

Constraints on Regge models from perturbation theory

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Our aim

- *We will consider the vector-vector correlator*

$$\Pi_V^{\mu\nu}(q) \equiv (q^\mu q^\nu - g^{\mu\nu} q^2) \Pi_V(q) \equiv i \int d^4x e^{iqx} \langle vac | J_V^\mu(x) J_V^\nu(0) | vac \rangle$$

where

$$J_V^\mu = \sum_f Q_f \bar{\psi}_f \gamma^\mu \psi_f$$

- *In particular we will focus on the Adler function*

$$\mathcal{A}(Q^2) \equiv -Q^2 \frac{d}{dQ^2} \Pi_V(Q^2) = Q^2 \int_0^\infty \frac{1}{(Q^2 + t)^2} \frac{1}{\pi} \text{Im} \Pi_V(t)$$

Our aim

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Perturbative calculation

Non-perturbative calculation

- *We want to address the constraints that the knowledge of the perturbative calculation (in particular the perturbative expansion in $\alpha_s(Q^2)$) imposes on the hadronic spectrum.*

Large N_c limit + Regge theory

- *In the large N_c limit the meson spectrum consists of infinitely narrow resonances*

$$A(Q^2) = Q^2 \sum_{n=0}^{\infty} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2}$$

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- *High excitations of the QCD spectrum are believed to follow Regge trajectories*

$$\lim_{n \rightarrow \infty} \frac{M_V^2(n)}{n} = \text{constant}$$

Large N_c limit + Regge theory

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- *At leading order*

$$M_V^2(n) = B_V n$$

This is consistent with perturbation theory in the Euclidean region at leading order in α_s if

$$F_V^2(n) = \text{Const.}$$

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- *Going further...*

$$M_V^2(n) = \sum_{s=-1}^{\infty} B_V^{(-s)} n^{(-s)} = B_V n + A_V + \frac{C_V}{n} + \dots$$

Large N_c limit + Regge theory

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$$A(Q^2) = Q^2 \sum_{n=0}^{\infty} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2}$$

- *Going further...*

$$F_V^2(n) = \sum_{s=0}^{\infty} F_{V,s}^2(n) \frac{1}{n^s} = F_{V,0}^2(n) + \frac{F_{V,1}^2(n)}{n} + \frac{F_{V,2}^2(n)}{n^2} + \dots$$

where

$$F_{V,s}^2(n) = \sum_{r=0}^{\infty} C_{V,s}^{(r)}(n) \frac{1}{\ln^r n}$$

Matching with OPE

$$\mathcal{A}(Q^2) = Q^2 \sum_{n=0}^{\infty} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2}$$

$$\begin{aligned} \mathcal{A}_{OPE}(Q^2) = & \sum_f Q_f^2 \left[\frac{4}{3} \frac{N_c}{16\pi^2} \left(1 + \frac{3}{8} N_c \frac{\alpha_s(Q^2)}{\pi} + \dots \right) \right. \\ & \left. + \frac{C(\alpha_s(Q^2))}{Q^4} \beta(\alpha_s(\nu)) \langle vac | G^2(\nu) | vac \rangle + \mathcal{O} \left(\frac{1}{Q^6} \right) \right] \end{aligned}$$

Matching with OPE

- *Use the Euler-MacLaurin formula*

$$\mathcal{A}(Q^2) = Q^2 \left(\sum_{n=0}^{n^*-1} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} + \sum_{n=n^*}^{\infty} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} \right)$$

n^* arbitrary but formally large, $\Lambda_{QCD} n^* \ll Q$

Matching with OPE

- *Use the Euler-MacLaurin formula*

$$\begin{aligned} \mathcal{A}(Q^2) = & Q^2 \int_0^\infty dn \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} + Q^2 \left[\sum_{n=0}^{n^*-1} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} - \int_0^{n^*} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} \right] \\ & + \frac{Q^2}{2} \frac{F_V^2(n^*)}{(Q^2 + M_V^2(n^*))^2} + Q^2 \sum_{k=1}^{\infty} (-1)^k \frac{|B_{2k}|}{(2k)!} \frac{d^{(2k-1)}}{dn^{(2k-1)}} \frac{F_V^2(n)}{(Q^2 + M_V^2(n))^2} \Bigg|_{n=n^*} \end{aligned}$$

n^ arbitrary but formally large, $\Lambda_{QCD} n^* \ll Q$*

With this expression, we will be able to perform the matching for the logarithmic terms in the OPE

Matching at LO

- *The only term in $\mathcal{A}(Q^2)$ that can produce Logs of Q^2 that are not suppressed by $1/Q^2$ is*

$$Q^2 \int_0^\infty dn \frac{F_{V,0}^2(n)}{(Q^2 + B_V n)^2}$$

- *This must match the leading term in the OPE (the perturbative calculation)*

$$Q^2 \int_0^\infty \frac{1}{(Q^2 + t)^2} \frac{1}{\pi} \text{Im} \Pi_V^{pert.}(t)$$

Matching at LO

- *The condition to be fulfilled is*

$$\frac{F_{V,0}^2(n)}{B_V} = \frac{1}{\pi} \text{Im} \Pi_V^{pert.}(B_V n)$$

Matching at LO

- *The condition to be fulfilled is*

$$\frac{F_{V,LO}^2(n)}{|dM_{V,LO}^2(n)/dn|} = \frac{1}{\pi} \text{Im} \Pi_V^{\text{pert.}}(M_{V,LO}^2(n))$$

Matching at LO

- *The condition to be fulfilled is*

$$\frac{F_{V,LO}^2(n)}{|dM_{V,LO}^2(n)/dn|} = \frac{1}{\pi} \text{Im} \Pi_V^{\text{pert.}}(M_{V,LO}^2(n))$$

- *By using the 3-loop expression of $\text{Im} \Pi_V^{\text{pert.}}$ we obtain*

$$\begin{aligned} F_{V,0}^2(n) = & B_V \frac{4}{3} \frac{N_c}{16\pi^2} \sum_f Q_f^2 \left\{ 1 + \frac{3}{8\pi} N_c \alpha_s(nB_V) + \frac{243 - 176 \zeta(3)}{128\pi^2} N_c^2 \alpha_s^2(nB_V) \right. \\ & \left. + \frac{346201 - 2904\pi^2 - 324528 \zeta(3) + 63360 \zeta(5)}{27648\pi^3} N_c^3 \alpha_s^3(nB_V) + \mathcal{O}(\alpha_s^4(nB_V)) \right\} \end{aligned}$$

Matching at LO

- *The condition to be fulfilled is*

$$\frac{F_{V,LO}^2(n)}{|dM_{V,LO}^2(n)/dn|} = \frac{1}{\pi} \text{Im} \Pi_V^{\text{pert.}}(M_{V,LO}^2(n))$$

$$\begin{aligned} F_{V,0}^2(n) = & B_V \frac{4}{3} \frac{N_c}{16\pi^2} \sum_f Q_f^2 \left\{ 1 + \frac{9}{22} \frac{1}{\ln \tilde{n}} + \frac{1}{\ln^2 \tilde{n}} \left[-\frac{459}{1331} \ln \ln \tilde{n} + \frac{144}{121} \left(\frac{243}{128} - \frac{11}{8} \zeta(3) \right) \right] \right. \\ & + \frac{1}{\ln^3 \tilde{n}} \left[\frac{46818}{161051} \ln^2 \ln \tilde{n} + \frac{459}{322102} (-2877 + 1936 \zeta(3)) \ln \ln \tilde{n} + \frac{42272605}{2576816} \right. \\ & \left. \left. - \frac{3\pi^2}{22} - \frac{20283 \zeta(3)}{1331} + \frac{360 \zeta(5)}{121} \right] + \mathcal{O} \left(\frac{1}{\ln^4 n} \right) \right\} \end{aligned}$$

with $\tilde{n} = n B_V / \Lambda_{\overline{\text{MS}}}$

Matching at NLO

- *We expand the integral from the Euler-Maclaurin expression of $A(Q^2)$ to NLO*

$$Q^2 \int_0^\infty dn \frac{F_{V,0}^2(n) + \frac{F_{V,1}^2(n)}{n}}{(Q^2 + A_V + B_V n)^2} = Q^2 \int_0^\infty dn \frac{F_{V,0}^2(n)}{(Q^2 + B_V n)^2} + Q^2 \int_0^\infty dn \frac{-2A_V F_{V,0}^2(n)}{(Q^2 + B_V n)^3} + Q^2 \int_0^\infty dn \frac{\frac{F_{V,1}^2(n)}{n}}{(Q^2 + B_V n)^2} + \dots$$

Matching at NLO

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- *There is no $1/Q^2$ term in the OPE, so all logarithmic (and finite) terms at this order must vanish*

$$Q^2 \int_0^\infty dn \frac{-2A_V F_{V,0}^2(n)}{(Q^2 + B_V n)^3} + Q^2 \int_0^\infty dn \frac{\frac{F_{V,1}^2(n)}{n}}{(Q^2 + B_V n)^2} = 0$$

Matching at NLO

- *We find*

$$\begin{aligned} \frac{F_{V,1}^2(n)}{n} = & A_V \frac{4}{3} \frac{N_c}{16\pi^2} \sum_f Q_f^2 \frac{1}{n} \left\{ -\frac{11}{32\pi^2} N_c^2 \alpha_s^2(nB_V) - \frac{2877 - 1936 \zeta(3)}{768\pi^3} N_c^3 \alpha_s^3(nB_V) \right. \\ & \left. - \frac{11(376357 - 2904\pi^2 - 344112 \zeta(3) + 63360 \zeta(5))}{110592\pi^4} N_c^4 \alpha_s^4(nB_V) + \mathcal{O}(\alpha_s^5(nB_V)) \right\} \end{aligned}$$

Matching at NNLO

$$\frac{35}{121} \frac{\alpha_s(Q^2)}{4\pi} \frac{\beta(\alpha_s(\nu)) \langle vac | G^2(\nu) | vac \rangle}{Q^4}$$

$$\doteq Q^2 \int_{n^*}^{\infty} \frac{dn}{(Q^2 + B_V n)^2} \left[\frac{F_{V,2}^2(n)}{n^2} - \frac{1}{B_V} \frac{d}{dn} \left(\frac{C_V F_{V,0}^2(n)}{n} + \frac{A_V F_{V,1}^2(n)}{2n} \right) \right]$$

Using

$$\frac{1}{Q^4} \alpha_s(Q^2) \doteq Q^2 \int_{n^*}^{\infty} \frac{dn}{(Q^2 + B_V n)^2} \frac{1}{B_V n^2} \frac{\beta_0}{8\pi} \alpha_s^2(n B_V) ,$$

we find

Matching at NNLO

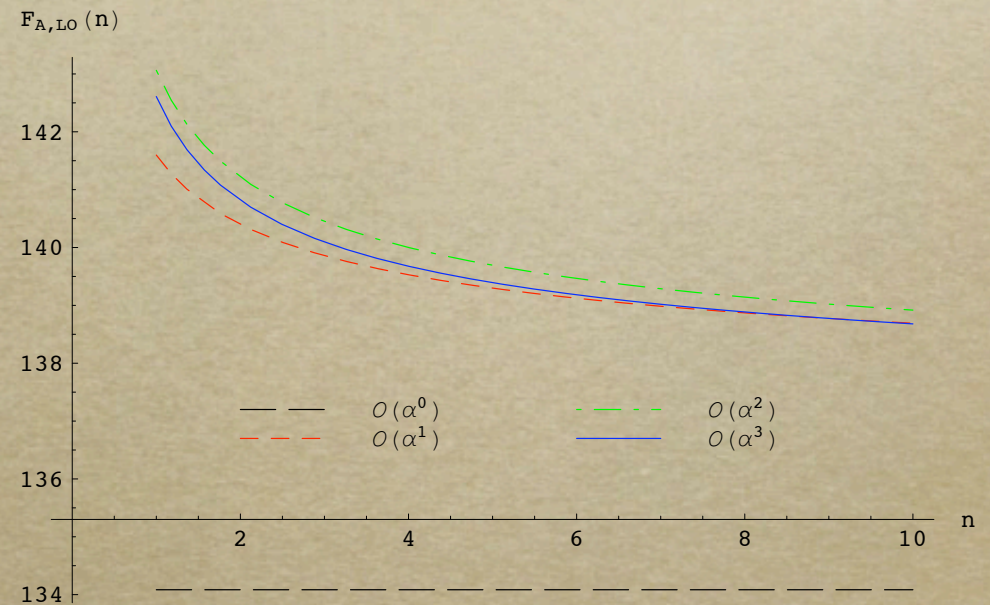
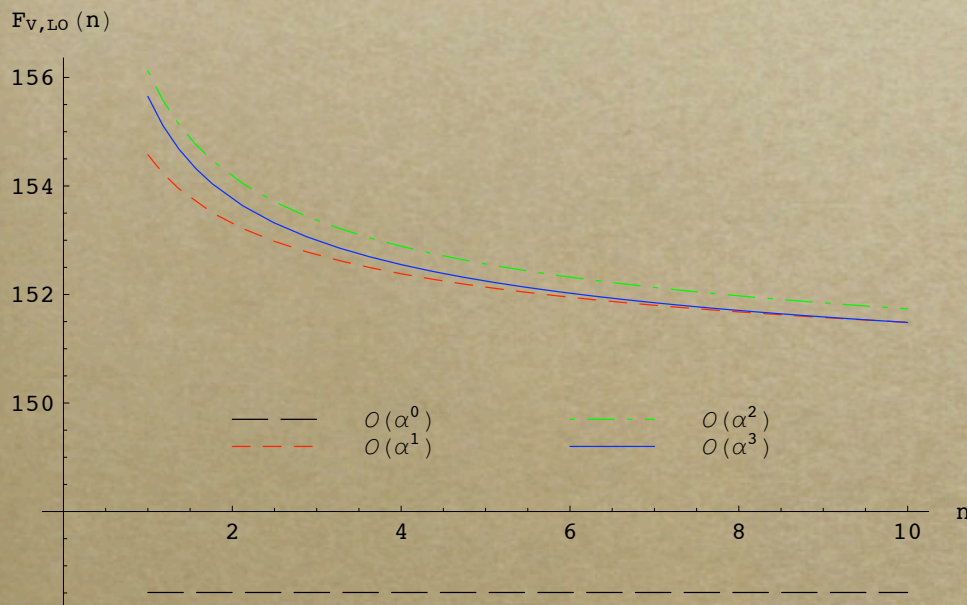
$$F_{V,2}^2(n) = -C_V \frac{4}{3} \frac{N_c}{16\pi^2} \sum_f Q_f^2 \left\{ 1 + \frac{3}{8\pi} N_c \alpha_s(nB_V) \right. \\ \left. + \left[\frac{287 - 176 \zeta(3)}{128\pi^2} - \frac{11A_V^2}{64\pi^2 B_V C_V} - \frac{35}{88} \frac{\beta(\alpha_s(\nu)) \langle vac | G^2(\nu) | vac \rangle}{B_V C_V N_c^2} \right] N_c^2 \alpha_s^2(nB_V) \right. \\ \left. + \mathcal{O}(\alpha_s^3(nB_V)) \right\}$$

Some numerics...

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
M_ρ	781.3 (775.5 $\pm$ 0.4)	1440.2 (1459 $\pm$ 11)	1891.8 (1870 $\pm$ 20)	2257 (2265 $\pm$ 40)
M_{a1}	1235.6 (1230 $\pm$ 40)	1621.7 (1647 $\pm$ 22)	1962 (1930 $^{+30}_{-70}$)	2257.8 (2270 $^{+55}_{-40}$)
F_V	152 (156 $\pm$ 1)	153	153	152
F_A	121 (122 $\pm$ 24)	136	138	138

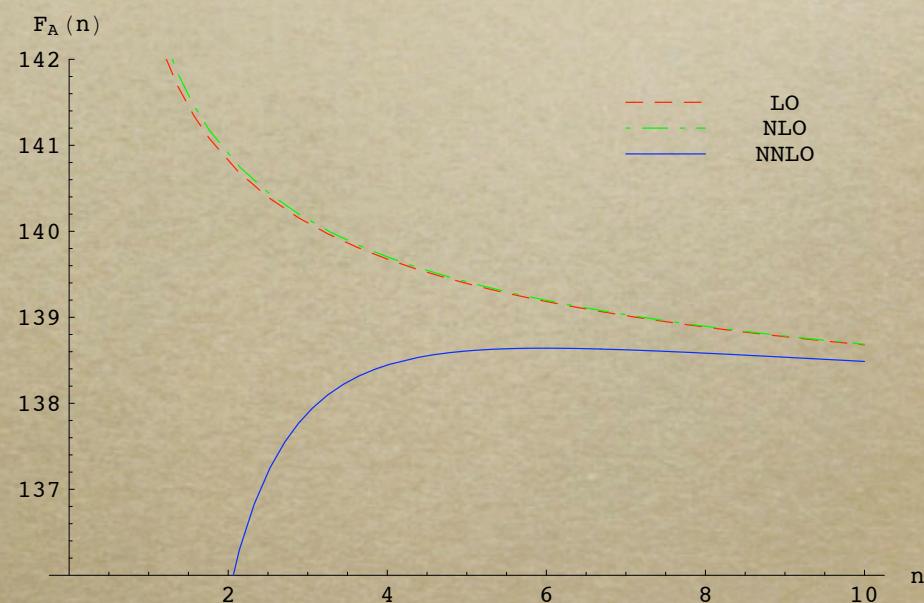
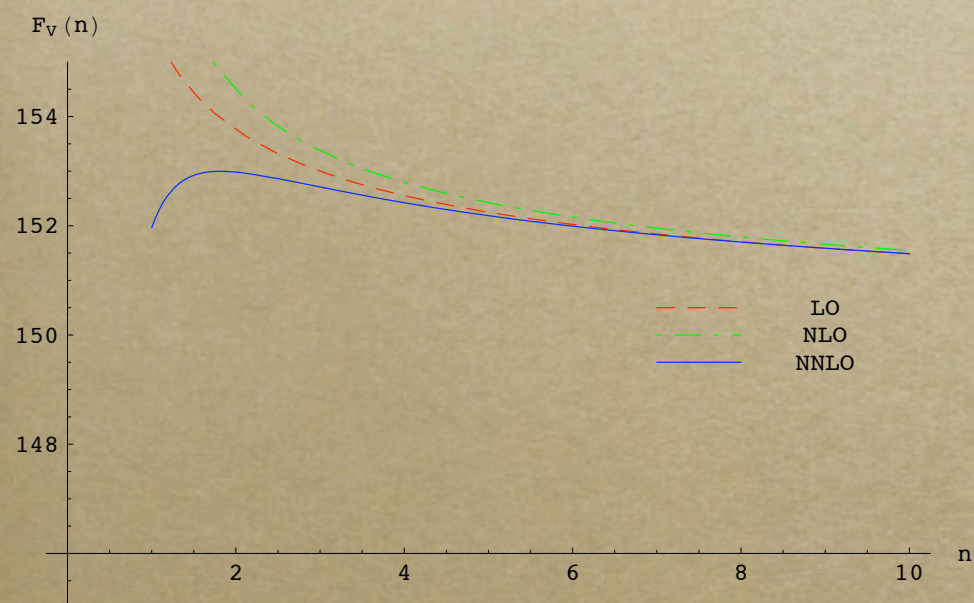
taking $\alpha_s(1\text{GeV}) = 0.33$ and $\beta\langle G^2 \rangle = -(352\text{MeV})^4$

Some numerics...



$$F_{X,0}^2(n) = \sum_{r=0}^{\infty} C_{X,0}^{(r)}(n) \frac{1}{\ln^r n}$$

Some numerics...



$$F_{V,LO}^2(n) = F_{V,0}^2(n)$$

$$F_{V,NLO}^2(n) = F_{V,0}^2(n) + \frac{F_{V,1}^2(n)}{n}$$

$$F_{V,NNLO}^2(n) = F_{V,0}^2(n) + \frac{F_{V,1}^2(n)}{n} + \frac{F_{V,2}^2(n)}{n^2}$$

Conclusions

- *We have studied the constraints that perturbation theory imposes on a model for the meson spectrum inspired on the large N_c limit plus Regge theory, focusing on current-current correlators.*
- *Introducing $1/n$ corrections to the mass spectrum and the decay constants, we can match the hadronic calculation with the perturbative calculation at 3-loop order in α_s .*
- *A numerical analysis shows that the corrections are small, but significant (more so in the case of the axial-vector mesons). Although numerically unstable and with considerable uncertainties, our results compare favorably with existing experimental data.*
- *We can have a different large n behavior for the vector and the axial-vector mesons complying with the OPE.*