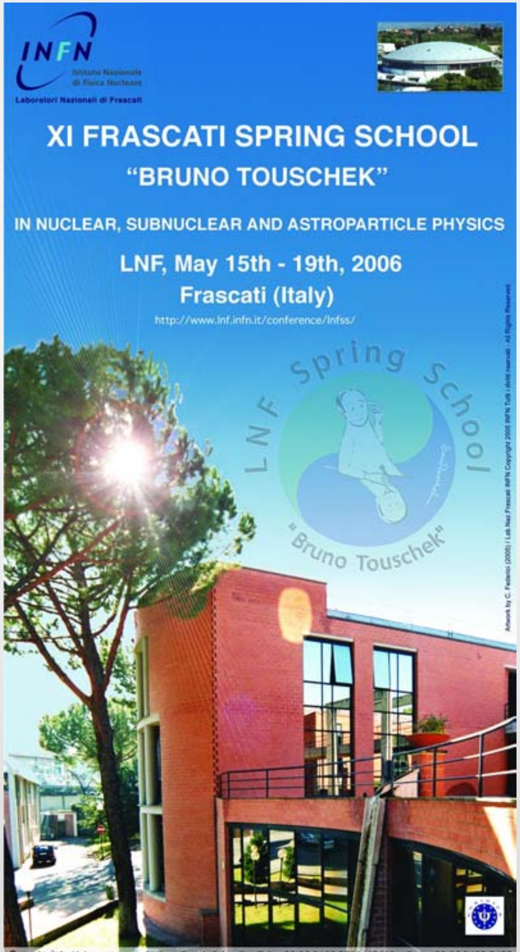
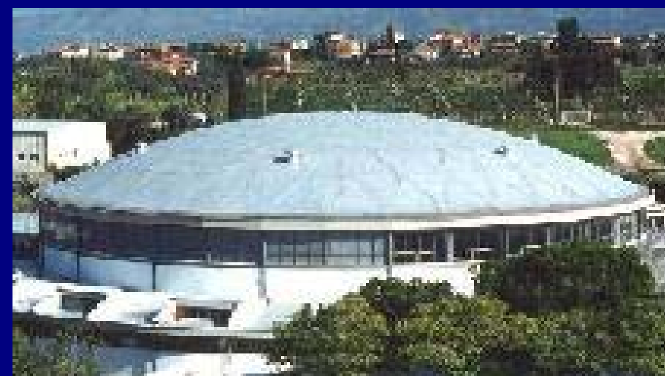
FLAVOUR PHYSICS AND LATTICE QCD IN 2006



Vittorio Lubicz



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1. FLAVOUR PHYSICS AND ITS MOTIVATIONS

- Problems in the Standard Model and evidence for New Physics
- Open issues in Flavour Physics
- A simple model of Flavour Physics

2. LATTICE QCD AND FLAVOUR PHYSICS

- Introduction to Lattice QCD
- Systematic errors in lattice calculations
- Lattice QCD and quark masses
- Lattice QCD and the Unitarity Triangle Analysis (UTA)

  [New measurements: Δm_s (CDF) and $BR(B \to \tau \nu_\tau)$ (Belle)]
- The UTA beyond the Standard Model

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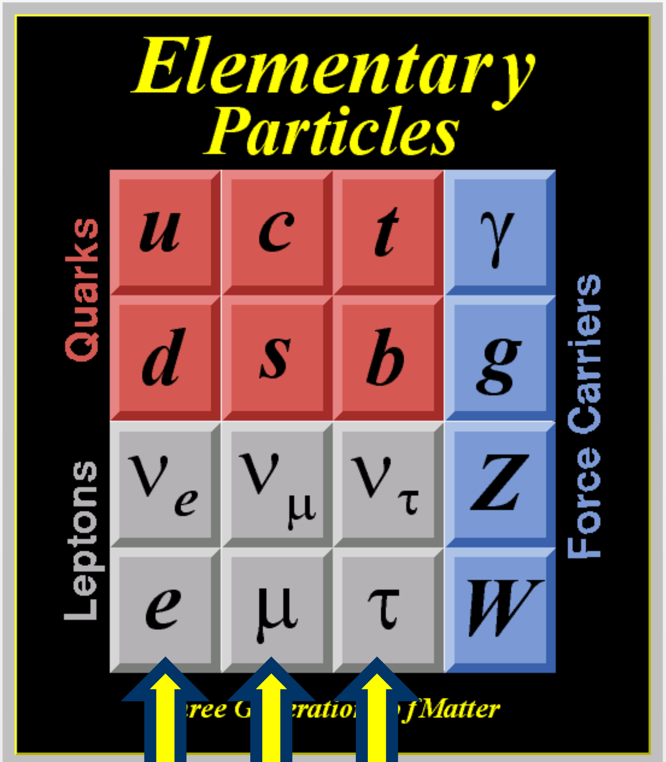
FLAVOR PHYSICS AND ITS MOTIVATIONS

FLAVOUR PHYSICS



6 Flavours
3 Families

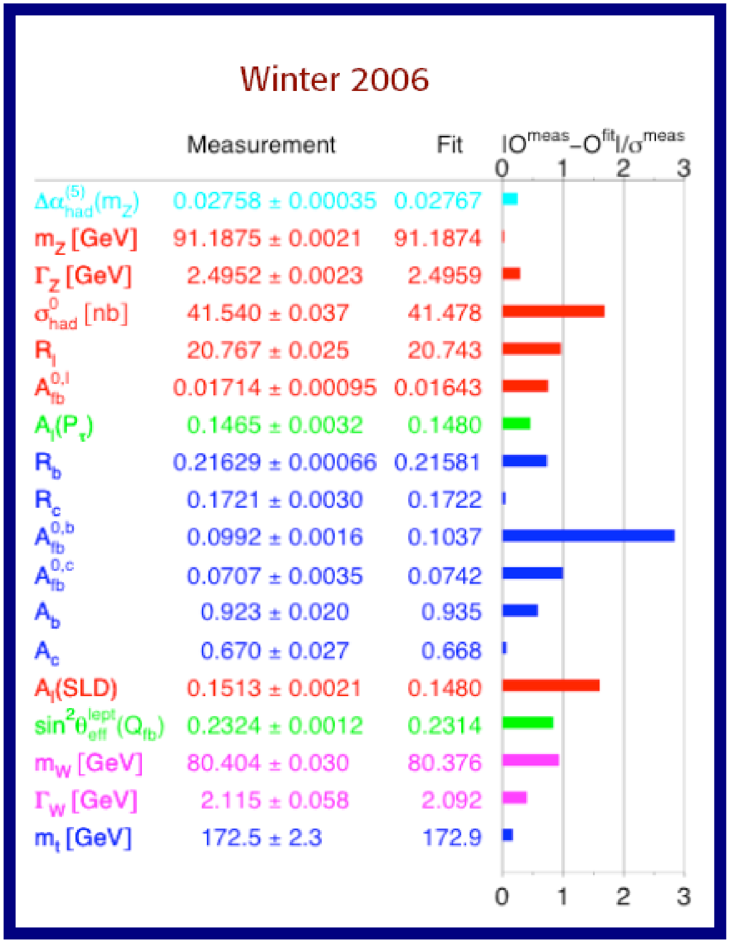
- **FLAVOUR**: elementary fermions (matter particles) are 6 flavours of quarks and 6 of leptons

- **MASSES**: Quarks and leptons come in 3 families which only differ for particle masses

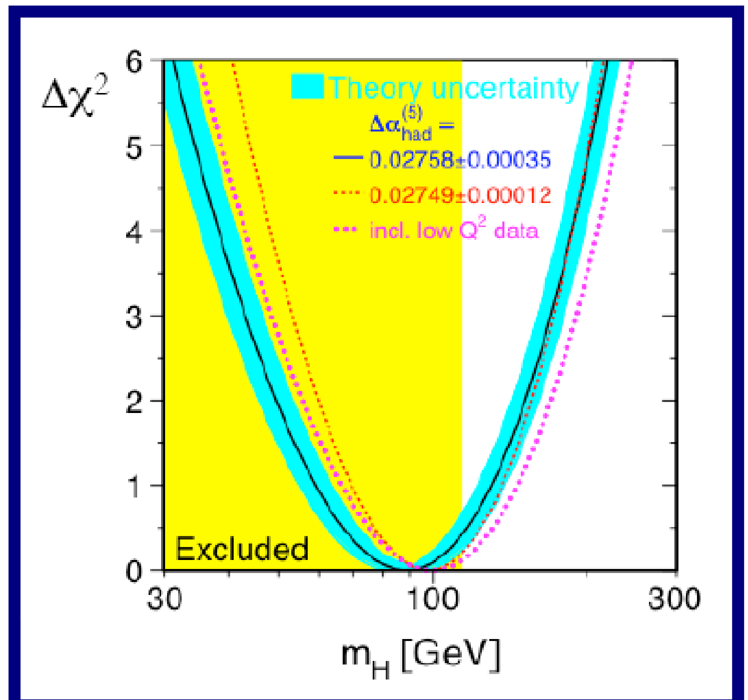
- **MIXING**: Flavour is conserved by e.m. and strong interactions. Only weak interactions (charged currents) change flavour ⟶ CKM matrix and CP violation

PROBLEMS IN THE STANDARD MODEL AND EVIDENCE FOR NEW PHYSICS

Experiments show that the Standard Model provides an extremely successful description of electro-magnetic, weak and strong interactions (the gauge sector), at least up to the Fermi scale



EW precision tests support the SM with a light Higgs



TWO OPEN QUESTIONS:

1) Which is the mechanism of gauge symmetry breaking ?

2) Which is the origin of flavor symmetry breaking ?

Fermion masses are generated by gauge symmetry breaking →

GAUGE SYMMETRY BREAKING AND FLAVOR PHYSICS ARE CLOSELY RELATED

THE STANDARD MODEL: A LOW ENERGY EFFECTIVE THEORY

Conceptual and phenomenological problems:

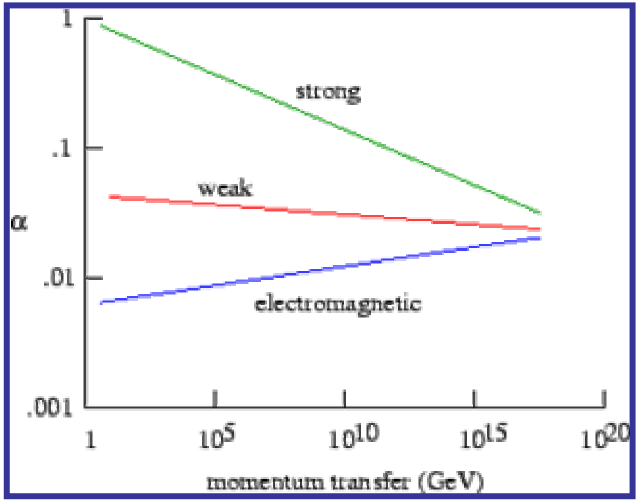
- Gravity $(M_{Planck} = (\hbar c/G_N)^{1/2} \approx 10^{19}$ GeV)
- Hierarchy $(M_{Higgs} << M_{Planck})$
- Unification of couplings ($M_{GUT} \approx 10^{15}$-10^{16} GeV)
- Neutrino masses $(M \approx M_{GUT})$
- Dark matter $(\Omega_M \approx 0.3)$ and vacuum energy $(\Omega_\Lambda \approx 0.7)$
- Baryogenesis
- Inflation
- …

Unification of Couplings

The running of gauge couplings provides strong indication of unification. However:

precise unification **fails** in the SM [$\alpha_s(M_Z) \approx 0.073$].

(Well compatible in SUSY)



Grand Unification Theories (GUT) are very appealing for several reasons:

• Unity of forces • Unity of quark and leptons (different directions in G) • Family Q-numbers (in SO(10) a whole family in 16) • Charge quantization ($Q_d = -1/Nc = -1/3$) • B and L non conservation • ....

Neutrino Masses

The existence of neutrino masses and mixings is well established. But **neutrinos are massless in the SM**.

Neutrino masses are really special: $m_t / (\Delta m_{atm}) \sim 10^{12}$

➡ The simple extension of the SM with the inclusion of ν_R looks very unnatural

A natural solution: ν's are Majorana particles and get masses through L violating interactions suppressed by a large scale M

$$m_\nu \sim \frac{m^2}{M}$$

For $m_\nu \sim 0.05$ eV and $m \sim v \sim 200$ GeV ➡

$$M \sim 10^{15}\,\text{GeV} \sim M_{GUT}$$

Energy Density of the Universe

$$\Omega_{tot} = \Omega_{vac} + \Omega_{mat} + \Omega_{rad} + \ldots$$

$\Omega_i \equiv \rho_i / \rho_c$

$\rho_c = 3H^2/8\pi G_N \approx$

$\approx 5 \cdot 10^{-6}\ \mathrm{GeV\,cm^{-3}}$

$\Omega_{tot} > 1 \rightarrow k = +1$ closed universe

$\Omega_{tot} < 1 \rightarrow k = -1$ open universe

$\Omega_{tot} = 1 \rightarrow k = 0$ flat universe

$k \equiv$ curvature constant

$\Omega_{tot} = 1.02 \pm 0.02$

Inflation: $\Omega_{tot} = 1$

Flat Universe

$\Omega_{rad} \approx 10^{-5}$

Ω_{rel} negligible

$\Omega_{mat} \approx 0.3\ ,\ \Omega_{vac} \approx 0.7$

Both problematic !

Dark Matter

$$\Omega_{mat} = \Omega_b + \Omega_{dm}$$

Baryonic matter

Dark matter (i.e. non-luminous and non-absorbing)

$$\Omega_{mat} \approx 0.3 \;,\; \Omega_b \approx 0.04$$

→ more than 80% of matter is dark matter !!

Cold DM ≡ non relativistic at the onset of galaxy formation

Primordial black holes, axions, WIMP, ...

(SUSY neutralino)

Hot DM ≡ relativistic at the onset of galaxy formation

Could be ν's but

$\Omega_\nu < 0.015$ (WMAP)

Most of DM should be cold ← All hot DM would have not permitted galaxies to form

Vacuum Energy

$$\Omega_{vac} \approx 0.7$$

The scale of the cosmological constant is a big mystery

- In QFT the energy density of the vacuum receives an infinite contribution from the zero-point energies of the various modes of oscillation. For a bosonic scalar field:

$$H_b = \sum_{p} (a_p^\dagger a_p + \tfrac{1}{2})\varepsilon_p \quad \Rightarrow \quad \langle 0|\, H_b \,|0\rangle = \tfrac{1}{2} \sum_{p} \varepsilon_p$$

Fermionic s=1/2 fields give a negative contribution:

$$H_f = \sum_{p} (b_p^\dagger b_p + c_p^\dagger c_p - 1)\varepsilon_p \quad \Rightarrow \quad \langle 0|\, H_f \,|0\rangle = -\sum_{p} \varepsilon_p$$

• The scale of the zero-point energy density is provided by the cutoff:

$$\rho_{vac} = \frac{1}{V}\langle 0|\,H\,|0\rangle \sim \frac{1}{V}\sum_{\varepsilon_p<\Lambda_{cut}} \varepsilon_{\mathbf{p}} \overset{(\varepsilon_p = cp)}{\approx} \Lambda_{cut}^4/(\hbar c)^3$$

If $\Lambda_{cut} \sim M_{Planck}$ ⟹ $\rho_{vac} \sim \mathbf{10^{123}}\,\rho_{vac}^{obs}$

• Exact SUSY would solve the problem:

$$\langle 0|\,H\,|0\rangle = \left(\tfrac{1}{2}n_b - n_f\right)\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}} = 0$$

But SUSY is broken. Assuming $\Lambda_{SUSY} \approx 1$ TeV:

$$\rho_{vac} \approx \Lambda_{SUSY}/(\hbar c)^3 \sim \mathbf{10^{59}}\,\rho_{vac}^{obs}$$

So far, the problem of the scale of the cosmological constant has found no solution

Baryogenesis:
Matter-Antimatter asymmetry

- So far, no primordial anitimatter has been observed in the Universe. Up to distances of order 100 Mpc – 1 Gpc the Universe consists only of matter.

(1Mpc = 3.2 10^6 light years. Observable universe : $H_0^{-1} \sim 10$ Gpc)

- A very plausible assumption is that the big bang produces an equal number of quarks and antiquarks

WHEN AND WHY ANTIMATTER DISAPPEARED ?

THE SAKHAROV CONDITIONS: (1967)

1) Baryon number violation

2) C and CP violation

3) Departure from thermal equilibrium

In the SM:

Istanton process

Weak interactions

Electro-weak phase transition

In the SM, for $m_H \geq 80$ GeV, the e.w. phase transition is not "strong" enough: it does not provide enough termal instability necessary for baryogenesis

CP violation generated by the CKM mechanism is irrelevant for baryogenesis ⟹ Non-standard CP violation is a necessary ingredient for baryiogenesis

MOST OF "BEYOND STANDARD MODEL PHYSICS" CAN BE EXPLAINED BY NEW PHYSICS MODELS (SUSY, GUT, EXTRA-DIM,…)

- Gravity ☹ NO!!
- Hierarchy ($M_{Higgs} << M_{Planck}$) ☺ Yes
- Unification of couplings ☺ Yes
- Neutrino masses ☺ Yes
- Dark matter ☺ Yes
- Vacuum energy ☹ NO!!
- Baryogenesis ☺ Yes

Flavor Physics could allow us to discriminate among various New Physics scenarios

OPEN ISSUES IN FLAVOUR PHYSICS

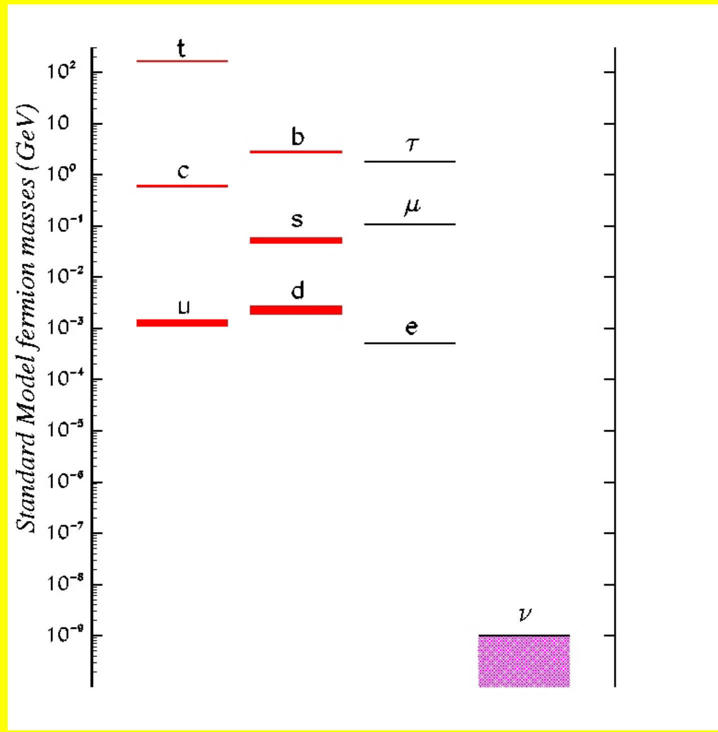
Flavor physics is (well) described but not explained in the Standard Model:

A large number of free parameters in the flavor sector (10 parameters in the quark sector only, 6 m_q + 4 CKM)

- Why 3 families?

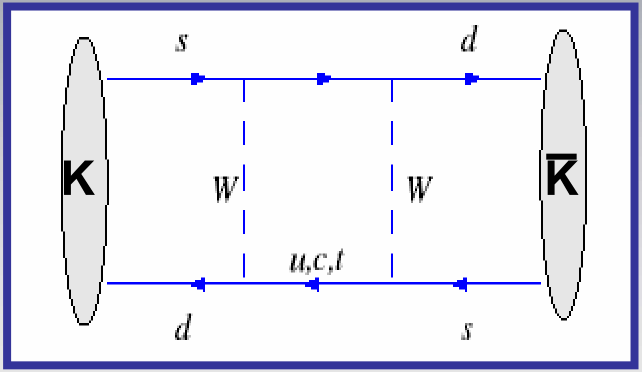
- Why the spectrum of quarks and leptons covers 5 orders of magnitude? ($m_q \sim v \sim G_F^{-1/2}$...)

- What give rise to the pattern of quark mixing and the magnitude of CP violation?

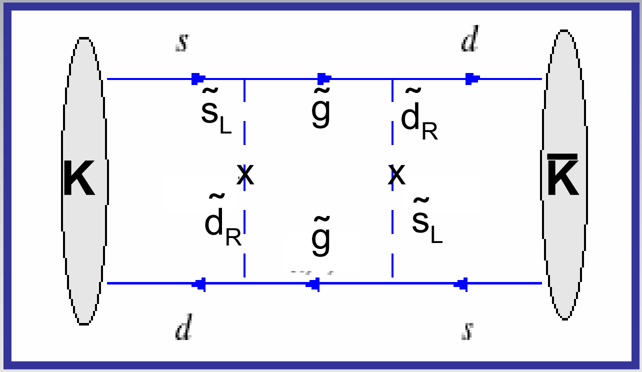


Flavor physics is an open window on physics beyond the Standard Model

New Physics enters through quantum loops:



Standard Model



New Physics

New Physics can be conveniently described in terms of a **low energy effective theory**:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda_{NP}} O_i^{(5)} + \sum_i \frac{c_i}{\Lambda_{NP}^2} O_i^{(6)} + \ldots$$

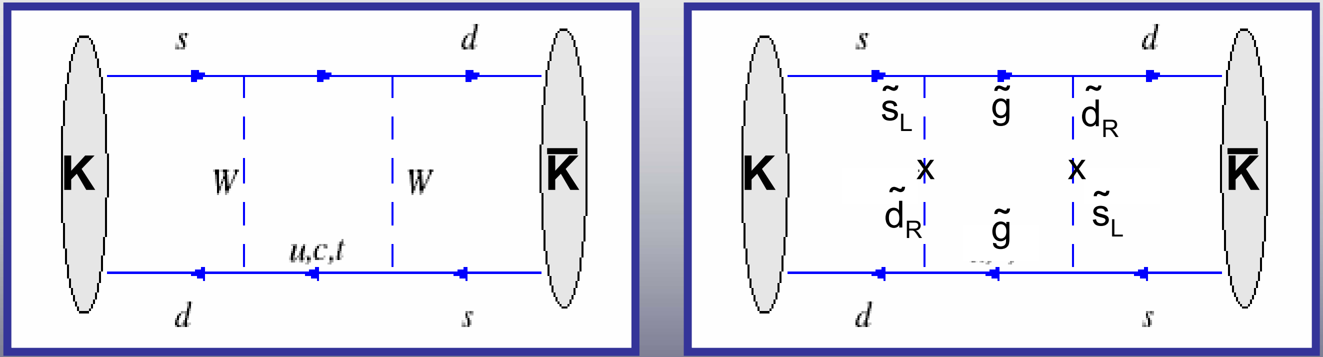
THE "FLAVOR PROBLEM"

The "natural" cut-off

NEW PHYSICS MUST BE VERY "SPECIAL"

$$\delta m_H^2 = \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 \Lambda^2 \approx (0.3\,\Lambda)^2 \longrightarrow \Lambda = O(1\text{ TeV})$$

From higher dimensional operator in the flavor sector



$$\Lambda_{K^0\text{-}\overline{K}^0} \approx O(100\text{ TeV})$$

The flavor problem

A SIMPLE MODEL OF FLAVOUR PHYSICS

THE QUARK MASS MATRICES

$$L^{Yukawa} = - \sum_{i,k} [\bar{Q}^i_L Y^d_{ik} D^k_R H + \bar{Q}^i_L Y^u_{ik} U^k_R H^C] + + \text{ h.c.}$$

Gauge symmetry breaking

$$L^{mass} = - \sum_{i,k} [\bar{d}^i_L m^d_{ik} d^k_R + \bar{u}^i_L m^u_{ik} u^k_R] + \text{h.c.}$$

$$m^q = Y^q v / \sqrt{2}$$

$\longleftrightarrow$ $M_W = gv/2$

Why $m^q \neq O(M_W)$??

DIAGONALIZATION OF THE MASS MATRIX

The mass matrices m^q are not Hermitean. Up to singular cases, they can be diagonalized by 2 unitary transformations:

$$U_L^\dagger \, m \, U_R = m_D$$

$$\begin{cases} U_L^\dagger \, m \, m^\dagger \, U_L = m_D \, m_D^\dagger \\ U_R^\dagger \, m^\dagger \, m \, U_R = m_D^\dagger \, m_D \end{cases}$$

i

$$(U_L^\dagger)_{ik} \, q_L^k \rightarrow q_L^i \, , \quad (U_R^\dagger)_{ik} \, q_R^k \rightarrow q_R^i$$

($U_{L,R}$ different for u^k and d^k)

$$L^{mass} = - [m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + \ldots] + h.c.$$

With respect to:

$$(U_L^\dagger)_{ik} q_L^k \to q_L^i \;, \quad (U_R^\dagger)_{ik} q_R^k \to q_R^i$$

neutral currents $\bar{q}_L^i \gamma_\mu q_L^i$ and $\bar{q}_R^i \gamma_\mu q_R^i$ are invariant: quark kinetic terms, QCD couplings with gluons, QED couplings with photons, weak couplings with Z^0

No flavor changing neutral currents (FCNC) at tree level

The only effect is in the weak charged currents:

$$\bar{u}_L^i \gamma_\mu d_L^i \cdot W^\mu \;\to\; \bar{u}_L^k \gamma_\mu (U_L^{u\dagger} U_L^d)_{kj} d_L^j \cdot W^\mu$$

$$V_{CKM} = U_L^{u\dagger} U_L^d$$

$$V_{CKM} V_{CKM}^\dagger = 1$$

THERE IS A CLEAR CORRELATION BETWEEN MASSES AND MIXINGS ANGLES

In the first 2 generations:

$$\left(\frac{m_d}{m_s}\right)^{1/2} \approx 0.24 \qquad \left(\frac{m_u}{m_c}\right)^{1/4} \approx 0.22$$

$$\Rightarrow \quad \left(\frac{m_d}{m_s}\right)^{1/2} \approx \left(\frac{m_u}{m_c}\right)^{1/4} \approx V_{us}$$

Can we explain this relation ?

MASS TEXTURES

Two generations: Gatto et al.

$$\mathbf{m^d} = m_s \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix} \qquad \mathbf{m^u} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}$$

$$\mathrm{diag}(\mathbf{m^d}) = m_s(x, 1) \quad \Rightarrow \quad x = m_d / m_s$$

Diagonalization:

$$\begin{cases} U_L^\dagger m\, m^\dagger U_L = m_D\, m_D^\dagger \\ U_R^\dagger m^\dagger m\, U_R = m_D^\dagger\, m_D \end{cases} \qquad U_L^\dagger m\, U_R = m_D \qquad V_{CKM} = U_L^{u\dagger} U_L^d$$

$$V_{CKM} = U_L^{u\dagger} U_L^d = U_L^d \approx \begin{pmatrix} 1 - x/2 & \sqrt{x} \\ -\sqrt{x} & 1 - x/2 \end{pmatrix}$$

$$V_{us} = \sin\theta_C = \sqrt{x} = \sqrt{m_d / m_s} \approx 0.22$$

Which theory of flavor generates this texture?

HORIZONTAL SYMMETRIES

Example: **Horizontal U(2)** (Barbieri, Hall, ...)

$q^a \to U_{ab}\, q^b$, $U \in U(2)$ $a,b = 1,2$ (Generation indices)

$$L = \frac{1}{M_F}\, \phi_{ab}\, q^a\, q^b\, H$$

Non-renorm. interaction
M_F = flavor scale

ϕ_{ab}: "Flavon" field

H: Higgs field (U(2) scalar)

$$\phi_{ab} = S_{ab} + A_{ab}$$

S_{ab}: Symmetric tensor

A_{ab}: Anti-symmetric tensor

$$U(2) \xrightarrow{S_{ab}} U(1) \xrightarrow{A_{ab}} \{1\}$$

$$\langle S_{ab} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & V \end{pmatrix} \qquad \langle A_{ab} \rangle = \begin{pmatrix} 0 & -v \\ v & 0 \end{pmatrix}$$

$$L = \frac{1}{M_F}(S_{ab} + A_{ab})\, q^a q^b H$$ → **Flavor symm. breaking**

$$\rightarrow \frac{V}{M_F} q^2 q^2 H + \frac{v}{M_F}(q^2 q^1 - q^1 q^2) H \equiv q^a Y_{ab} q^b H$$

Yukawa matrix (Y_{ab})

$$Y_{ab} = \begin{pmatrix} 0 & -v/M_F \\ v/M_F & V/M_F \end{pmatrix}$$

$v/M_F = \sqrt{x}$

$V/M_F = 1+x$

Is the Gatto's texture

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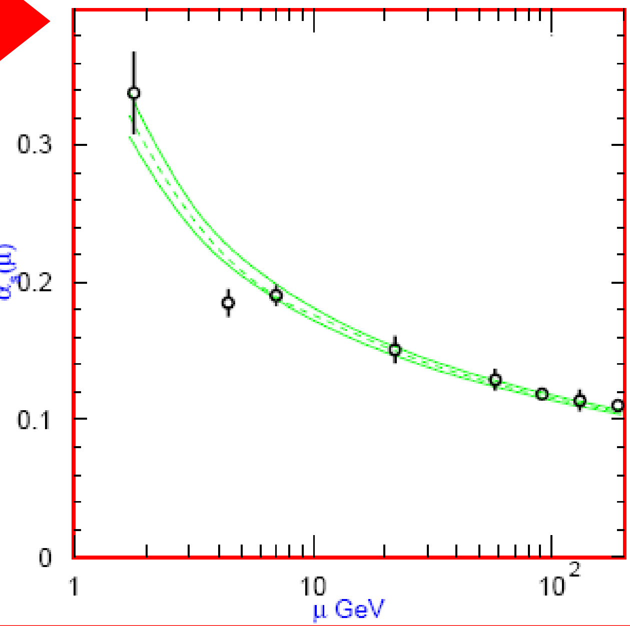
LATTICE QCD AND FLAVOUR PHYSICS

Lattice QCD

Strong interactions are non-perturbative at low energies

Confinement



Asymptotic freedom

LQCD is a non-perturbative approach

INTRODUCTION TO LATTICE QCD

The Functional Integral

The Green Functions can be written in terms of Functional Integrals over classical fields:

$$G(x_1, x_2, x_3, x_4) = \langle\, \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\, \rangle \equiv$$

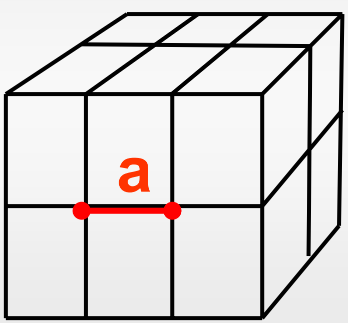
$$Z^{-1} \int [d\phi]\, \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\, e^{-S(\phi)}$$

The functional integral is defined by discretizing the space-time on a hypercubic 4-dimensional lattice

$$\phi(x) \rightarrow \phi(a\, n)$$

$$n = (\, n_x, n_y, n_z, n_t\,)$$

$$\partial_\mu \phi(x) \rightarrow \nabla_\mu \phi(x) = [\, \phi(x + a\, n_\mu) - \phi(x)\,] / a$$



The Lattice regularization

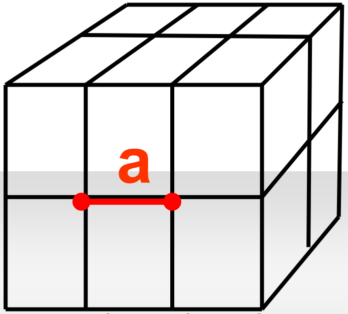
The functional integral is a formal definition because of the infrared and ultraviolet divergences. These are cured by introducing an infrared and an ultraviolet cutoff

1) The ultraviolet cutoff: $|p| \leq \pi/a$

The momentum p is cutoff at the first Brioullin zone

2) The infrared cutoff: $p_{min}\, a = 2\pi/L$

The lattice is defined in a finite volume



The physical theory is obtained in the limit

$a \to 0$ Continuum limit ; $L \to \infty$ Thermodinamic limit

Montecarlo techniques

$$Z^{-1} \int [d\phi]\, O(x_1) \ldots O(x_k)\, e^{-S(\phi)}$$

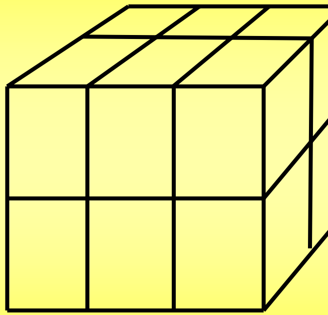
With a finite lattice spacing (**a**) and on a finite volume (**L**) this is now an integral on $(L/a)^4$ real variables.

For the 3d Ising model: $2^N = 2^{L^3} \approx 10^{301}$ for $L = 10$!!

IMPORTANT SAMPLING TECHNIQUES:

The fields are extracted with weight $e^{-S(\phi)}$

$$\langle O(x_1) \ldots O(x_k) \rangle \approx N^{-1} \sum_{\{\phi(x)\}_n} O_n(x_1) \ldots O_n(x_k)$$



→ Statistical errors

The theoretical calculation can be performed only numerically. In order to achieve the required precision, about

1 billion of billions operations

are necessary, corresponding to an integral over 50 millions degrees of freedom and to the inversion of thousands of matrices with 100 millions of elements

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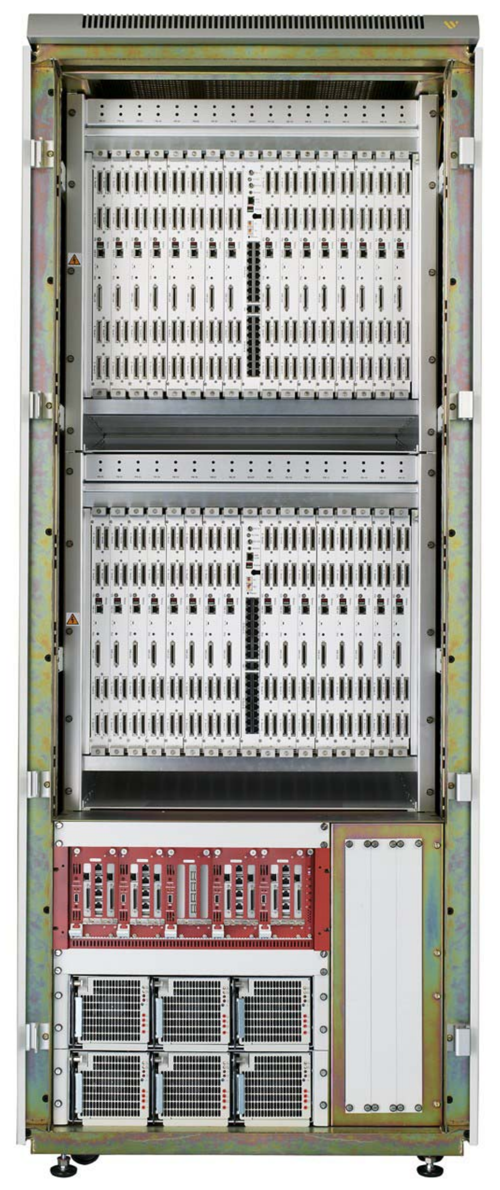
The apeNEXT super-computer (~1 Tflop)



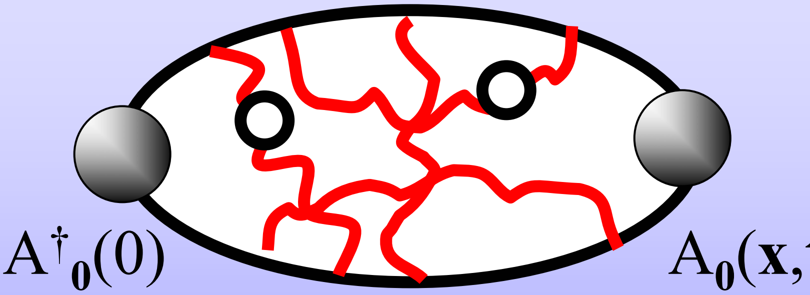
Hadron masses and matrix elements

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^\dagger_0(\mathbf{0},0) \rangle =$$

The operator A_0 can excite 1-π, 3-π etc. states

$$= \sum_{\mathbf{x}} \sum_n \frac{\langle 0 | e^{iPx} A_0(0) e^{-iPx} | n \rangle \langle n | A^\dagger_0(0) | 0 \rangle}{2 E_n}$$

$$= \sum_n \frac{|\langle 0 | A_0 | n \rangle|^2}{2 m_n} \exp[-m_n t]$$

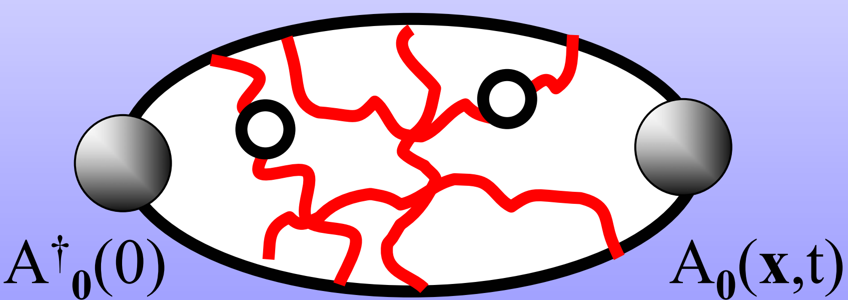


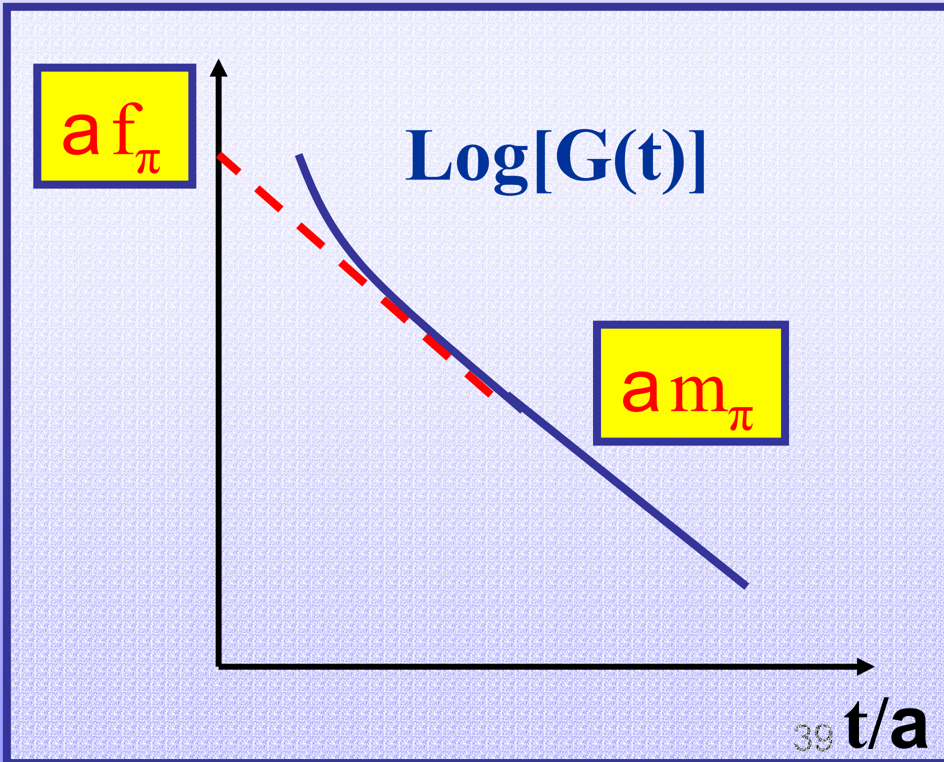
$$\xrightarrow{t \to \infty} \frac{|\langle 0 | A_0 | \pi \rangle|^2}{2 m_\pi} \exp[-m_\pi t] = \frac{f_\pi^2 m_\pi}{2} \exp[-m_\pi t]$$

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^\dagger{}_0(\mathbf{0},0) \rangle \rightarrow$$

$$\rightarrow \frac{|\langle 0|A_0|\pi\rangle|^2}{2\,m_\pi} \exp[-m_\pi t] = \frac{f_\pi^2\, m_\pi}{2} \exp[-m_\pi t]$$

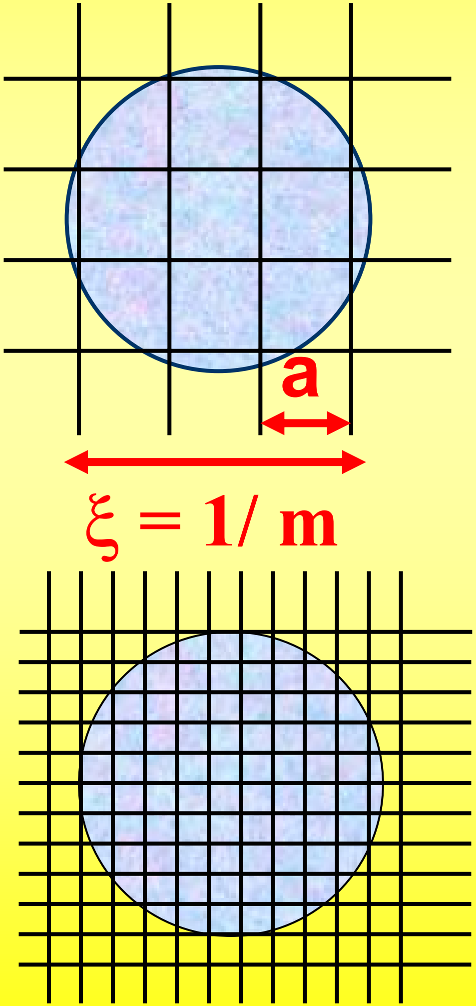
Hadron mass and $\langle 0|A|h\rangle$ matrix elements from the 2-point correlation function





SYSTEMATIC ERRORS IN LATTICE CALCULATIONS

DISCRETIZATION ERRORS
(THE ULTRAVIOLET PROBLEM)

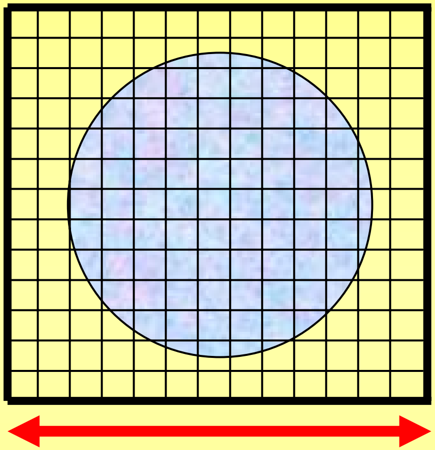


$\xi = 1/m$ is the Compton wave length (the size) of the hadron

If $\xi \sim a \Longrightarrow m\,a \sim 1$ the size of the object is comparable to the lattice spacing

$$C_{LATT} = C_{CONT}\,[\,1 + O(am,\ ap,\ a\Lambda_{QCD})\,]$$

FINITE VOLUME EFFECTS
(THE INFRARED PROBLEM)



BOX SIZE

$$L \gg \xi = 1/m$$

to avoid finite size effects

For a large class of important physical amplitudes finite size effects are not really a problem:

$O(\exp[-\xi/L])$ ⟶ $L \geq 4 \div 5\, \xi$ is sufficient

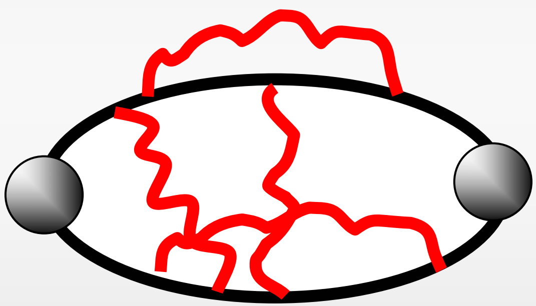
But there are more problematic cases, e.g. non-leptonic decays…

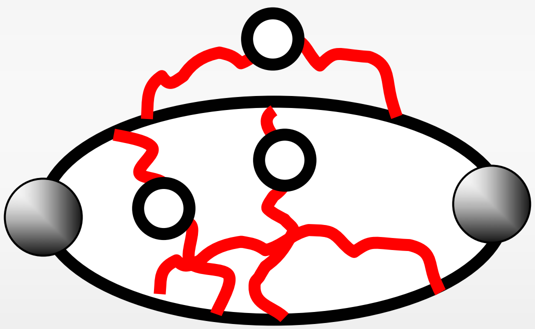
THE QUENCHED APPROXIMATION

$$\int [dU]\,[d\psi\, d\bar{\psi}]\, \exp[-\,S_g - \bar{\psi} M \psi] = \int [dU]\, \det M\, \exp[-S_g\,]$$

QUENCHED APPROXIMATION

det M = cost.



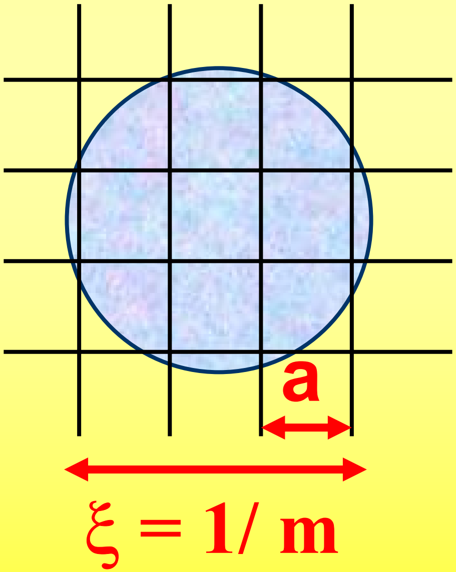


QA is not used in most recent calculations

EXTRAPOLATIONS IN QUARK MASSES

1) HEAVY QUARK MASSES

DISCRETIZATION ERRORS, THE ULTRAVIOLET PROBLEM



$1/M_H >> a \longleftrightarrow a\, M_H << 1$

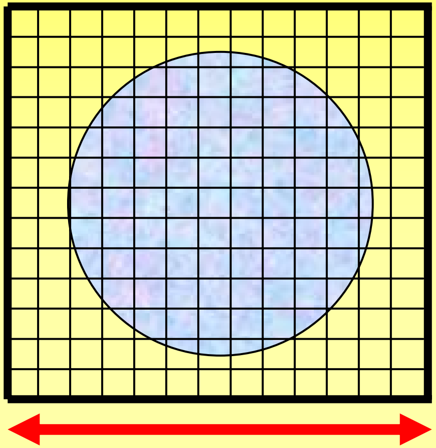
Typically $a^{-1} \sim 2 \div 5$ GeV

$m_{charm} \sim 1.3$ GeV $\quad m_{charm}\, a \sim 0.3$

$m_{bottom} \sim 4.5$ GeV $\quad m_{bottom}\, a \sim 1$

Or use effective theories (HQET, NRQCD,…)

2) LIGHT QUARK MASSES



$$1/M_\ell << L \longleftrightarrow L\, M_\ell >> 1$$

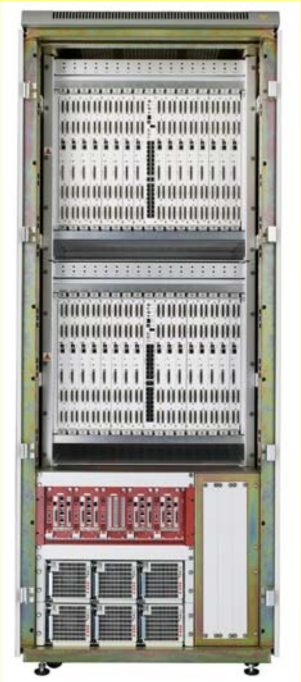
BECAUSE OF LIMITATIONS IN COMPUTER RESOURCES, VOLUMES CANNOT BE LARGE ENOUGH TO WORK AT THE PHYSICAL LIGHT QUARK MASSES

Lowest quark masses: $M_\pi \approx 300$ MeV

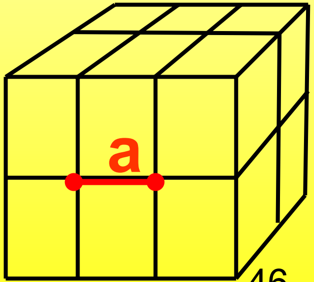
An extrapolation in m_{light} to the physical point is necessary. Chiral Perturbation Theory may help in the extrapolation.

There are several sources of systematic errors in lattice QCD simulations but:

- The accuracy can be systematically improved in time by increasing the computer resources



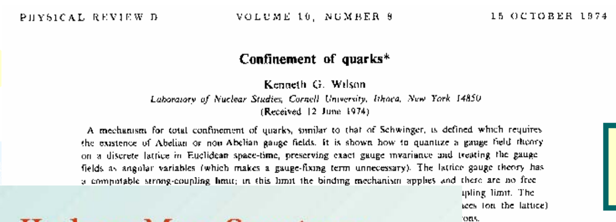
- Lattice QCD is the only non-perturbative approach to QCD which does not contain any additional free parameter besides those of the fundamental theory



From S. Hashimoto
ICHEP 2004

30 years of lattice QCD

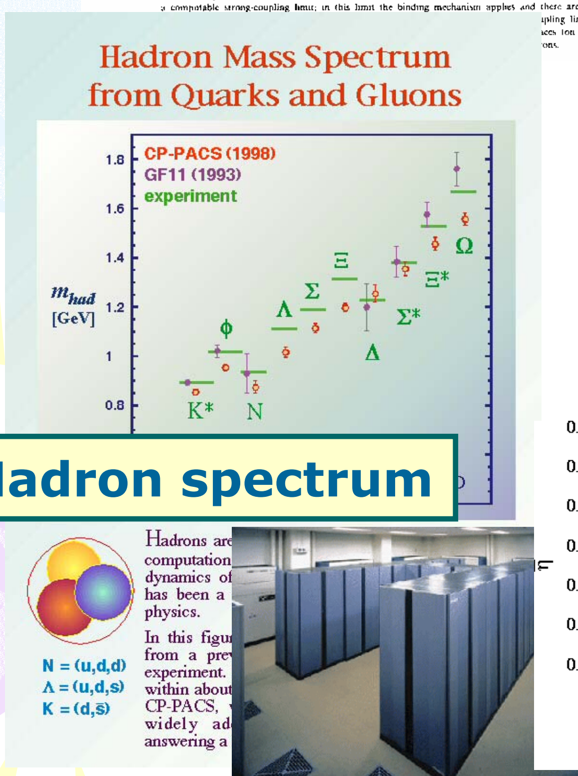
K. Wilson (1974)


PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

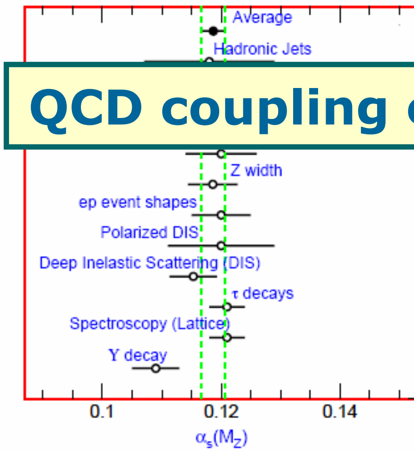
Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

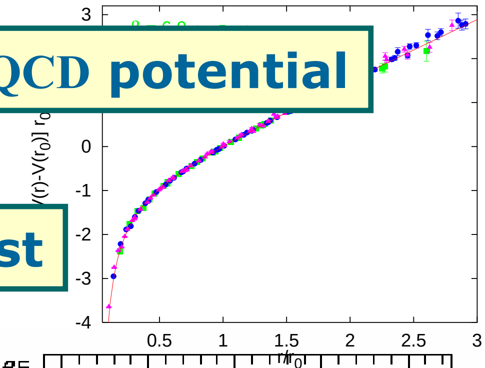
A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free ...



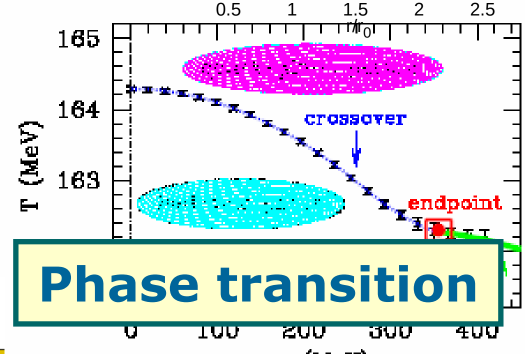
Hadron spectrum



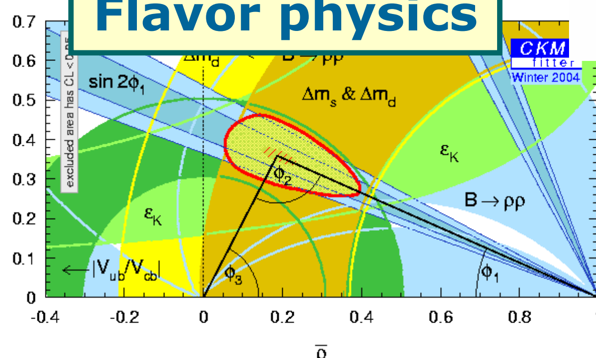
QCD coupling const



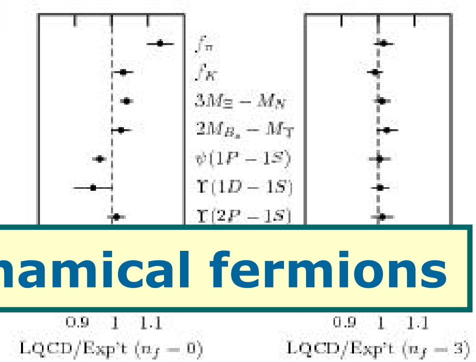
QCD potential



Phase transition



Flavor physics



Dynamical fermions

THE PRECISION ERA OF FLAVOR PHYSICS

EXPERIMENTS

ε_K = 2.280 10^{-3} ± 0.6%

Δm_d = 0.502 ps^{-1} ± 1%

sin(2β) = 0.687 ± 5%

………

THEORY

We need to control the theoretical input parameters at a comparable level of accuracy !!

Challenge for LATTICE QCD

LATTICE QCD AND QUARK MASSES

♦ QUARK MASSES CANNOT BE DIRECTLY MEASURED IN THE EXPERIMENTS, BECAUSE QUARKS ARE CONFINED INSIDE HADRONS

♦ BEING FUNDAMENTAL PARAMETERS OF THE STANDARD MODEL, QUARK MASSES CANNOT BE DETERMINED BY THEORETICAL CONSIDERATIONS ONLY.

→ QUARK MASSES CAN BE DETERMINED BY COMBINING TOGETHER A THEORETICAL AND AN EXPERIMENTAL INPUT. E.G.:

$$[M_{HAD}(\Lambda_{QCD}, m_q)]^{TH.} = [M_{HAD}]^{EXP.}$$

LATTICE QCD

LATTICE DETERMINATION OF QUARK MASSES

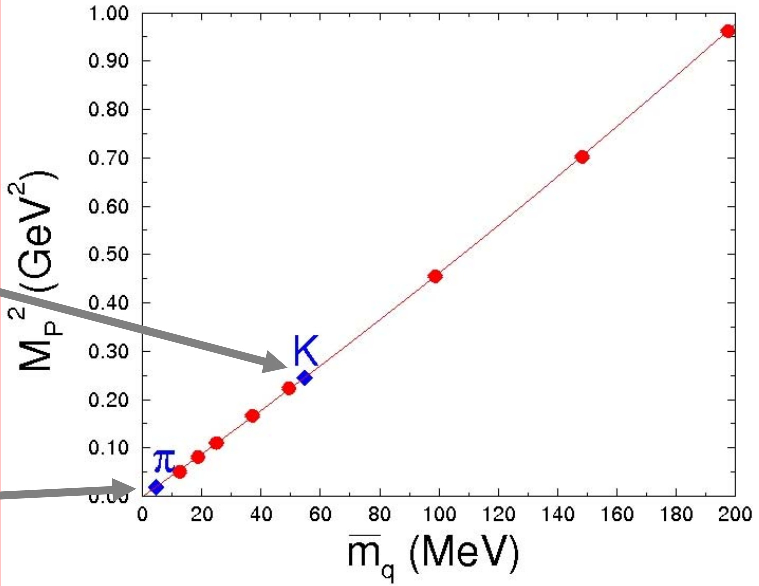
$$\hat{m}_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY OR
NON-PERTURBATIVE METHODS

Extrapolation
to $m = m_s$

Extrapolation
to $m = m_{u,d}$



SYSTEMATIC ERRORS

$$m_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL $M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY

O(a)

O(α^2)

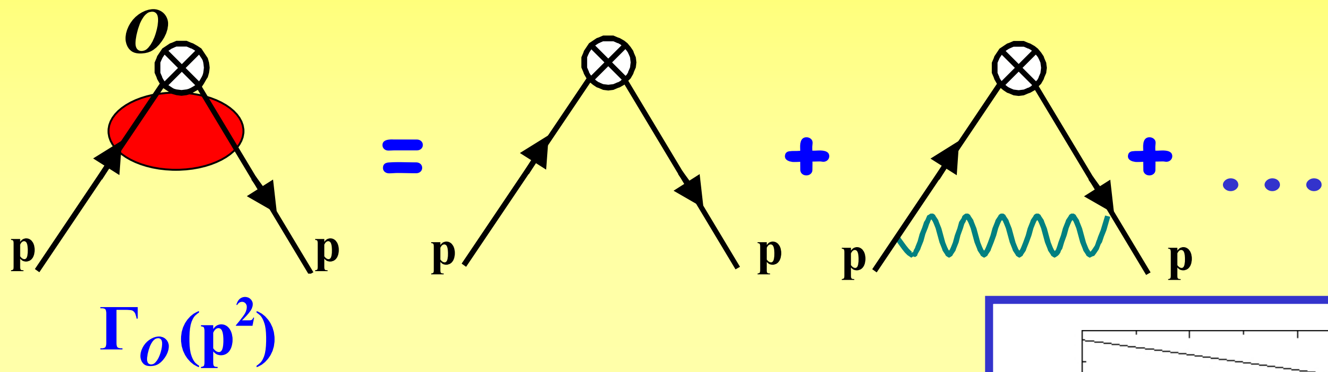
IMPROVED ACTIONS:

$$S_{LATT} = S_{QCD} + \cancel{a\, S_1} + a^2 S_2 + \ldots$$

NON-PERTURBATIVE RENORMALIZATION

TWO IMPORTANT THEORETICAL TOOLS

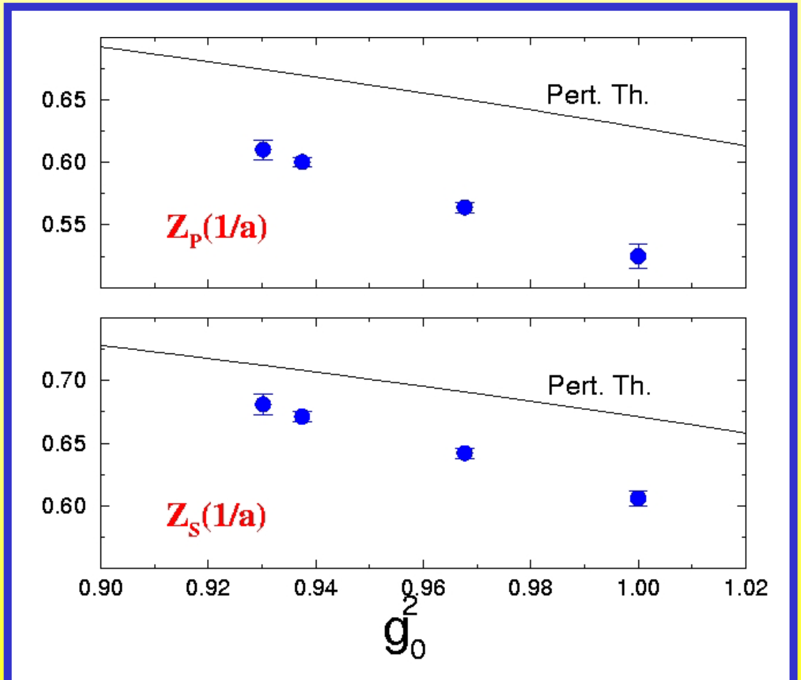
NON-PERTURBATIVE RENORMALIZATION THE RI-MOM METHOD



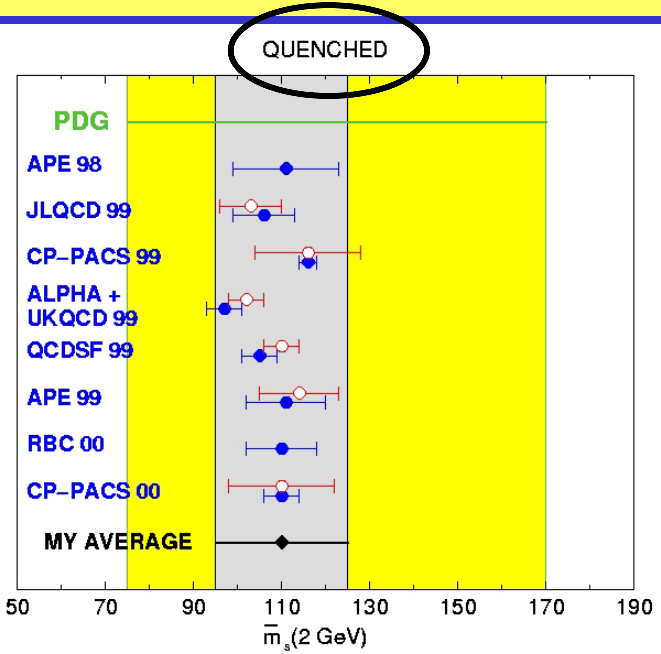
The (non-perturbative) renormalization condition:

$$Z_O(a\mu)\, \Gamma_O(p^2)|_{p^2=\mu^2} = \Gamma_{\text{Tree-Level}}$$

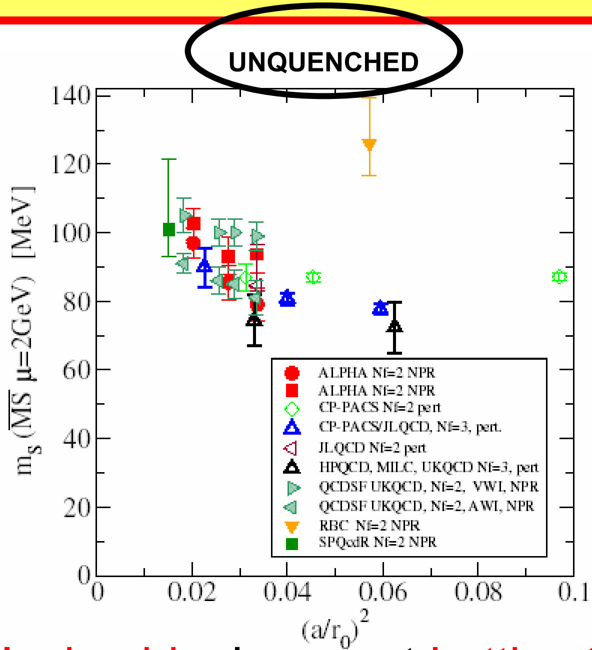
Several NPR techniques have been developed: Ward Identities, Schrodinger functional, X-space



THE STRANGE QUARK MASS



V.Lubicz, plenary at Lattice 2000



T.Izubuchi, plenary at Lattice 2005

$\overline{m}_s$ (2 GeV)

2000: $\overline{m}_s$=(120 ± 50) MeV

2002: $\overline{m}_s$=(120 ± 40) MeV

2004: $\overline{m}_s$=(105 ± 25) MeV

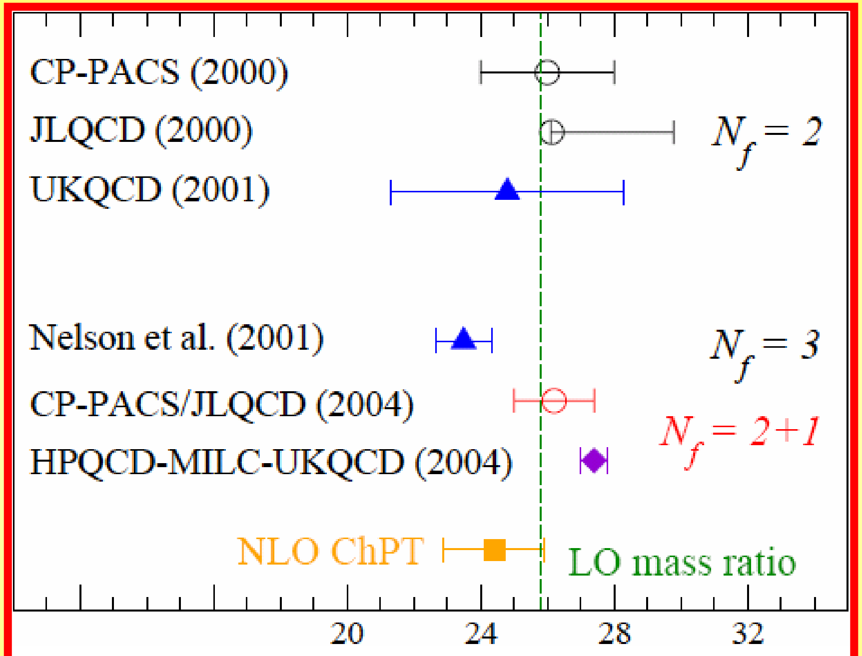
THE AVERAGE UP/DOWN QUARK MASS

RATIOS OF LIGHT QUARK MASSES ARE PREDICTED ALSO BY CHIRAL PERTURBATION THEORY:

$$\frac{m_u}{m_d} = 0.553 \pm 0.043$$

$$\frac{m_s}{(m_u + m_d)/2} = 24.4 \pm 1.5$$

From S. Hashimoto ICHEP 2004

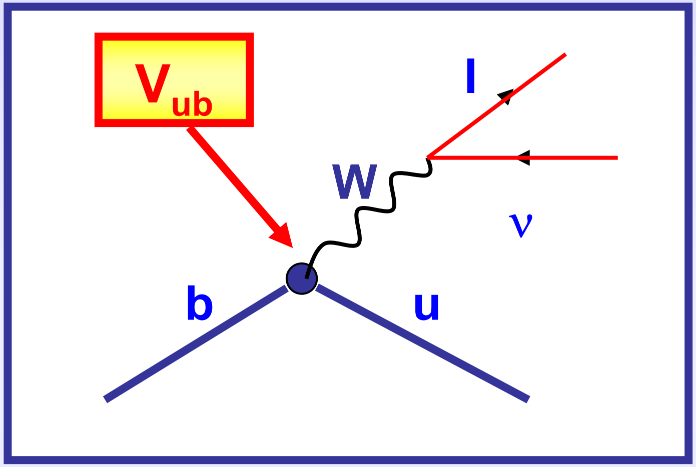


$2\, m_s/(m_u+m_d)$

Good agreement with the ChPT prediction

LATTICE QCD AND THE UNITARITY TRIANGLE ANALISYS

THE CKM MATRIX



$$L_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu W_\mu^+ (1-\gamma^5) d_j + \text{h.c.}$$

3 FAMILIES: 3 angles and 1 phase

Only one parameter for CP VIOLATION

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \approx \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

(Lincoln Wolfenstein)

THE UNITARITY TRIANGLES

(Bjorken-Jarlskog)

Unitarity relations:

$$V^{\dagger}V = 1 \longrightarrow \sum_k V^*_{ki} V_{kj} = \delta_{ij}$$

9 constraints,
6 triangular relations

Only 2 triangles have all sides with length of the same $O(\lambda^3)$:

$$V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0$$

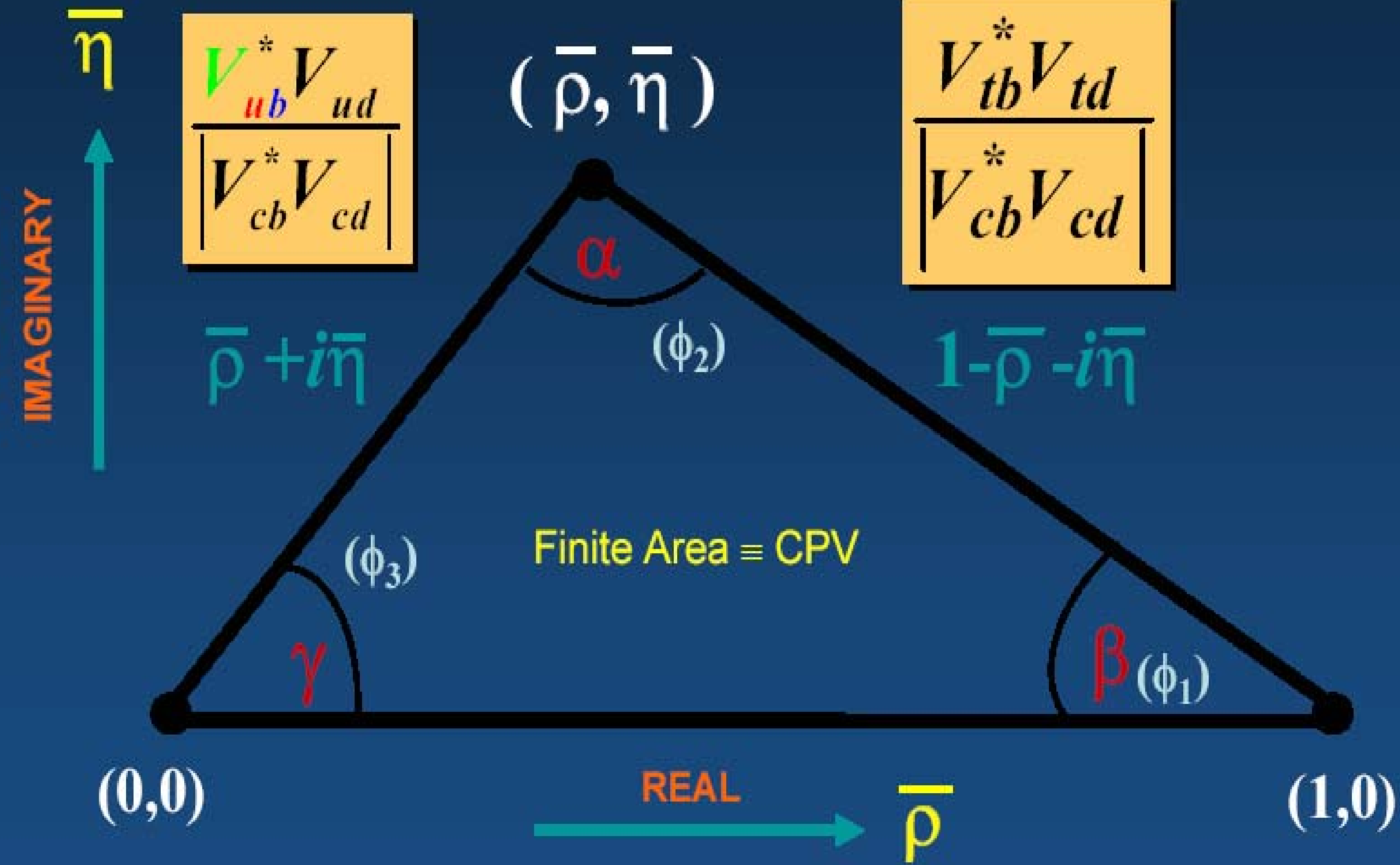
$$V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0$$

(They are equivalent at order λ^3)

Only the orientation of the triangles depends on the phase convention. The area and ~~CP~~ are proportional to:

$$J = c_{12}\, c^2_{13}\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta \approx A^2\lambda^6\eta \sim 10^{-5}$$

Unitarity: $V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$



UTfit Collaboration

M.Bona, M.Ciuchini, E.Franco,

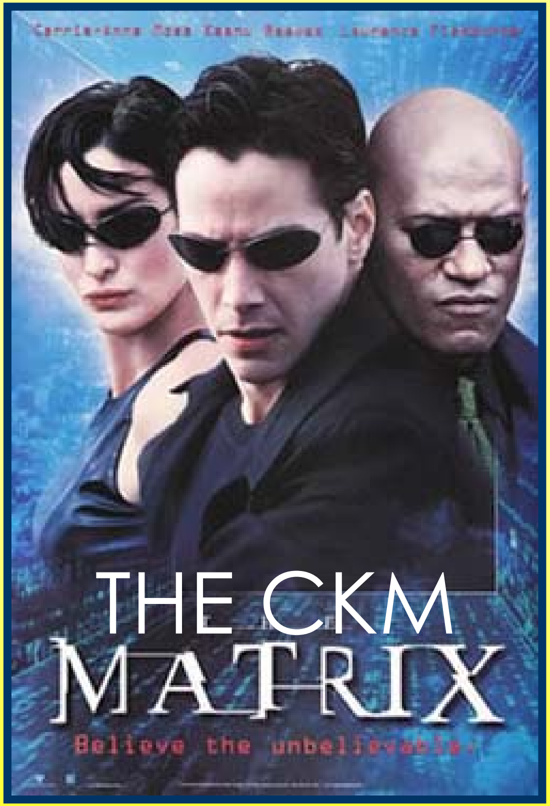
V.L., G.Martinelli, F.Parodi,

M.Pierini, P.Roudeau, C.Schiavi,

L.Silvestrini, A.Stocchi, V.Vagnoni

Roma, Genova, Torino,
Orsay, Bologna

www.utfit.org

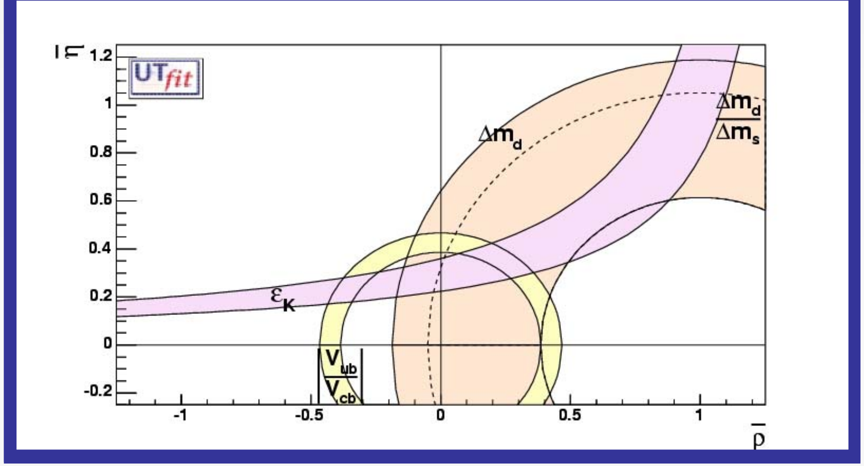


THE "CLASSICAL" FIT

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho - i\bar\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CP violation

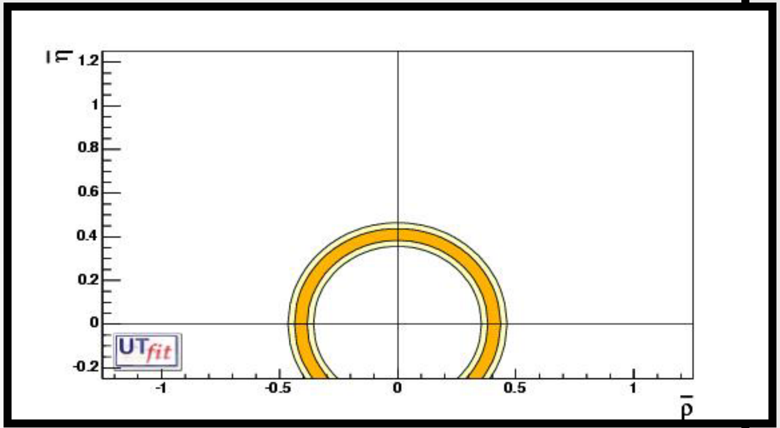


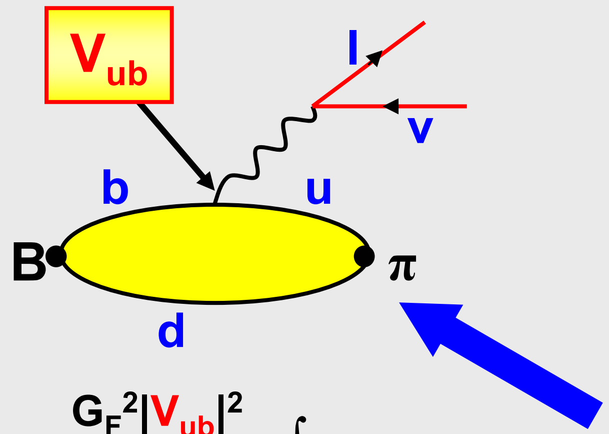
(b→u)/(b→c)	$\bar\rho^2 + \bar\eta^2$	f_+,F(1),…
Δm_d	$(1-\bar\rho)^2 + \bar\eta^2$	$f^2_{B_d} B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1-\bar\rho)^2 + \bar\eta^2$	ξ
ε_K	$\bar\eta[(1-\bar\rho) + P]$	B_K

4 CONSTRAINTS

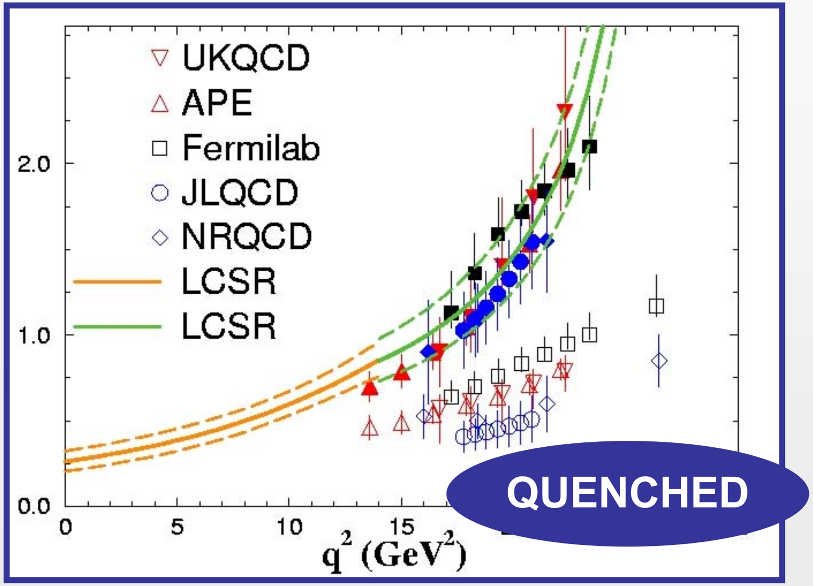
Hadronic matrix elements from LATTICE QCD

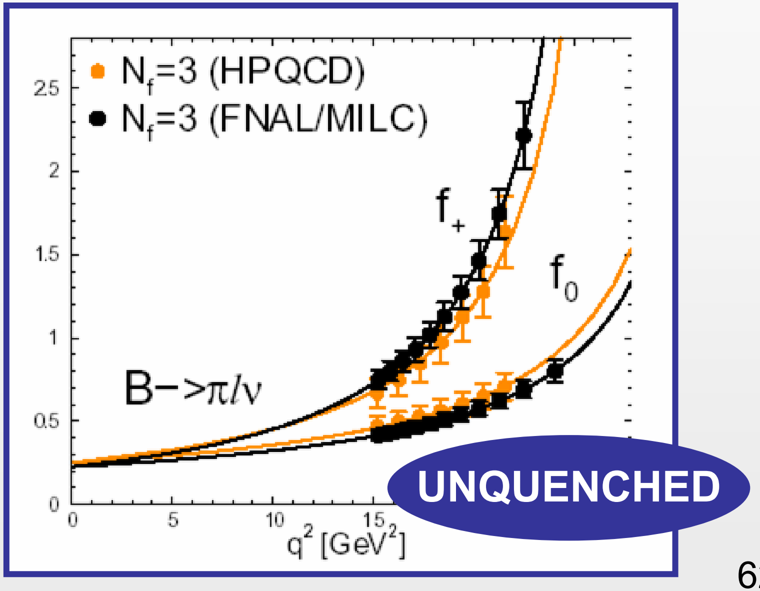
V_{ub} and V_{cb} from semileptonic decays



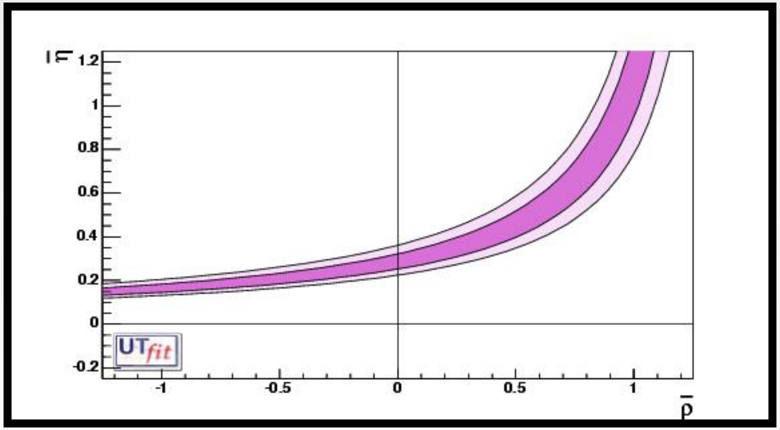


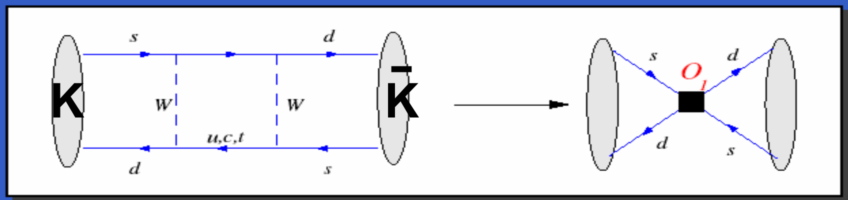
$$\Gamma(B \rightarrow \pi l v) = \frac{G_F^2 |V_{ub}|^2}{192\,\pi^3} \int dq^2\, \lambda(q^2)^{3/2}\, |f_+(q^2)|^2$$





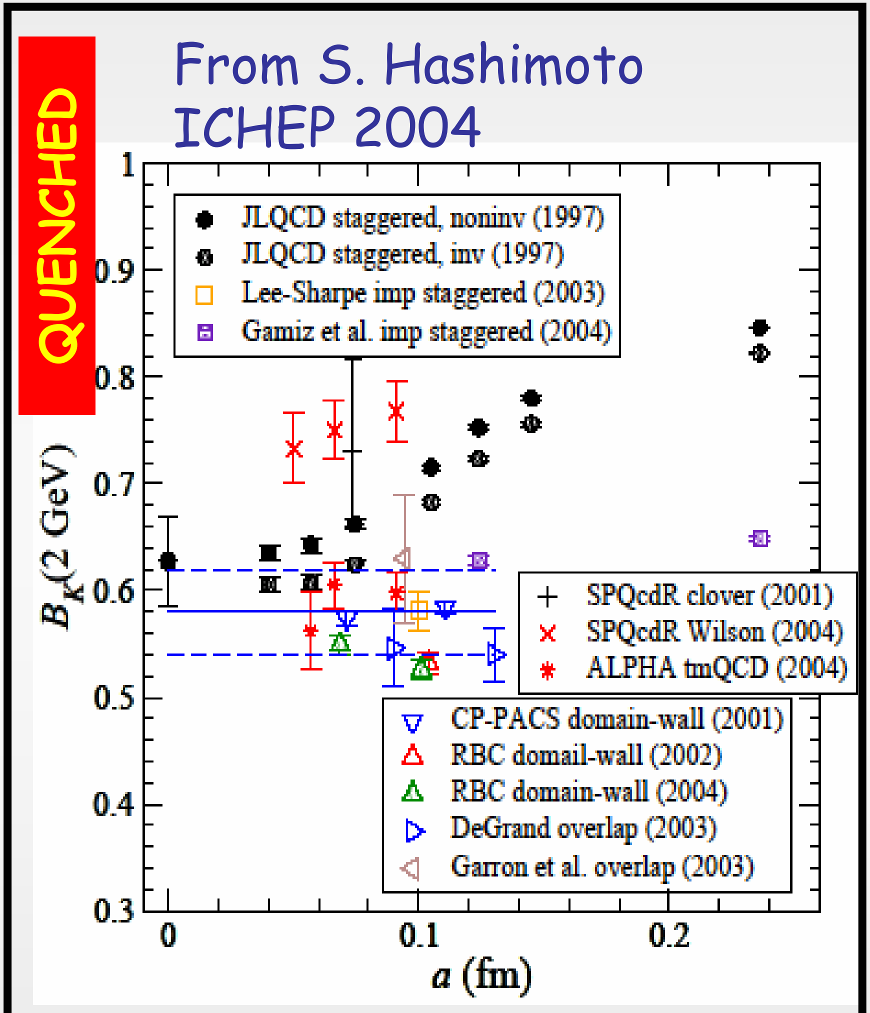
$K-\bar{K}$ mixing: ε_K and B_K





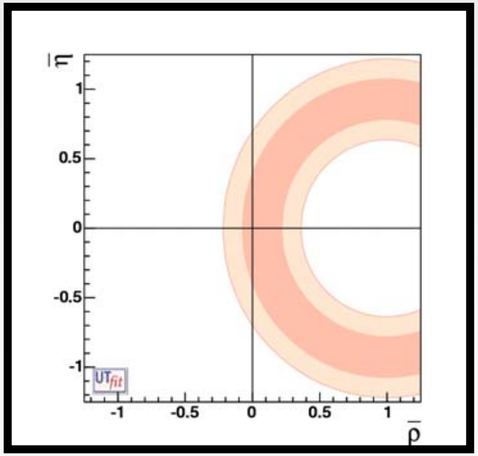
$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

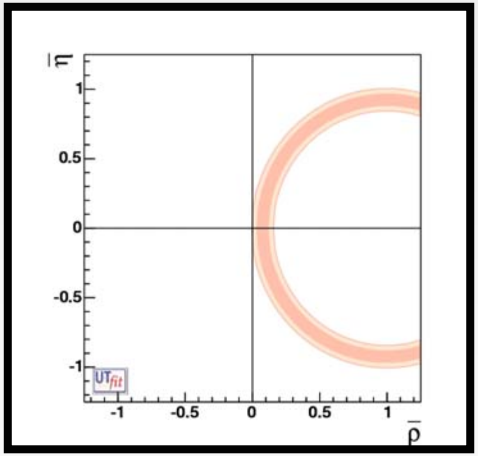
$\hat{B}_K = 0.79 \pm 0.04 \pm 0.09$



LATTICE PREDICTION (!) $\hat{B}_K = 0.90 \pm 0.20$ [Gavela et al., 1987]

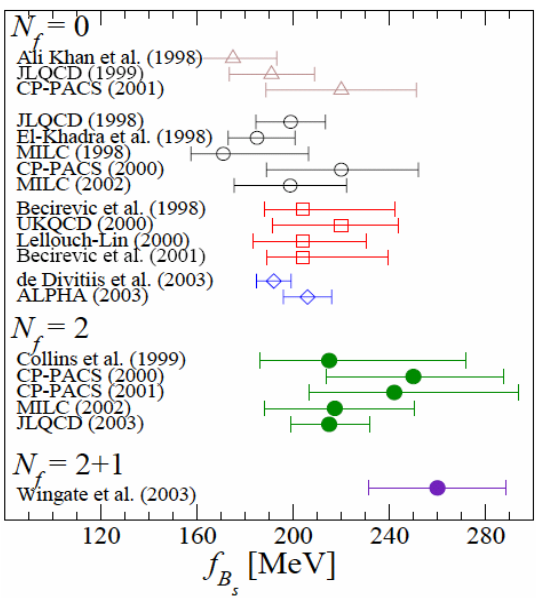
B_d and B_s mixing: $f_B\sqrt{B_B}$



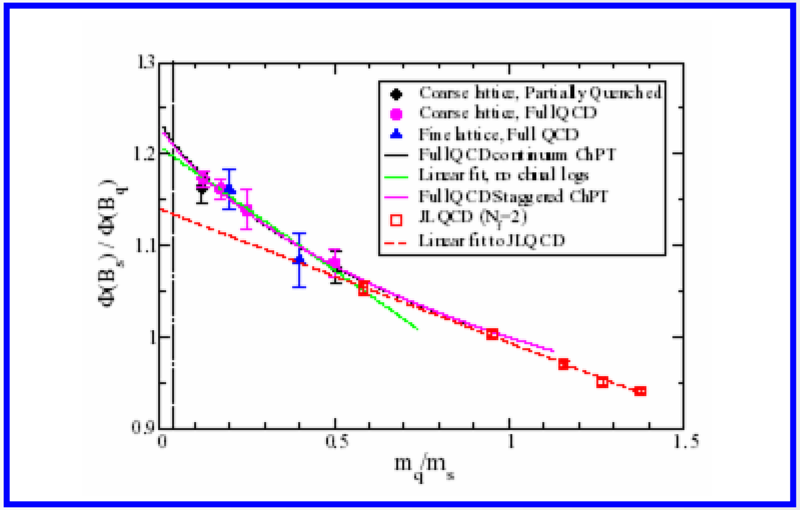


$f_{Bs}\sqrt{B_{Bs}}$ = 262 ± 35 MeV,

ξ = 1.23 ± 0.06



From S. Hashimoto ICHEP 2004



PREDICTION OF Sin2β

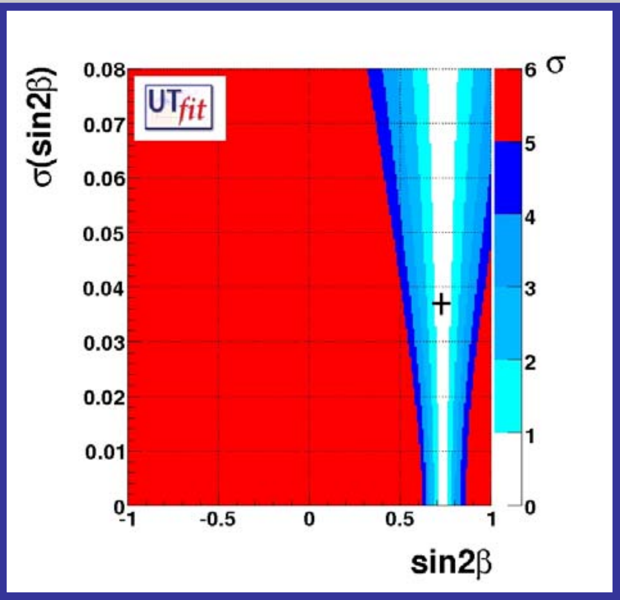
The first observation of ~~CP~~ in B-physics after the discovery in the kaon sector (1964)

Ciuchini et al., 2000:

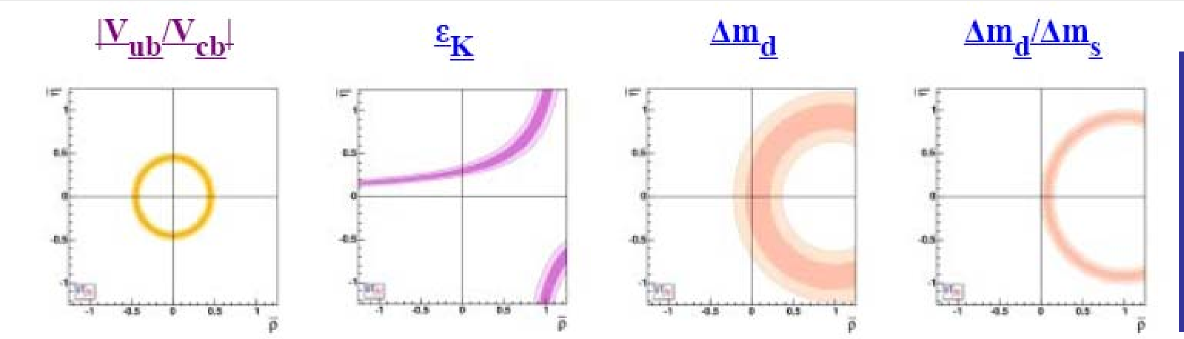
$\mathrm{Sin}2\beta_{UTFit} = 0.698 \pm 0.066$

B-factories:

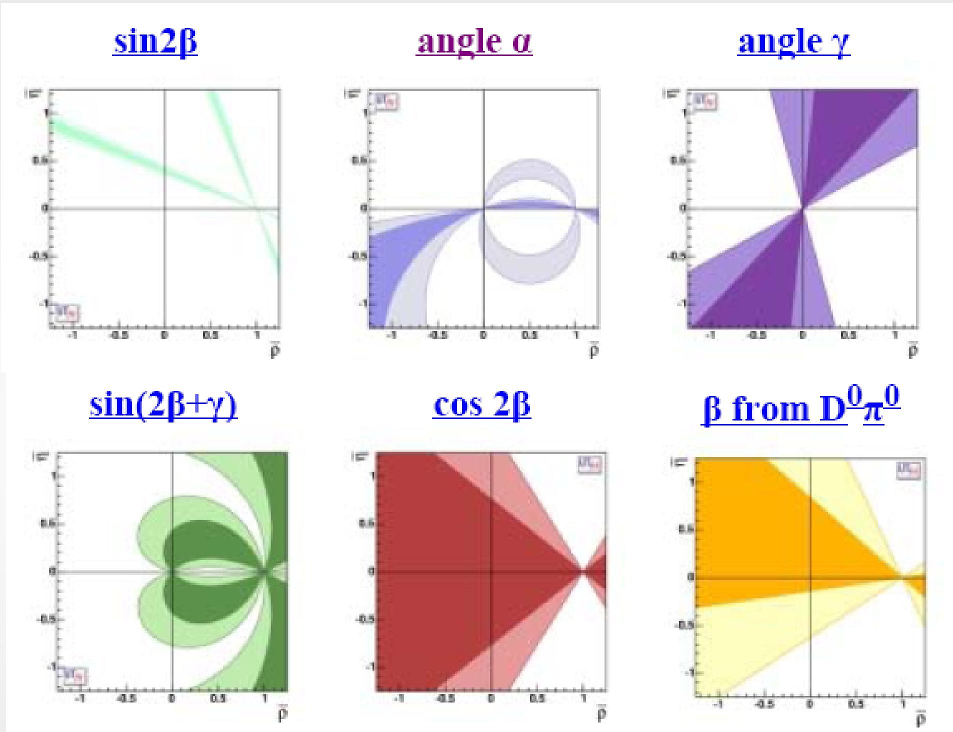
$\mathrm{Sin}2\beta_{J/\psi\ Ks} = 0.687 \pm 0.032$



A success of (quenched) LQCD calculations !!



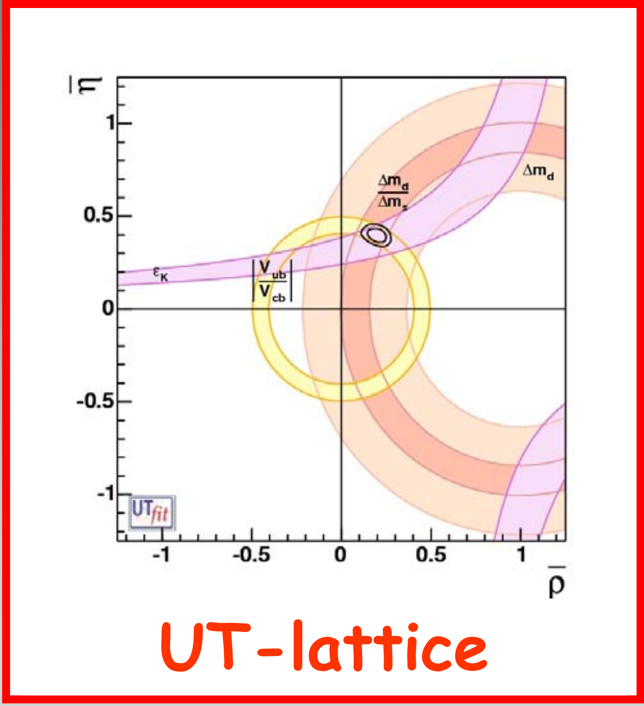
THE CLASSICAL FIT "UT-lattice"

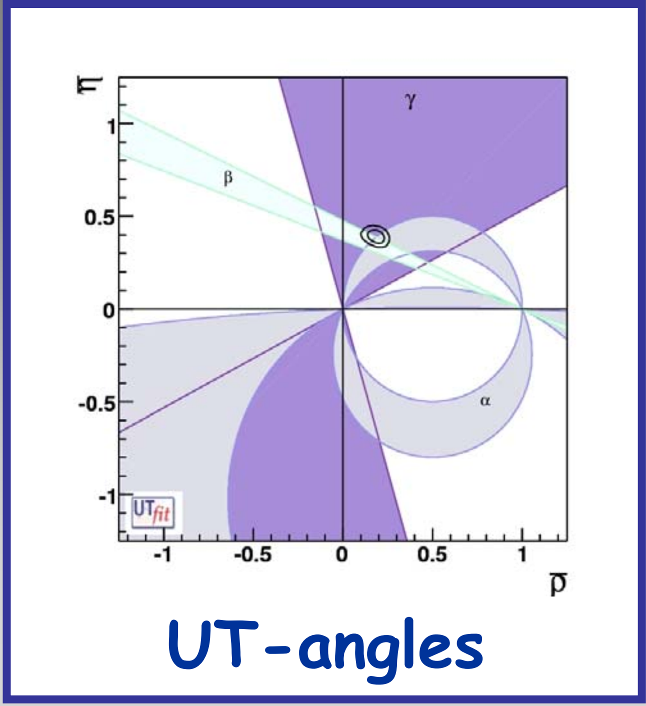


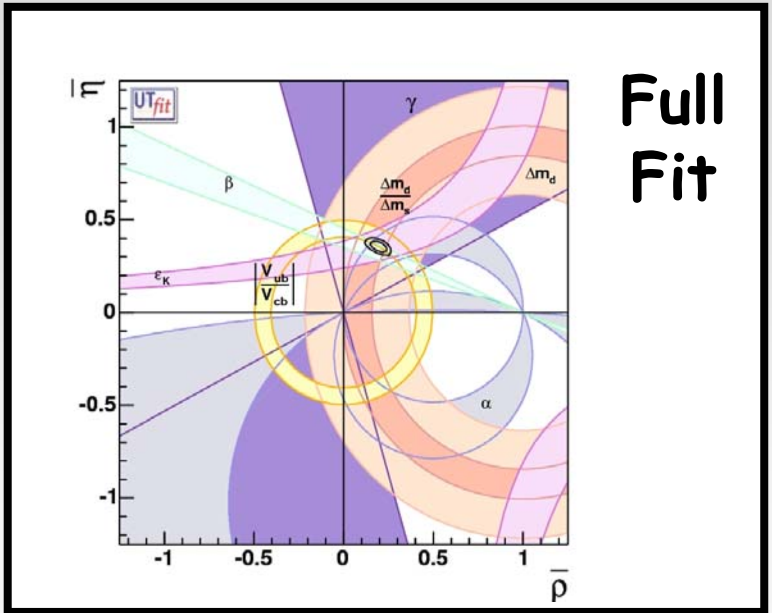
UTfit

THE ANGLE FIT "UT-angles"

Several determinations of UT angles are now available, thanks to the results coming from the B-factories experiments







- UT-lattice and UT-angles fits are in good agreement
- The errors have comparable sizes
- The UT-angles fit does not rely at all on theoretical calculations

PREDICTION FOR Δm_s

Ciuchini et al., 2000:

$\Delta m_s = (16.3 \pm 3.4)\ ps^{-1}$

UTfit Collaboration, 2005:

$\Delta m_s = (21.2 \pm 3.2)\ ps^{-1}$

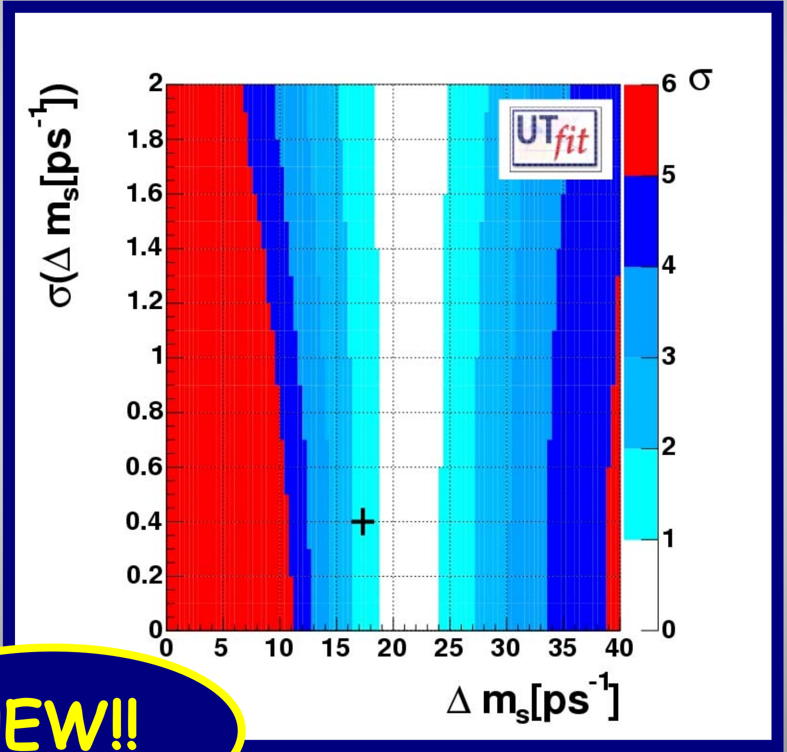
CDF, 2006:

$\Delta m_s = (17.33\,^{+0.42}_{-0.21} \pm 0.07)\ ps^{-1}$

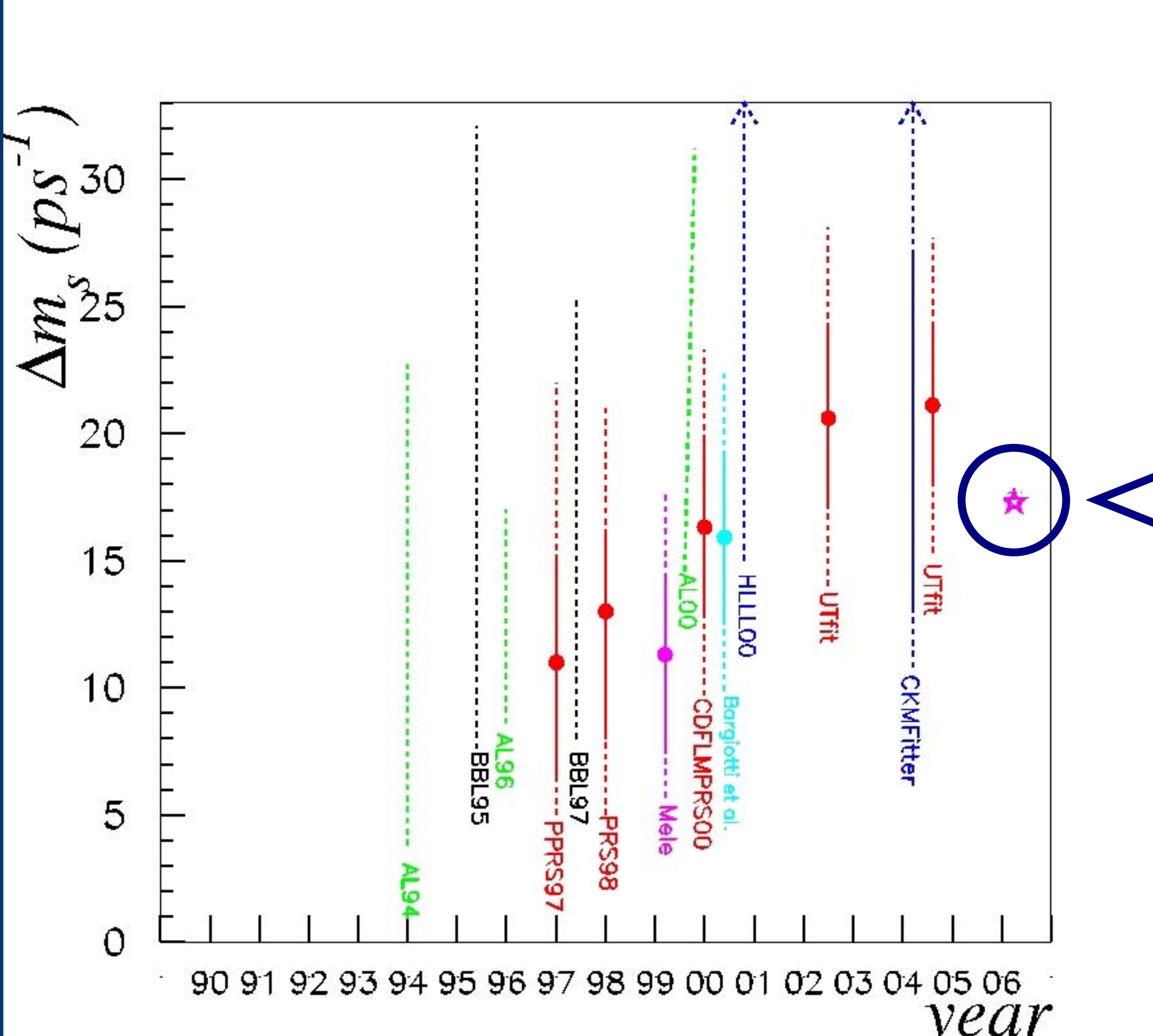
CDF+D0+LEP combined:

$\Delta m_s = (17.35 \pm 0.25)\ ps^{-1}$

NEW!!

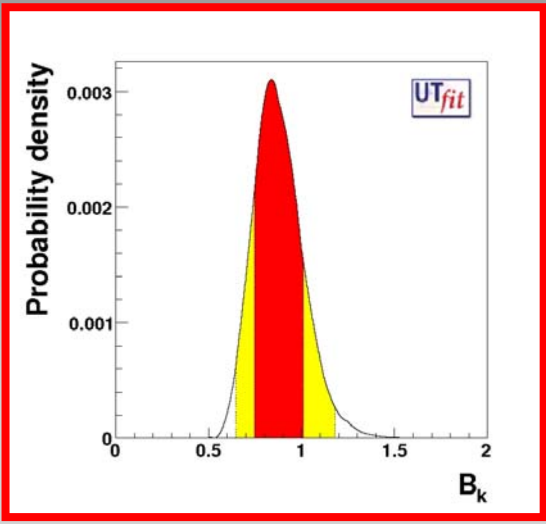


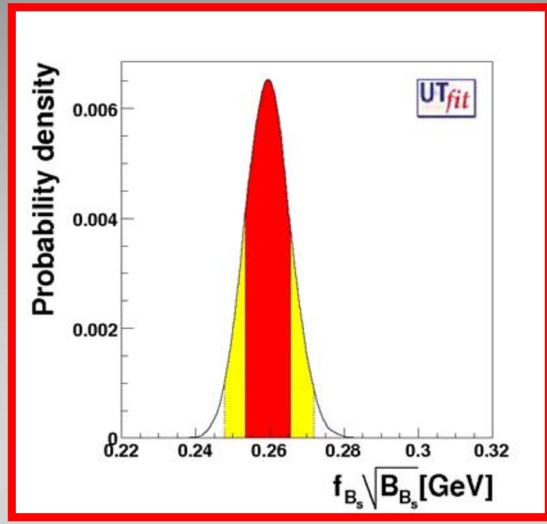
HISTORY OF Δm_s PREDICTIONS

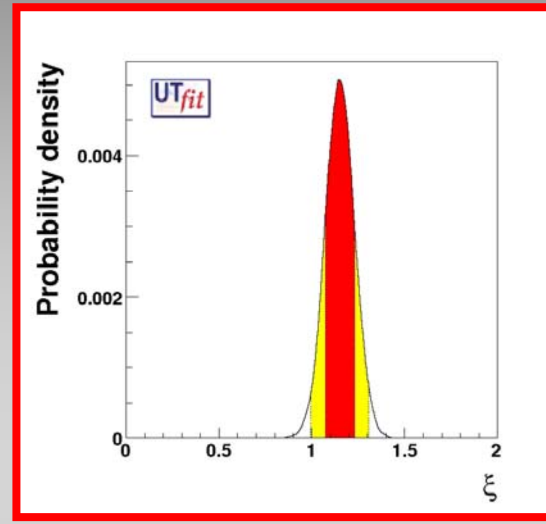


Measurement

LATTICE QCD vs UT FITS

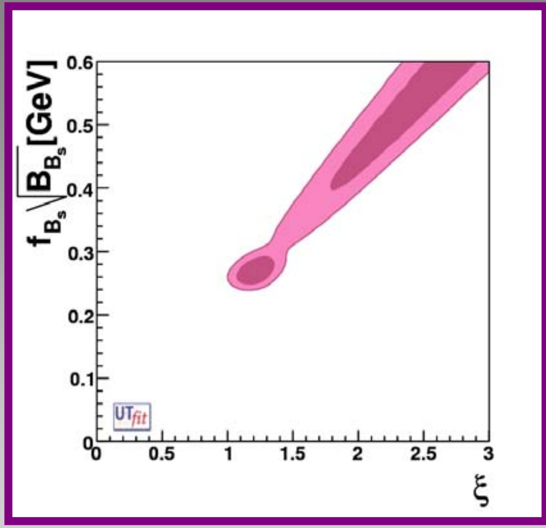




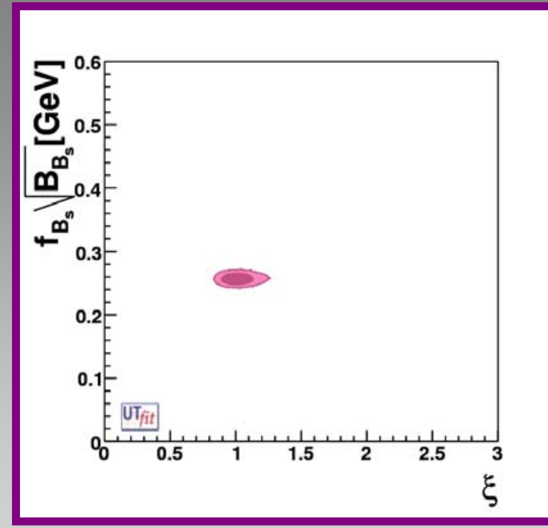


	Lattice QCD	UT Fits
$\hat{B}_K$	0.79 ± 0.04 ± 0.09	0.88 ± 0.13
$f_{Bs}\sqrt{B_{Bs}}$	262 ± 35 MeV	259 ± 6 MeV
ξ	1.23 ± 0.06	1.13 ± 0.08

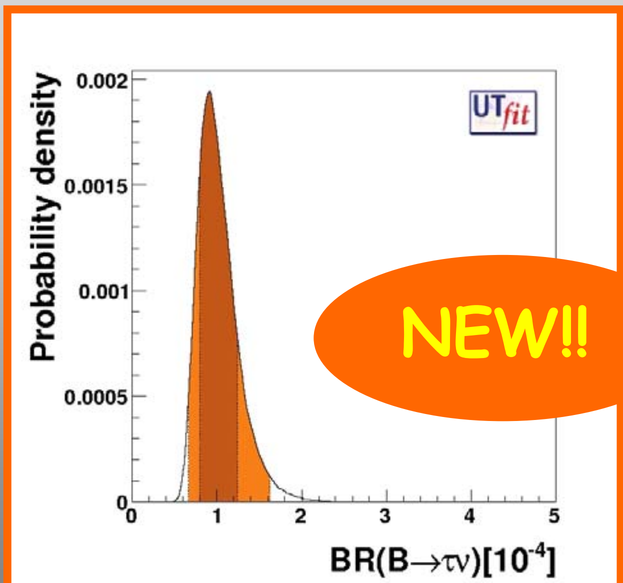
The accuracy of LQCD calculations must be improved



Before the measurement of Δm_s



After the measurement of Δm_s



NEW!!

Belle, 2006:

$$BR(B\to\tau\nu_\tau) = (1.06\ {}^{+0.34}_{-0.28}\ {}^{+0.18}_{-0.16}) \times 10^{-4}$$

f_{Bd} = 192 ± 26 ± 9 MeV, LQCD

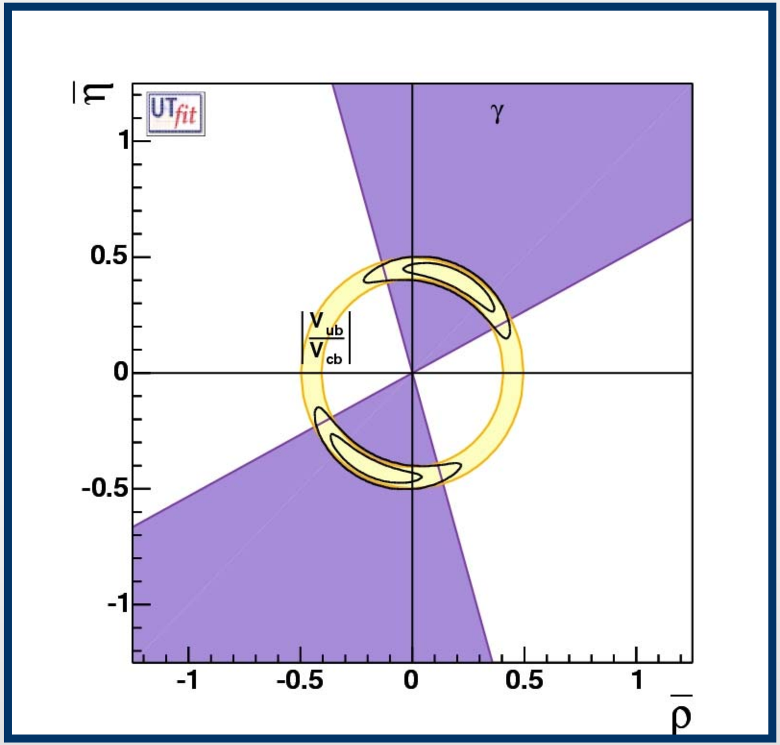
f_{Bd} = 180 ± 17 MeV , UTfit

THE UTA BEYOND THE STANDARD MODEL

Given the present theoretical and experimental constraints, to which extent the UTA can still be affected by New Physics contributions?

UTFIT FROM TREE-LEVEL PROCESSES

Two constraints are now available, which are almost unaffected by the presence of NP:



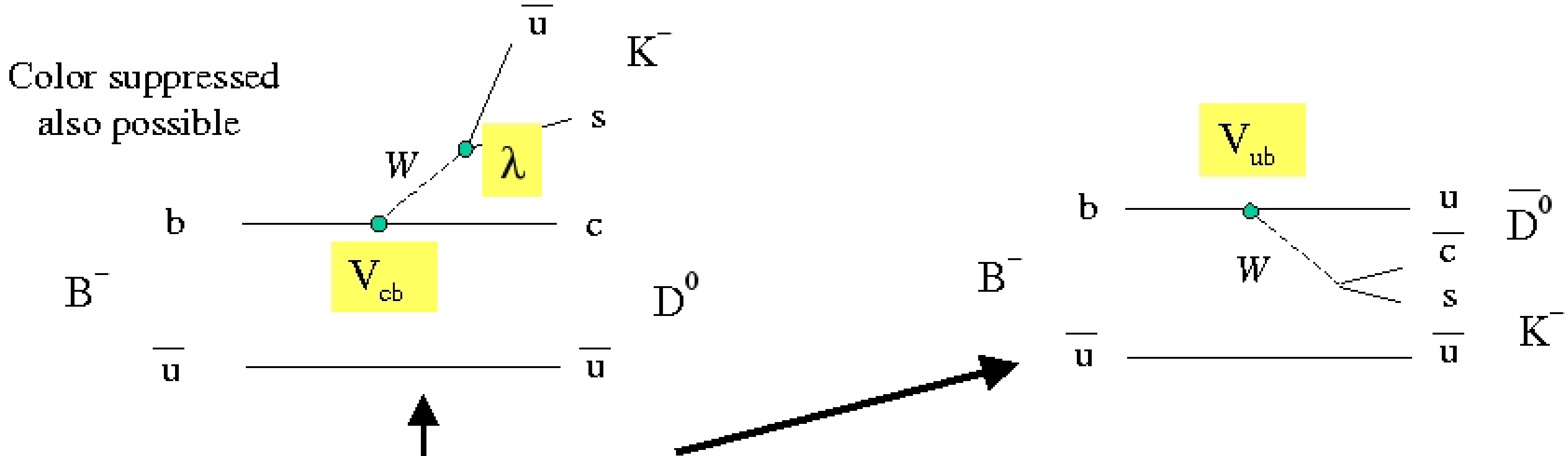
1) $|V_{ub}/V_{cb}|$ from semileptonic B decays

2) The angle γ from $B \to D^{(*)} K$ decays

2 solutions

It's now lunchtime (13:35) on Buras' unitarity clock

THE ANGLE γ FROM $B \to D^{(*)} K$ DECAYS



$$A_-(B^- \to f) = v_1 A_1 e^{i\alpha_1} + v_2 A_2 e^{i\alpha_2} \quad , \quad A_+(B^+ \to \bar{f}) = v_1^* A_1 e^{i\alpha_1} + v_2^* A_2 e^{i\alpha_2}$$

If neutral D mesons in a CP eigenstate ($D^0 \pm \bar{D}^0$) are considered in the final states, the two amplitudes interferes, and the relative weak phase γ can be determined

A MODEL INDEPENDENT ANALYSIS

New Physics in $\Delta F = 2$ amplitudes can be parameterized in a simple general form:

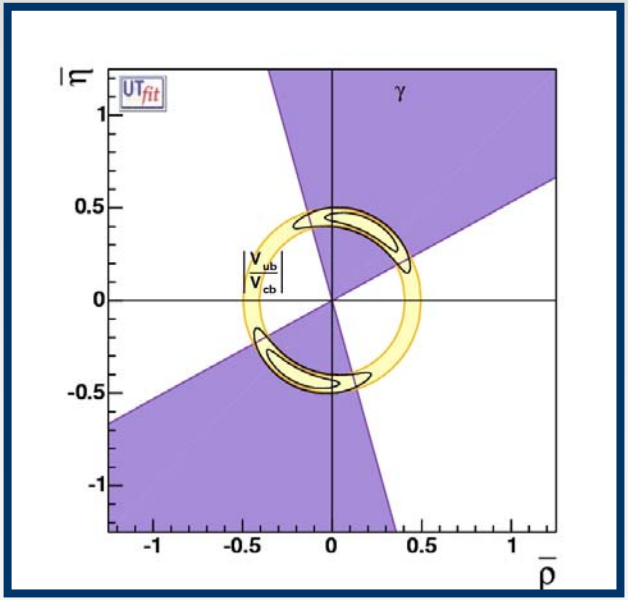
$$C_{B_q} e^{2i\phi_{B_q}} = \frac{\langle B_q^0 | H_{\text{eff}}^{\text{full}} | \bar{B}_q^0 \rangle}{\langle B_q^0 | H_{\text{eff}}^{\text{SM}} | \bar{B}_q^0 \rangle}, \quad C_{\varepsilon_K} = \frac{\text{Im}\left[\langle K^0 | H_{\text{eff}}^{\text{full}} | \bar{K}^0 \rangle\right]}{\text{Im}\left[\langle K^0 | H_{\text{eff}}^{\text{SM}} | \bar{K}^0 \rangle\right]}$$

$(q = d, s)$

E.g.: $(\Delta m_d)^{exp} = C_{Bd} (\Delta m_d)^{SM}$, $\sin 2\beta^{exp} = \sin 2(\beta^{SM} + \varphi_{Bd})$

and similarly for $\Delta F = 1$.

In the Standard Model: $C_{xx} = 1$, $\varphi_{xx} = 0$



Since:

$$(\Delta m_d)^{exp} = C_{Bd}\, \Delta m_d(\rho,\eta)^{SM}$$

$$\sin 2\beta^{exp} = \sin[2\beta(\rho,\eta)^{SM} + 2\varphi_{Bd}]$$

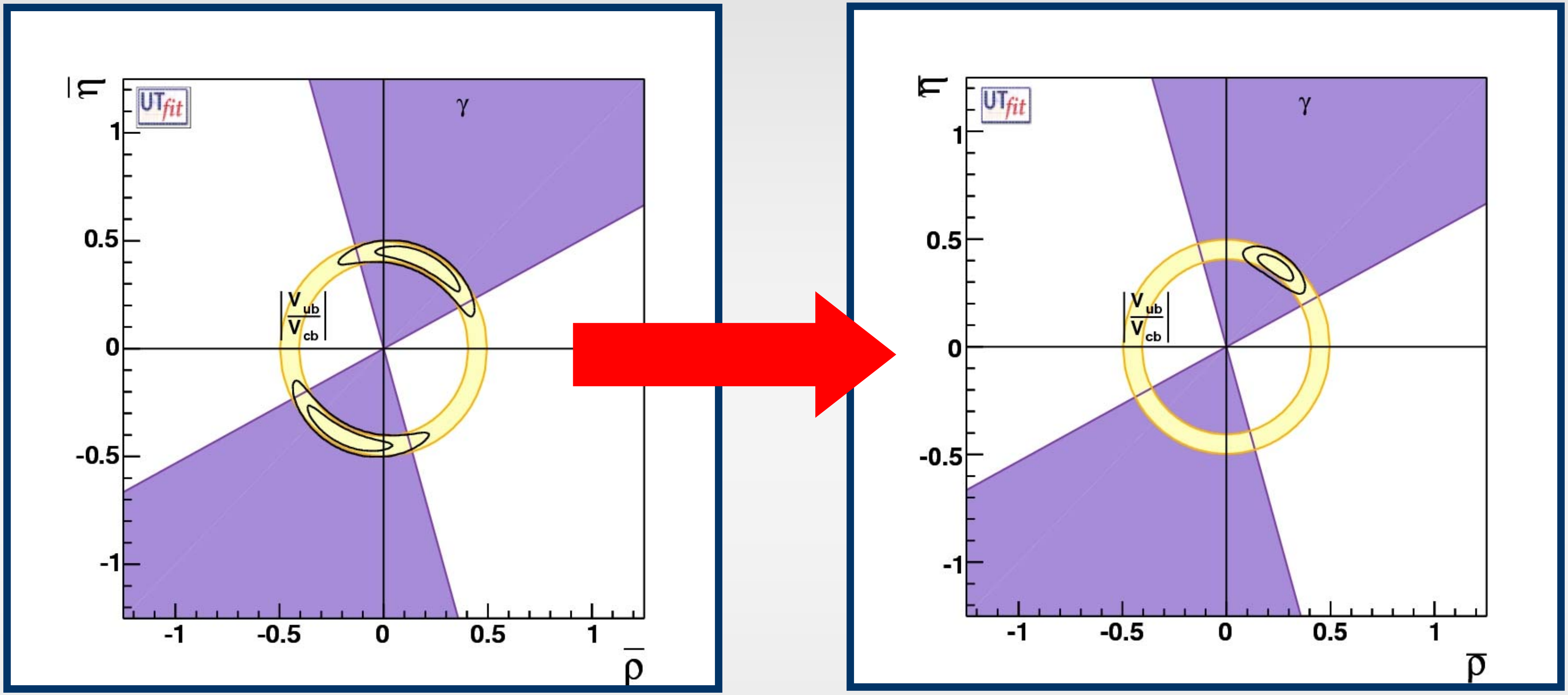
observables which depend simultaneously on ρ,η and the two NP parameters C_{Bd} and φ_{Bd} are needed in order to discriminate between the 2 solutions

The semileptonic asymmetries: (H=M-iΓ)

$$A_{SL}^{q} = \frac{\Gamma(\bar{B}_q^0 \to \ell^+ X) - \Gamma(B_q^0 \to \ell^- X)}{\Gamma(\bar{B}_q^0 \to \ell^+ X) + \Gamma(B_q^0 \to \ell^- X)} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) =$$

$$= -\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right)^{SM} \frac{\sin 2\phi_{B_q}}{C_{B_q}} + \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)^{SM} \frac{\cos 2\phi_{B_q}}{C_{B_q}}$$

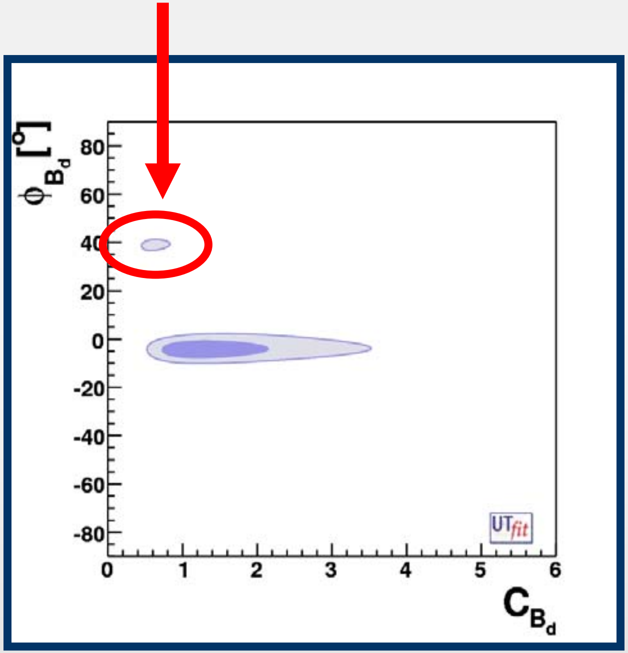
Including all the constraints :

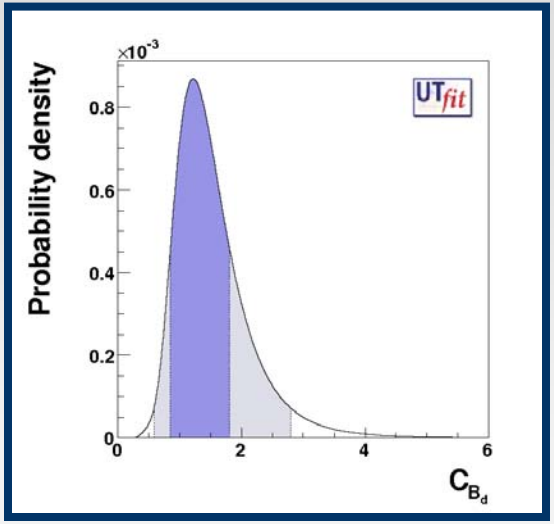


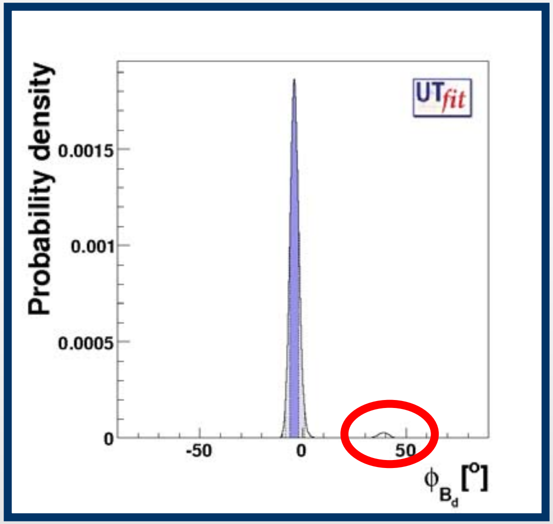
the **non-standard solution disappears** at the 95% C.L.

NP in B_d-$\overline{B}_d$ mixing

The non-standard solution has a probability below 5%





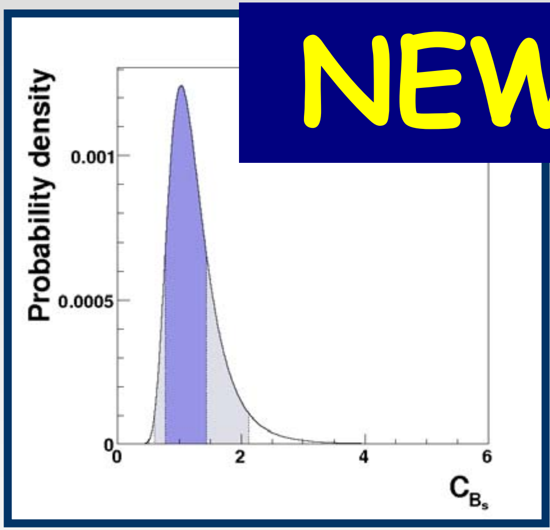


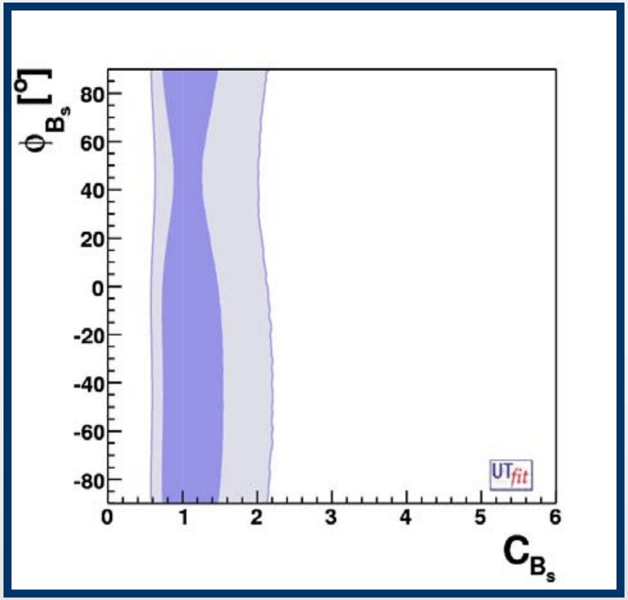
The results point to models of Minimal Flavour Violation

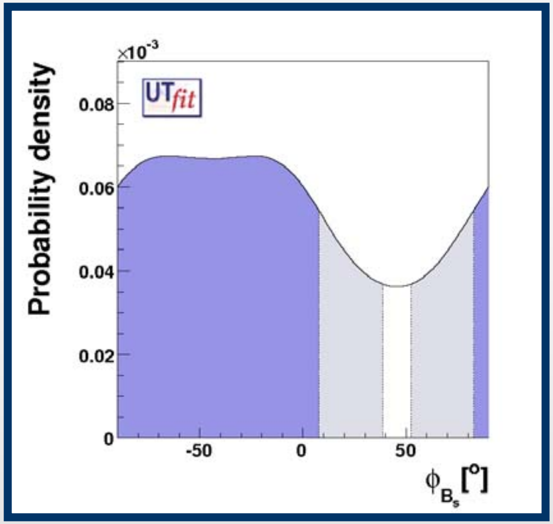
NP in B_s-$\overline{B}_s$ mixing

NEW!!

C_{Bs} from Δm_s [CDF], φ_{Bs} from A_{CH}, the CP asymmetry in dimuon events [D0]

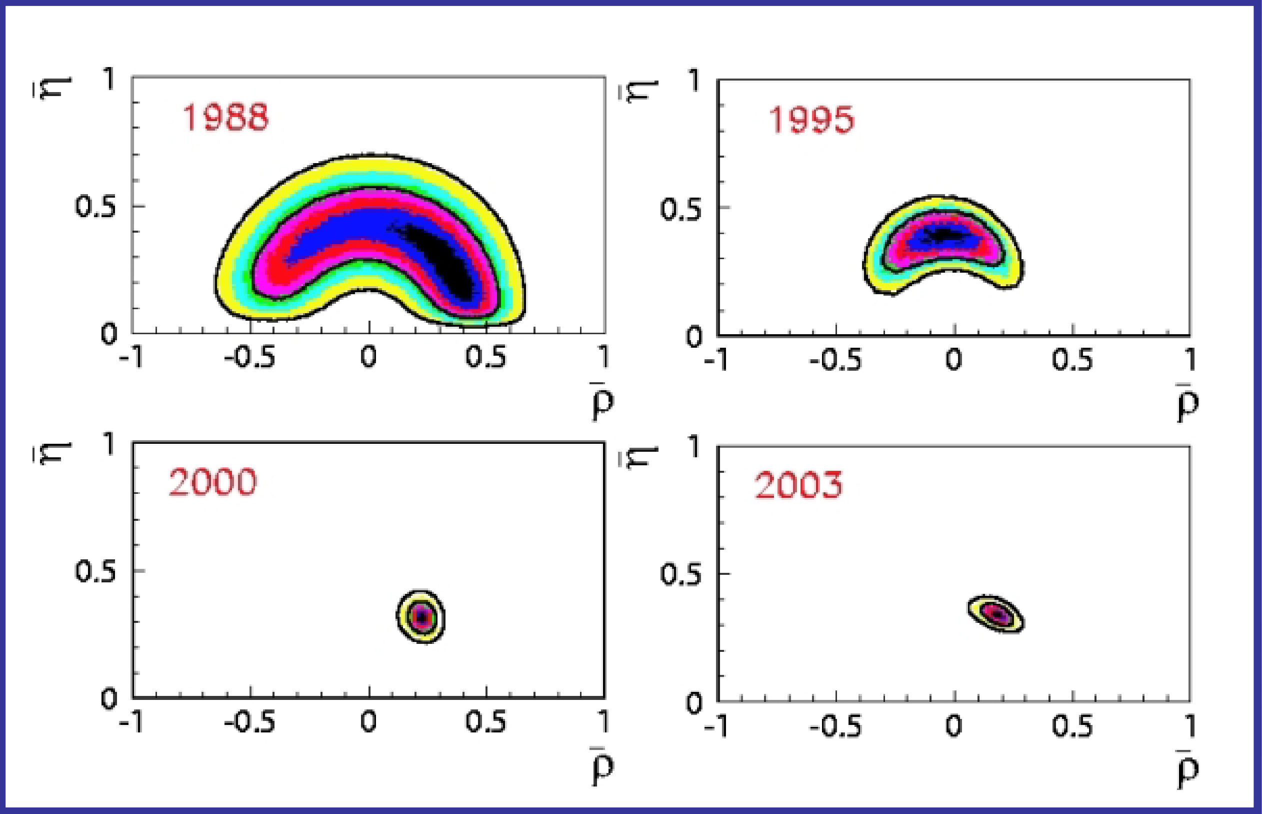






The allowed range of φ_{Bs} is still large. Non-standard values of $A(B_s \to J/\psi\,\varphi)$ can still be observed at LHCb

15 YEARS OF ($\bar{\rho}$-$\bar{\eta}$) DETERMINATIONS



The result of a remarkable experimental and theoretical progress