Strangeness conservation and pair correlations of neutral kaons with close momenta produced in inclusive multiparticle processes

V. L. Lyuboshitz, Valery V. Lyuboshitz
(Joint Institute for Nuclear Research, Dubna, Russian Federation)

XI Frascati Spring School in Nuclear, Subnuclear and Astroparticle Physics – „Physics in the LHC era"
LNF, Frascati, Italy, May 15–19, 2006

1. Consequences of strangeness conservation in inclusive processes.

[V. L. Lyuboshitz. Yad. Fiz. 23, 1266 (1976) [Sov. J. Nucl. Phys. 23, 673 (1976)]
V. L. Lyuboshitz and M. I. Podgoretsky. Yad. Fiz. 30, 789 (1979) [Sov. J. Nucl. Phys. 30, 407 (1979)]]

Pairs $|K^0\rangle^{(\vec{p}_1)} \otimes |K^0\rangle^{(\vec{p}_2)}$, $|\bar{K}^0\rangle^{(\vec{p}_1)} \otimes |\bar{K}^0\rangle^{(\vec{p}_2)}$, $|K^0\rangle^{(\vec{p}_1)} \otimes |\bar{K}^0\rangle^{(\vec{p}_2)}$, $|\bar{K}^0\rangle^{(\vec{p}_1)} \otimes |K^0\rangle^{(\vec{p}_2)}$.

Density matrix in the representation of states with definite strangeness: $(\rho_{\bar{K}^0K^0 \to K^0\bar{K}^0} = \rho^*_{K^0\bar{K}^0 \to \bar{K}^0K^0})$

$$\begin{pmatrix} f_{K^0K^0} & 0 & 0 & 0 \\ 0 & f_{\bar{K}^0\bar{K}^0} & 0 & 0 \\ 0 & 0 & f_{K^0\bar{K}^0} & \rho_{K^0\bar{K}^0 \to \bar{K}^0K^0} \\ 0 & 0 & \rho_{\bar{K}^0K^0 \to K^0\bar{K}^0} & f_{\bar{K}^0K^0} \end{pmatrix}$$

Non-diagonal elements of the density matrix → correspond to the transitions $K^0\bar{K}^0 \to \bar{K}^0K^0$, $\bar{K}^0K^0 \to K^0\bar{K}^0$

$f_{K^0K^0}, f_{\bar{K}^0\bar{K}^0}, f_{K^0\bar{K}^0}, f_{\bar{K}^0K^0} \Rightarrow$ two-particle structure functions in the representation of states with definite strangeness.

(~ double inclusive cross-sections)

Due to the strangeness conservation

→ Incoherent production of the pairs of neutral kaons

$$K^0K^0(S=+2);\ \bar{K}^0\bar{K}^0(S=-2);\ K^0\bar{K}^0(S=0).$$

Internal states of two identical neutral kaons and two non-identical neutral kaons ⟹ superpositions of the states $|K_S^0\rangle|K_S^0\rangle$, $|K_L^0\rangle|K_L^0\rangle$, $|K_S^0\rangle|K_L^0\rangle$.

$$|K^0\rangle=\frac{1}{\sqrt{2}}\left(|K_S^0\rangle+|K_L^0\rangle\right);\quad |\bar{K}^0\rangle=\frac{1}{\sqrt{2}}\left(|K_S^0\rangle-|K_L^0\rangle\right)$$

$|K_S^0\rangle \rightarrow$ short-lived neutral kaon ($CP=+1$)

$|K_L^0\rangle \rightarrow$ long-lived neutral kaon ($CP=-1$)

$$|K^0\rangle^{(\vec{p}_1)}\otimes|K^0\rangle^{(\vec{p}_2)}=\frac{1}{2}\Big(|K_S^0\rangle^{(\vec{p}_1)}\otimes|K_S^0\rangle^{(\vec{p}_2)}+|K_L^0\rangle^{(\vec{p}_1)}\otimes|K_L^0\rangle^{(\vec{p}_2)}+$$
$$+|K_S^0\rangle^{(\vec{p}_1)}\otimes|K_L^0\rangle^{(\vec{p}_2)}+|K_L^0\rangle^{(\vec{p}_1)}\otimes|K_S^0\rangle^{(\vec{p}_2)}\Big)$$

$$|\bar{K}^0\rangle^{(\vec{p}_1)}\otimes|\bar{K}^0\rangle^{(\vec{p}_2)}=\frac{1}{2}\Big(|K_S^0\rangle^{(\vec{p}_1)}\otimes|K_S^0\rangle^{(\vec{p}_2)}+|K_L^0\rangle^{(\vec{p}_1)}\otimes|K_L^0\rangle^{(\vec{p}_2)}-$$
$$-|K_S^0\rangle^{(\vec{p}_1)}\otimes|K_L^0\rangle^{(\vec{p}_2)}-|K_L^0\rangle^{(\vec{p}_1)}\otimes|K_S^0\rangle^{(\vec{p}_2)}\Big).$$

Pair $K^0\bar{K}^0 \rightarrow$ always $CP=+1$ ($C=(-1)^L$, $P=(-1)^L$, $L \rightarrow$ orbital momentum)

System of two non-identical neutral kaons $K^0\bar{K}^0$ in the symmetric internal state (even orbital momenta) $\Rightarrow$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle^{(\vec{p}_1)} \otimes |\bar{K}^0\rangle^{(\vec{p}_2)} + |\bar{K}^0\rangle^{(\vec{p}_1)} \otimes |K^0\rangle^{(\vec{p}_2)}\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|K_S^0\rangle^{(\vec{p}_1)} \otimes |K_S^0\rangle^{(\vec{p}_2)} - |K_L^0\rangle^{(\vec{p}_1)} \otimes |K_L^0\rangle^{(\vec{p}_2)}\right). \qquad C=+1$$

System $K^0\bar{K}^0$ in the antisymmetric internal state (odd orbital momenta) $\Rightarrow$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle^{(\vec{p}_1)} \otimes |\bar{K}^0\rangle^{(\vec{p}_2)} - |\bar{K}^0\rangle^{(\vec{p}_1)} \otimes |K^0\rangle^{(\vec{p}_2)}\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|K_S^0\rangle^{(\vec{p}_1)} \otimes |K_L^0\rangle^{(\vec{p}_2)} - |K_L^0\rangle^{(\vec{p}_1)} \otimes |K_S^0\rangle^{(\vec{p}_2)}\right). \qquad C=-1$$

Consequences of strangeness conservation

1) Invariance of two-particle structure functions (in the representation of states with definite CP-parity) under the replacement of K_S^0 by K_L^0 and vice versa:

$$f_{SS}(\vec{p}_1,\vec{p}_2) = f_{LL}(\vec{p}_1,\vec{p}_2) =$$

$$= \frac{1}{4}\left(f_{K^0K^0}(\vec{p}_1,\vec{p}_2) + f_{\bar{K}^0\bar{K}^0}(\vec{p}_1,\vec{p}_2) + f_{K^0\bar{K}^0}(\vec{p}_1,\vec{p}_2) + f_{\bar{K}^0K^0}(\vec{p}_1,\vec{p}_2)\right) +$$

$$+ \frac{1}{2}\,\mathrm{Re}\,\rho_{K^0\bar{K}^0 \rightarrow \bar{K}^0K^0}(\vec{p}_1,\vec{p}_2);$$

$$f_{SL}(\vec{p}_1,\vec{p}_2) = f_{LS}(\vec{p}_1,\vec{p}_2) =$$

$$= \frac{1}{4}\left(f_{K^0K^0}(\vec{p}_1,\vec{p}_2) + f_{\bar{K}^0\bar{K}^0}(\vec{p}_1,\vec{p}_2) + f_{K^0\bar{K}^0}(\vec{p}_1,\vec{p}_2) + f_{\bar{K}^0K^0}(\vec{p}_1,\vec{p}_2)\right) -$$

$$- \frac{1}{2} \operatorname{Re} \rho_{K^0\bar{K}^0 \to \bar{K}^0K^0}(\vec{p}_1,\vec{p}_2)$$

(It is well seen that the difference between f_{SS} and f_{SL} is connected with the non-diagonal elements of the density matrix)

2) $f_{SS}(\vec{p}_1,\vec{p}_2) = f_{SS}(\vec{p}_2,\vec{p}_1)$; $f_{SL}(\vec{p}_1,\vec{p}_2) = f_{SL}(\vec{p}_2,\vec{p}_1)$;
$f_{LL}(\vec{p}_1,\vec{p}_2) = f_{LL}(\vec{p}_2,\vec{p}_1)$;

(All double inclusive cross-sections of production of pairs $K_S^0K_S^0$, $K_L^0K_L^0$, $K_S^0K_L^0$ → symmetric with respect to the permutation $\vec{p}_1 \to \vec{p}_2$, $\vec{p}_2 \to \vec{p}_1$; $f(\vec{p}_1,\vec{p}_2) = E_1E_2 \frac{d^6\sigma}{d^3\vec{p}_1 d^3\vec{p}_2}$)

3) Mutual equality of the average multiplicities of the K_S^0- and K_L^0-states and the average squares of multiplicities (after integrating the previous relations):

$$\langle n_S \rangle = \langle n_L \rangle; \quad \langle n_S^2 \rangle = \langle n_L^2 \rangle.$$

2 Structure of pair correlations of identical and non-identical neutral kaons with close momenta

The model of independent one-particle sources

[R. Lednicky, V. L. Lyuboshitz, Yad. Fiz. 35, 1316 (1982)
[Sov. J. Nucl. Phys. 35, 770 (1982)]

1) Identical states $K_S^0 K_S^0$ and $K_L^0 K_L^0$.

Expressions for the correlation functions $R_{SS}(\vec{k})$, $R_{LL}(\vec{k})$:

$$R_{SS}(\vec{k}) = R_{LL}(\vec{k}) = \lambda_{K^0K^0}\left(1 + F_{K^0}(2\vec{k}) + 2\, b_{int}(\vec{k})\right) + \\ + \lambda_{\bar{K}^0\bar{K}^0}\left(1 + F_{\bar{K}^0}(2\vec{k}) + 2\, \tilde{b}_{int}(\vec{k})\right) + \\ + \lambda_{K^0\bar{K}^0}\left(1 + F_{K^0\bar{K}^0}(2\vec{k}) + 2\, B_{int}(\vec{k})\right)$$

$\vec{k} \to$ momentum of one of the kaons in the c.m. frame of the pair;
(normalization to unity at large $\vec{k}$)

$\lambda_{K^0K^0}$, $\lambda_{\bar{K}^0\bar{K}^0}$ and $\lambda_{K^0\bar{K}^0} \to$ relative fractions of the average numbers of produced pairs K^0K^0, $\bar{K}^0\bar{K}^0$ and $K^0\bar{K}^0$, respectively

$$\left(\lambda_{K^0K^0} = \frac{N_{K^0K^0}}{N},\ \lambda_{\bar{K}^0\bar{K}^0} = \frac{N_{\bar{K}^0\bar{K}^0}}{N},\ \lambda_{K^0\bar{K}^0} = \frac{N_{K^0\bar{K}^0}}{N}; \right.$$

$$\left. \lambda_{K^0K^0} + \lambda_{\bar{K}^0\bar{K}^0} + \lambda_{K^0\bar{K}^0} = 1 \right)$$

„Formfactors" $F_{K^0}(2\vec{k})$, $F_{\bar{K}^0}(2\vec{k})$, $F_{K^0\bar{K}^0}(2\vec{k})$ $\Rightarrow$ appear due to the contribution of Bose-statistics:

$$F_{K^0}(2\vec{k}) = \int W_{K^0}(\vec{r}) \cos 2\vec{k}\vec{r}\, d^3\vec{r};$$

$$F_{\bar{K}^0}(2\vec{k}) = \int W_{\bar{K}^0}(\vec{r}) \cos 2\vec{k}\vec{r}\, d^3\vec{r};$$

$$F_{K^0\bar{K}^0}(2\vec{k}) = \int W_{K^0\bar{K}^0}(\vec{r}) \cos 2\vec{k}\vec{r}\, d^3\vec{r}.$$

($W_{K^0}(\vec{r})$, $W_{\bar{K}^0}(\vec{r})$, $W_{K^0\bar{K}^0}(\vec{r})$ $\Rightarrow$ probability distributions of distances between two sources of K^0-mesons, two sources of $\bar{K}^0$-mesons and between the sources of K^0-mesons and $\bar{K}^0$-mesons, respectively, in the c.m. frame of the kaon pair).

Contribution of the s-wave strong final-state interaction:

$b_{int}(\vec{k}) \to$ for two K^0-mesons;

$\tilde{b}_{int}(\vec{k}) \to$ for two $\bar{K}^0$-mesons;

$B_{int}(\vec{k}) \to$ for the K^0-meson and $\bar{K}^0$-meson.

General structure of b_{int}, $\tilde{b}_{int}$ and B_{int}: due to CP-invariance

$$b_{int}(\vec{k}) = \int W_{K^0}(\vec{r})\, b(\vec{k},\vec{r})\, d^3\vec{r}, \quad \tilde{b}_{int}(\vec{k}) = \int W_{\bar{K}^0}(\vec{r})\, b(\vec{k},\vec{r})\, d^3\vec{r}.$$

$$B_{int}(\vec{k}) = \int W_{K^0\bar{K}^0}(\vec{r})\, B(\vec{k},\vec{r})\, d^3\vec{r}.$$

(For $b_{int}(\vec{k})$ and $\tilde{b}_{int}(\vec{k}) \to$ the function $b(\vec{k},\vec{r})$ is the same;
for $B_{int}(\vec{k}) \to B(\vec{k},\vec{r}) \neq b(\vec{k},\vec{r})$).

Contributions b_{int}, $\tilde{b}_{int}$, B_{int} $\to$ depend upon the space-time parameters of the emission region;

at high $q = 2|\vec{k}|$ $\left\{\begin{matrix} b_{int} \\ \tilde{b}_{int} \\ B_{int} \end{matrix}\right. \to 0.$

2) Pairs of non-identical kaon states K_S^0 and K_L^0.

Expression for the correlation function $R_{SL} = R_{LS}$:

$$R_{SL}(\vec{k}) = R_{LS}(\vec{k}) = \lambda_{K^0K^0}\left(1 + F_{K^0}(2\vec{k}) + 2b_{int}(\vec{k})\right) +$$
$$+ \lambda_{\bar{K}^0\bar{K}^0}\left(1 + F_{\bar{K}^0}(2\vec{k}) + 2\tilde{b}_{int}(\vec{k})\right) +$$
$$+ \lambda_{K^0\bar{K}^0}\left(1 - F_{K^0\bar{K}^0}(2\vec{k})\right)$$

(in the same notations).

Thus, the correlation functions for pairs of neutral kaons with close momenta, created in inclusive processes, obey the following relation:

$$R_{SS}(\vec{k}) + R_{LL}(\vec{k}) - R_{SL}(\vec{k}) - R_{LS}(\vec{k}) = 2\left(R_{SS}(\vec{k}) - R_{SL}(\vec{k})\right) =$$
$$= 4\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2\vec{k}) + B_{int}(\vec{k})\right]$$

In principle, the p-wave resonance (φ-meson, $M = 1021\ MeV/c^2$, $\Gamma = 4\ MeV$) may influence the $K_S^0 K_L^0$-correlations additionally. But: this influence manifests itself only in the narrow region of comparatively large relative momenta $2k \sim 220\ MeV/c$, and it is strongly suppressed at low relative momenta (may be taken into account analytically, in principle).

Meantime, $B_{int}(\vec{k}) = \int W_{K^0\bar{K}^0}(\vec{r}) B(\vec{k},\vec{r})\, d^3\vec{r}$;

Function $B(\vec{k},\vec{r})$, describing the contribution of final-state $K^0\bar{K}^0$-interaction in the $K_S^0 K_S^0$-correlations and in the difference $(R_{SS} - R_{SL})$ => may be expressed analytically using the approximation of superposition of plane and spherical waves, if characteristic distances between sources of K^0- and $\bar{K}^0$-mesons $r > d_0$ ($d_0 \to$ radius of short-range $K^0\bar{K}^0$-interaction):

$$B(\vec{k},\vec{r}) \approx \left|A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k)\right|^2 \frac{1}{r^2} + 2\,\mathrm{Re}\left(A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k)\, \frac{\exp(ikr)\cos \vec{k}\vec{r}}{r}\right)$$

$A_{K^0\bar{K}^0 \to K^0\bar{K}^0}(k) \to$ amplitude of s-wave $K^0\bar{K}^0$-scattering

Generation of K^+K^--pairs $\Rightarrow$ transitions $K^+K^- \to K^0\bar{K}^0$ should be also taken into account

[s-wave multichannel scattering;
R. Lednicky, V.L. Lyuboshitz, V.V. Lyuboshitz, Yad. Fiz. 61, 2161 (1998)]

Assuming that pairs K^+K^- and $K^0\bar{K}^0$ are emitted by the same pairs of sources with equal probabilities (isotopically unpolarized sources) $\Rightarrow$

$$B(\vec{k},\vec{r}) \to B(\vec{k},\vec{r}) + \Delta B^{(K^+K^-)}(\vec{k},\vec{r});$$

$$\Delta B^{(K^+K^-)}(\vec{k},\vec{r}) \approx \left| A^{(c)}_{K^+K^- \to K^0\bar{K}^0}(k)\right|^2 \frac{\cos^2 \tilde{k}r + C(\tilde{k}a_c)\sin^2 \tilde{k}r}{r^2}$$

Here $\tilde{k} = \sqrt{k^2 + 2m_K \Delta m} \to$ momentum of the charged kaon in the c.m. frame of the pairs K^+K^- and $K^0\bar{K}^0$

$\left(m_K = \frac{1}{2}(m^{(K^+)} + m^{(K^0)}) \approx 495.6 \ \mathrm{MeV}/c^2; \right.$

$\left. \Delta m = m^{(K^0)} - m^{(K^+)} \approx 4 \ \mathrm{MeV}/c^2 \right)$

$a_c = \dfrac{2\hbar^2}{m^{(K^+)}e^2} \approx 108.5 \ \mathrm{Fm} \to$ Bohr radius of the K^+K^--system

$$C(\tilde{k}a_c) = \frac{2\pi/\tilde{k}a_c}{1 - \exp(-2\pi/\tilde{k}a_c)} \to \text{Coulomb factor corresponding to attraction}$$

$A^{(c)}_{K^+K^-\to K^0\bar{K}^0}(k) \to$ effective amplitude;

$A_{K^+K^-\to K^0\bar{K}^0}(k) = \sqrt{C(\tilde{k}a_c)}\, A^{(c)}_{K^+K^-\to K^0\bar{K}^0}(k) \to$ s-wave amplitude of the process $K^+K^-\to K^0\bar{K}^0$

(effective cross-section $\sigma_{K^+K^-\to K^0\bar{K}^0} = 4\pi \left| A_{K^+K^-\to K^0\bar{K}^0}(k)\right|^2 \frac{k}{\tilde{k}}$)

For $k=0 \Rightarrow \tilde{k} = k_0 = \sqrt{2m_K \Delta m} \approx 62.8\ \mathrm{MeV}/c$;

$C(k_0 a_c) = 1.0934 \Rightarrow 1.0934 \geq C(\tilde{k}a_c) \geq 1$

Thus, with precision $\lesssim 10\%$

$$\Delta B^{(K^+K^-)}(\vec{k},\vec{r}) \approx \left| A_{K^+K^-\to K^0\bar{K}^0}(k)\right|^2 \frac{1}{r^2}$$

Amplitudes $A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k)$ and $A_{K^+K^-\to K^0\bar{K}^0}(k) \Rightarrow$ determined by contribution of the sub-threshold s-wave resonances $f_0(980)\ (T=0)$ and $a_0(980)\ (T=1)$:

$$A_{K^0\bar{K}^0\to K^0\bar{K}^0}(k) = \frac{1}{2}\left(A^{(0)}(k) + A^{(1)}(k)\right),$$

$$A_{K^+K^-\to K^0\bar{K}^0}(k) = \frac{1}{2}\left(A^{(0)}(k) - A^{(1)}(k)\right).$$

So, in the approximation $C(\tilde{k}a_c) \approx 1$:

$$B(\vec{k},\vec{r}) + \Delta B^{(K^+K^-)}(\vec{k},\vec{r}) \approx \left(|A^{(0)}(k)|^2 + |A^{(1)}(k)|^2\right)\frac{1}{2r^2} + $$
$$+ \mathrm{Re}\left(\left(A^{(0)}(k) + A^{(1)}(k)\right)\frac{\exp(ikr)\cos\vec{k}\vec{r}}{r}\right)$$

One should note also that, for isotopically unpolarized sources of K^+K^- and $K^0\bar{K}^0$-pairs, the average numbers of produced pairs K^+K^- and $K^0\bar{K}^0$ are approximately equal $\Rightarrow$ $\lambda_{K^0\bar{K}^0}$ may be assessed from the multiplicities of K^+K^--generation.

Finally: the difference between the correlation functions of the pairs $K^0_S K^0_S$ (identical neutral kaons) and $K^0_S K^0_L$ (non-identical neutral kaons) is conditioned exclusively by the generation of $K^0\bar{K}^0$-pairs with zero strangeness.

So, using the experimental values of the correlation function for $K^0_S K^0_S$ and knowing the parameters of $K^0\bar{K}^0$-interaction, one may estimate, in principle, the correlation function R_{SL} for $K^0_S K^0_L$ (which is still practically inaccessible for direct measurement over the decays $K^0_S \to 2\pi$, $K^0_L \to 3\pi$ at present colliders)

3. Summary

1. The phenomenological structure of inclusive cross-sections of the production of two neutral K-mesons in collisions of hadrons and nuclei is investigated taking into account the strangeness conservation in strong and electromagnetic interactions. It is demonstrated that, as follows directly from strangeness conservation, the double inclusive cross-sections of production of two K_S^0-mesons and two K_L^0-mesons are mutually equal.

2. Within the model of one-particle sources, the phenomenological formulas for the correlation functions $R_{SS} = R_{LL}$ and $R_{SL} = R_{LS}$, involving the contributions of Bose-statistics and s-wave strong final-state interaction for two K^0-($\bar{K}^0$-) mesons as well as for K^0 and $\bar{K}^0$, and depending on the relative fractions of generated pairs K^0K^0, $\bar{K}^0\bar{K}^0$ and $K^0\bar{K}^0$, have been derived.

3. It is shown that namely the generation of $K^0\bar{K}^0$-pairs with zero strangeness gives rise to the difference between the correlation functions R_{SS} and R_{SL} of two neutral kaons.