

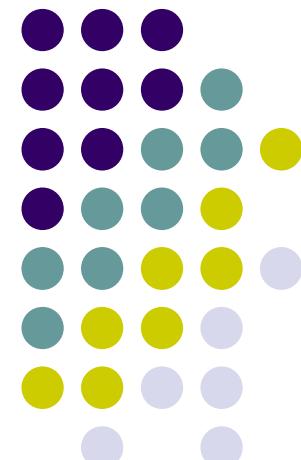
QCD condensates for the light quark V-A correlator

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SUMMARY

- QCD condensates: a few words;
- V-A correlator;
- QCD Sum Rules;
- Previous results and motivation;
- Laplace Sum Rules;
- Finite Energy Sum Rules;
- Conclusion;



QCD CONDENSATES

- What are they?
→ Quantities that parametrize the non-perturbative character of QCD.



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- What are they?
→ Quantities that parametrize the non-perturbative character of QCD.
- Why are they interesting?
→ They describe the QCD vacuum;
→ We need to know them to make accurate predictions;
- How do we extract them?
→ QCD: we don't know;
→ Some attempts with Lattice, Instantons Models, ...;
→ The QCD SUM RULES represent still the most reliable method;



V-A correlator

$$\Pi_{V-A}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T(J_L^\mu(x) J_R^\nu(0)^\dagger) | 0 \rangle = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi(q^2)$$

$$J_L^\mu = \bar{u} \gamma^\mu (1 - \gamma_5) d$$

$$J_R^\mu = \bar{u} \gamma^\mu (1 + \gamma_5) d$$

V+A

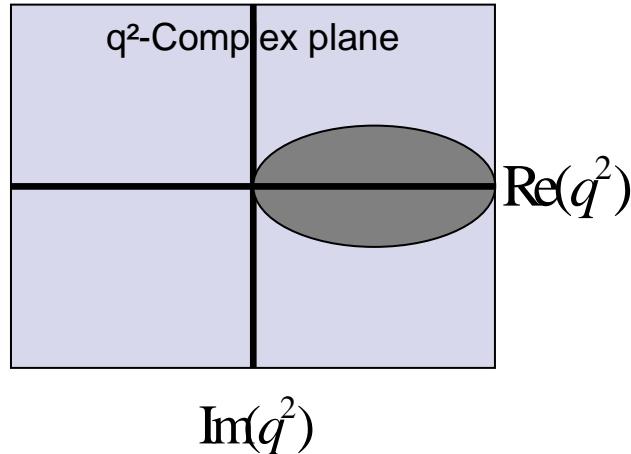


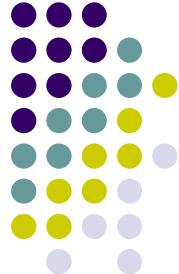
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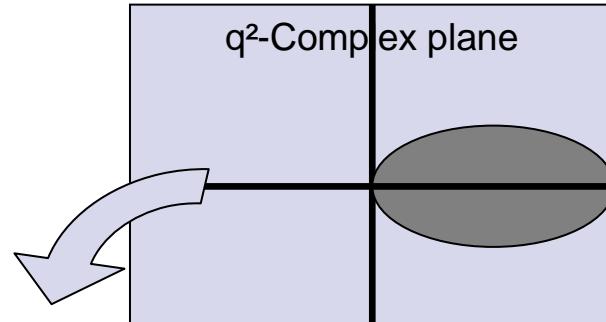


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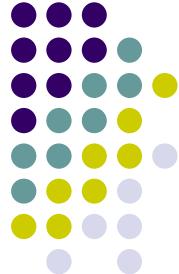
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Here we can calculate using the OPE
(Wilson, 1967):

$$\Pi_{V-A}(q^2) = \Pi_{V-A}^{pert}(q^2) + \frac{\mathcal{O}_4}{(-q^2)^2} + \frac{\mathcal{O}_6}{(-q^2)^3} + \frac{\mathcal{O}_8}{(-q^2)^4} + \dots$$

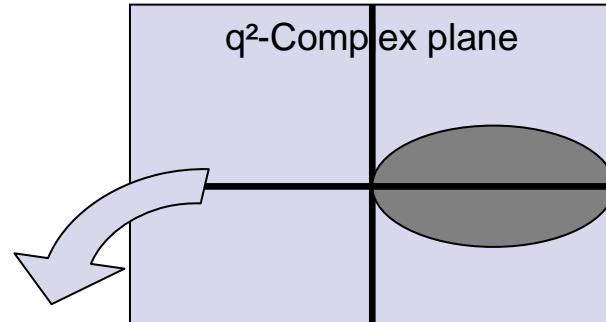


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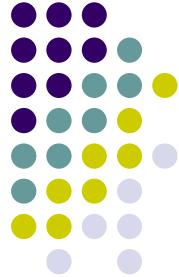
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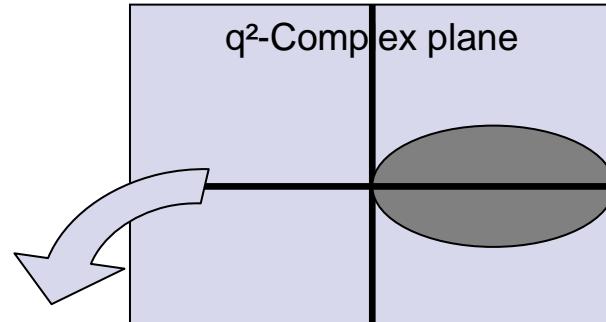


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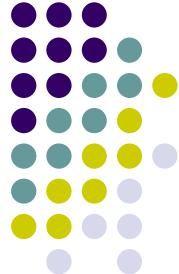
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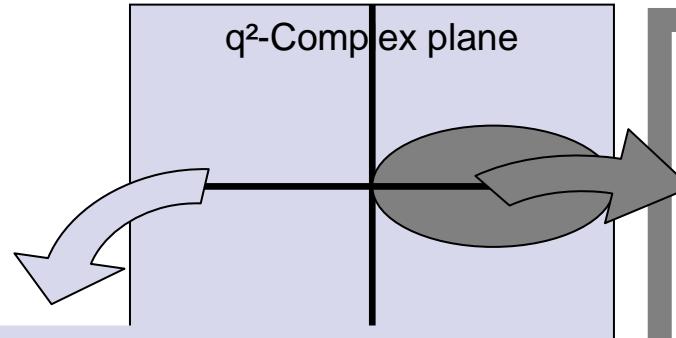


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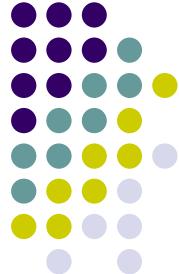


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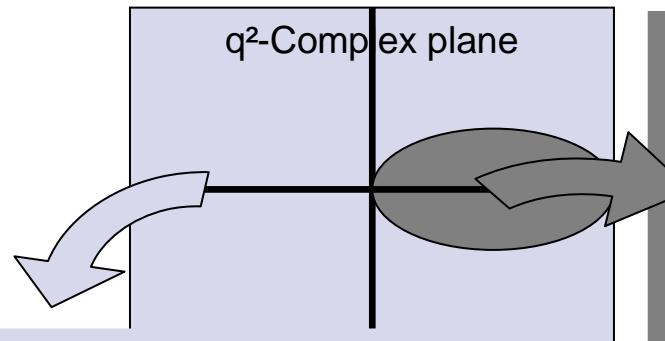


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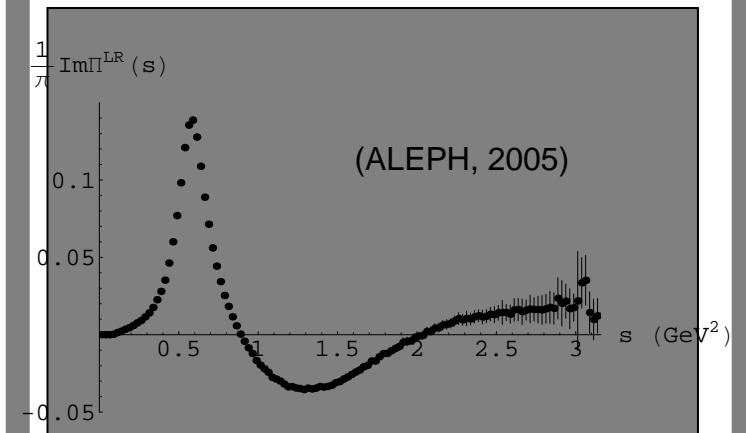


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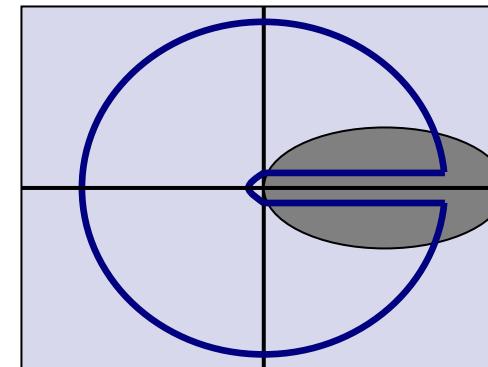
QCD Sum Rules.

They exploit the analytical properties of the correlator and establish a connection between the OPE side (QCD, i.e. quarks and gluons) and the experimental side (hadrons).

Let's apply Cauchy's Theorem to the function:

$w(z)\Pi_{V-A}(z)$ in the contour of the figure.

Weight function





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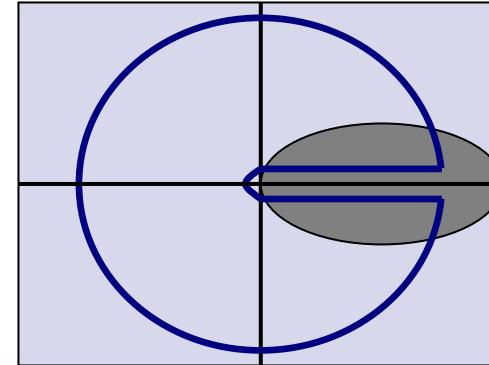
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$$\frac{1}{2\pi i} \int_0^R ds \frac{1}{\pi} \text{Im} \Pi_{V-A}(s) w(s) = -\frac{1}{2\pi i} \oint_{|z|=R} dz \Pi_{V-A}(z) w(z) \approx -\frac{1}{2\pi i} \oint_{|z|=R} dz \Pi_{V-A}^{OPE}(z) w(z)$$



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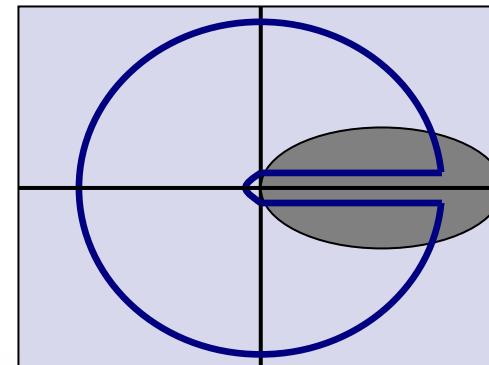
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Hadronic side
(Experimentally accessible)

← →
FIT

QCD side (involves the condensates)



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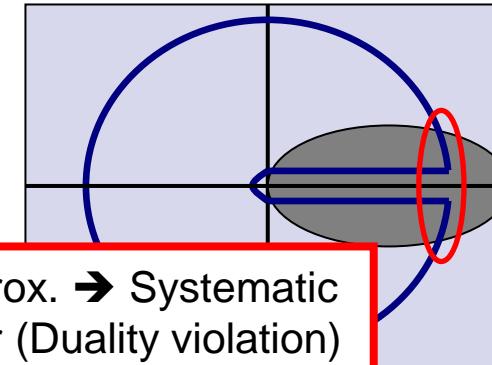
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$$w(z)\Pi_{V-A}(z) \quad \text{in the complex } s\text{-plane}$$

Two kind of errors:

1. Experimental error;
2. No data for $s > 3.15 \text{ GeV}^2$.

Therefore...



Approx. \rightarrow Systematic error (Duality violation)

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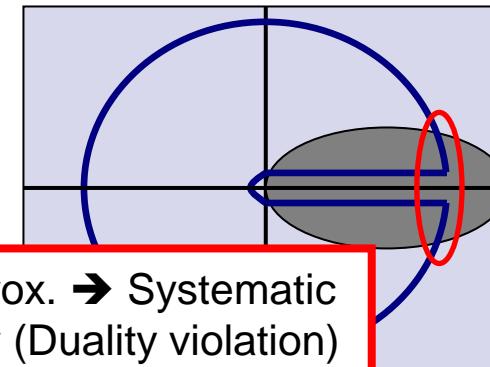
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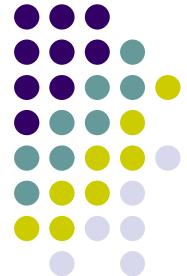
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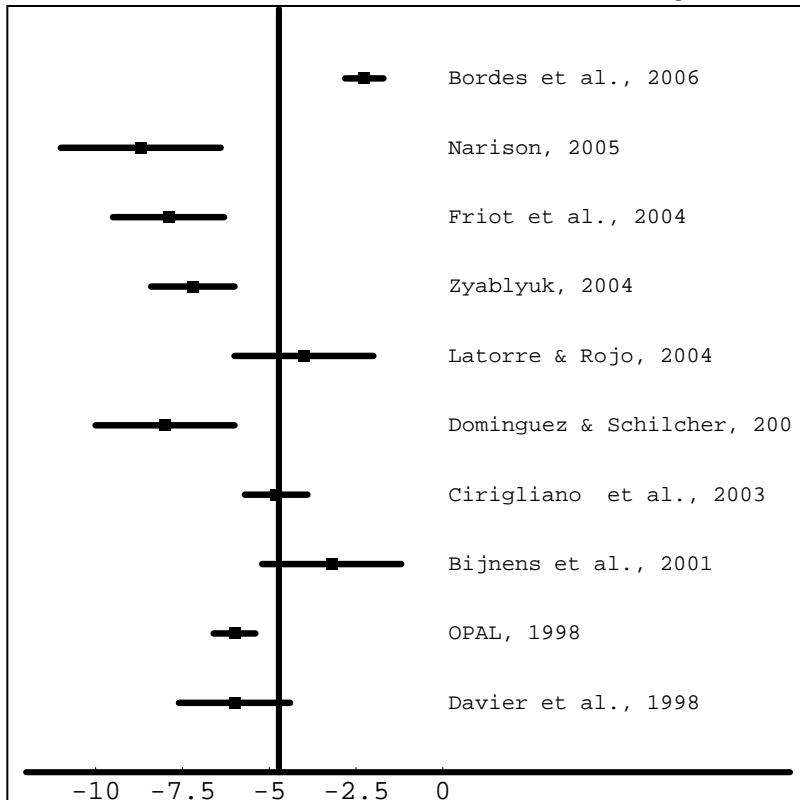
In order to minimize the errors there are several different strategies, based mainly on the choice of the weight function and the radius R .

LET'S SHOW THE DIFFERENT RESULTS OBTAINED...



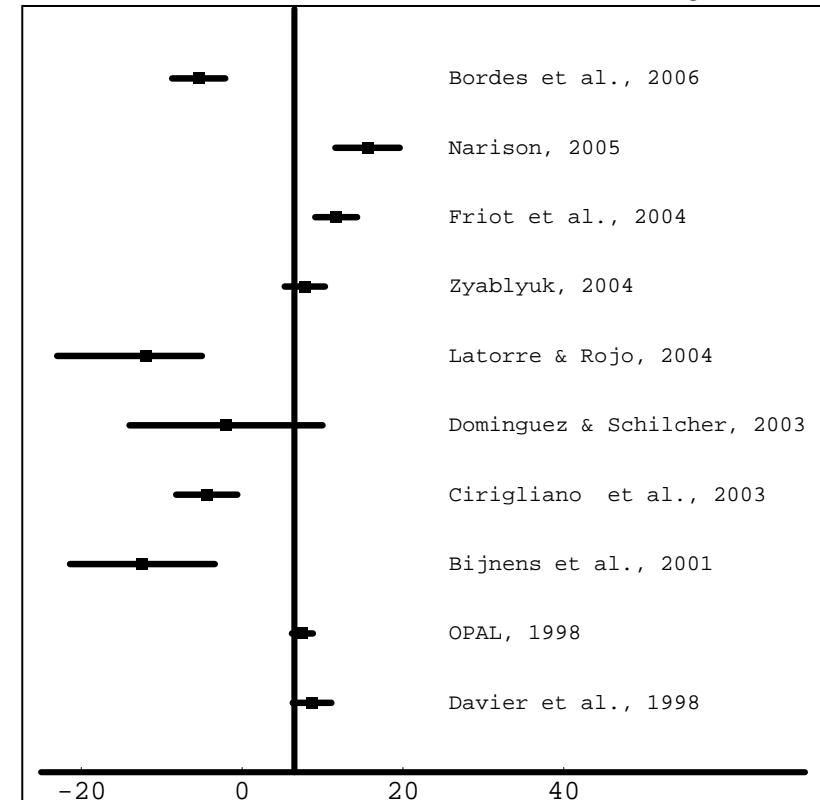
Previous Results and Motivation.

Different estimations of O_6



$O_6 (10^{-3} \text{ GeV}^6)$

Different estimations of O_8

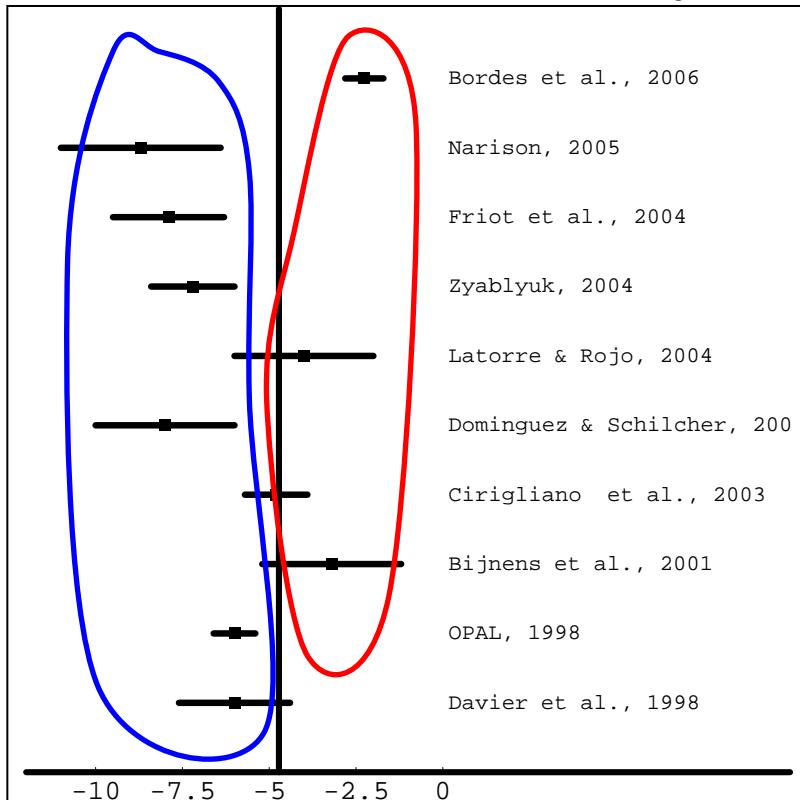


$O_8 (10^{-3} \text{ GeV}^8)$

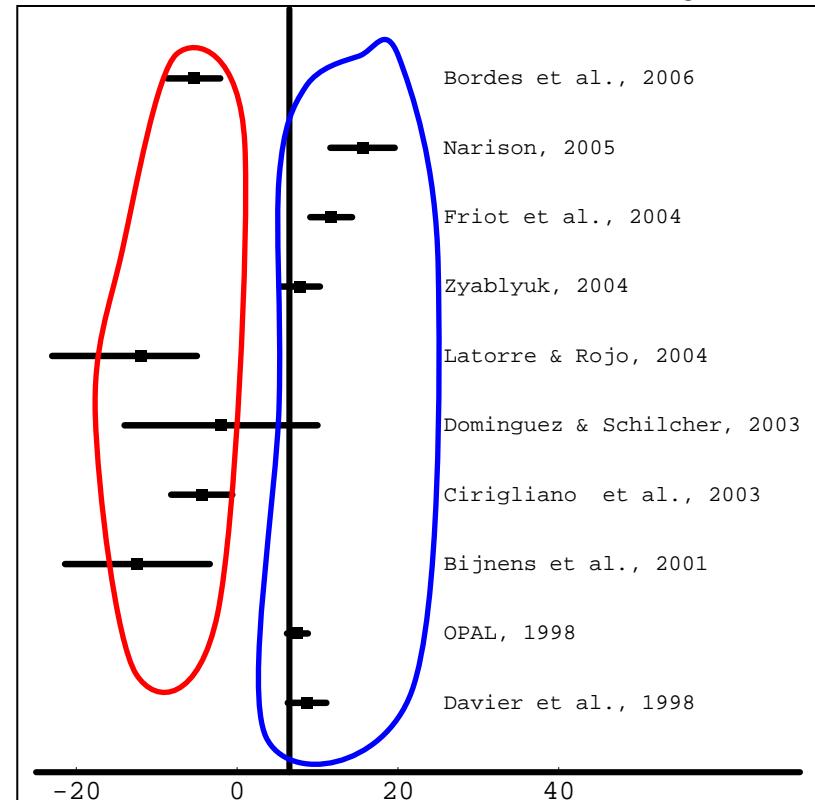


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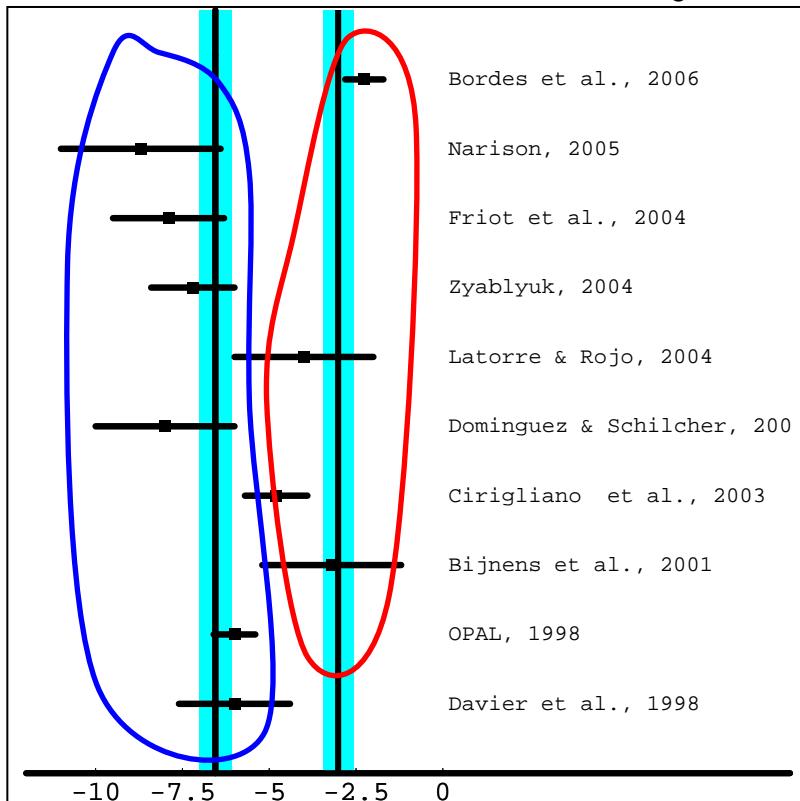
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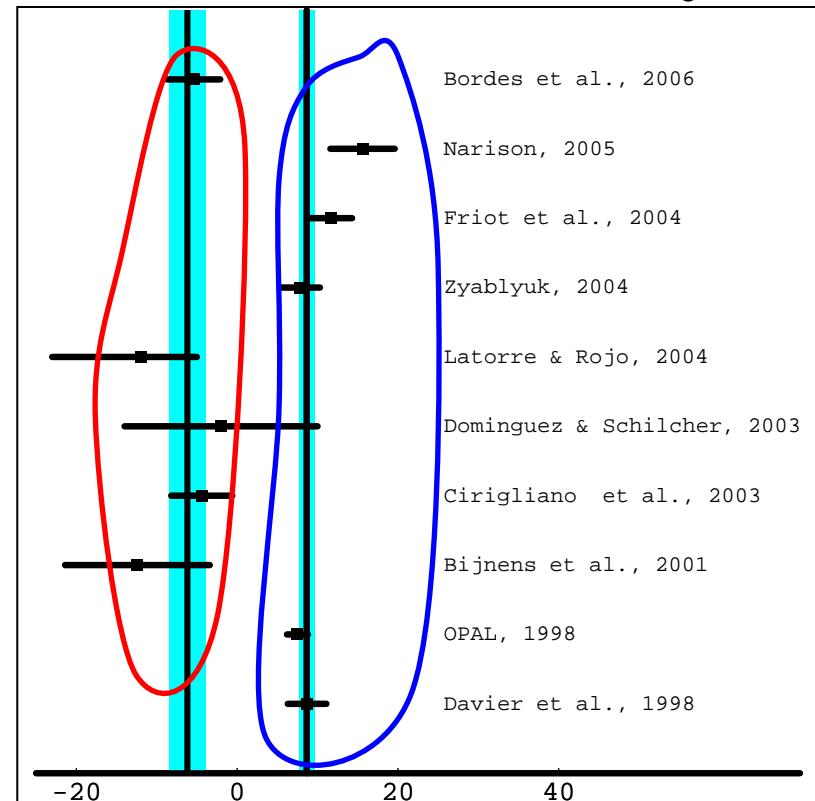


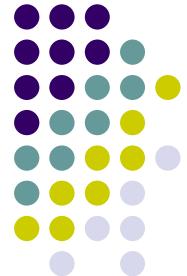
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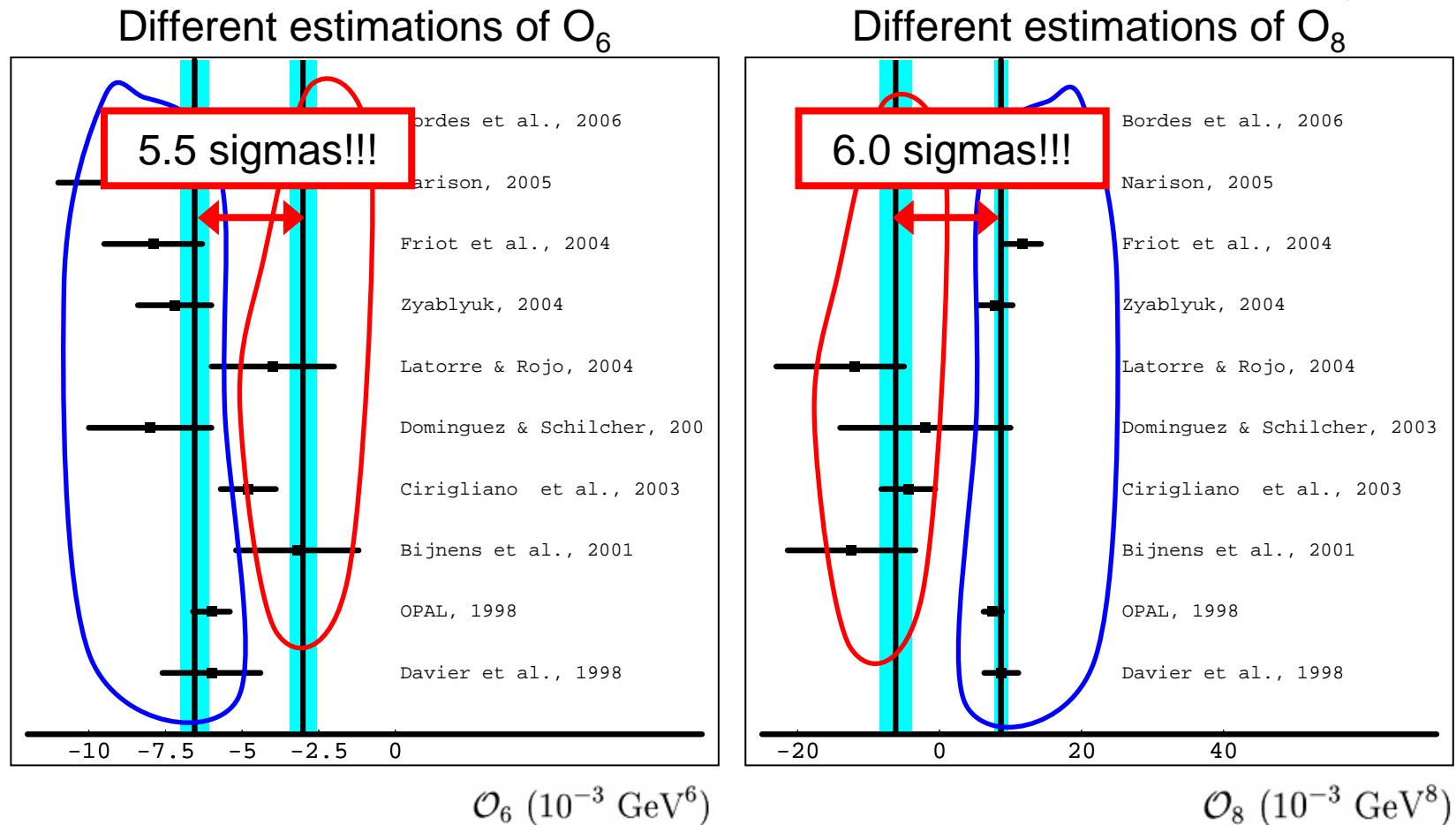


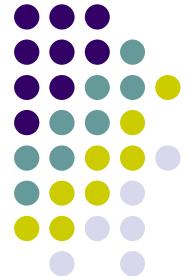
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Previous Results and Motivation.





A deeper look: 1. Laplace Sum Rules.

$$w_i = e^{-s \cdot \tau} s^i$$

We take the weights:

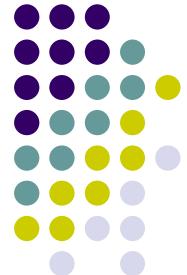
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EXPONENTIAL WEIGHTS



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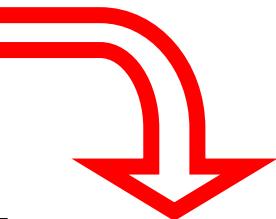
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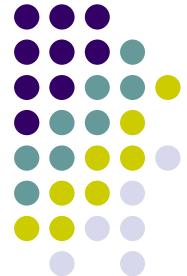
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Laplace Sum Rules

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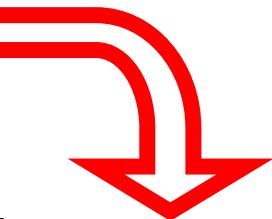
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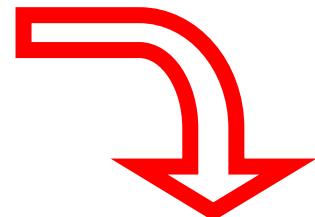
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Once we neglect condensates with $\text{dim} > 2N+2 \dots$

We can obtain the condensates iteratively.



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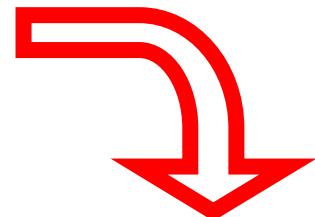
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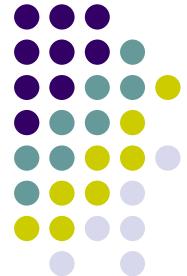
Once we neglect condensates with $\text{dim} > 2N+2\dots$

We can obtain the condensates iteratively.

What value of N?

(Narison, 2005) → N=8

therefore they calculate
 O_4, O_6, \dots, O_{18} .



A deeper look: 1.Laplace Sum Rules.

$$w_i = e^{-s \cdot \tau} s^i$$

So, this is the process (let's take **N=8**)...

- Calculate the Laplace transform of order 8,
and use the first LSR to extract \mathcal{O}_{18} .

$$\mathcal{L}_8(\tau) = \mathcal{O}_{18}$$

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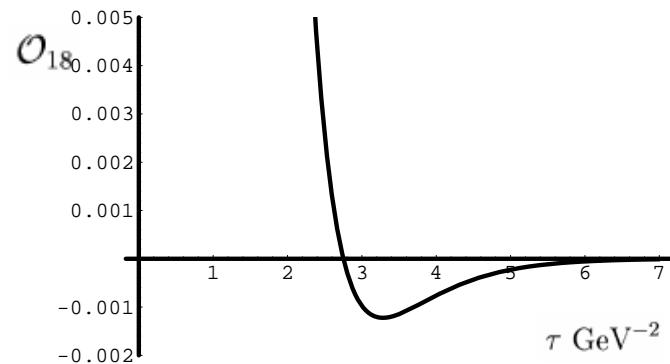
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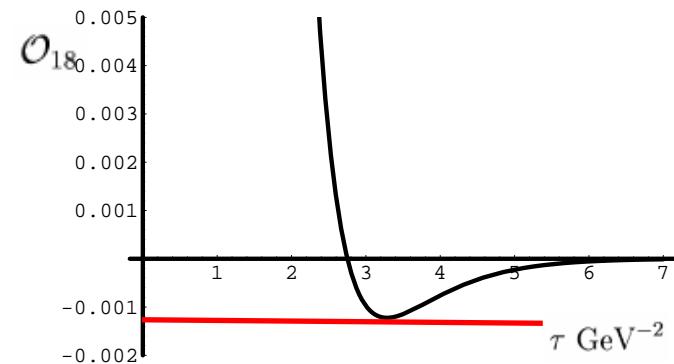
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- Calculate the Laplace transform of order 8, and use the first LSR to extract O_{18} .

$$\mathcal{L}_8(\tau) = O_{18} \rightarrow O_{18} = -(1 \pm 0,9) 10^{-3} \text{ GeV}^{18}$$



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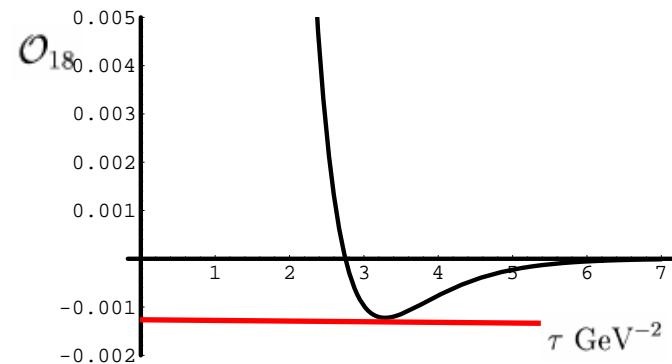
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- Calculate the Laplace transform of order 7, and use the second LSR to extract O_{16} .

$$\mathcal{L}_7(\tau) = -O_{16} - \tau O_{18}$$



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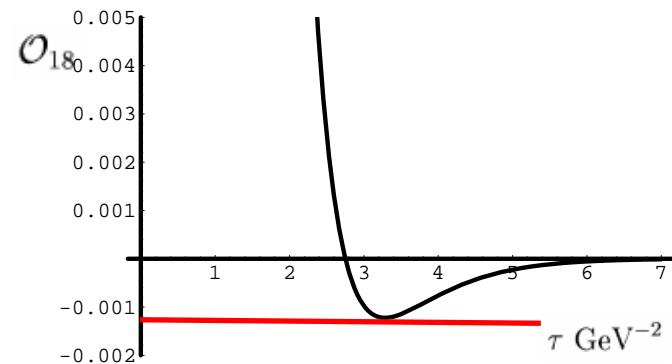
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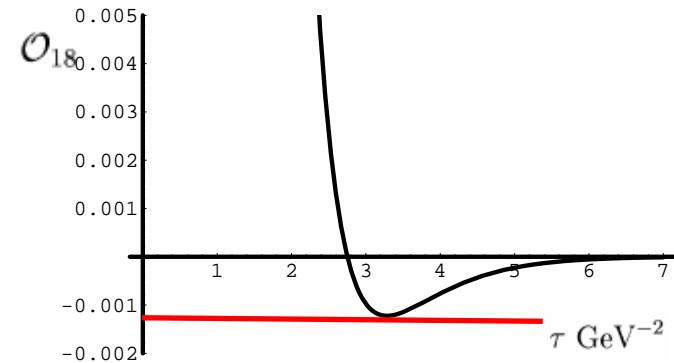
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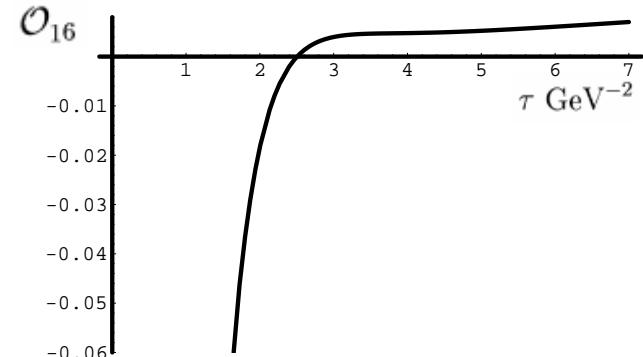
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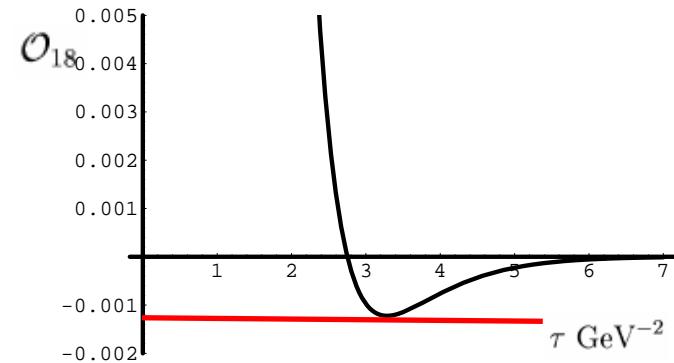
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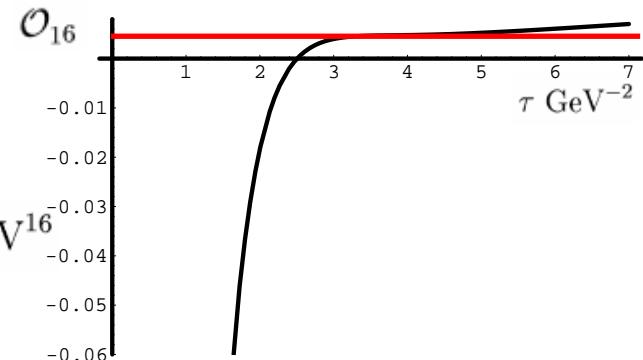
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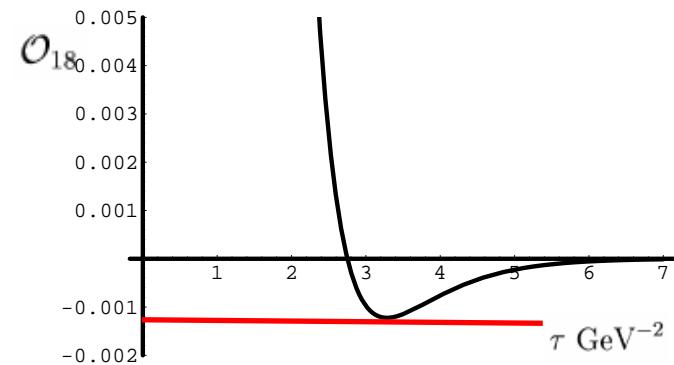
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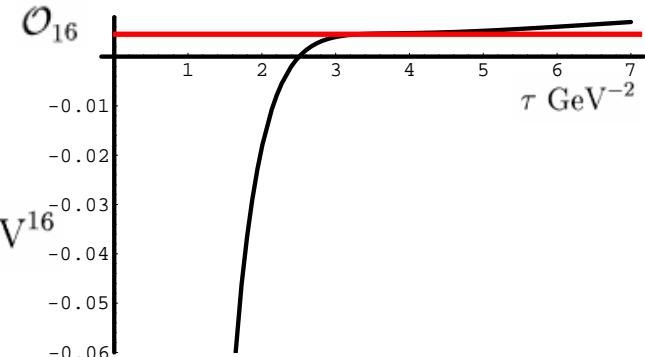


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- ...



A deeper look: 1.Laplace Sum Rules.

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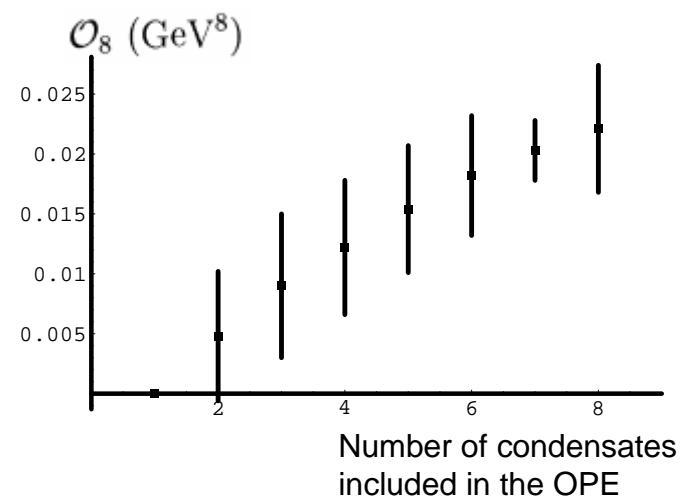
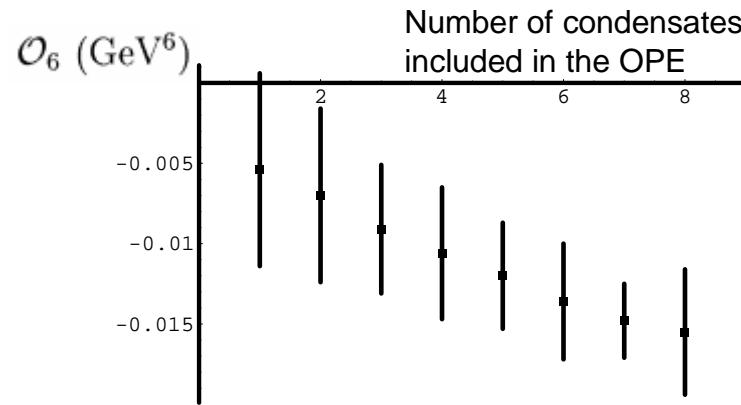
But we wondered... What happens if we take a different N?

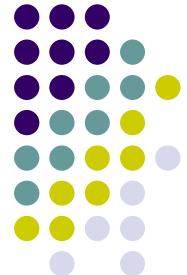
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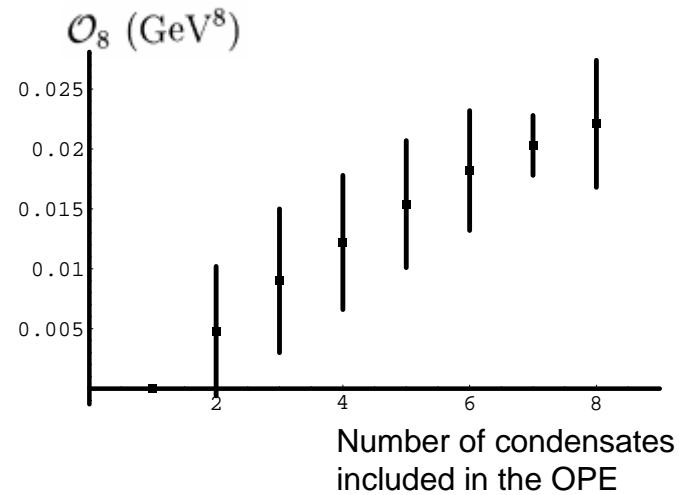
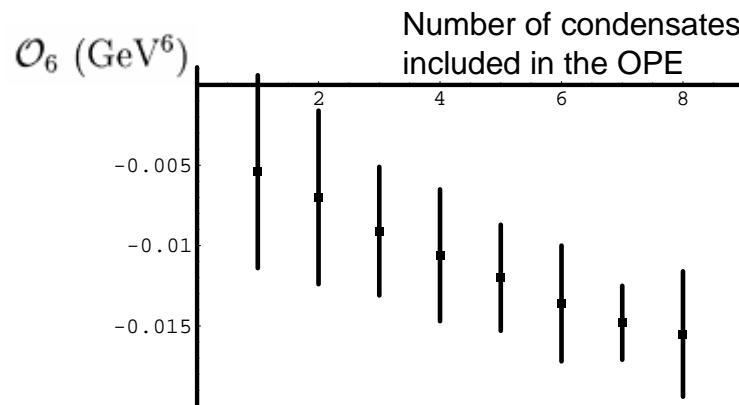




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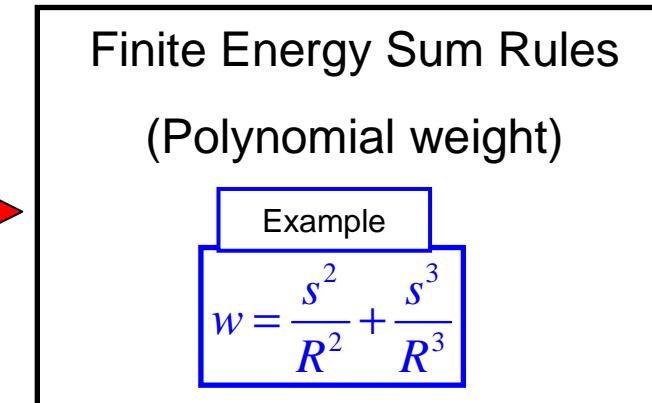
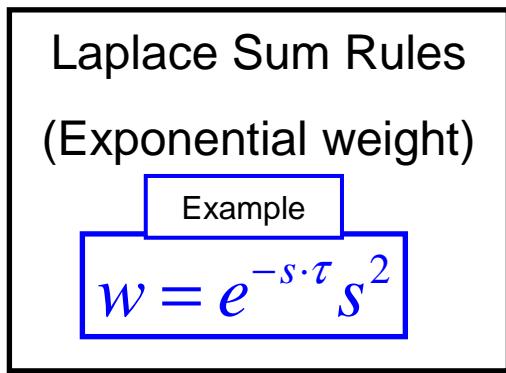


CONCLUSION:

The results depends strongly on the value of N, what indicates that the approaches based on this method underestimate the systematic error, and therefore are not appropriate.

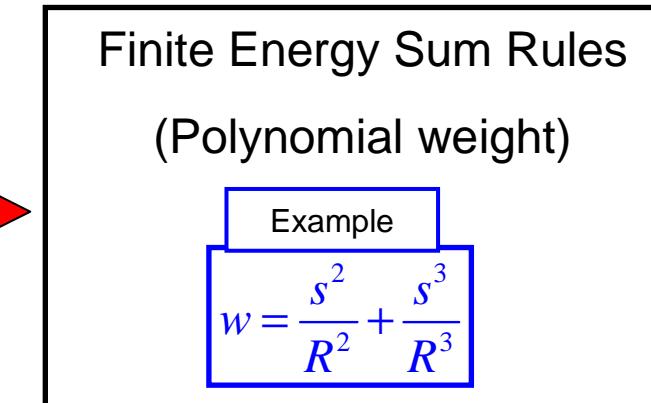
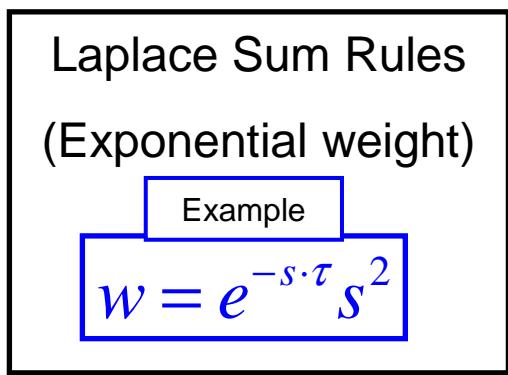


A deeper look: 2.“pinched weight” FESR.





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- Number of condensates involved in the analysis:

Infinite



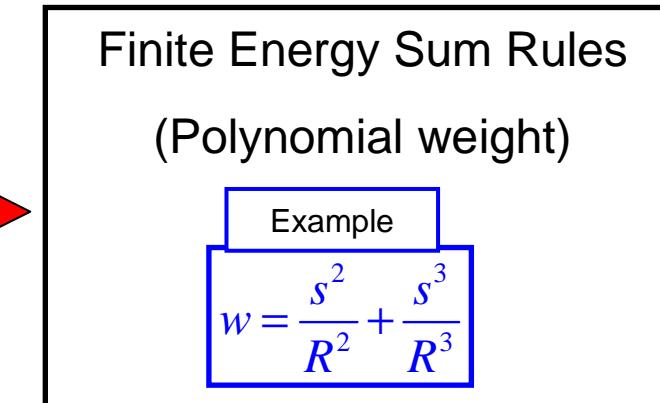
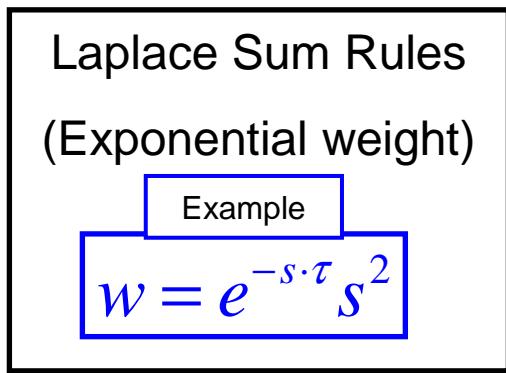
Finite



Source of systematic error!



A deeper look: 2.“pinched weight” FESR.



- Number of condensates involved in the analysis:

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Finite



Source of systematic error!

- High energy region:

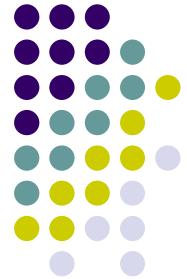
Exponentially suppressed



Not suppressed



Source of systematic error!



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EXAMPLE: $w(s) = s$ (2nd Weinberg Sum Rule)

$$\int_{s_{th}}^R \frac{1}{\pi} \text{Im}\Pi(s) s ds = - \mathcal{O}_4$$

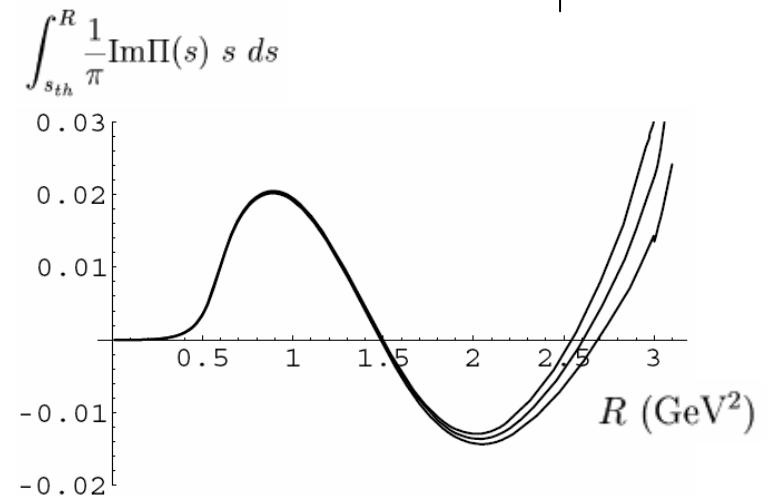


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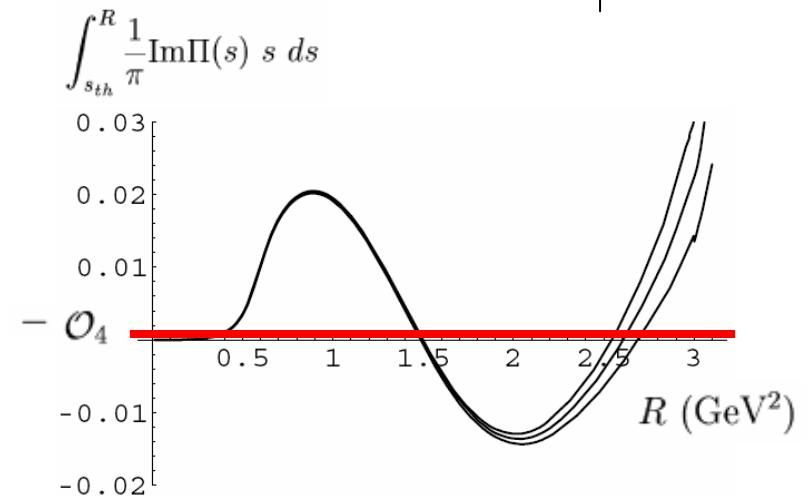


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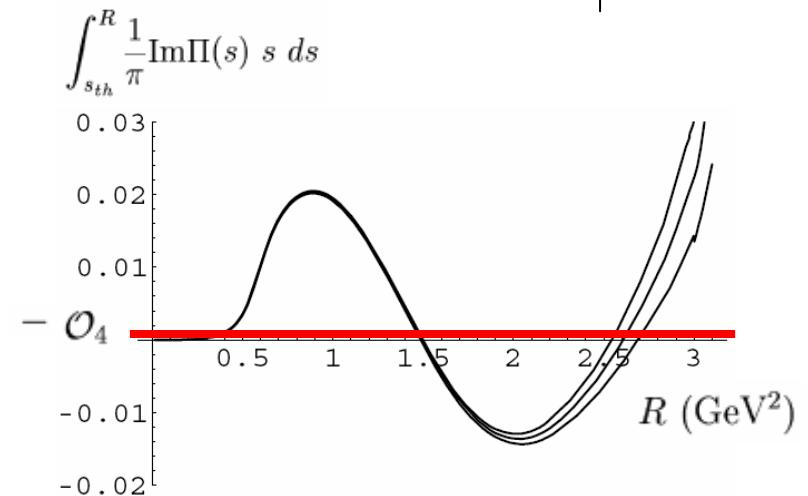


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A possible solution was given by (*Cirigliano et al., 2003*)...

“Pinched weights”: weights with (at least) a double zero at $s=R$.

We are working on the applicability of this method!



SUMMARIZING...

- I've introduced the QCD condensates;
- We've seen how the Sum Rules let us estimate them from the V-A correlator;
- I've motivated the work showing the current disagreement in the value of O_6 and O_8 .
- We've analyzed the Laplace Sum Rules method used in (Narison, 2005) and we have concluded that there is a big systematic error that spoiled the method.
- And we have introduce a possible solution: the finite energy sum rules.



**That's all folks!
Thank you !**