LATTICE QCD AND FLAVOUR PHYSICS

LNF Spring School "Bruno Touschek"

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European Twisted Mass Collaboration

INFN

ALPHA Collaboration

Heavy Flavours

HQE theories: basics

- we have seen that it is impossible to discretize the bottom fermion because its Compton wavelength ($1/m_B \sim 0.04$ fm) is smaller than present day lattice spacings
- formulate a theory which separates this degree of freedom from low-energy ones
- light quark action is as before
- heavy quark action is the **static** one (describing a heavy hadron at rest) + corrections in powers of $1/m_B$
- in the static limit we distinguish the heavy quark degrees of freedom from the antiquark ones; the former travel forwards in time, the latter backwards

$$\psi_{\rm h} = P_+ \psi \, , \quad \overline{\psi}_{\rm h} = \overline{\psi} P_+ ,$$

$$\psi_{\bar{\rm h}} = P_- \psi \, , \quad \overline{\psi}_{\bar{\rm h}} = \overline{\psi} P_-$$

HQE theories: basics

- in terms of these fields we have the tree level action:

$$\mathcal{L} \quad = \quad \mathcal{L}_{\rm h}^{\rm stat} + \mathcal{L}_{\bar{\rm h}}^{\rm stat} + \left\{ \mathcal{L}_{\rm h}^{(1)} + \mathcal{L}_{\bar{\rm h}}^{(1)} + \mathcal{L}_{{\rm h}\bar{\rm h}}^{(1)} \right\} + {\rm O}(\tfrac{1}{m^2})$$

- **static** terms: lowest order terms in heavy quark mass expansion

$$\mathcal{L}_{\rm h}^{\rm stat} \quad = \quad \overline{\psi}_{\rm h}(D_0 + m)\psi_{\rm h} \qquad\qquad \mathcal{L}_{\bar{\rm h}}^{\rm stat} = \overline{\psi}_{\bar{\rm h}}(-D_0 + m)\psi_{\bar{\rm h}}$$

- they describe a static quark which only moves forward in time without movement in space (do spatial derivatives)
- eventually the heavy mass “factors out” through a redefinition of the fermion field
- the quark propagator is a Wilson (Polyakov) line
- the “static” B-meson (heavy-light quark particle) propagates as follows:

$$[\ \vec{\mathbf{x}}\,;\ \mathbf{0}\ ] \longrightarrow [\ \vec{\mathbf{x}}\,;\ \mathbf{t}\ ]$$

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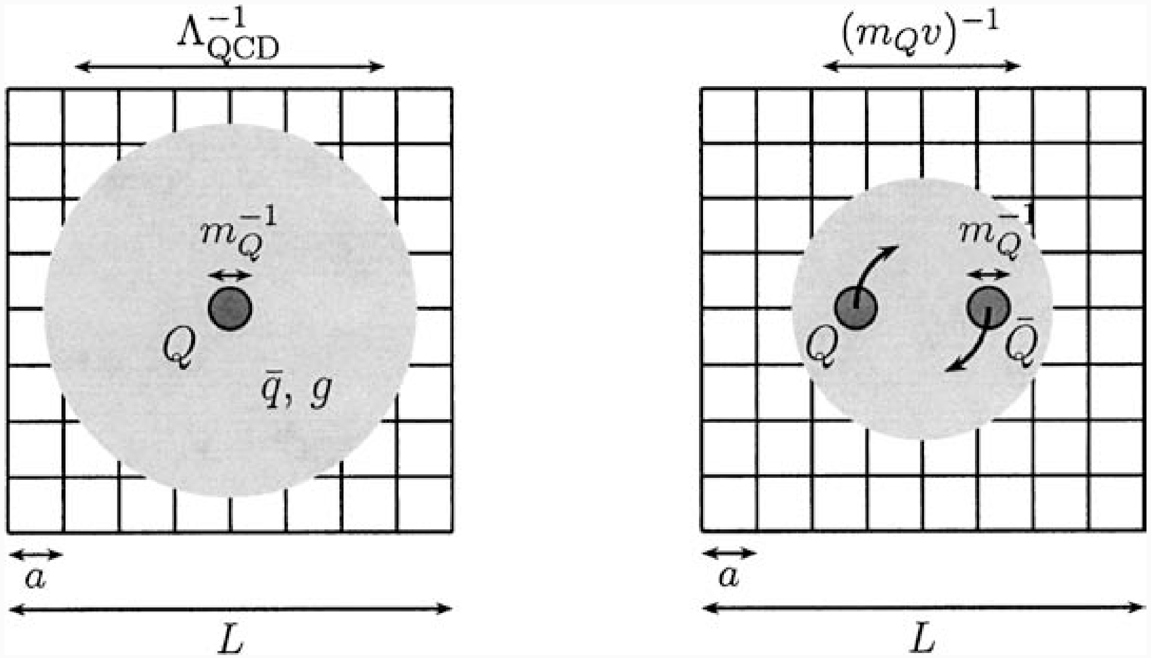
- the *1/m* correction has terms like

$$\begin{aligned}
\mathcal{L}_{\rm h}^{(1)} \quad &= \quad -\frac{1}{2m}(\mathcal{O}_{\rm kin} + \mathcal{O}_{\rm spin}) \, , \\
\mathcal{O}_{\rm kin} \quad &= \quad \overline{\psi}_{\rm h} \, D_k D_k \, \psi_{\rm h} = \overline{\psi}_{\rm h} \, \mathbf{D}^2 \, \psi_{\rm h} \, , \\
\mathcal{O}_{\rm spin} \quad &= \quad \overline{\psi}_{\rm h} \, \frac{1}{2i} F_{kl} \, \sigma_{kl} \, \psi_{\rm h} = \overline{\psi}_{\rm h} \, \sigma \cdot \mathbf{B} \, \psi_{\rm h}
\end{aligned}$$

- two distinct physical situations: the physics of a heavy-light quark meson and that of the bottonium (heavy quark-antiquark pair)

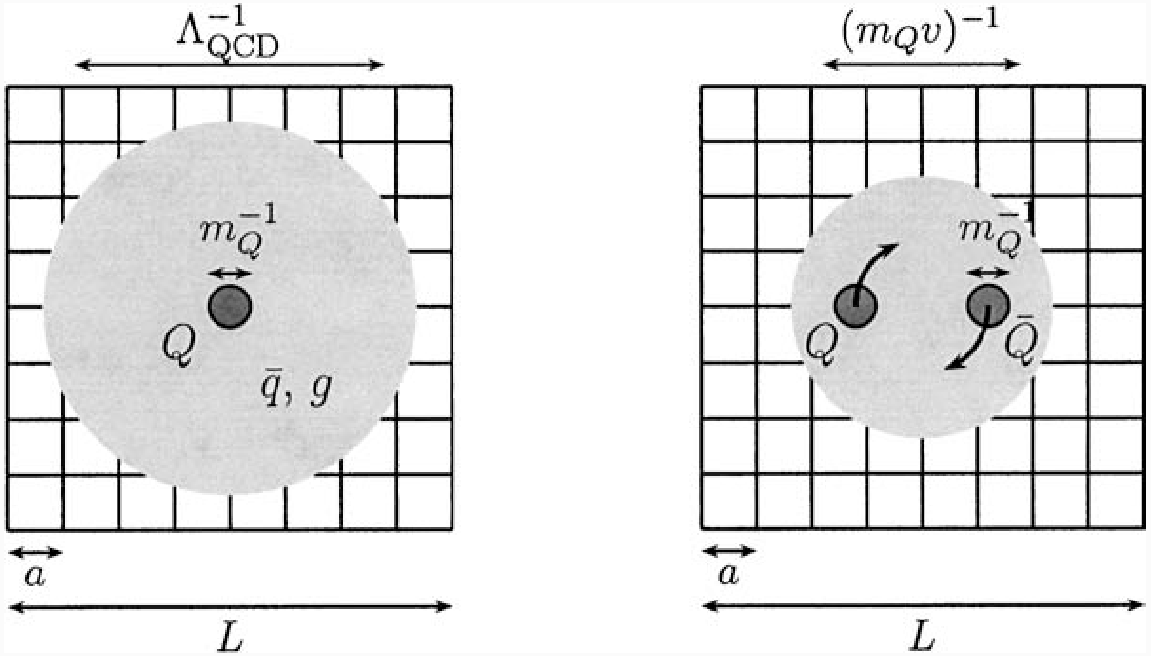
HQE theories: basics



- heavy quark almost at rest, with motion suppressed as Λ_{QCD}/m_Q
- described by HQET: systematic expansion in Λ_{QCD}/m_Q

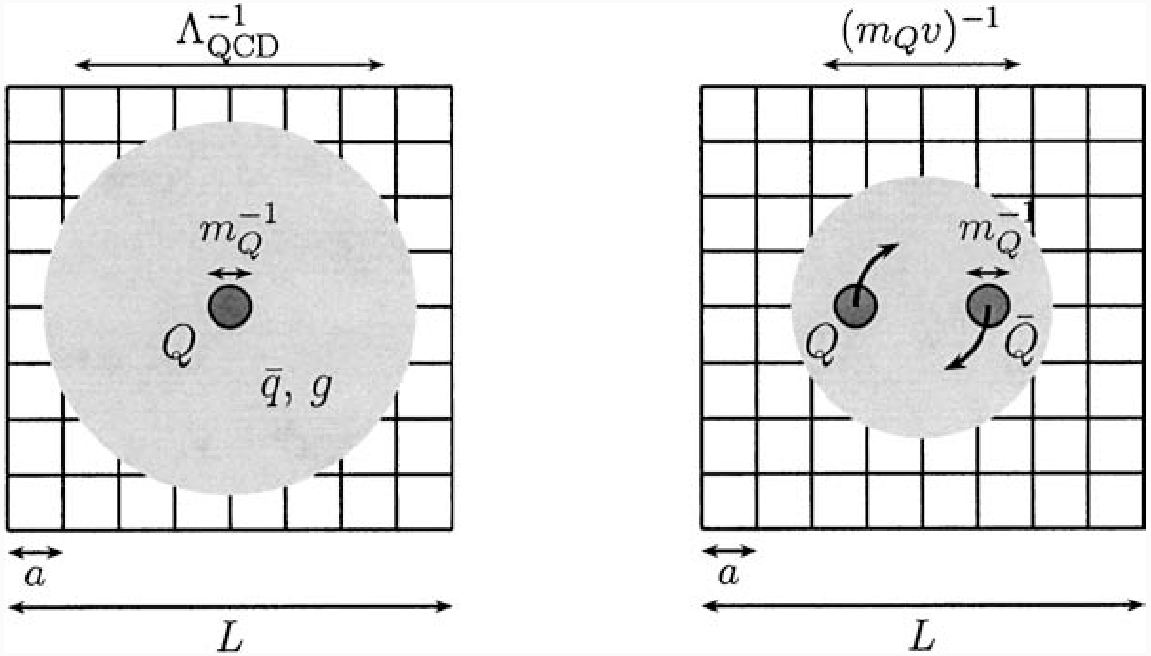
- heavy quarks move around each other in the meson rest frame

HQE theories: basics



- balanced kinetic and potential energy $$\frac{<p^2>}{2m_Q} \sim -\frac{4}{3}\alpha_s \frac{1}{<r>}$$
- uncertainty relation $$<p> \sim \frac{1}{<r>}$$

HQE theories: basics



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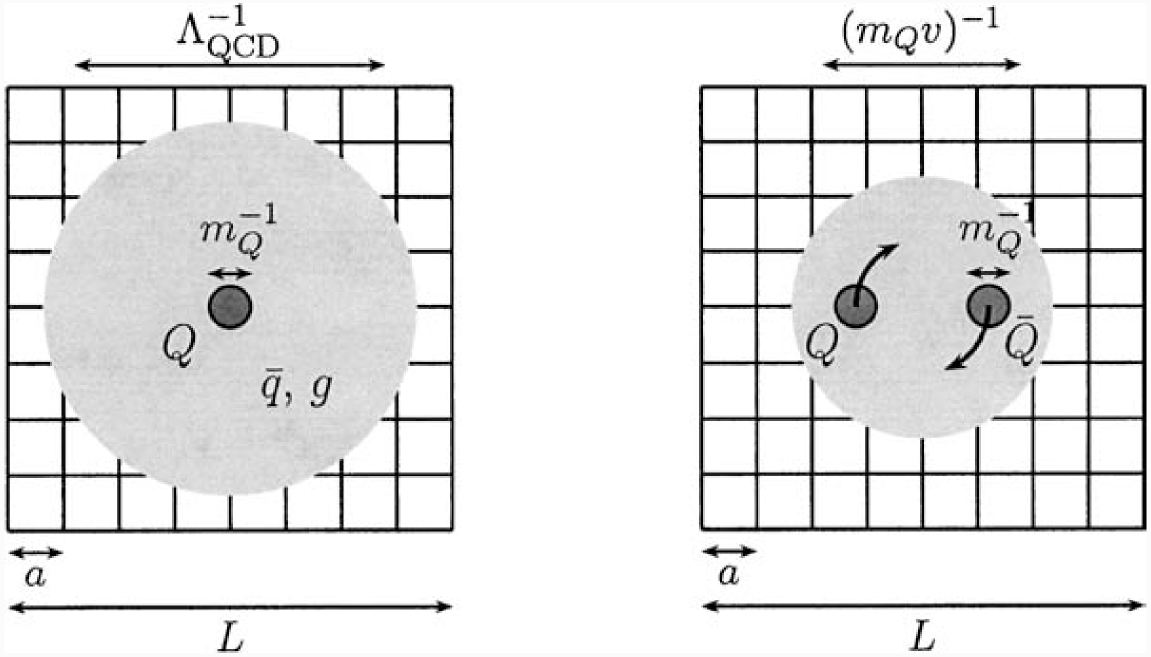

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$$< p > \sim \alpha_s m_Q$$

average velocity $$v \sim < p > / m_Q \sim \alpha_s$$

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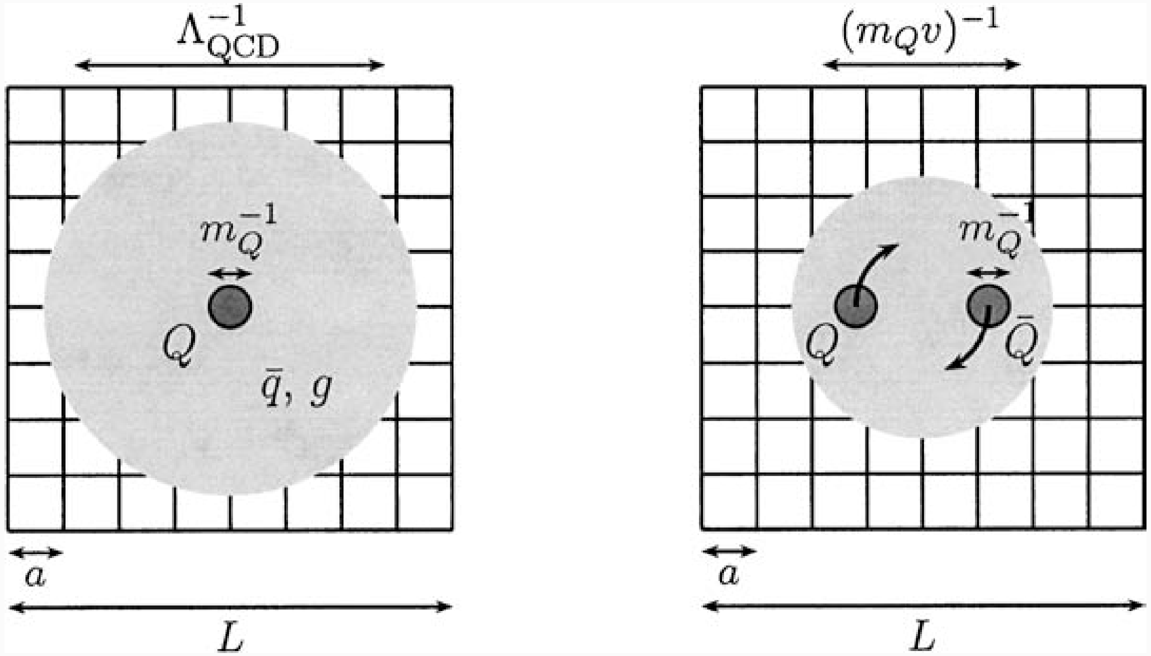

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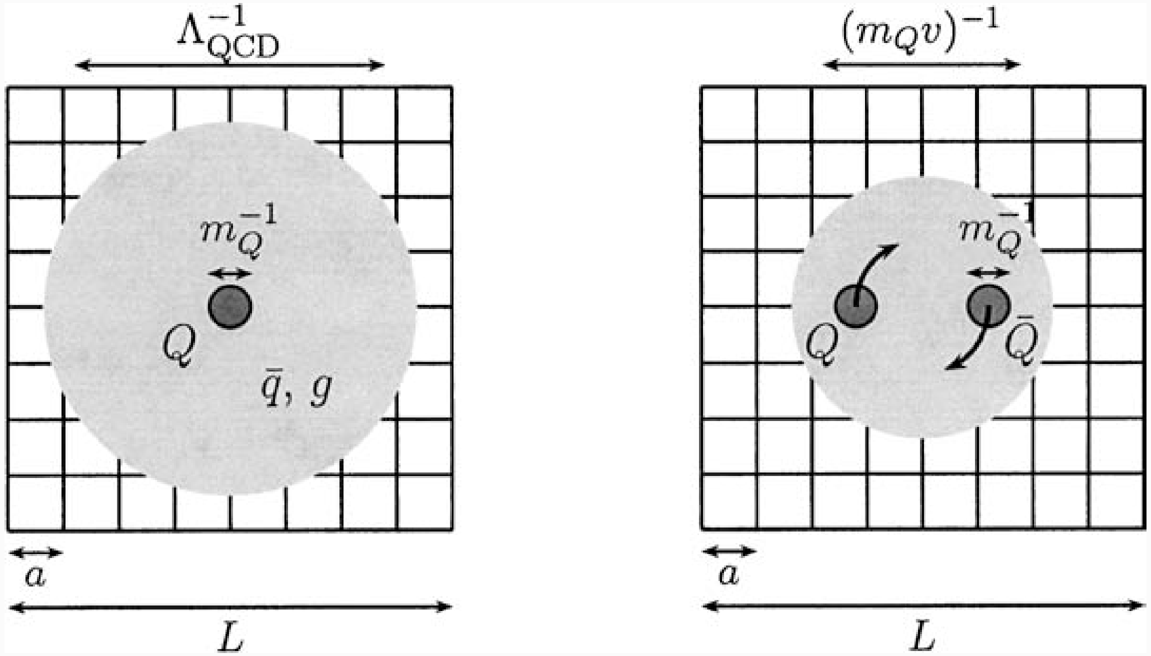
- HQET: systematic expansion in
  - Λ_{QCD}/m_Q

- heavy quarks move around each other in the meson rest frame
- NRQCD: three well separated scales
  - quark mass m_Q
  - spatial momentum $\langle p \rangle \sim m_Q v$
  - binding energy $\langle p^2 \rangle / m_Q \sim m_Q v^2$

HQE theories: basics



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HQET leading order terms are *O(1)* and subleading are *O(1/m)*

NRQCD leading order terms are O(1) and subleading are O(v^2)

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- beyond classical level we regularize the theory on the lattice (e.g. Wilson fermions)
- the static (LO) contribution acquires a mass counterterm
- the HQET terms (NLO) have less trivial *O(1/m)* coefficients

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$\rightarrow$

$$\mathcal{L}_{\rm h}^{(1)}(x) = -(\omega_{\rm kin}\, \mathcal{O}_{\rm kin}(x) + \omega_{\rm spin}\, \mathcal{O}_{\rm spin}(x))\,.$$

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- NB: the *O(1/m)* terms are dim=5 operators; thus both HQET & NRQCD are non-renormalizable theories, while the static theory is OK

- they have no continuum limit

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- some lattice calculations adopt a "phenomenological approach", working at fixed lattice spacing, with *ma* not too small

- way out: since static theory is renormalizable, you can consider the static term as the "theory's action" and expand the *O(1/m)* terms as part of the "observable":

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- expand the action exponential to *O(1/m)*

$$\exp\left[-S^{\rm stat} \;-\; S^{(1)}\right] \;=\; \exp\left[-S^{\rm stat}\right]\left[1 \;-\; S^{(1)}\right]$$

renormalizable action

part of the observale

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- example I: B-meson mass:

$$m_{\rm B} = m_{\rm b} + \widehat{\delta m} + E_{\rm stat} + \omega_{\rm kin} E_{\rm kin} + \omega_{\rm spin} E_{\rm spin} \,,$$

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- example II: B-meson decay constant from the WME $\langle B | A_0^{\rm HQET} | 0 \rangle$:

$$A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET} \, [A_0^{\rm stat}(x) + c_{\rm A}^{\rm HQET} \delta A_0^{\rm stat}(x)]$$

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- leading *O(1)* terms
- NB: Z_A^{HQET} may be computed either in PT or NP (better NP)

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$$E_{\rm spin} = -\langle B | a^3 \sum_{\mathbf{z}} \mathcal{O}_{\rm spin}(0, \mathbf{z}) | B \rangle_{\rm stat}$$

- example II: B-meson decay constant from the WME $\langle B|\, A_0^{\rm HQET} |0\rangle$:

$$A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET}\, [A_0^{\rm stat}(x) + c_{\rm A}^{\rm HQET} \delta A_0^{\rm stat}(x)]$$

$$\delta A_0^{\rm stat}(x) = \overline{\psi}_{\rm l}(x) \frac{1}{2} (\overleftarrow{\nabla}_i + \overleftarrow{\nabla}_i^*) \gamma_i \gamma_5 \psi_{\rm h}(x) \,.$$

- subleading *O(1/m)* terms
- once these terms are included, everything **must** be computed NP-ly !!

HQE theories: quantum theory

$$A_0^{\mathrm{HQET}}(x) \;=\; Z_{\mathrm{A}}^{\mathrm{HQET}}\,[A_0^{\mathrm{stat}}(x) + c_{\mathrm{A}}^{\mathrm{HQET}}\,\delta A_0^{\mathrm{stat}}(x)]$$

- suppose Z_A^{HQET} is known from 1-loop PT; this estimate has an error:

$$\Delta Z_A^{\mathrm{HQET}} \;\propto\; g_R^4(m_b) \;\sim\; \frac{1}{\ln(m_b/\Lambda_{\mathrm{QCD}})}$$

- if the power correction, i.e. the subleading $O(1/m)$ term is also known:

$$c_A^{\mathrm{HQET}} \;\propto\; \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

- as m_b is increased, the LO error dominates the power correction
- several lattice HQET computations adopt this phenomenological approach, where it is assumed that the LO correction comes with a coefficient which is small compared to the power subtraction, in the mass interval of the simulation
- the only theoretically consistent approach is the NP one

Step scaling functions for HQET

- we need to determine several HQET renormalization constants; e.g. for the HQET determination of the decay constant f_B (which involves the axial current) we need to know :
  - LO δm, Z_A^{HQET}
  - NLO δm, ω_{kin}, ω_{kin}, c_A
- their determination goes through matching of several correlations, involving the axial current, in lattice QCD and in HQET

$$\Phi_k^{\mathrm{HQET}}(L_1, M_{\mathrm{b}}) = \Phi_k^{\mathrm{QCD}}(L_1, M_{\mathrm{b}})$$

- if the matching could be done in physical regimes (e.g. large volumes), there would be no point in doing HQET in the first place
- can do matching in small volumes L_1
- ALPHA: at small volumes, match HQET to QCD and compute renormalization constants, define a SSF (a new one!), compute it at several a/L, extrapolate it in the continuum and use matching and iterative techniques to scale up to physical volumes

Step scaling functions for heavy WME

- SSF can also be used for the computation of WMEs (rather than renormalization)

M. Guagnelli, F.Palombi, R.Petronzio, & N.Tantalo, Phys. Lett. B546(2002)237

- first compute the physical quantity (say f_B) on a small volume with good resolution
- result is unphysical due to strong finite size effects
- use finite-volume SSF to move to higher volumes:

$$f_{h\ell}(L_\infty) = f_{h\ell}(L_0)\frac{f_{h\ell}(L_1)}{f_{h\ell}(L_0)}\frac{f_{h\ell}(L_2)}{f_{h\ell}(L_1)}\cdots, \qquad L_0 < L_1 < L_2 < \cdots$$

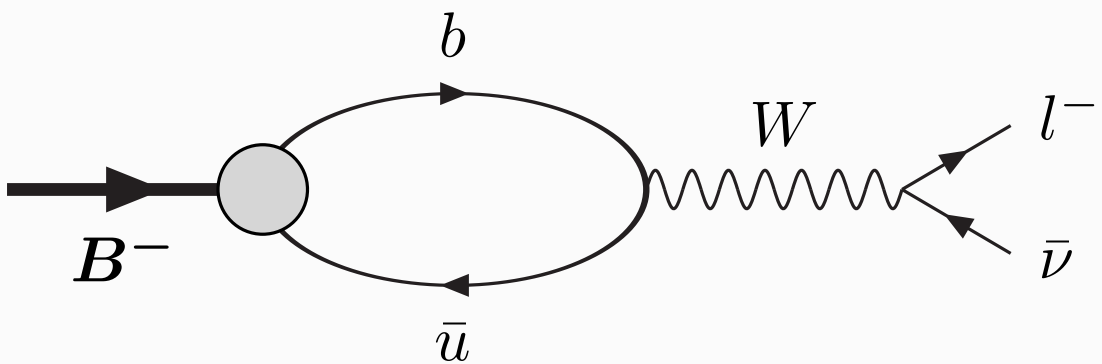
$$\sigma(m_\ell, m_h, L_{k-1}) = \left.\frac{f_{h\ell}(m_\ell, m_h, L_k)}{f_{h\ell}(m_\ell, m_h, L_{k-1})}\right|_{L_k = sL_{k-1}}$$

- method works because σ has a slighter dependence on $1/m_h$ than f_B (cancellations between numerator and denominator)
- the continuum SSF is obtained by extrapolation at several resolutions a/L of the discrete SSF

Decay Constants

Basics

QCD effects in leptonic decays are parametrized in terms of a single parameter f_M



$$< 0|A_\mu(0)|B(p) > = f_B \, p_\mu$$

Vacuum-to-meson matrix element of axial current $A_\mu = \bar{b}\, \gamma_\mu \gamma_5 \, u$

Knowledge of f_B allows prediction of corresponding decay rate

$$\Gamma(B \to l\nu_l + l\nu_l\gamma) = \frac{G_F^2 V_{ub}^2}{8\pi} f_B^2 m_l^2 m_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2 (1 + O(\alpha))$$

Basics

- Also interested in the decay constants of
  - pion (u-d quarks) f_π
  - K-meson f_K (s-d quarks)
  - D_d (c-d) and D_s (c-s) mesons f_{Dd} , f_{Ds}
  - B_d (b-d) and B_s (b-s) mesons f_{Bd} , f_{Bs}
- f_π monitors the chiral behaviour of QCD as predicted by chiral PT
- $f_\pi = 132$ MeV can also be used to calibrate lattice spacing
- f_K can be a postdiction or a way to calibrate the strange quark mass
- f_{Bd} and f_{Bs} are part of the computation of neutral B-meson oscillations (later)
- similarly for f_{Dd} - f_{Ds} (cf. recent experiments on D-meson oscillations)

f_K: recent quenched results

Using tmQCD the Wilson fermion computation has acquired even better precision:

Alpha P. Dimopoulos et al. hep-lat/0702017

- several lattice spacings 0.04 fm < a < 0.09 fm enable control of continuum limit
- lattice volumes adequate at L ~ 2 fm
- with tmQCD no axial current normalization Z_A needed
- two variants of tmQCD enable combined fit to continuum
- realisitc masses allowed by tmQCD (m_K ~ 490 MeV with degenerate quarks)

$$f_K = 165 \pm 3 \text{ MeV}$$

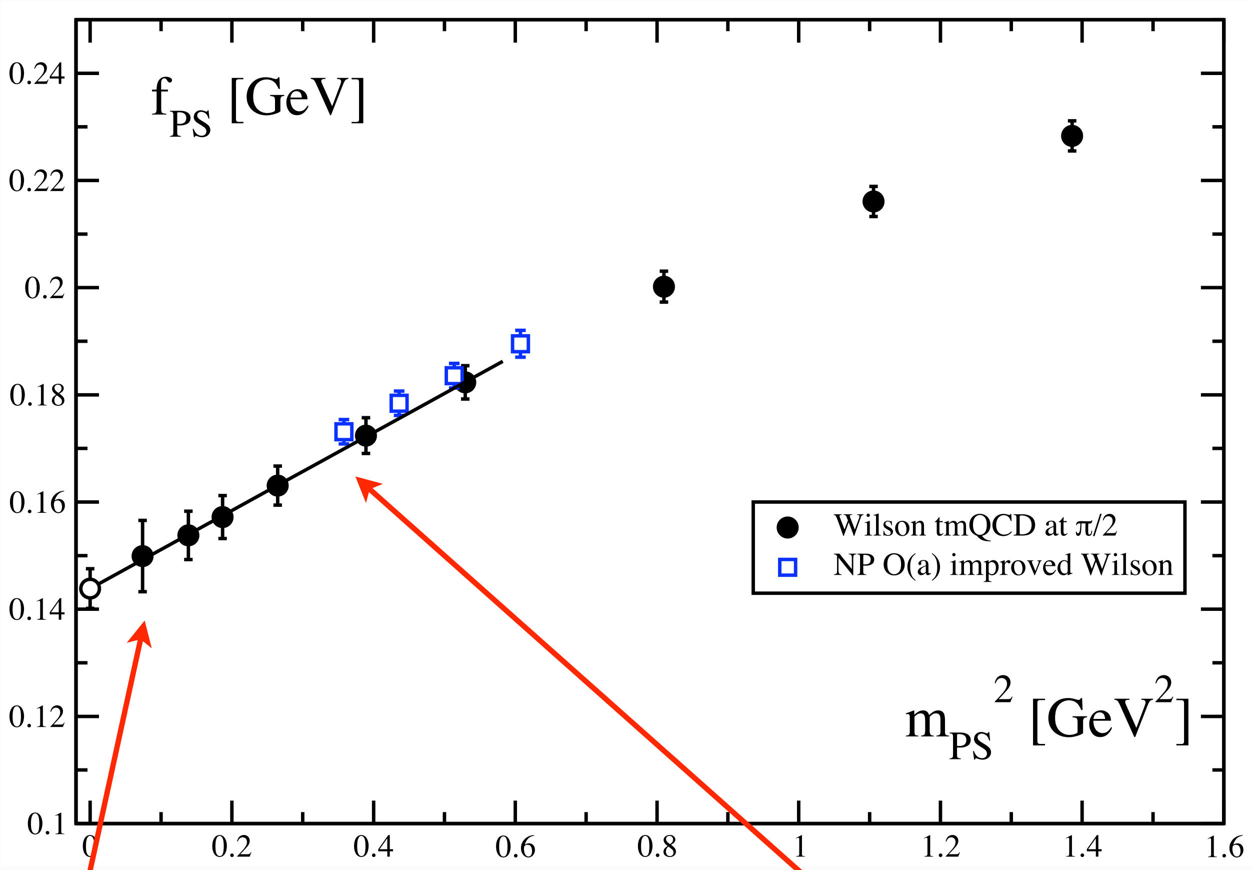
$$f_K = 164 \pm 4 \text{ MeV}$$

Alpha J. Garden et al. Nucl.Phys.B571(2000)237

$$f_K = 162 \pm 4 \text{ MeV}$$

χLF K. Jansen et al. JHEP09(2005)071

f_K: recent quenched results



$$\frac{f_K}{f_\pi} = 1.11 \pm 0.04$$

$$\frac{f_K}{f_\pi} = 1.22 \qquad \text{exp}$$

10% quenching error

m_π = 270 MeV

m_π = 550 MeV

χLF K. Jansen et al. JHEP09(2005)071

Alpha J. Garden et al. Nucl.Phys.B571(2000)237

f_K: recent unquenched results

Using staggered fermions with $N_f = 2$ the MILC collaboration reports:

MILC C. Bernard et al. PoS(LAT2006)163

- four lattice spacings 0.06 fm < a < 0.12 fm enable control of continuum limit
- lattice volumes adequate at 2 fm < L < 2.4 fm
- with staggered no axial current normalization Z_A needed
- light quark masses (sea) m_q ~ 11 MeV (m_π ~ 240 MeV)
- strange quark masses m_K ~ 490 MeV
- each physical flavour accompanied by 3 "tastes"; determinant rooting !!

$$
\begin{aligned}
f_\pi &= 128.6 \ \pm 0.4 \ \pm 3.0 \text{ MeV} \\
f_K &= 155.3 \ \pm 0.4 \ \pm 3.1 \text{ MeV} \\
\frac{f_K}{f_\pi} &= 1.208 \ \pm 0.02 \ {}^{+0.07}_{-0.14}
\end{aligned}
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statistical

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systematic

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$$\frac{f_K}{f_\pi} = 1.208 \pm 0.02 \;{}^{+0.07}_{-0.14}$$

rate $K \rightarrow \pi \, \mu \, \nu$

V_{ud}

$$\rightarrow \quad |V_{us}| = 0.2223\left({}^{+26}_{-14}\right)$$

W.J. Marciano Phys. Rev. Lett.93(2004)231803

Particle Data Group 2006 $\longrightarrow$ $V_{us} = 0.2257(21)$

f_K: recent unquenched results

Using DW fermions with N_f = 2+1 the RBC-UKQCD collaboration reports:

RBC-MILC C. Allton et al. hep-lat/0701013

- one lattice spacing $a \sim 0.12$ fm (the coarsest MILC); one lattice volume $L \sim 2$ fm
- with DW ($L_5 = 16$) good chirality (?); axial current normalization Z_A present
- light quark masses (sea) $0.33\, m_s < m_q < 0.85\, m_s$ and m_s physical

$$
\begin{aligned}
f_K &= 127 \pm 4 \text{ MeV} \\
f_\pi &= 157 \pm 5 \text{ MeV} \\
\frac{f_K}{f_\pi} &= 1.24 \pm 0.02
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statistical

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Exotic alternatives: staggered sea with DW valence! $\frac{f_K}{f_\pi} = 1.218 \pm 0.002\,^{+0.011}_{-0.024}$

NPLQCD S.R.Beane et al. hep-lat/0606023

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systematic

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NPLQCD S.R.Beane et al. hep-lat/0606023

$$\frac{f_K}{f_\pi} = 1.218 \pm 0.002 {}^{+0.011}_{-0.024}$$

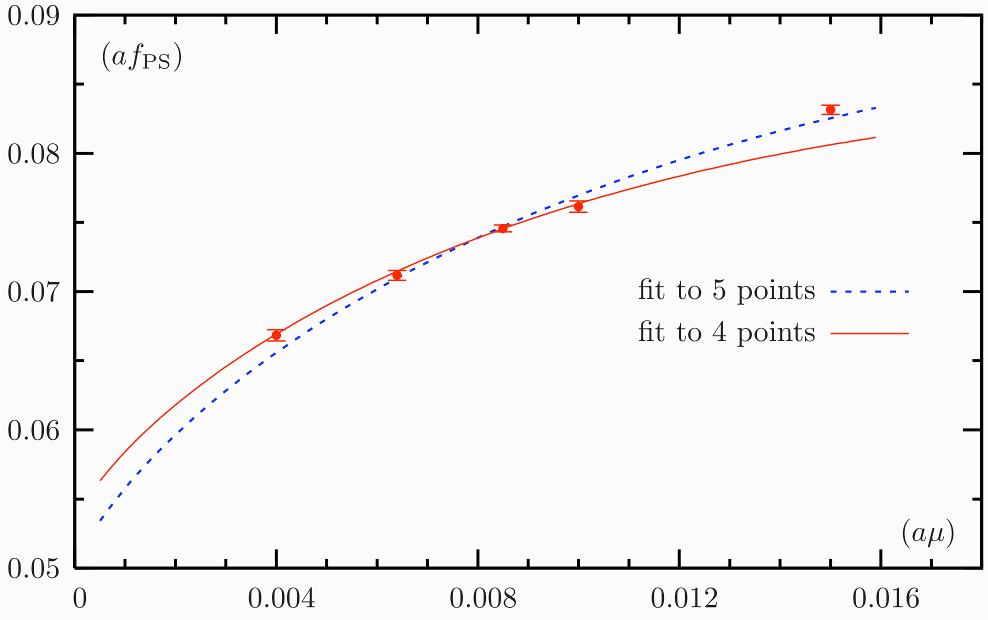
f_π: recent unquenched results

Using tmQCD fermions with $N_f = 2$ the ETM-Collaboration reports:

ETMC Ph. Boucaud et al. hep-lat/0701012

- one lattice spacing $a \sim 0.1$ fm (the coarsest MILC); one lattice volume $L \sim 2.4$ fm
- light quark masses (sea) 300 MeV < m_π < 550 MeV
- no axial current (and no Z_A) is needed, due to tmQCD Ward identity
- due to tmQCD @ twist angle $\alpha = \pi/2$, we have automatic $O(a)$ improvement

$$f_\chi = 121.3 \pm 0.7$$



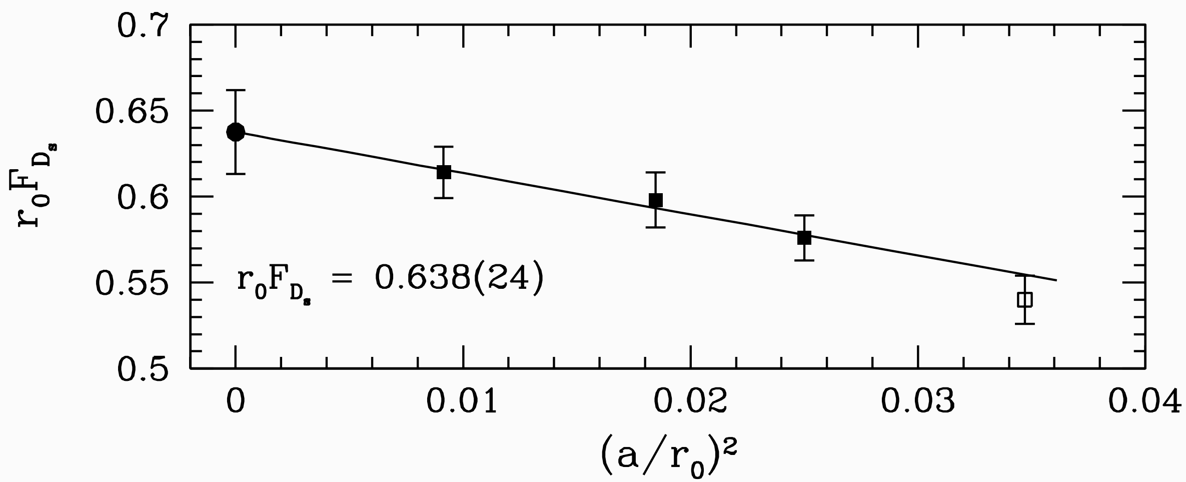
f_D: recent quenched results

Using W-Clover fermions the Alpha collaboration reports:

Alpha A. Juttner and J. Rolf Phys.Lett.B560(2003)59

- four lattice spacings 0.04 fm < a < 0.09 fm ; lattice volume L ~ 1.5 fm
- O(a)-improvement, renormalization etc well under control

$$f_{D_s} = 252 \pm 9\text{MeV}$$



PDG 2002

$$f_{D_s} = 285 \pm 19 \pm 40 \text{ MeV}$$

f_D - f_B: recent quenched results

Using W-Clover fermions a Rome 2 collaboration reports:

G.M. de Divitiis et al. Nucl.Phys.B672(2003)372

- finite volume step scaling method $L \sim 0.4, 0.8, 1.6$ fm
- several lattice spacings $0.06 \leq a \leq 0.13$ control continuum limit
- compute SSF around the charm quark mass, extrapolate it to bottom region

$$
\begin{aligned}
f_{D_s} &= 240 \pm 5 \pm 5 \text{ MeV} \\
f_{B_s} &= 192 \pm 6 \pm 4 \text{ MeV}
\end{aligned}
$$

f_D - f_B: recent quenched results

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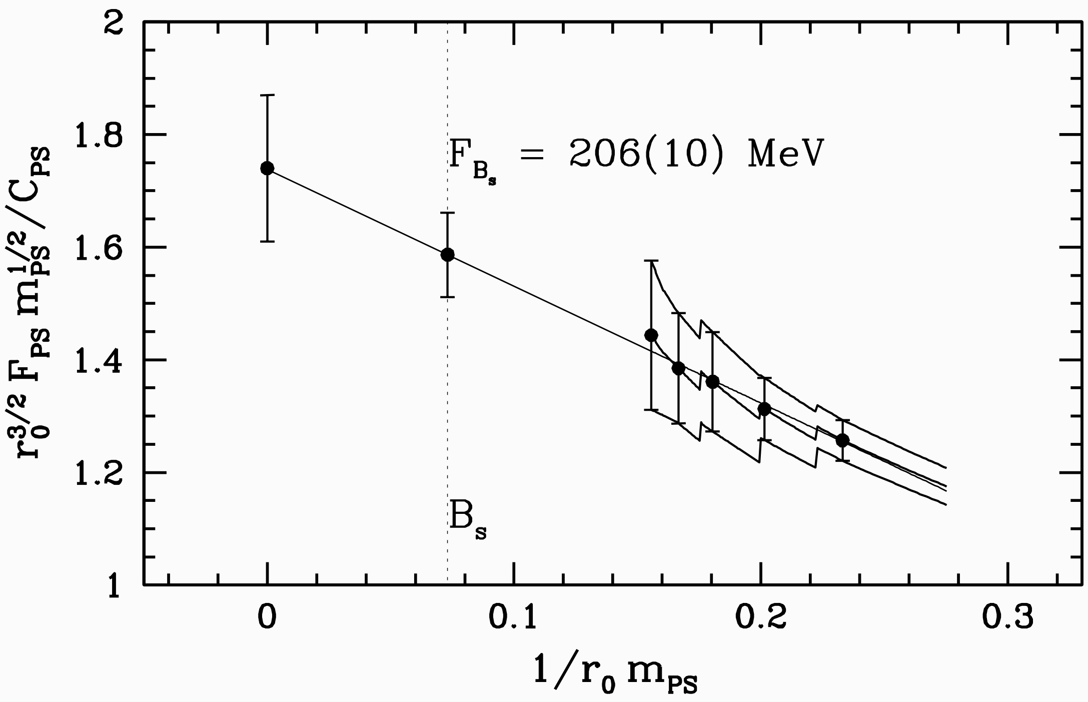
systematic

f_D - f_B: recent quenched results

Using W-Clover fermions the Alpha collaboration reports:

Alpha J. Heitger et al. Phys.Lett.B581(2004)93

- finite volume step scaling method for NP renormalization for static case
- compute f_D around the charm quark mass, f_B-static and INTERPOLATE for f_B



$$f_{B_s} = 206 \pm 10 \text{ MeV}$$

f_D - f_B: recent quenched results

Using W-Clover fermions the Alpha/Rome 2 collaboration reports:

D.Guazzini, R.Somer, N.Tantalo PoS(Lat2006)084

- combination of methods and data
- compute step scaling function around the charm quark mass and in the static limit; then INTERPOLATE for step scaling function in bottom
- use these step scaling functions and finite volume Rome 2 method to get f_B

$$f_{B_s} = 191 \pm 6 \text{ MeV}$$

f_D: recent quenched results

Using DW fermions the RBC collaboration reports:

H.W. Lin et al., Phys.Rev.D749(2006)114506

- one lattice spacing $a \sim 0.065$ fm and one volume $L \sim 1.6$ fm
- quark mass range $m_s/4 < m_q < 5m_s/4$

$$f_{D_s} = 254 \pm 4 \pm 12 \text{ MeV}$$

T.W.Chiu, Phys.Lett.B624(2005)31

- one lattice spacing $a \sim 0.09$ fm
- thirty quark masses ranging 70 MeV $< m_q < 180$ MeV

$$f_{D_s} = 266 \pm 10 \pm 18 \text{ MeV}$$

f_D - f_B : recent unquenched result

FNAL/MILC collaboration reports at $N_f = 2+1$:

C. Bernard et al., PoS(LAT2006)094

- staggered light quarks with Fermilab heavy quarks
- three lattice spacings $a \sim 0.09$ fm, 0.12 fm, 0.15 fm
- work in progress

$$f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV}$$

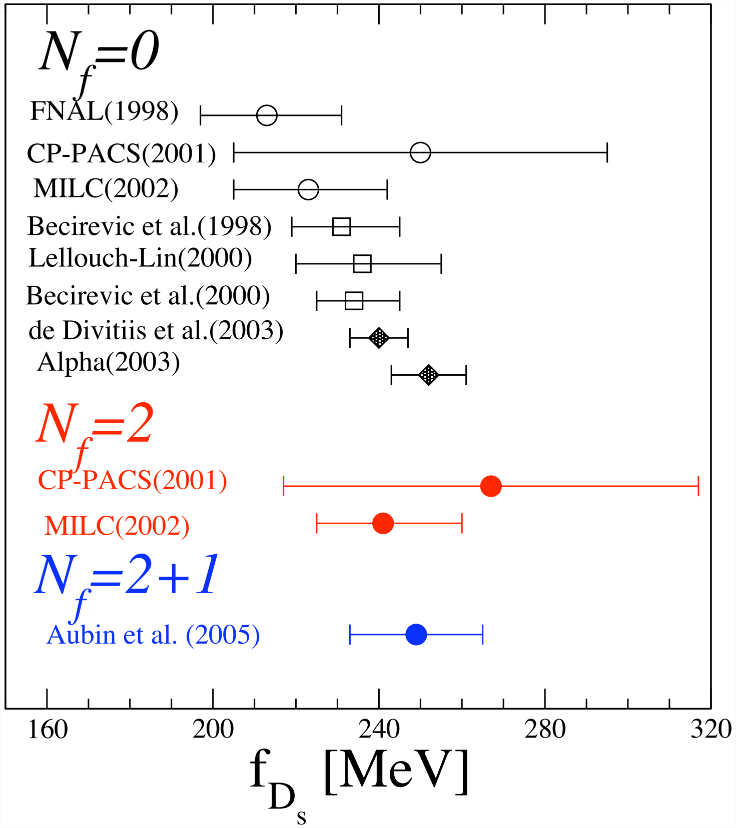
CLEO:

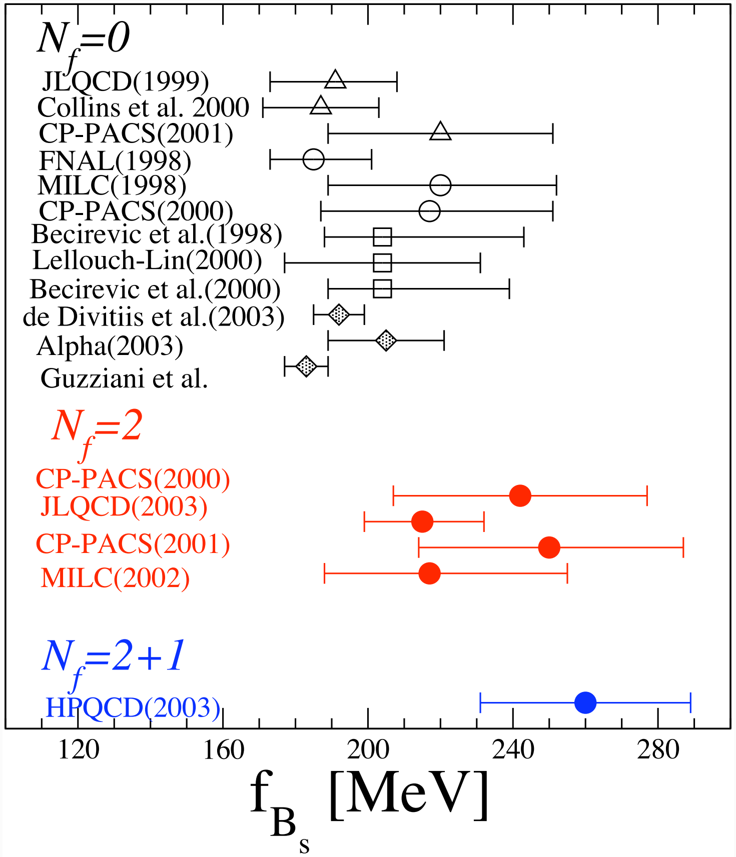
$$f_{D_s} = 282 \pm 16 \pm 7 \text{ MeV}$$

$$\frac{f_{B_s}}{f_{D_s}} = 0.99 \pm 0.02 \pm 0.06$$

f_D - f_B : summary

T. Onogy PoS(LAT2006)017





- due to step scaling function methods, quenched Wilson results are the best
- unquenched results suggest a 10-15% increase in the f_B values
- all error bars are not equally reliable; all results are not on equal footing

B_K

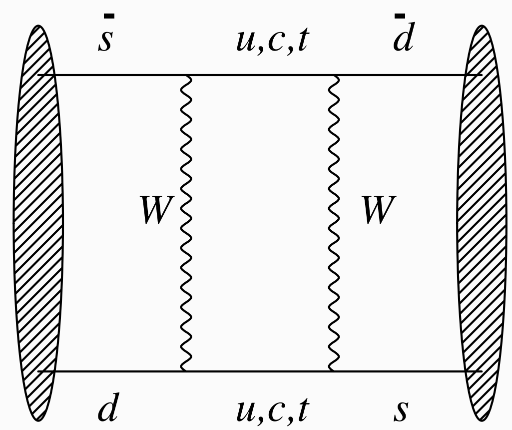
ΔS=2 transitions: ϵ_K

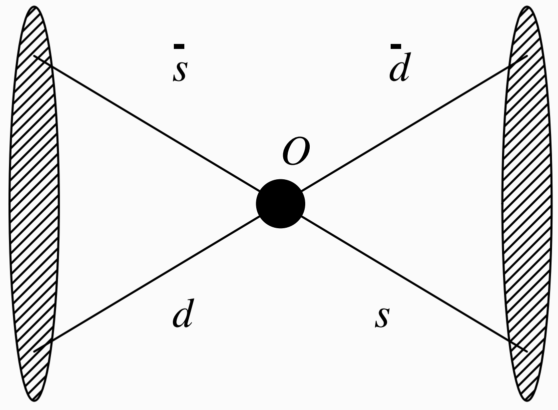
indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\, \exp(i\pi/4)$$

can also be expressed in terms of K^0 - K^0 mixing
dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1\, S_0(x_c) - \eta_3\, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2\, S_0(x_t)]\}$$





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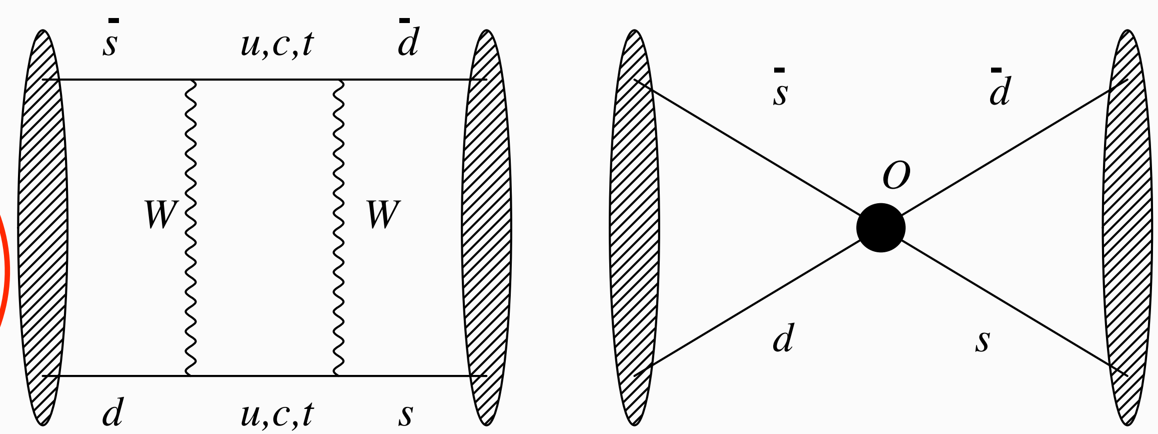
$$C_\epsilon = \frac{G_F^2 f_K^2 m_K M_W^2}{6\sqrt{2}\pi^2(\Delta M_K)} = 3.78 \times 10^4$$

$$\Delta M_K = M_{K_L} - M_{K_S}$$



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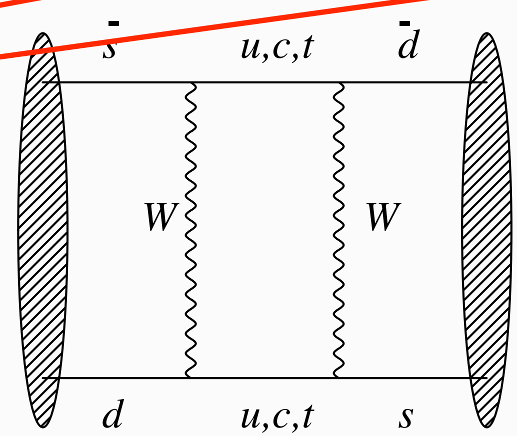
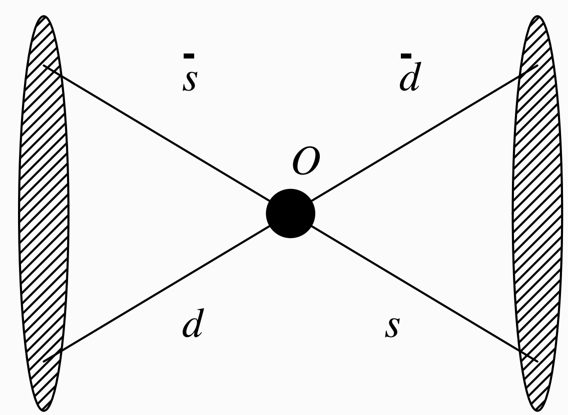
known functions from EW calculation (no QCD)

$$x_k = m_k^2/M_W^2$$





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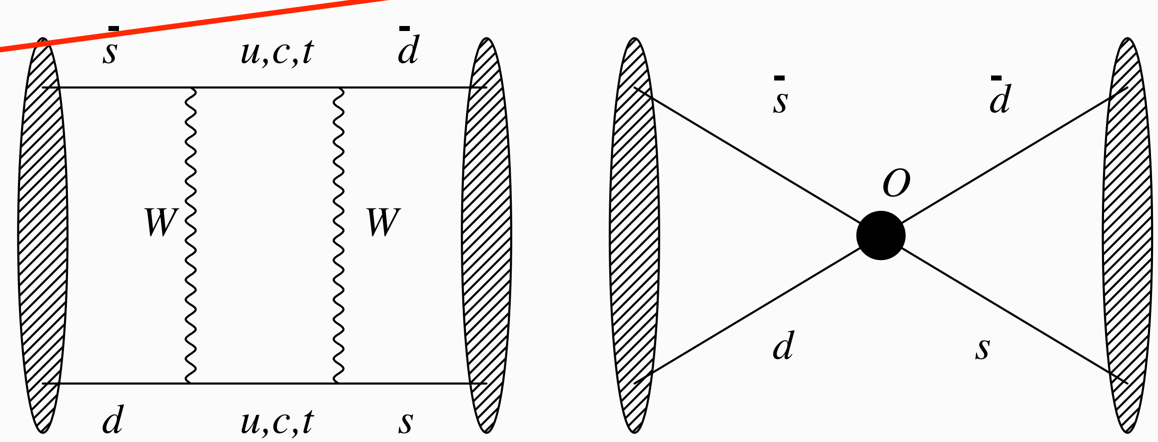
from NLO PT (with QCD)



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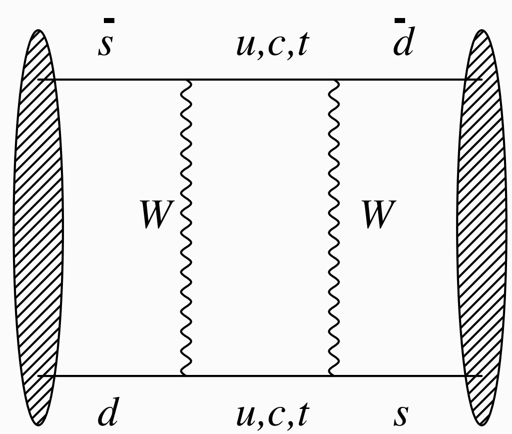
can also be expressed in terms of K^0 - K^0 mixing

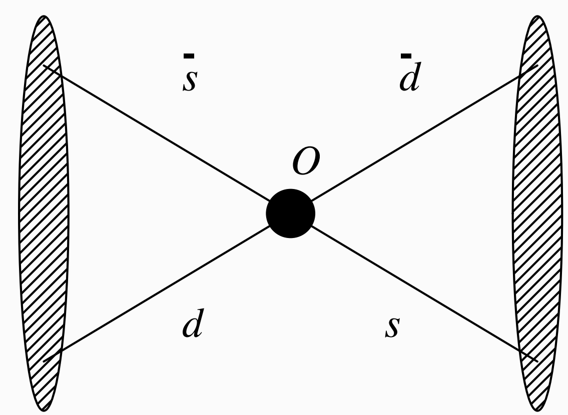
dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1\, S_0(x_c) \,-\, \eta_3\, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2\, S_0(x_t)]\}$$

long distance NP

$$\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}$$





$$\hat{O}^{\Delta S=2} = [\bar{s}\gamma_\mu^L d]\ [\bar{s}\gamma_\mu^L d]$$

ΔS=2 transitions: ϵ_K

indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\exp(i\pi/4)$$

can also be expressed in terms of K^0 - K^0 mixing
dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon \hat{B}_K \,\mathrm{Im}\{V_{td}^* V_{ts}\} \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2 S_0(x_t)]\}$$

long distance NP

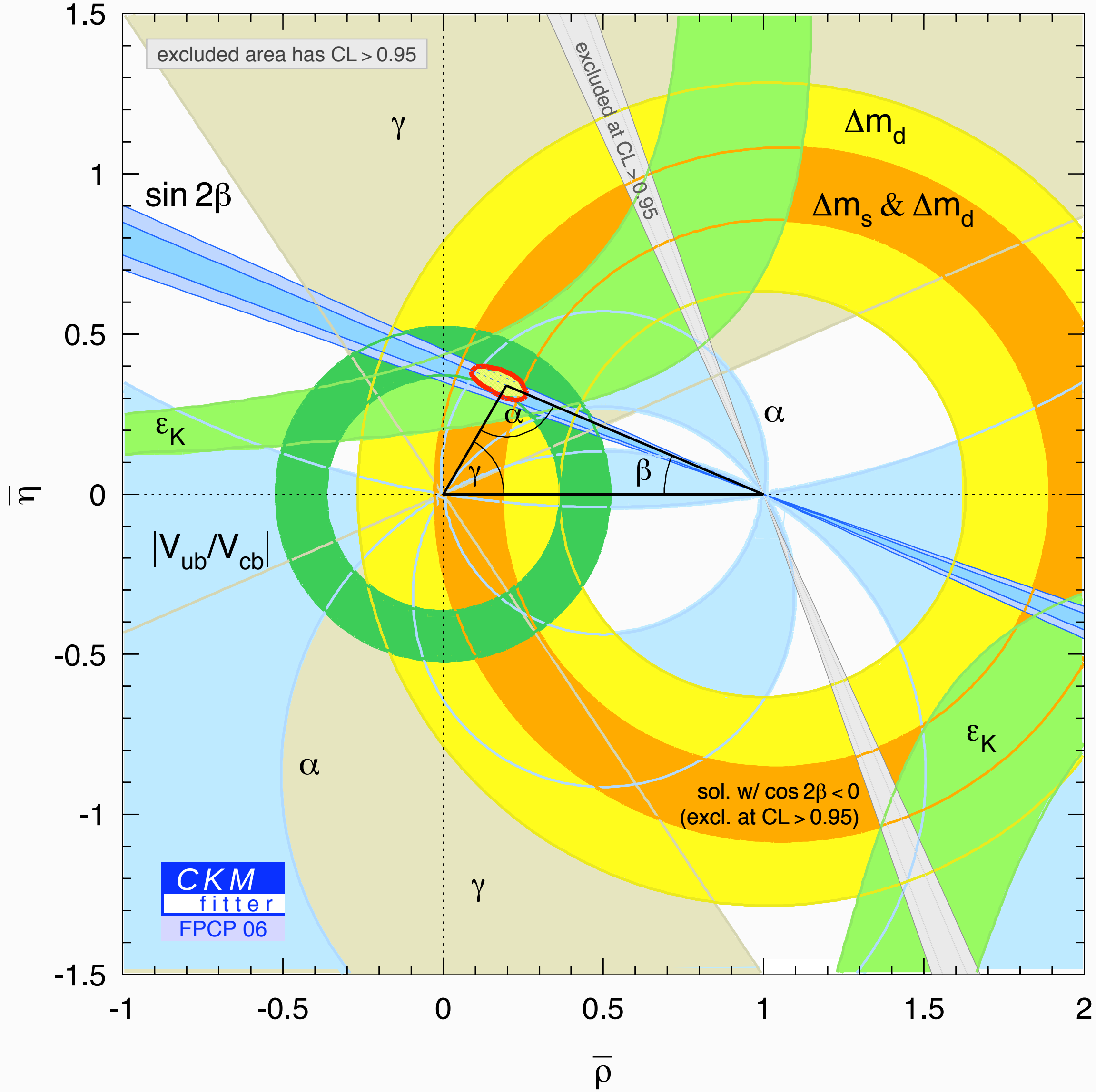
Put in NLO PT + Cabibbo angle + A + $m_{c,t}$:

$$\bar{\eta}(1.4 - \bar{\rho})\,\hat{B}_K \approx 0.40$$

$$\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2 m_K^2}$$

ΔS=2 transitions: ϵ_K

$$|\epsilon_K| = \frac{\mathcal{A}(K_L \to (\pi\pi)_{I=0})}{\mathcal{A}(K_S \to (\pi\pi)_{I=0})} \overset{\text{exp}}{=} [2.282(17) \times 10^{-3}]\, e^{i\pi/4}$$



B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987, 1998; Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} - \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)]}_{O_{\mathrm{VA+AV}}}$$

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$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4}\Delta_k(g_0^2)O_k(a)\right]$$

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Vanishes if chiral symmetry is preserved (at least partially)

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Vanishes if chiral symmetry is preserved (at least partially)

Vanishes for staggered, GW, DW fermions

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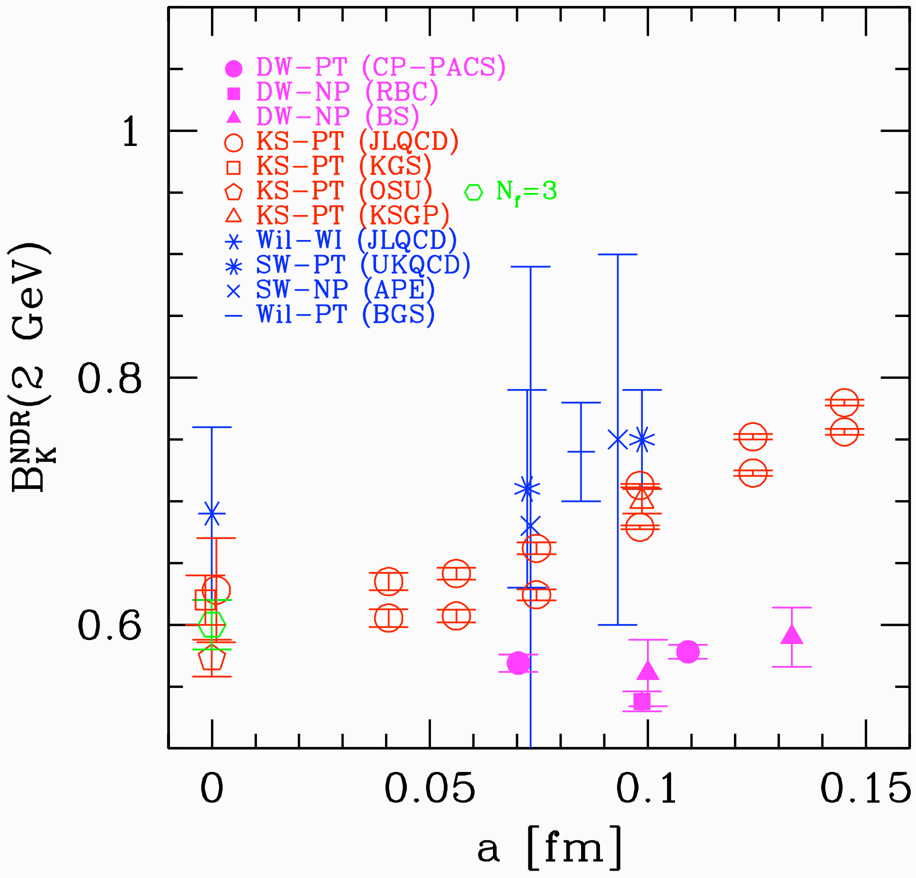
$$\bar{O}_{\mathrm{VA+AV}} = \lim_{a\to 0} Z_{\mathrm{VA+AV}}(g_0^2, a\mu)\, O_{\mathrm{VA+AV}}(a)$$

Protected from mixing by discrete symmetries *C P S(s ↔ d)*

B_K – a renormalisation classic

Subtractions flaw the quality of Wilson fermion results

L. Lellouch Nucl.Phys.Proc.Suppl.94(2001)142



Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly <u>exactly</u>.

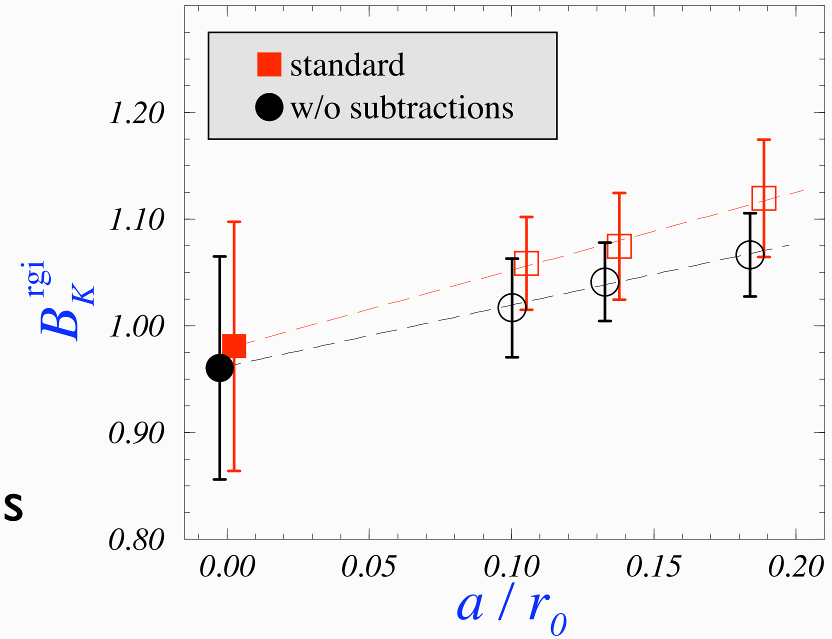
- Wilson I: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV})

D.Becirevic et al. Phys.Lett.B487(2000)74; Eur.Phys.J.C37(2004)315

$$< \bar{K}^0 | \, \delta O_R \, | K^0 > \quad = \quad < \bar{K}^0 | \, O_R \, [\partial_\mu A_\mu - 2mP] \, | K^0 >$$

$$\delta O_R = [O_{VV+AA}]_R$$

$$O_R = [O_{VA+AV}]_R$$



- subtractions traded off for fluctuations

ALPHA Collaboration quenched computation of B_K

Guagnelli, Heitger, Pena, Sint, A.V. JHEP 03 (2006) 088

Palombi, Pena, Sint JHEP 03 (2006) 089

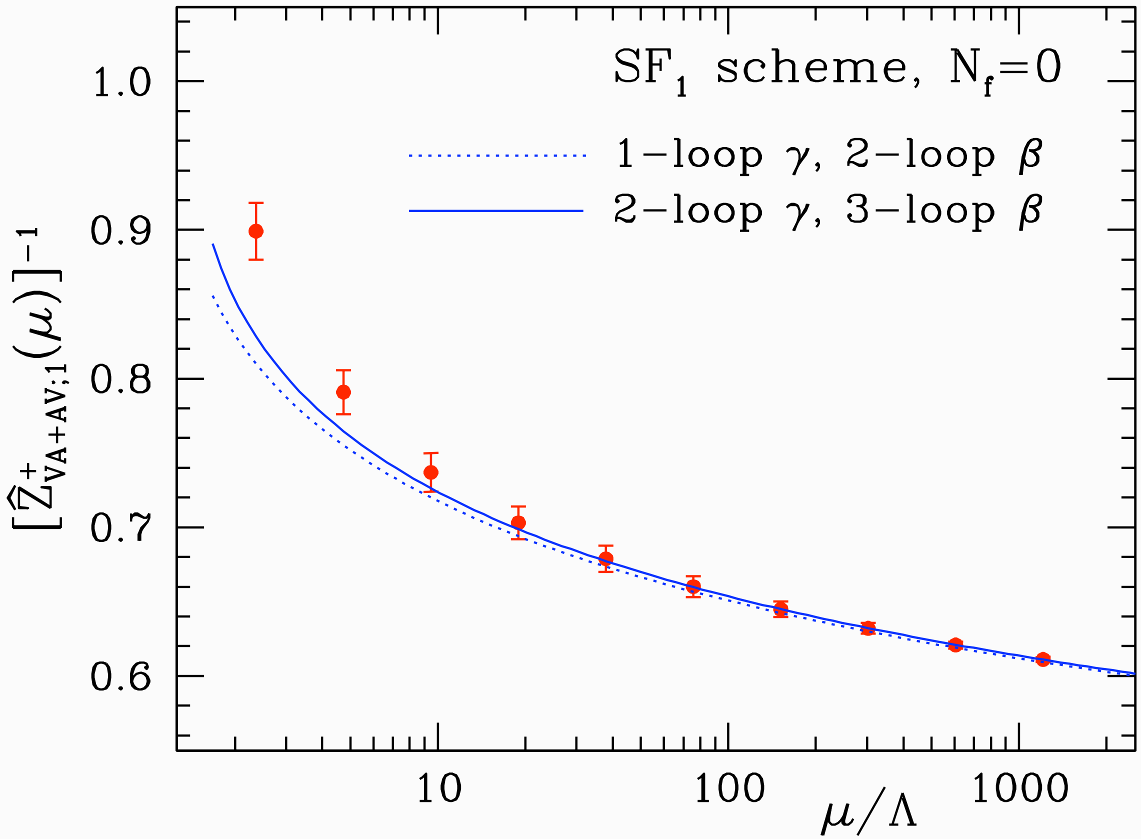
Dimopoulos, Heitger, Palombi, Pena, Sint, A.V. NPB 749 (2006) 69

- tmQCD → no operator mixing (no exceptional configurations).
- SF non-perturbative renormalisation.
- Various physical volumes: check control of finite volume effects.
- Two different regularisations: check control of the continuum limit.
- N.B.: action is O(a) improved, but four-fermion operator is *not* ⇒ continuum limit approached linearly in a.

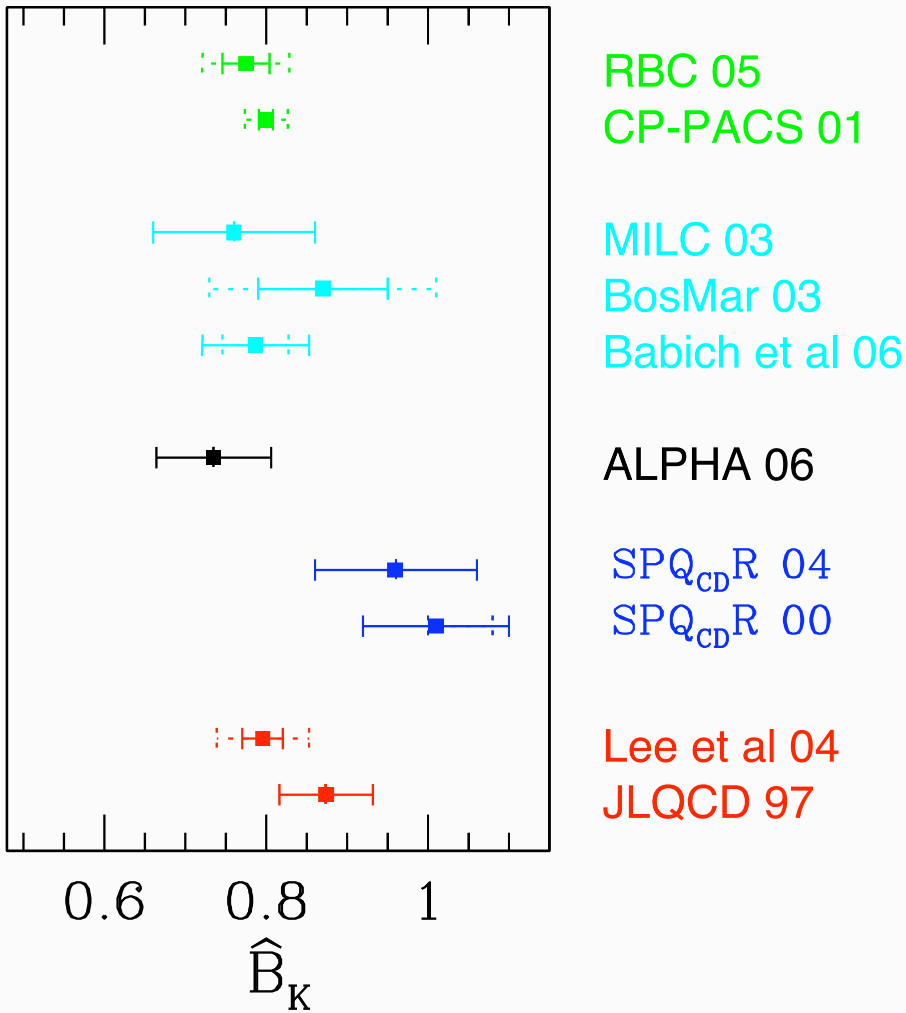
Approach to continuum: non-perturbative renormalisation

ALPHA, Guagnelli et al., JHEP 03 (2006) 088 ALPHA, Palombi et al., JHEP 03 (2006) 089

- SF technique via finite size scaling: split renormalisation into
  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
  - NP running to very high scales (~100 GeV) where contact with PT is made.



Comparison with quenched literature



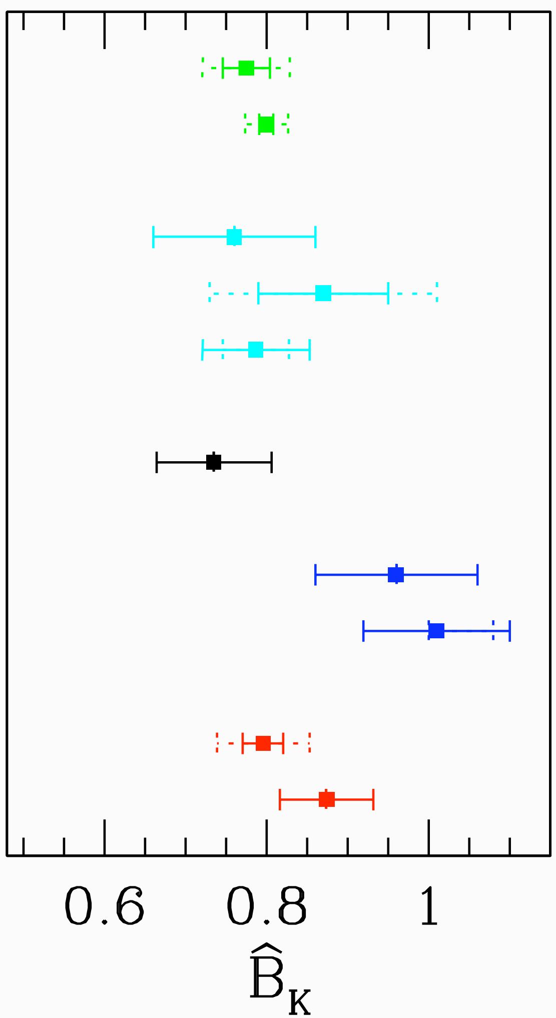
$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.534(52)$$

Difference with other Wilson fermion computations mainly due to method employed to extract B_K.

Comparison with quenched literature

C. Pena, PoS(Lat2006)019



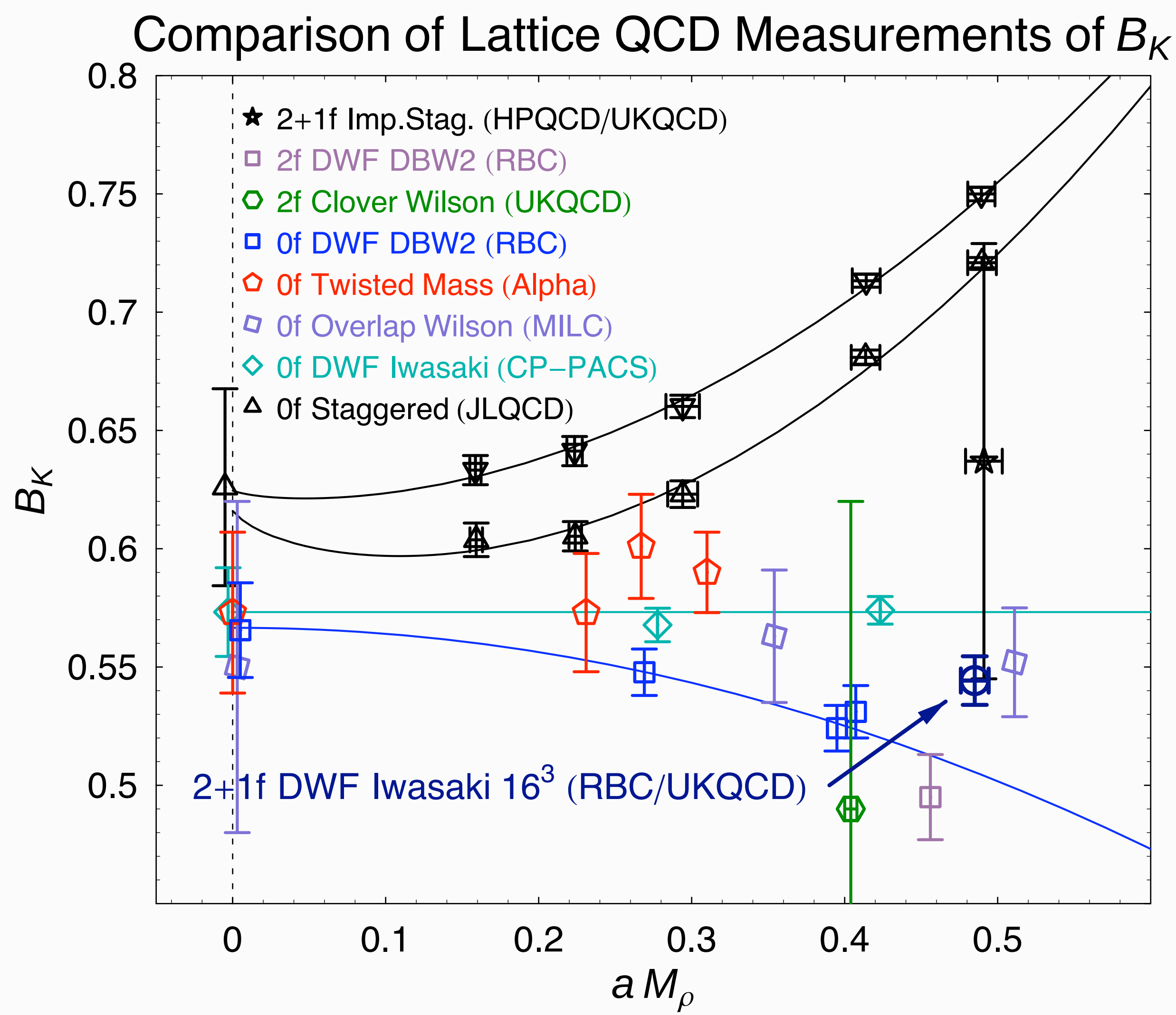
	no mass extrap	NP renormalisation	NP RG running	test FV effects	UV cutoff dep
RBC 05	● (green)	● (green)	● (red)	● (red)	● (orange)
CP-PACS 01	● (green)	● (red)	● (red)	● (green)	● (orange)
MILC 03	● (green)	● (red)	● (red)	● (red)	● (orange)
BosMar 03	● (green)	● (green)	● (red)	● (red)	● (red)
Babich et al 06	● (green)	● (green)	● (red)	● (red)	● (orange)
ALPHA 06	● (orange)	● (green)	● (green)	● (green)	● (green)
$SPQ_{CD}R$ 04	● (red)	● (green)	● (red)	● (red)	● (green)
$SPQ_{CD}R$ 00	● (red)	● (green)	● (red)	● (red)	● (green)
Lee et al 04	● (green)	● (red)	● (red)	● (red)	● (red)
JLQCD 97	● (green)	● (red)	● (red)	● (green)	● (green)

Recent unquenched result

RBC & UKQCD D.J. Antonio et al hep-ph/0702042

DW computation, N_f = 2+1; a = 0.12 fm

$$B_K^{\overline{\rm MS}}(2\rm GeV) \; = \; 0.557(12)(16)$$



B_K summary

collaboration	$\hat{B}_K$	N_f
JLQCD97 [12]	0.868(59)	0
Becirevic00 [20]	1.01(9)	0
CP-PACS01 [13]	0.795(29)	0
SPQCDR02 [10]	0.91(9)	0
BosMar03 [14]	0.87(8)	0
MILC03 [15]	0.79(9)	0
Babich06 [16]	0.79(8)	0
ALPHA06 [18]	0.735(71)	0
RBC03 [21]	0.697(33)	2
UKQCD04 [19]	0.67(18)	2
SPQCDR05 [11]	1.02(25)	2
RBC05 [17]	0.78(7)	2
RBC-UKQCD06 [22]	0.778(36)	2+1
HPQCD-UKQCD06 [23]	0.85(12)	2+1

N.Tantalo, CKM2006, hep-ph/0703241

- averaging is difficult: different groups use different approaches which suffer from different systematics
- keep only the latest unquenched results from each group (unless they change N_f etc.)

$$\hat{B}_K = 0.78 \pm 0.02 \pm 0.09$$

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weighted average

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semi-dispersion

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N.Tantalo, CKM2006, hep-ph/0703241

- compare to result obtained fom UT-fit : $\hat{B}_K = 0.68 \pm 0.10$

$$\hat{B}_K = 0.78 \pm 0.02 \pm 0.09$$

B_B summary

collaboration	$B_{B_s}(m_b)$	$B_B(m_b)$	N_f
UKQCD00 [32]	0.90(4)	0.91(6)	0
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JLQCD02 [35]	0.86(5)	0.84(6)	0
JLQCD03 [36]	0.850(64)	0.836(68)	2
Gadiyak05 [50]	0.864(76)	0.812(82)	2
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N.Tantalo, CKM2006, hep-ph/0703241

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N.Tantalo, CKM2006, hep-ph/0703241

Wilson-clover action
no static point

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N.Tantalo, CKM2006, hep-ph/0703241

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NRQCD

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N.Tantalo, CKM2006, hep-ph/0703241

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N.Tantalo, CKM2006, hep-ph/0703241

GW fermions
static

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N.Tantalo, CKM2006, hep-ph/0703241

staggered action
NRQCD

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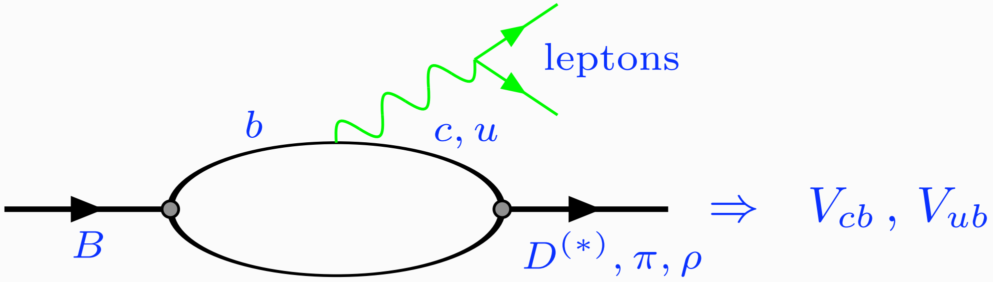
Semileptonic decays

$K \rightarrow \pi \, l \, \nu$

$B \rightarrow \pi \, l \, \nu$



K → π l ν decays

- decay rate

$$\Gamma = \frac{G_F^2}{192\pi^3} m_K^5 \cdot |V_{us}|^2 \cdot C^2 \cdot |f_+(0)|^2 \cdot I \cdot (1+\delta)$$

- gives the combination

$$|V_{us}|\ f_+(0)\ =\ 0.2173\ \pm\ 0.0008$$

- Cabibbo angle requests knowledge of *$f_+(0)$* with accuracy within *1%*

- it is a form factor of the neutral Kaon decay:

$$< \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|K(p_K)> =\ (p_\pi + p_K)_\mu\ f_+(q^2)\ + q_\mu\ f_-(q^2) \qquad q = p_K - p_\pi$$

- in principle one uses above to extract the form factors at several momenta transfers and extrapolate to the *$q = 0$* point (using various Ansätze)

- NB: on the lattice momenta are discretized and only the low ones ($p = 0, 2\pi/L$) are useful (higher ones introduce fluctuations, unwanted systematic effects etc.)

- the requested high accuracy requires use of “clever” ratios of correlation functions, in order to cancel fluctuations, unwanted chiral effects etc.

$K \rightarrow \pi\, l\, \nu$ decays

- recent accurate quenched result D. Becirevic et al., Nucl.Phys.B705(2005)339
- one lattice spacing a = 0.066 fm, several masses and momenta

$$f_+(0) \;=\; 0.960 \pm 0.005 \pm 0.007$$

- encouraging comparison with χPT calculation

H. Leutwyler & M.Roos, Z.Phys. C25 (1984) 91

$$f_+(0) \;=\; 0.961 \pm 0.008$$

- unquenched computations have begun:
    - DW fermions, $N_f = 2$, $a = 0.12$ fm, $L = 1.9$ fm

RBC Collab. C. Dawson et al., Phys.Rev. D74 (2006) 114502

K → π l ν decays

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$$f_+(0) \; = \; 0.961 \pm 0.008$$

- unquenched computations have begun:
  - Clover-Wilson fermions, N_f = 2, a = 0.09 fm, L = 1.8 fm
  - improved action but not operator (current)

JLQCD Collab. N.Tsutsui et al., PoS LAT2005 357

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- unquenched computations have begun:
  - N_f = 2+1 staggered light quarks, Clover-Wilson strange

HPQCD/FNAL/MILC Collab. M.Okamoto hep-lat/0412044

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- one lattice spacing a = 0.066 fm, several masses and momenta

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$$f_+(0) \;=\; 0.961 \pm 0.008$$

- unquenched computations have begun:

collaboration	$f_+(0)$
RBC	$0.968(9)(6)\ (N_F = 2)$
HPQCD/FNAL	$0.962(6)(9)\ (N_F = 2+1)$
JLQCD	$0.952(6)(-)\ (N_F = 2)$

$B \to \pi\, l\, \nu$ decays

- similar approach reviewed by T.Onogi, PoS(LAT2006)017
- the physics:

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{k}_\pi|^3|f^+(q^2)|^2.$$

- the form factors:

$$\langle\pi(k_\pi)|\bar{q}\gamma^\mu b|B(p_B)\rangle = f^+(q^2)\left[(p_B+k_\pi)^\mu - \frac{m_B^2-m_\pi^2}{q^2}q^\mu\right] + f^0(q^2)\frac{m_B^2-m_\pi^2}{q^2}q^\mu$$

- unquenched N_F=2+1 computations have begun:
  - FNAL/MILC: M.Okamoto et al., Nucl.hys.B(PS)140(2005)461
  - staggered light flavours, HQET(Fermilab) heavy flavours
  - HPQCD/MILC: E.Gulez et al., Phys.Rev.D73(2006)074502
  - staggered light flavours, NRQCD heavy flavours

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- unquenched N_F=2+1 computations have begun:
  - FNAL/MILC: HQET(Fermilab) heavy flavours

$$|V_{ub}| = [3.76 \pm 0.25 \pm 0.65] \times 10^{-3}$$

  - HPQCD/MILC: NRQCD heavy flavours

$$|V_{ub}| = [4.22 \pm 0.30 \pm 0.51] \times 10^{-3}$$

- NB: same light quark ensemble

$$\Delta I = 1/2$$

$$K \rightarrow \pi\ \pi$$

$K \to \pi\pi$ decays in a nutshell

- If CP is conserved the eigenstates of the Hamiltonian are $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle)$,
  CP violation in the SM leads to mixing:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_1\rangle + \bar{\varepsilon}|K_2\rangle) \quad |K_L\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_2\rangle + \bar{\varepsilon}|K_1\rangle) \qquad \bar{\varepsilon} = \frac{p-q}{p+q}$$

- CP violation parameters accessible via decay amplitudes into two pions:

$$-iT[K^0 \to (\pi\pi)_I] = A_I\, e^{i\delta_I} \qquad T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2e^{i\delta_I}\sin\delta_I$$

$$\varepsilon = \frac{T[K_L \to (\pi\pi)_0]}{T[K_S \to (\pi\pi)_0]} \simeq \bar{\varepsilon} + i\frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}$$

$$\varepsilon' = \frac{\varepsilon}{\sqrt{2}}\left(\frac{T[K_L \to (\pi\pi)_2]}{T[K_L \to (\pi\pi)_0]} - \frac{T[K_S \to (\pi\pi)_2]}{T[K_S \to (\pi\pi)_0]}\right) \simeq \frac{1}{\sqrt{2}}e^{i(\delta_2-\delta_0+\pi/2)}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

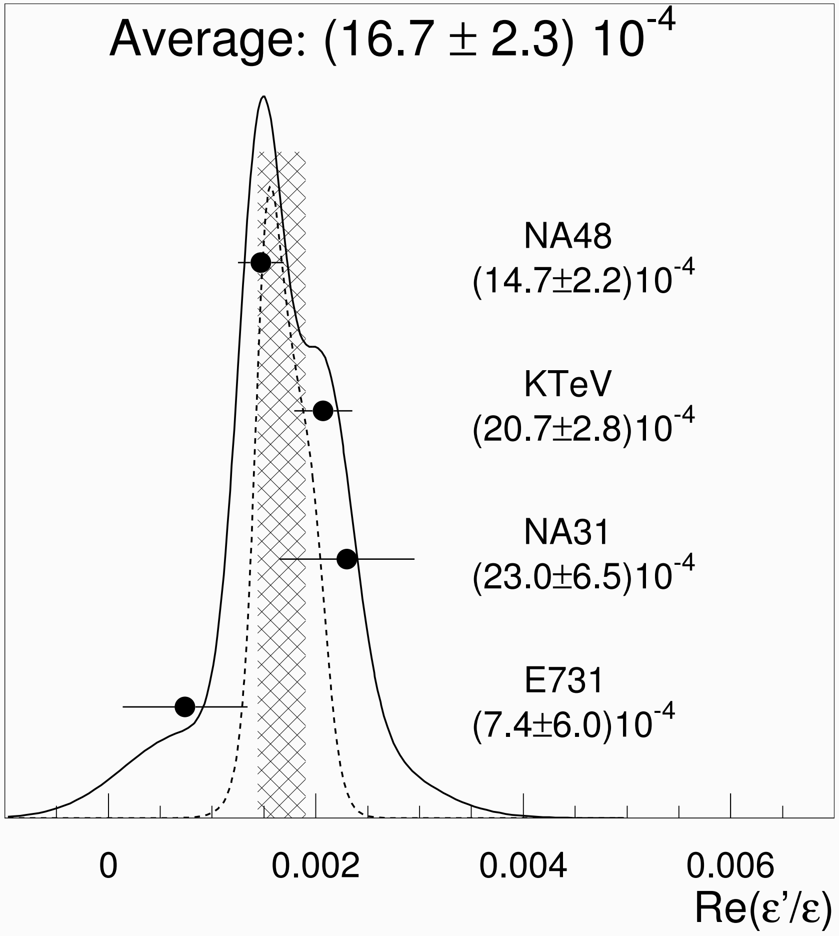
$K \to \pi\pi$ decays in a nutshell

Experiment:

$$\left|\frac{A_0}{A_2}\right| \simeq 22.1$$

$$|\varepsilon| = (2.282 \pm 0.017) \times 10^{-3}$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (16.7 \pm 2.3) \times 10^{-4}$$



The $\Delta I=1/2$ rule for kaon decays

$$T(K \to (\pi\pi)_\alpha) = i A_\alpha e^{i\delta_\alpha}, \quad \alpha = 0, 2 \qquad\qquad |A_0/A_2| = 22.1$$

- Bulk of enhancement in the SM must come from long-distance strong interaction effects ...

  Gaillard & Lee, PRL 33 (1974) 108
  Altarelli & Maiani, PLB 52 (1974) 351

- ... that have to be addressed non-perturbatively.

  Cabibbo, Martinelli & Petronzio, NPB 244 (1984) 381
  Brower, Maturana, Gavela & Gupta, PRL 53 (1984) 1318

- Lattice QCD studies hampered by no-go theorems on chiral fermions and multiparticle decays, almost no activity in the '90s.

- Theoretical breakthroughs in late '90s (mainly chiral lattice fermions) have led to a renewed interest and some "rough" lattice results.

  CP-PACS & RBC Collaborations

- Still far from having an understanding of the mechanism(s) behind the enhancement.

Effective Weak Hamiltonian

$$\mathcal{A}(i \to f) \approx \langle f | H_{\mathrm{W}}^{\mathrm{eff}} | i \rangle$$

$$H_{\mathrm{W}}^{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_k f_k(V_{\mathrm{CKM}}) C_k(\mu/M_W) \bar{O}_k(\mu)$$

Effective Weak Hamiltonian

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CKM parameters

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Wilson coefficients — high energy, NLO computation

Composite operators — low energy (hadronic) scales

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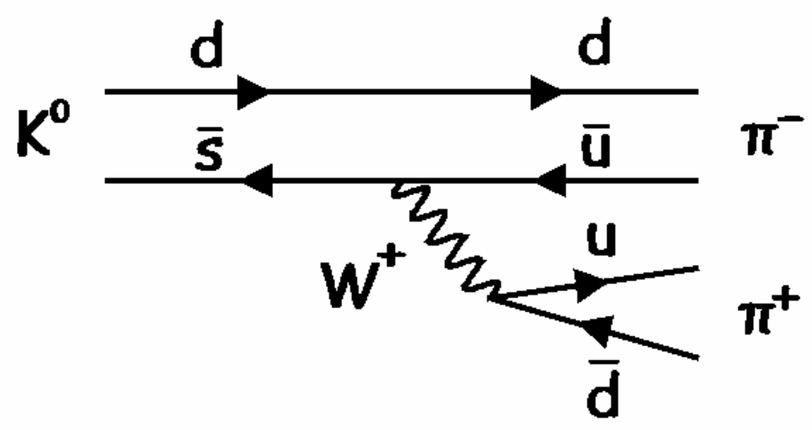
Composite operators — low energy (hadronic) scales



Effective Weak Hamiltonian

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$$H_{\mathrm{W}}^{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_k f_k(V_{\mathrm{CKM}}) C_k(\mu / M_{\mathrm{W}}) \bar{O}_k(\mu)$$

With an active charm quark (CP-violating effects neglected):

$$H_{\mathrm{w}} = \frac{g_{\mathrm{w}}^2}{2M_{\mathrm{W}}^2} (V_{us})^* (V_{ud}) \sum_{\sigma=\pm} \{ k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2 \}$$

$$\mathcal{Q}_1^\pm = (\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u) \; - \; [u \; \rightarrow \; c]$$

$$\mathcal{Q}_2^\pm = (m_u^2 - m_c^2) \{ m_d(\bar{s}P_+ d) \; + \; m_s(\bar{s}P_- d) \}$$

$\mathcal{Q}_1^\pm$ transform according to irreps of d=84 (+) and d=20 (-) of SU(4).

$\mathcal{Q}_2^\pm$ do not contribute to the physical K→ππ transition.

Effective Weak Hamiltonian

$$\mathcal{A}(i \to f) \approx \langle f | H_{\mathrm{W}}^{\mathrm{eff}} | i \rangle$$

$$H_{\mathrm{W}}^{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_k f_k(V_{\mathrm{CKM}}) C_k(\mu / M_W) \bar{O}_k(\mu)$$

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(+): ΔI=3/2, 1/2 (-): ΔI=1/2 only

Effective Weak Hamiltonian

$$H_{\rm w} = \frac{g_{\rm w}^2}{2M_W^2}(V_{us})^*(V_{ud}) \sum_{\sigma=\pm} \{k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2\}$$

$$\mathcal{Q}_1^\pm = (\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u) \ - \ [u \ \rightarrow \ c]$$

$$\mathcal{Q}_2^\pm = (m_u^2 - m_c^2)\,\{m_d(\bar{s}P_+ d) \ + \ m_s(\bar{s}P_- d)\}$$

$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)}\,\frac{\langle(\pi\pi)_{I=0}|\hat{Q}_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|\hat{Q}_1^+|K\rangle} \qquad\qquad \frac{k_1^-(M_W)}{k_1^+(M_W)} = 2.8 \ \sim O(1)$$

Enhancement dominated by matrix elements of effective interaction vertices (long-distance regime of the strong interaction).

Well, let's compute the matrix elements ...

A tale of various scales

M_W $\quad \mathcal{H}_{SM} \to \mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (k_+ Q_+ + k_- Q_-)$

$$Q_\pm \equiv [\bar{s}u]_{V-A}[\bar{u}d]_{V-A} \pm [\bar{s}d]_{V-A}[\bar{u}u]_{V-A} - (u \leftrightarrow c)$$

$SU(4)_L \times SU(4)_R$: $Q_+ \to (84,1) \quad Q_- \to (20,1)$

m_c $\quad \mathcal{H}^{N_f=4}_{\Delta S=1} \to \mathcal{H}^{N_f=3}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} \sum_{\sigma=1,10} C_\sigma Q_\sigma$

$$Q_\sigma : \ldots, [\bar{s}d]_{V-A}[\bar{q}q]_{V+A}, \ldots$$

$SU(3)_L \times SU(3)_R$: $(27,1) \to A_2, A_0, (8,1) \to A_0$

Λ_χ $\quad \mathcal{H}^{N_f=3}_{\Delta S=1} \quad \to \quad \mathcal{H}^{N_f=3}_{\chi PT}$

A tale of various scales

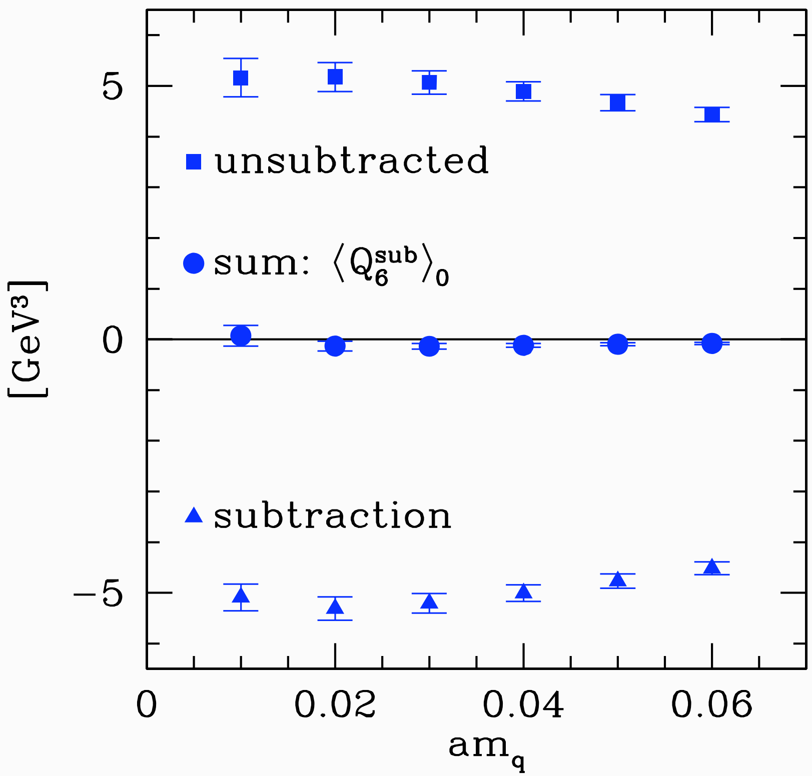
The standard [?] lore:

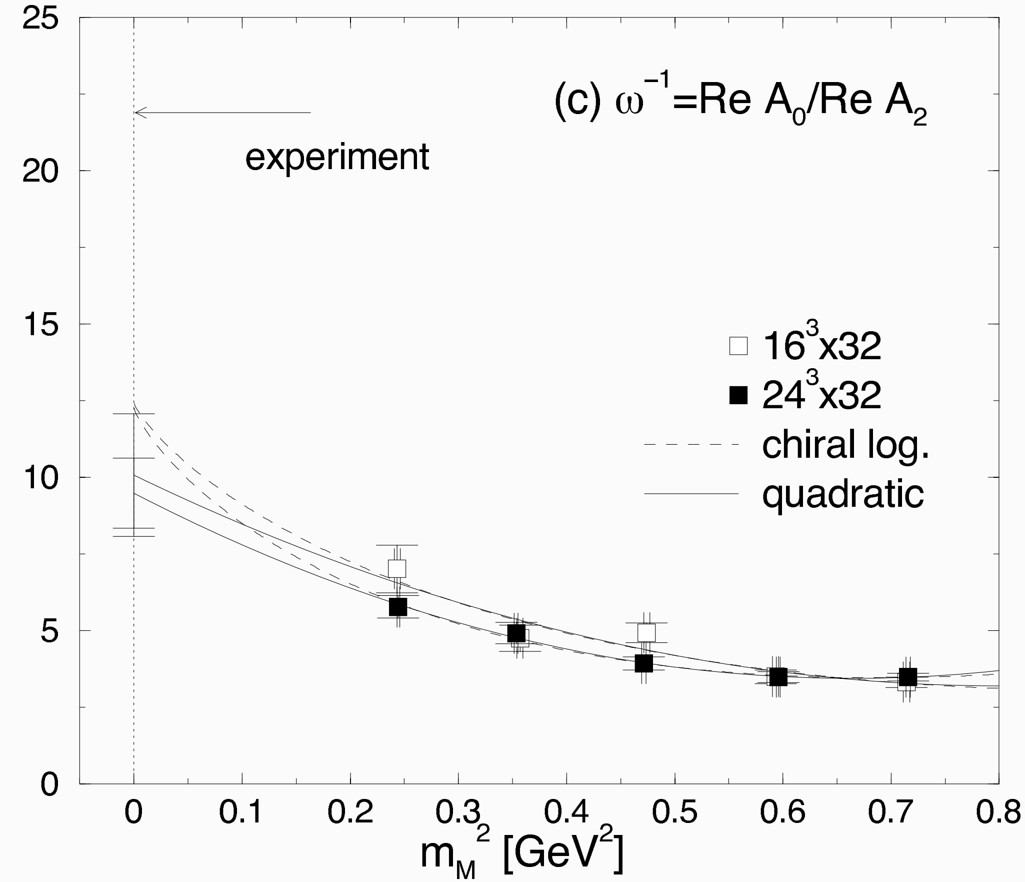
- Resummation of $O(1/N)\,\log(\mu/M_W)$ up to $\mu > m_c$ gives a moderate enhancement.
- Charm threshold: $\mu < m_c \longrightarrow$ penguins.
- Penguin matrix elements can be large compared to that of left-left operators.

Still to be verified/discarded via an explicit computation ...

Shifman, Vainshtein, Zakharov 1977; Bardeen, Buras, Gerard 1986

Existing results for A_0, A_2?





Lightest pion mass around 495 MeV.

CP-PACS Collaboration (Ali Khan et al.) 01

New strategy to reveal the role of the charm

Disentangle several possible origins/contributions:

- Physics at the charm scale (via penguins).
- Physics at intrinsic QCD scale ~200-300 MeV.
- Final state interactions.
- All of the above (no dominating "mechanism").

Separate "intrinsic QCD" effects from physics at the charm scale:

Consider effective weak Hamiltonian with an active charm and study A_0, A_2 as a function of m_c.

$$m_u = m_d = m_s = m_c$$

$$\downarrow$$

$$m_u = m_d = m_s \ll m_c$$

Giusti, Hernández, Laine, Weisz & Wittig, JHEP 11 (2004) 016

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Effective low-energy description

Dynamics of Goldstone bosons @ LO:

$$\mathcal{L}_{\rm E} = \tfrac{1}{4}F^2{\rm Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] - \tfrac{1}{2}\Sigma{\rm Tr}\left[U M^\dagger e^{i\theta/N_{\rm f}} + \text{ h.c.}\right]$$

$$U \in {\rm SU}(4)\,, \quad M = \text{ mass matrix}$$

Low-energy counterpart of the weak effective Hamiltonian @ LO:

$$\mathcal{H}_{\rm w}^{\chi{\rm PT}} = \frac{g_w^2}{2M_W^2}(V_{us})^*(V_{ud})\sum_{\sigma=\pm} g_1^\sigma \left\{[\widehat{\mathcal{O}}_1^\sigma]_{suud} - [\widehat{\mathcal{O}}_1^\sigma]_{sccd}\right\}$$

$$[\widehat{\mathcal{O}}_1]_{\alpha\beta\gamma\delta} = \tfrac{1}{4}F^4(U\partial_\mu U^\dagger)_{\gamma\alpha}(U\partial_\mu U^\dagger)_{\delta\beta}$$

Relation of LEC's to $K\rightarrow\pi\pi$ transition amplitudes @ LO in χPT:

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3}{2}\frac{g_1^-}{g_1^+}\right)$$

⇒ Determine LEC's using lattice QCD

Matching QCD to the chiral expansion

$$R^{\pm}(x_0,y_0) = \frac{C^{\pm}(x_0,y_0)}{C(x_0)C(y_0)}$$

$$C^{\pm}(x_0,y_0) = \sum_{\mathbf{x},\mathbf{y}} \langle [J_0(x)]_{du} [Q_1^{\pm}(0)] [J_0(y)]_{us} \rangle$$

$$C(x_0) = \sum_{\mathbf{x}} \langle [J_0(x)]_{ds} [J_0(0)]_{sd} \rangle$$

QCD →

$$k^{\pm}_{\mathrm{RGI}} \left[\frac{Z^{\pm}}{Z_{\mathrm{A}}^2}\right]_{\mathrm{RGI}} R^{\pm} = g^{\pm} \mathcal{R}^{\pm}(m, V, \mathrm{LECs})$$

← **χPT**

$$\widehat{\mathcal{C}}^{\pm}(x_0,y_0) = \int \mathrm{d}^3x\, \mathrm{d}^3y\, \langle \mathcal{J}_0(x)\, \mathcal{O}_1^{\pm}(0) \mathcal{J}_0(y) \rangle$$

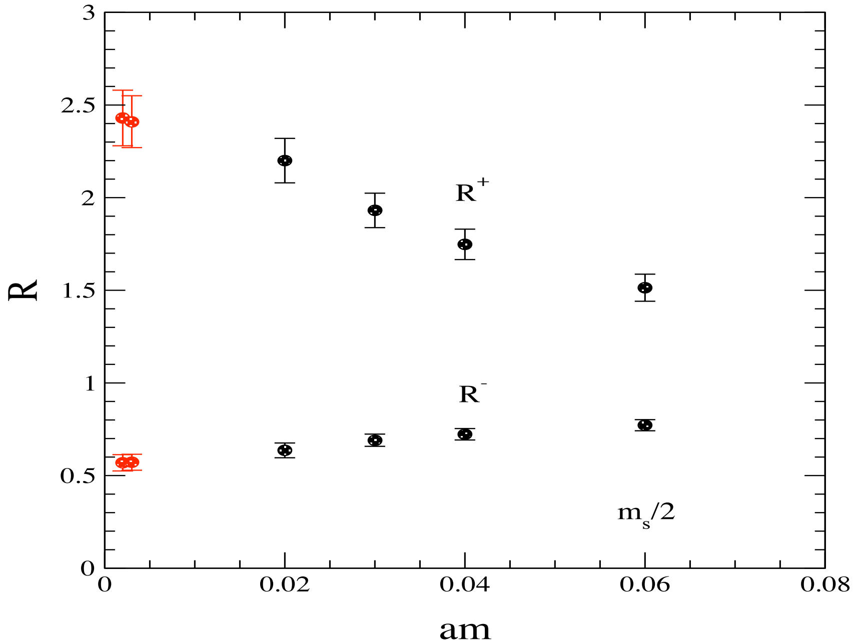
$$\mathcal{C}(x_0) = \int \mathrm{d}^3x\, \langle \mathcal{J}_0(x)\, \mathcal{J}_0(0) \rangle$$

$$\mathcal{R}^{\pm}(x_0,y_0) = \frac{\widehat{\mathcal{C}}^{\pm}(x_0,y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

- p-regime: new LECs appear at NLO
- ε-regime: no additional $\Delta S=1$ interaction terms at $O(\varepsilon^2)$ ⇒ enables matching at NLO!

Results: $K \to \pi\pi$ amplitudes in the chiral limit

Giusti, Hernández, Laine, Pena, Wennekers, Wittig 2006



	g^+	g^-
This work	$0.51(3)(5)(6)$	$2.6(1)(3)(3)$
"Exp"	~ 0.5	~ 10.4
Large N_c	1	1

- $\Delta I=3/2$ comes in the right ballpark (N.B.: charm effects enter only via quark loops).
- $\Delta I=1/2$ channel and amplitude ratio are a factor ~4 too small.
- Enhancement of the $\Delta I=1/2$ channel already present with an unphysically light charm quark ($A_0/A_2 \sim 6$): "pure no-penguin" effect.

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Conclusions

- The lattice is a rigorously defined regularization of QCD (the only one?).
- As such, it enables non-pertrubative computations at low energies, from first principles, without any model assumptions.
- The price to pay is the presence of a plethora of systematic effects. They can be kept under control and are being systematically reduced.
- The control of these effects is not just the result of better hardware an software, but principally stems from a better theoretical understanding of non-perturbative QFT at fixed UV cutoff.
- We are currently moving away from uncontrolled approximations (quenching) and approach a realistic situation of $N_f = 2 + 1 + 1$. Moreover, we are approaching the most "critical" areas of the QCD parameter space (chiral limit, heavy flavours).
- The result of this progress is that lattice QCD is a mature field, capable of providing reliably some missing puzzles in Standard Model phenomenology.