Static quark-antiquark pairs at finite temperature

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Outline

- Brief introduction to QGP, heavy quark bound states and Thermal Field Theory
- Potentials in two different temperature regions
- Conclusions
- Sources:

The Quark-Gluon Plasma

- A new state of matter
- It forms at high temperatures/densities $T_C \approx 175\text{MeV}$
- Screening of the non-abelian charge, deconfined phase
- Heavy ion collisions (SPS, BNL, ALICE at LHC)
The J/Psi as a QGP probe

- Proposed in 1986 by Matsui and Satz
- Clean leptonic decay
- Hypothesis of thermal dissociation above the critical temperature
- The study of the potential in different temperature regions can provide useful information, without resorting to models
Potentials

- Potential models
  - Phenomenological nature
  - Introduced early (70’s)
  - Example: Cornell $V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r$
- Effective field theories: modern and rigorous definition of the potential (last 15 years): NRQCD, pNRQCD
Heavy quarks bound states

- Non-relativistic treatment: possible due to the large mass of the heavy quarks

- Hierarchy: \( m \gg m v \gg m v^2 \)

- Another scale in QCD: \( \Lambda_{QCD} \)

- Many scales, EFT more suitable

- We consider the static limit \( m \to \infty \)
Temperature: more scales

\[ m \gg mv \gg mv^2 \]

\[ T \gg gT \gg g^2T \ldots \]

\[ m_D \sim gT \]

Debye scale

Screening scale

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\[ mv \sim \frac{1}{r} \]

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\[ m_D \sim \frac{1}{r} \]

\[ \frac{1}{r} \gg T \gg m_D \gg \Delta V \]

- We study \[ T \gg m_D \gg \frac{1}{r} \]
- Perturbative study
Real-time formalism

- Evolution along this path
- Limit $t_i \rightarrow -\infty$
- Doubling of the degrees of freedom

Figure 3.1: The time path $C$ for the real-time formalism. We call $C_1$ the horizontal leg from $t_i$ to $-t_i$, $C_3$ the vertical leg from $-t_i$ to $-t_i - i \sigma$, $C_2$ the horizontal, backward leg from $-t_i - i \sigma$ to $t_i - i \sigma$ and finally $C_4$ the last leg from $t_i - i \sigma$ to $t_i - i \beta$. It can be shown that physical observables are independent of $\sigma$ as long as $0 < \sigma < \beta$. We will choose $\sigma = 0$ throughout this chapter.

Real particles (1)

Virtual particles (2)
Potential for \( T \gg \frac{1}{r} \sim m_D \)

- Integration of the scale \( T \): the longitudinal gluon propagator (Coulomb gauge) acquires a mass (the Debye mass):

\[
\frac{i}{q^2} \rightarrow \frac{i}{q^2 + m_D^2} \quad m_D^2 = g^2 T^2 \left( \frac{N_C}{3} + \frac{N_f}{6} \right)
\]

- Integration of the scale \( \frac{1}{r} \): we obtain the potential
Potential for $T \gg \frac{1}{r} \sim m_D$

$$V(r) = -\frac{g^2 C_F}{4\pi} \left( \frac{e^{-m_D r}}{r} - \frac{2iT}{m_D r} \int_0^\infty dz \frac{\sin(m_D r z)}{(z^2 + 1)^2} \right)$$

- **Real part**: Debye potential screening
- **Imaginary part**: decay Landau damping

We have obtained the result of

Potential for $\frac{1}{r} \gg T \gg m_D \gg \Delta V$

$m_D \sim gT \Rightarrow \frac{1}{r} \gg T \gg m_D$

- It is necessary to compute the vacuum polarizations at finite temperature.
- The computation is carried out without any hierarchy assumption.
- Only at the end of the computation we can consider the contribution from the various momentum regions.
Potential for \( \frac{1}{r} \gg T \gg m_D \gg \Delta V \)

\[
V(r) = -C_F \frac{\alpha_V (1/r)}{r} \left[ \pi C_F C_A \alpha_s^2 T^2 r + \frac{C_F}{6} \alpha_s^3 T \right] \left[ \frac{\pi C_F}{9} \alpha_s r^2 T m_D^2 - \frac{3}{2} \right] \left( \frac{2\gamma_E - \log \frac{T^2}{m_D^2}}{m_D^2} - 1 - 4 \log 2 - \frac{2 \zeta'(2)}{\zeta(2)} \right) + \left[ \frac{C_F}{9} \log 2 C_A C_F \alpha_s^2 r^2 T^3 \right] + \ldots
\]

- Contributions from the scale \( k \sim \frac{1}{r} \)
- Contributions from the scale \( k \sim T \)
- Contributions from the scale \( k \sim m_D \)
- Imaginary part
- Singlet->octet thermal dissociation
Singlet to octet thermal dissociation

- Energetically forbidden at $T=0$
Conclusions

- Rigorous QCD study of the potential
- New results in the regime $\frac{1}{r} \gg T$
- New thermal dissociation process
- Possible predictions on J/Psi and $\Upsilon$ phenomenology in the QGP