3-point Green's functions

One loop corrections to order parameters

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Contents

Contents

- Motivation

- Definitions

Contents

Contents

- Motivation
- Definitions
- Feynman Graphs
- Disguised Infrared singularities

Contents

- Motivation
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- Feynman Graphs
- Disguised Infrared singularities
- Renormalization Group equations

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- Feynman Graphs
- Disguised Infrared singularities
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- Conclusions

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- V.M., A.Pich and M. Jamin, work in preparation

Definitions

External Sources (composite operators)

$$
\begin{aligned}
V^{a\mu}(x) &= \; : \bar{q}(x)\gamma^\mu \frac{\lambda^a}{2} q(x) : \\
A^{a\mu}(x) &= \; : \bar{q}(x)\gamma^\mu \gamma_5 \frac{\lambda^a}{2} q(x) : \\
S^{a}(x) &= \; : \bar{q}(x)\, \lambda^a q(x) : \\
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Green's Functions

$$\left(\Pi^{\mu\nu}_{VVP}\right)^{abc}\left(q^2,p^2,r^2\right) = \int \mathrm{d}^4x\,\mathrm{d}^4y\,e^{i(qx+py)}\,\langle 0|\,T\{V^{a\mu}(x)V^{b\nu}(y)P^c(0)\}\,|0\rangle$$
$$\left(\Pi^{\mu\nu}_{AAP}\right)^{abc}\left(q^2,p^2,r^2\right) = \int \mathrm{d}^4x\,\mathrm{d}^4y\,e^{i(qx+py)}\,\langle 0|\,T\{A^{a\mu}(x)A^{b\nu}(y)P^c(0)\}\,|0\rangle$$
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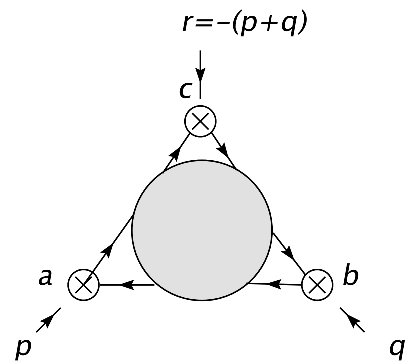
$$\Pi^{abc}_{SSS}\left(q^2,p^2,r^2\right) = \int \mathrm{d}^4x\,\mathrm{d}^4y\,e^{i(qx+py)}\,\langle 0|\,T\{S^a(x)S^b(y)S^c(0)\}\,|0\rangle$$
$$\Pi^{abc}_{SPP}\left(q^2,p^2,r^2\right) = \int \mathrm{d}^4x\,\mathrm{d}^4y\,e^{i(qx+py)}\,\langle 0|\,T\{S^a(x)P^b(y)P^c(0)\}\,|0\rangle$$

Feynman representation of the 3-point Green's Functions



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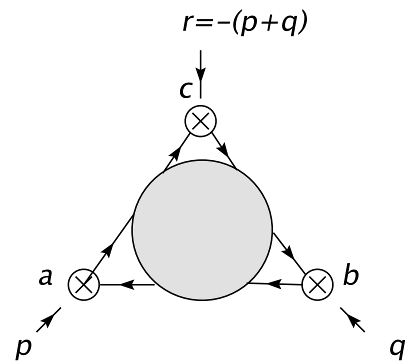
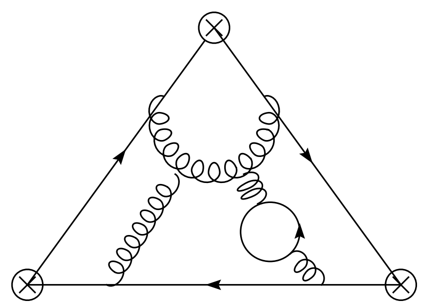


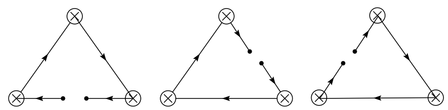
Perturbative terms



Perturbative contribution $\to 0$. Need for quark condensates $\langle \bar{q}q \rangle$!

Feynman Graphs

Tree level contributions with one quark condensate $\langle \bar{q}q \rangle$



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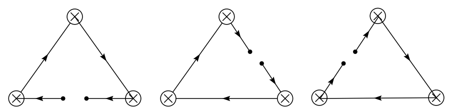
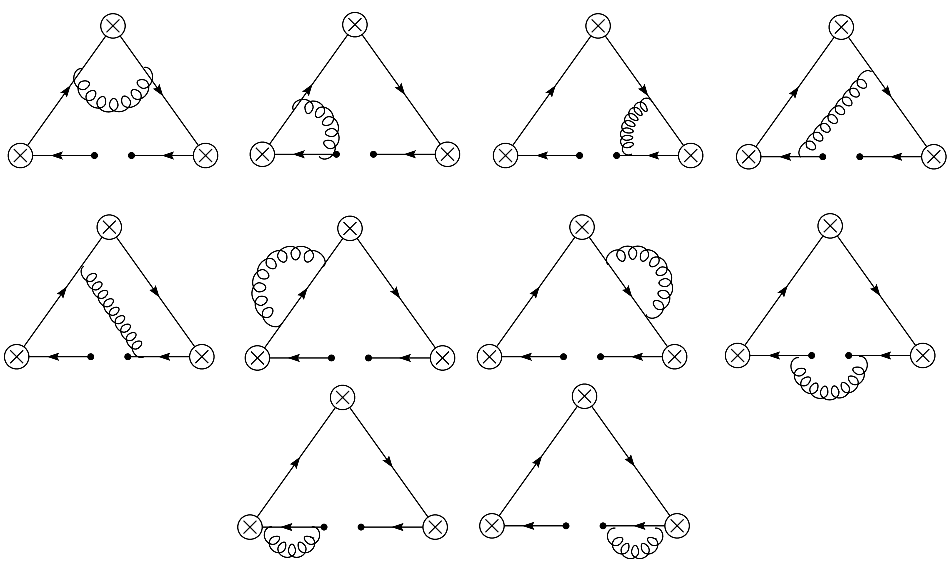
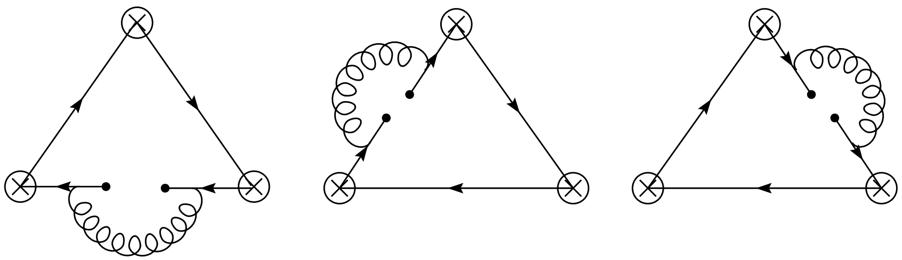


One loop contributions



Disguised Infrared singularities



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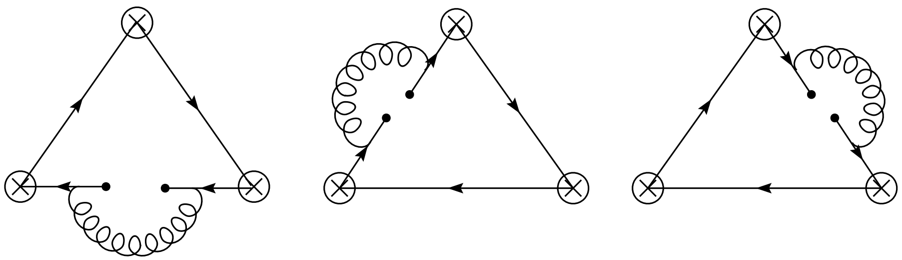

IR integrals $\left\{ \begin{array}{l} \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^4 (l-q)^2} \\ \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^4 (l-q)^2 (l-q)^2} \end{array} \right.$ new integral!

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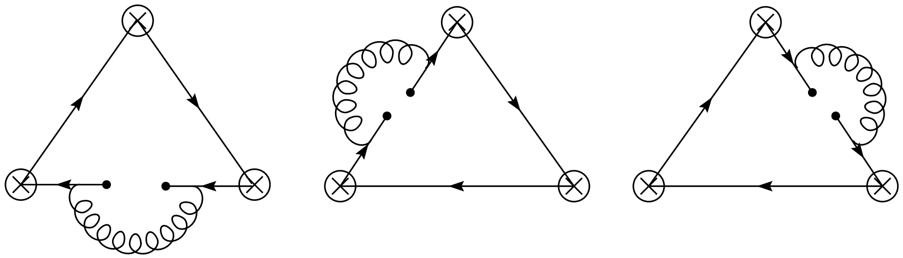

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Can be regulated by means of: $\begin{cases} & \text{small quark mass} \\ \bigstar & \text{dimensional regularization} \end{cases}$

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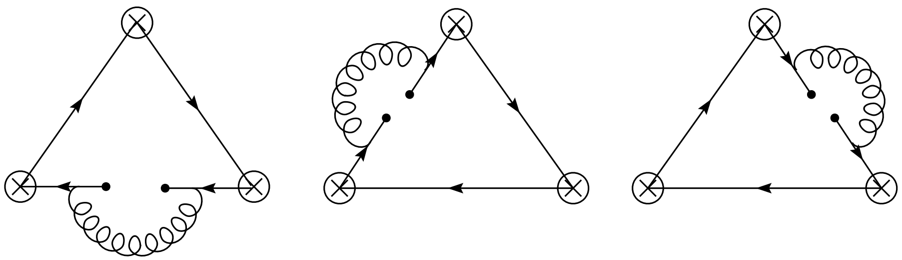

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$$\langle \bar{q}_R q_R \rangle \left(1 - \frac{(3+a)}{4} \frac{\alpha_s}{\pi} C_F \right) = \langle \bar{q}_R q_R \rangle \, Z_m Z_{2F}$$

$$= \langle \bar{q}_B q_B \rangle \, Z_m = \frac{m_B}{m_R(\mu)} \langle \bar{q}_B q_B \rangle = \langle \bar{q} q \rangle_R (\mu)$$

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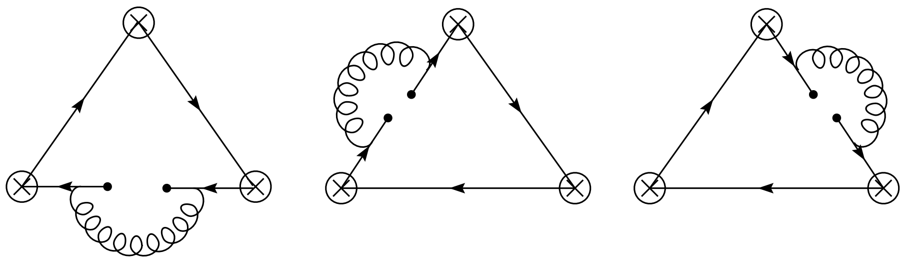

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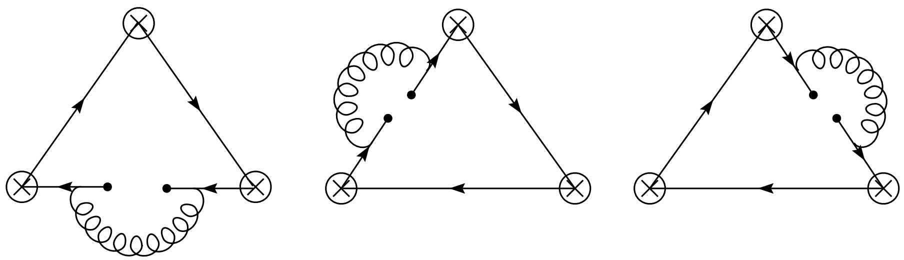

IR integrals
$$\begin{cases} \int \frac{d^D l}{(2\pi)^D}\left(\frac{1}{l^4(l-q)^2} - \frac{1}{q^2 l^4}\right) = \frac{1}{q^2}\int \frac{d^D l}{(2\pi)^D}\frac{2lq-l^2}{l^4(l-q)^2} \\ \int \frac{d^D l}{(2\pi)^D}\frac{1}{l^4(l-q)^2(l-q)^2} \quad \text{new integral!} \end{cases}$$

Can be regulated by means of:
- small quark mass
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Renorm.Group equations

General result

$$\Pi^{abc}_{\mu\nu}\left(q^2,p^2,r^2\right) = \mathcal{L}_{\mu\nu}\mathcal{G}^{abc}\frac{\langle\bar{q}q\rangle(\mu)}{p^2q^2r^2}\left[\mathcal{T}+\frac{\alpha_s}{\pi}\frac{C_F}{4}\left(A+B\log\mu\right)\right]$$

$$\mathcal{T} = \text{Tree}-\text{level} = \text{polynomial}$$

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$$\frac{1}{\langle\bar{q}q\rangle(\mu)}\mu\frac{d\langle\bar{q}q\rangle(\mu)}{d\mu} = -\gamma_{\langle\bar{q}q\rangle} = \gamma_m$$

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$$\left[2\gamma_m(\alpha_s)-\mu\frac{\partial}{\partial\mu}\right]\left\langle S^aS^bS^c\right\rangle(\mu) = \left[2\gamma_m(\alpha_s)-\mu\frac{\partial}{\partial\mu}\right]\left\langle S^aP^bP^c\right\rangle(\mu) = 0$$

$$\frac{\partial}{\partial\mu}\left\langle V^{\mu a}V^{\nu b}P^c\right\rangle(\mu) = \frac{\partial}{\partial\mu}\left\langle A^{\mu a}A^{\nu b}P^c\right\rangle(\mu) = \frac{\partial}{\partial\mu}\left\langle V^{\mu a}A^{\nu b}P^c\right\rangle(\mu) = 0$$

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- For explicit expressions, please contact the authors