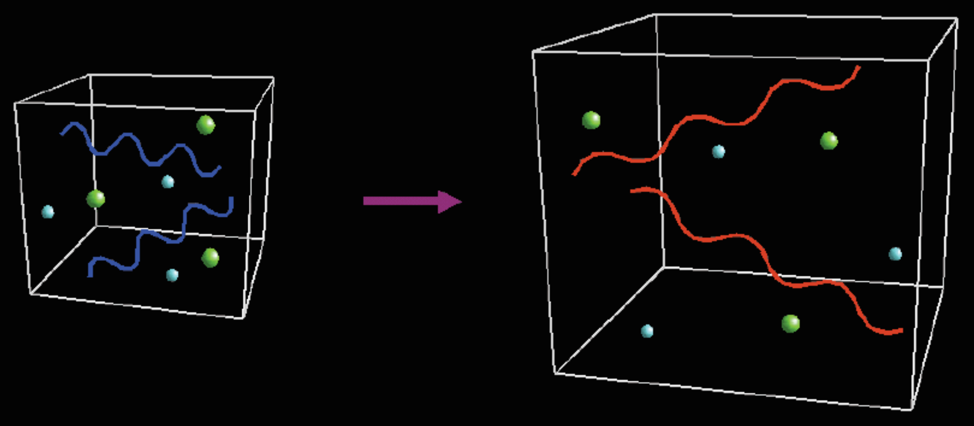
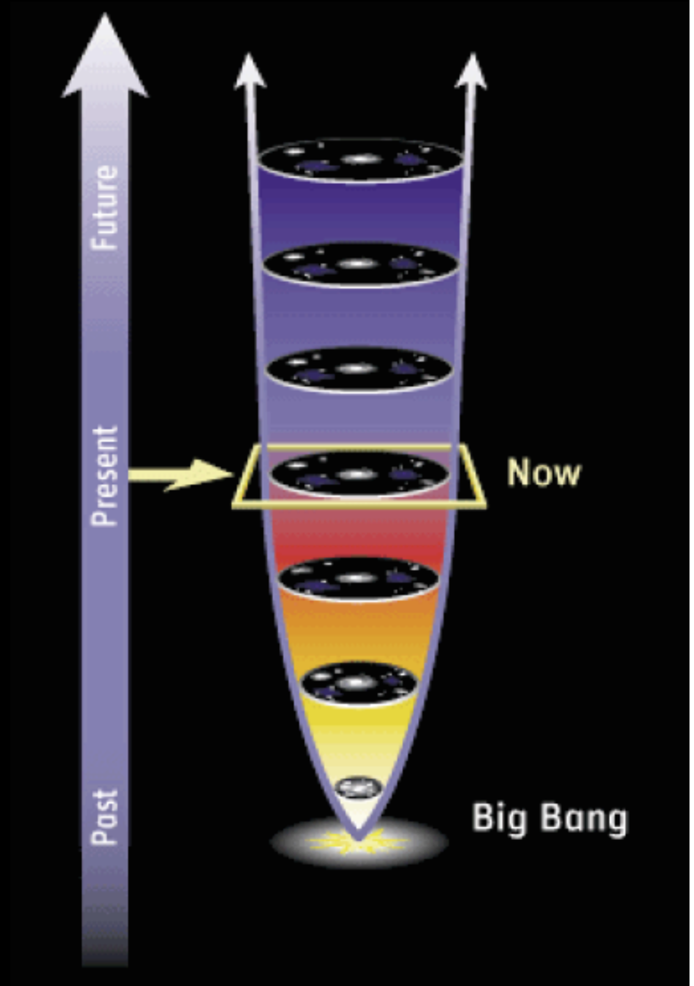


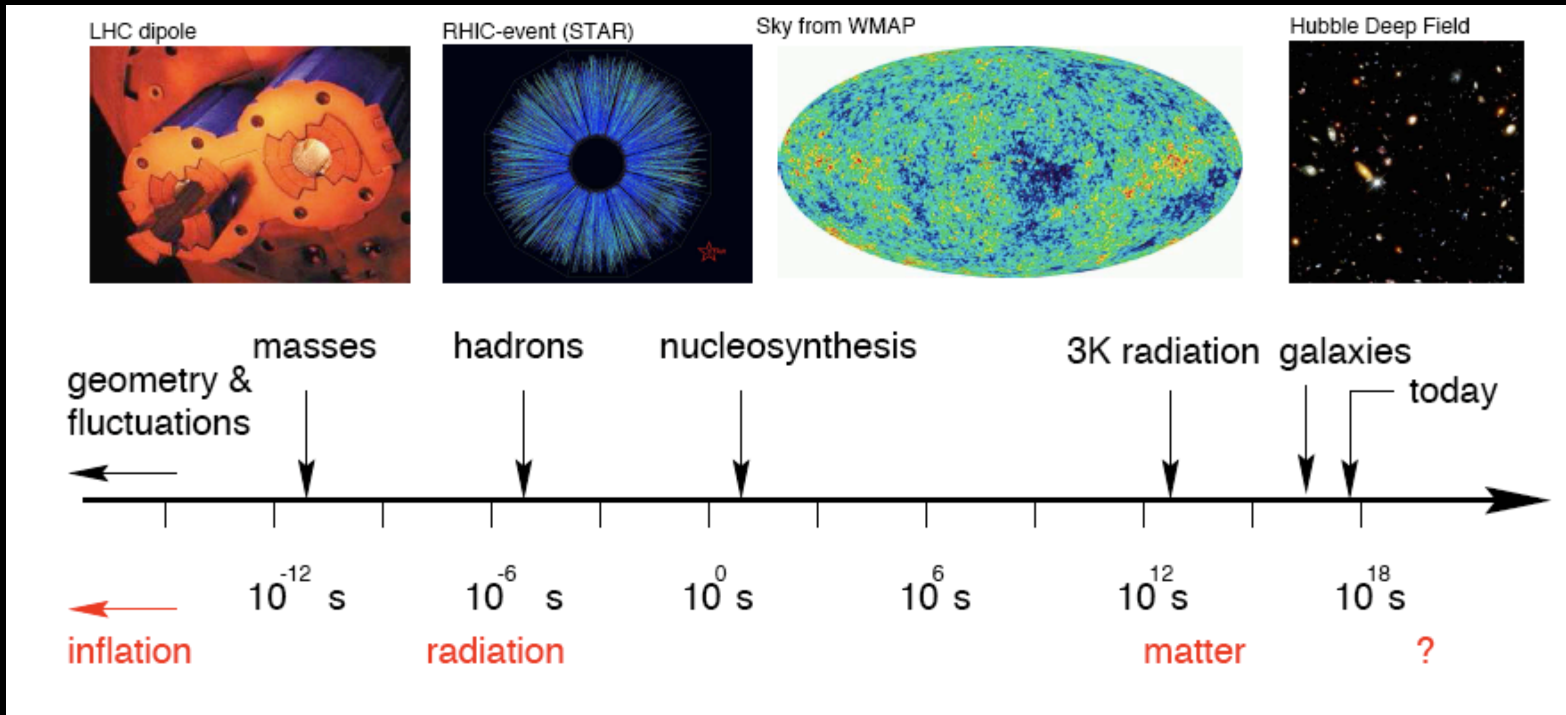
FIG. 4: The positron spectrum and positron fraction from the sum of contributions from B0656+14, Geminga, and all pulsars farther than 500 parsecs from the Solar System.

The past




Future
Present
Past
Now
Big Bang

Brief History of the Universe



At high temperatures, the Universe is expected to be Radiation Dominated

IF equilibrium holds, then

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \quad (T \gg m)$$

$$g_* = \sum_{\text{bosons}} g_{\text{b}} + \frac{7}{8} \sum_{\text{fermions}} g_{\text{f}}$$

$$8\pi G_N = \frac{1}{M_p^2}$$

$$\boxed{H \simeq 1.66\, g_*^{1/2} \frac{T^2}{M_p}, \;\; M_p \simeq 1.2 \times 10^{19}\,\text{GeV}}$$

$$\frac{t}{\mathrm{sec}} \sim \left(\frac{\mathrm{MeV}}{T}\right)^2$$

$$\boxed{t_{\mathrm{LHC}} \sim 10^{-14}\,\mathrm{sec}}$$

Entropy Density

$$\boxed{s = \frac{\rho_R + P_R}{T} = \frac{4}{3}\frac{\rho_R}{T} = \frac{2\pi^2}{45} g_* T^3}$$

If expansion is adiabatic:

$$S \equiv s \times V = \text{constant} \Rightarrow g_* (Ta)^3 = \text{constant}$$

$$\boxed{T \propto \frac{1}{g_*^{1/3} a}}$$

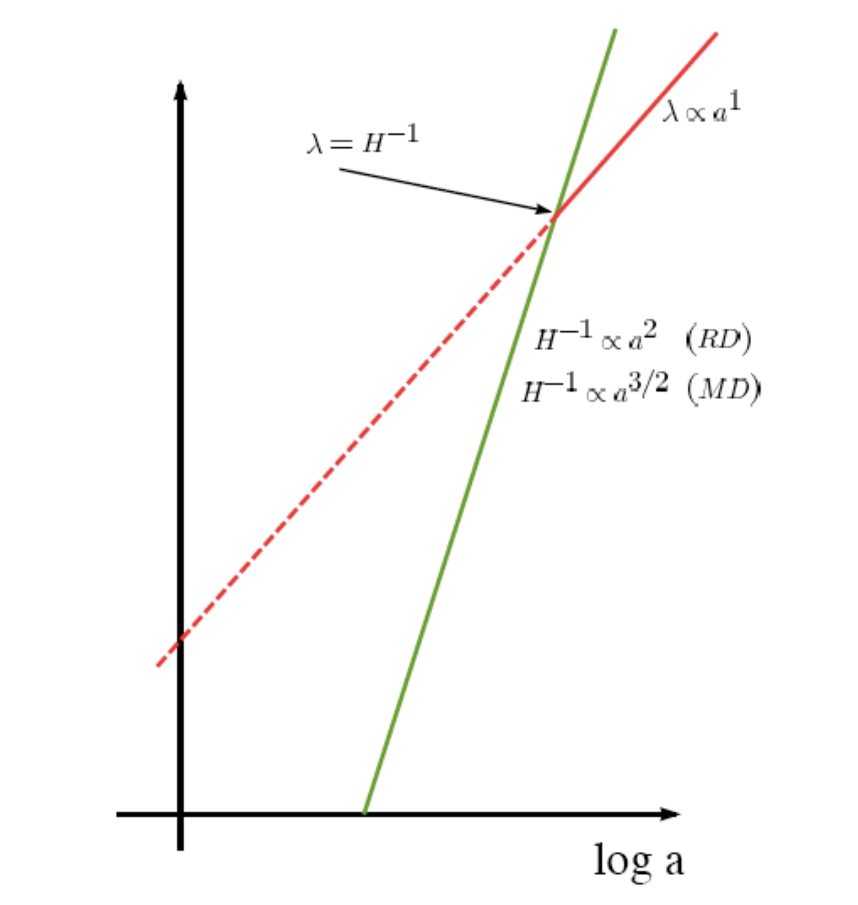
The Particle Horizon

It is the maximum distance travelled by light in an expanding Universe within a given time t

$$ds = 0 \Rightarrow dl = \frac{dt}{a} \qquad\qquad R_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

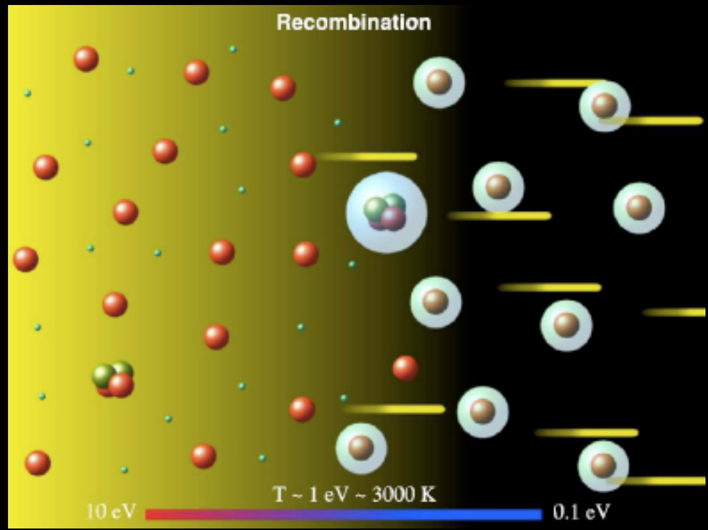
$$a(t) \propto t^n \Rightarrow R_H(t) \simeq \frac{1}{1-n} t^{-1} \sim H^{-1}(t)$$

Standard Cosmology and the Horizon Problem



$$R_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} \simeq \frac{a(t)}{\dot{a}(t)} = H^{-1}(t)$$

Hydrogen Recombination & Last Scattering Surface

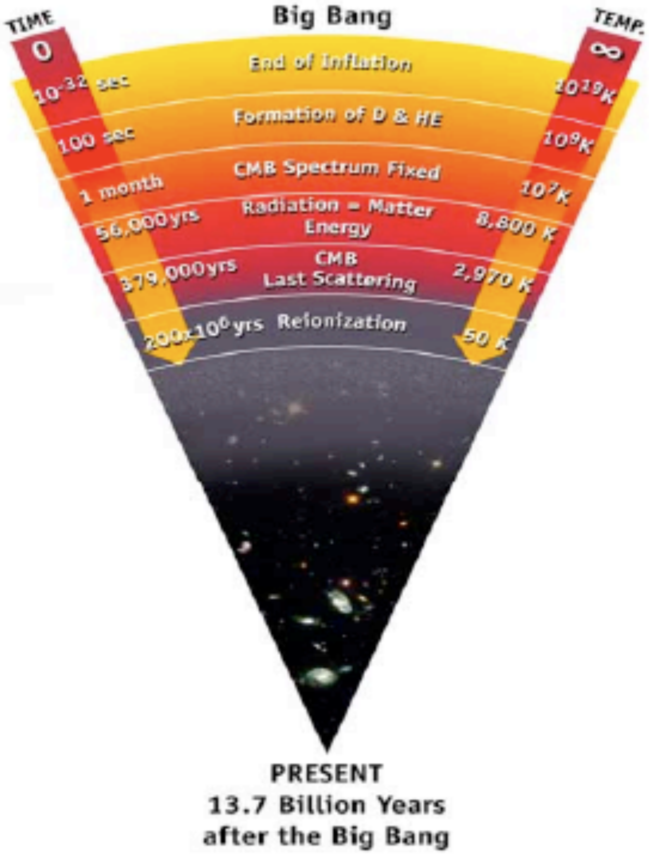


Matter is ionized at temperatures higher than the hydrogen ionization energy of 13.6 eV

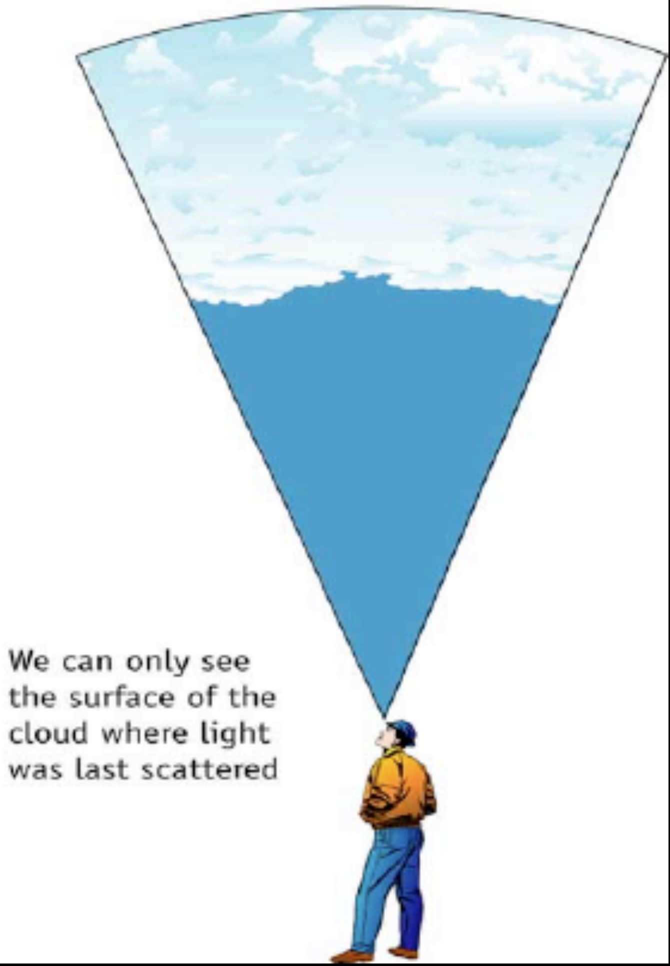
$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-E_{\rm ion}/T}$$

The Universe becomes transparent to photons when

$$(\sigma_{e\gamma} n_e)^{-1} \sim t,\ \sigma_{e\gamma} = 8\pi\alpha^2/3m_e^2,\ T_{\rm LS} \simeq 0.26\ \mathrm{eV}$$

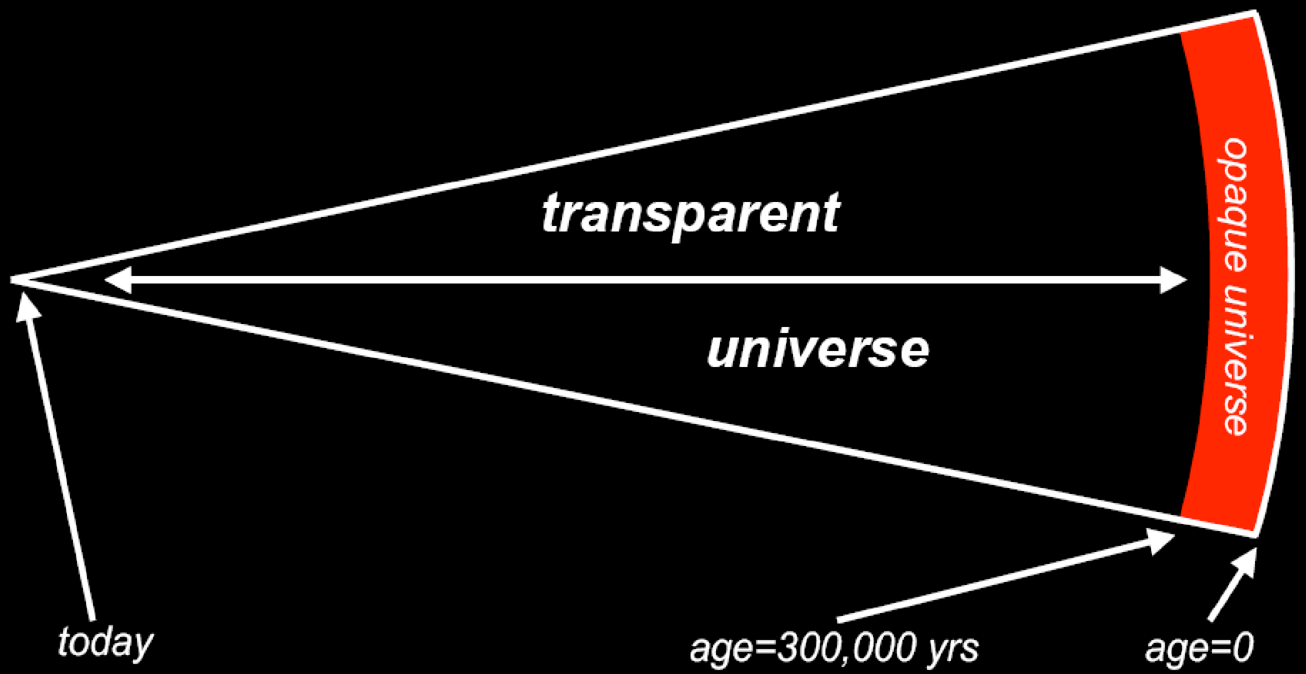


The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

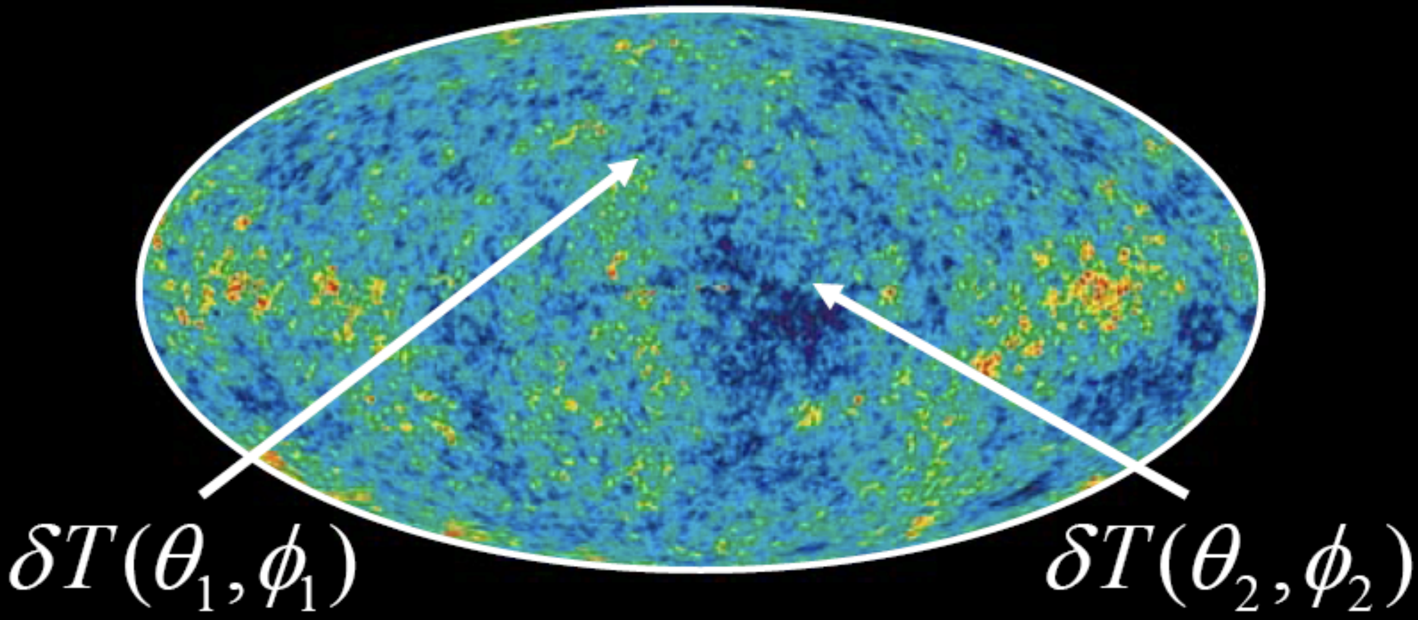


Cosmic background radiation

looking out in space is looking back in time



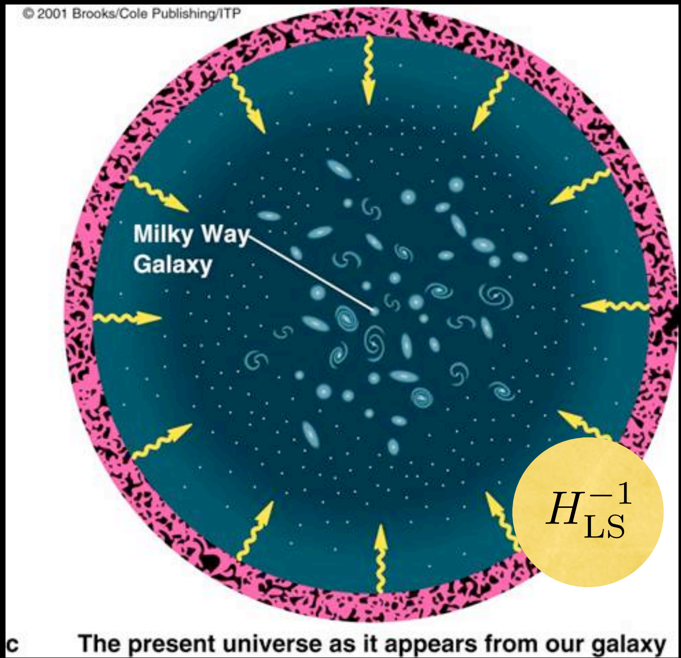
CMB anisotropy



$$
\begin{aligned}
\frac{\Delta T}{T}(x_0, \tau_0, \mathbf{n}) &= \sum_{\ell m} a_{\ell m}(x_0) Y_{\ell m}(\mathbf{n}) \\
\langle a_{\ell m} a_{\ell' m'} \rangle &= \delta_{\ell \ell'} \delta_{m m'} C_\ell \\
\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle &= \sum_{\ell} \frac{(2\ell + 1)}{4\pi} C_\ell P_\ell(\mathbf{n} \cdot \mathbf{n}')
\end{aligned}
$$

(ensemble averages)

Horizon at Last Scattering



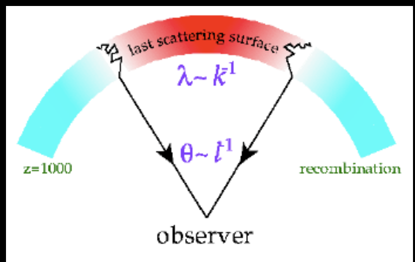
Comoving distance between us and the last scattering surface

$$d\tau = dt/a$$

$$\int_{t_{\rm LS}}^{t_0} \frac{dt}{a} = \int_{\tau_{\rm LS}}^{\tau_0} d\tau = (\tau_0 - \tau_{\rm LS})$$

Angle subtended by a given comoving length scale

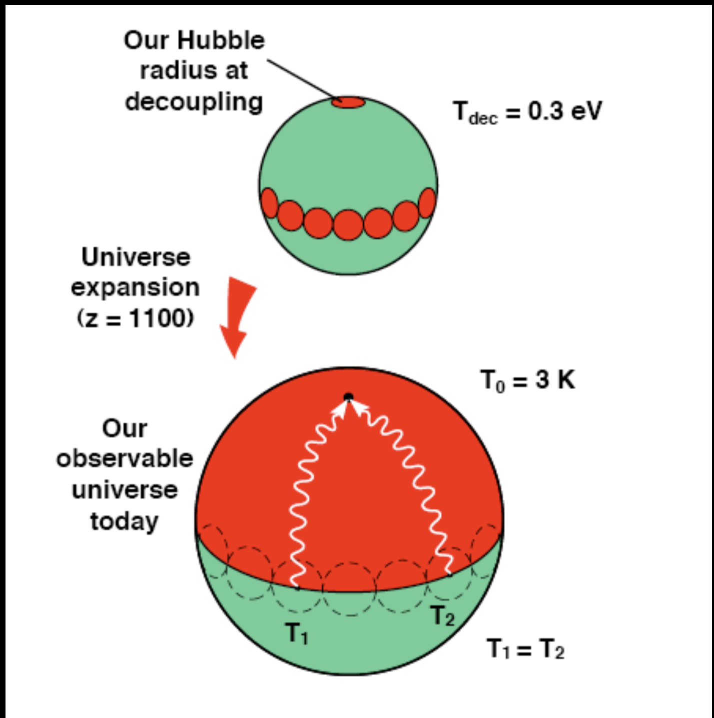
$$\theta \simeq \frac{\lambda}{(\tau_0 - \tau_{\rm LS})}$$

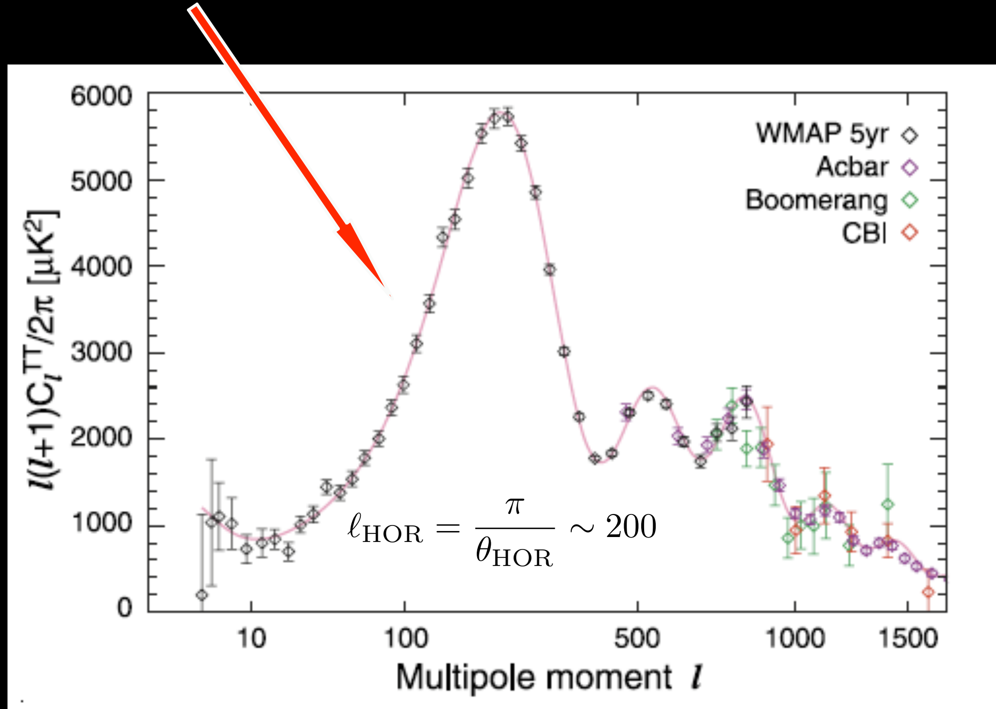


Sound Horizon

$$\theta_{\rm HOR} \simeq c_s \frac{\tau_{\rm LS}}{(\tau_0 - \tau_{\rm LS})} \simeq c_s \frac{\tau_{\rm LS}}{\tau_0} \simeq c_s \left(\frac{T_0}{T_{\rm LS}}\right)^{1/2} \simeq 1^{\rm o}$$

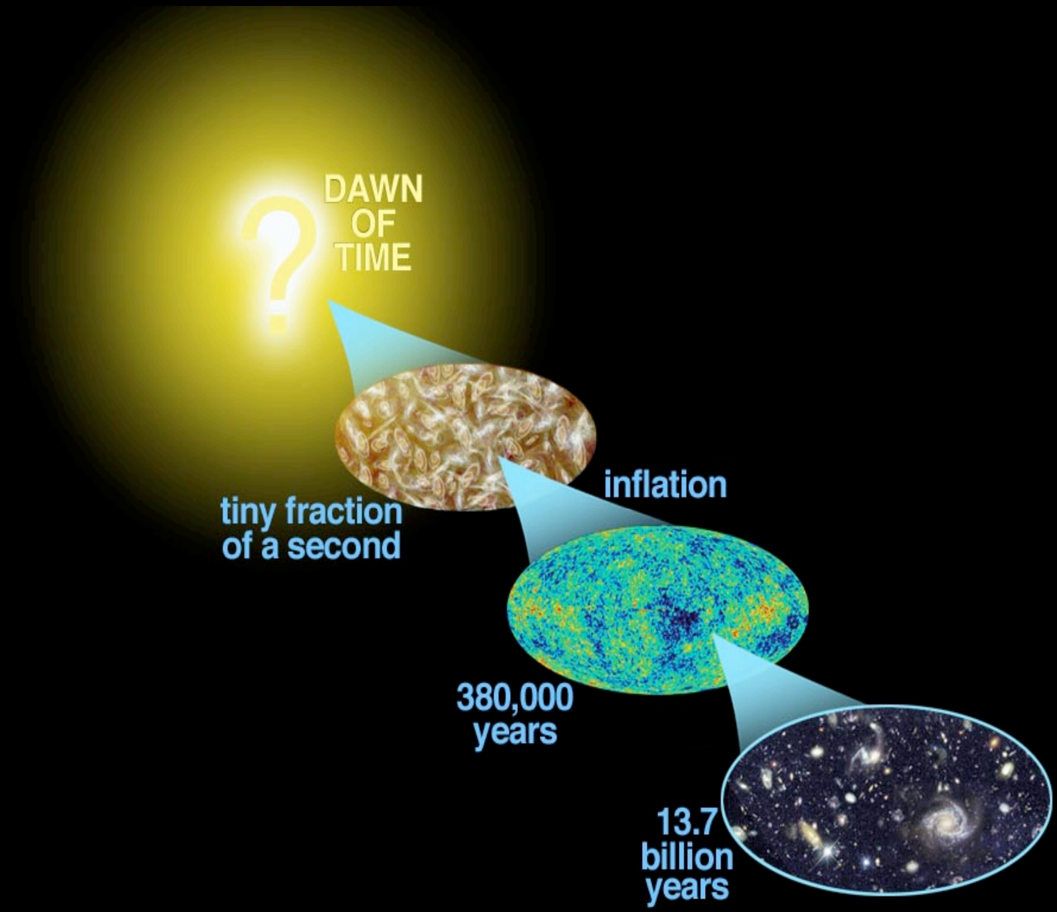
Super-Horizon mode detected in the CMB anisotropy



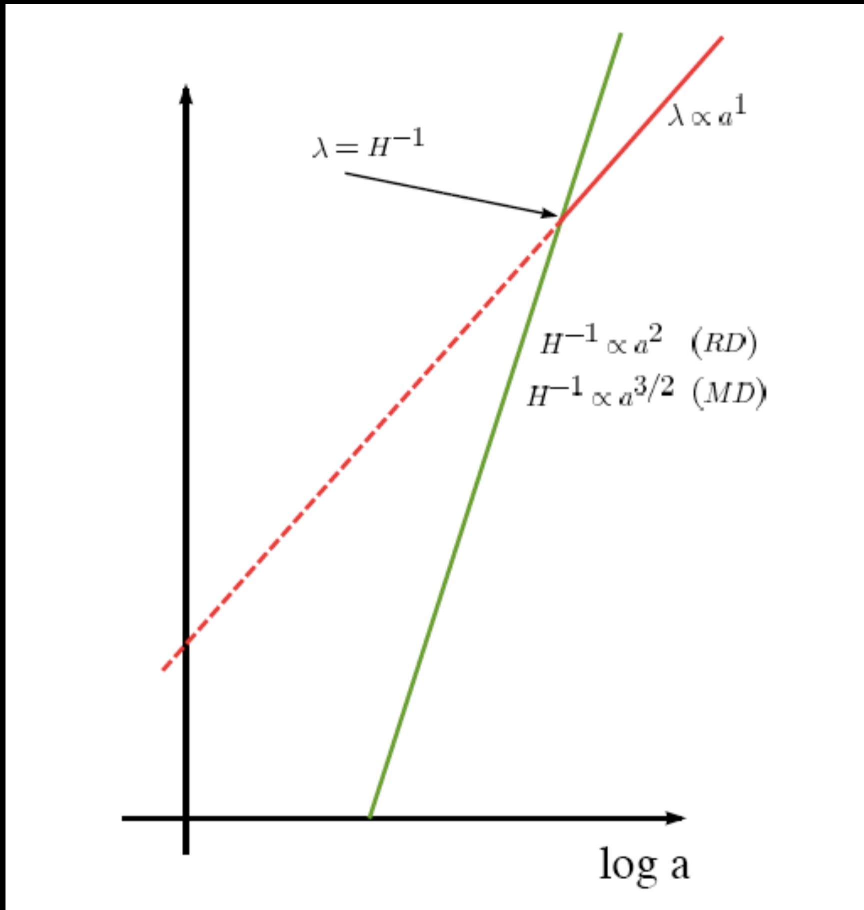


Why is the Universe so
homogeneous and isotropic
if, back in time, it was a collection
of separated Universes?

The Inflationary Cosmology

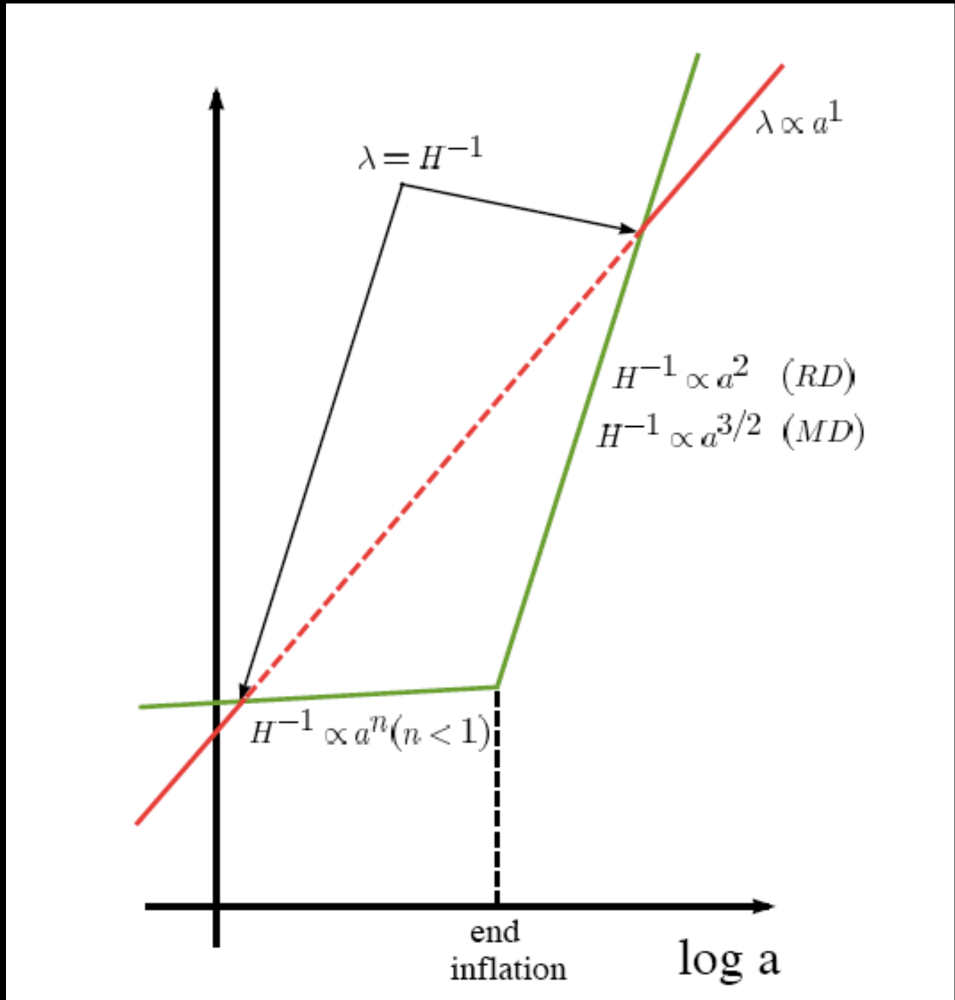


Standard Cosmology and the Horizon Problem



$$R_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \simeq \frac{a(t)}{\dot{a}(t)} = H^{-1}(t)$$

Inflationary Cosmology



$$\left(\frac{\lambda}{H^{-1}}\right)^{\cdot} = \ddot{a} > 0 \Leftrightarrow \text{Inflation}$$

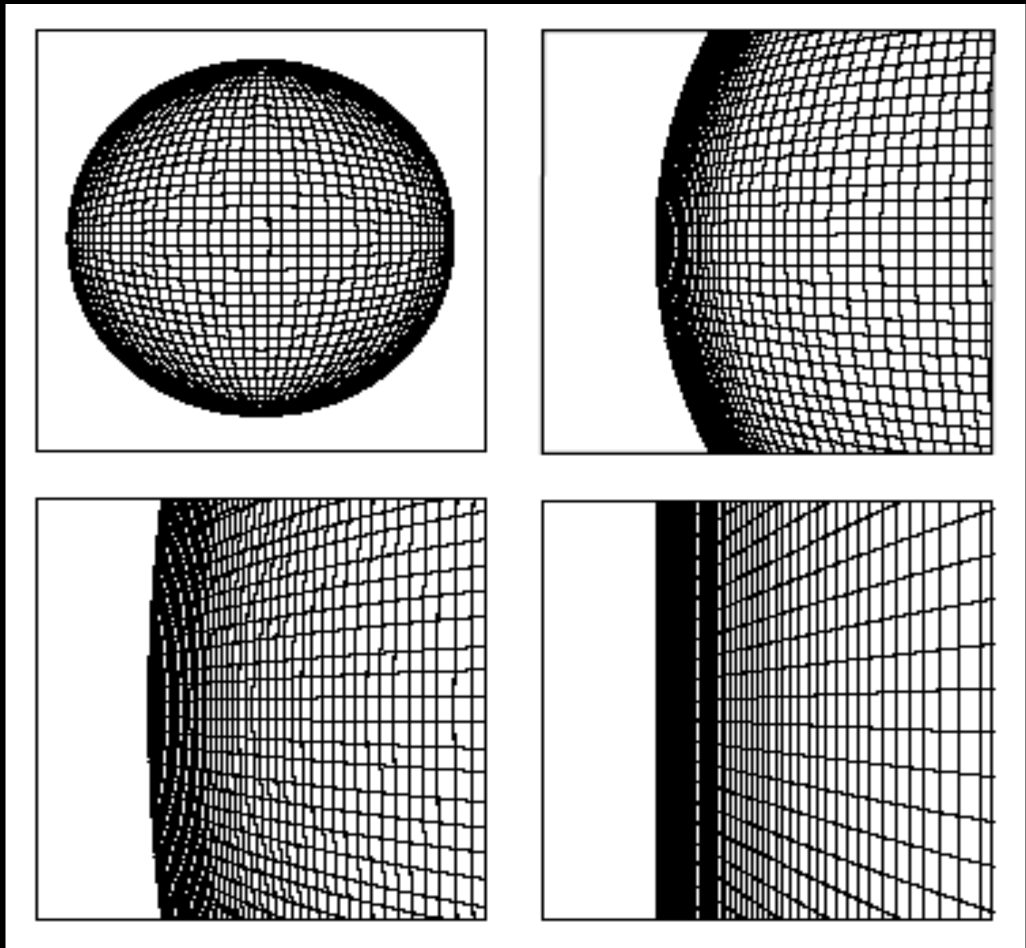
Suppose there is a period during which the Hubble rate is constant (pure de Sitter epoch)

$$H = \text{constant} = \frac{\dot{a}}{a} \Rightarrow a = a_i e^{H_*(t-t_i)} \equiv a_i e^N$$

$$N = \text{number of efolds}$$

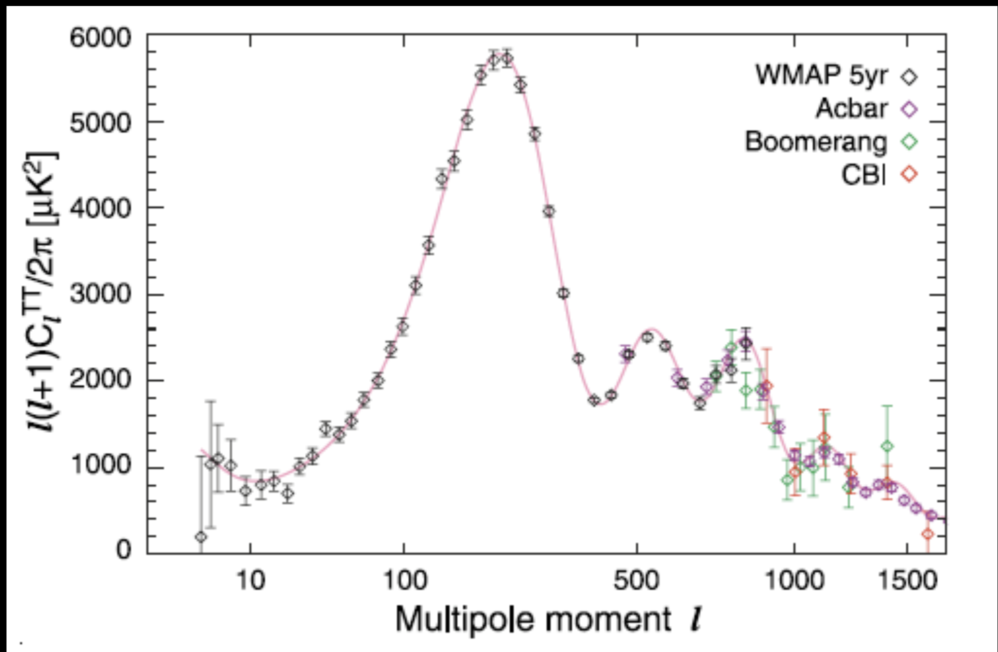
$$\text{In conformal time } a(\tau) = -\frac{1}{H\tau}\ (\tau < 0)$$

Inflation does NOT change the global structure of space, but LOCALLY it makes it flat

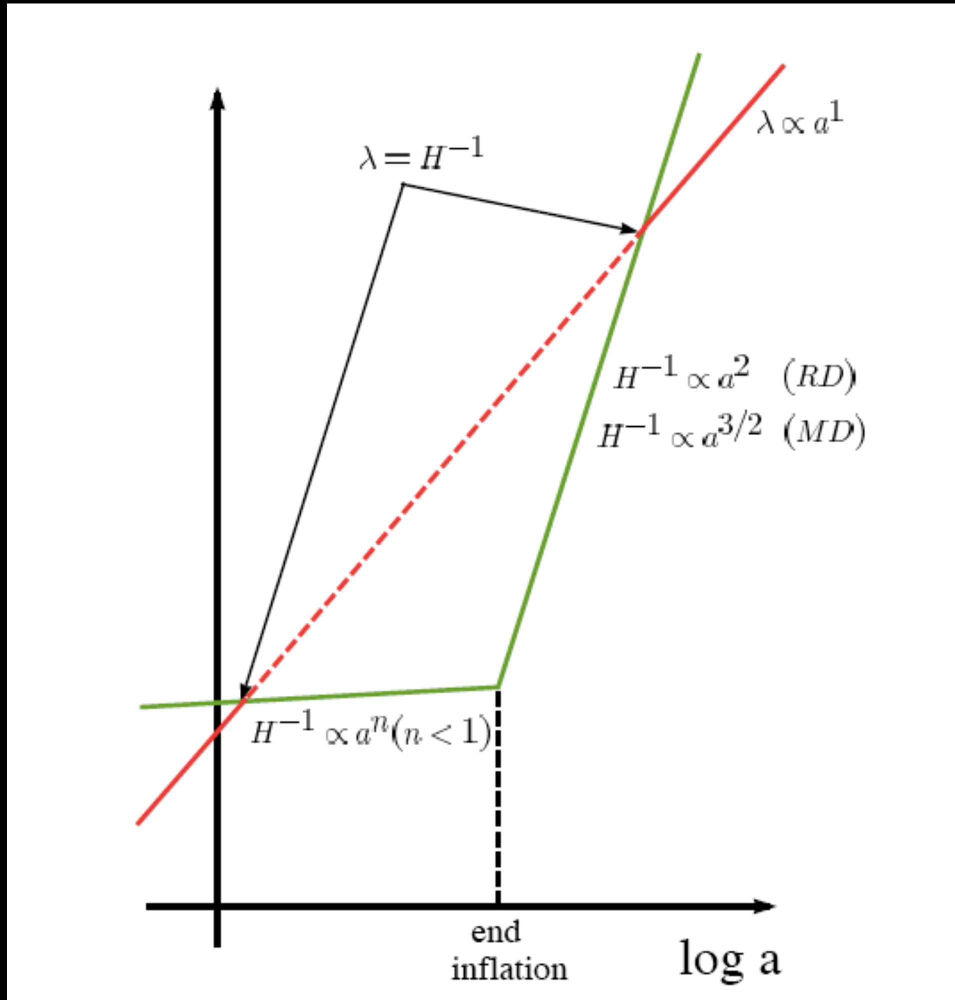


$$\text{IF } N \gg 60 \Rightarrow \Omega_0 = 1 + \mathcal{O}\left(e^{60-N}\right)$$

Lecture two: the cosmological perturbations and CMB anisotropy



Inflationary Cosmology



$$\left(\frac{\lambda}{H^{-1}}\right)^{\cdot} = \ddot{a} > 0 \Leftrightarrow \text{Inflation}$$

Suppose there is a period during which the Hubble rate is constant (pure de Sitter epoch)

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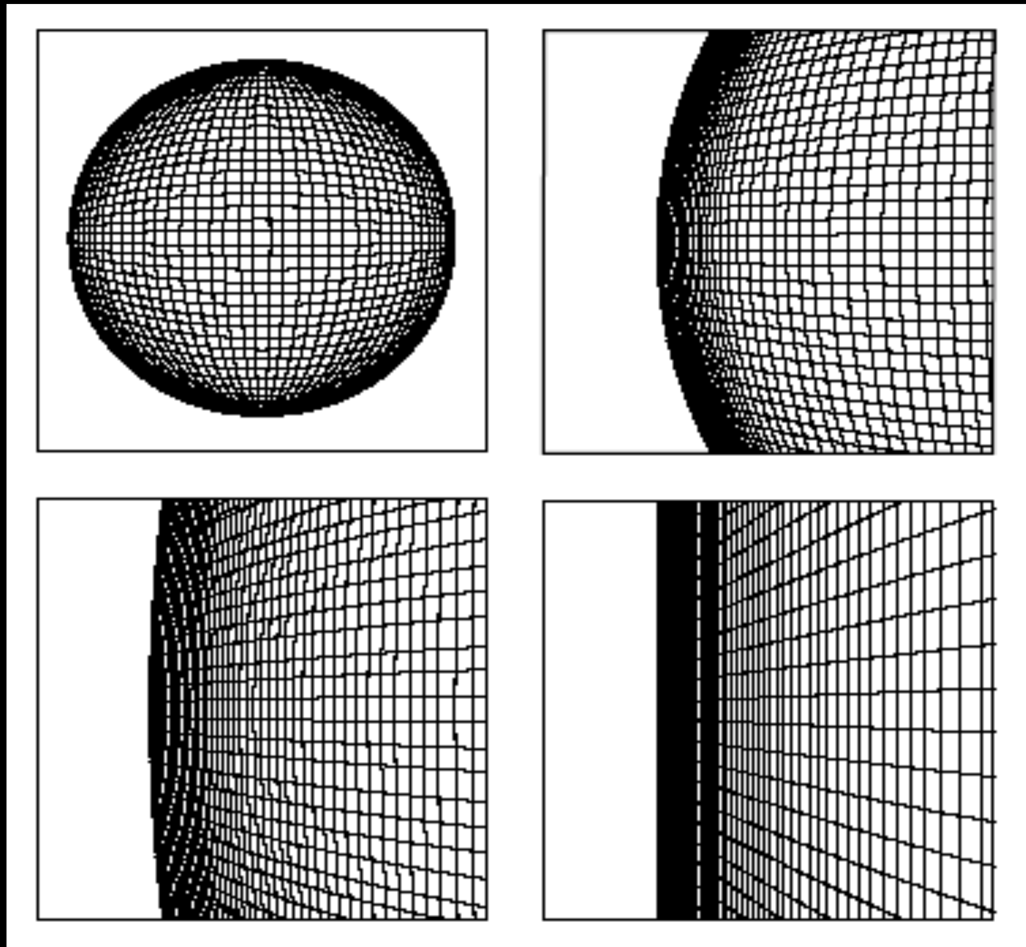
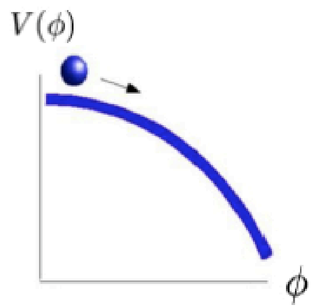


$$\text{IF } N \gg 60 \Rightarrow \Omega_0 = 1 + \mathcal{O}\left(e^{60-N}\right)$$

How to get Inflation

Inflation

For a review,see
D.H. Lyth and A.R.,
Phys. Rept. 314
(1999) 1



Potential → Vacuum Energy → Inflation

Friedmann equation:

slow roll

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\left[\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right] \simeq \mathrm{const.}$$

Scalar field equation of motion:

$$\cancel{\ddot{\phi}} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} + V'(\phi) = 0 \qquad a(t) \propto e^{\int H dt} \equiv e^N$$

How to get Inflation

Slow Roll Parameters

$\epsilon(\phi)$ Parameterizes equation of state:

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 \simeq \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

$$\boxed{p = \rho\left(\frac{2}{3}\epsilon - 1\right)}$$

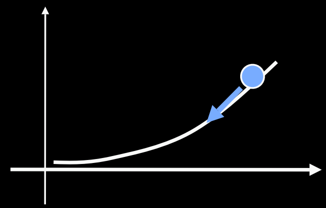
Inflation $\longleftrightarrow$ $\epsilon(\phi) < 1$

Second slow roll parameter:

$$\eta \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H''(\phi)}{H(\phi)}\right] \simeq \frac{m_{\rm Pl}^2}{8\pi}\left[\frac{V''(\phi)}{V(\phi)}\right] - \frac{m_{\rm Pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]$$

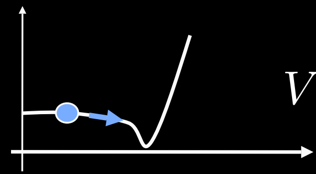
Standard scenario = one-single field (slow-roll) models

1. **large field**
   e.g. chaotic inflation



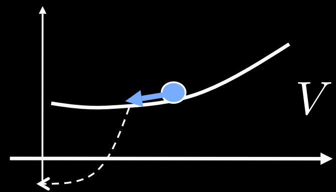
$$V \sim \mu^{4-p}\phi^p$$

2. **small field**
   e.g. new or natural inflation



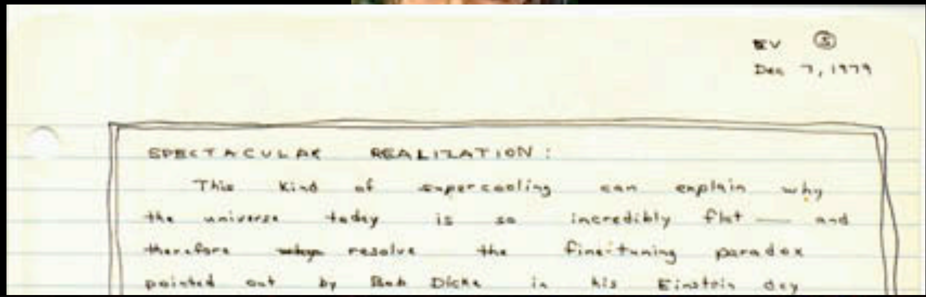
$$V \sim V_0 - \mu^{4-p}\phi^p$$

3. **hybrid inflation**
   e.g., Susy or Sugra models



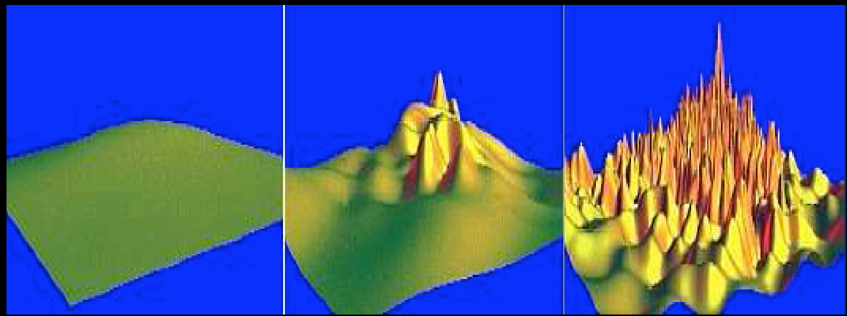
$$V \sim V_0 + \mu^{4-p}\phi^p$$

The Universe is NOT homogeneous and isotropic

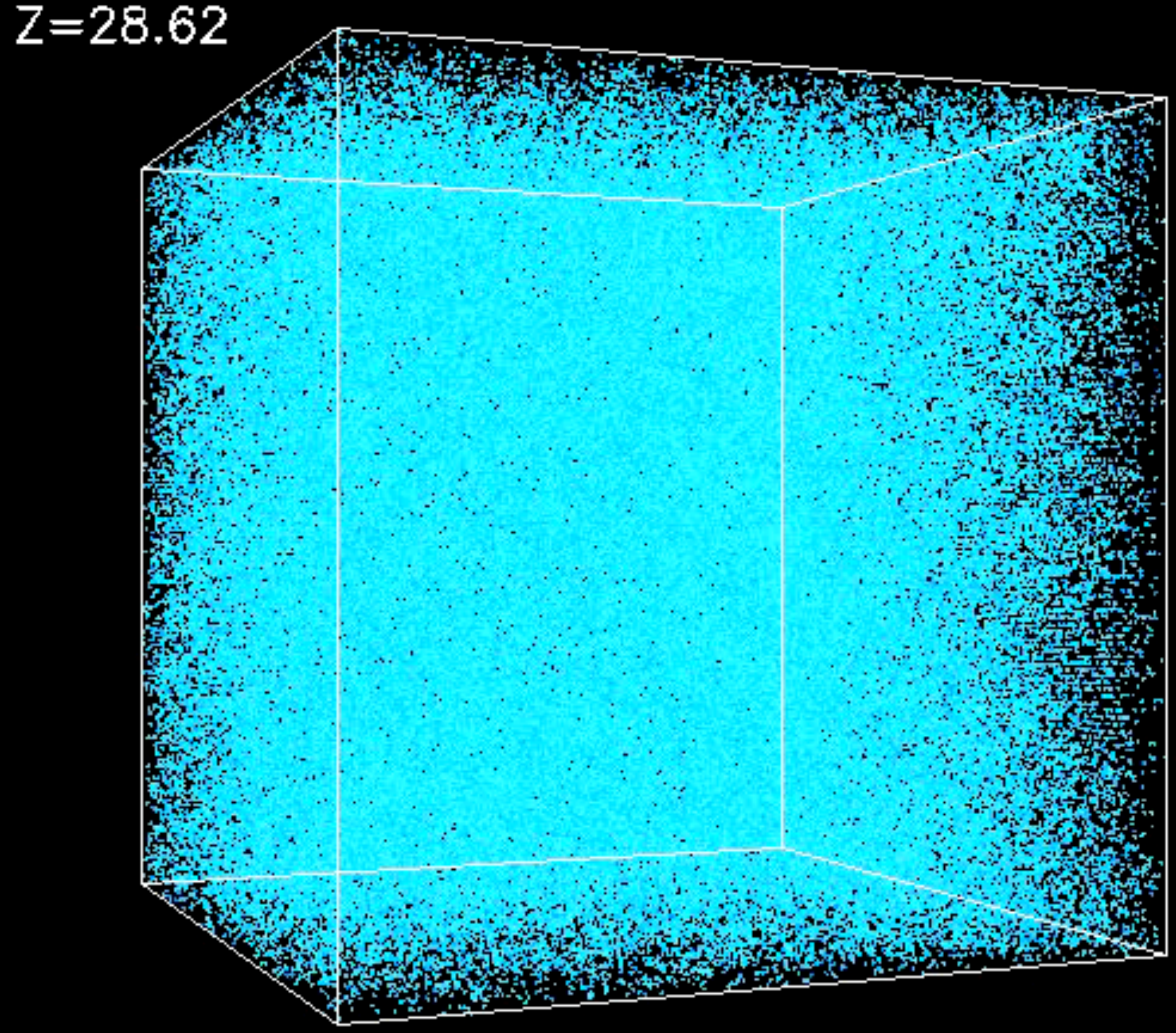


EV ⑤
Dec 7, 1979

SPECTACULAR REALIZATION:
This kind of supercooling can explain why the universe today is so incredibly flat — and therefore resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day



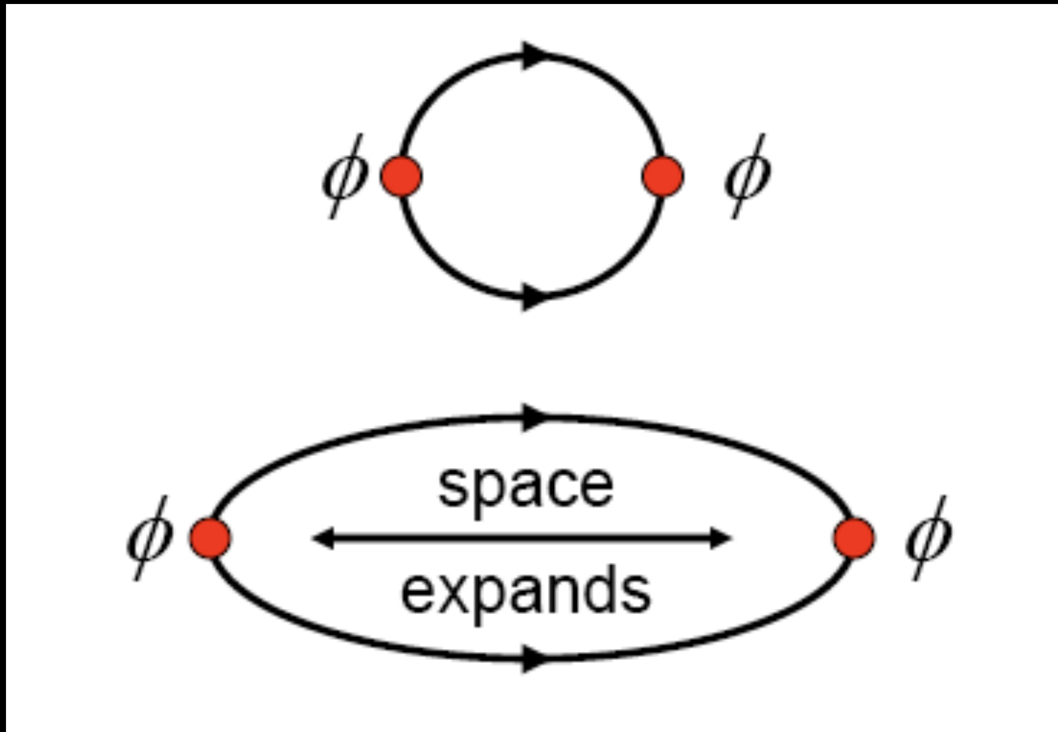
From Quantum Fluctuations to the Large Scale Structure



The Millenium Simulation Project:

http://www.mpa-garching.mpg.de/galform/virgo/millennium/

Particle production in an expanding Universe



Take now perturbations of the inflaton field: heuristic explanation of why the inflaton field is perturbed

$$
\begin{aligned}
\phi(\mathbf{x},t) &= \phi_0(t) + \delta\phi(\mathbf{x},t) \\
\delta\ddot{\phi} + 3H\delta\dot{\phi} &- \cancel{\frac{\nabla^2\delta\phi}{a^2}} + V''\delta\phi = 0 \\
\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) &= 0 \Rightarrow \dddot{\phi}_0 + 3H\ddot{\phi}_0 + V''\dot{\phi}_0 = 0
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\delta\phi &= \dot{\phi}_0\tau(\mathbf{x}) \\
\phi(\mathbf{x},t) &= \phi_0(t+\tau(\mathbf{x}))
\end{aligned}
}
$$

The inflaton field has different classical values at different points in space

All massless scalar fields are excited during Inflation

Linear Theory

$$
\begin{aligned}
\sigma(\mathbf{x},\tau) &= \sigma_0(\tau) + \delta\sigma(\mathbf{x},\tau), \\
u_k(\tau) &= a(\tau)\delta\sigma_k(\tau), \\
d\tau &= \frac{dt}{a}
\end{aligned}
$$

$$
\boxed{u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0}
$$

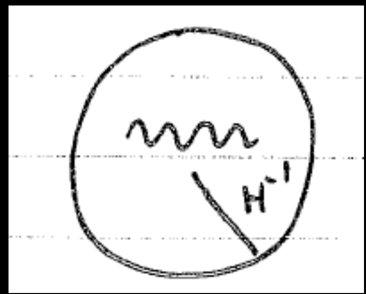
$$
\left(\delta\ddot{\sigma}_k + 3H\delta\dot{\sigma}_k + \frac{k^2}{a^2}\delta\sigma_k = 0\right)
$$

Oscillator with time-dependent frequency

a) For modes with wavelengths inside the horizon:

$$\lambda_{\rm phys} \ll H^{-1} \Rightarrow k/a \gg H \Rightarrow (-k\tau) \gg 1$$

$$a''/a = 2/\tau^2 \Rightarrow (-k\tau) \gg 1 \Rightarrow k^2 \gg a''/a$$



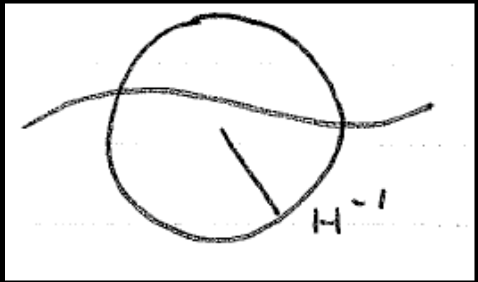
$$\boxed{u_k'' + k^2 u_k = 0 \Rightarrow u_k = A(k)\frac{e^{-ik\tau}}{\sqrt{2k}} + B(k)\frac{e^{ik\tau}}{\sqrt{2k}}}$$

Set of independent plane waves: locally Minkowski, no curvature seen by the waves

b) For modes with wavelengths outside the horizon:

$$\lambda_{\rm phys} \gg H^{-1} \Rightarrow k/a \ll H \Rightarrow (-k\tau) \ll 1$$

$$a''/a = 2/\tau^2 \Rightarrow (-k\tau) \ll 1 \Rightarrow k^2 \ll a''/a$$



$$\boxed{u_k'' - \frac{a''}{a} u_k = 0 \Rightarrow u_k = C(k) a(\tau) \Rightarrow \delta\sigma_k = C(k)}$$

Superhorizon perturbations do not evolve in time

Exact solution exists:

$$u_k(\tau) = A(k)\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + B(k)\frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)$$

Choose the boundary conditions in the far UV such that the solution is a plane wave propagating with positive frequency (Bunch-Davies vacuum)

$$\begin{aligned}
(-k\tau) \gg 1 \quad &\Rightarrow \quad A(k) = 1,\ B(k) = 0 \\
u_k(\tau) \quad &= \quad \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right), \\
\delta\sigma_k \quad &= \quad \frac{u_k}{a} = (-H\tau)\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)
\end{aligned}$$

Power Spectrum

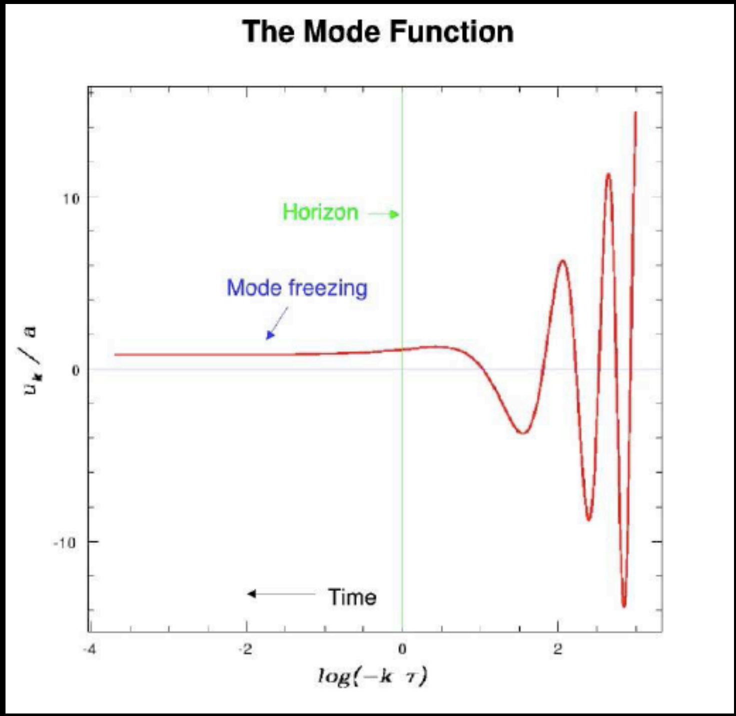
$$\langle 0 \left| (\delta\sigma(\mathbf{x},t))^2 \right| \rangle = \int \frac{d^3k}{(2\pi)^3} \left|\delta\sigma_k\right|^2$$

$$\equiv \int \frac{dk}{k} \mathcal{P}_{\delta\sigma}(k)$$

$$\boxed{\mathcal{P}_{\delta\sigma}(k) = \frac{k^3}{2\pi^2} \left|\delta\sigma_k\right|^2}$$

$$\mathcal{P}_{\delta\sigma}(k) = \mathcal{A}^2 \left(\frac{k}{aH}\right)^{n-1}$$

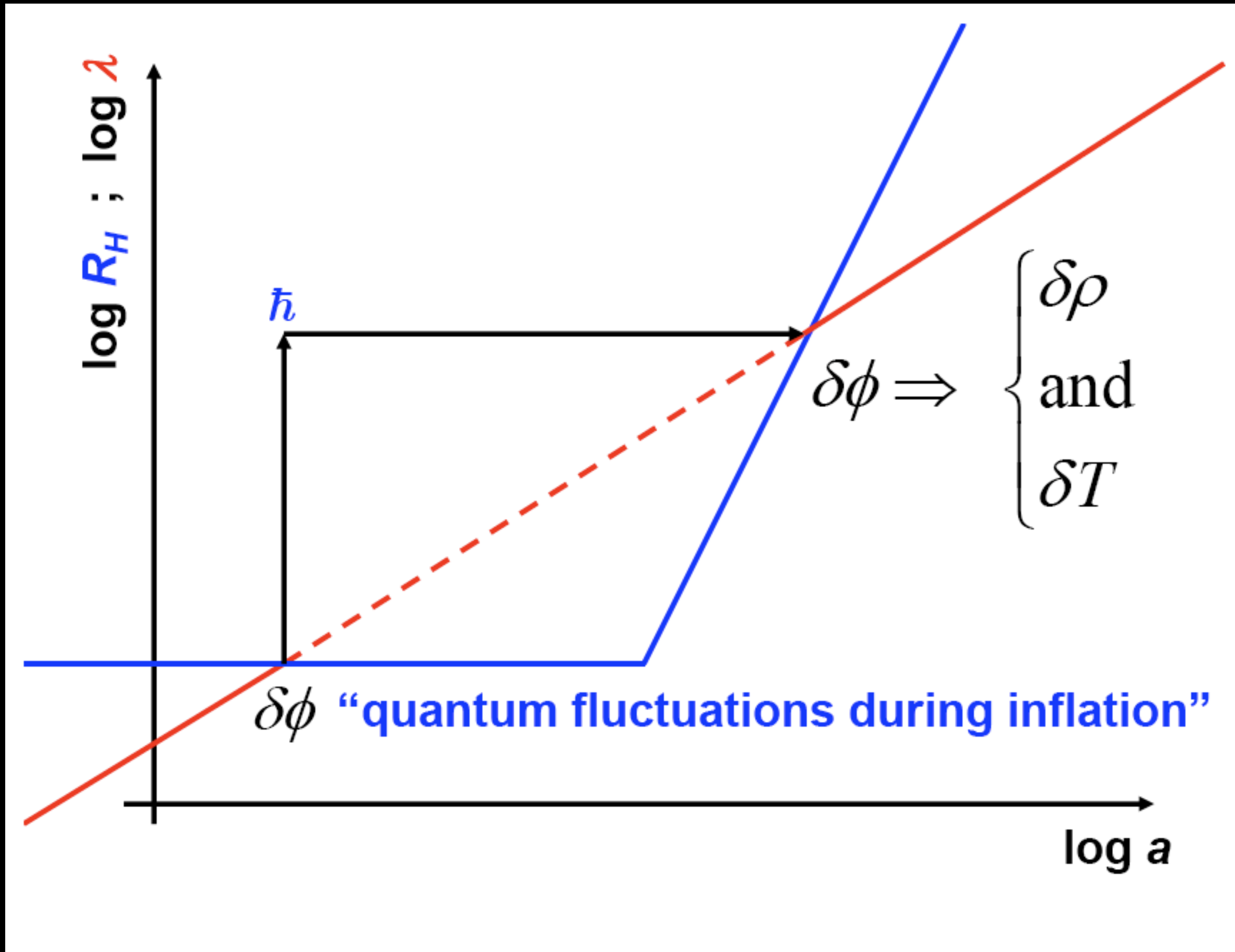
Perturbations of a (nearly) massless scalar field
are born as plane waves with wavelengths below the horizon.
As inflation proceeds, their wavelenghts are stretched
outside the horizon and get frozen



$$\mathcal{P}_{\delta\sigma} = \frac{k^3}{2\pi^2}\left|\delta\sigma_k\right|^2 = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n-1}$$

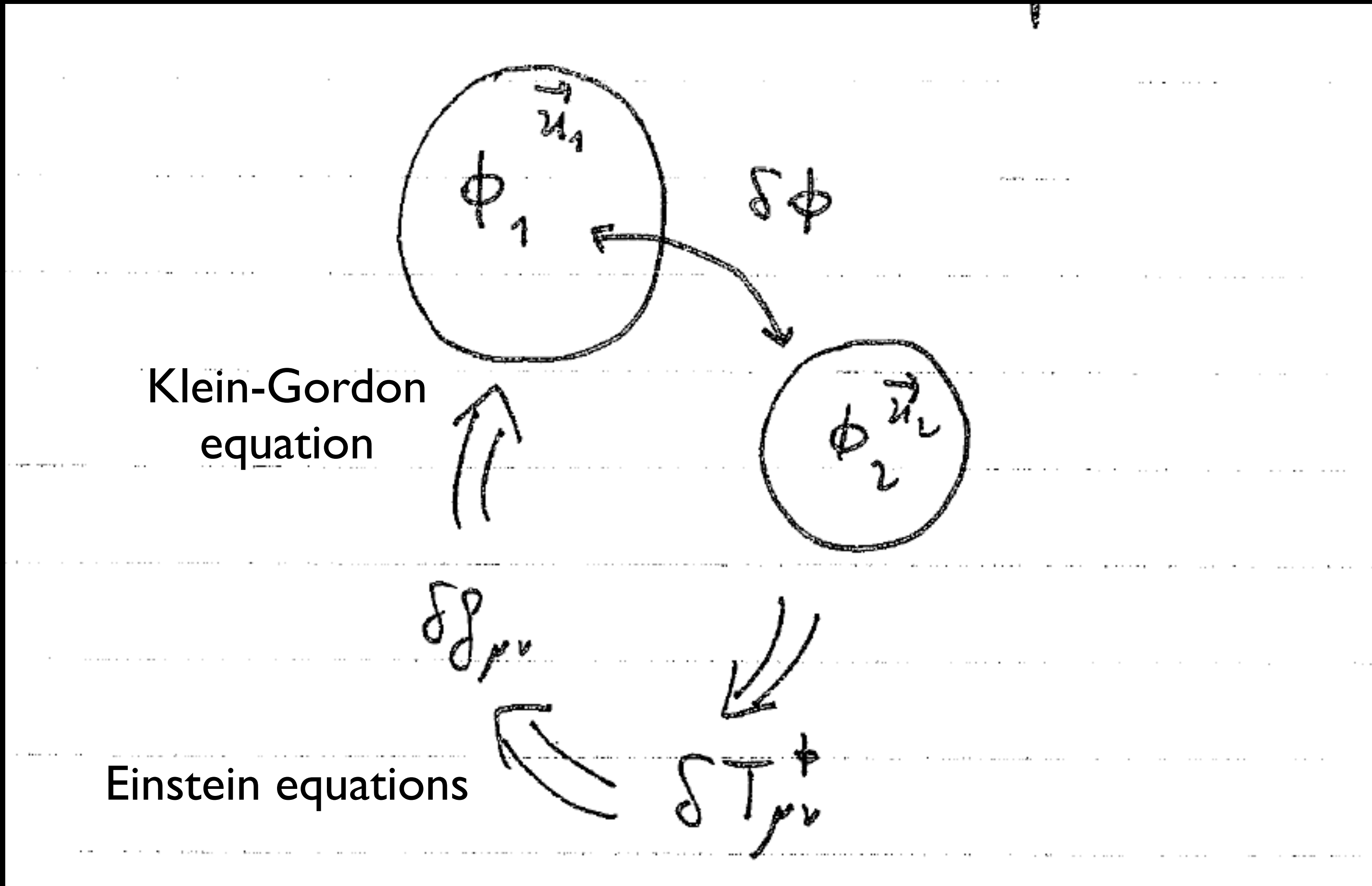
All massless scalar fields during a period of exponential inflation (pure de Sitter) are quantum mechanically excited with a power spectrum which is constant and flat on superhorizon scales (independent from the wavelength)

Perturbations are GAUSSIAN:
it is linear perturbation theory
and all oscillators evolve independently from each other



log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a

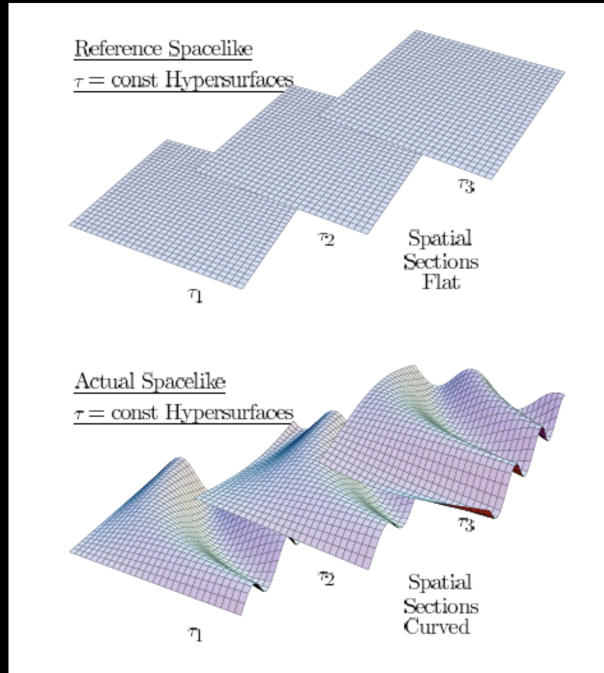
Have to include gravity



Including gravity

$$ds^2 = \left[(1+2\Phi)dt^2 - a^2(1-2\psi)d\mathbf{x}^2\right]$$

Need to define a gauge invariant quantity upon general coordinate transformations



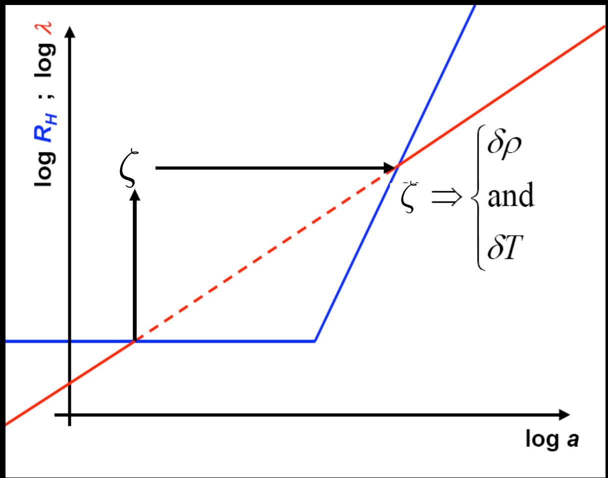
$$\begin{aligned} t &\rightarrow t + \delta t, \\ \psi &\rightarrow \psi + H\,\delta t, \\ \delta\rho &\rightarrow \delta\rho - \dot\rho\,\delta t \end{aligned}$$

Comoving curvature perturbation

$$\zeta = -\psi - H\,\frac{\delta\rho}{\dot\rho}$$

Gravitational potential

Comoving curvature perturbation generated by the one-single (slow-roll) field driving inflation



Quantum fluctuations on spatially flat hypersurfaces during inflation

$$\zeta = -\left(H\frac{\delta\rho}{\dot\rho}\right)_{k=aH} = -\left(H\frac{\delta\phi}{\dot\phi}\right)_{k=aH}$$

Curvature perturbation generated during inflation

$$\boxed{\begin{aligned} \mathcal{P}_\zeta &= \frac{1}{2}\left(\frac{H}{2\pi M_P \epsilon^{1/2}}\right)^2\left(\frac{k}{aH}\right)^{n_\zeta - 1}, \\ n_\zeta &= 1 + 2\eta - 6\epsilon \end{aligned}}$$

$$n_\zeta - 1 = \frac{d\ln\mathcal{P}_\zeta}{d\ln k} = \frac{d\ln H_k^4}{d\ln k} - \frac{d\ln\dot\phi_k^2}{d\ln k} = -4\epsilon + (2\eta - 2\epsilon) = 2\eta - 6\epsilon$$

Tensor perturbations

$$ds^2 = dt^2 - a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$\mathcal{L}_h = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \frac{1}{2} \partial_\sigma h_{ij} \partial^\sigma h^{ij}$$

$$v_k = \frac{aM_P}{\sqrt{2}} h_k$$

Massless scalar field

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\boxed{\begin{aligned} \mathcal{P}_T(k) &= \frac{8}{M_P^2} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n_T} \\ n_T &= -2\epsilon \end{aligned}}$$

Comments:

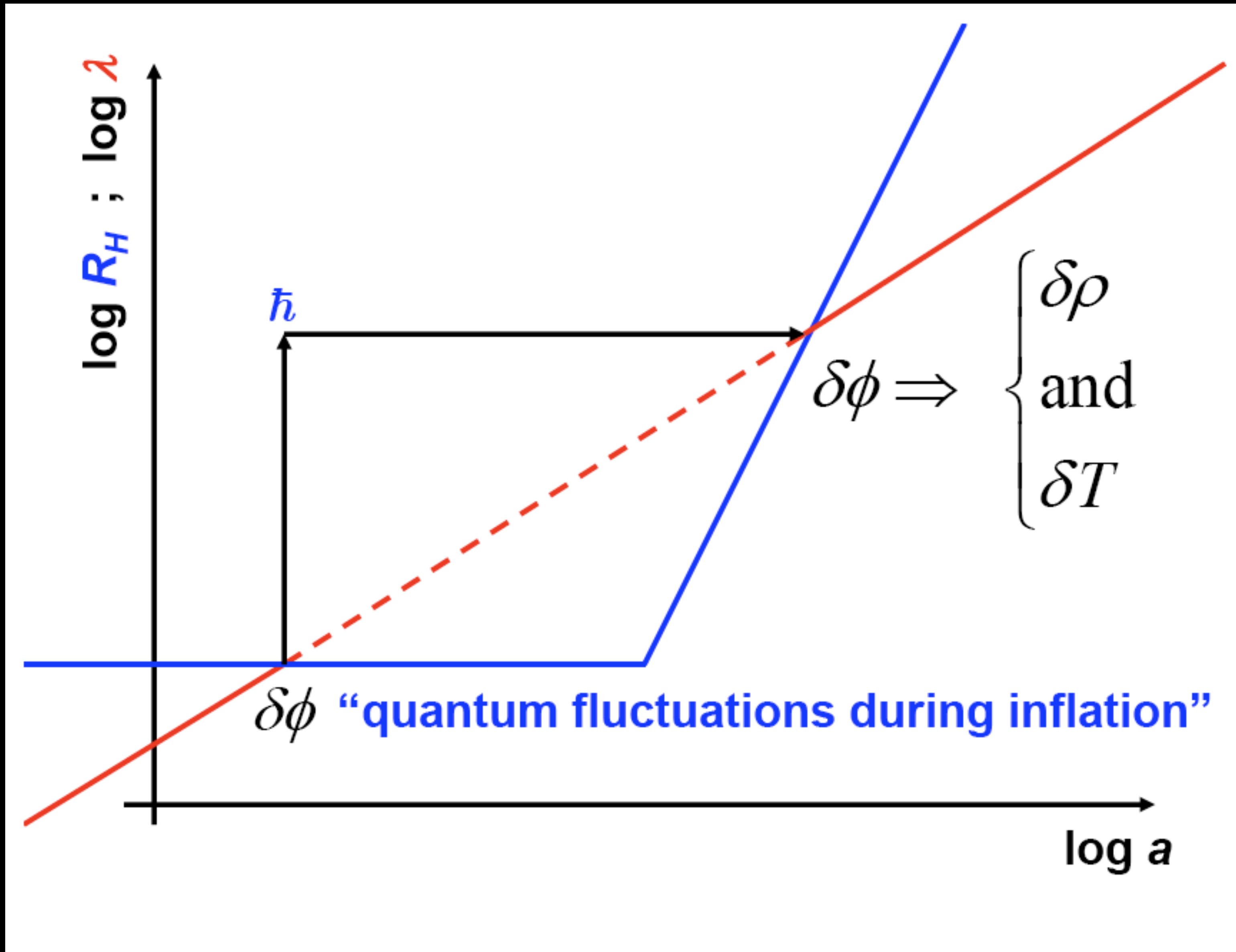
1) The amplitude of the tensor modes is proportional to the energy density of the inflaton field

2) For one-single field models of inflation there exists a CONSISTENCY RELATION

$$\boxed{r \equiv \frac{\frac{1}{100}\mathcal{P}_T}{\frac{4}{25}\mathcal{P}_\zeta} = \epsilon = -\frac{n_T}{2}}$$

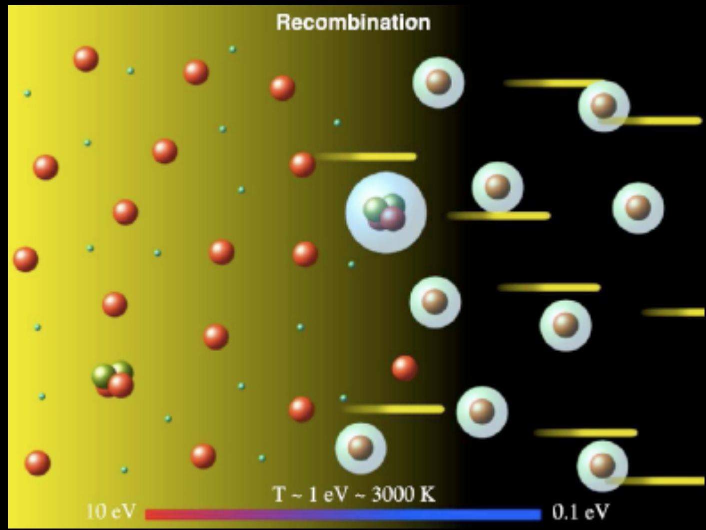
The standard slow-roll scenario predicts:

- A (nearly) exact power law
- spectrum of (nearly) Gaussian
- super-Hubble radius
- scalar perturbations (seeds of structure) &
- tensor perturbations (gravitational waves)
- in their growing mode
- in a spatially flat universe



log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a

Hydrogen Recombination & Last Scattering Surface

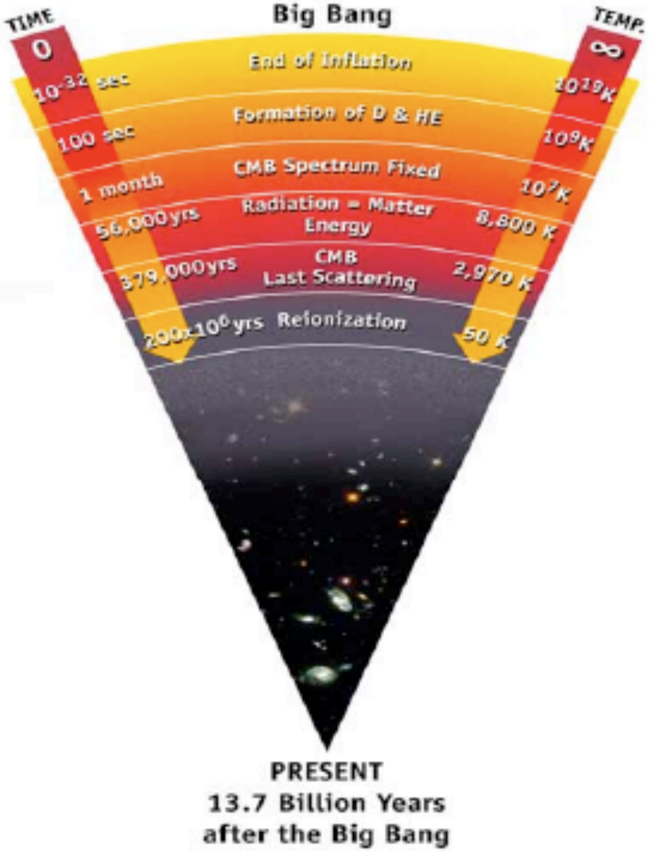


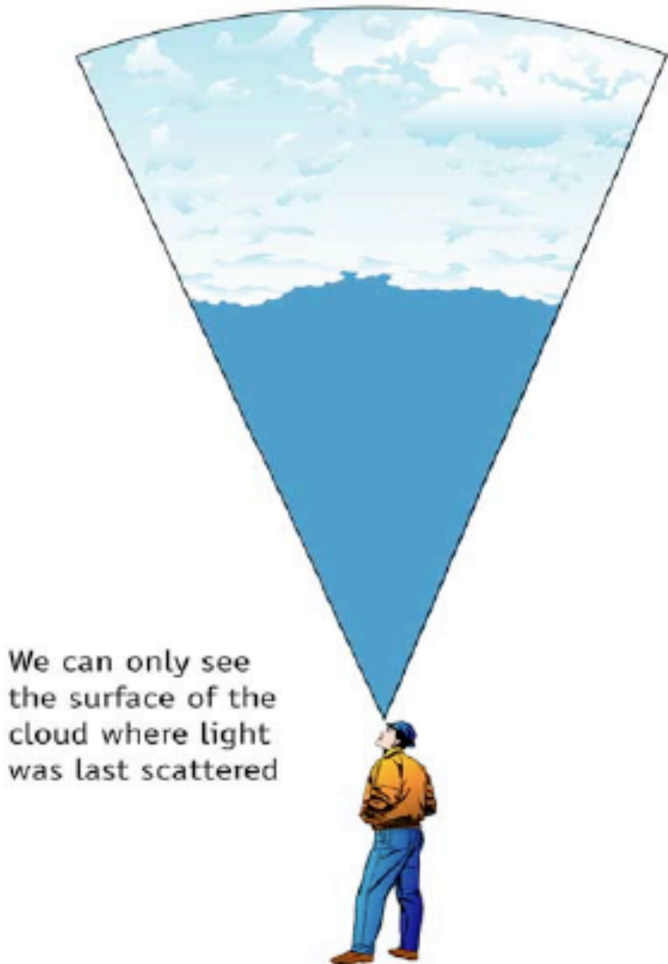
Matter is ionized at temperatures higher than the hydrogen ionization energy of 13.6 eV

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-E_{\rm ion}/T}$$

The Universe becomes transparent to photons when

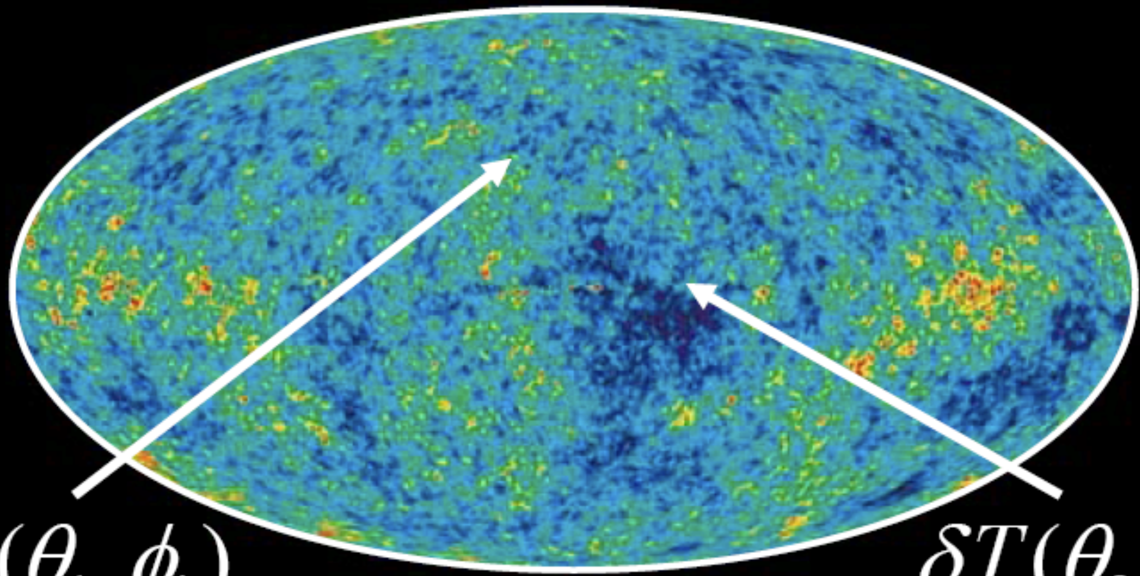
$$(\sigma_{e\gamma} n_e)^{-1} \sim t,\ \sigma_{e\gamma} = 8\pi\alpha^2/3m_e^2,\ T_{\rm LS} \simeq 0.26 \text{ eV}$$





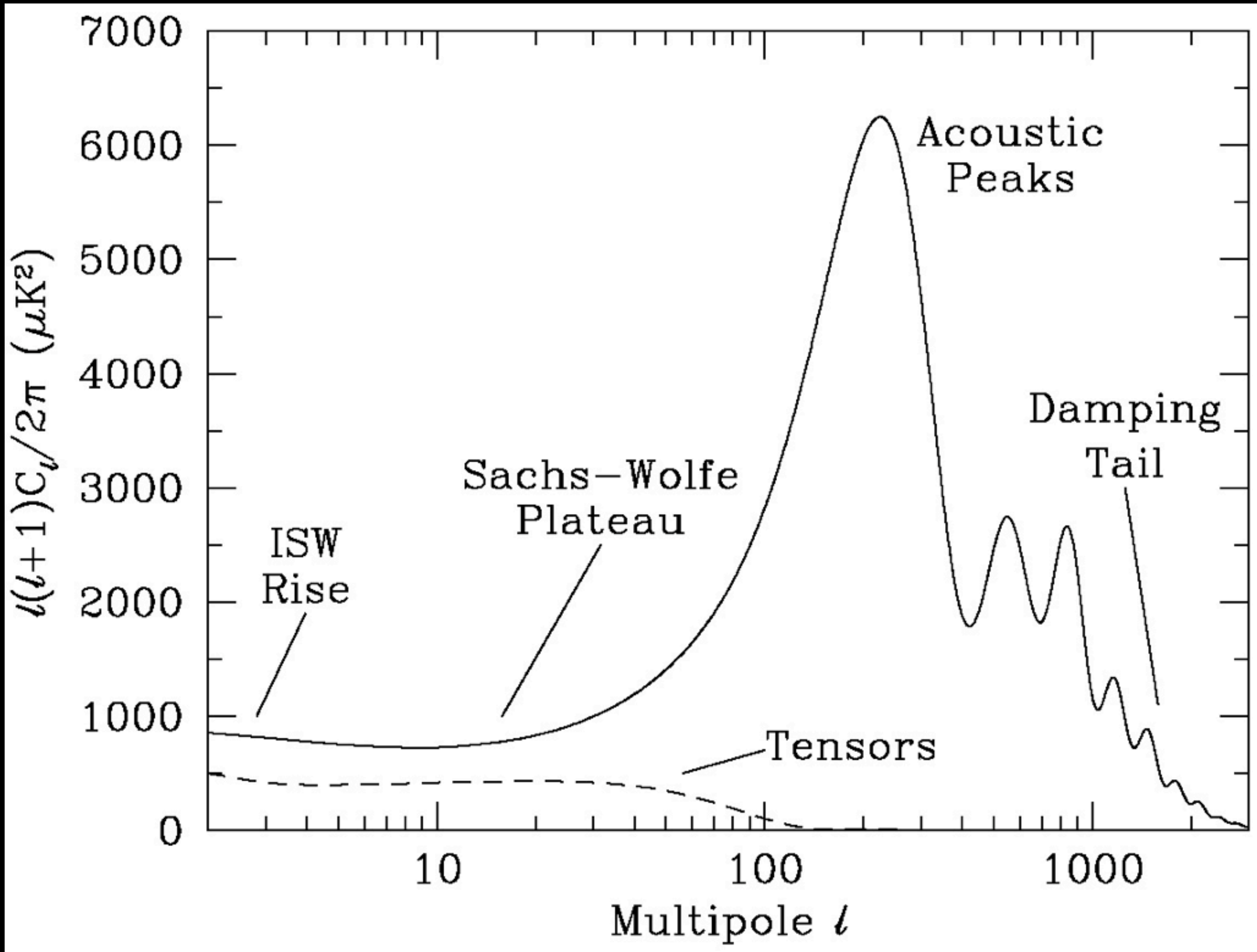
The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

CMB anisotropy



$$
\begin{aligned}
\frac{\Delta T}{T}(x_0, \tau_0, \mathbf{n}) &= \sum_{\ell m} a_{\ell m}(x_0) Y_{\ell m}(\mathbf{n}) \\
\langle a_{\ell m} a_{\ell' m'} \rangle &= \delta_{\ell \ell'} \delta_{m m'} C_\ell \\
\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle &= \sum_{\ell} \frac{(2\ell + 1)}{4\pi} C_\ell P_\ell(\mathbf{n} \cdot \mathbf{n}')
\end{aligned}
$$

(ensemble averages)



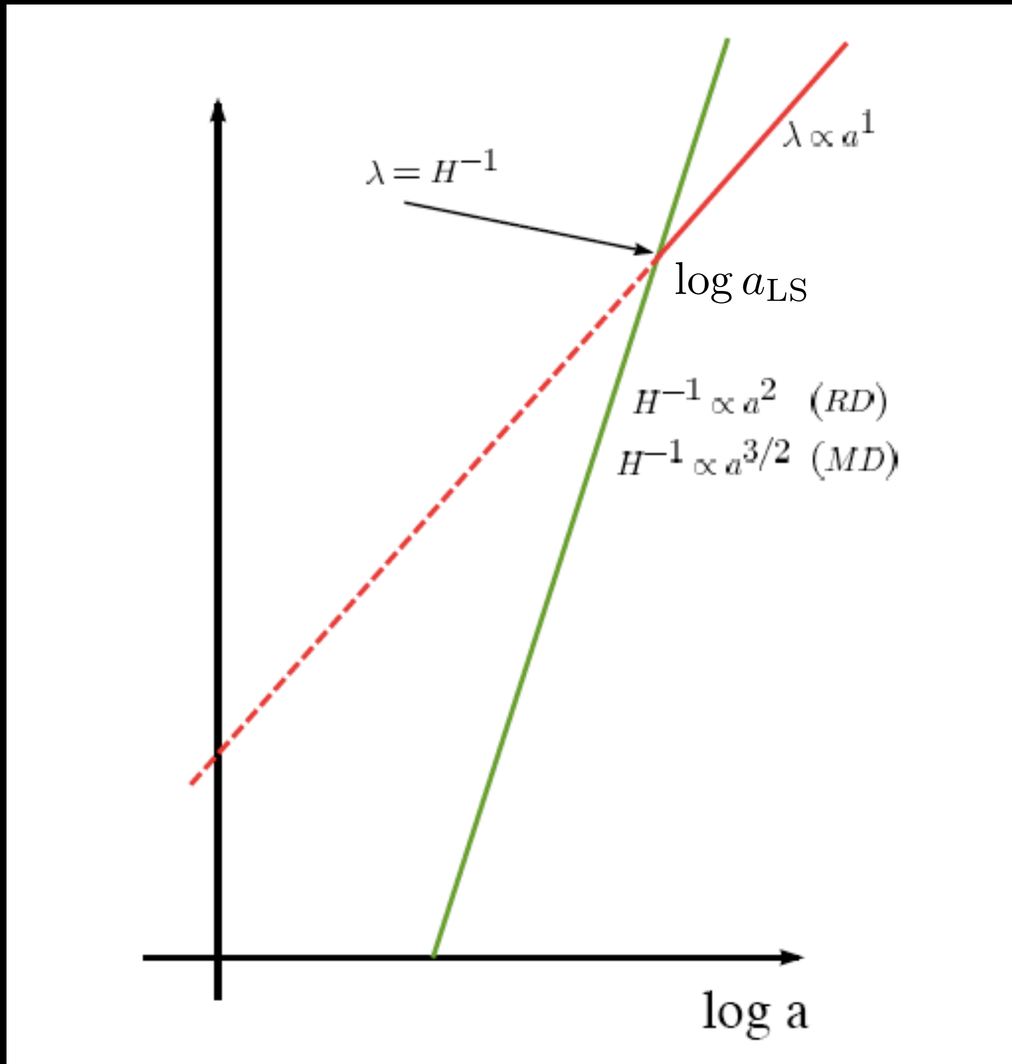
7000
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
10
100
1000
Multipole l
Acoustic Peaks
Damping Tail
Sachs–Wolfe Plateau
ISW Rise
Tensors

The total CMB anisotropy

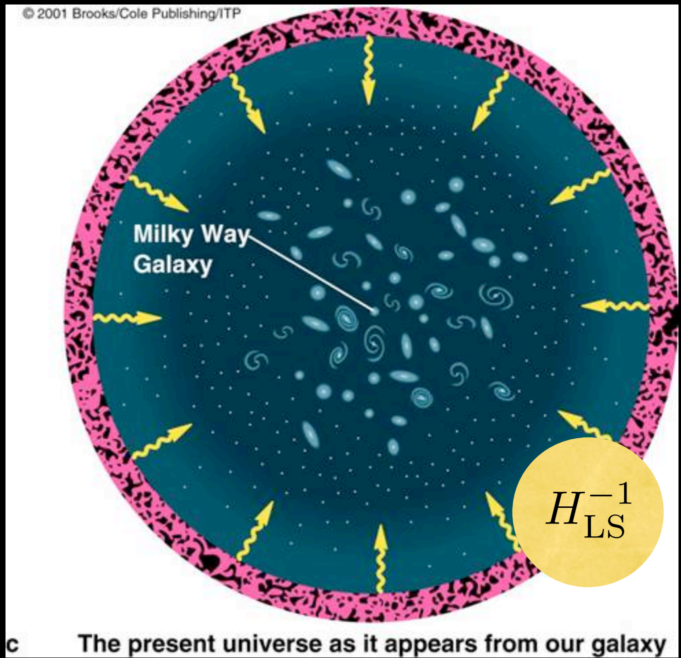
$$\Delta(\mathbf{k},\mathbf{n},\eta) = \underbrace{(\Delta_0 + 4\Phi}_{\text{Sachs-Wolfe effect}} + \underbrace{4\mathbf{v}\cdot\mathbf{n})}_{\text{Doppler effect}} + 4\underbrace{\int_0^{\eta_0}(\Phi+\psi)'}_{\text{Integrated Sachs-Wolfe effect}}$$

$$\Delta = \frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}$$

Φ and ψ are gravitational potentials


$\lambda = H^{-1}$
$\lambda \propto a^1$
$\log a_{\rm LS}$
$H^{-1} \propto a^2 \quad (RD)$
$H^{-1} \propto a^{3/2} \quad (MD)$
log a

Horizon at Last Scattering



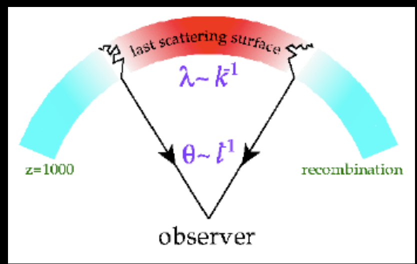
Comoving distance between us and the last scattering surface

$$d\tau = dt/a$$

$$\int_{t_{\rm LS}}^{t_0} \frac{dt}{a} = \int_{\tau_{\rm LS}}^{\tau_0} d\tau = (\tau_0 - \tau_{\rm LS})$$

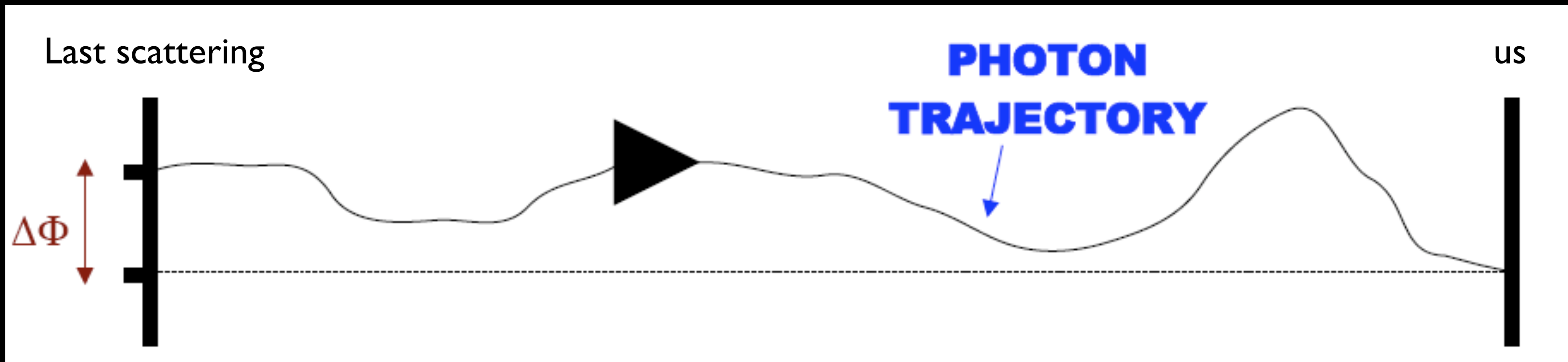
Angle subtended by a given comoving length scale

$$\theta \simeq \frac{\lambda}{(\tau_0 - \tau_{\rm LS})}$$



Sound Horizon

$$\theta_{\rm HOR} \simeq c_s \frac{\tau_{\rm LS}}{(\tau_0 - \tau_{\rm LS})} \simeq c_s \frac{\tau_{\rm LS}}{\tau_0} \simeq c_s \left(\frac{T_0}{T_{\rm LS}}\right)^{1/2} \simeq 1^{\rm o}$$


Last scattering
PHOTON
TRAJECTORY
us
ΔΦ

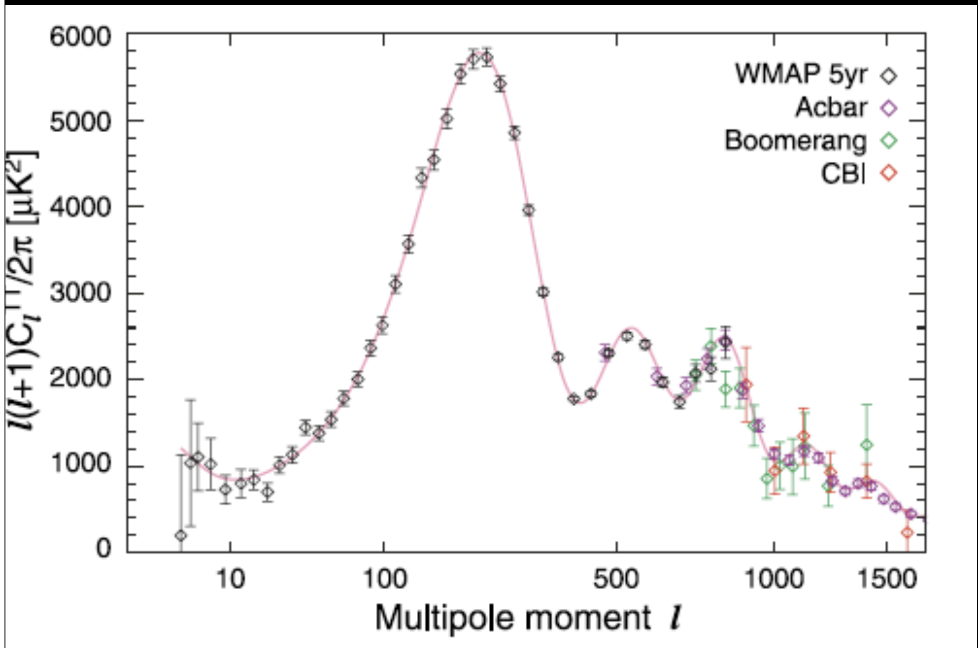
Sachs-Wolfe Plateau

For modes beyond the horizon at last scattering and adiabatic conditions:

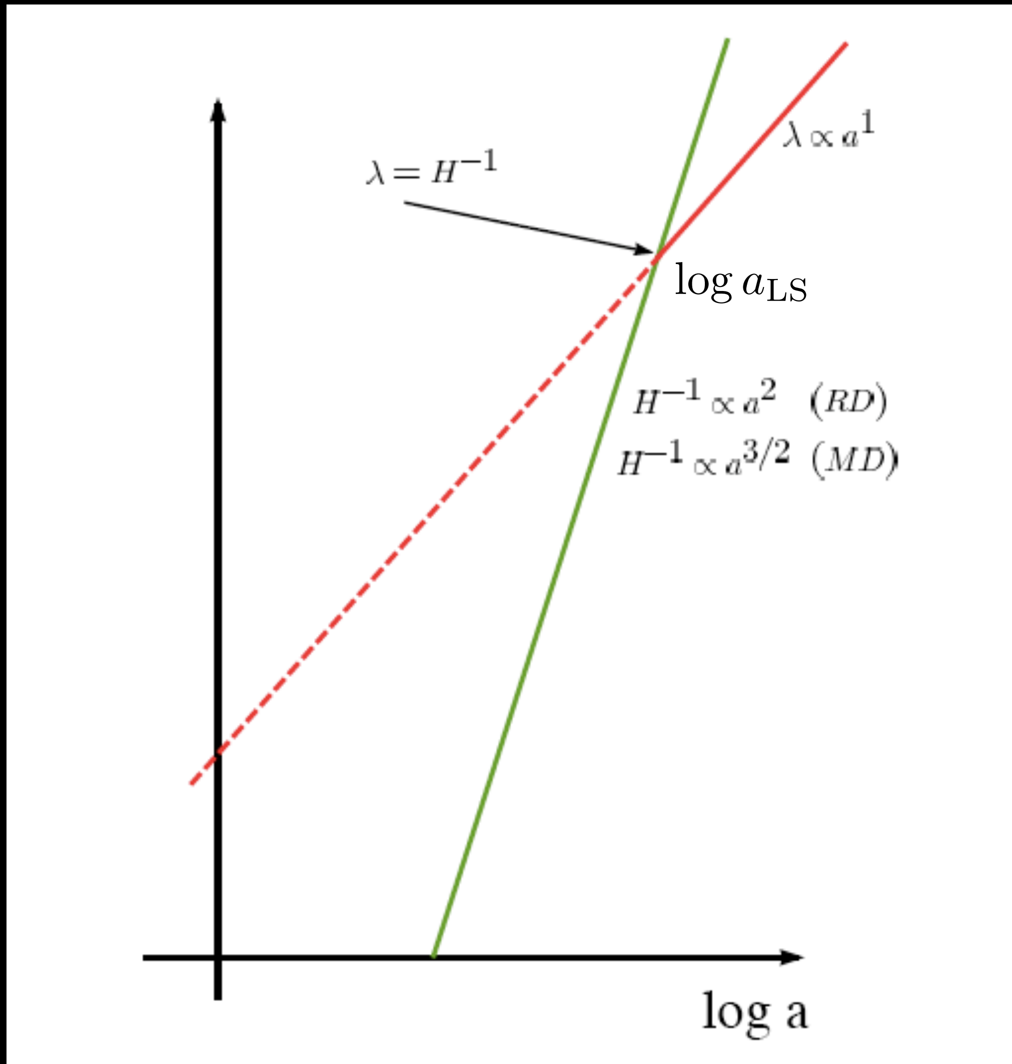
$$\begin{aligned}
\frac{\delta T(\mathbf{n})}{T} &= \frac{\Delta(\mathbf{n})}{4} = \left(\frac{\Delta}{4} + \Phi\right)(\eta_{\mathrm{LS}}) \\
&= \left(\frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} + \Phi\right)(\eta_{\mathrm{LS}}) = \left(\frac{1}{3}\frac{\delta\rho_m}{\rho_m} + \Phi\right)(\eta_{\mathrm{LS}}) \\
&= \left(-\frac{2}{3}\Phi + \Phi\right)(\eta_{\mathrm{LS}}) = \frac{1}{3}\Phi(\eta_{\mathrm{LS}}) \\
&= -\frac{1}{5}\zeta_{\mathrm{inf}}
\end{aligned}$$

$$C_\ell = \frac{2}{\pi} \int \frac{dk}{k} \langle \frac{1}{25} |\zeta_k|^2 \rangle k^3 \mathrm{j}_\ell^2(k(\eta_0 - \eta_{\mathrm{LS}}))$$

$$\pi\ell(\ell+1)C_\ell = \frac{1}{50} \frac{1}{M_P^2} \left(\frac{H}{2\pi\epsilon^{1/2}} \right)^2$$



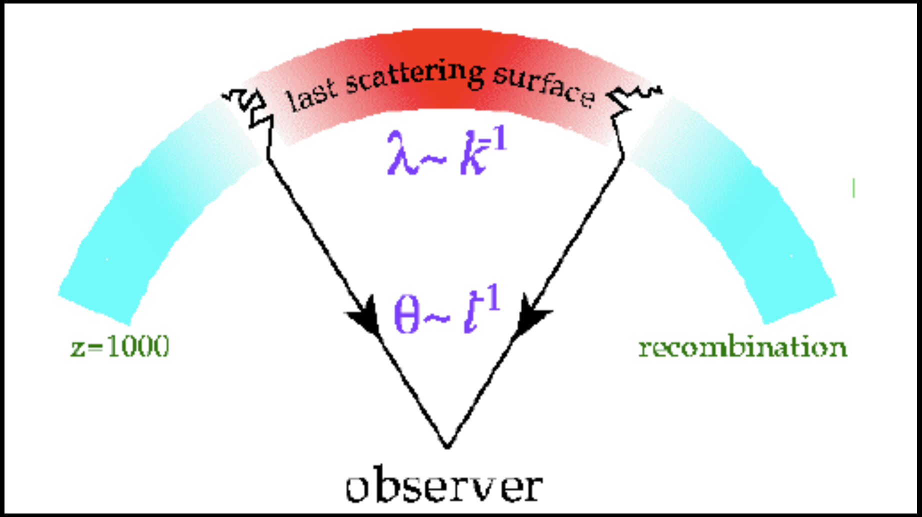
$$\boxed{\left(\frac{V}{\epsilon}\right)^{1/4} \simeq 6.7 \times 10^{16}\,\mathrm{GeV}}$$



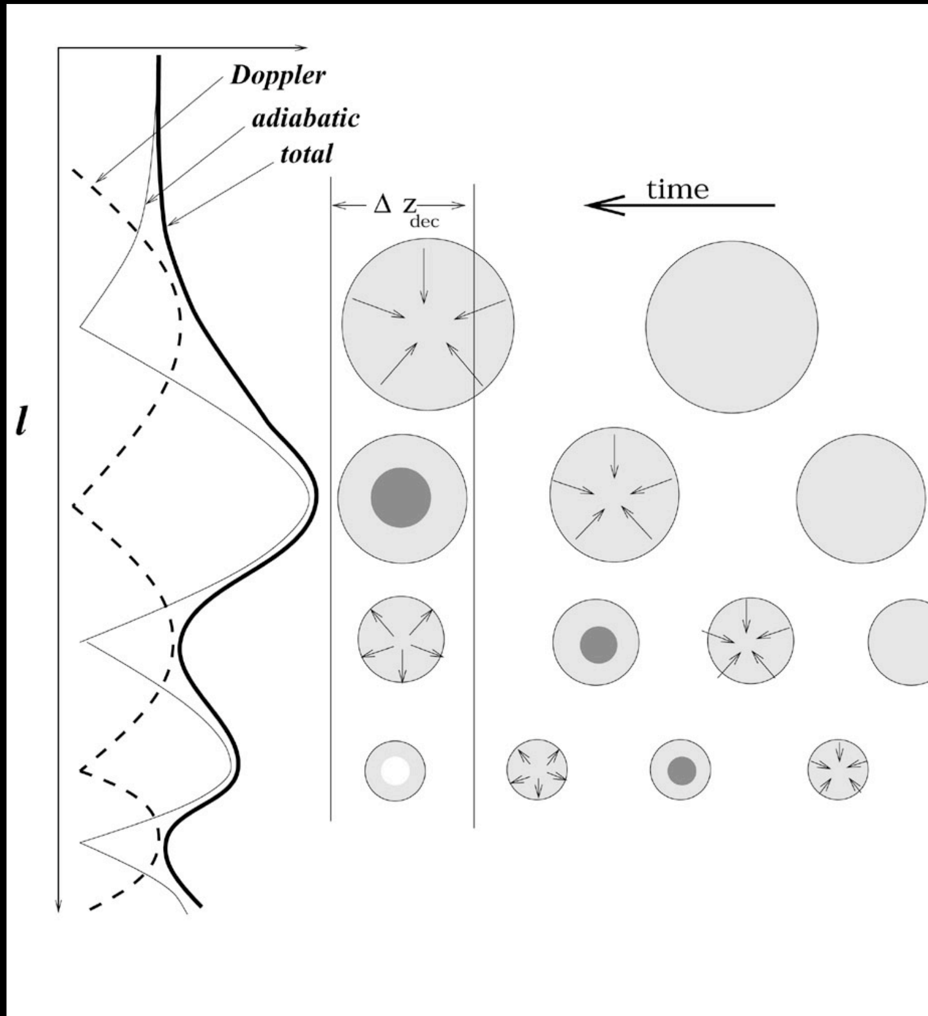
$\lambda = H^{-1}$
$\lambda \propto a^1$
$\log a_{\rm LS}$
$H^{-1} \propto a^2 \quad (RD)$
$H^{-1} \propto a^{3/2} \quad (MD)$
log a

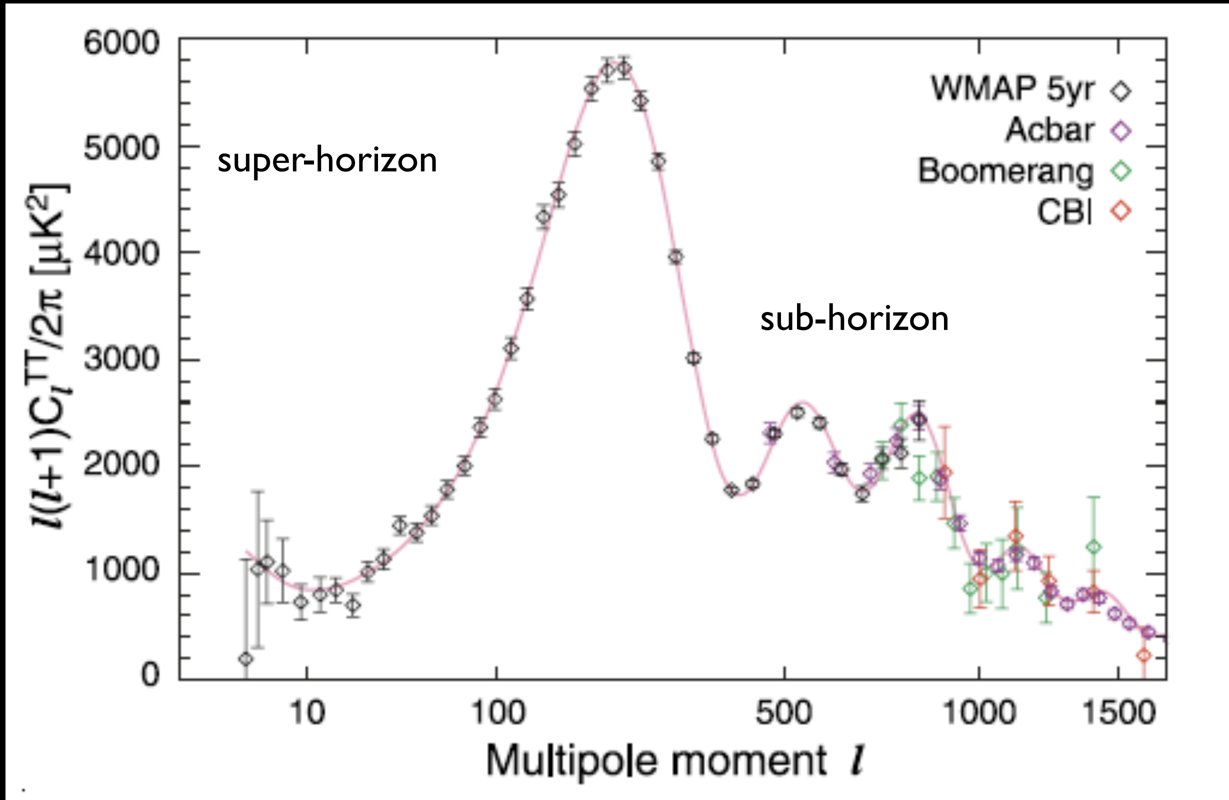
Acoustic peaks

- At recombination, baryon-photon fluid undergoes "acoustic oscillations" $A \cos c_s k\eta + B \sin c_s k\eta$
- Compressions and rarefactions change the temperature T_γ
- Peaks in ΔT_γ corresponds to extrema of compressions and rarefactions



Acoustic peaks

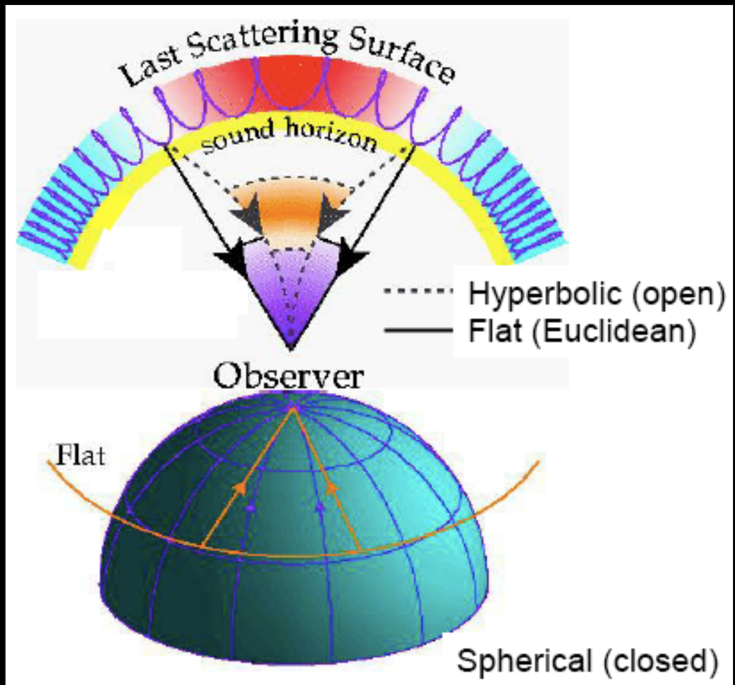




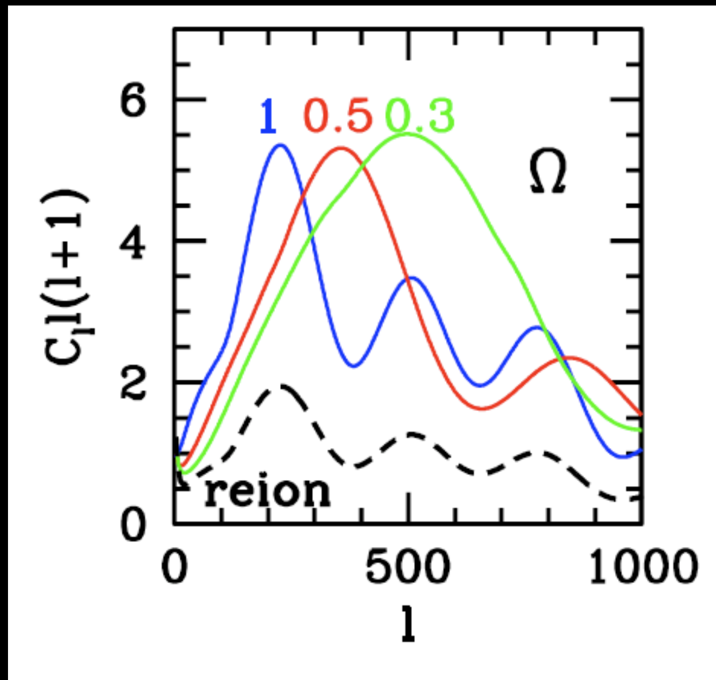
6000
5000
4000
3000
2000
1000
0
l(l+1)C_l^TT/2π [μK²]
super-horizon
sub-horizon
WMAP 5yr
Acbar
Boomerang
CBI
10
100
500
1000
1500
Multipole moment l

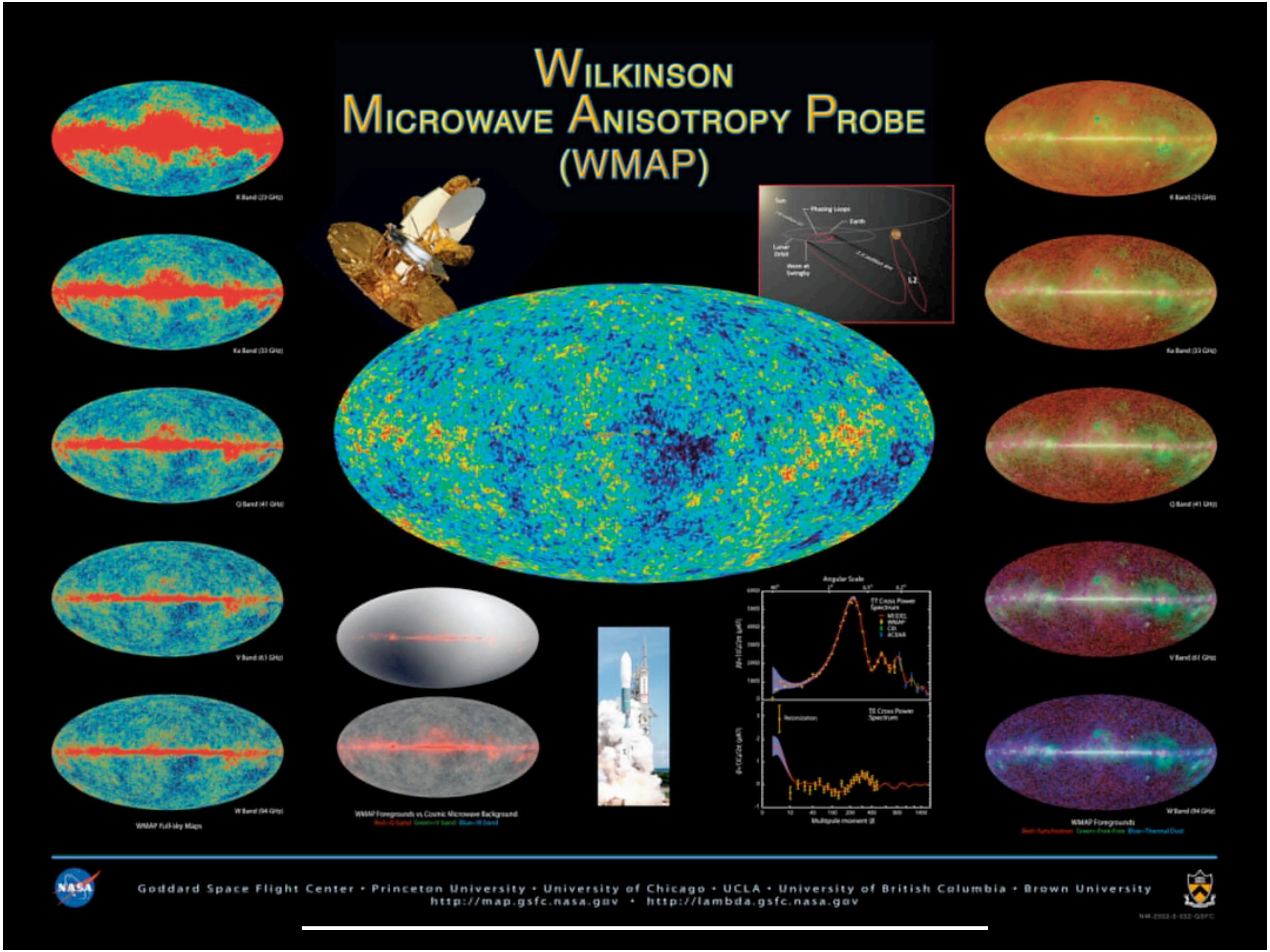
Position of the first peak

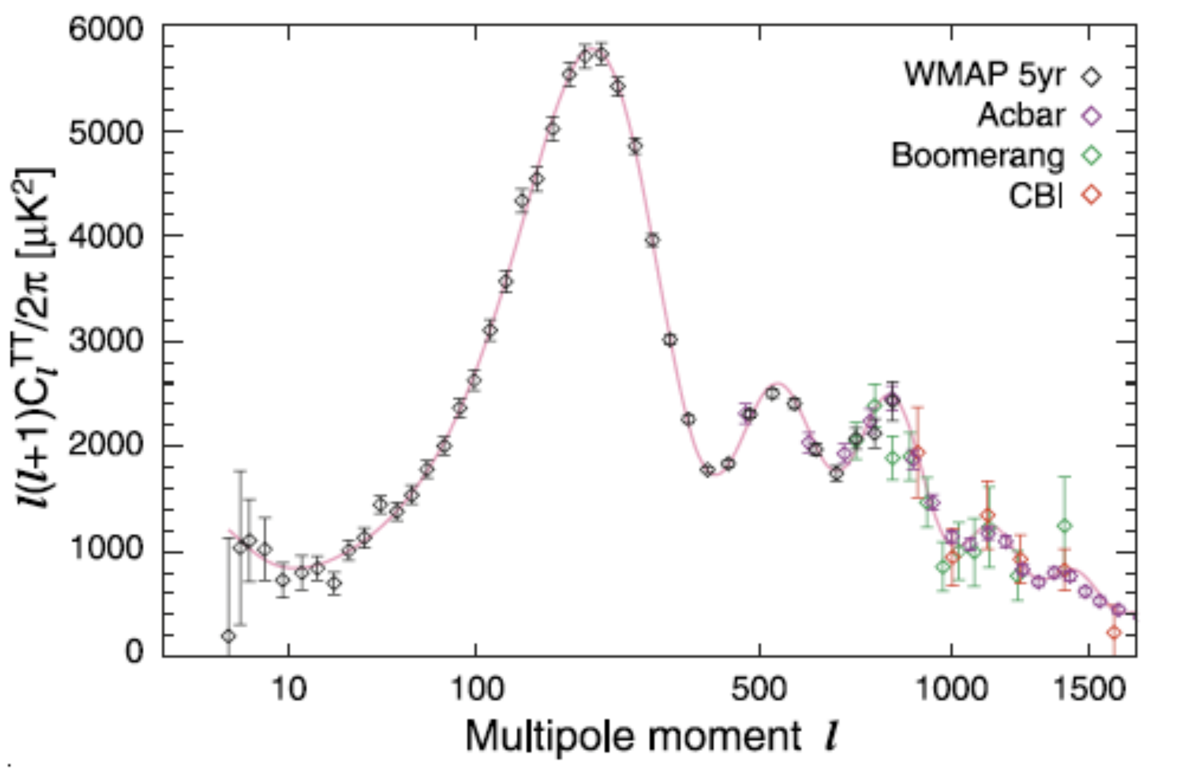
Modes caught in the extrema of their oscillation at recombination will have enhanced fluctuations, yielding a fundemental scale or frequency related to the Universe sound horizon



$$\ell_{\text{first peak}} \simeq \frac{220}{\sqrt{\Omega_{\text{tot}}}}$$




Wilkinson
Microwave Anisotropy Probe
(WMAP)
Angular Scale
TT Cross Power Spectrum
TE Cross Power Spectrum
Multipole moment (ℓ)
W Band (94 GHz)
WMAP Full-sky Maps
WMAP Foregrounds vs Cosmic Microwave Background
WMAP Foregrounds
Red=Synchrotron Green=Free-Free Blue=Thermal Dust
NASA
Goddard Space Flight Center • Princeton University • University of Chicago • UCLA • University of British Columbia • Brown University
http://map.gsfc.nasa.gov • http://lambda.gsfc.nasa.gov



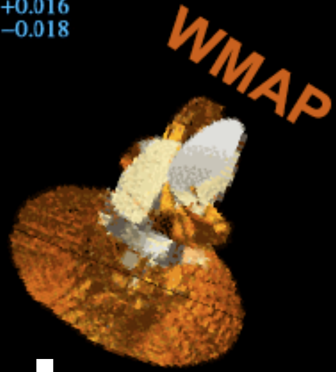
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l^{TT}/2\pi$ [μK²]
WMAP 5yr
Acbar
Boomerang
CBI
10
100
500
1000
1500
Multipole moment l

Precision Cosmology

INFLATION

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$
$w < -0.78$ (95% CL)
$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$
$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$
$\Omega_b = 0.044^{+0.004}_{-0.004}$
$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}
$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$
$\Omega_m = 0.27^{+0.04}_{-0.04}$
$\Omega_\nu h^2 < 0.0076$ (95% CL)
$m_\nu < 0.23$ eV (95% CL)
$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K
$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}
$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$
$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$
$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc
$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$
$A = 0.833^{+0.086}_{-0.083}$

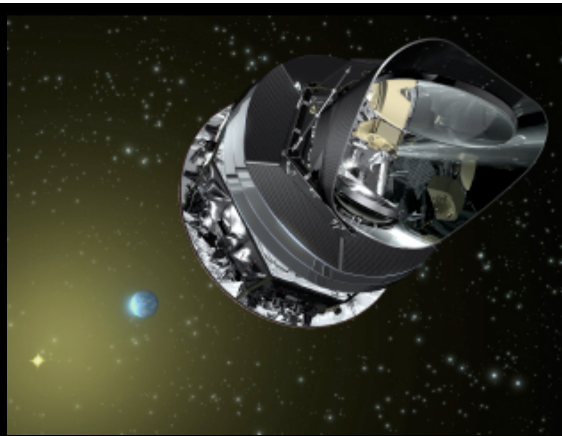
$n_s = 0.93^{+0.03}_{-0.03}$
$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$
$r < 0.71$ (95% CL)
$z_{dec} = 1089^{+1}_{-1}$
$\Delta z_{dec} = 195^{+2}_{-2}$
$h = 0.71^{+0.04}_{-0.03}$
$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr
$t_{dec} = 379^{+8}_{-7}$ kyr
$t_r = 180^{+220}_{-80}$ Myr (95% CL)
$\Delta t_{dec} = 118^{+3}_{-2}$ kyr
$z_{eq} = 3233^{+194}_{-210}$
$\tau = 0.17^{+0.04}_{-0.04}$
$z_r = 20^{+10}_{-9}$ (95% CL)
$\theta_A = 0.^\circ598^{+0.002}_{-0.002}$
$d_A = 14.0^{+0.2}_{-0.3}$ Gpc
$l_A = 301^{+1}_{-1}$
$r_s = 147^{+2}_{-2}$ Mpc



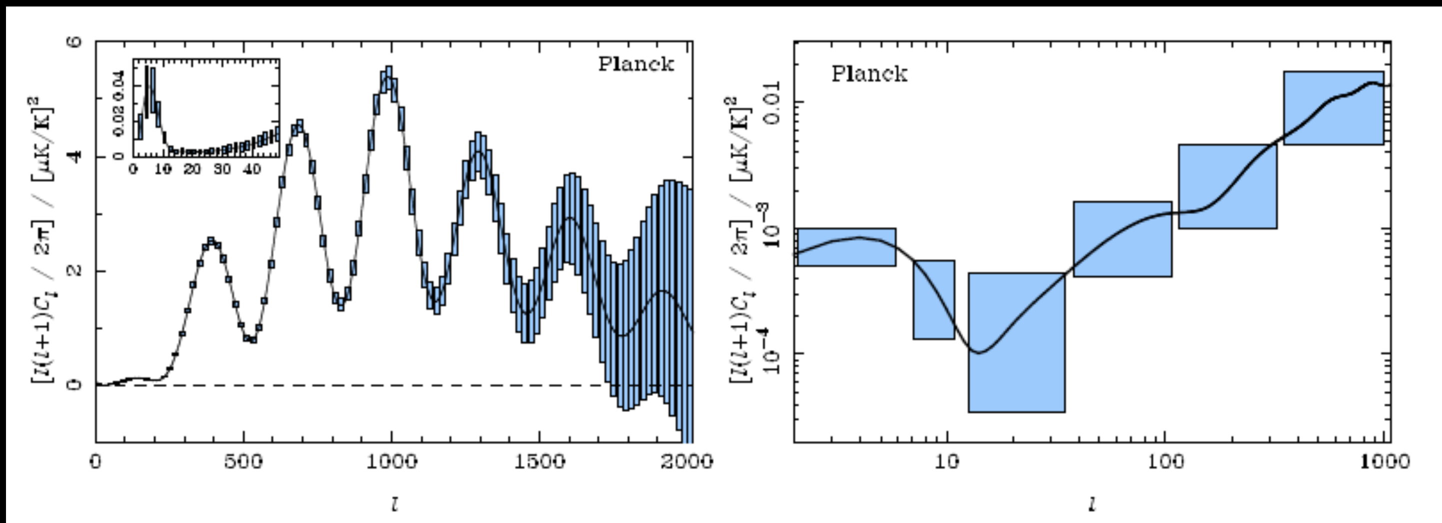
The Future

- CMB polarization
- Non-Gaussianity

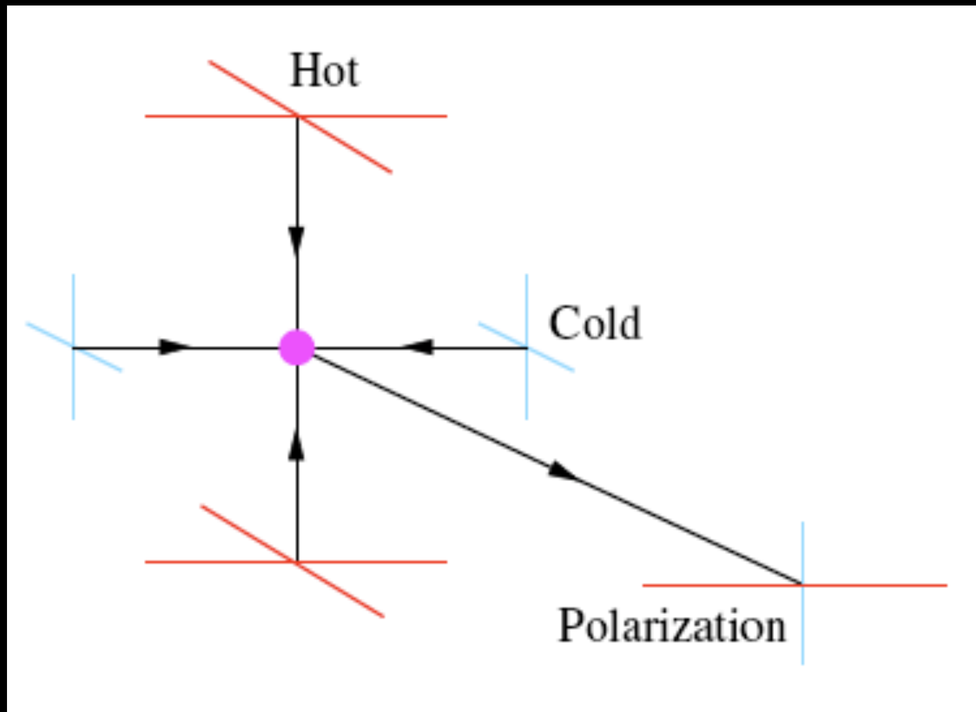
Planck



- Lunch in April 29, 2009
- Fully sky imaging from L2 in nine frequency bands (30-587 GHz)
- Polarization may be sensitive to $r \sim 0.1$



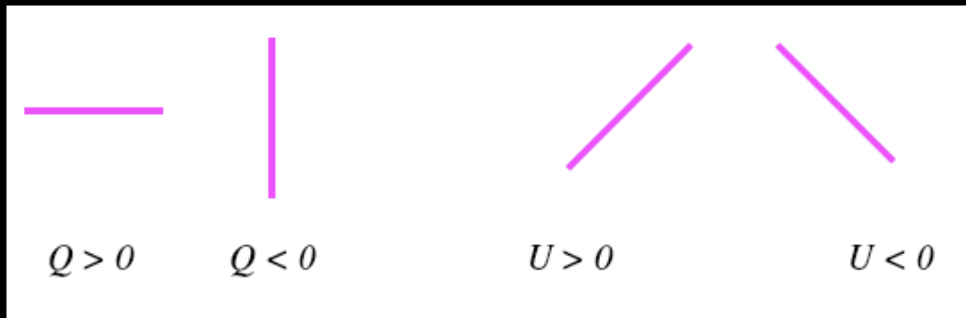
CMB anisotropy is polarized



CMB Polarization

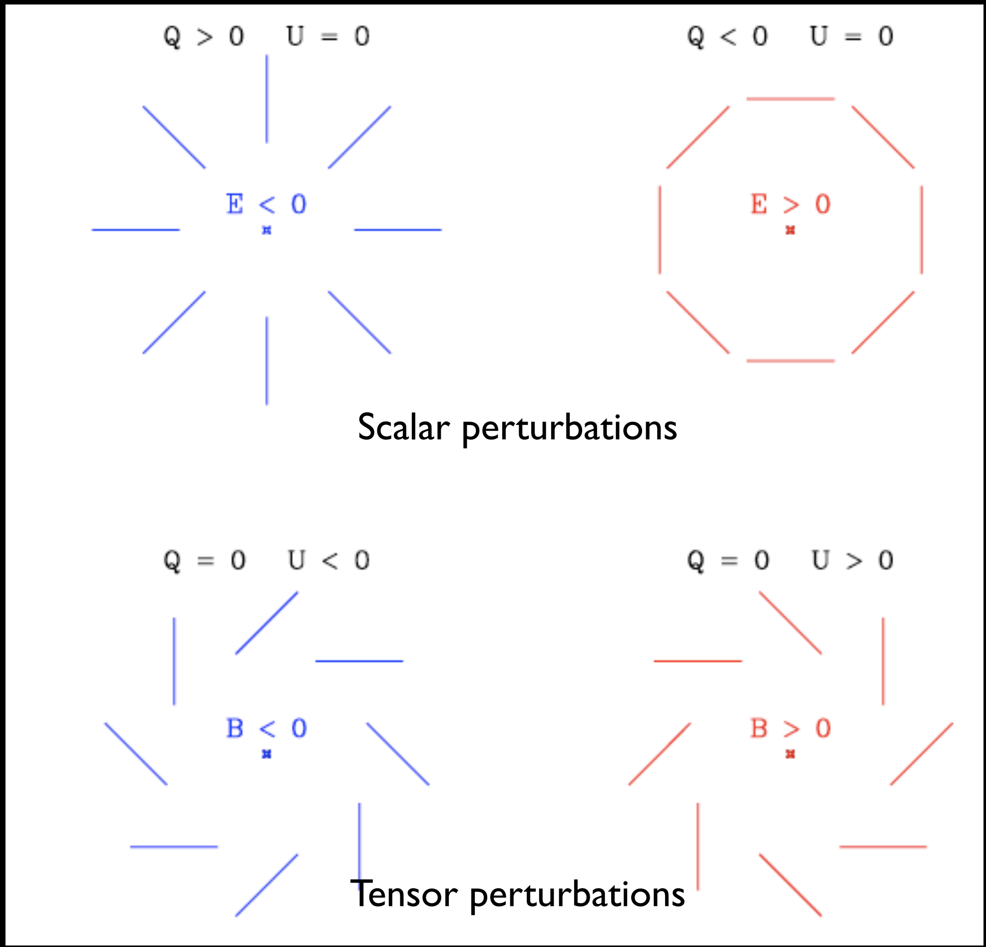
For a plane wave along the z-direction, symmetric trace-free (STF) correlation tensor of electric field defines (transverse) linear polarization tensor:

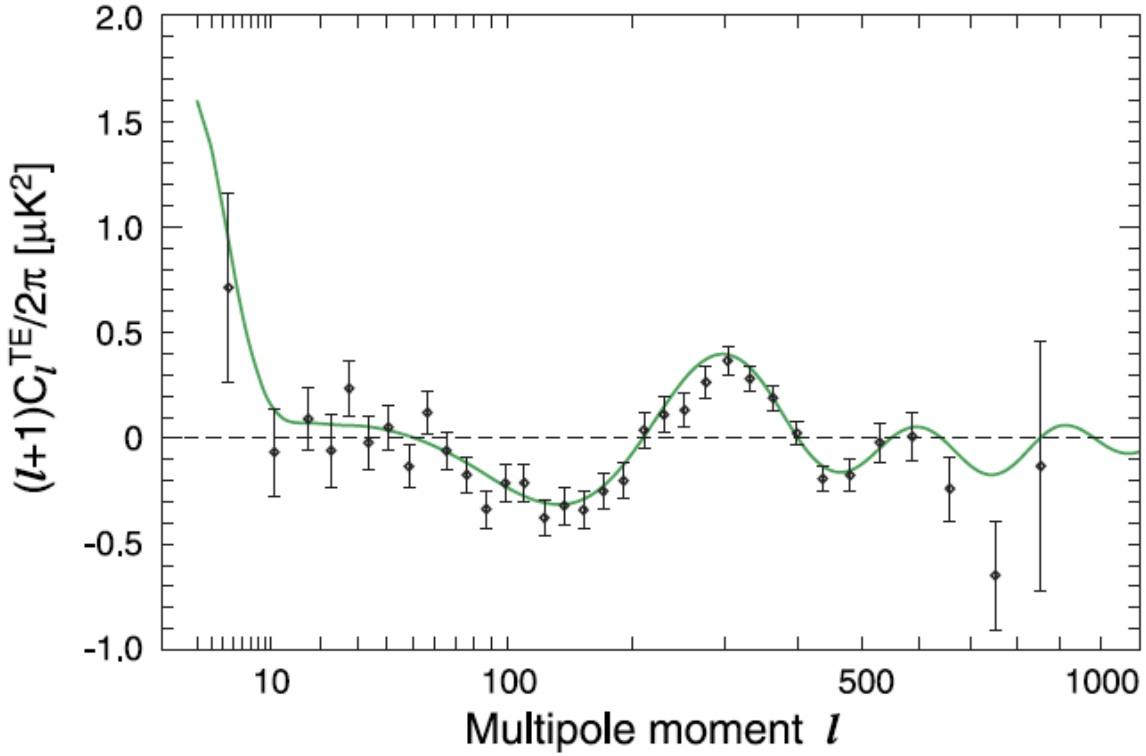
$$\mathcal{P}_a \equiv \begin{pmatrix} \frac{1}{2}\langle E_x^2 - E_y^2 \rangle & \langle E_x E_y \rangle \\ \langle E_x E_y \rangle & -\frac{1}{2}\langle E_x^2 - E_y^2 \rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



Under a rotation in the (x-y)-plane

$$Q \pm iU \to (Q \pm iU)e^{-\mp\alpha} \Rightarrow (Q + iU) \text{ is spin 2}$$

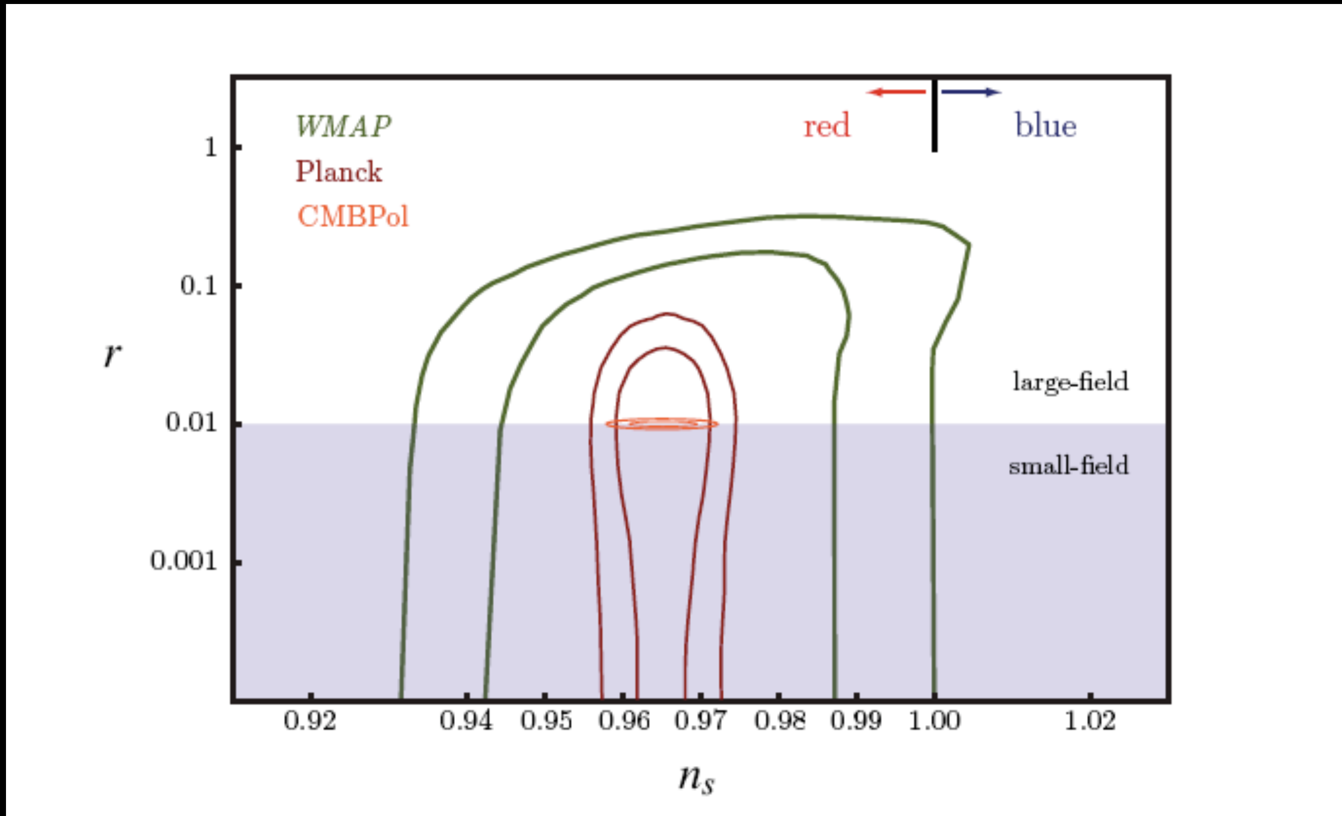

Q > 0 U = 0
E < 0
Q < 0 U = 0
E > 0
Scalar perturbations
Q = 0 U < 0
B < 0
Q = 0 U > 0
B > 0
Tensor perturbations



2.0
1.5
1.0
0.5
0
-0.5
-1.0
$(l+1)C_l^{TE}/2\pi$ [μK^2]
10
100
500
1000
Multipole moment l

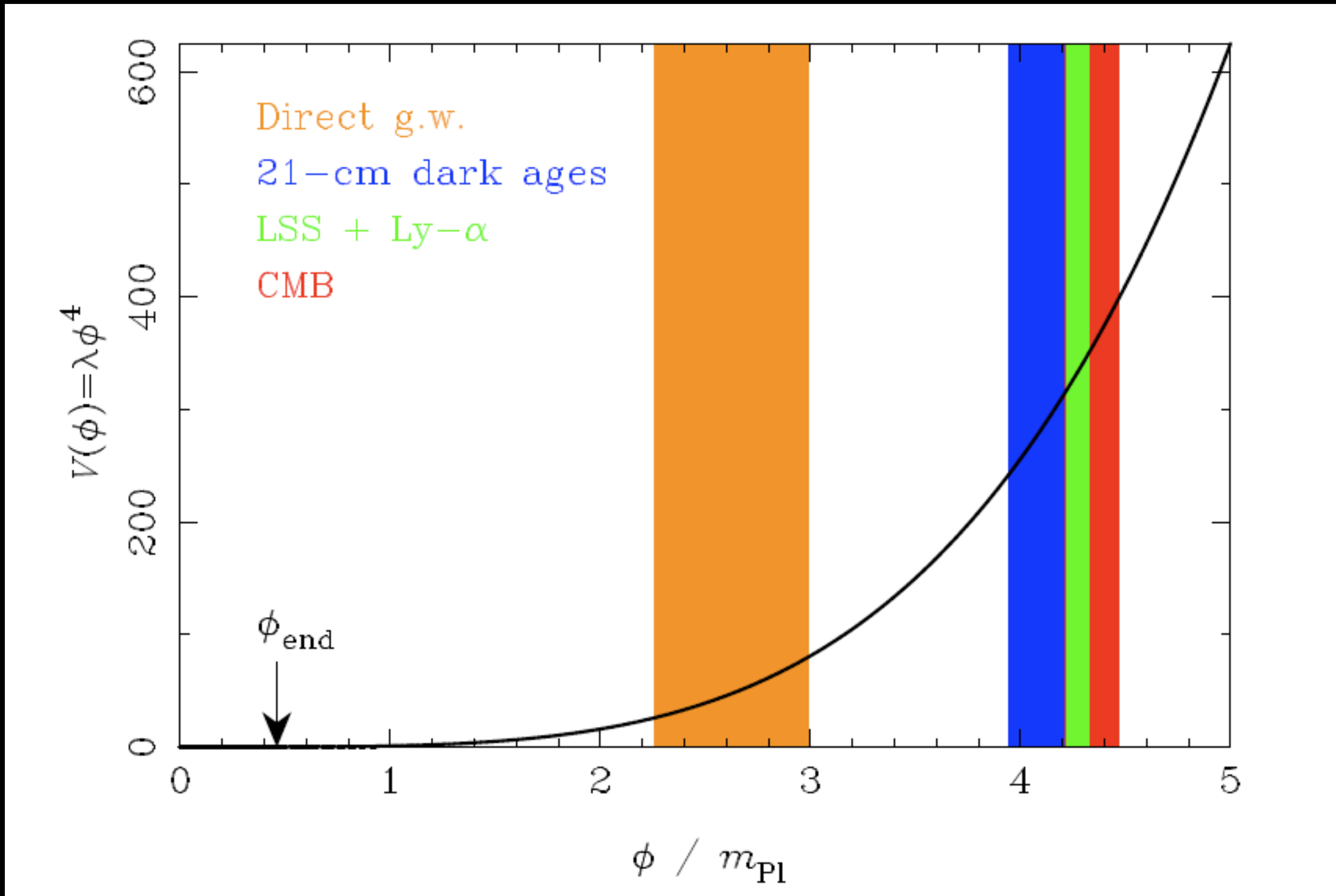
Testing the energy scale of Inflation

CMBpol: approved by NASA on Feb. 18, 2008,
http://astro.fnal.gov/cmb/, Weiss document



$$\boxed{[\ell(\ell+1)C_{B\ell}/2\pi]^{1/2} \simeq 0.024(E_{\rm inf}/10^{16}\,{\rm GeV})\,\mu{\rm K}}$$

Interesting from the theoretical point of view: Planckian excursus of the inflaton field



Observation of the B-mode polarization from inflationary gravity waves requires

$$\boxed{r \simeq 10^{-2}\left(\frac{\Delta\phi}{m_{\mathrm{Pl}}}\right) > 10^{-2}}$$

Non-Gaussianity

Characterizing the cosmological perturbations

- The WMAP data are telling us that primordial fluctuations are very close to being Gaussian.

- It may not be so easy to explain that CMB is Gaussian unless we have a compelling early universe model that predicts Gaussian primordial fluctuations: Inflation

What if we discover
in the future
that perturbations
are non-Gaussian?

Gaussian ⟺ free (i.e. non-interacting) field, linear theory

- Collection of independent harmonic oscillators (no mode-mode coupling)

- NG requires more than linear theory

"*... the linear perturbations are so surprisingly simple that a perturbation analysis accurate to second order may be feasible ...*" (Sachs & Wolfe 1967)

Why do we expect some NG, i.e. some Non-Linearity ?

- The observed sky is NG: astrophysical sources (point sources and galactic emission, low level contaminaton of galactic foreground leads to detectable NG, but negligible effects in the angular power spectrum)
- Secondary anisotropies (lensing, SZ, .etc: known to exist)
- Variance of the noise is spatially variable, increasing the variance of the NG estimator
- Gravity itself is nonlinear
- Primordial contribution

How large is the predicted value of NG ?

It depends on the primordial contribution: it is the contribution generated either during or after inflation, when the comoving curvature perturbation becomes finally constant (in time) on super-horizon scales

It is the real science driver

Phenomenlogical approach:

$$\boxed{\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_g^2(x) - \langle \zeta_g^2 \rangle\right)}$$

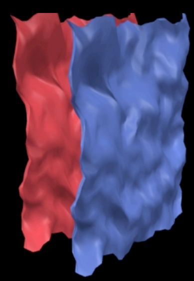
The expanding parameter is roughly $f_{\rm NL}\zeta_g$

The non-linear parameter is usually momentum dependent

$\rightarrow$ It is not directly connected to the measurable quantity, the CMB anisotropy

Second scenario: Inflation is non-standard (DBI, ghost inflation,....)
Third scenario: inflation does not take place, instead ekpyrotic,....

	Canonical	Non canonical
One field	• Single field inflation with canonical kinetic term $f_{\rm NL}^{\rm local,equil} = O(\varepsilon, \eta) \sim 0.01$	• K-inflation, DBI-inflation,… $f_{\rm NL}^{\rm equil} \sim 1/c_s^2 \sim 100$ • Break in slow-roll…
More fields	• Multi-field inflation $f_{\rm NL}^{\rm local} \sim \frac{1}{16} r + {\rm nl\ evol}$ ~ 0.01	• DBI-multi-field inflation $f_{\rm NL}^{\rm equil/local} \sim 1/c_s^2$ • Curvaton-like models $f_{\rm NL}^{\rm local} \sim \frac{5}{4}\left(\rho/\rho_{\rm curvaton}\right)_{\rm dec} > 1$ • New ekpyrotic $f_{\rm NL}^{\rm local} > (n_s - 1)^{-1} \quad (n_s > 1)$

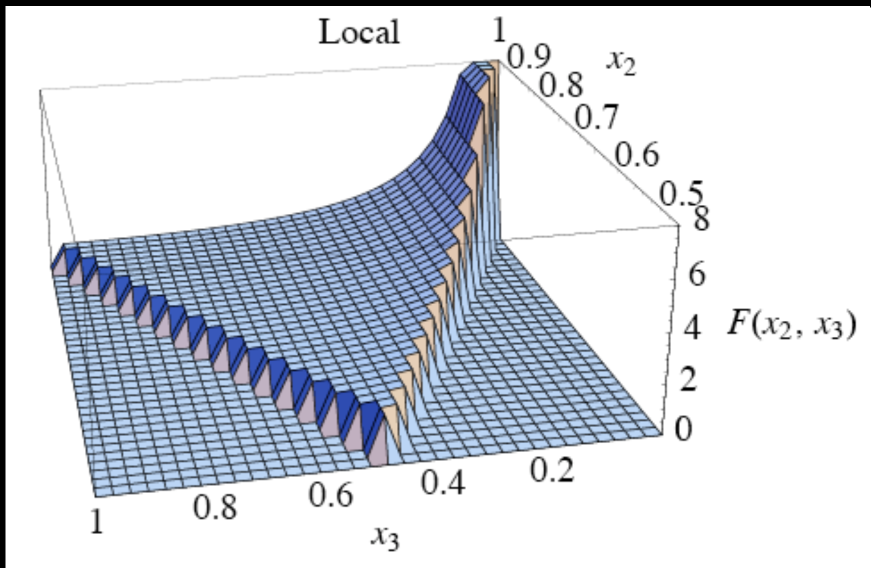


N. Bartolo, E. Komatsu, S. Matarrese and A.R., Phys. Rept. 402, 103 (2004)

The Bispectrum

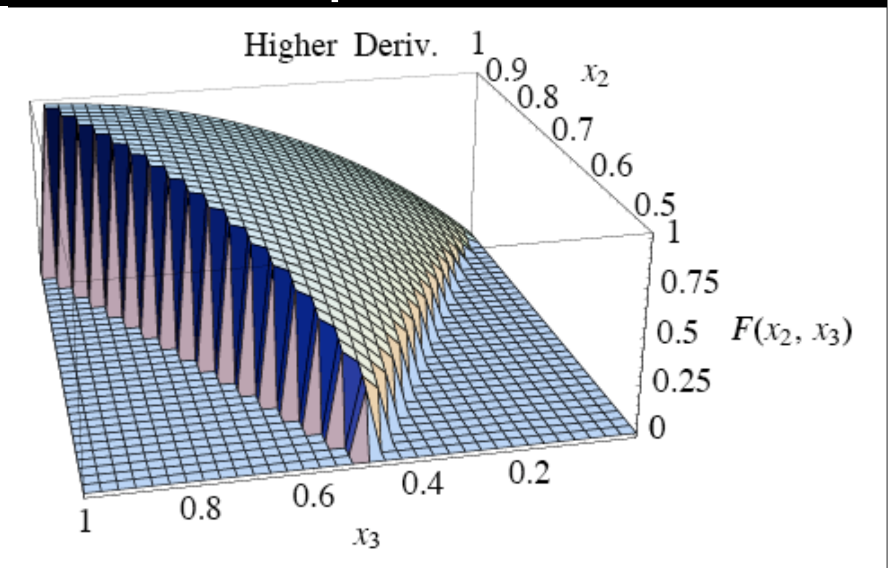
$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \delta \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 \right) B_\zeta(k_1, k_2, k_3)$$

Local



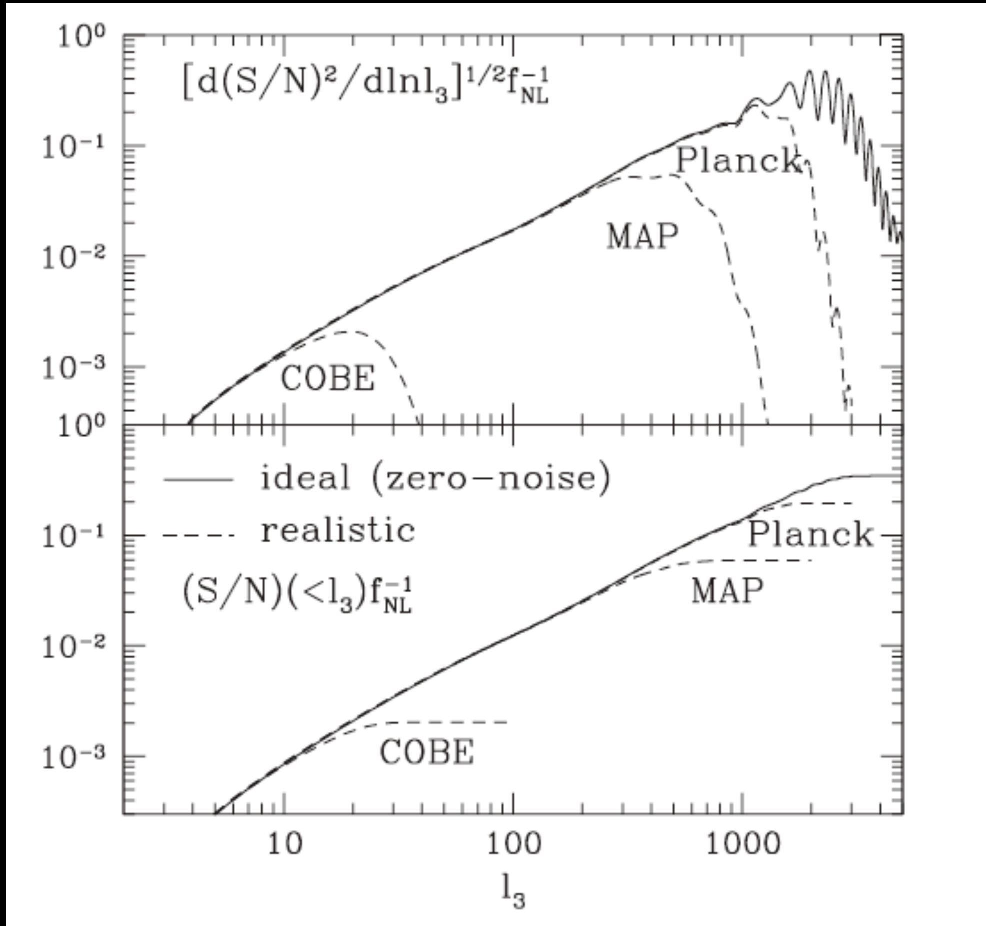
$$B_\zeta(k_1, k_2, k_3) \propto f_{\rm NL} \left[P(k_1) P(k_2) + \text{perm.} \right]$$
$$k_1 \ll k_2, k_3$$

Equilateral



D. Babich et al., (2005)

$$\left(\frac{S}{N}\right)_{\rm prim} \sim 10^{-4} f_{\rm NL}\,\ell$$



$$\Delta f_{\rm NL} \sim 20,\ \ell_{\rm max} \sim 500\ ({\rm WMAP})$$
$$\Delta f_{\rm NL} \sim 3,\ \ell_{\rm max} \sim 3000\ ({\rm Planck})$$
$$\Delta f_{\rm NL} \sim 2,\ ({\rm ideal\ \ experiment})$$

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Phys. Rept. 402, 103 (2004)