Towards a holographic description of QCD

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Introduction

QCD: a difficult theory to deal with

Two regimes of QCD and tools for theoretical analysis

Perturbative

• Perturbation theory

Non-Perturbative

Lattice

QCD sum rules

• . . .

Old dream: "QCD: a solvable theory"



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Holographic QCD \rightarrow new way of approaching QCD \rightarrow new powerful tool

What is holography?

How to construct a holographic description of QCD?

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What is holography? How to construct a holographic description of QCD?

AdS/CFT Link to QCD

AdS/CFT correspondence

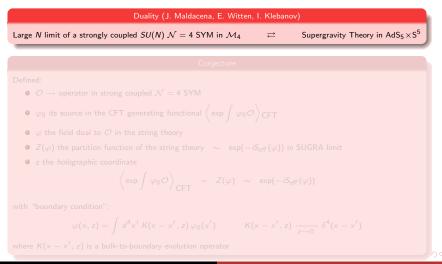
• AdS₅ symmetry group isomorphic to four dimensional conformal group

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• AdS₅ symmetry group isomorphic to four dimensional conformal group

Duality (J. Maldacena, E. Witten, I. Klebanov)
arge N limit of a strongly coupled $SU(N)$ $\mathcal{N} = 4$ SYM in \mathcal{M}_4 \rightleftharpoons Supergravity Theory in $AdS_5 \times S^5$
Contraction
Conjecture
Defined:
• $\mathcal{O} \rightarrow$ operator in strong coupled $\mathcal{N} = 4$ SYM
• φ_0 its source in the CFT generating functional $\left\langle \exp \int \varphi_0 \mathcal{O} \right\rangle_{\sf CFT}$
• $arphi$ the field dual to ${\cal O}$ in the string theory
• $Z(\varphi)$ the partition function of the string theory $\sim \exp(-iS_{eff}(\varphi))$ in SUGRA limit
• z the holographic coordinate
$\left\langle \exp \int arphi_0 \mathcal{O} ight angle_{CFT} = Z(arphi) ~\sim~ \exp(-iS_{eff}(arphi))$
vith "boundary condition":
$\varphi(x,z) = \int d^4x' \mathcal{K}(x-x',z) \varphi_0(x') \qquad \qquad \mathcal{K}(x-x',z) \xrightarrow[z \to 0]{} \delta^4(x-x')$
where $K(x - x', z)$ is a bulk-to-boundary evolution operator

AdS/CFT Link to QCD

Link to QCD

Problems with QCD

- $\mathcal{N} = 4$ SYM is a conformal field theory while QCD is not
- $\mathcal{N} = 4$ SYM is a supersymmetric theory while QCD is not
- $\mathcal{N} = 4$ SYM has no S-matrix while QCD has

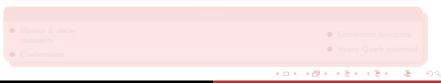
Proposals

QCD "nearly conformal" when all m = 0 and when α_s does not run Supposing the existence of a "gravity dual" of QCD:

Top-to-bottom approach

- Starting by string theory
- Trying to reproduce QCD

- Bottom-to-top approach
 - Starting by QCD
 - Trying to construct the dual theory



AdS/CFT Link to QCD

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Stefano Nicotri Towards a holographic description of QCD

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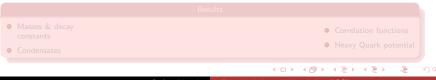
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Results		
constants Powerful too		Correlation functionsHeavy Quark potential
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Hard wall Soft wall - Dilator

Bottom-to-top approach: hard wall model

 $\label{eq:starsest} \begin{array}{l} \mbox{Framework (J. Polchinski, M. Strassler)} \end{array}$ Bulk = AdS5 cut at $z=z_m\sim 1/\Lambda_{QCD}$ \Leftrightarrow $ds^2=rac{1}{z^2}(\eta_{\mu\nu}dx^{\mu}dx^{\nu}+dz^2)$ $0\leqslant z\leqslant z_m$

QCD lives on the boundary z = 0

How to construct the action

• QCD operator $\mathcal{O}_{\mu\nu\dots\beta}$ of order p and dim $\Delta \leftrightarrow$ free field in the bulk $B_{\mu\nu\dots\beta}$

•
$$m_5^2 = (\Delta - p)(\Delta + p - 4)$$

- Global symmetry in the boundary \leftrightarrow local (gauge) symmetry in the bulk

Example: Scalar operator ${\cal O}$.

Action & EOM:

$$S = -\frac{1}{2k} \int d^5 x \sqrt{|g|} \left(g^{MN} \partial_M X \partial_N X + m_5^2 X^2 \right)$$

$$\partial_M \left[\sqrt{|g|} g^{MN} \partial_N X(x, z) \right] - m_5^2 X(x, z) = 0$$

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- QFT in the boundary ↔ classical theory in the bulk

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Bottom-to-top approach: soft wall - dilaton model

Framework (A. Karch, E. Katz, D. Son, M. Stephanov)

Bulk = asymptotically AdS₅ with a background "dilaton" field $\phi(z)$

 $ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dz^2)$

$$S = -\frac{1}{2k} \int d^5 x \sqrt{|g|} e^{-\phi(z)} (g^{MN} \partial_M X \partial_N X + m_5^2 X^2)$$

$$\partial_{M}\left[\sqrt{|g|} e^{-\phi(z)}g^{MN} \partial_{N}X(x,z)\right] - m_{5}^{2}X(x,z) = 0$$

With:

- $\phi A \xrightarrow[z \to \infty]{z \to \infty} c^2 z^2$ • $\phi - A \xrightarrow[z \to 0]{z \to 0} - \ln z$
- A(z) cannot contain powers z^{β} with $\beta \ge 2$

This model reproduces Regge behaviour of vector-mesons: $m_{\pi}^2 \sim m_{\pi}^2$

Simplest choice: • $\phi = c^2 z^2$

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Masses

Example of a calculation: glueball masses

	0 ⁺ glueball
 QCD Described by tr(G²) = Tr(G_{μν} G^{μν}) Dimension Δ = 4 Scalar (p = 0) 	 AdS₅ Described by a scalar field X(x, z) Mass m₅² = (Δ - p)(Δ + p - 4) = 0

Massless free scalar field

Action & EOM

$$\begin{split} S &= -\frac{1}{2k} \int d^5 x \sqrt{|g|} \, e^{-\phi(z)} g^{MN} \, \partial_M X \, \partial_N X \\ \partial_M \left[\sqrt{|g|} \, e^{-\phi(z)} g^{MN} \, \partial_N X(x,z) \right] &= 0 \end{split}$$

How to compute masses?

- 4-dim Fourier transformation of the field: $X(x,z) = \int d^4x \, e^{iq \cdot x} \widetilde{X}(q,z)$
- Fields on shell $\rightarrow q^2 = -m^2$

Eigenvalues m_p^2 of the EOM \rightarrow 0⁺ glueball spectrum

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 $m_n^2 = 4c^2(n+2)$ $m_{0+}^2 = 2m_\rho^2$

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 QCD Described by Tr(G(DG)G) Dimension Δ = 7 Vector (p = 1) 	 AdS₅ Described by a vector field V_M(x, z) Mass m²₅ = (Δ - p)(Δ + p - 4) = 24

Massive free vector field

Action & EOM:

$$S = -\frac{1}{2k} \int d^5 x \sqrt{|\mathbf{g}|} e^{-\phi(z)} \left(\frac{1}{2} g^{MN} g^{ST} F_{MS} F_{NT} + 24 g^{ST} V_S V_T\right)$$

$$\partial_M \left| \sqrt{|g|} e^{-\phi(z)} g^{MN} g^{ST} F_{MS}(x, z) \right| - 24 \sqrt{|g|} e^{-\phi(z)} g^{ST} V_S(x, z) = 0$$

where:

$$F_{MN} = \partial_M V_N - \partial_M V_N$$



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