First order formalism in Resonance chiral theory

Jaroslav Trnka

Institute of Nuclear and Particle Physics,
Charles University in Prague

Based on articles:

Kampf, Novotny, Trnka, hep-ph/0608051, Eur.Phys.J.C50:385-403,2007

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- χPT
- Resonance chiral theory
- First order formalism
- Toy example of computation
- Problems and future plans

Chiral perturbation theory

Chiral perturbation theory

- Effective theory for QCD at low energies [Weinberg 1969; Gasser, Leutwyler 1985]
- Degrees of freedom: only octet of pseudoscalar mesons are present

$$\phi(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & K^0 & -\frac{1}{\sqrt{3}}\eta \end{pmatrix}$$

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- Lagrangian can be expanded in terms of momenta

$$\mathcal{L}_\chi = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

- At the leading order $\mathcal{O}(p^2)$

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_- \rangle$$

- NLO and NNLO for SU(3)

$\mathcal{O}(p^4)$	$\mathcal{L}_4$	$\approx$	10 terms	[Gasser, Leutwyler 1985]
$\mathcal{O}(p^6)$	$\mathcal{L}_6$	$\approx$	100 terms	[Bijnens, Colangelo, Ecker 1999]

- Chiral building blocks consist of $\phi(x)$ and external sources s, p, v^μ and a^μ.

Resonance chiral theory

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- At low energies, $E \ll \Lambda \simeq 1\,\mathrm{GeV} \Rightarrow \chi$PT works
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- Two traditional ways how to describe vector resonances

  V^{μ} ... vector fields

  $R^{\mu\nu}$ ... antisymmetric tensor fields
- Large N_C inspired phenomenological Lagrangians

  [Ecker, Gasser, Pich, de Rafael 1989]

$$\mathcal{L}_T = \mathcal{L}_T^{(4)} + \mathcal{L}_T^{(6)} + \mathcal{L}_T^{(8)} + \ldots, \qquad \mathcal{L}_V = \mathcal{L}_V^{(6)} + \mathcal{L}_V^{(8)} + \ldots$$

where $\quad V = \mathcal{O}(p^3), \quad R = \mathcal{O}(p^2)$

$D^{\mu} = \mathcal{O}(p), \quad u^{\mu} = \mathcal{O}(p), \quad f_{\pm} = \mathcal{O}(p^2), \quad \chi_{\pm} = \mathcal{O}(p^2), \quad h_{\mu\nu} = \mathcal{O}(p^2)$ - contain $\phi(x)$ and external sources

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- Examples of Lagrangians: [Ecker, Gasser, Pich, de Rafael 1989]

$$\mathcal{L}_V^{(6)} = \frac{\mathrm{i} f_V}{2\sqrt{2}} \langle (D^\mu V^\nu - D^\nu V^\mu) f_{+\mu\nu} \rangle, \qquad \mathcal{L}_T^{(4)} = \frac{\mathrm{i} F_V}{2\sqrt{2}} \langle R^{\mu\nu} f_{+\mu\nu} \rangle$$

- Antisymmetric tensor formalism is richer than vector up to given chiral order
- The study of basis of terms and calculations of Green functions:
  - vector formalism - [Prades 1994], [Knecht, Nyffeler 2001]
  - antisymmetric tensor formalism - [Ruiz-Femenia, Pich, Portoles 2003], [Cirigliano, Ecker, Eidemuller, Kaiser, Pich and Portoles 2006]
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- Resonance saturation of LEC = integrating out resonances

$$\int \mathcal{D}V \exp\left(\mathrm{i}\int d^4x \mathcal{L}_V\right) \to \mathcal{L}_{\chi,V}$$

- Problem: $\mathcal{L}_{\chi,V}$ and $\mathcal{L}_{\chi,R}$ start at different chiral orders ($\mathcal{O}(p^6)$ vs. $\mathcal{O}(p^4)$)
  $\Rightarrow$ expressing one set of constants in terms of another does not ensure equivalence

Resonance chiral theory

Resonance chiral theory

- Study of equivalence of resonance Lagrangians
  $\Rightarrow$ introducing of dummy variable and shifting

$$
\begin{aligned}
Z &= \int \mathcal{D}V \exp\left(\mathrm{i}\int d^4x \mathcal{L}_V\right) = \frac{\int \mathcal{D}V \int \mathcal{D}R \exp\left(\mathrm{i}\int d^4x(\mathcal{L}_V + \frac{1}{4}R_{\mu\nu}R^{\mu\nu})\right)}{\int \mathcal{D}R \exp\left(\mathrm{i}\int d^4x \frac{1}{4}R_{\mu\nu}R^{\mu\nu}\right)} \\
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- Example of calculations with $\mathcal{L}_V$ and $\mathcal{L}_T$ - $\langle VVP \rangle$ correlator
  $\Rightarrow$ vector formalism [Knecht, Nyffeler 2001]
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  Does the result satisfy short distance constraints with contact terms up to $\mathcal{O}(p^6)$?
- Which of formalisms is generally "better"?
  None, each has problems in some cases.
- What are the examples of these bad behaviors?
  VVP for vector formalism
  Compton scattering for antisymmetric tensor formalism

First order formalism

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    $\Rightarrow$ contains both types of propagators $\langle VV\rangle$, $\langle RR\rangle$
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  - interaction Lagrangians $\mathcal{L}_{VT}$ is a sum of $\mathcal{L}_V$ and $\mathcal{L}_T$ plus additional mixed terms
- Relation with vector and antisymmetric tensor formalisms

$$\begin{aligned} Z &= \exp\left(\mathrm{i}\int d^4x\mathcal{L}_{\chi,VT}\right) = \int\mathcal{D}V\int\mathcal{D}R\exp\left(\mathrm{i}\int d^4x\mathcal{L}_{VT}\right) \\ &= \int\mathcal{D}V\exp\left(\mathrm{i}\int d^4x\mathcal{L}_V^{\mathrm{eff}}\right) = \int\mathcal{D}R\exp\left(\mathrm{i}\int d^4x\mathcal{L}_T^{\mathrm{eff}}\right) \end{aligned}$$

where $\mathcal{L}_{\chi,VT}$ contains all possible contributions.

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  [Kampf, Novotny, Trnka 2006 and 2007]

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    We are searching for such an evidence.
  - Is it necessary to add some contact terms in first order formalism in order to satisfy short-distance constraints?
    More systematic study of short-distance constraints is still missing.

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- Definition

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- Ward identities determines the structure

$$\Pi(p)^{ab}_{\mu\nu} = i\delta^{ab}(p^2 g_{\mu\nu} - p_\mu p_\nu)\mathcal{F}(p^2)$$

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- Diagram: only one with interchange of resonance 

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  - vector formalism:

$$\mathcal{F}_V(p^2) = \frac{f_V^2 p^2}{M^2 - p^2}$$

  - antisymmetric tensor formalism:

$$\mathcal{F}_R(p^2) = \frac{1}{M^2 - p^2}(F_V - 2\sqrt{2}\lambda_{22}^V p^2)^2$$

  - first order formalism:

$$\mathcal{F}_{RV}(p^2) \quad = \quad \mathcal{F}_R(p^2) + \mathcal{F}_V(p^2) - \frac{2 f_V p^2}{M(M^2 - p^2)}(F_V - 2\sqrt{2}\lambda_{22}^V p^2)$$

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$$\mathcal{F}_{RV}(p^2) \quad = \quad \mathcal{F}_R(p^2) + \mathcal{F}_V(p^2) - \frac{2 f_V p^2}{M(M^2 - p^2)}(F_V - 2\sqrt{2}\lambda_{22}^V p^2)$$
- Short distance constraints:
$$\lim_{\lambda \to \infty} \mathcal{F}(\lambda^2 p^2) = \frac{1}{24\pi^2} \ln \lambda^2 + \mathcal{O}(\lambda^0)$$
  - vector formalism: automatically satisfied $\rightarrow$ no conditions
  - antisymmetric tensor formalism: $\lambda_{22}^V = 0$
  - first order formalism: $\lambda_{22}^V = 0$ or $\lambda_{22}^V = -\frac{f_V}{\sqrt{2}M}$

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    $$\mathcal{L}_{ct} = \alpha D^{\alpha} R^{\mu\nu} D_{\alpha} R_{\mu\nu} + \dots$$

    where

    $$\alpha = \alpha_r(\mu) + \lambda_{\infty} \frac{5M^2 d_1^2}{12\pi^2 F^2} + \dots = \mathcal{O}\left(\frac{1}{N_C}\right)$$

    $\Rightarrow$ new negative norm state with mass $m = \mathcal{O}(N_C)$
  - We continue in studying this problem.

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THANK YOU!