Towards a numerical solution of the "1/2 vs. 3/2" puzzle

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[D. Bećirević *et al*, *Phys. Lett.* B **609**, 298 (2005)]

[B. B. *et al*, *Phys. Lett.* B **632**, 319 (2006)]

[I. Bigi *et al*, hep-ph/0512270]

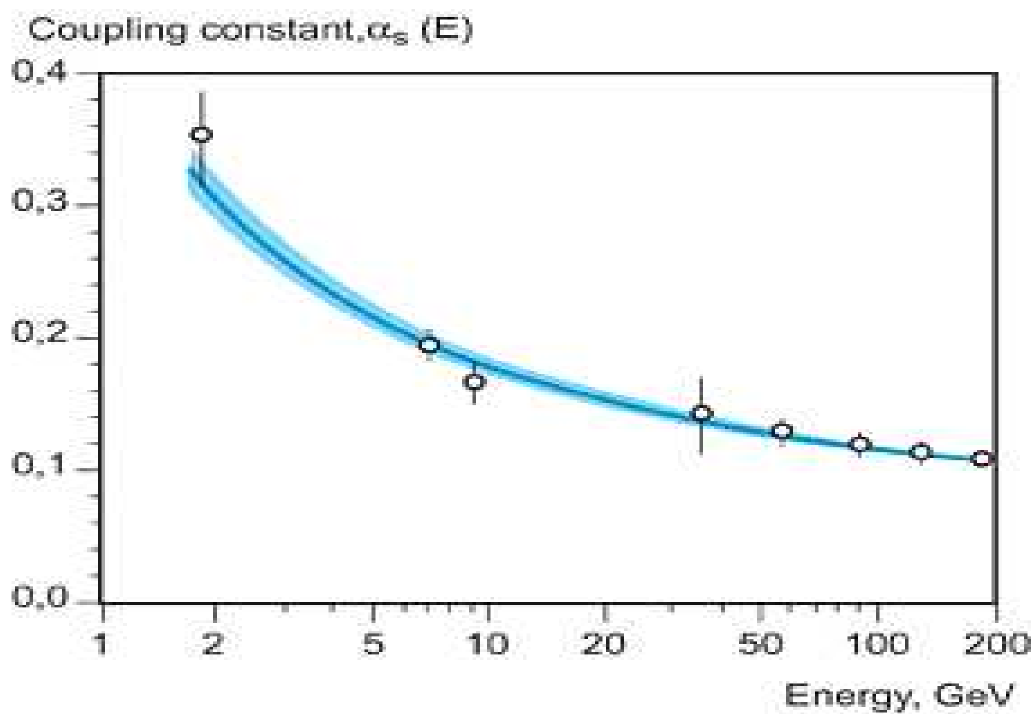
Motivations

Experimental data are well described by the CKM picture in the flavor sector.

Precision tests are made by CDF, D0, B factories and LHCb:

-☺ hope of detecting physics beyond the SM in the flavor sector, especially in transitions between the 2^{nd} and 3^{rd} flavor family

-☹ at present the uncertainty on fundamental parameters is dominated by theoretical errors: one needs a deep understanding of the current theory before using a theory beyond the SM to analyze data



What is the composition of the hadronic final state X_c in $\bar{B} \to X_c l \bar{\nu}$?

		Mass (MeV)	Width (MeV)	J^P
S: $D^{(*)}$	$D^{\pm}$	1869±0.5	-	0^-
	$D^{*\pm}$	2010±0.4	96±25	1^-
P: D^{**}	D_0^*	2352± 50	261 ± 50	0^+
	D_1^*	2427± 26 ± 25	$384^{+107}_{-75} \pm 74$	1^+
	D_1	2421.8 ± 1.3	$20.8^{+3.3}_{-2.8}$	1^+
	D_2^*	2461.1± 1.6	32± 4	2^+

$D^{**} \to D^{(*)}\pi$ is the main decay channel: parity and orbital momentum conservation $\Longrightarrow$ the decay occurs with the pion in a S wave or in a D wave

$D^*_{0,1} \to D^{(*)}\pi$: S wave $D_2^* \to D^{(*)}\pi$: D wave
$D_1 \to D^*\pi$: S and D wave are *a priori* allowed; however the S wave is forbidden by HQS

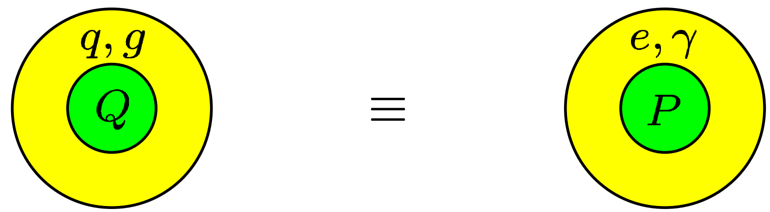
An appropriate framework to study $b \to c$ transitions is the Heavy Quark Effective Theory (HQET): it describes soft interactions between a heavy quark and the light cloud of quarks and gluons within heavy-light hadrons.

Heavy Quark Effective Theory

HQET is an effective theory whose the hard cut-off is m_b.

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(iv\cdot D)h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}^{\rm stat}_{\rm HQET} + \mathcal{O}(\Lambda_{\rm QCD}/m_Q) \qquad p_Q = m_Q\, v + k$$

Symmetry $\mathrm{SU(2N_h)}$ for $\mathcal{L}^{\rm stat}_{\rm HQET}$: flavor $\times$ spin



Heavy-light meson   Atom of hydrogen

Angular momentum: $J = \frac{1}{2} \oplus j_l$.

Spectroscopy: heavy-light mesons are put together in doublets.

$H = B, D$:

j_l^P	J^P	orbital excitation
$\frac{1}{2}^-$	0^- 1^-	H H^*
$\frac{1}{2}^+$	0^+ 1^+	H_0^* H_1^*
$\frac{3}{2}^+$	1^+ 2^+	H_1 H_2^*

$E(j_l^P) = m_Q + \Lambda_{j_l^P} - \frac{\lambda_1(j_l^P) - 2(J^2 - 1/4 - j_l^2)\lambda_2(j_l^P)}{2m_Q}$:

$\Lambda_{j_l^P}$, $\lambda_1(j_l^P)$ and $\lambda_2(j_l^P) \ll m_Q$ are defined in terms of HQET hadronic matrix elements.

$m_{B^*} - m_B \sim 46$ MeV $m_{D^*} - m_D \sim 142$ MeV
$m^2_{B^*} - m^2_B \sim 0.49\ \mathrm{GeV}^2$ $m^2_{D^*} - m^2_D \sim 0.55\ \mathrm{GeV}^2$

- With the *trace formalism*, transitions $H_v^{j_l,J} \to H_{v'}^{j'_l,J'}$ are expressed in terms of universal form factors: the Isgur-Wise functions $\Xi(w \equiv v \cdot v')$.

- Thanks to SU($2\mathrm{N_h}$) symmetry, their number is small:
  $\xi(w)$ parameterizes the elastic transition $H_v^{\frac{1}{2}^-} \to H_{v'}^{\frac{1}{2}^-}$: $\xi(1) = 1$
  $\langle H_v^{0^+} | \bar{h}_{v'} \gamma^\mu \gamma^5 h_v | H_{v'}^{0^-} \rangle \equiv \tau_{\frac{1}{2}}(\mu, w)(v - v')^\mu$
  $\langle H_{v'}^{2^+} | \bar{h}_{v'} \gamma^\mu \gamma^5 h_v | H_v^{0^-} \rangle \equiv \sqrt{3}\, \tau_{\frac{3}{2}}(\mu, w)[(w+1)\epsilon^{*\mu\alpha} v_\alpha - \epsilon^*_{\alpha\beta} v^\alpha v^\beta v'^\mu]$

- $\tau_{\frac{1}{2}}$ and $\tau_{\frac{3}{2}}$ are not normalised at zero recoil; however, $\tau_{\frac{1}{2},\frac{3}{2}}(\mu, 1) \equiv \tau_{\frac{1}{2},\frac{3}{2}}(1)$

Theoretical expectations

- Various quark models (*à la* Bakamijan-Thomas, null plane, harmonic oscillator wave functions in a particular frame and in the $m_Q \to \infty$ limit, models with approximate Lorentz boosts and using wave functions with small components) predict hierarchies

$$\tau_{\frac{1}{2}}(1) < \tau_{\frac{3}{2}}(1) \quad \Gamma(\bar{B} \to D\left(\frac{1}{2}\right) l\bar{\nu}) \ll \Gamma(\bar{B} \to D\left(\frac{3}{2}\right) l\bar{\nu})$$

- $1/m_Q$ contributions to $\bar{B} \to D^{**} l\bar{\nu}$ do not change this picture

- Sum rules obtained by performing the heavy quark expansion of a B meson 2-pts function (Bjorken,Uraltsev, Voloshin, momenta) lead to the same hierarchy:

$$\tau_{\frac{1}{2}}(1) < \tau_{\frac{3}{2}}(1)$$

- Sum rules *à la* SVZ: $\tau_{\frac{1}{2}}(1) = 0.34 \pm 0.09$
  [P. Colangelo *et al*, *Phys. Rev.* D **58**, 116005 (1998)]

Experimental measurements

- ALEPH and DELPHI data indicate a big component of broad states in $\bar{B} \to D^{**} l\bar{\nu}$; the latter lead to the hierarchy

$$\Gamma(\bar{B} \to D\left(\frac{1}{2}\right) l\bar{\nu}) \gg \Gamma(\bar{B} \to D\left(\frac{3}{2}\right) l\bar{\nu})$$

  In contradiction with quark models predictions and with the OPE result: '1/2 vs. 3/2' puzzle [V. Morénas et al, hep-ph/0110372, N. Uraltsev, hep-ph/0409125].

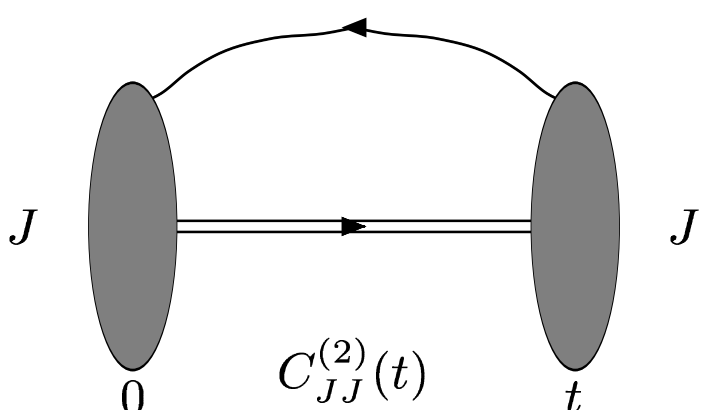
  One could fit the broad $D^{(*)}\pi$ system as radial excitations or non resonant combinations.

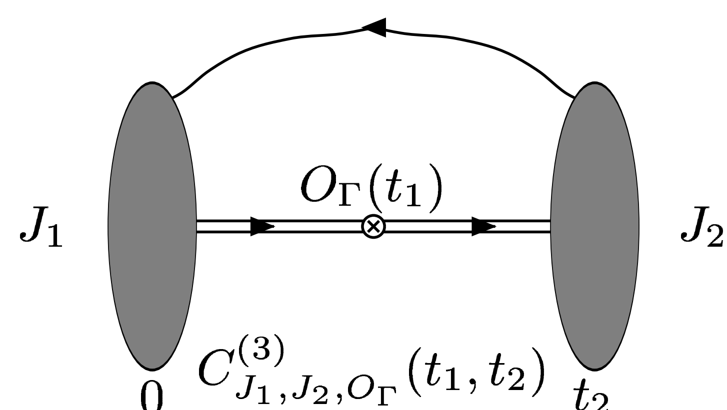
- Measurements by BELLE are qualitatively compatible with quark models predictions ($D^*\pi$ states dominate $D\pi$ in the decay). States $D^{(*)}$ and D^{**} do not seem to saturate $\bar{B} \to X_c l\bar{\nu}$ ($\neq$ DELPHI).
  Theoretically this saturation is not expected as well because of radiative corrections. A recent measurement by BABAR seems to confirm that hypothesis: however, a determination of the resonant component of the invariant mass distribution is hoped.

As $\bar{B} \to D^{**} l\bar{\nu}$ is mainly governed by $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$, their computation on the lattice can confirm (infirm) theoretical expectations of the hierarchy between $\Gamma(\bar{B} \to D\left(\frac{1}{2}\right) l\bar{\nu})$ and $\Gamma(\bar{B} \to D\left(\frac{3}{2}\right) l\bar{\nu})$.

Numerical computation of $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$

- One can not extract the required form factors from the computation of $\lim_{v\to v'}\, _{v'}\langle H^{**}|[\bar{h}_{v'}\Gamma_l h_v](x)|H^{(*)}\rangle_v$ (linear cancellation in $v - v'$)
- However, thanks to HQET equation of motion, one can extract $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$ from the computation of $_v\langle H^{**}|[\bar{h}_v\Gamma_l D^m h_v](x)|H^{(*)}\rangle_v \equiv t_l^m \tau_j(1)\,(M_{H^{**}} - M_H)$
[A. K. Leibovich *et al*, *Phys. Rev.* D **57**, 308 (1998)]
- Extraction of $M_{H_0^*} - M_H$ and $M_{H_2^*} - M_H$ (no $1/a$ divergence) by the computation of $C^{(2)}_{JJ}(t) = \langle 0|O_{0^-}(t)O^\dagger_{0^-}(0)|0\rangle$, $\langle 0|O_{P^\pm}(t)O^\dagger_{P^\pm}(0)|0\rangle$
- Computation of $C^{(3)}_{P^\pm,0^-,O_\Gamma}(t_1,t_2) = \langle 0|O_{P^\pm}(t_2)O_{\Gamma,j_l=1\pm 1/2}(t_1)O^\dagger_{0^-}(0)|0\rangle$ and the ratios $\left(M_{H_J^{j_l=1\pm1/2}} - M_H\right)\sqrt{2j_l}\,\tau_{j_l=1\pm1/2}(1) = \frac{\mathcal{Z}_{0^-}\mathcal{Z}_{P^\pm} C^{(3)}_{P^\pm,0^-,O_\Gamma}(t_1,t_2)}{C^{(2)}_{0^-0^-}(t_1)C^{(2)}_{P^\pm P^\pm}(t_2-t_1)}$
- Renormalisation of the operator $O_{\Gamma,j_l=1\pm1/2}$: $\langle O\rangle^{\text{DR},\overline{\text{MS}}} = Z(a)\langle O\rangle^{\text{lat}}(a)$





$$\langle 2^+, F|2^+, G\rangle = \left\langle \tfrac{1}{3}\sum_i A_i(t,F)A_i^\dagger(0,G) - \tfrac{1}{6}\sum_{i\neq j} A_i(t,F)A_j^\dagger(0,G)\right\rangle$$

$$\langle 0^+, F|0^+, G\rangle = \left\langle \tfrac{1}{3}\sum_i A_i(t,F)A_i^\dagger(0,G) + \tfrac{1}{3}\sum_{i\neq j} A_i(t,F)A_j^\dagger(0,G)\right\rangle$$

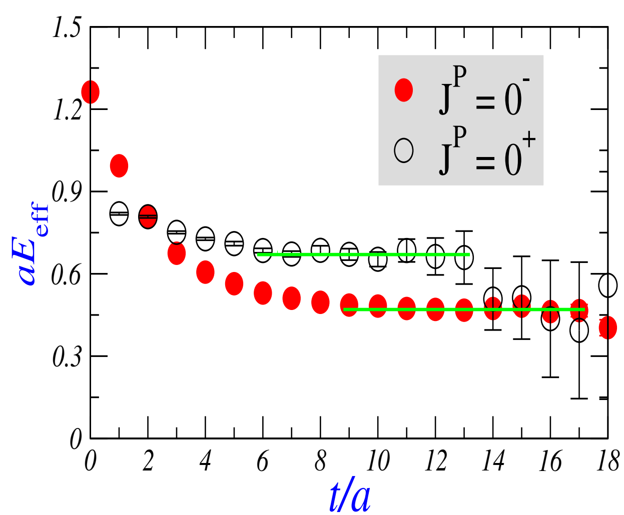
$$\langle 2^+, F|O_{j_l=3/2}|0^-\rangle = \left\langle \tfrac{1}{3}\sum_i A_i(t_2,F)B_i(t_1)C(0) - \tfrac{1}{6}\sum_{i\neq j} A_i(t_2,F)B_j(t_1)C(0)\right\rangle$$

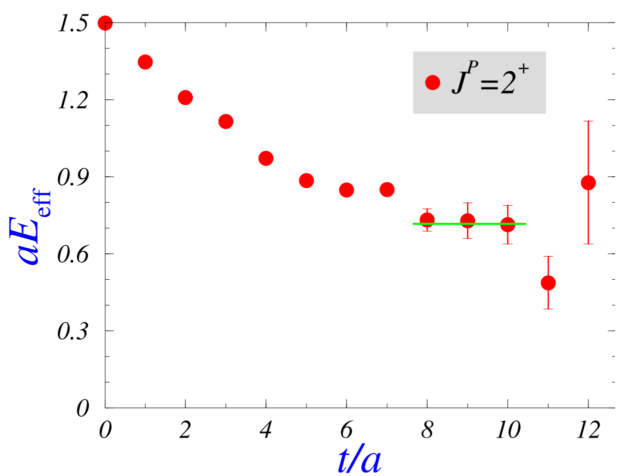
$$\langle 0^+, F|O_{j_l=1/2}|0^-\rangle = \left\langle \tfrac{1}{3}\sum_i A_i(t_2,F)B_i(t_1)C(0) + \tfrac{1}{3}\sum_{i\neq j} A_i(t_2,F)B_j(t_1)C(0)\right\rangle$$

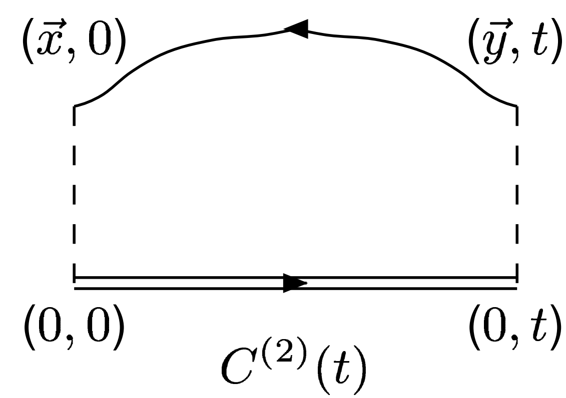
$$A_i(t,O) = [\bar{q}\gamma_i\hat{r}_i h](t) \quad A_i(t,E) = [\bar{q}h](t) \quad B_i(t) = [\bar{h}\gamma_i D_i h](t) \quad C(t) = [\bar{h}\gamma^5 q](0)$$

The mass splitting 0^+-2^+ comes from a spin-orbit coupling:

$$\langle l\cdot s_l\rangle = \tfrac{1}{2}(j_l(j_l+1) - l(l+1) - s_l(s_l+1)) \;=\; 1/2 \quad (j_l = 3/2)$$
$$=\; -1 \quad (j_l = 1/2)$$



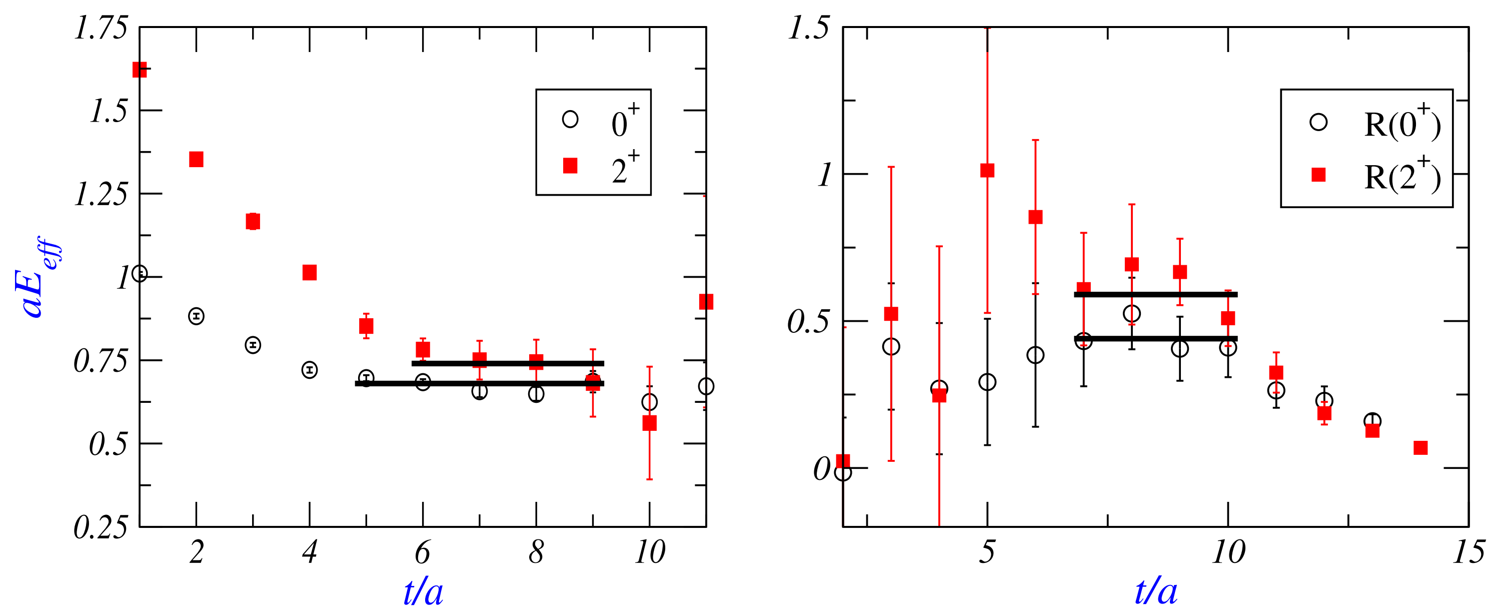




☹ Plateaux of the 2^+ state effective mass and of $\tau_{\frac{1}{2},\frac{3}{2}}(1)$ are very short despite the use of HYP links and smeared interpolating fields. Further improvements are needed.

Reduction of the coupling of radial excitations to the vacuum

- Smearing of the interpolating field O_Γ: $O_{\Gamma,r} = \bar{h}(\vec{0})\Gamma\Phi(r)q(\vec{r})$
- $\psi^n(r) = \langle 0|O_{\Gamma,r}|n\rangle$: $C^{(2)}_{r,r'}(t) = \langle O_{\Gamma,r}(t)|O_{\Gamma,r'}(0)\rangle = \sum_n \psi^n(r)\psi^n(r')e^{-E_n t}$
- $O^{(0)}_{\Gamma,r} = \psi^\perp(r)O_{\Gamma,r} \quad \sum_r \psi^\perp(r)\psi^n(r) = A\delta_{n0}$



Plateaux are still very short.

Preliminary results: $\tau_{\frac{1}{2}}(1) \lesssim \tau_{\frac{3}{2}}(1) \; \tau_{\frac{1}{2}}(1) = 0.44(9) \quad \tau_{\frac{3}{2}}(1) = 0.59(9)$

Conclusion and outlook

- The composition of the final state X_c in $\bar{B} \to X_c l \bar{\nu}$ has received some attention since 10 years.
- Theoretically it is expected that states D, D^* and the 4 P wave states D^{**} do not saturate the total width. Moreover, covariant quark models and sum rules extracted from the OPE in the heavy quark limit lead to the hierarchies
$[\Gamma(\bar{B} \to D\left(\frac{1}{2}\right) l\bar{\nu}) \ll \Gamma(\bar{B} \to D\left(\frac{3}{2}\right) l\bar{\nu})$ and $\tau_{\frac{1}{2}}(1) < \tau_{\frac{3}{2}}(1)]^{\mathrm{TH}}$.
- Experimentally it was found at LEP that the total width is saturated by D, D^*, D^{**} and the measured branching ratios read $[\Gamma(\bar{B} \to D\left(\frac{1}{2}\right) l\bar{\nu}) \gg \Gamma(\bar{B} \to D\left(\frac{3}{2}\right) l\bar{\nu})]^{\mathrm{EXP}}$.
- We have proposed a method to compute $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$ on the lattice. We obtained as a preliminary result $[\tau_{\frac{1}{2}}(1) < \tau_{\frac{3}{2}}(1)]^{\mathrm{LAT}}$. Unfortunately the noise pollutes significantly correlation functions of heavy-light meson orbital excitations and the coupling of radial excitations to the vacuum is important.
- Improvements to reduce them have been implemented but they are not sufficient yet.
- The search of methods to highly increase statistics is underway: one possibility is to compute economically the entire light propagator $S(x, y)$ instead of a part $S(x, 0)$.
- To take account of all $1/m_c$ corrections, we plan to compute form factors associated with $\bar{B} \to D^{(*,**)} l\bar{\nu}$: the c quark is described by QCD and $a \sim 0.05$ fm to reduce $\mathcal{O}(am_c)$ discretisation effects.