LUND UNIVERSITY

LNF Spring School “Bruno Touschek”
Entering the LHC Era
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Event Generators for LHC

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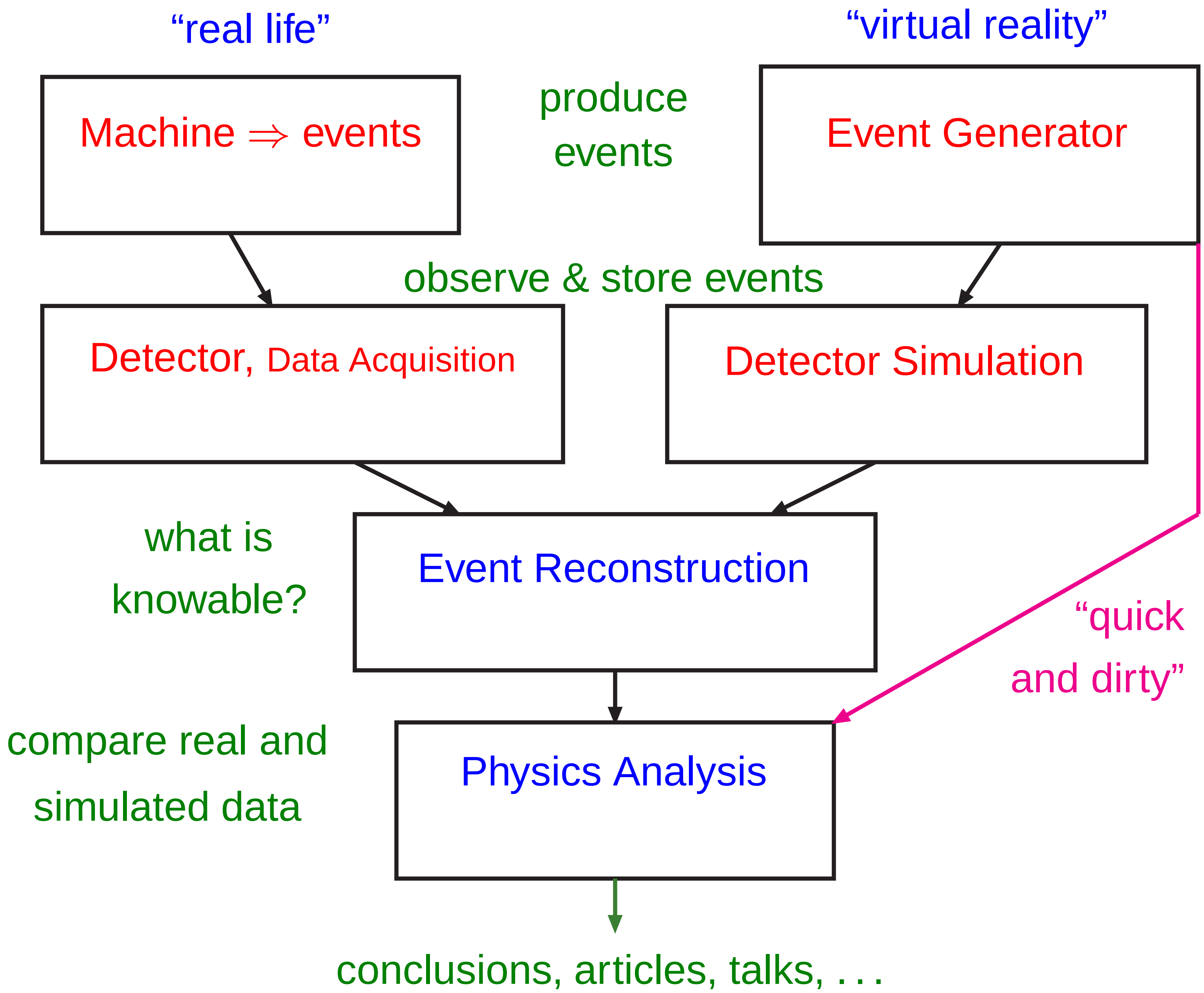
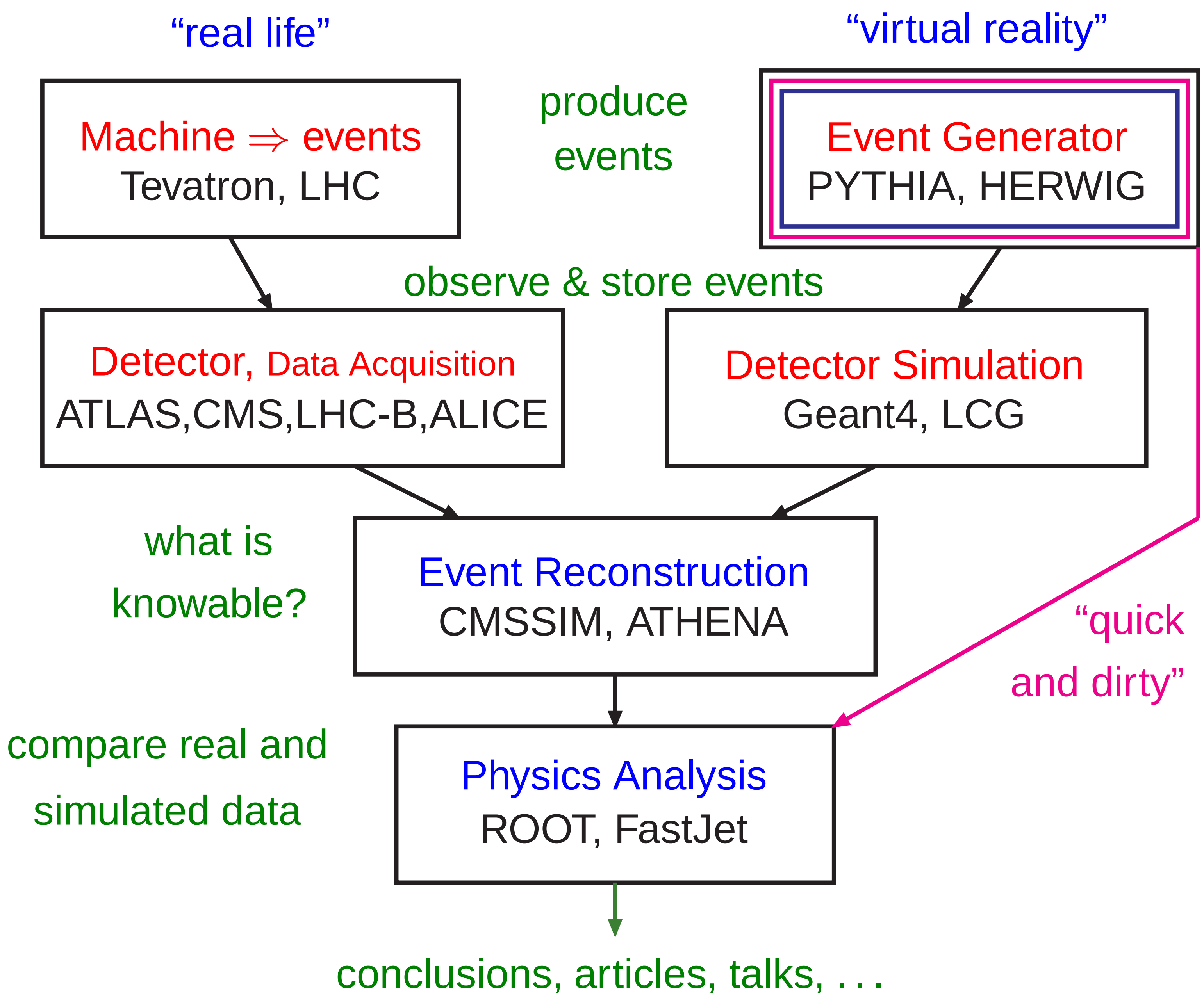
1. (today) Introduction and Overview;
Parton Showers; Matching Issues

2. (tomorrow) Multiple Interactions;
Hadronization; Generator News & Conclusions

Event Generator Position



Event Generator Position



Why Generators?

- Allow studies of *complex* multiparticle physics
- Large flexibility in physical quantities that can be addressed
- Vehicle of ideology to disseminate ideas

Can be used to

- predict event rates and topologies $\Rightarrow$ estimate feasibility
- simulate possible backgrounds $\Rightarrow$ devise analysis strategies
- study detector requirements $\Rightarrow$ optimize detector/trigger design
- study detector imperfections $\Rightarrow$ evaluate acceptance corrections

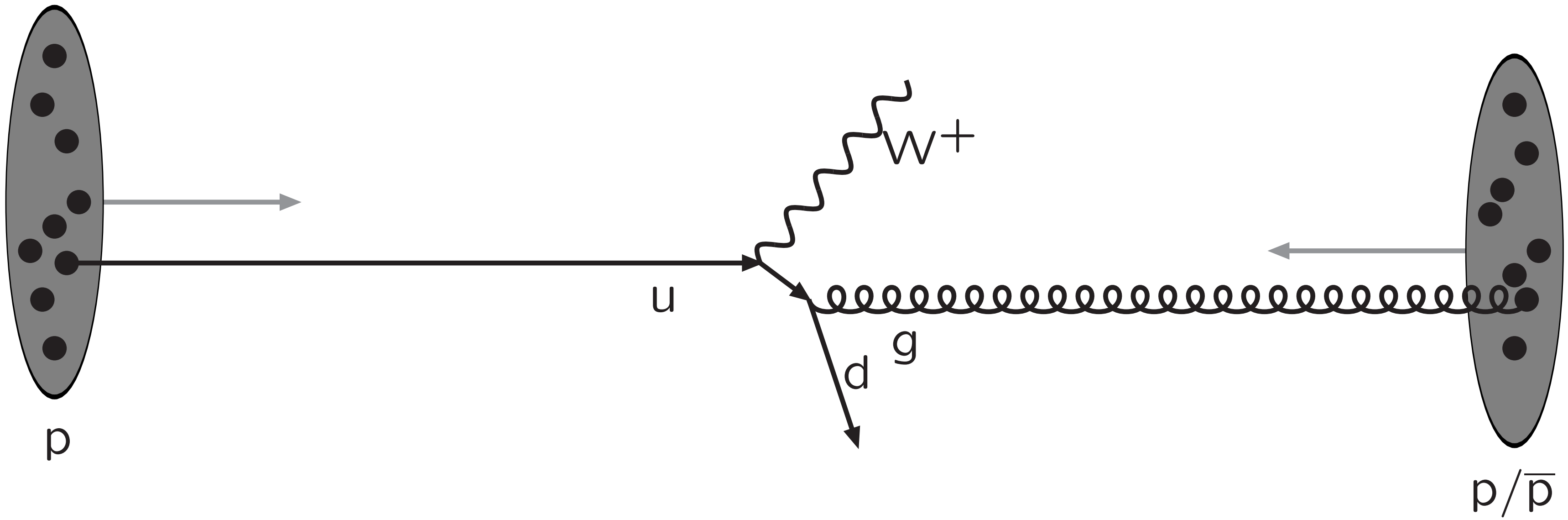
Monte Carlo method convenient because Einstein was wrong:
God does throw dice!

Quantum mechanics: amplitudes $\Longrightarrow$ probabilities
Anything that possibly can happen, will! (but more or less often)

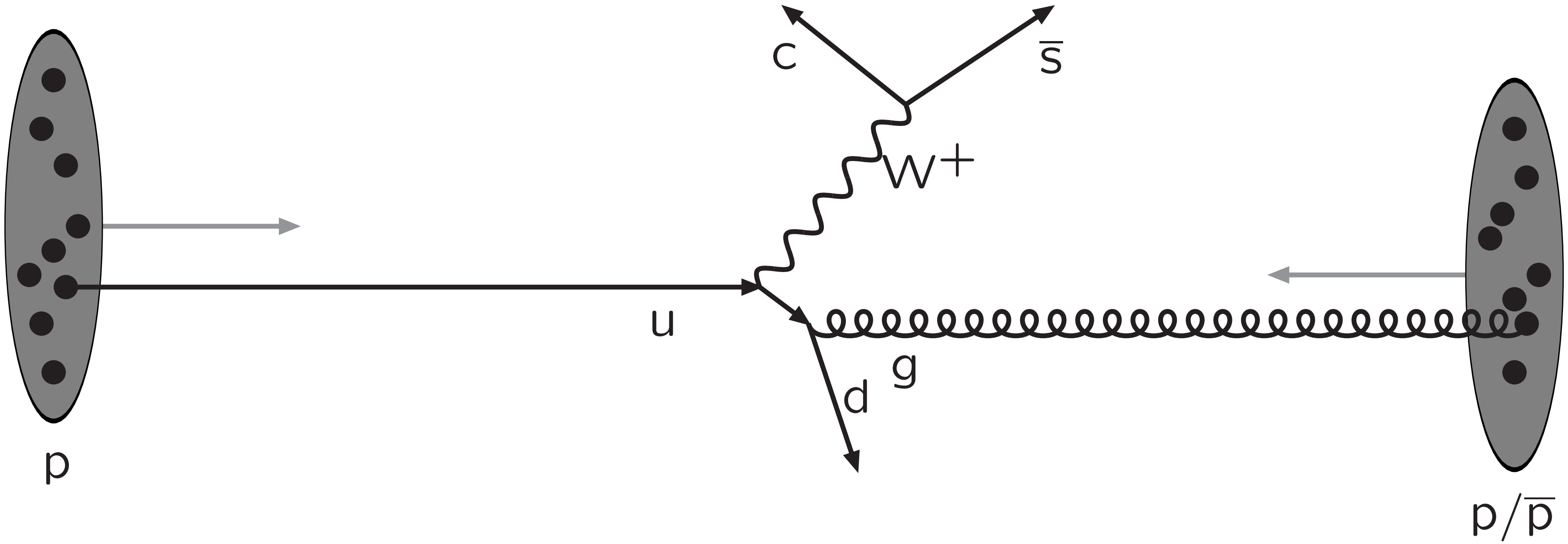
The structure of an event

Warning: schematic only, everything simplified, nothing to scale, . . .

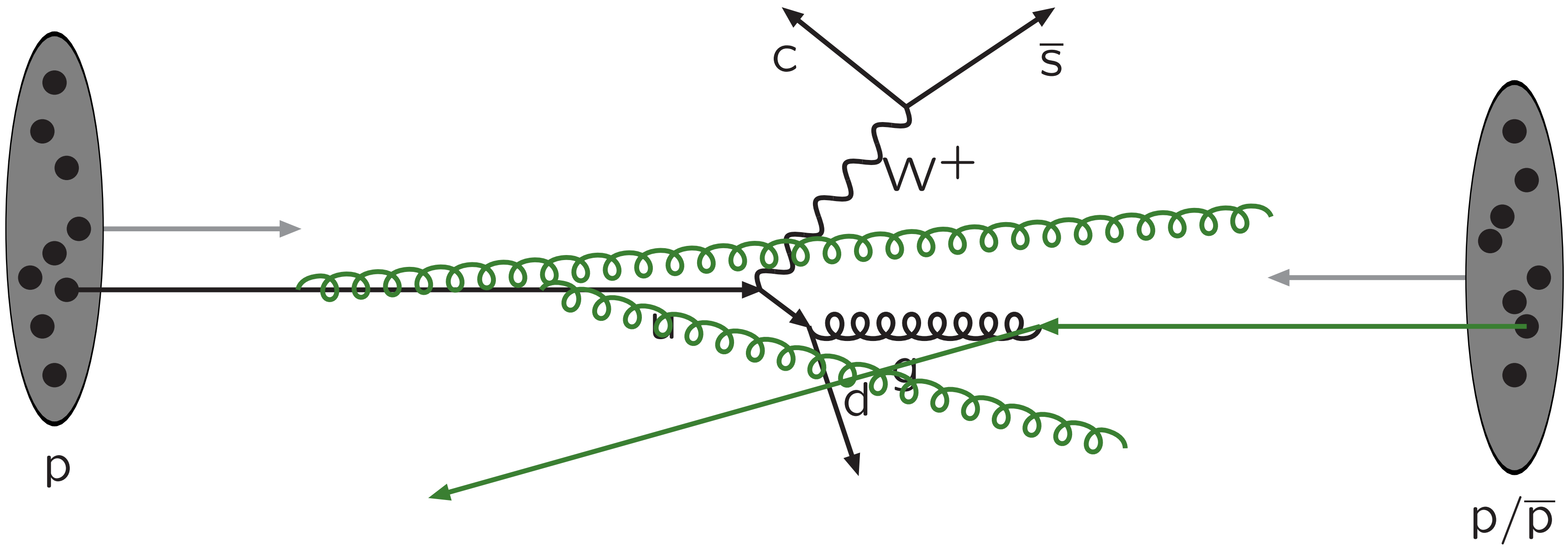
Incoming beams: parton densities



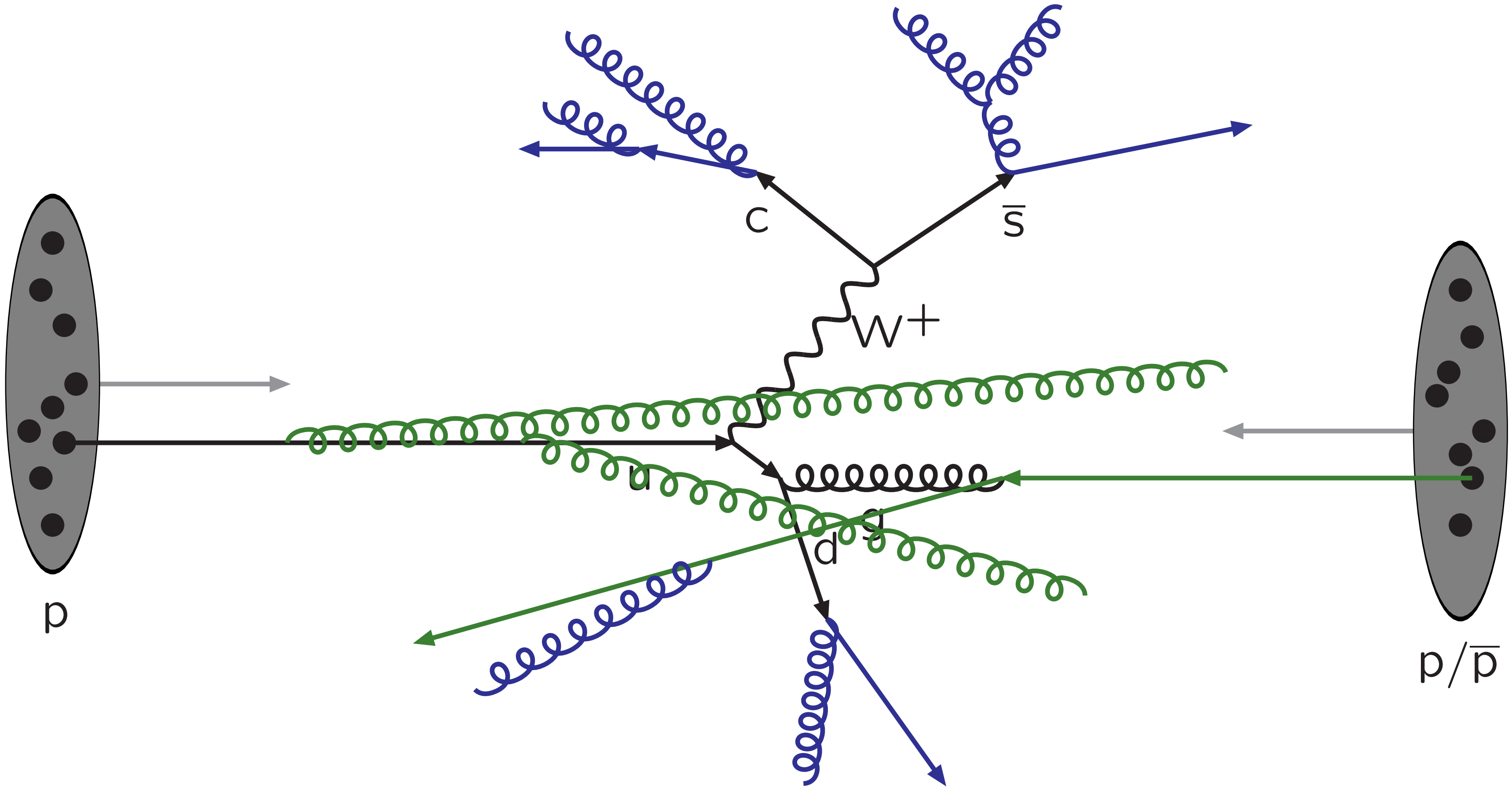
Hard subprocess: described by matrix elements



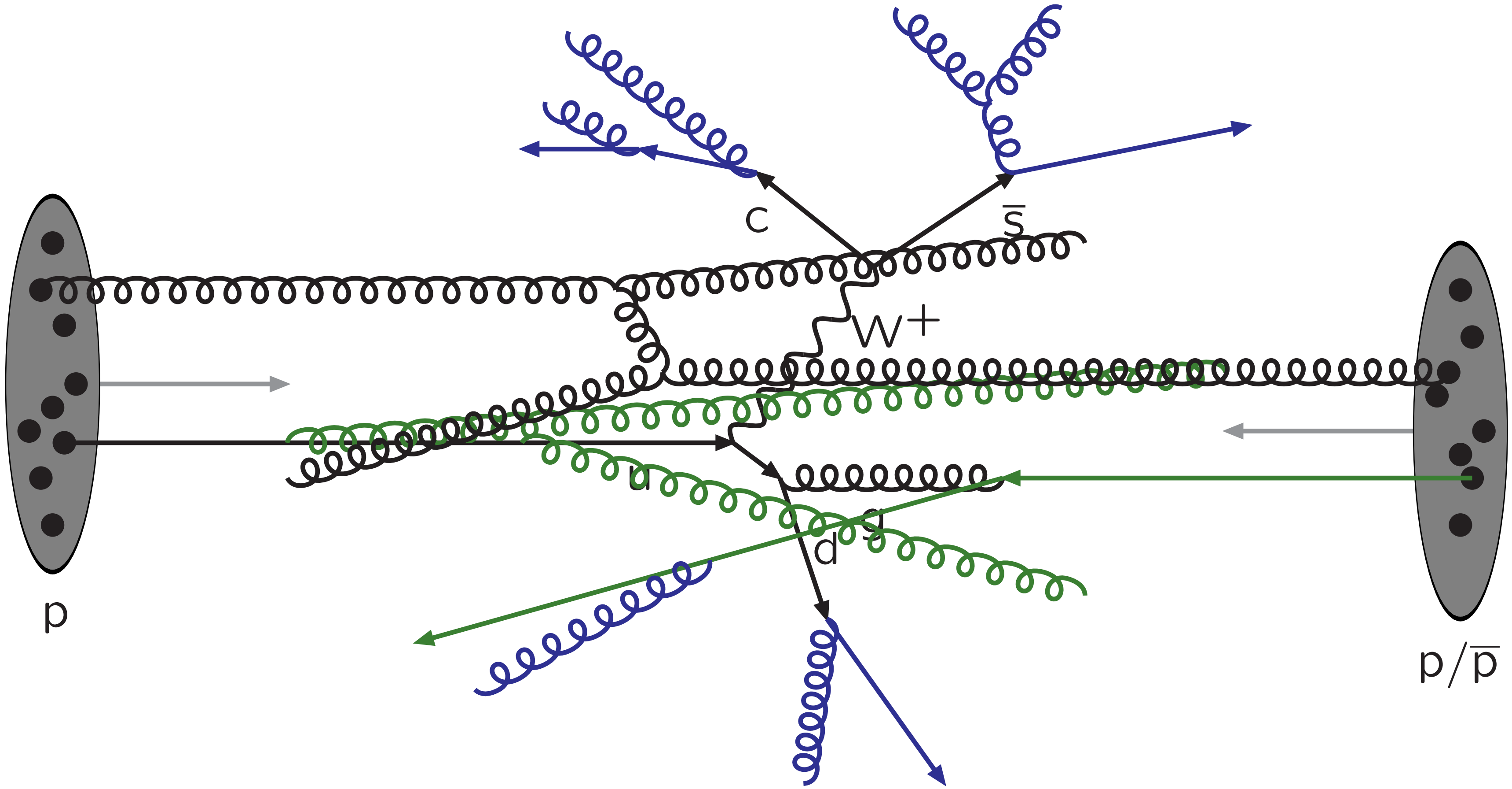
Resonance decays: correlated with hard subprocess



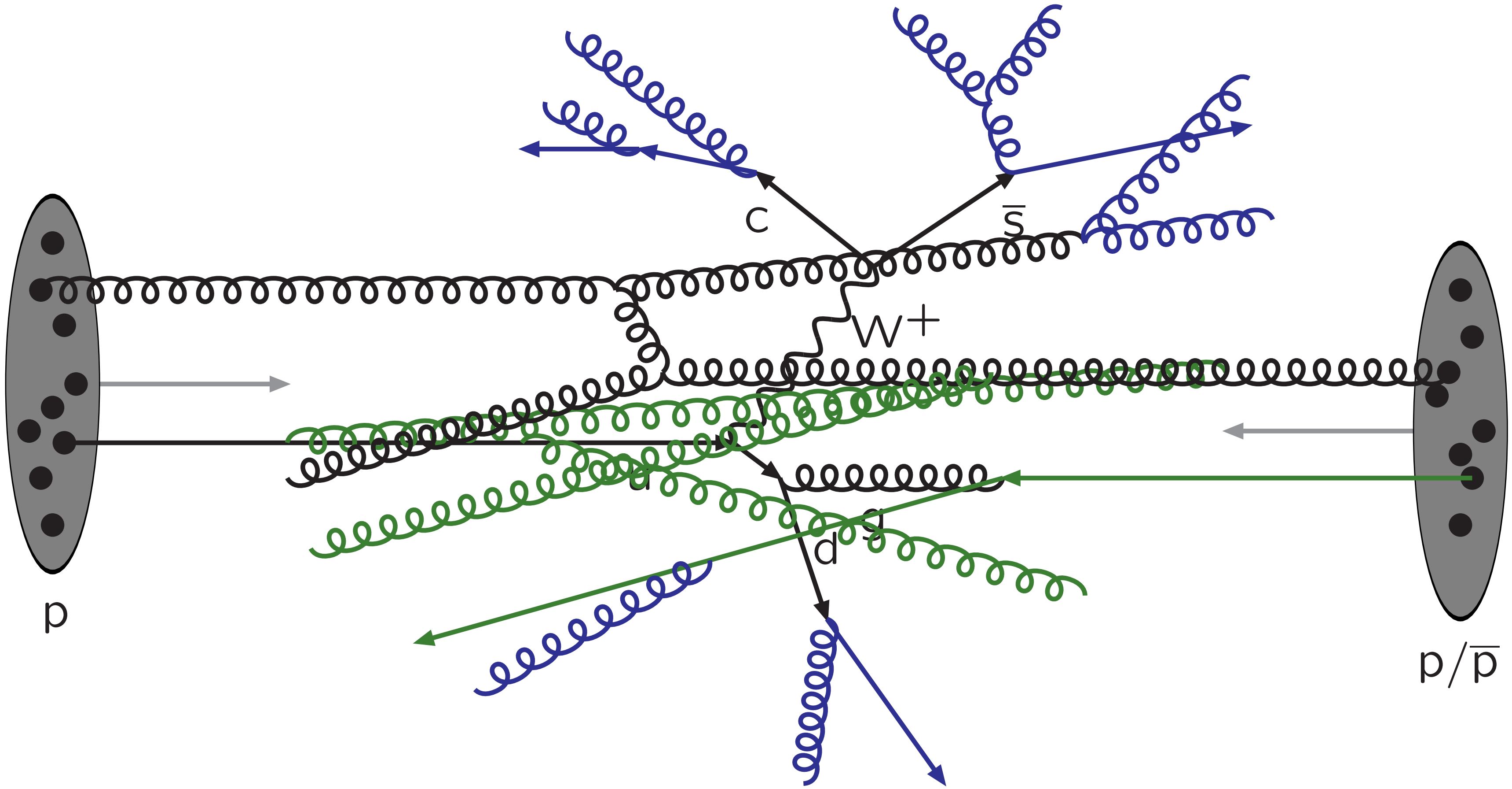
Initial-state radiation: spacelike parton showers



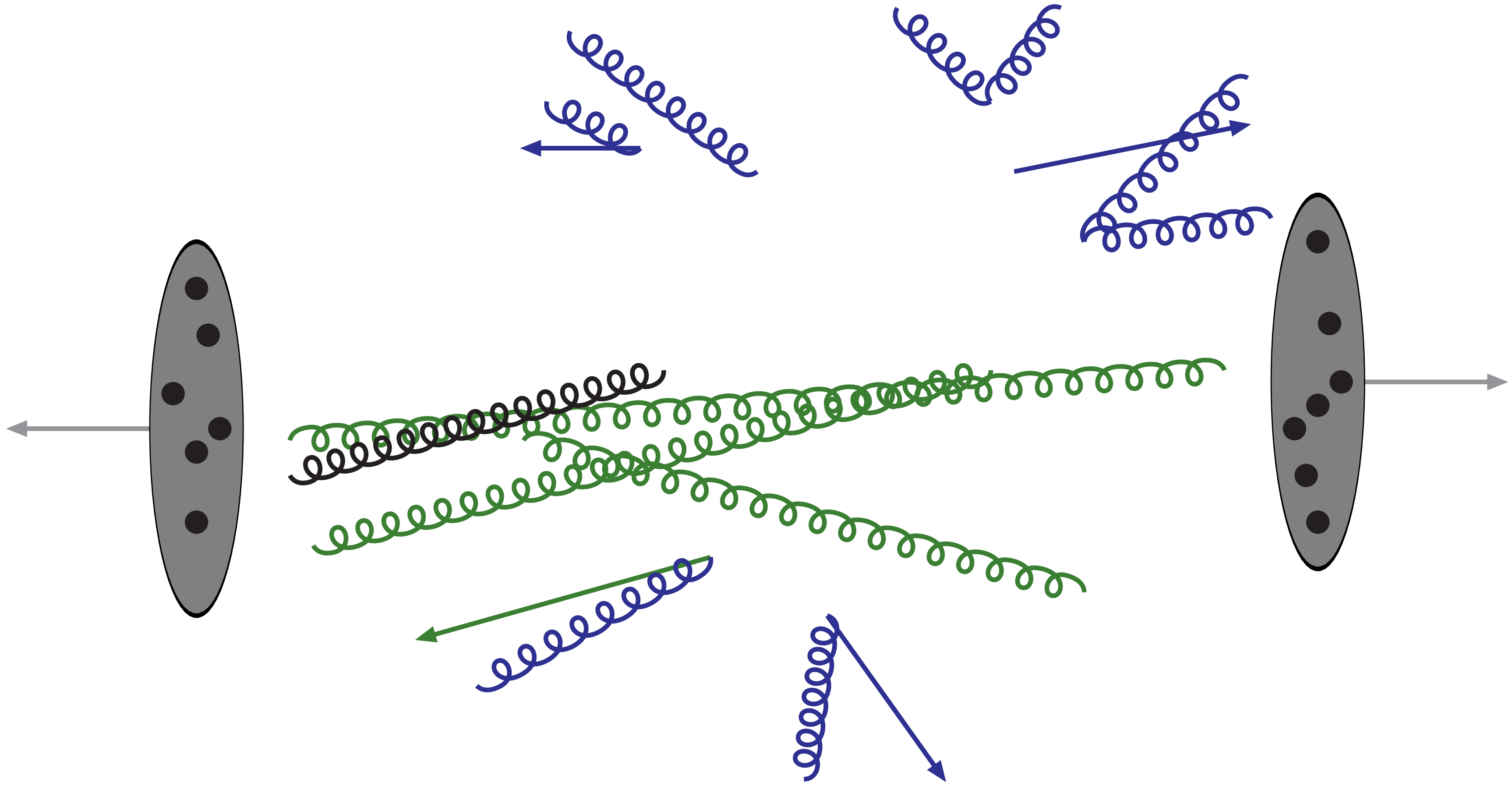
Final-state radiation: timelike parton showers



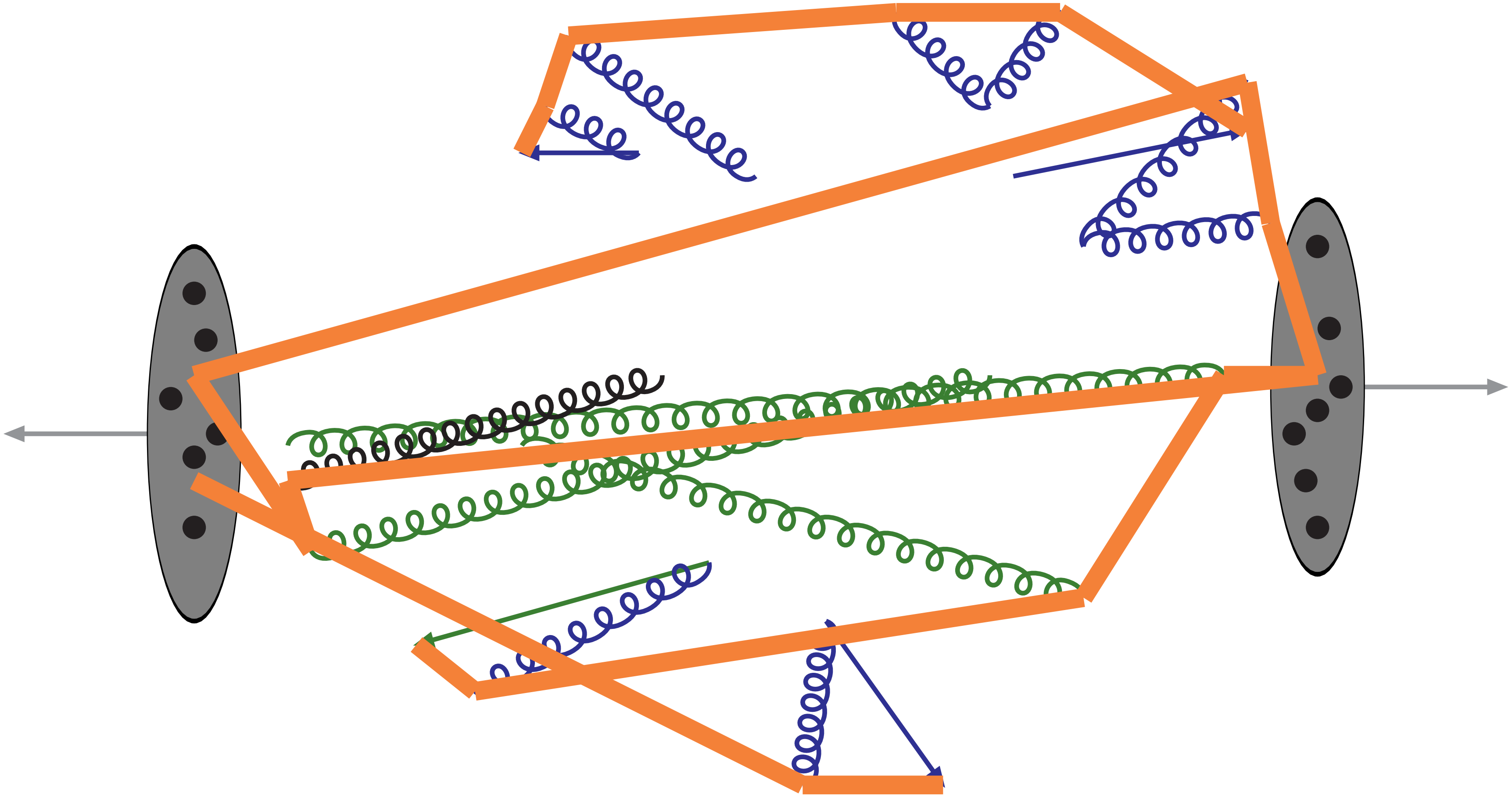
Multiple parton–parton interactions . . .



. . . with its initial- and final-state radiation

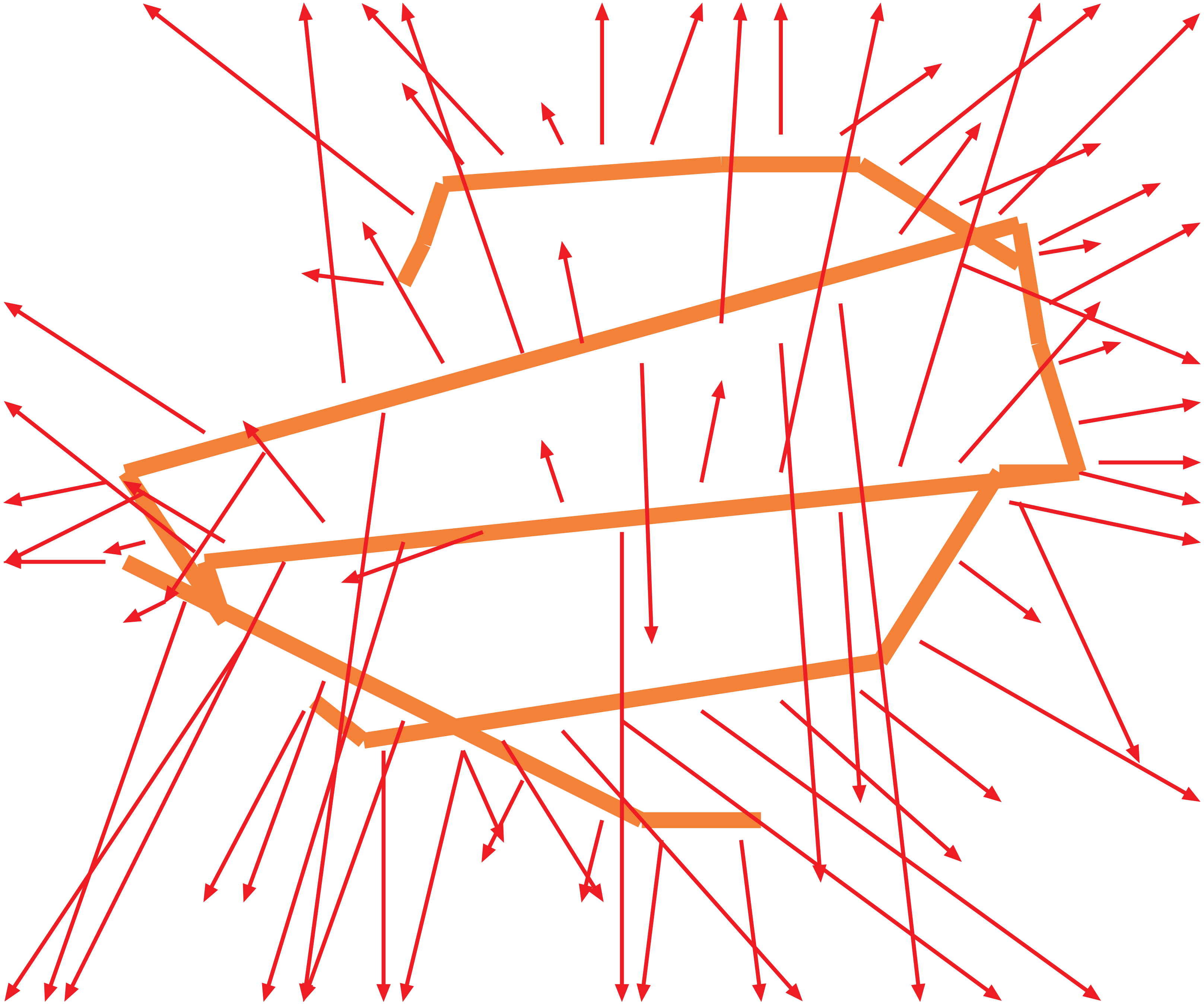


Beam remnants and other outgoing partons

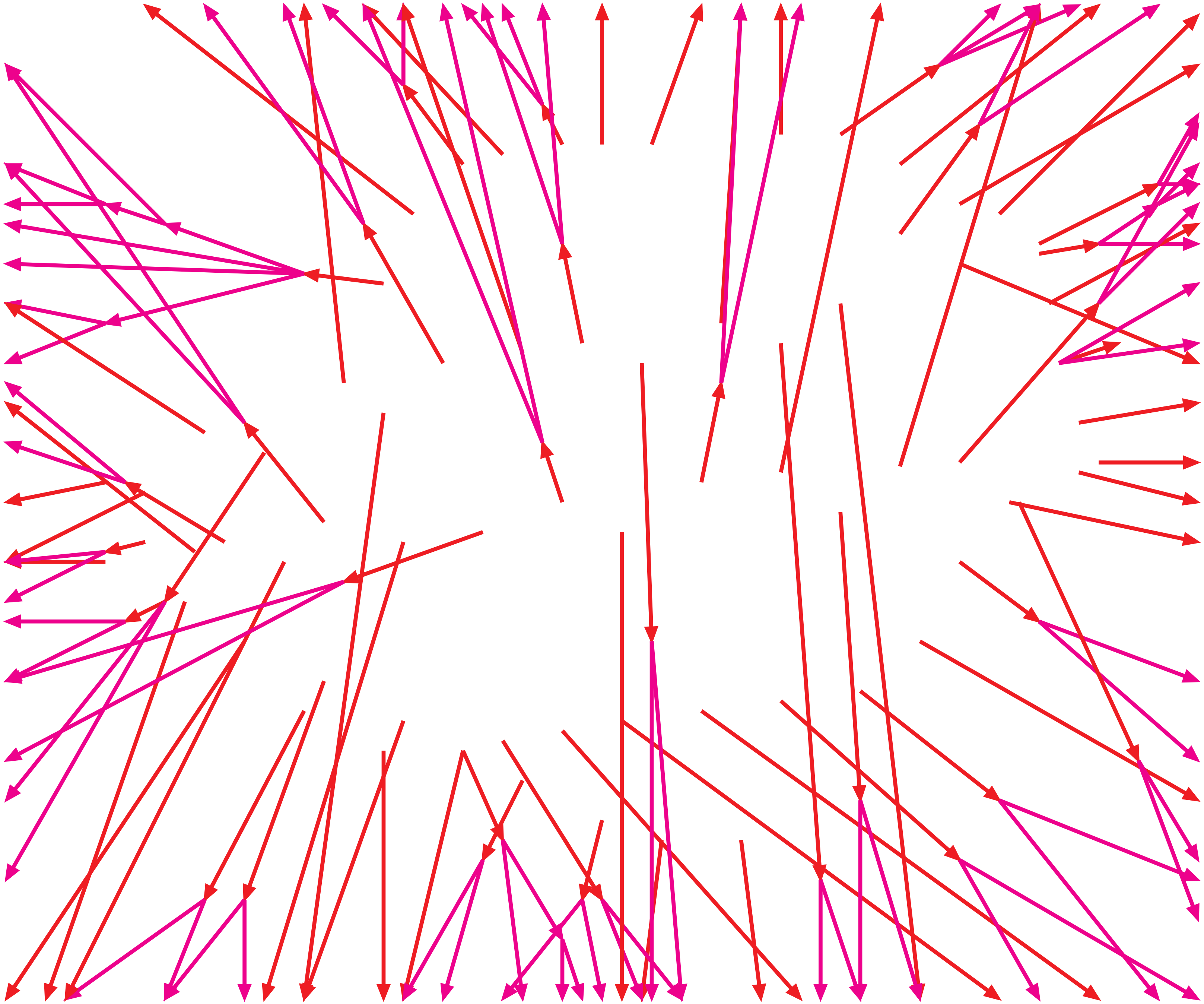


Everything is connected by colour confinement strings

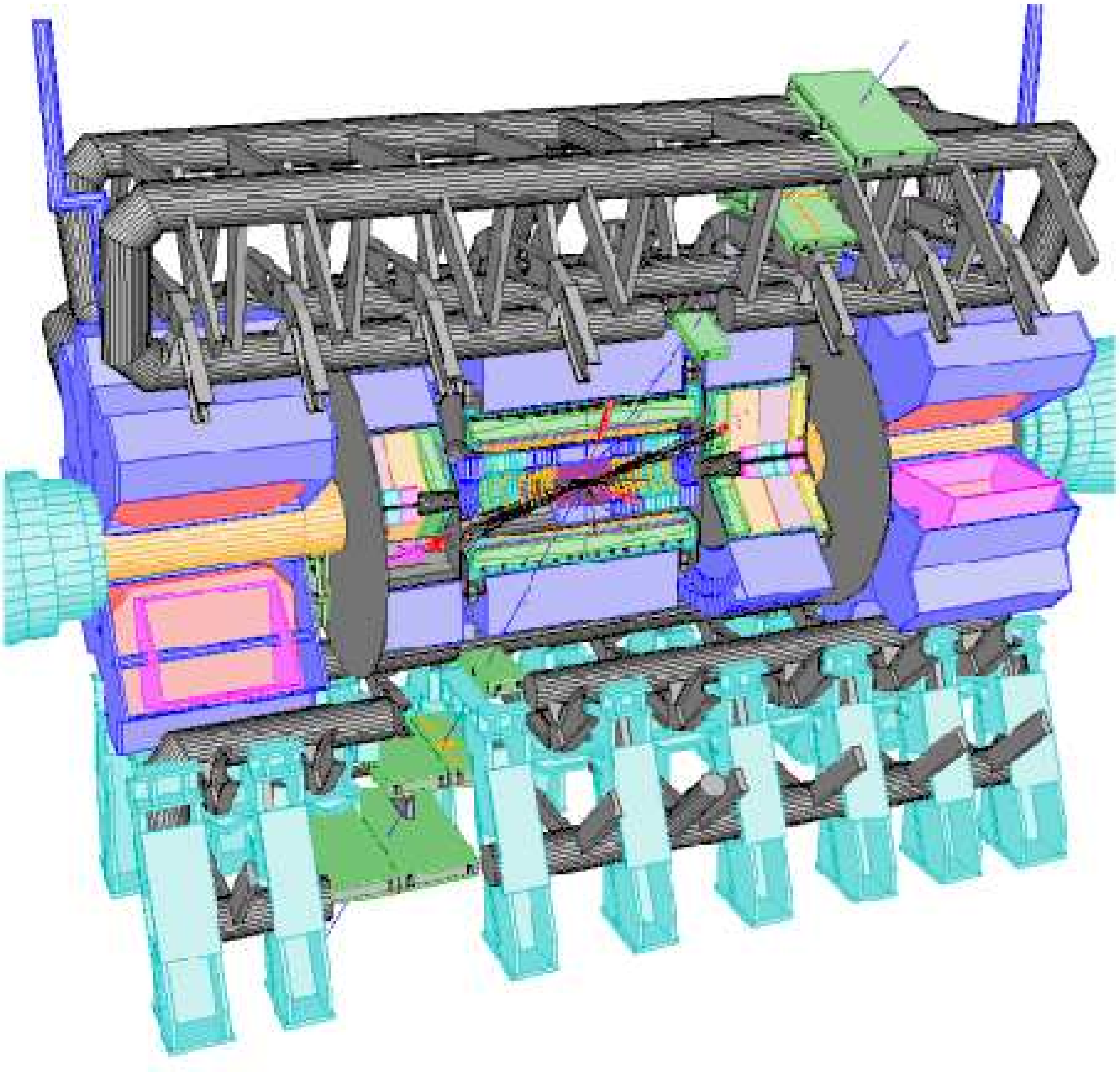
Recall! Not to scale: strings are of hadronic widths



The strings fragment to produce primary hadrons



Many hadrons are unstable and decay further



These are the particles that hit the detector

The Monte Carlo method

Want to generate events in as much detail as Mother Nature
$\Longrightarrow$ get average *and* fluctutations right
$\Longrightarrow$ make random choices, $\sim$ as in nature

$$\sigma_{\text{final state}} = \sigma_{\text{hard process}} \mathcal{P}_{\text{tot,hard process} \to \text{final state}}$$

(appropriately summed & integrated over non-distinguished final states)

where $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{res}} \mathcal{P}_{\text{ISR}} \mathcal{P}_{\text{FSR}} \mathcal{P}_{\text{MI}} \mathcal{P}_{\text{remnants}} \mathcal{P}_{\text{hadronization}} \mathcal{P}_{\text{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \ldots$ in its turn

$\Longrightarrow$ **divide and conquer**

an event with n particles involves $\mathcal{O}(10n)$ random choices,
(flavour, mass, momentum, spin, production vertex, lifetime, . . .)
LHC: $\sim$ 100 charged and $\sim$ 200 neutral ($+$ intermediate stages)
$\Longrightarrow$ several thousand choices
(of $\mathcal{O}(100)$ different kinds)

Generator Landscape

	General-Purpose	Specialized
Hard Processes	**HERWIG** **PYTHIA** SHERPA ISAJET	a lot
Resonance Decays		HDECAY, . . .
Parton Showers		Ariadne/LDC, NLLjet
Underlying Event		DPMJET
Hadronization		none (?)
Ordinary Decays		TAUOLA, EvtGen

specialized often best at given task, but need General-Purpose core

Matrix-Elements Programs

Wide spectrum from "general-purpose" to "one-issue", see e.g.

`http://www.cedar.ac.uk/hepcode/`

Free for all as long as Les-Houches-compliant output.

I) General-purpose, leading-order:

- MadGraph/MadEvent (amplitude-based, $\leq$ 7 outgoing partons):

  `http://madgraph.physics.uiuc.edu/`
- CompHEP (matrix-elements-based, $\sim\leq$ 4 outgoing partons)
- AMEGIC++: part of SHERPA ($\sim$ MadGraph $\rightarrow$ Behrends-Giele)
- HELAC–PHEGAS (Dyson-Schwinger)

II) Special processes, leading-order:

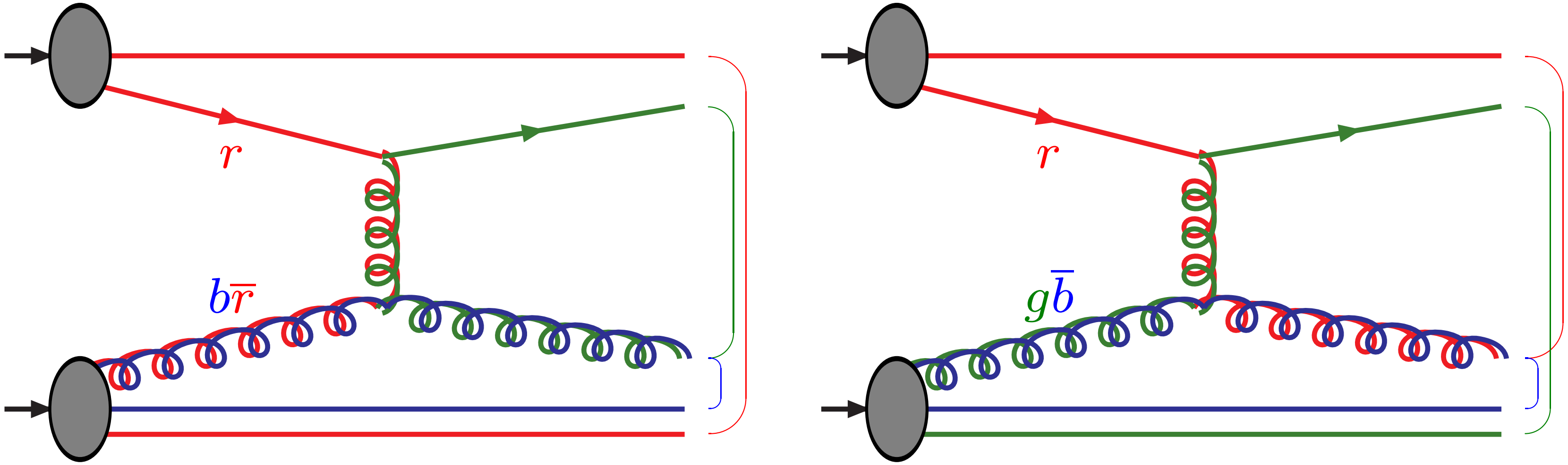
- ALPGEN: $\mathrm{W}/\mathrm{Z}+\leq 6\mathrm{j}$, $n\mathrm{W}+m\mathrm{Z}+k\mathrm{H}+\leq 3\mathrm{j}$, $\ldots$
- AcerMC: $\mathrm{t}\overline{\mathrm{t}}\mathrm{b}\overline{\mathrm{b}}$, $\ldots$
- VECBOS: $\mathrm{W}/\mathrm{Z}+\leq 4\mathrm{j}$

III) Special processes, next-to-leading-order:

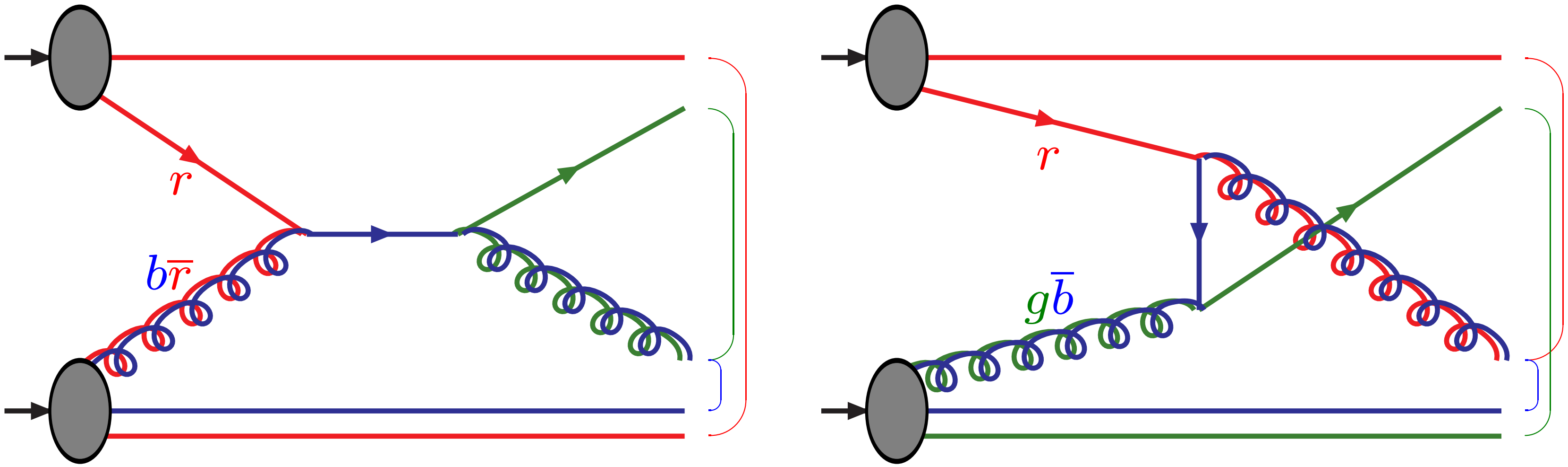
- MCFM: NLO $\mathrm{W}/\mathrm{Z}+\leq 2\mathrm{j}$, WZ, WH, $\mathrm{H}+\leq 1\mathrm{j}$
- GRACE+Bases/Spring

Colour flow in hard processes

One Feynman graph can correspond to several possible colour flows, e.g. for $qg \to qg$:



while other $qg \to qg$ graphs only admit one colour flow:



so nontrivial mix of kinematics variables $(\hat{s}, \hat{t})$
and colour flow topologies I, II:

$$\begin{aligned} |\mathcal{A}(\hat{s},\hat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t}) + \mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 \\ &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 + 2\,\mathcal{R}e\,(\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})\mathcal{A}^*_{\mathrm{II}}(\hat{s},\hat{t})) \end{aligned}$$

with $\mathcal{R}e\left(\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})\mathcal{A}^*_{\mathrm{II}}(\hat{s},\hat{t})\right) \neq 0$
$\Rightarrow$ indeterminate colour flow, while
- showers *should* know it (coherence),
- hadronization *must* know it (hadrons singlets).

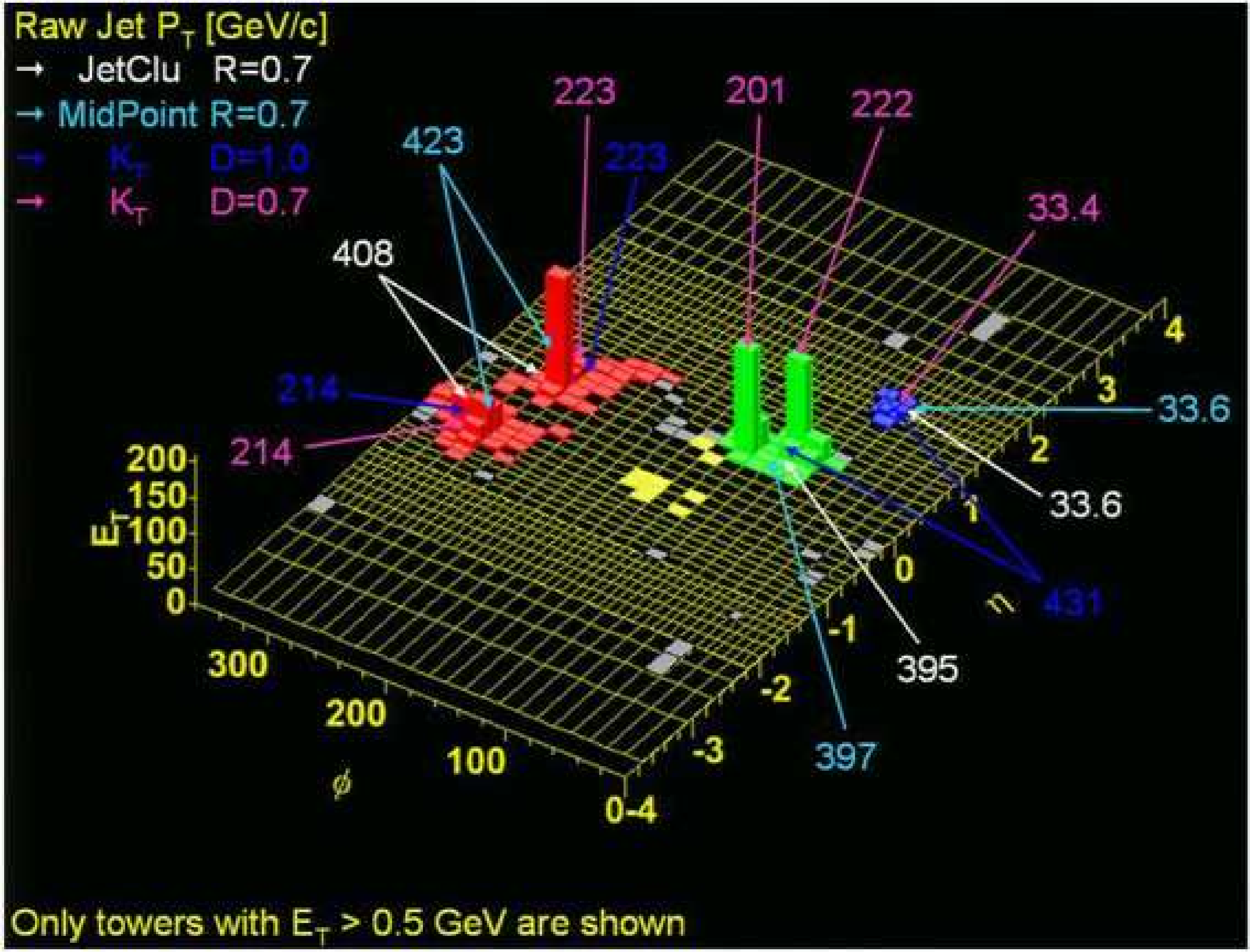
Normal solution:

$$\frac{\text{interference}}{\text{total}} \propto \frac{1}{N_{\mathrm{C}}^2 - 1}$$

so split I : II according to proportions in the $N_{\mathrm{C}} \to \infty$ limit, i.e.

$$\begin{aligned} |\mathcal{A}(\hat{s},\hat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2_{\mathrm{mod}} + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2_{\mathrm{mod}} \\ |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2_{\mathrm{mod}} &= |\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t}) + \mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2 \left(\frac{|\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2}{|\mathcal{A}_{\mathrm{I}}(\hat{s},\hat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2}\right)_{N_{\mathrm{C}}\to\infty} \\ |\mathcal{A}_{\mathrm{II}}(\hat{s},\hat{t})|^2_{\mathrm{mod}} &= \ldots \end{aligned}$$

Parton Showers

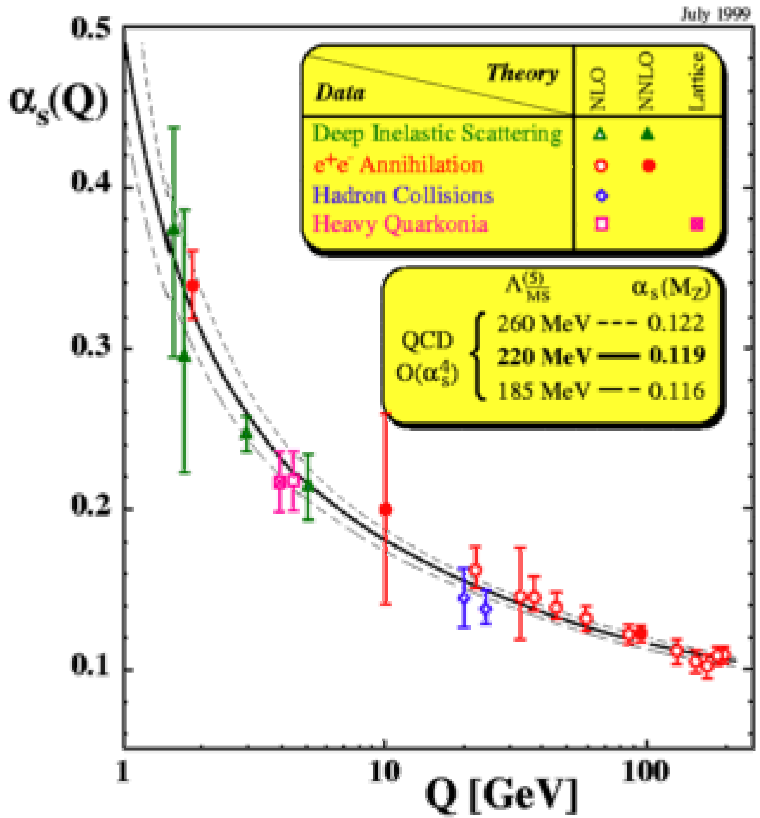


- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
- Matching to Matrix Elements

Divergences

Emission rate q $\longrightarrow$ qg diverges when
- collinear: opening angle $\theta_{\mathrm{qg}} \to 0$
- soft: gluon energy $E_{\mathrm{g}} \to 0$

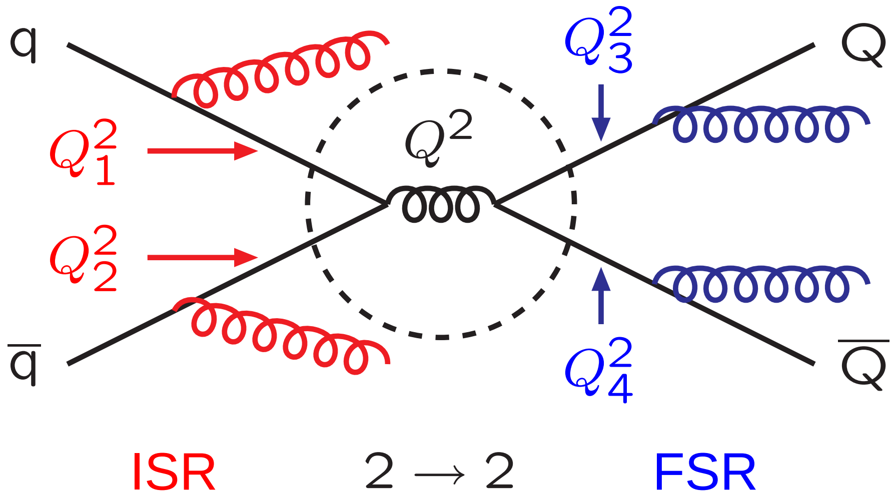
Almost identical to e $\longrightarrow$ eγ (“bremsstrahlung”),
but QCD is non-Abelian so additionally
- g $\longrightarrow$ gg similarly divergent
- $\alpha_{\mathrm{s}}(Q^2)$ diverges for $Q^2 \to 0$
  (actually for $Q^2 \to \Lambda^2_{\mathrm{QCD}}$)



Big probability for one emission $\Longrightarrow$ also big for several
$\Longrightarrow$ with ME’s need to calculate to high order **and** with many loops
$\Longrightarrow$ extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.
Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \to n \;=\; (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$



FSR = Final-State Rad.;
timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing

ISR = Initial-State Rad.;
spacelike shower
$Q_i^2 \sim -m^2 > 0$ increasing

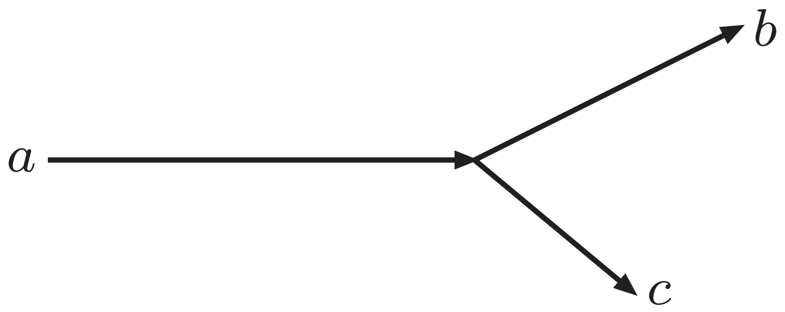
$2 \to 2$ = hard scattering (on-shell):

$$\sigma = \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \; f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Shower evolution is viewed as a probabilistic process,
which occurs with unit total probability:
the cross section is not directly affected,
but indirectly it is, via the changed event shape

Technical aside: why timelike/spacelike?

Consider four-momentum conservation in a branching $a \to b\,c$



$$\mathbf{p}_{\perp a} = 0 \quad \Rightarrow \quad \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b}$$
$$p_+ = E + p_\mathrm{L} \quad \Rightarrow \quad p_{+a} = p_{+b} + p_{+c}$$
$$p_- = E - p_\mathrm{L} \quad \Rightarrow \quad p_{-a} = p_{-b} + p_{-c}$$

Define $p_{+b} = z\,p_{+a}, \quad p_{+c} = (1-z)\,p_{+a}$

Use $p_+ p_- = E^2 - p_\mathrm{L}^2 = m^2 + p_\perp^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z\,p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1-z)\,p_{+a}}$$

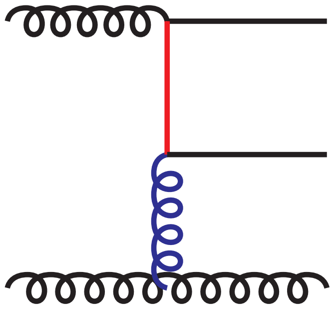
$$\Rightarrow \quad m_a^2 = \frac{m_b^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1-z} = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_\perp^2}{z(1-z)}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_\perp^2}{z(1-z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_\perp^2}{1-z} < 0 \Rightarrow$ spacelike

Doublecounting

A $2 \to n$ graph can be "simplified" to $2 \to 2$ in different ways:



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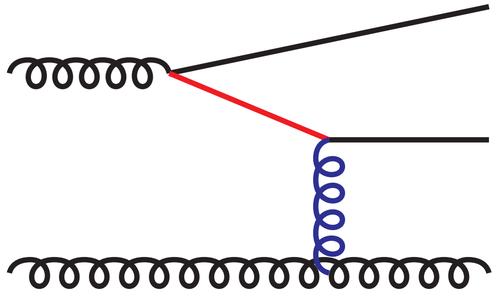
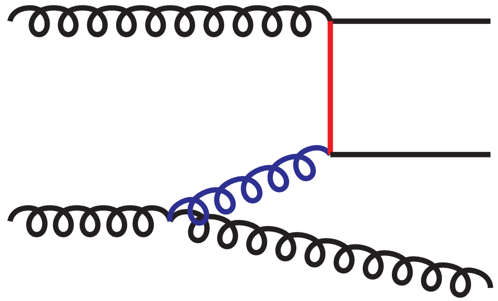


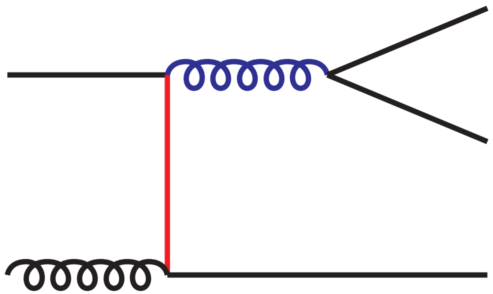
$g \to q\bar{q} \oplus qg \to qg$

or



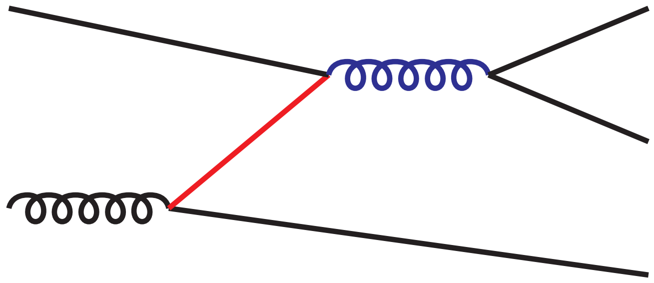
$g \to gg \oplus gg \to q\bar{q}$

or deform



FSR

to

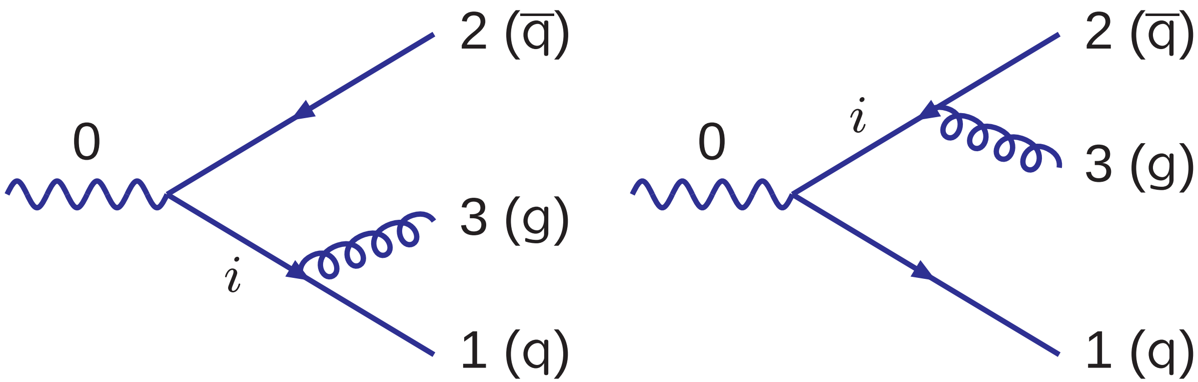


ISR

Do not doublecount: $2 \to 2$ = most virtual = shortest distance

Conflict: theory derivations often assume virtualities strongly ordered; interesting physics often in regions where this is not true!

From Matrix Elements to Parton Showers



$e^+e^- \to q\bar{q}g$

$x_j = 2E_j/E_{cm} \Rightarrow$

$x_1 + x_2 + x_3 = 2$

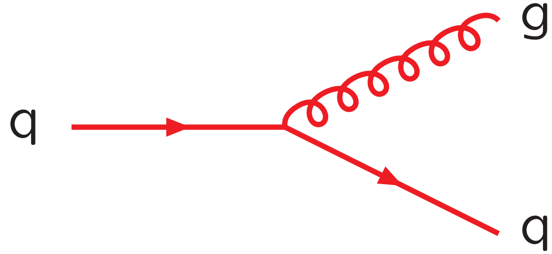
$$m_q = 0: \quad \frac{d\sigma_{ME}}{\sigma_0} = \frac{\alpha_s}{2\pi}\frac{4}{3}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\, dx_1\, dx_2$$

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{cm}^2} = \frac{Q^2}{E_{cm}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{cm}^2}$$

$$x_1 \approx z \Rightarrow dx_1 \approx dz$$

$$x_3 \approx 1 - z$$



$$\Rightarrow d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi}\frac{dx_2}{(1-x_2)}\frac{4}{3}\frac{x_2^2 + x_1^2}{(1-x_1)}\, dx_1 \approx \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}\frac{4}{3}\frac{1+z^2}{1-z}\, dz$$

Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

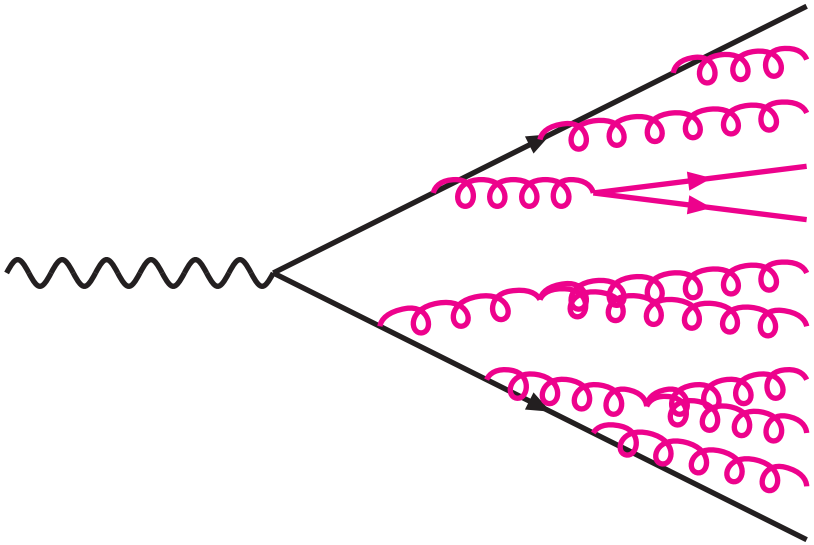
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z$$

$$P_{\mathrm{q}\to\mathrm{qg}} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{\mathrm{g}\to\mathrm{gg}} = 3\,\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{\mathrm{g}\to\mathrm{q\bar{q}}} = \frac{n_f}{2}(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs to stay away from nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \mathrm{min}(m_{ij}) \approx 1$ GeV,

$z_{\mathrm{min}}(E,Q) < z < z_{\mathrm{max}}(E,Q)$

or $p_\perp > p_{\perp\mathrm{min}} \approx 0.5$ GeV

The Sudakov Form Factor

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

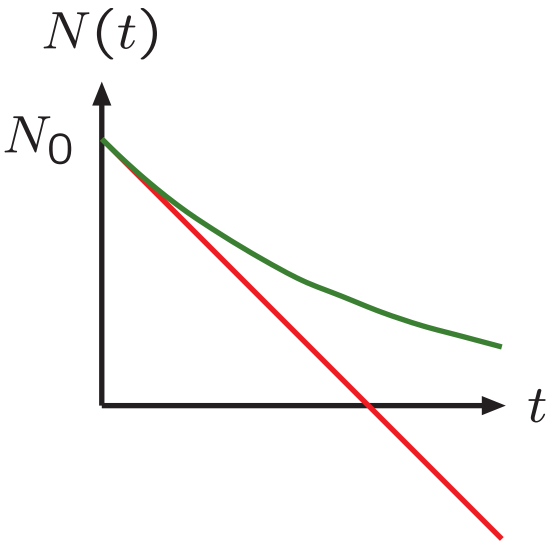
$\mathcal{P}_{\text{nothing}}(0 < t \le T) = \mathcal{P}_{\text{nothing}}(0 < t \le T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \le T)$

Subdivide further, with $T_i = (i/n)T$, $0 \le i \le n$:

$$\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \le T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \le T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(- \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t} \mathrm{d}t\right)
\end{aligned}$$

$$\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T) \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t} \mathrm{d}t\right)$$

Example: radioactive decay of nucleus



naively: $\frac{dN}{dt} = -cN_0 \Rightarrow N(t) = N_0\,(1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{dN}{dt} = -cN(t) \Rightarrow N(t) = N_0\,\exp(-ct)$

generalizes to: $N(t) = N_0\,\exp\left(-\int_0^t c(t')dt'\right)$

or: $\frac{dN(t)}{dt} = -c(t)\,N_0\,\exp\left(-\int_0^t c(t')dt'\right)$

sequence allowed: $\text{nucleus}_1 \to \text{nucleus}_2 \to \text{nucleus}_3 \to \ldots$

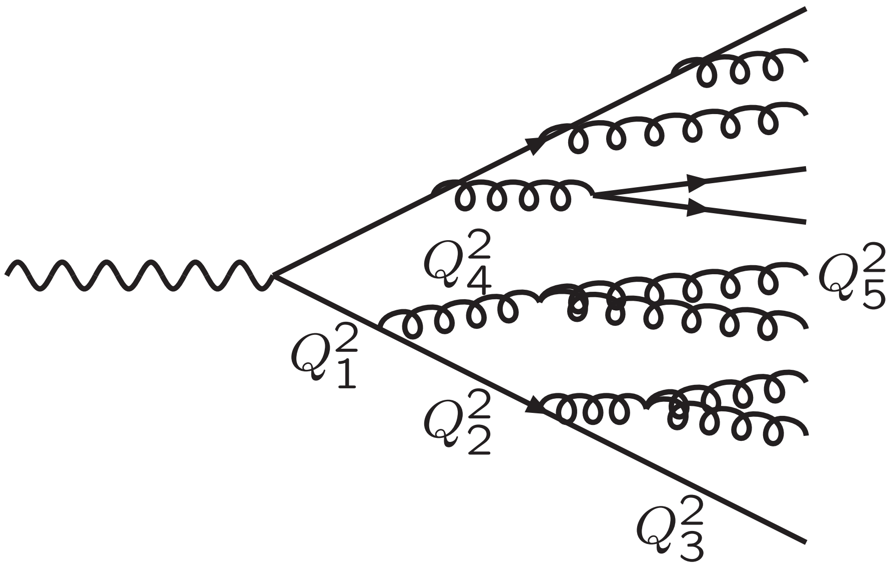
Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}P_{a\to bc}(z)\,dz\;\exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_{max}}\frac{dQ'^2}{Q'^2}\int\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\,dz'\right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

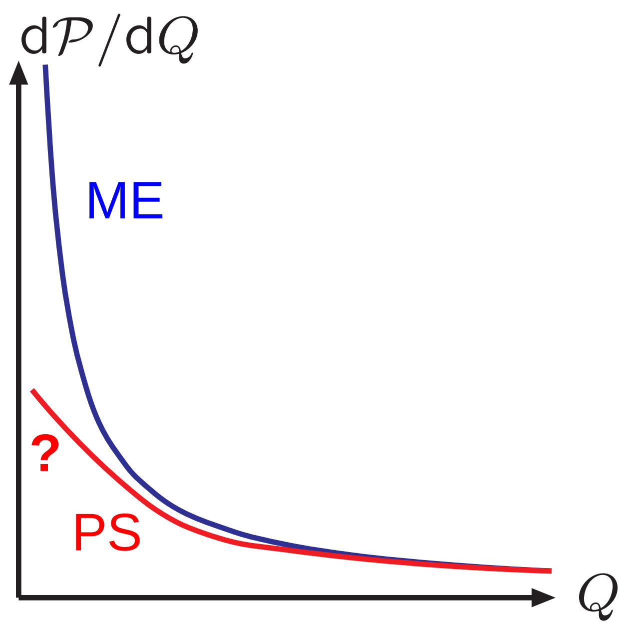
Note that $\sum_{b,c}\int\int\, d\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*
($\equiv 1$ if extended over whole phase space, else possibly nothing happens)



Sudakov form factor provides
"time" ordering of shower:
lower Q^2 $\Longleftrightarrow$ longer times

$Q_1^2 > Q_2^2 > Q_3^2$
$Q_1^2 > Q_4^2 > Q_5^2$
etc.

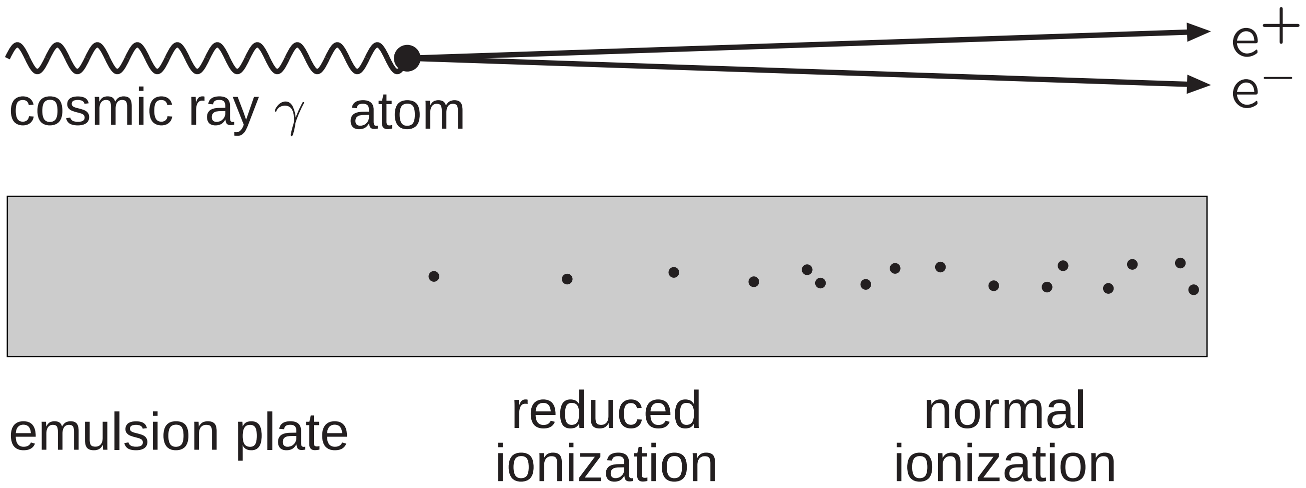
Sudakov regulates singularity for *first* emission . . .



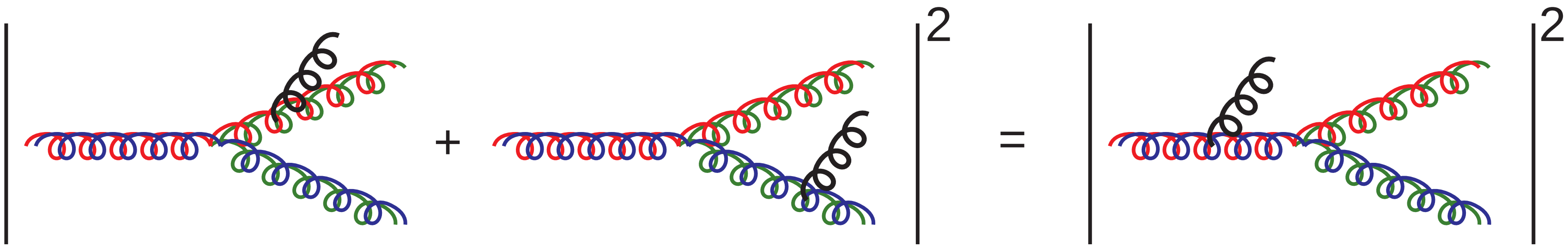
. . . but in limit of *repeated soft* emissions $\mathrm{q} \to \mathrm{qg}$
($\mathrm{g} \to \mathrm{gg}$, $\mathrm{g} \to \mathrm{q\overline{q}}$ not considered)
one obtains the same inclusive Q emission spectrum as for ME,
i.e. divergent ME spectrum
$\Longleftrightarrow$ infinite number of PS emissions

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



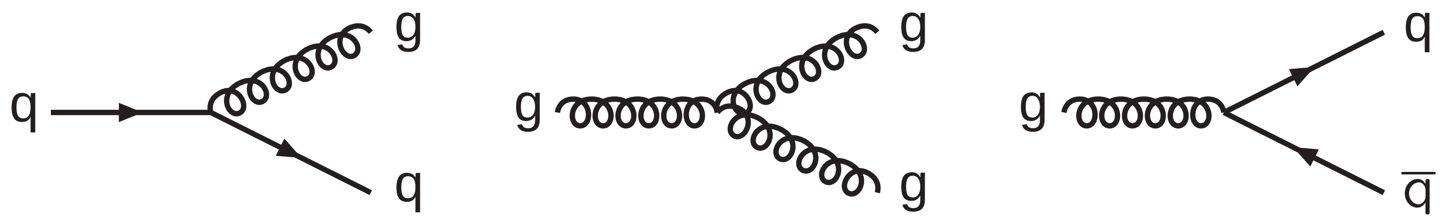
solved by • requiring emission angles to be decreasing

or • requiring transverse momenta to be decreasing

The Common Showering Algorithms

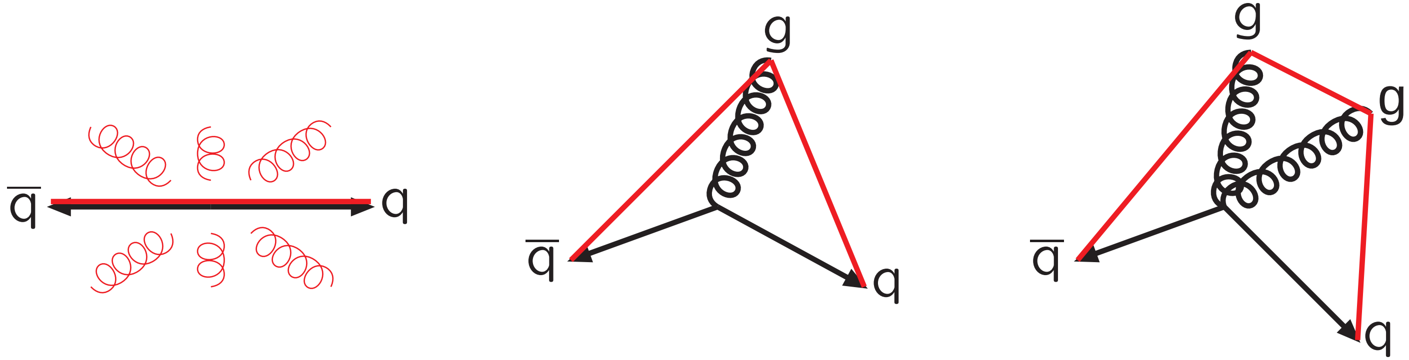
Three main approaches to showering in common use:

Two are based on the standard shower language
of $a \to bc$ successive branchings:



HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

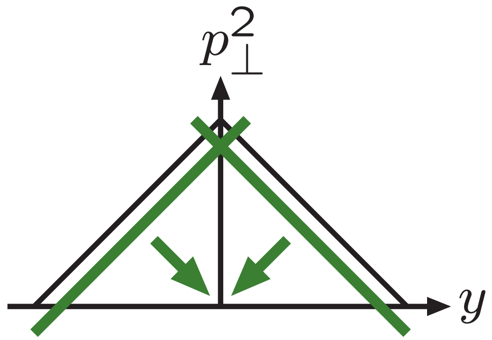
One is based on a picture of dipole emission $ab \to cde$:



ARIADNE: $Q^2 = p_\perp^2$; FSR mainly, ISR is primitive;
there instead LDCMC: sophisticated but complicated

Ordering variables in final-state radiation

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ “hardness” ordered

coherence brute force

covers phase space

ME merging simple

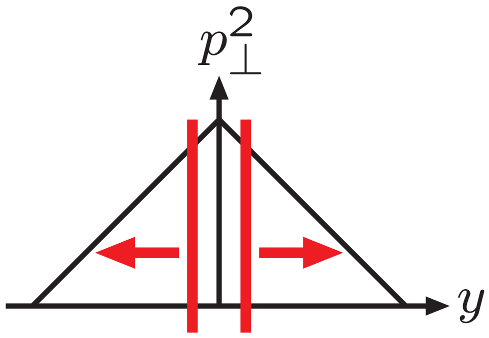
g $\rightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \rightarrow -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

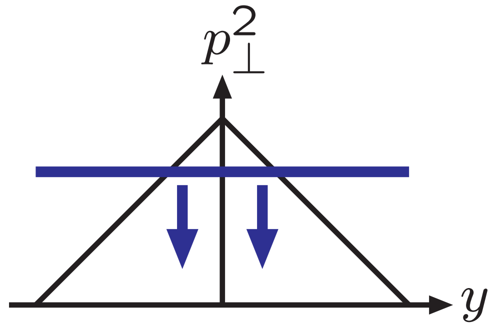
g $\rightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \rightarrow \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ “hardness” ordered

coherence inherent

covers phase space

ME merging simple

g $\rightarrow$ q$\overline{\text{q}}$ messy

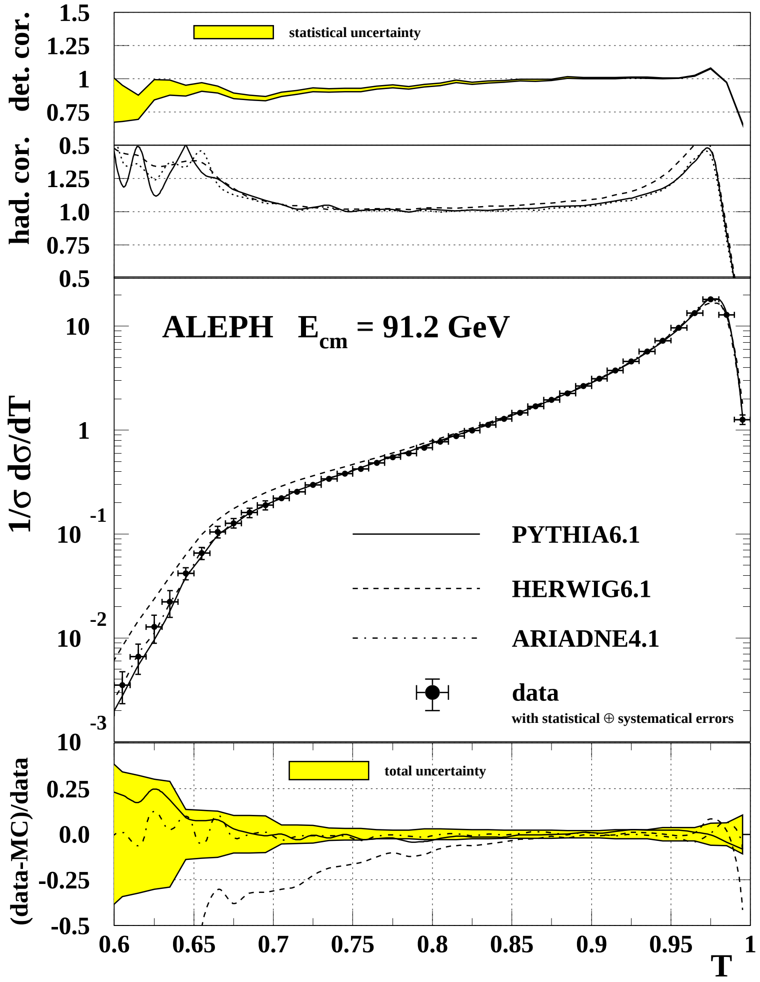
Lorentz invariant

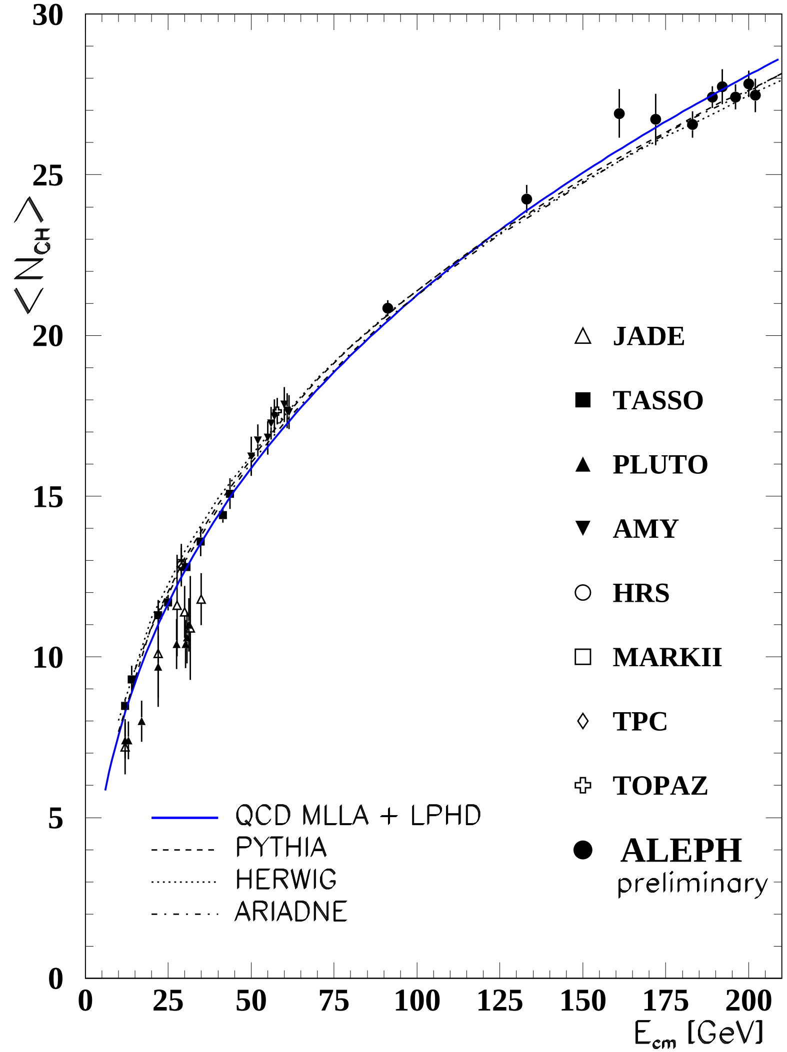
can stop/restart

ISR: more messy

Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically ARIADNE ($p_\perp^2$) $>$ PYTHIA (m^2) $>$ HERWIG (θ)





. . . and programs evolve to do even better . . .

Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{qg}} \approx \int \frac{\mathrm{d}Q^2}{Q^2} \int \mathrm{d}z \, \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z}$$

$$\approx \alpha_{\mathrm{s}} \ln\left(\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}}\right) \frac{8}{3} \ln\left(\frac{1-z_{\mathrm{min}}}{1-z_{\mathrm{max}}}\right) \sim \alpha_{\mathrm{s}} \ln^2$$

Rate for n emissions is of form:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{q}n\mathrm{g}} \sim (\mathcal{P}_{\mathrm{q}\to\mathrm{qg}})^n \sim \alpha_{\mathrm{s}}^n \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type $\alpha_{\mathrm{s}}^n \ln^{2n-1}$

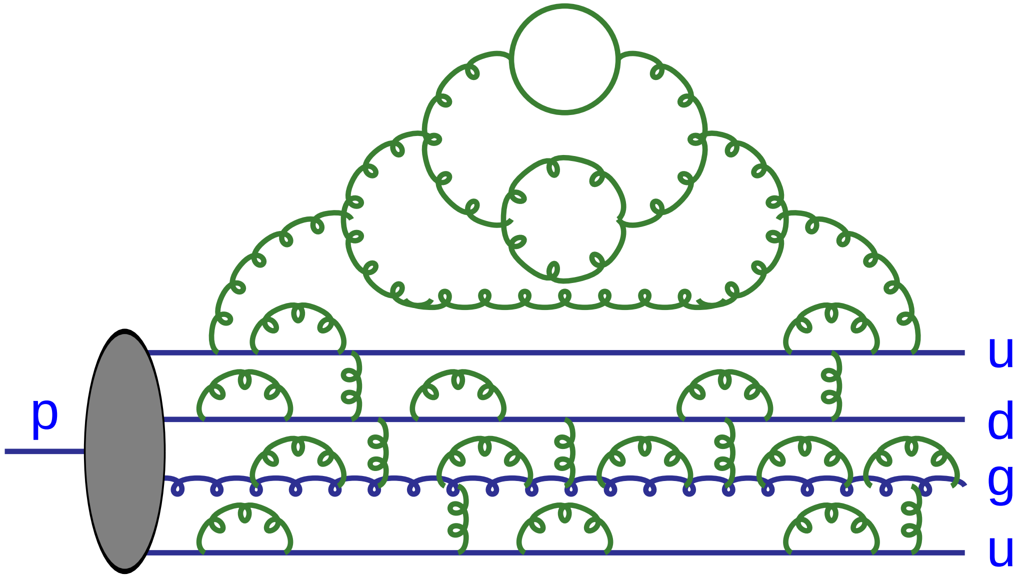
No existing $\mathrm{pp}/\mathrm{p\overline{p}}$ generator completely NLL, but

- energy-momentum conservation (and “recoil” effects)
- coherence
- $2/(1-z) \to (1+z^2)/(1-z)$
- scale choice $\alpha_{\mathrm{s}}(p_{\perp}^2)$ absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_{\mathrm{s}}^2)$ splitting kernels $P_{\mathrm{q}\to\mathrm{qg}}$ and $P_{\mathrm{g}\to\mathrm{gg}}$
- ...

$\Rightarrow$ far better than naive, analytical LL

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2) =$ number density of partons i
at momentum fraction x and probing scale Q^2.

Linguistics (example):

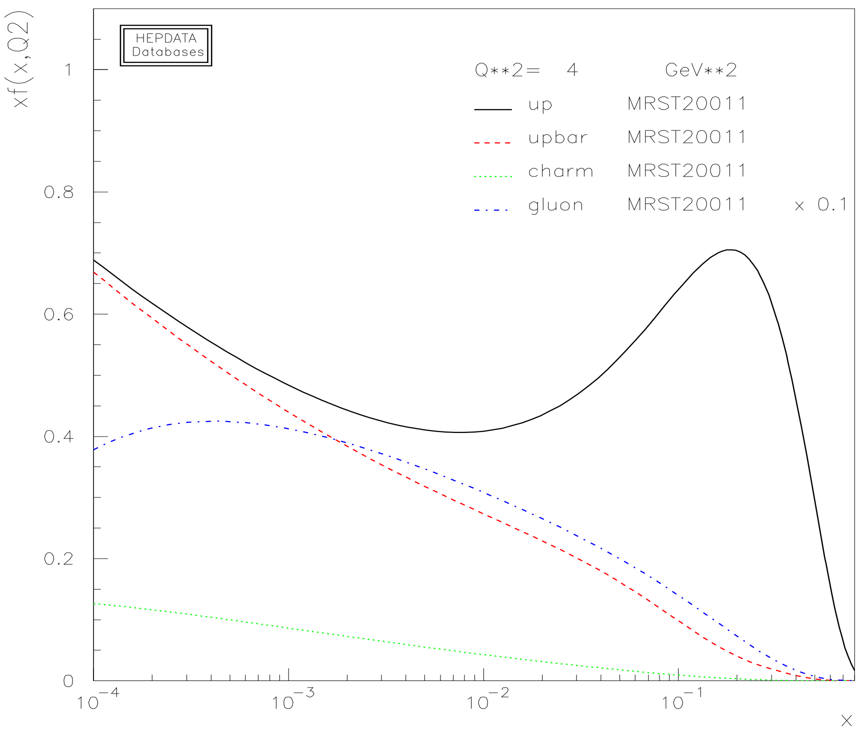
$$F_2(x, Q^2) \;=\; \sum_i e_i^2 \, x f_i(x, Q^2)$$

structure function   parton distributions

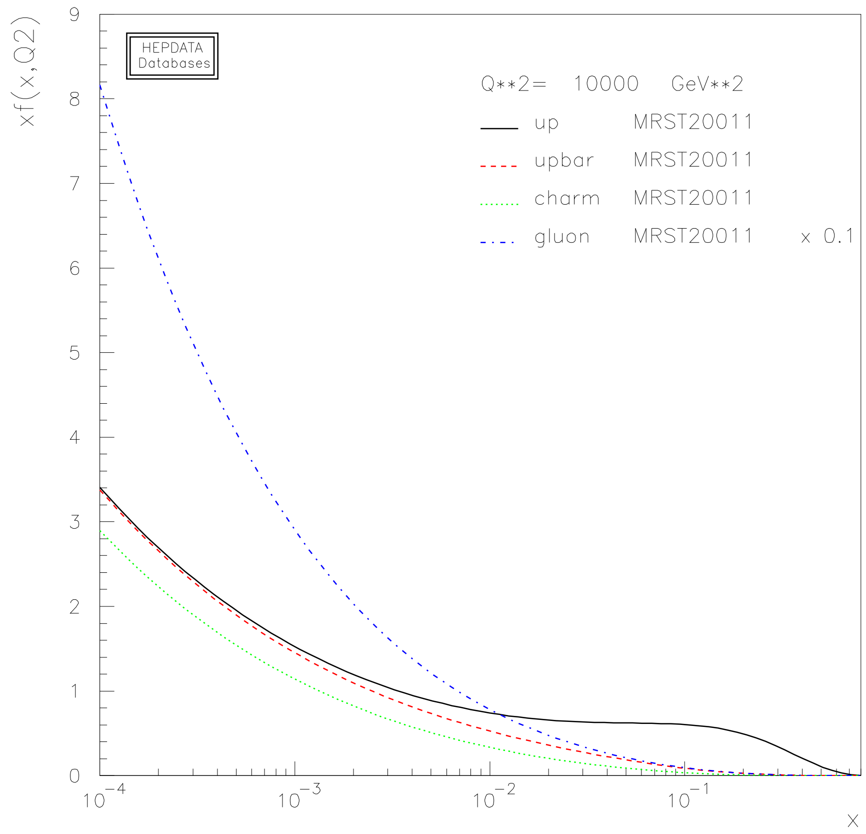
Absolute normalization at small Q_0^2 unknown.
Resolution dependence by DGLAP:

$$\frac{\mathrm{d} f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}\left(z=\frac{x}{x'}\right)$$

$Q^2 = 4\ \mathrm{GeV}^2$



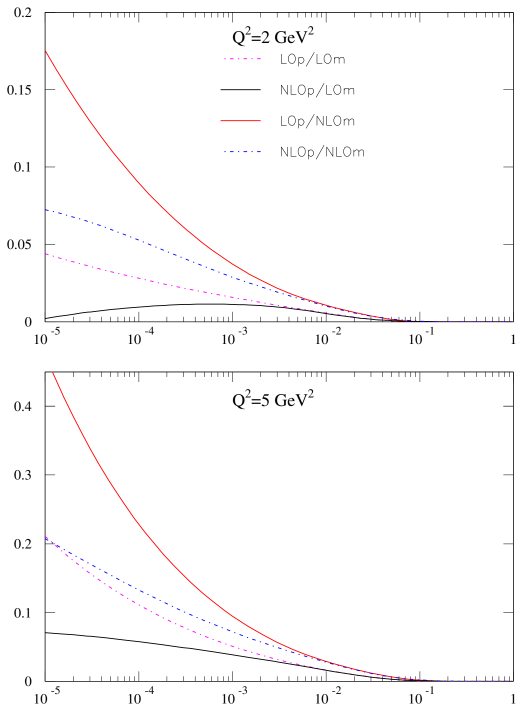
$Q^2 = 10000\ \mathrm{GeV}^2$

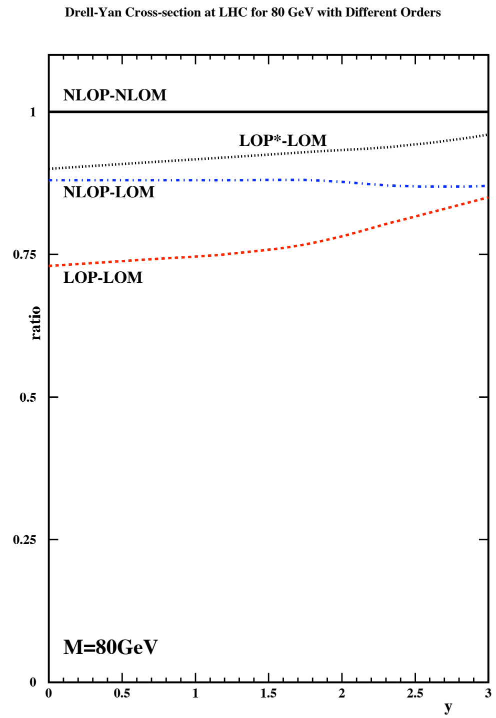


For cross section calculations NLO PDF's are combined with NLO σ's.
Gives significantly better description of data than LO can.

But NLO $\Rightarrow$ parton model not valid, e.g $g(x, Q^2)$ can be negative.
Not convenient for LO showers, nor for many LO ME's.

Recent revived interest in modified LO sets, e.g. by Thorne & Sherstnev:
allow $\sum_i \int_0^1 x f_i(x, Q^2)\,\mathrm{d}x > 1$; around ~ 1.15





$pp \to jj$

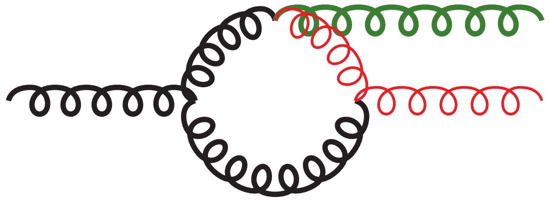
pdf type	matrix element	σ (μb)	K-factor
NLO	NLO	183.2	
LO	LO	149.8	1.22
NLO	LO	115.7	1.58
LO*	LO	177.5	1.03

$pp \to H$

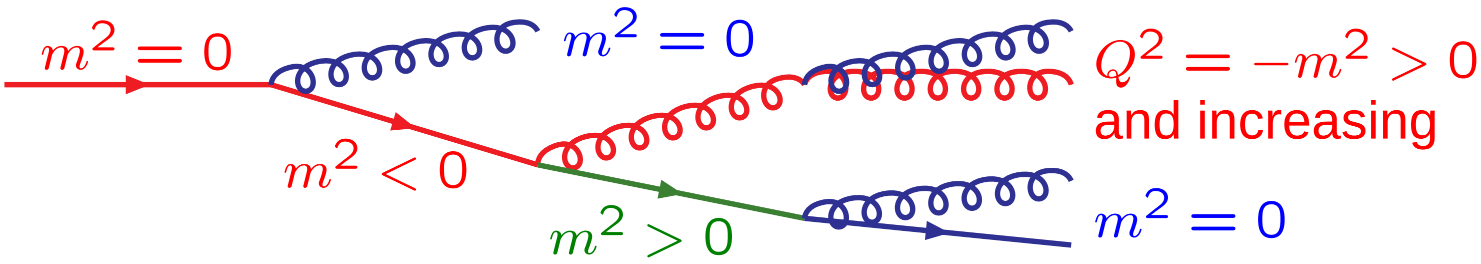
pdf type	matrix element	σ (pb)	K-factor
NLO	NLO	38.0	
LO	LO	22.4	1.70
NLO	LO	20.3	1.87
LO*	LO	32.4	1.17

Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



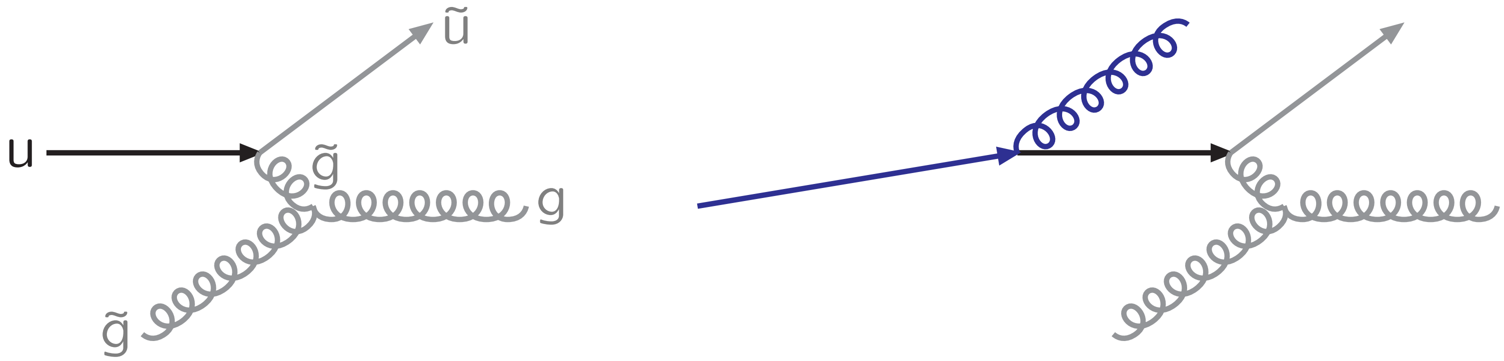
- Convenient reinterpretation:



Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low Q_0 and evolve, see what happens.
Inefficient:
1) have to evolve and check for *all* potential collisions, but 99.9. . . % inert
2) impossible to steer the production e.g. of a narrow resonance (Higgs)

Backwards evolution

Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened “before”



Monte Carlo approach, based on *conditional probability*: recast

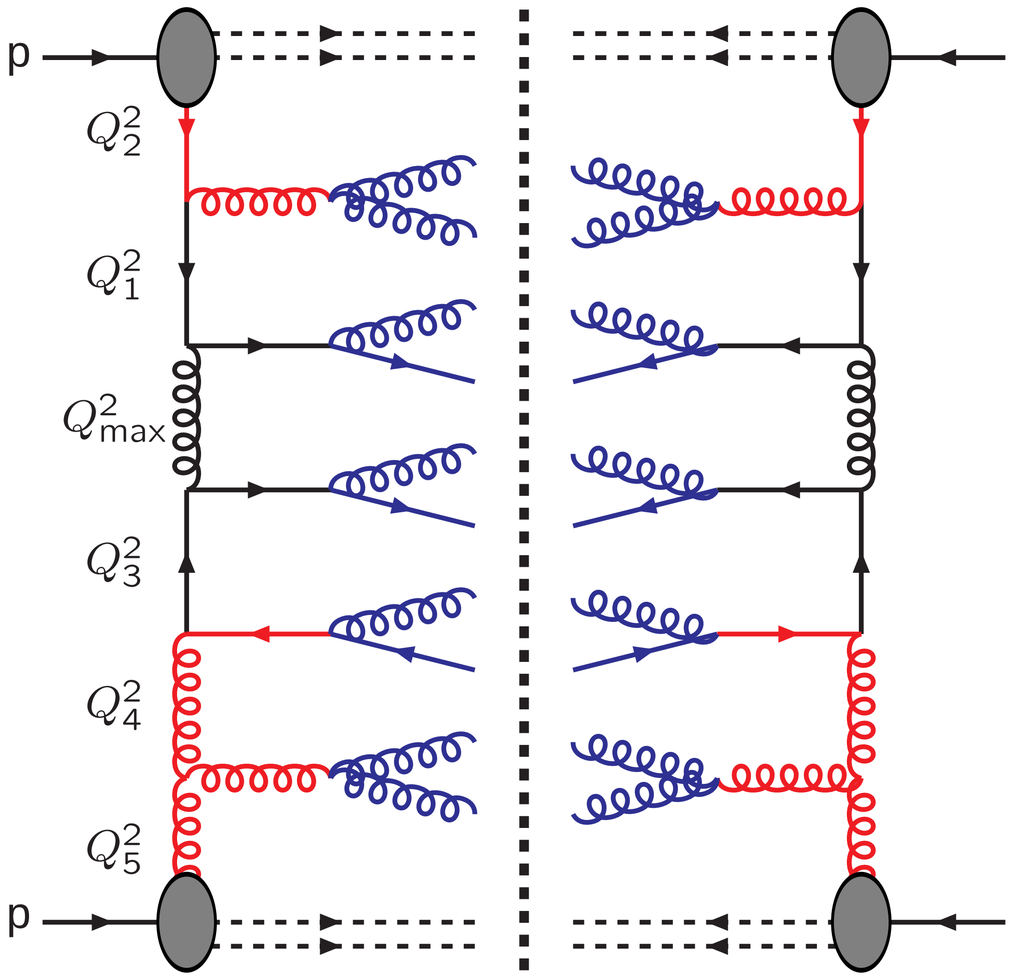
$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \frac{x' f_a(x',t)}{x f_b(x,t)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

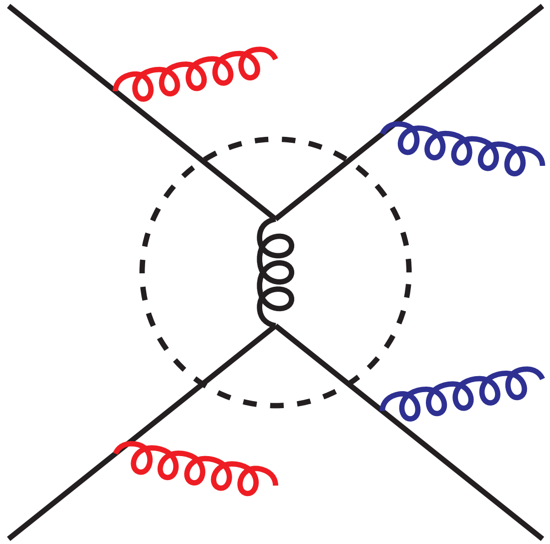
Ladder representation combines whole event:



DGLAP: $Q^2_{\mathrm{max}} > Q_1^2 > Q_2^2 \sim Q_0^2$

$Q^2_{\mathrm{max}} > Q_3^2 > Q_4^2 > Q_5^2 \sim Q_0^2$

cf. previously:



One possible

Monte Carlo order:

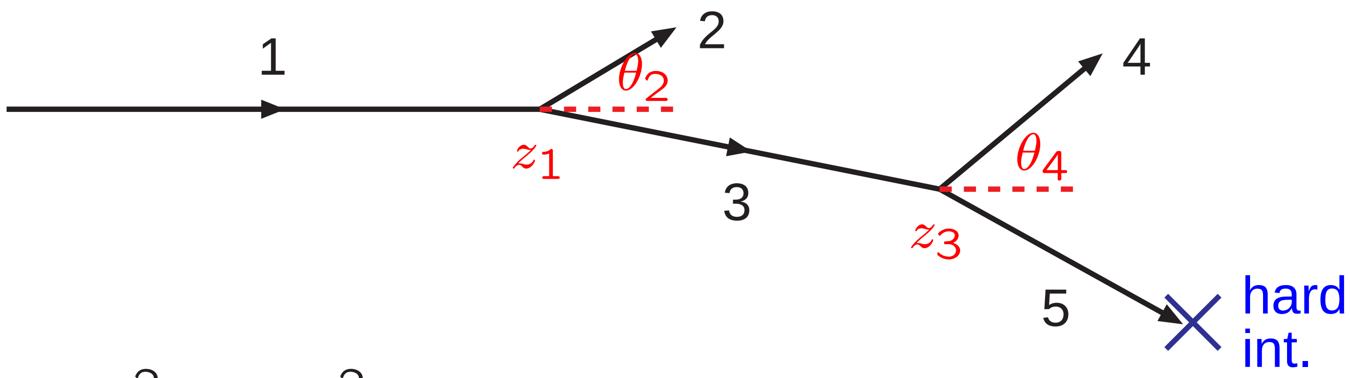
1) Hard scattering

2) Initial-state shower

from center outwards

3) Final-state showers

Coherence in spacelike showers



$z_1 = E_3/E_1$
$z_3 = E_5/E_3$
$\theta_2 = \theta_{12}$
$\theta_4 = \theta_{14}$!!

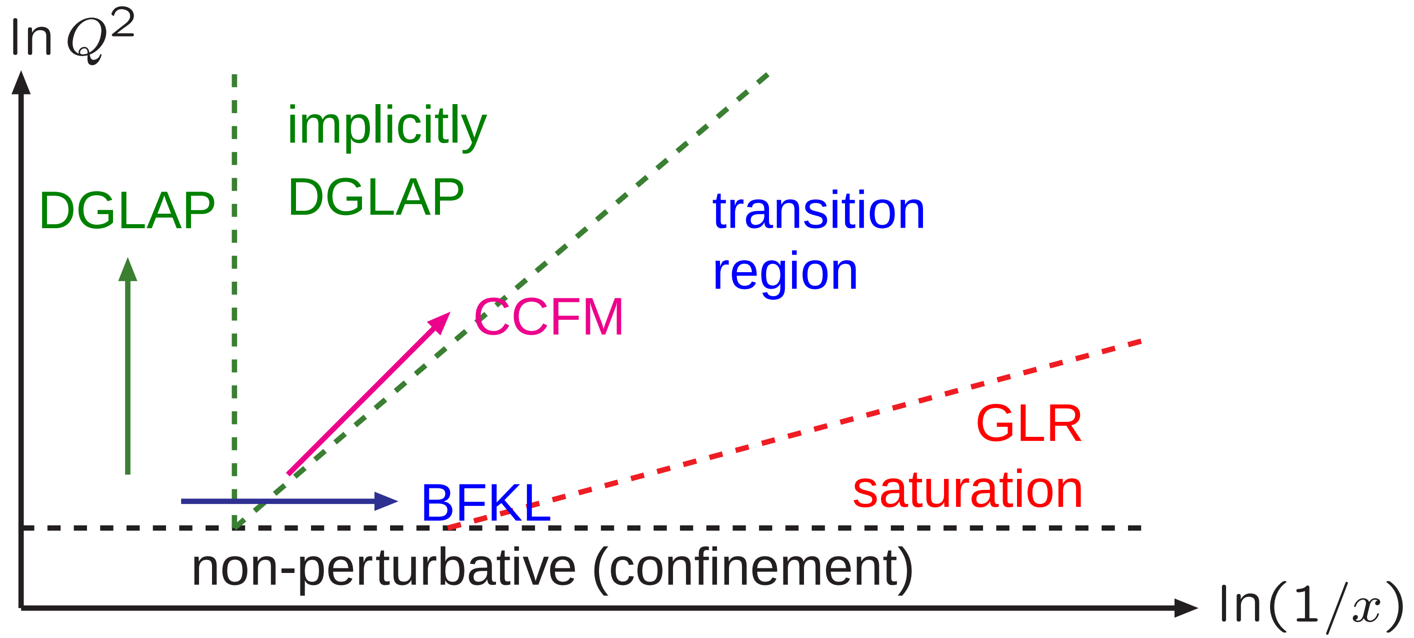
with $Q^2 = -m^2 =$ spacelike virtuality

- kinematics only:
  $Q_3^2 > z_1 Q_1^2$, $Q_5^2 > z_3 Q_3^2$, ...
  i.e. Q_i^2 need not even be ordered

- coherence of leading collinear singularities:
  $Q_5^2 > Q_3^2 > Q_1^2$, i.e. Q^2 ordered

- coherence of leading soft singularities (more messy):
  $E_3\theta_4 > E_1\theta_2$, i.e. $z_1\theta_4 > \theta_2$
  $z \ll 1$: $E_1\theta_2 \approx p_{\perp 2}^2 \approx Q_3^2$, $E_3\theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$
  i.e. reduces to Q^2 ordering as above
  $z \approx 1$: $\theta_4 > \theta_2$, i.e. angular ordering of soft gluons
  $\Longrightarrow$ reduced phase space

Evolution procedures



DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger Q^2 and (implicitly) towards smaller x

BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller x (with small, unordered Q^2)

CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL

GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch

Initial-State Shower Comparison

Two(?) CCFM Generators:

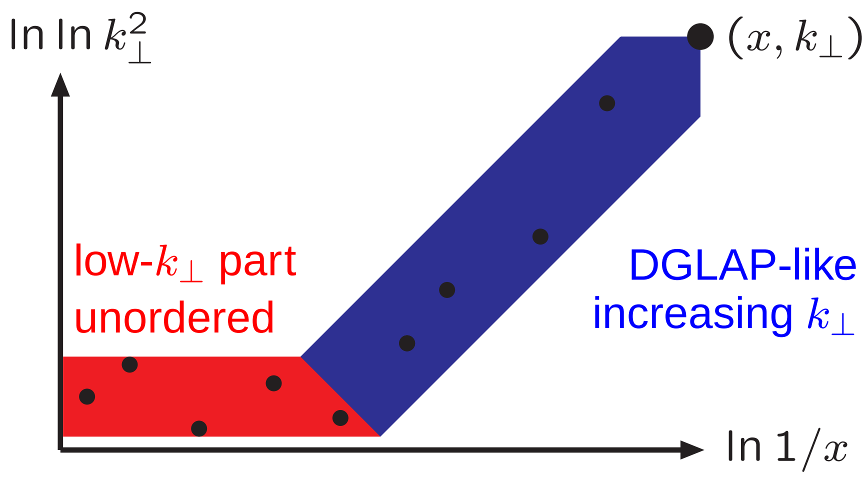
(SMALLX (Marchesini, Webber))

CASCADE (Jung, Salam)

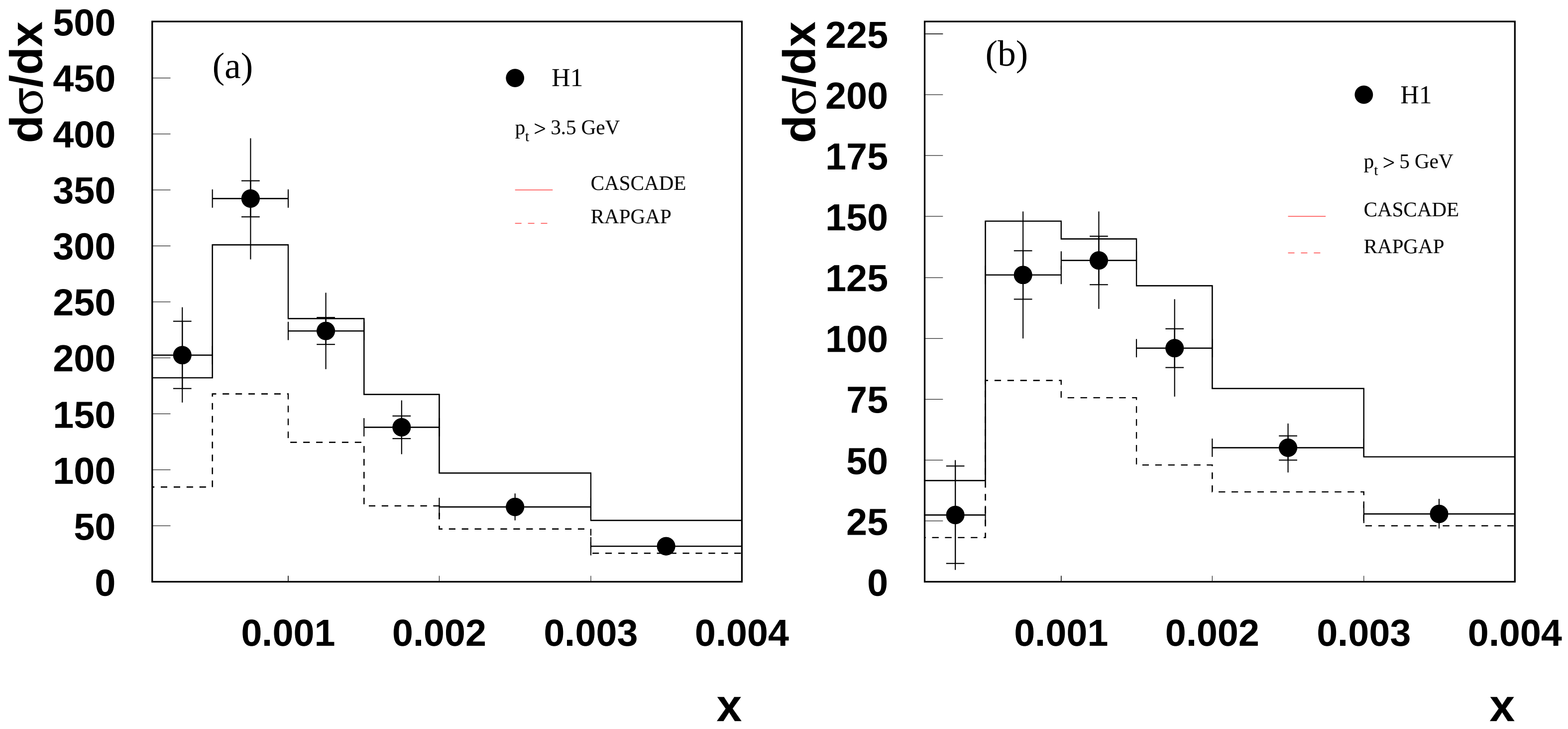
LDC (Gustafson, Lönnblad):

reformulated initial/final rad.

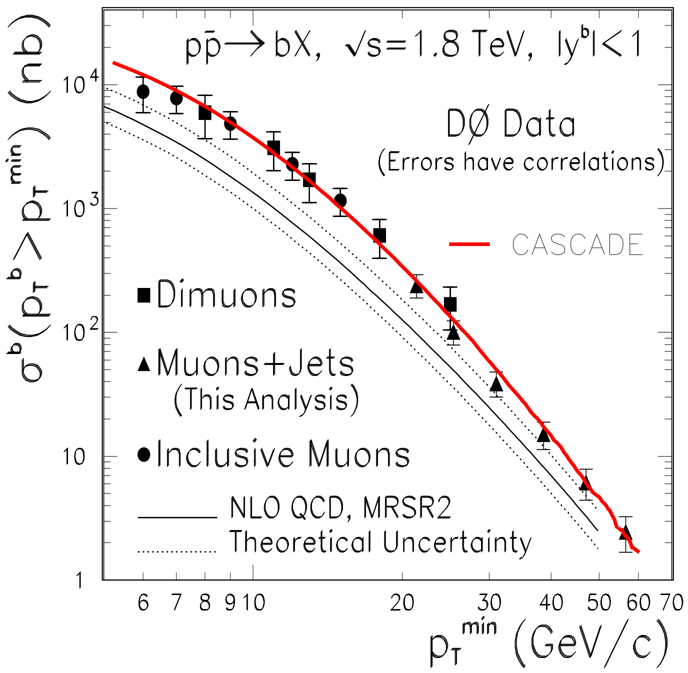
$\Longrightarrow$ eliminate non-Sudakov

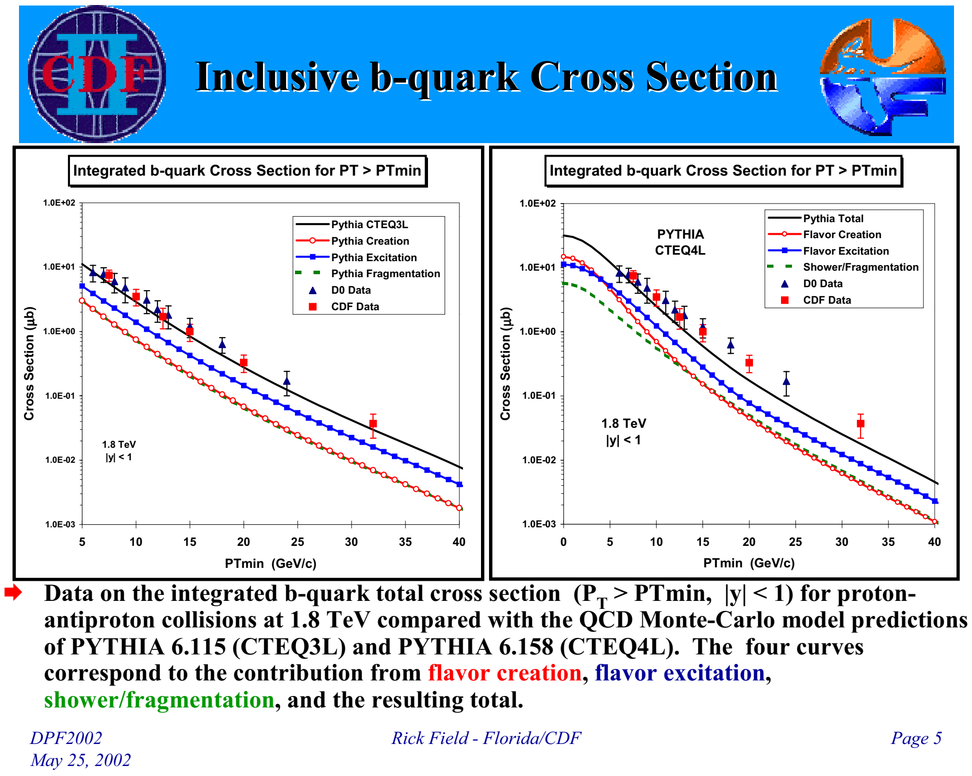


Test 1) forward (= p direction) jet activity at HERA



2) Heavy flavour production





but also explained by DGLAP with leading order pair creation
+ flavour excitation (≈ unordered chains)
+ gluon splitting (final-state radiation)

CCFM requires off-shell ME's + unintegrated parton densities

$$F(x,Q^2) = \int^{Q^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathcal{F}(x,k_\perp^2) + (\text{suppressed with } k_\perp^2 > Q^2)$$

so not ready for prime time in pp

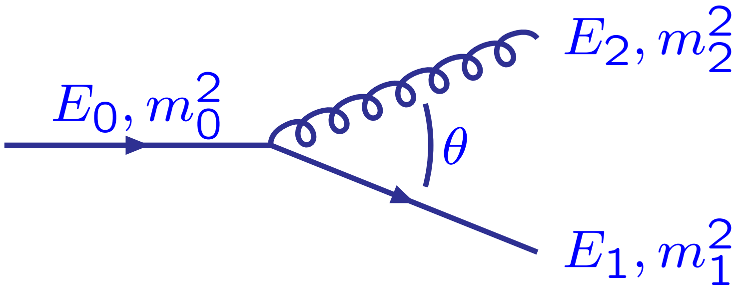
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z \cdot (\text{Sudakov})$$

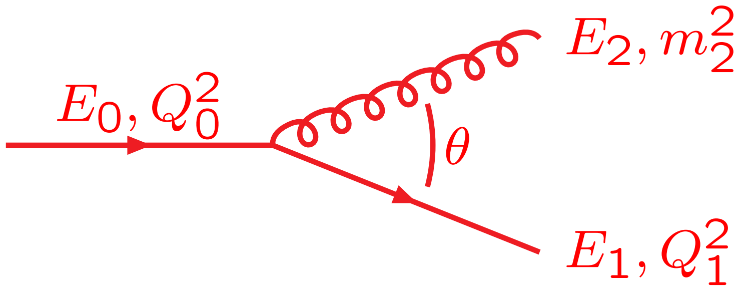
but

Final-state showers:
Q^2 timelike ($\sim m^2$)



decreasing E, m^2, θ
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ “minor” variations:
Q^2, shower vs. dipole, . . .

Initial-state showers:
Q^2 spacelike ($\approx -m^2$)



decreasing E, increasing Q^2, θ
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, . . .

Future of showers

Showers still evolving:

HERWIG has new evolution variable better suited for heavy particles

$$\tilde{q}^2 = \frac{\mathbf{q}^2}{z^2(1-z)^2} + \frac{m^2}{z^2} \quad \text{for } \mathrm{q} \to \mathrm{qg}$$

Gives smooth coverage of soft-gluon region, no overlapping regions in FSR phase space, but larger dead region.

PYTHIA has moved (but not yet users?) to $p_\perp$-ordered showers (borrowing some of ARIADNE dipole approach, but still showers)

$p^2_{\perp\mathrm{evol}} = z(1-z)Q^2 = z(1-z)M^2$ for FSR

$p^2_{\perp\mathrm{evol}} = (1-z)Q^2 = (1-z)(-M^2)$ for ISR

Guarantees better coherence for FSR, hopefully also better for ISR.

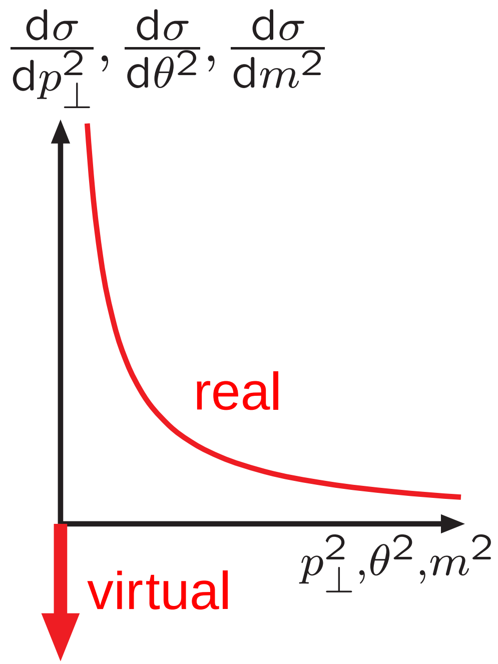
SHERPA moves from mass-ordered (∼PYTHIA) showers to $p_\perp$-ordered (Catani-Seymour) dipoles

However, main evolution is *matching to matrix elements*

Matrix Elements vs. Parton Showers

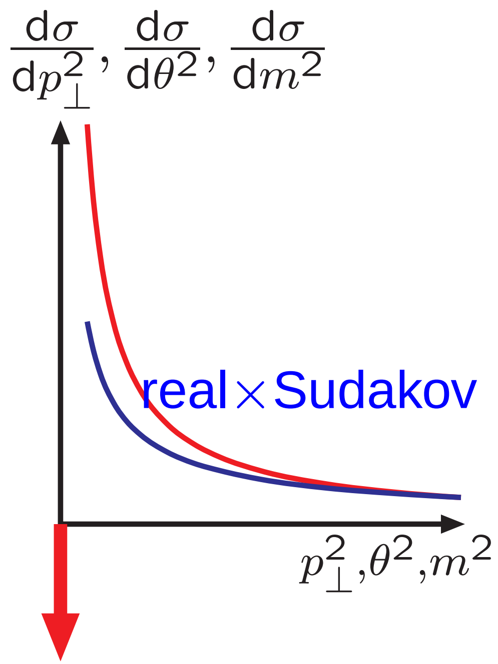
ME : Matrix Elements

+ systematic expansion in α_s (‘*exact*’)
+ powerful for multiparton Born level
+ flexible phase space cuts
– loop calculations very tough
– negative cross section in collinear regions
  $\Rightarrow$ unpredictive jet/event structure
– *no easy match to hadronization*

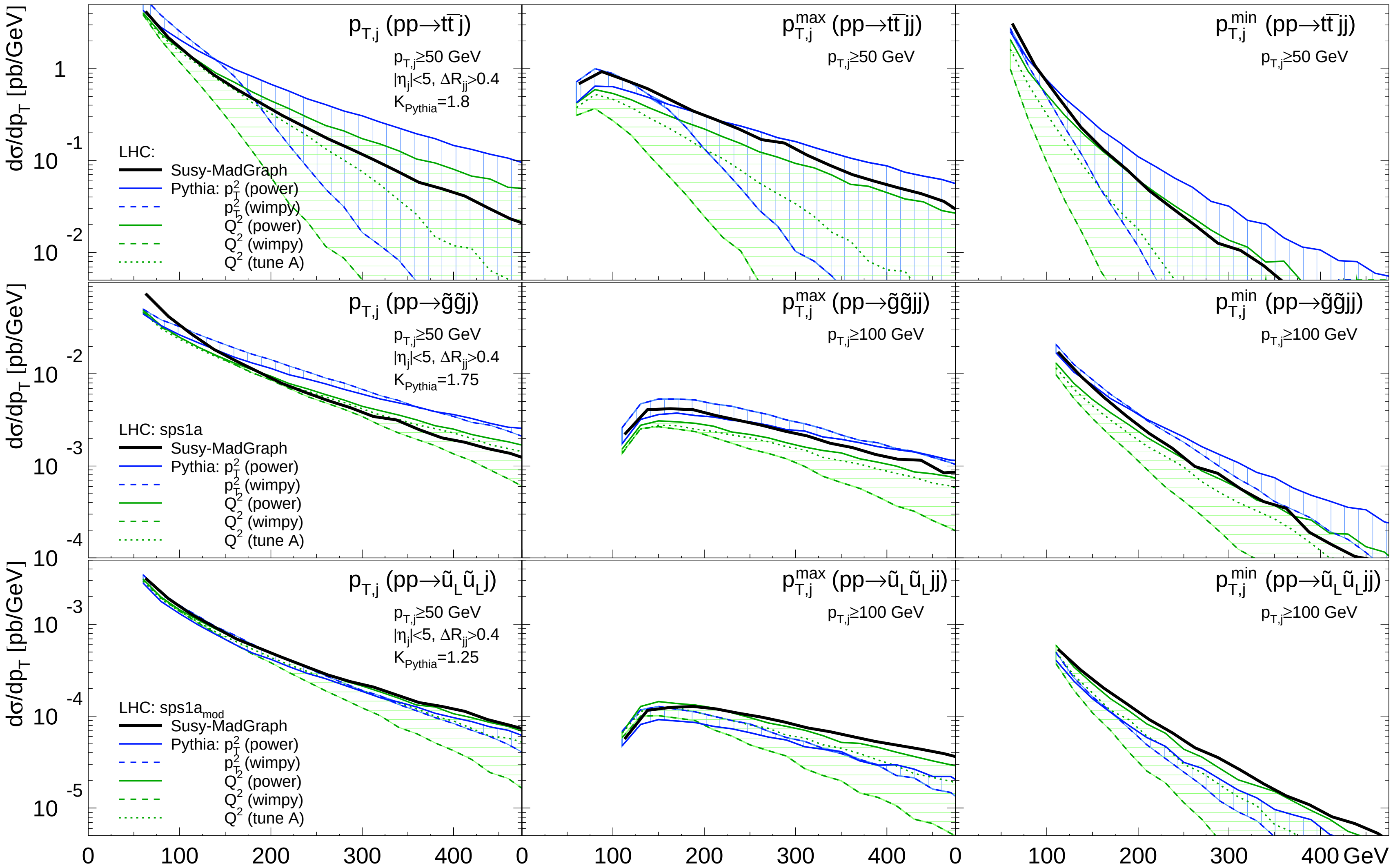


PS : Parton Showers

– approximate, to LL (or NLL)
– main topology not predetermined
  $\Rightarrow$ inefficient for exclusive states
+ process-generic $\Rightarrow$ simple multiparton
+ Sudakov form factors/resummation
  $\Rightarrow$ sensible jet/event structure
+ *easy to match to hadronization*



$p_\perp$(1 jet) $p_\perp^{max}$(2 jets) $p_\perp^{min}$(2 jets)



power: $Q^2_{max} = s$; wimpy: $Q^2_{max} = m^2_\perp$; tune A: $Q^2_{max} = 4m^2_\perp$

$m_t = 175$ GeV, $m_{\tilde{g}} = 608$ GeV, $m_{\tilde{u}_L} = 567$ GeV

(T. Plehn, D. Rainwater, P. Skands)

Matrix Elements and Parton Showers

Recall complementary strengths:

- ME's good for well separated jets
- PS's good for structure inside jets

Marriage desirable! But how?

Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing $\Longrightarrow$ no established orthodoxy

Three main areas, in ascending order of complication:

1) Match to lowest-order nontrivial process — merging

2) Combine leading-order multiparton process — vetoed parton showers

3) Match to next-to-leading order process — MC@NLO

Merging

= cover full phase space with smooth transition ME/PS

Want to reproduce $W^{\mathrm{ME}} = \frac{1}{\sigma(\mathrm{LO})} \frac{\mathrm{d}\sigma(\mathrm{LO}+\mathrm{g})}{\mathrm{d(phasespace)}}$

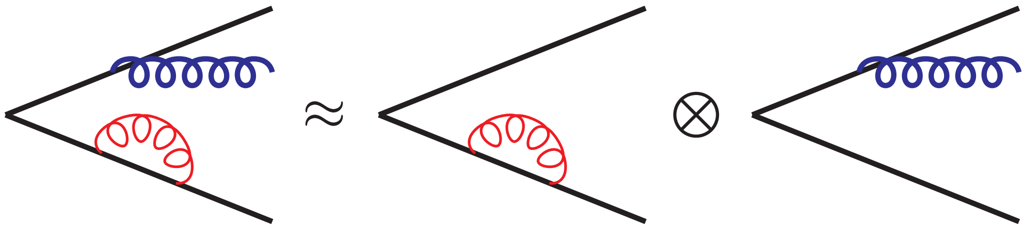
by shower generation + correction procedure

$$\underbrace{W^{\mathrm{ME}}}_{\text{wanted}} = \underbrace{W^{\mathrm{PS}}}_{\text{generated}} \; \overbrace{\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}}^{\text{correction}}$$

- Exponentiate ME correction by shower Sudakov form factor:

$$W^{\mathrm{PS}}_{\mathrm{actual}}(Q^2) = W^{\mathrm{ME}}(Q^2)\,\exp\left(-\int_{Q^2}^{Q^2_{\mathrm{max}}} W^{\mathrm{ME}}(Q'^2)\,\mathrm{d}Q'^2\right)$$

- Do not normalize W^{ME} to $\sigma(\mathrm{NLO})$ (error $\mathcal{O}(\alpha_s^2)$ either way)



$$\mathrm{d}\sigma = K\,\sigma_0\,\mathrm{d}W^{\mathrm{PS}}$$

with $K = 1+\mathcal{O}(\alpha_s)$ and $\int \mathrm{d}W^{\mathrm{PS}} = 1$

- Normally several shower histories $\Rightarrow$ ~equivalent approaches

Final-State Shower Merging

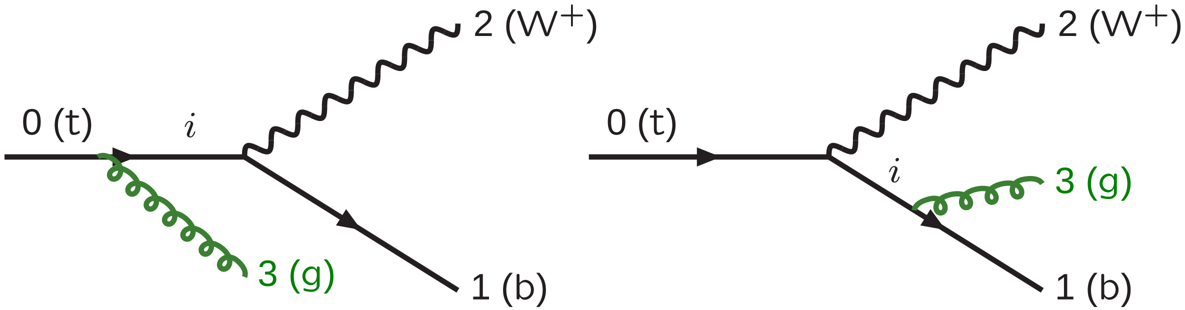
Merging with $\gamma^*/\mathrm{Z}^0 \to \mathrm{q\bar{q}g}$ for $m_\mathrm{q} = 0$ since long

(M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)

For $m_\mathrm{q} > 0$ pick $Q_i^2 = m_i^2 - m_{i,\mathrm{onshell}}^2$ as evolution variable since

$$W^\mathrm{ME} = \frac{(\ldots)}{Q_1^2 Q_2^2} - \frac{(\ldots)}{Q_1^4} - \frac{(\ldots)}{Q_2^4}$$

Coloured decaying particle also radiates:



ME $\frac{1}{Q_0^2 Q_1^2}$ matches PS $\mathrm{b} \to \mathrm{bg}$

$\Rightarrow$ can merge PS with generic $a \to bc\mathrm{g}$ ME

(E. Norrbin & TS, NPB603 (2001) 297)

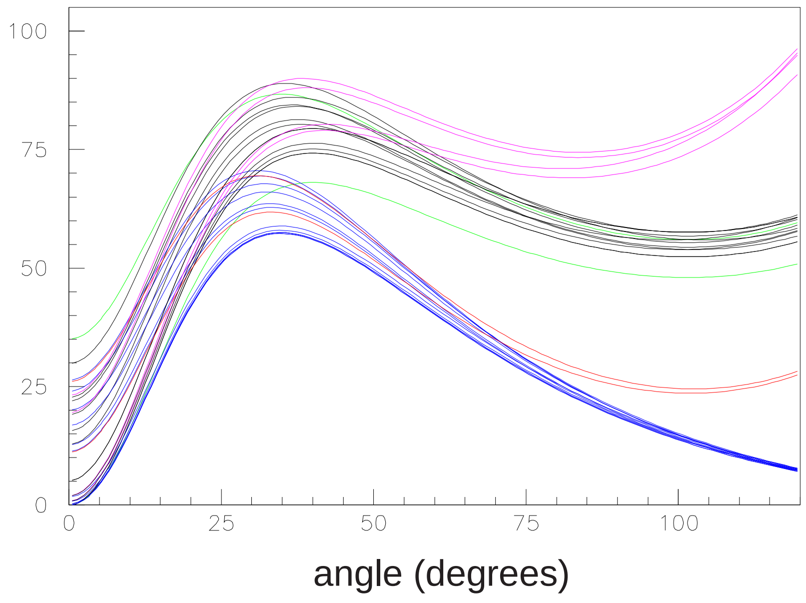
Subsequent branchings $\mathrm{q} \to \mathrm{qg}$: also matched to ME, with reduced energy of system

PYTHIA performs merging with generic FSR $a \to bc$g ME,

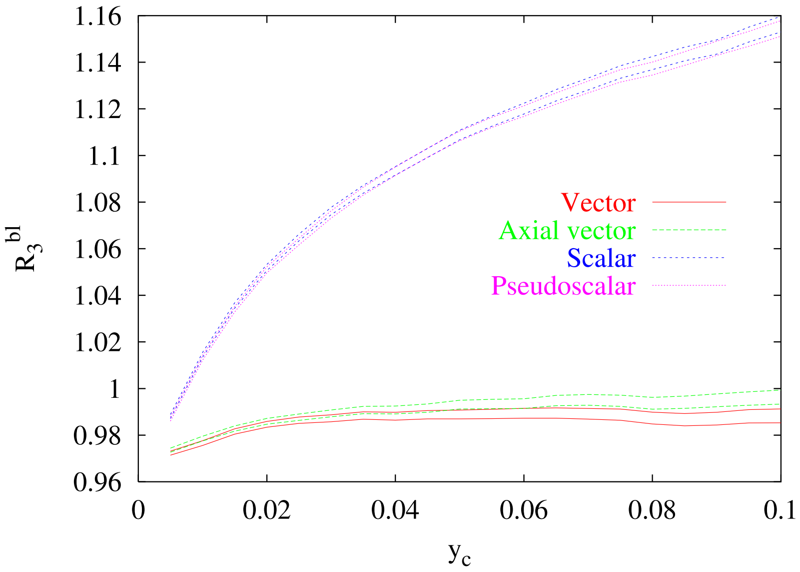
in SM: $\gamma^*/\mathrm{Z}^0/\mathrm{W}^\pm \to \mathrm{q}\overline{\mathrm{q}}$, $\mathrm{t} \to \mathrm{bW}^+$, $\mathrm{H}^0 \to \mathrm{q}\overline{\mathrm{q}}$,

and MSSM: $\mathrm{t} \to \mathrm{bH}^+$, $\mathrm{Z}^0 \to \tilde{\mathrm{q}}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{W}^+$, $\mathrm{H}^0 \to \tilde{\mathrm{q}}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{H}^+$, $\chi \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\chi \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\chi$, $\mathrm{t} \to \tilde{\mathrm{t}}\chi$, $\tilde{\mathrm{g}} \to \mathrm{q}\overline{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\tilde{\mathrm{g}}$, $\mathrm{t} \to \tilde{\mathrm{t}}\tilde{\mathrm{g}}$

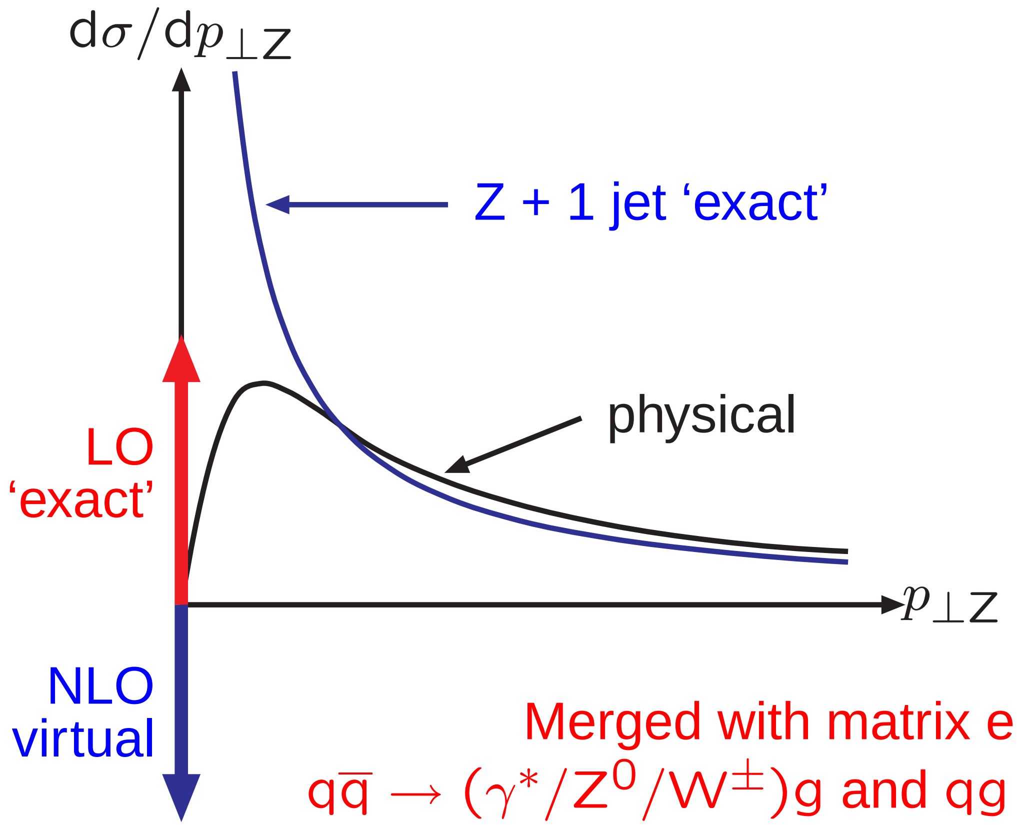
g emission for different colour, spin and parity:



$R_3^{\mathrm{bl}}(y_c)$: mass effects in Higgs decay:



Initial-State Shower Merging



resummation:
physical $p_{\perp Z}$ spectrum

shower: ditto
+ accompanying
jets (exclusive)

Merged with matrix elements for
$q\overline{q} \to (\gamma^*/Z^0/W^{\pm})g$ and $qg \to (\gamma^*/Z^0/W^{\pm})q'$:

(G. Miu & TS, PLB449 (1999) 313)

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{q\overline{q}' \to gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1$$

$$\left(\frac{W^{ME}}{W^{PS}}\right)_{qg \to q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3$$

with $Q^2 = -m^2$
and $z = m_W^2/\hat{s}$

Merging in HERWIG

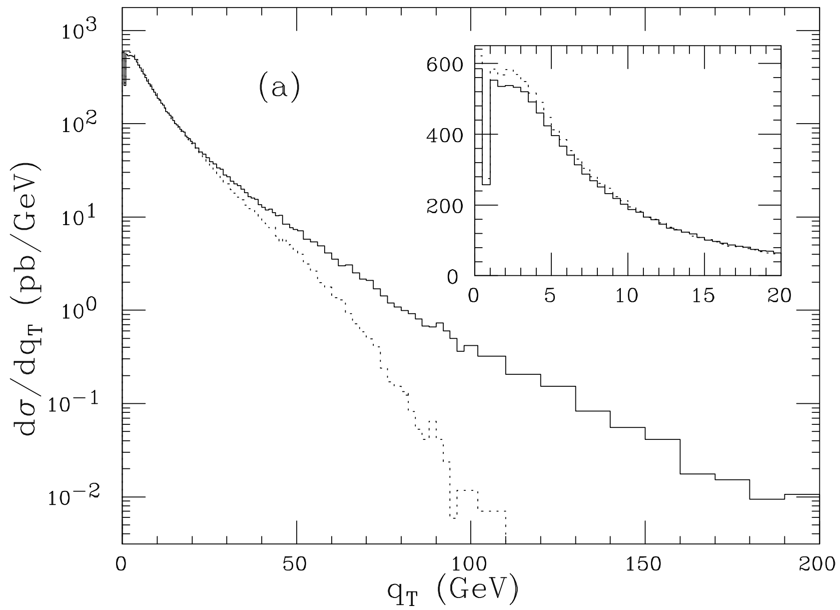
HERWIG also contains merging, for

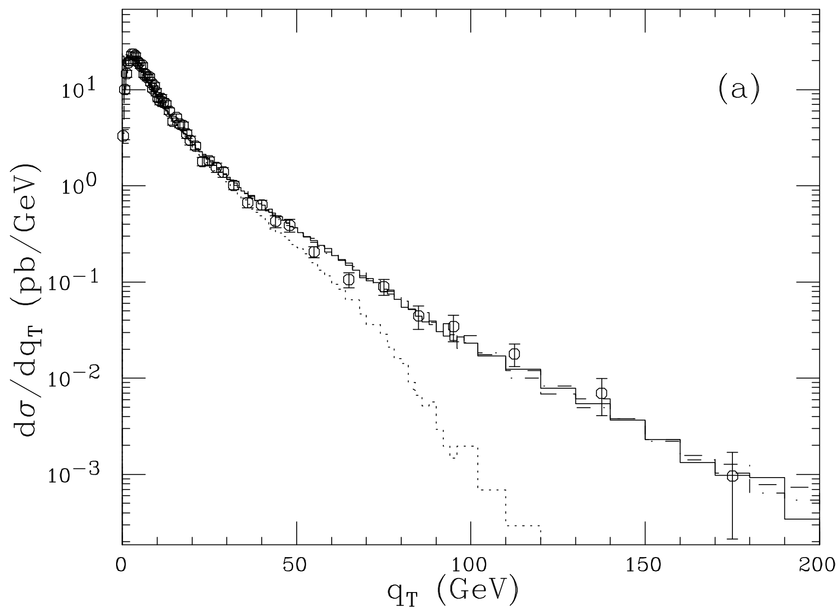
- $Z^0 \to q\bar{q}$
- $t \to bW^+$
- $q\bar{q} \to Z^0$

and some more

Special problem:
angular ordering does not cover full phase space; so
(1) fill in “dead zone” with ME
(2) apply ME correction
   in allowed region

Important for agreement with data:





Vetoed Parton Showers

S. Catani, F. Krauss, R. Kuhn, B.R. Webber, JHEP 0111 (2001) 063; L. Lönnblad, JHEP0205 (2002) 046;
F. Krauss, JHEP 0208 (2002) 015; S. Mrenna, P. Richardson, JHEP0405 (2004) 040;
S. Höche et al., hep-ph/0602031

Generic method to combine ME's of several different orders to NLL accuracy; will be a 'standard tool' in the future

Basic idea:

- consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
- σ_i, $i \geq 1$, are divergent in soft/collinear limits
- absent virtual corrections would have ensured "detailed balance", i.e. an emission that adds to σ_{i+1} subtracts from σ_i
- such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
- so use shower routines to provide missing virtual corrections $\Rightarrow$ rejection of events (especially) in soft/collinear regions

Veto scheme:

1) Pick hard process, mixing according to $\sigma_0 : \sigma_1 : \sigma_2 : \ldots$,

above some ME cutoff (e.g. all $p_{\perp i} > p_{\perp 0}$, all $R_{ij} > R_0$),

with large fixed α_{s0}

2) Reconstruct imagined shower history (in different ways)

3) Weight $W_\alpha = \prod_{\text{branchings}} (\alpha_s(k^2_{\perp i})/\alpha_{s0}) \Rightarrow$ accept/reject

CKKW-L:

4) Sudakov factor for non-emission on all lines above ME cutoff

$W_{\text{Sud}} = \prod_{\text{“propagators”}} \text{Sudakov}(k^2_{\perp\text{beg}}, k^2_{\perp\text{end}})$

4a) CKKW : use NLL Sudakovs

4b) L: use trial showers

5) $W_{\text{Sud}} \Rightarrow$ accept/reject

6) do shower, vetoing emissions above cutoff

MLM:

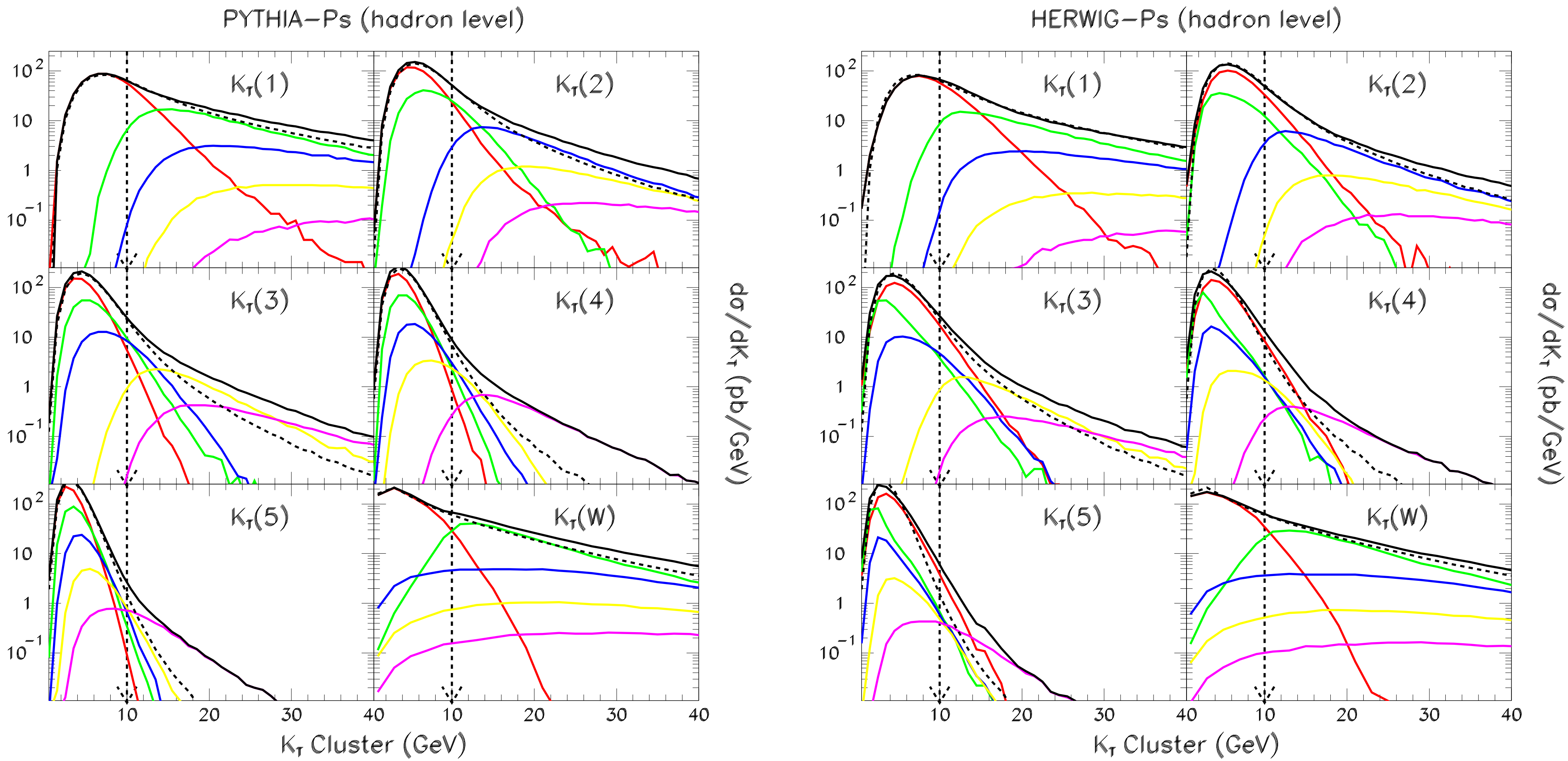
4) do parton showers

5) (cone-)cluster showered event

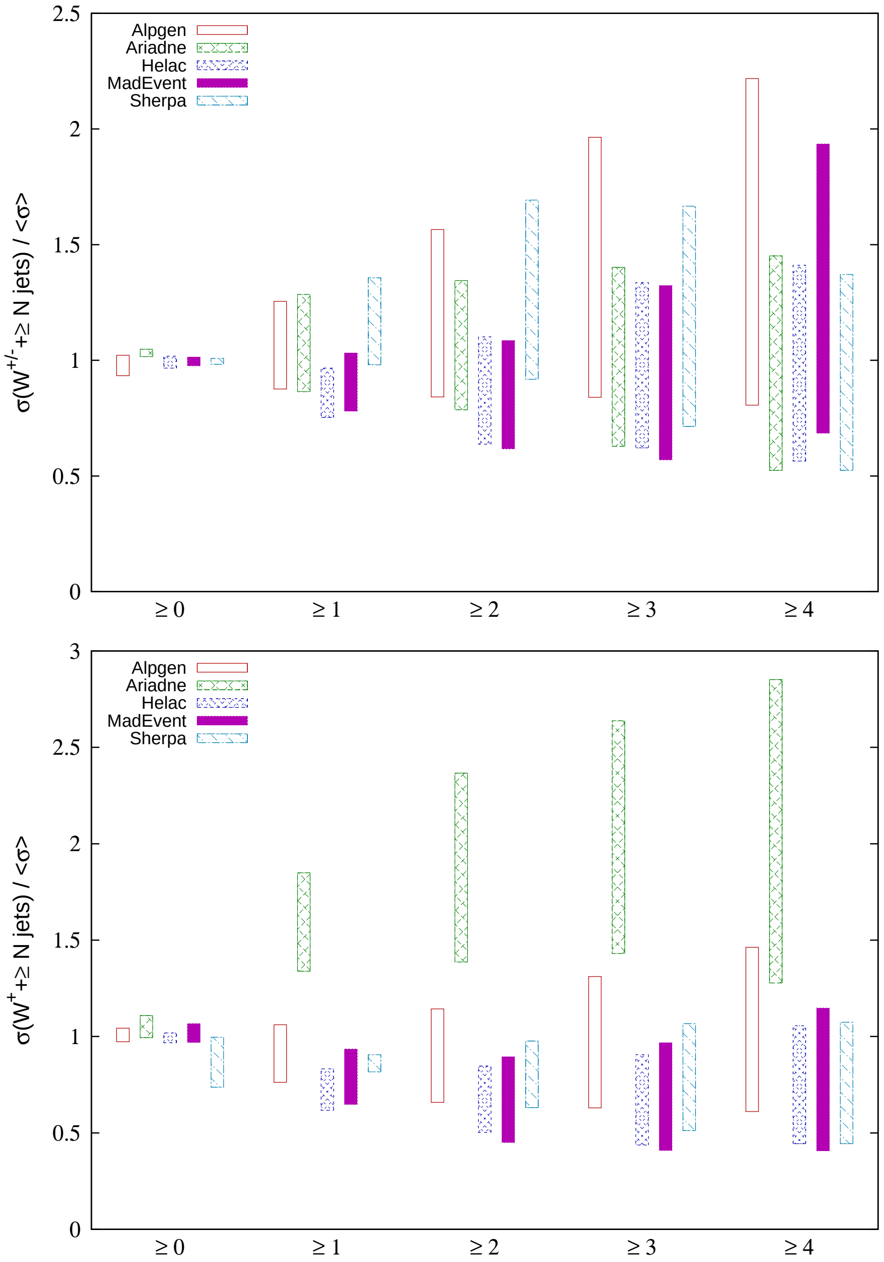
6) match partons and jets

7) if all partons are matched, and $n_{\text{jet}} = n_{\text{parton}}$, keep the event, else discard it

CKKW mix of $W + (0, 1, 2, 3, 4)$ partons,
hadronized and clustered to jets:



(S.Mrenna, P. Richardson)



Spread of W + jets rate for different matching schemes + showers,

top: Tevatron,

bottom: LHC.

ALPGEN: MLM + HERWIG

ARIADNE: CKKW-L + ARIADNE

HELAC: MLM + PYTHIA

MADEVENT: MLM/CKKW + PYTHIA

SHERPA: CKKW + SHERPA

model varation: α_S, cuts, . . .

arXiv0706.2569 (Alwall et al.)

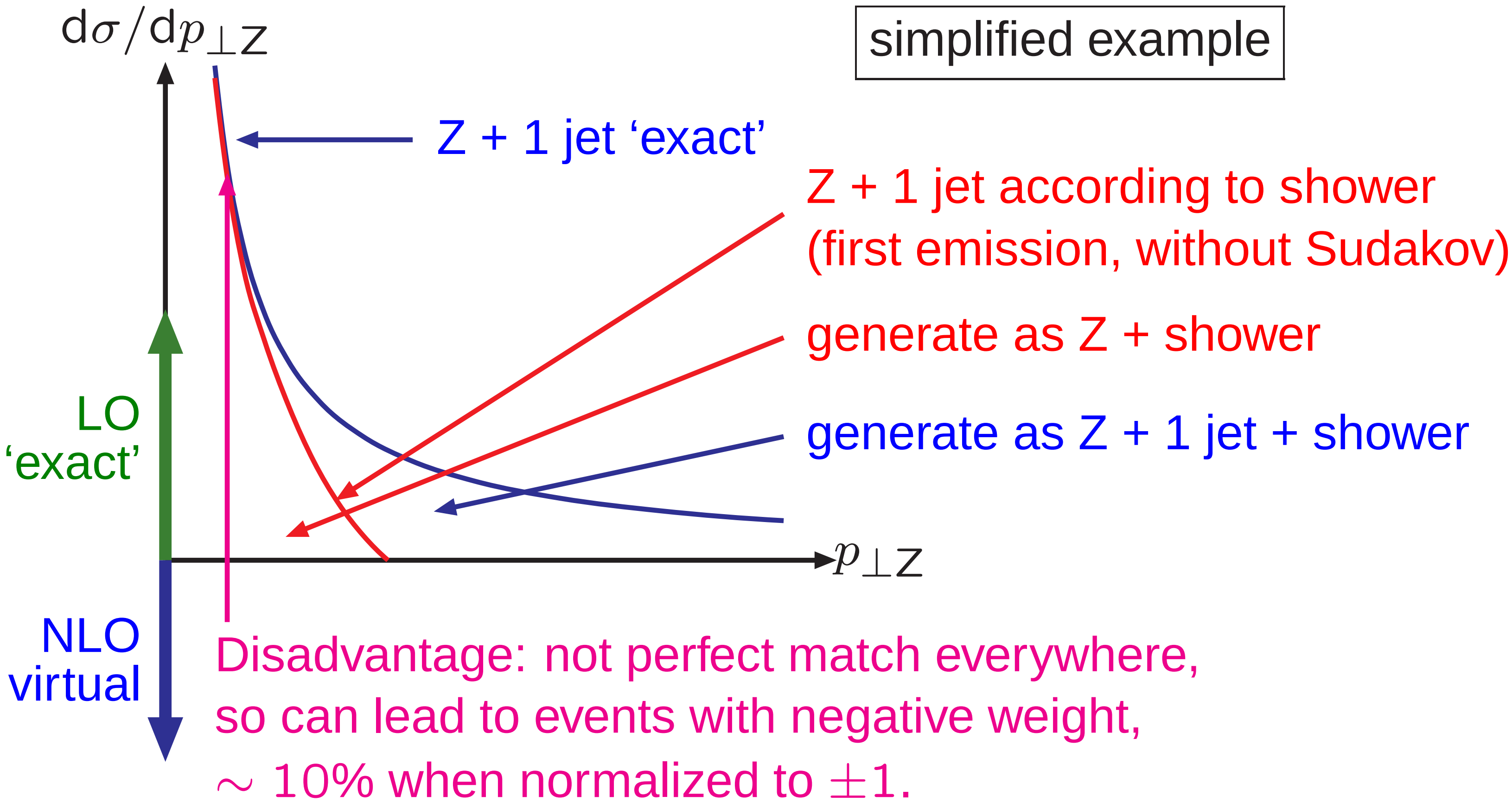
MC@NLO

Objectives:

- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of α_S.
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of “normal” events, that can be processed further.

Basic scheme (simplified!):

1) Calculate the NLO matrix element corrections to an n-body process (using the subtraction approach).
2) Calculate analytically (no Sudakov!) how the first shower emission off an n-body topology populates $(n+1)$-body phase space.
3) Subtract the shower expression from the $(n+1)$ ME to get the “true” $(n+1)$ events, and consider the rest of σ_{NLO} as n-body.
4) Add showers to both kinds of events.



MC@NLO in comparison:

- Superior with respect to "total" cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.

$\Rightarrow$ pick according to current task and availability.

MC@NLO 2.31 [hep-ph/0402116]

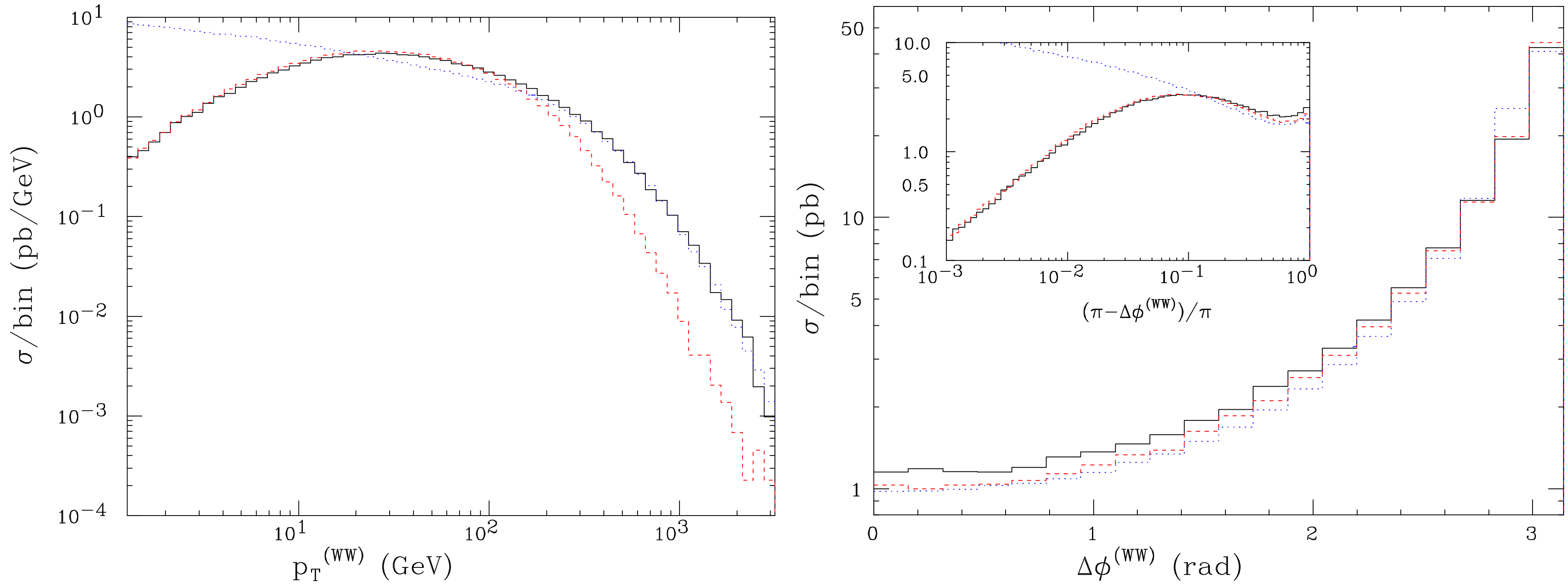
(Frixione, Webber)

IPROC	Process
–1350–IL	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1360–IL	$H_1H_2 \to (Z \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1370–IL	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1460–IL	$H_1H_2 \to (W^+ \to) l^+_{\rm IL}\nu_{\rm IL} + X$
–1470–IL	$H_1H_2 \to (W^- \to) l^-_{\rm IL}\bar{\nu}_{\rm IL} + X$
–1396	$H_1H_2 \to \gamma^*(\to \sum_i f_i\bar{f}_i) + X$
–1397	$H_1H_2 \to Z^0 + X$
–1497	$H_1H_2 \to W^+ + X$
–1498	$H_1H_2 \to W^- + X$
–1600–ID	$H_1H_2 \to H^0 + X$
–1705	$H_1H_2 \to b\bar{b} + X$
–1706	$H_1H_2 \to t\bar{t} + X$
–2850	$H_1H_2 \to W^+W^- + X$
–2860	$H_1H_2 \to Z^0Z^0 + X$
–2870	$H_1H_2 \to W^+Z^0 + X$
–2880	$H_1H_2 \to W^-Z^0 + X$

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented

Later additions: single top, $H^0W^\pm$, H^0Z^0

W^+W^- Observables



These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

Solid: MC@NLO

Dashed: HERWIG$\times \frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

POWHEG

Nason; Frixione, Oleari, Ridolfi (e.g. JHEP **0711** (2007) 070)
Alternative to MC@NLO:

$$\mathrm{d}\sigma = \bar{B}(v)\mathrm{d}\Phi_v \left[\frac{R(v,r)}{B(v)} \exp\left(- \int_{p_\perp} \frac{R(v,r')}{B(v)} \mathrm{d}\Phi'_r \right) \mathrm{d}\Phi_r \right]$$

where

$$\bar{B}(v) = B(v) + V(v) + \int \mathrm{d}\Phi_r [R(v,r) - C(v,r)] \, .$$

and

$v, \mathrm{d}\Phi_v$ Born-level n-body variables and differential phase space
$r, \mathrm{d}\Phi_r$ extra $n+1$-body variables and differential phase space
$B(v)$ Born-level cross section
$V(v)$ Virtual corrections
$R(v,r)$ Real-emission cross section
$C(v,r)$ Conterterms for collinear factorization of parton densities.

Basic idea:

- Pick the real emission with largest $p_\perp$ according to complete ME's, with NLO normalization.
- Let showers do subsequent evolution downwards from this $p_\perp$ scale.

Relative to MC@NLO:

$+$ no negative weights (except in regions with extreme virtual corrections)

$+$ clean separation to shower stage

$\pm$ optimal for $p_\perp$-ordered showers, messy for others

$\pm$ different higher-order terms

$-$ as of yet fewer processes than MC@NLO

$p_\perp$ spectrum of individual $\mathtt{t}$ quark and of $\mathtt{t}\bar{\mathtt{t}}$ pair: