Calibration of Satellites' Retro-reflectors Center

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Outline

1、Introduction

2、CoM for LAGEOS, Ajisai and Etalon

3、CoM for COMPASS IGSO satellites

4、Conclusion and Future Research

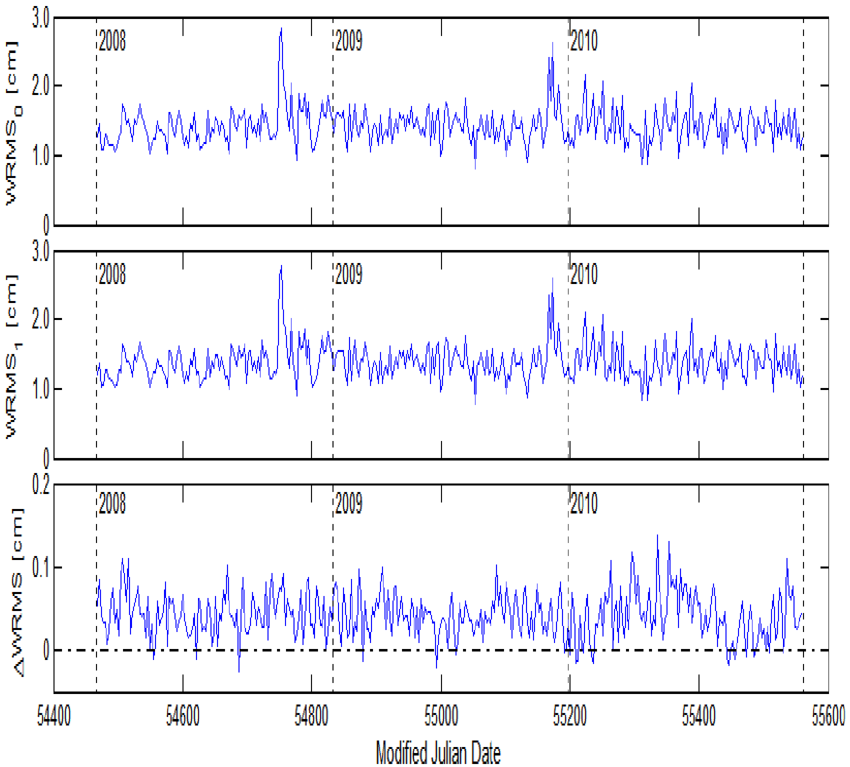
1、Introduction

- Satellite laser ranging system measures the distance from SLR station to satellite's Retro-reflectors center by the round-trip travel time of laser pulse. The position of retro-reflectors on satellites needs to be corrected which is helpful to improve the accuracy of precise orbit determination (POD) and geodetic parameters, especially the TRF and GM.
- Numerical simulations and theoretical analysis have demonstrated the Center-of-Mass (CoM) depend on system operating modes of SLR stations due to satellite signature effects.

1、Introduction

- By statistic analysis over long-time series it showed the short-arc orbit determination precision has indeed undergone general improvement comparing to the situation of traditional global uniform CoM correction. The mean precision improvement on residual RMS is approximately 0.4 mm for Lageos-1/2 and 0.6 mm for Etalon-1/2.
- As the current requirements on SLR data processing for relevant applications have achieved sub-centimeter even towards millimeter level, it is necessary to take in the effect of system-dependent CoM correction.

The mean precision improvement is about 0.4 mm for Lageos-1/2.



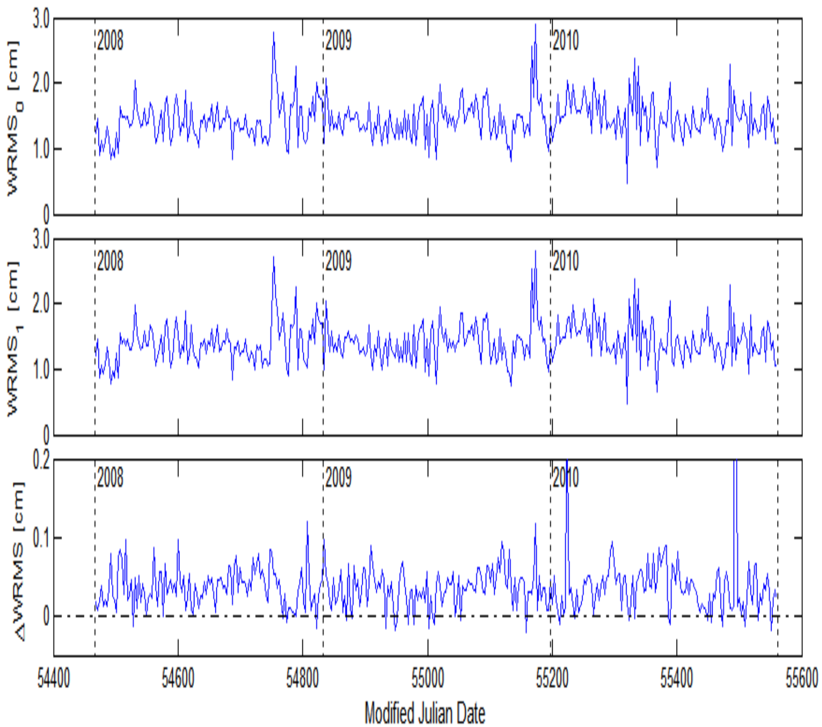


Fig.1 Precision of orbit determination for Lageos-1(left) and Lageos-2(right) by adopting global uniform CoM correction and system-dependent CoM correction, respectively. The bottom panel showed the difference between above two, i.e. the improvement of precision.

1、Introduction

The mean precision improvement is about 0.6mm for Etalon-1/2.

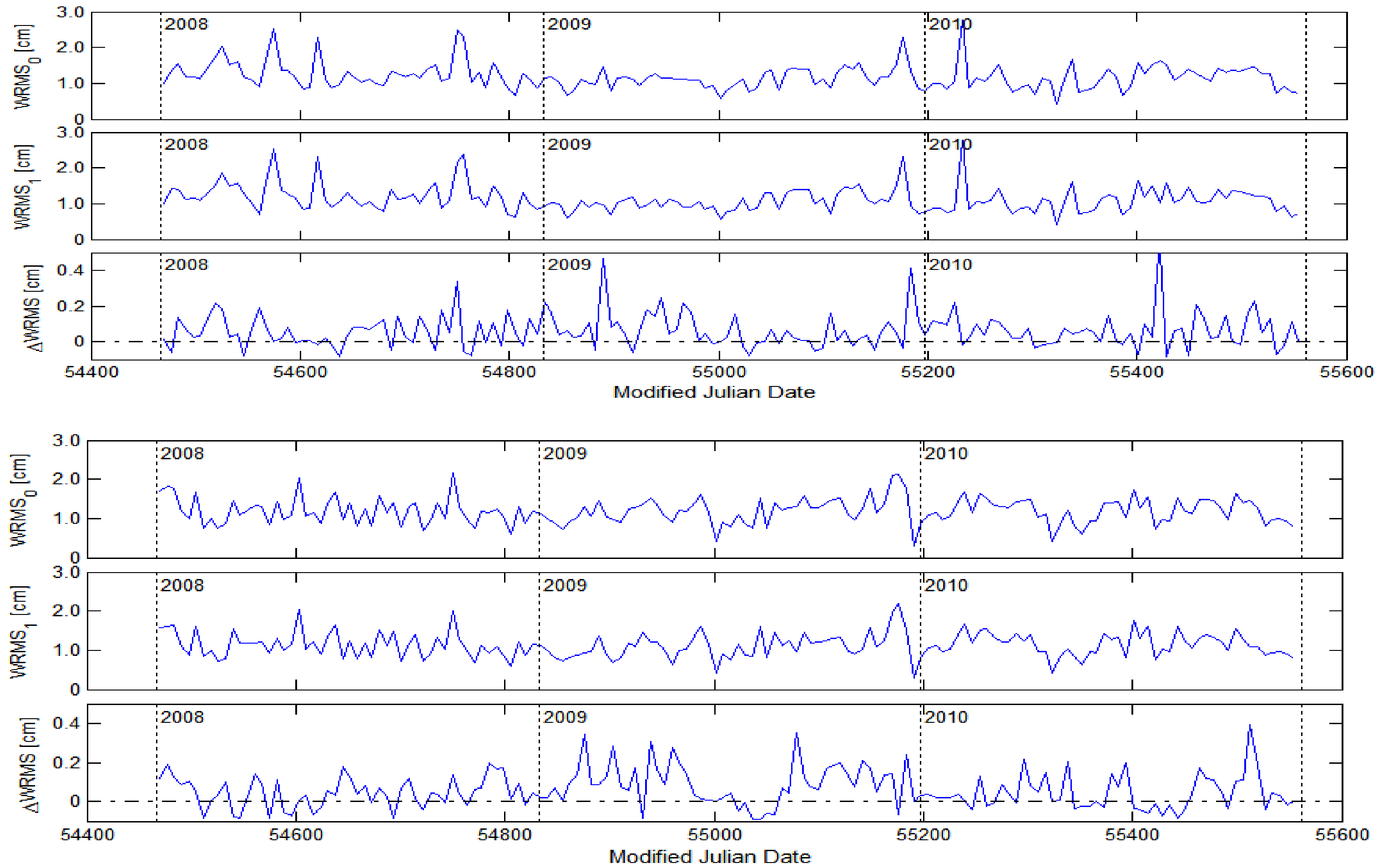


Fig.2 Same as Fig.1, but for Etalon-1(up) and Etalon-2(down).

1、Introduction

- CoM is related to incidence angle, structural alignment of retro-reflectors, station position and operating modes.
- Because the reflec probability of photons for retro-reflectors is proportional to the cross sections of retro-reflectors, we can get the probability model by calculation of the cross section area of corner reflectors.
- The CoMs of several spherical satellites are calculated. For the flat retro-reflector array, the CoMs of COMPASS are also tested and analyzed.

2、 CoM for LAGEOS, Ajisai and Etalon

Neubert's method

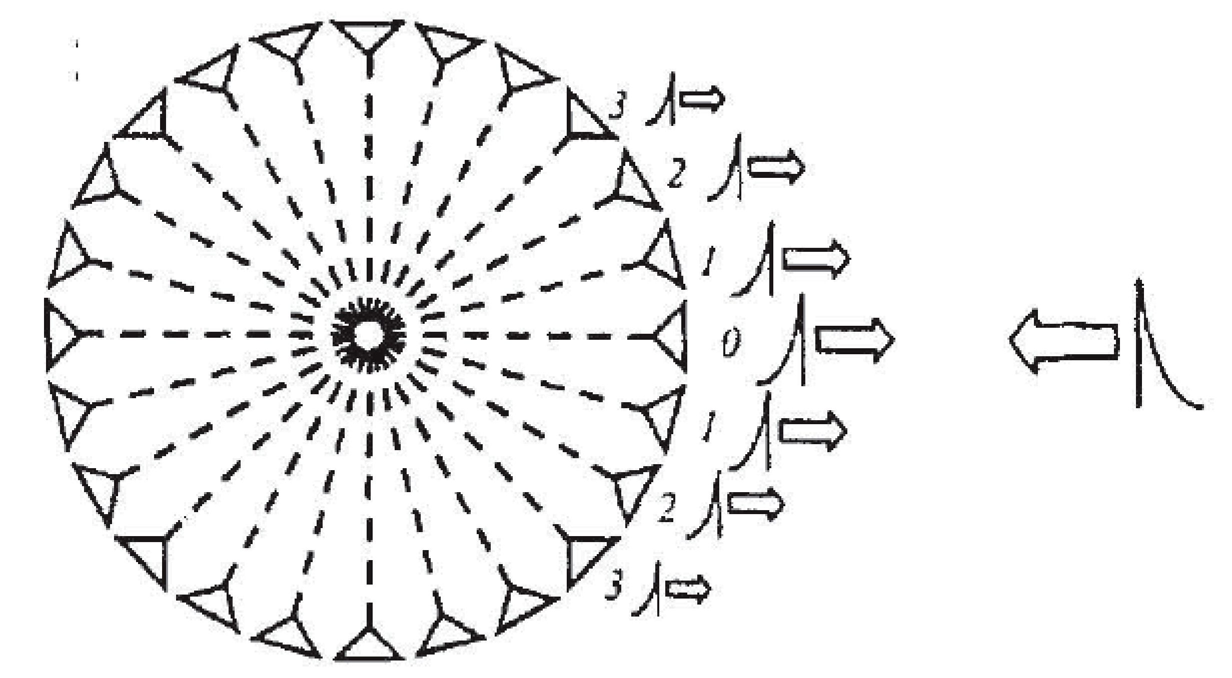


Fig.3.LAGEOS cube corner reflector distribution and its response on photon

2、 CoM for LAGEOS, Ajisai and Etalon

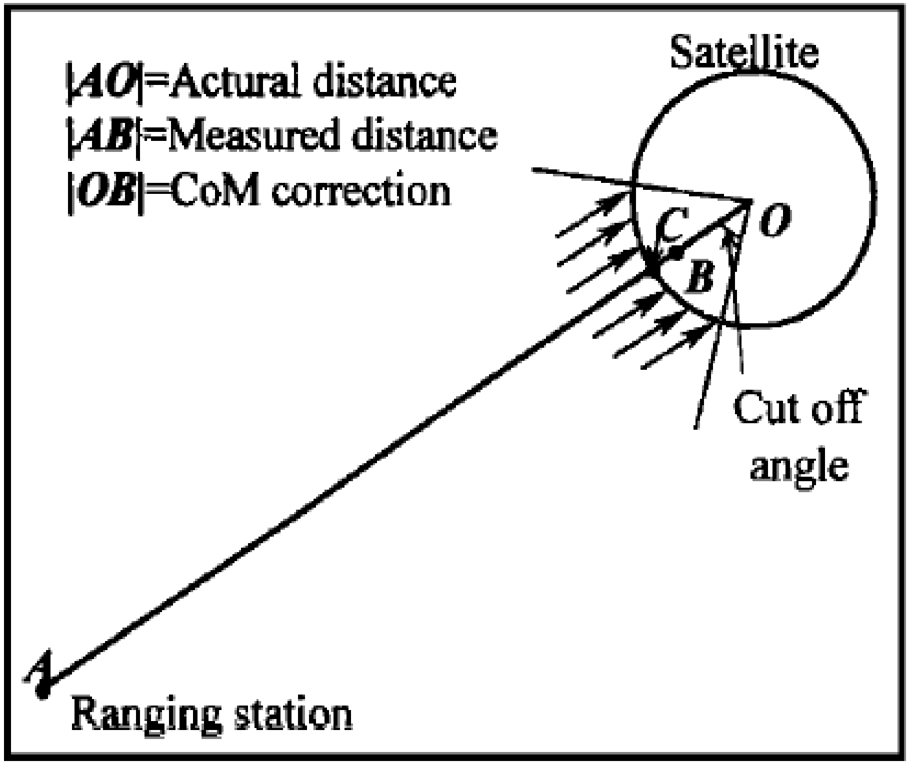


Fig.4. Relations of measured distance and CoM.

O: LAGEOS mass center;

C: cross point of laser pulse with satellite;

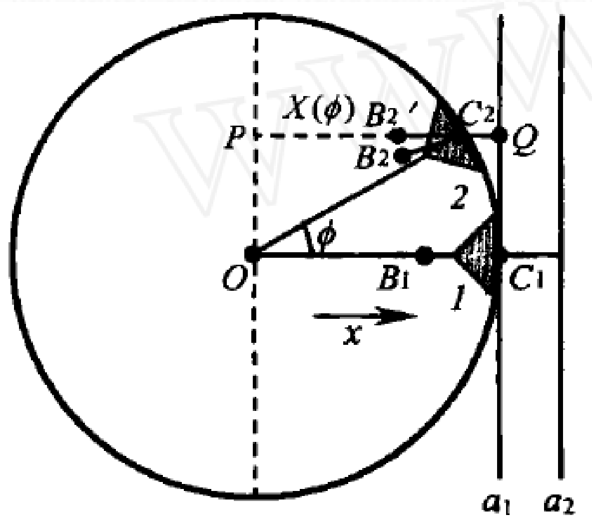
B: the reflection point;

|OB|: CoM value.

X is the probability density function. We can get the following relation：

$$X(\phi) = R_S \cos\phi - L\sqrt{n^2 - \sin^2\phi}$$

$$p_X(x) = \left.\frac{p(\phi)}{|dX(\phi)/d\phi|}\right|_{\phi=X^{-1}(x)} = \left.\frac{p_\Phi(\phi)}{(R_s - \frac{L\cos\phi}{\sqrt{n^2-\sin^2\phi}})\sin\phi}\right|_{\phi=X^{-1}(x)}$$



$X(\phi)$: the probability density function. Its mean value is the CoM；

ϕ: incidence angle；

R_S: the radius of satellite；

L: the distance from the reflector's vertex to the front face ；

n: : refractive index

$$\Rightarrow \quad CoM = E(X) = \int_{[X]} x p_X(x)dx = \int_0^{\phi_{\max}} X(\phi) p_\Phi(\phi) d\phi$$

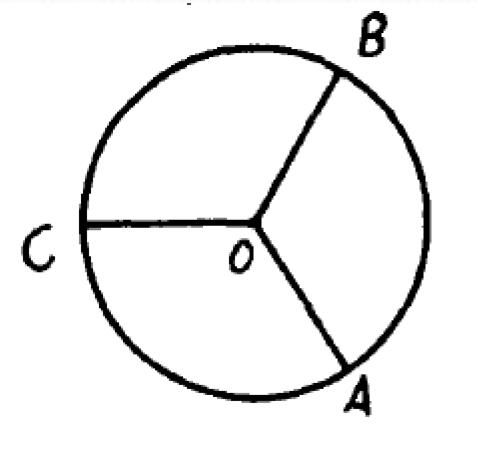
2、 CoM for LAGEOS, Ajisai and Etalon

Table.1. Initial results of CoM values of satellites from different models（**unit:mm**）.

Satellite	Standard Value	Neubert model	Our results
Lageos	251	242.79	242.26
Ajisai	1010	959.0	959.12
Etalon	576	579.0	579.44
Starlette	--	74.6	74.74
GFZ-1	--	59.4	58.96

3、 CoM for COMPASS IGSO satellites

During the actual use of corner reflector , the installation must be considered with every corner cut. Given the influence of the thermal effect, the bottom is cut by inscribed circle.



Effective reflection area for incidence angle $i = 0$ after the bottom is cut is as the following:

$$S_0 = \pi r^2$$

where r is the radius of the circle。 Relative effective area for incidence angle i is as the following:

$$A_s = \left[1 - \frac{2\sqrt{2}}{\pi}\tan i\sqrt{1 - 2\tan^2 i} - \frac{2}{\pi}\sin^{-1}\left(\sqrt{2}\tan i\right)\right]\cos i$$

3、 CoM for COMPASS IGSO satellites

The above formula is fit for empty corner reflectors. If it is filled we have to consider the refractivity of used materials. So $\tan i$ in the relative effective area is replaced by the following right item, i.e:

$$\tan i \Rightarrow \frac{\sin i}{\sqrt{n^2 - \sin^2 i}}$$

Actural effective reflection area S is as the following:

$$S = S_0 A_S$$

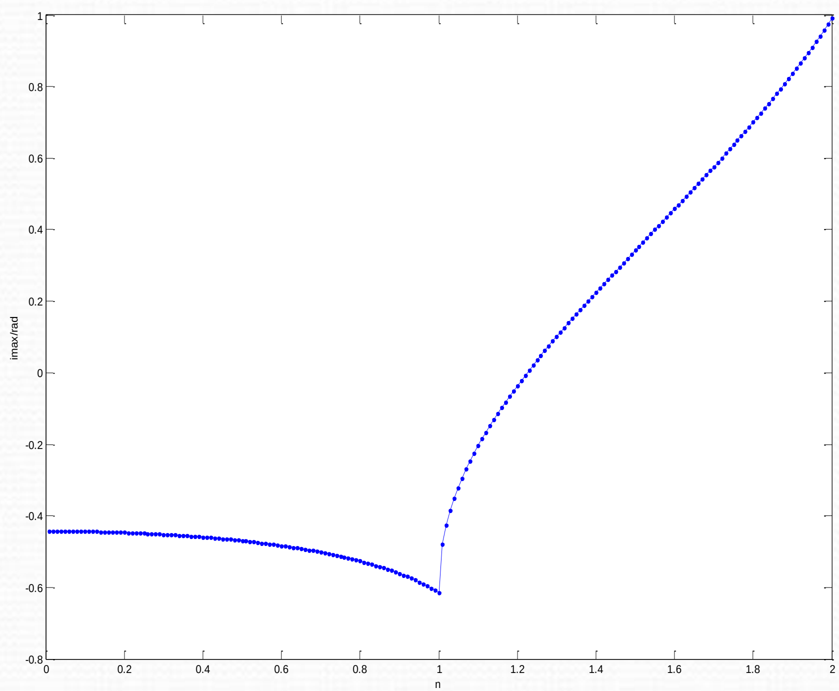
3、 CoM for COMPASS IGSO satellites

For uncoated corner reflector we can get the maximum incidence angle according to the snell's law and geometry as the following:

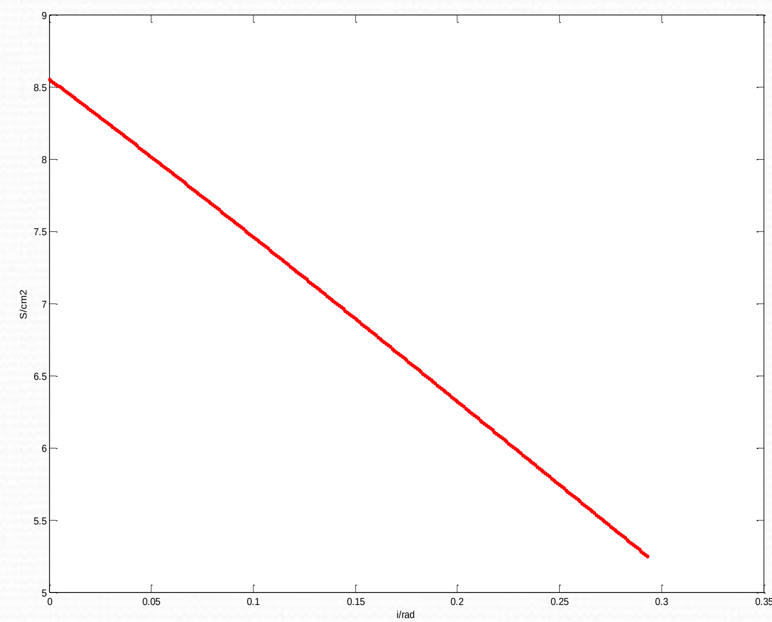
$$i_{\max} = \sin^{-1}\left[n \sin\left(\tan^{-1}\sqrt{2} - \sin^{-1}\frac{1}{n} \right) \right]$$

For IGSO : n=1.45843

$i_{\max}$ =16.8°



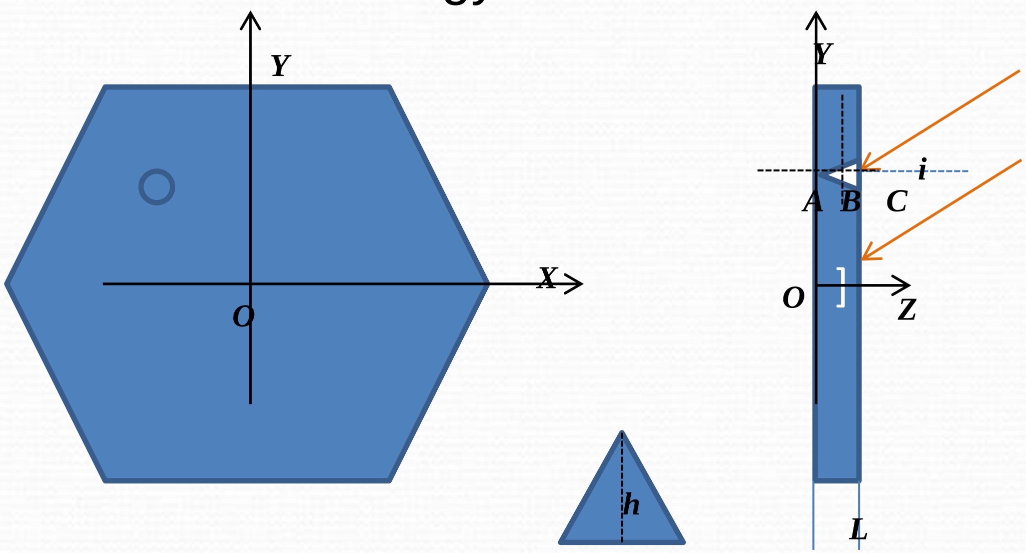
The relation of refractivity n and maximum incidence angle



The relation of incidence angle i and relative effective area S.

3、 CoM for COMPASS IGSO satellites

For COMPASS IGSO satellites the corner reflector array is symmetric. So the geometry center of the array plane is the horizontal component location of the energy center. We only need determine the thickness direction position of the energy center. We define the normal direction of the corner reflector bottom plane as Z axis. Our questions become to determine the energy center location at Z axis.



IGSO corner reflector

For each corner reflector we can get the following relations:

$$|BC| = h \cdot \sqrt{n^2 - \sin^2 i}$$

$$Z(i) = L - h \cdot \sqrt{n^2 - \sin^2 i}$$

$$\eta(i) = S = S_0 \cdot A_S$$

3、 CoM for COMPASS IGSO satellites

Actural effective reflection area of the corner reflector array with N corner reflectors is as the following:

$$N \cdot \eta(i)$$

So, we can get the probability density function of Z by the one of i, i.e.

$$p(i) = \frac{N \cdot \eta(i)}{\int_0^{\pi/2} N \cdot \eta(i) di} = \frac{\eta(i)}{\int_0^{i_{\max}} \eta(i) di}$$

$$\Rightarrow$$

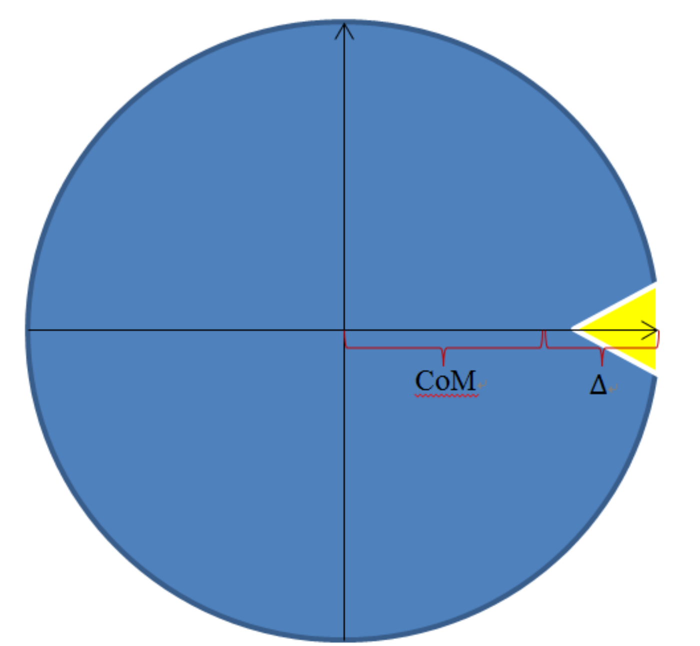
$$p(Z) = \left. \frac{p(i)}{\left| dZ(i) / di \right|} \right|_{i = Z^{-1}(z)} = \frac{p(i) \cdot \sqrt{n^2 - \sin^2 i}}{h \sin i \cos i}$$

3、 CoM for COMPASS IGSO satellites

So, we can get CoM of corner reflector array by the following formula:

$$CoM' = E(Z) = \int_{[z]} zP(z)dz$$

$$= \int_0^{i_{\max}} Z(i)p(i)di$$



Δ: the distance from the bottom of corner reflector to the energy center.

For IGSO : L=30mm，h=24mm，N=90，n=1.45843

$i_{\max}$ =16.8°

3、 CoM for COMPASS IGSO satellites

	R	h	i_{max}	CoM	Δ
IGSO	∞	24	0.29	-4.80	34.80
LAGEOS	298.00	27.84	0.75	242.26	55.74
Ajisai	1053.00	25.72	0.75	959.12	93.88
Etalon	641.50	19.10	0.75	579.44	62.06
GFZ-1	91.00	19.10	0.70	59.48	31.52

4. Conclusion and Future Research

- **Accurate CoMs should be provided not only for SLR geodesy satellites but also for navigation satellites. It is important for long term accuracy evaluation of microwave orbits and the system error calibration.**
- **System-dependent CoMs is helpful to improve the SLR orbit determination accuracy, mm-order TRF, GM and so on.**
- **Different methods give different CoMs. We need verify them.**

4. Conclusion and Future Research

Calibrate the System-dependent CoMs

- **Study Toshimichi and Graham system-dependent CoM model**
- **Solve range biases(a few station system-dependent CoMs have been obtained by long-term average of their range biases.)**

Import those CoM models to SLR data processing and verify them

Thanks