Calibration of Satellites' Retro-reflectors Center

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Abstract

Satellite laser ranging system measures the distance from SLR station to satellite's Retro-reflectors using the round-trip travel time of laser pulse. The position of retro-reflectors on satellites depends on system operating modes of SLR stations due to satellite signature effects. By statistic analysis over long-time series it showed the short-arc orbit determination precision has indeed undergone general improvement comparing to the situation of traditional global uniform CoM correction. As the current requirements on SLR data processing for relevant applications have achieved sub-centimeter even towards millimeter level, it is necessary to take in the effect of system-dependent CoM correction. Therefore, the correction models of CoMs of several spherical satellites are studied and tested. And then the CoMs of COMPASS are also established..

Keywords: Satellite Laser Ranging; Precise Orbit Determination; Center-of-Mass Correction; COMPASS

1 Introduction

Satellite laser ranging system measures the distance from SLR station to satellite's Retro-reflectors center by the round-trip travel time of laser pulse. The position of retro-reflectors on satellites needs to be corrected which is helpful to improve the accuracy of precise orbit determination (POD) and geodetic parameters, especially the TRF and GM. Numerical simulations and theoretical analysis have demonstrated the Center-of-Mass (CoM) depend on system operating modes of SLR stations due to satellite signature effects. By statistic analysis over long-time series it showed the short-arc orbit determination precision has indeed undergone general improvement comparing to the situation of traditional global uniform CoM correction. The mean precision improvement on residual RMS is approximately 0.4 mm for Lageos-1/2 and 0.6 mm for Etalon-1/2(see fig1 and fig2). As the current requirements on SLR data processing for relevant applications have achieved sub-centimeter even towards millimeter level, it is necessary to take in the effect of system-dependent CoM correction.

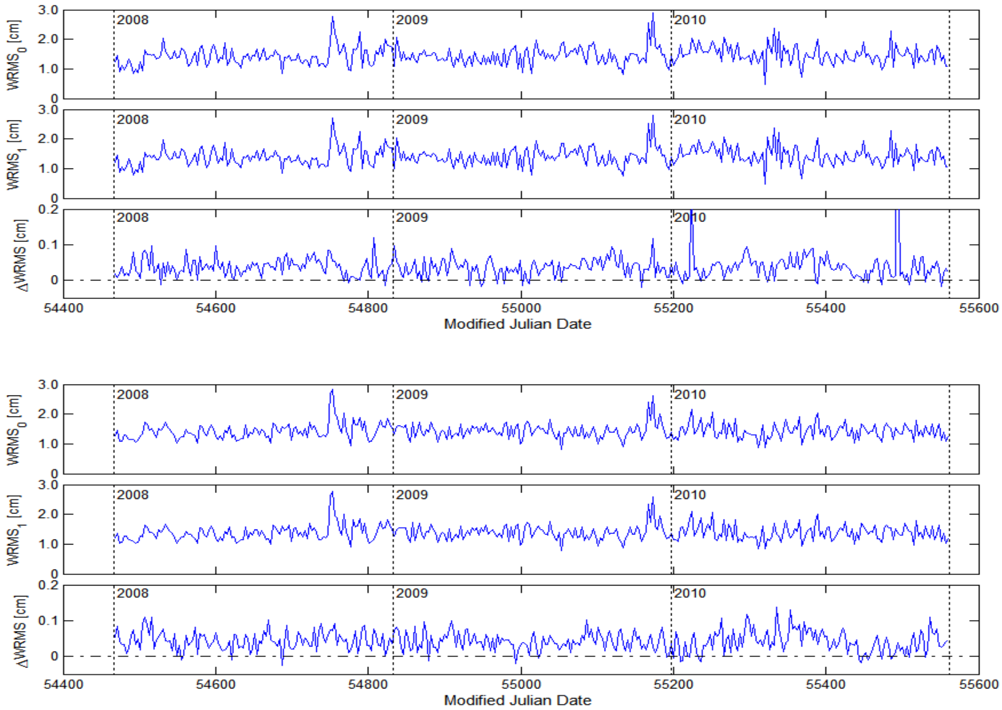

Fig.1 Precision of orbit determination for Lageos-1(up) and Lageos-2(down) by adopting global uniform CoM correction and system-dependent CoM correction, respectively. The bottom panel showed the difference between above two, i.e. the improvement of precision.

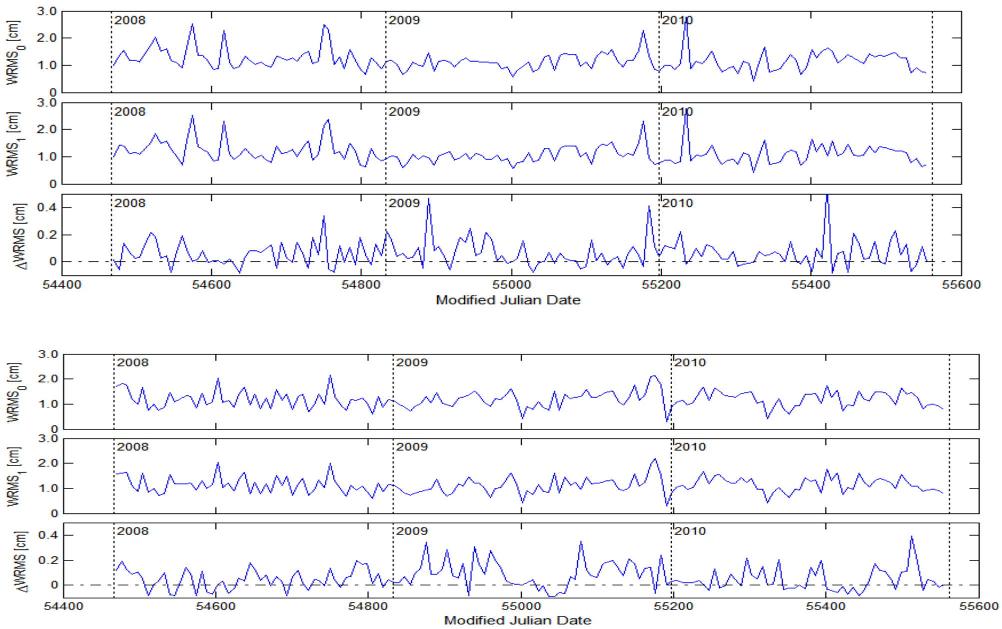

Fig.2 Same as Fig.1, but for Etalon-1(up) and Etalon-2(down).

CoM is related to incidence angle, structural alignment of retro-reflectors, station position and operating modes. The probability of photons for retro-reflectors is proportional to the cross sections of retro-reflectors. So, we can get the probability model by calculation of the cross section area of corner reflectors. The CoMs of several spherical satellites are calculated. For the flat retro-reflector array, the CoMs of COMPASS are also tested and analyzed.

2 CoMs for Laser Geodesy Satellites

We follow the Neubert's method to establish CoM models. Fig3 shows CoM theory. When a laser arrives at the cube corner reflector the reflected lights become wide. The real Retro-reflectors center is in the B point.

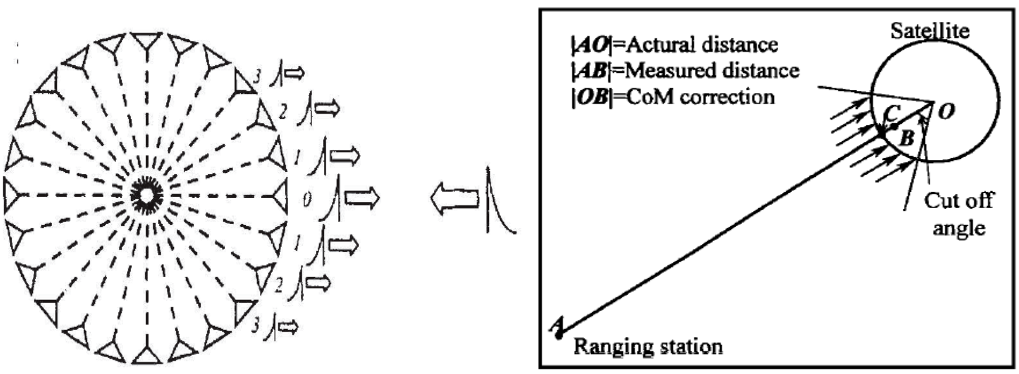

Fig.3.LAGEOS cube corner reflector distribution and its response on photon(left); Relations of measured distance and CoM (right). O: LAGEOS mass center; C:cross point of laser pulse with satellite; B: the reflection point; |OB|: CoM value.

Suppose X is the probability density function. We can get the following relation by fig4:

$$X(\phi) = R_S \cos\phi - L\sqrt{n^2 - \sin^2\phi}$$

$$p_X(x) = \frac{p(\phi)}{\left|dX(\phi)/d\phi\right|}\Bigg|_{\phi=X^{-1}(x)} = \frac{p_\Phi(\phi)}{(R_s - \frac{L\cos\phi}{\sqrt{n^2-\sin^2\phi}})\sin\phi}\Bigg|_{\phi=X^{-1}(x)}$$

$X(\phi)$: the probability density function. Its mean value is the CoM;
ϕ : incidence angle;
R_S : the radius of satellite;
L : the distance from the reflector's vertex to the front face ;
n : : refractive index

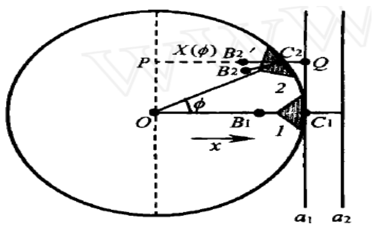

Fig4. The relation of the probability density function with incidence angle.

The CoM can be obtained by the following formula. By the above methods we calculated the CoMs of Lageos/Ajisai/Etalon/Starlette/GFZ-1. The results are showed in Table 1. From table 1 we can see our results is almost the same as Neubert model.

$$CoM = E(X) = \int_{[X]} x p_X(x) dx = \int_0^{\phi_{\max}} X(\phi) p_\Phi(\phi) d\phi$$

Table.1. Initial results of CoM values of satellites from different models（**unit:mm**）

Satellite	**Standard Value**	**Neubert model**	**Our results**
Lageos	251	242.79	242.26
Ajisai	1010	959.0	959.12
Etalon	576	579.0	579.44
Starlette	--	74.6	74.74
GFZ-1	--	59.4	58.96

3 CoM determination for COMPASS IGSO satellites

COMPASS is Chinese satellite navigation system. This system installs laser corner reflector. In order to get the CoMs of COMPASS we study the COMPASS IGSO satellite as an example. During the actual use of corner reflector, the installation must be considered with every corner cut. Given the influence of the thermal effect, the bottom is cut by inscribed circle. The effective reflection area for incidence angle $i = 0$ after the bottom is cut is as the following:

$$S_0 = \pi r^2$$

where r is the radius of the circle。Relative effective area for incidence angle i is as the following:

$$A_s = \left[1 - \frac{2\sqrt{2}}{\pi} \tan i \sqrt{1 - 2\tan^2 i} - \frac{2}{\pi} \sin^{-1}\left(\sqrt{2}\tan i\right)\right] \cos i$$

The above formula is fit for empty corner reflectors. If it is filled we have to consider the refractivity of used materials. So $\tan i$ in the relative effective area is replaced by the following item, i.e:

$$\frac{\sin i}{\sqrt{n^2 - \sin^2 i}}$$

Actural effective reflection area S is as the following: $S = S_0 A_S$

For uncoated corner reflector we can get the maximum incidence angle according to the snell's law and geometry as the following:

$$i_{\max} = \sin^{-1}\left[n \sin\left(\tan^{-1}\sqrt{2} - \sin^{-1}\frac{1}{n}\right)\right]$$

For IGSO n is 1.45843 we can get the following:

$$i_{\max} = 16.8^0$$

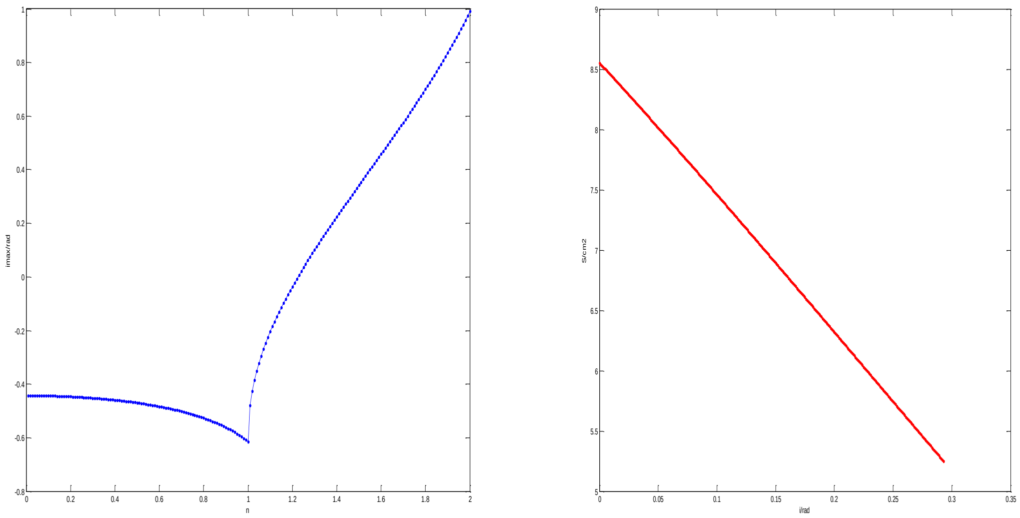

Fig 5. The relation of maximum incidence angle with refractivity n (left) and The relation of relative effective area S with incidence angle i (right).

For COMPASS IGSO satellites the corner reflector array is symmetric. So the geometry center of the array plane is the horizontal component location of the energy center. We only need determine the thickness direction position of the energy center. We define the normal direction of the corner reflector bottom plane as Z axis. Our questions become to determine the energy center location at Z axis as fig 6.

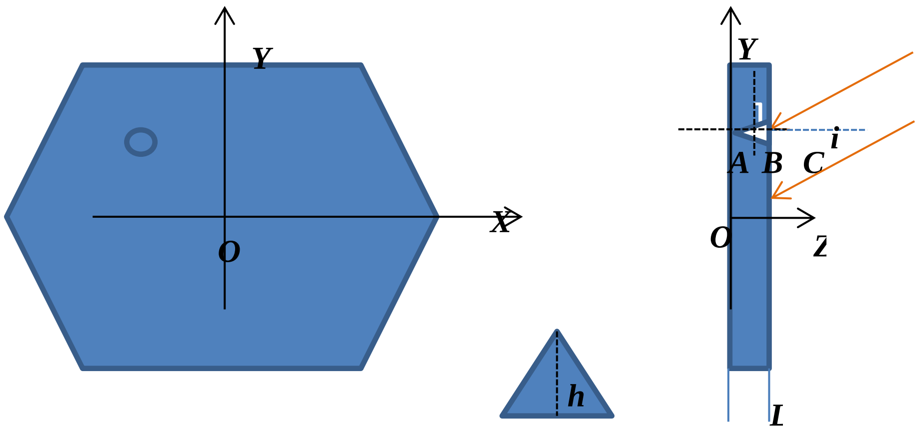

Fig 6. IGSO corner reflector

For each corner reflector we can get the following relations:

$$|BC| = h \cdot \sqrt{n^2 - \sin^2 i}$$

$$Z(i) = L - h \cdot \sqrt{n^2 - \sin^2 i}$$

$$\eta(i) = S = S_0 \cdot A_S$$

Actural effective reflection area of the corner reflector array with N corner reflectors is as the following:

$$N \cdot \eta(i)$$

So, we can get the probability density function of Z with i, i.e.

$$p(i) = \frac{N \cdot \eta(i)}{\int_0^{\pi/2} N \cdot \eta(i) di} = \frac{\eta(i)}{\int_0^{i_{\max}} \eta(i) di}$$

$$p(Z) = \left. \frac{p(i)}{\left| dZ(i) / di \right|} \right|_{i=Z^{-1}(z)} = \frac{p(i) \cdot \sqrt{n^2 - \sin^2 i}}{h \sin i \cos i}$$

So, we can get CoM of corner reflector array by the following formula:

$$CoM' = E(Z) = \int_{[z]} zP(z)dz = \int_0^{i_{\max}} Z(i)p(i)di$$

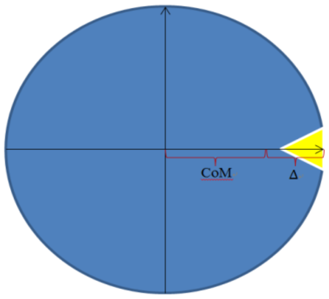

Fig 7. The distance from the bottom of corner reflector to the energy center.

For IGSO (L=30mm，h=24mm，N=90，n=1.45843) we can get the following result for COMPASS IGSO. Table 2 also shows the other parameters and results for other laser satellites. Fig 7 shows the relation of CoM and Δ**.** These results are only initial. We need verify them.

Table.1. Initial results of CoM values of satellites and other parameters(**unit:mm**）

	R	**h**	$i_{\max}$	**CoM**	**Δ**
IGSO	∞	24	0.29	-4.80	34.80
LAGEOS	298.00	27.84	0.75	242.26	55.74
Ajisai	1053.00	25.72	0.75	959.12	93.88
Etalon	641.50	19.10	0.75	579.44	62.06
GFZ-1	91.00	19.10	0.70	59.48	31.52

4 Conclusion and Future Research

Accurate CoMs should be provided not only for SLR geodesy satellites but also for navigation satellites. It is important for long term accuracy evaluation of microwave orbits and the system error calibration. System-dependent CoMs is helpful to improve the SLR orbit determination accuracy, mm-order TRF, GM and so on. Different methods give different CoMs. We need calibrate the System-dependent CoMs and verify them.