Divergence Estimation Procedure and Calculation

Dr. Ray Burris[1], Mr. Mark Davis[2],
Dr. Linda Thomas[1], Dr. Dave Huber[1]

ITLW 2012, Frascati, Italy

[1] U.S. Naval Research Laboratory, Code 8123, 4555 Overlook Ave., SW Washington, DC
[2] U.S. Naval Research Laboratory, Code 8125, 4555 Overlook Ave., SW Washington, DC

Abstract

Link budgets for many of the ILRS sites are estimated using divergence values that are derived from the site logs. Actual data for calculating the station divergence is often incomplete or very optimistic and based on diffraction theory from the full size of the primary mirror for monostatic systems or the full size of the Coude path and beam expander for bistatic systems.

Accurate divergence measurements and a standard method of measuring the divergence is needed by the ILRS for several reasons, including GNSS array requirements and performance prediction and reliable prediction of the energy density delivered on target for the entire ILRS network to deal with requests for information for potential new satellites.

A procedure was developed and presented at the last ILRS meeting for scanning over azimuth and elevation on satellites and using a graphical procedure to estimate the divergence. In this presentation, an equation has been derived from the laser radar equation for the number of photoelectrons detected which allows calculation of the $1/e^2$ divergence from the scan data directly without estimating from graphs. This method will reduce the subjectivity in the estimation and will also allow the measurement to be automated. Data from several stations which responded last year with divergence scan data has been used to test this method and results will be presented.

Why do we need Divergence information?

- Link budgets are Estimated using an implied value from the Site Logs
  - Theoretical Data is optimistic / incomplete
- Reliable differences between Day and Night
  - Useful for GNSS Array Requirements and performance prediction

- Missions WG is getting requests by potential new satellites
  - Need reliable W/cm^2 at the satellite for the whole ILRS network

- Estimation of divergence practices today
  - Estimated from diffraction theory from either
    - Full size of the primary mirror for monostatic systems
    - Full size of the coude path and beam expander

Procedure Setup

- Pick 2 satellites at > 40 deg elevation pass; similar seeing and similar sky region
  - Ajisai or Starlette/Stella (something very strong)
  - Lageos1/2
  - GNSS or Etalon
- Choose Night conditions
  - No daytime filters
  - No iris
  - No Clouds (in part of sky that's being used)
- Maximize signal
  - Turn off automatic attenuation (10% return rate matching)
  - Open received FOV spatial filter
  - Normal transmit energy
- Step size
  - Make it at least 10 steps
  - 1 arcsec (5 urad / 0.00028 deg) works well
- Repeat measurement on days or times with different seeing conditions
- ***Procedure requires data from two satellites at different ranges taken with identical system configuration, i.e. same pulse energy, divergence, etc.***

Scanning Procedure & Data Required

- Acquire
- Scan in azimuth - Initial
  - Find left and right boundary where the signal is ~ 0
    - go past by 2 or 3 steps to confirm
  - record offsets and center
  - Set to center
- Scan in elevation - Initial
  - Find upper and lower boundary where the signal is ~ 0
    - Go past by 2 or 3 steps to confirm
  - record offsets and center
  - Set to center
- Scan in azimuth (left and right) to Boundary where signal is ~ 0
  - Record offsets and center in azimuth
- Scan in elevation (up and down) to Boundary where signal is ~ 0
  - Record offsets and center in elevation
- Scan again in Azimuth to boundaries
  - These offsets are the azimuth measurement
  - Center in Azimuth
  - (right – left)* cos (elevation) is the reported measurement
- Scan again in Elevation to boundaries
  - These offsets are the elevation measurement

DATA REQUIRED:

1) Full scan width from 0 to 0 in # of steps with step size
2) Slant range while taking the scan
3) Elevation angle approx while taking scan
4) Same data from second satellite at different range
5) Two satellite scans should be done under similar seeing conditions

Factors affecting the measurements

- **Seasonal**
  - **Humidity (0-50, 50-70, 70-100%)**
  - **Where in the local pressure cycle (High vs Low)**
- **Sky Conditions (Jitter)**
  - **Expect 10 to 20 urad of short term (during the measurement)**
- **Local Seeing**
  - **If there is access to “waste light”**
  - **What is the diameter of a typical star (10^{th} Mv – doesn’t really matter – just be consistent) in the same part of the sky**
- **Thermal Gradients**
  - **+/- 1 hour of sunrise**
- **Laser Temperature**
  - **Thermal lensing effects in the amplifiers**
- **Sun Proximity to beam**
- **Operators**
  - **This is a subjective measurement**

How accurate is needed?
How will the numbers be used?

- The Real world will make these measurements vary from day to day
  - Need average
  - Need worst (biggest) beam for worst case link budget projections
  - Need best case (tightest) beam for MWG assessments
  - Are any stations doing active divergence control?

- Flux at satellite / array will vary as function of divergence

- Elevation dependence terms in the model
  - Telescope jitter
  - Atmosphere jitter
  - Divergences near these jitter limits need careful models

Divergence calculation from scan data

- Assume scans are done on two satellites at different ranges but **under very similar atmospheric conditions**. The scan is done in AZ and EL up to the points where NPE ~ 0 on both sides.
- At the peak of the return power, NPE for both satellites is proportional to σ/R^4 , but NPE_1 does not equal NPE_2. NPE refers to the number of photoelectrons, σ is the satellite cross section, and R is the slant range. At the half angle points of the scan, we still have NPE proportional to σ/R^4 but also with $NPE_1 = NPE_2 \sim 0$. Therefore by equating the expressions for NPE and assuming that most of the terms in the equation for NPE are approximately the same during the measurements, we can derive an expression for the divergence. This is why it is important to take the measurements fairly quickly under the same sky conditions and, if possible, in the same region of the sky.
- These relationships along with other assumptions which will be stated later, allow the development of an analytical expression to calculate the divergence from the satellite scan data.

NPE calculation[*]

$$NPE = \eta_e * (E_r * \lambda / hc) * \eta_t * G_t * \sigma * (1/4\pi R^2)^2 * A_r * \eta_r * T_a^2 * T_c^2$$

- η_e = detector quantum efficiency
- E_r = laser pulse energy
- λ = laser wavelength
- h = Planck’s constant
- c = speed of light
- η_t = transmit optics efficiency
- G_t = transmitter gain
- σ = satellite optical cross section
- R = slant range to target
- A_r = effective area of telescope receive aperture
- η_r = receive optics efficiency
- T_a = one-way atmospheric transmission
- T_c = one-way cirrus cloud transmission

[*] “Millimeter Accuracy Satellite Laser Ranging: A Review”, John J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology Geodynamics 25, American Geophysical Union, 1993.

Transmitter Gain*

$$G_t = (8/\theta_t^2) * \exp[-2(\theta/\theta_t)^2]$$

- θ_t = far field divergence half angle between beam center and $1/e^2$ intensity point
- θ = beam pointing error; or in this case, the half angle of the scan.

Assume that scan measurements are taken on two satellites in same region of sky quickly enough that all factors in the expression for NPE are ~ constant with the exception of σ, R, and G_t which changes due to pointing error change.

$$NPE = \eta_e * (E_r * \lambda/hc) * \eta_t * G_t * \sigma * (1/4\pi R^2)^2 * A_r * \eta_r * T_a^2 * T_c^2$$

Then the expression for NPE becomes:

$$NPE = K * (\sigma/R^4) * \exp[-2(\theta/\theta_t)^2]$$

Where K is an approximate constant

* "Millimeter Accuracy Satellite Laser Ranging: A Review", John J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology Geodynamics 25, American Geophysical Union, 1993.

Consider peak points where NPE = max

At the peak return rate, $\theta_1 = \theta_2 = 0$

$$NPE_{1max} = K*(\sigma_1/R_1^4)$$

$$NPE_{2max} = K*(\sigma_2/R_2^4)$$

$$NPE_{1max} \,/\, NPE_{2max} = (\sigma_1/R_1^4) \,/\, (\sigma_2/R_2^4)$$

If the satellite cross sections are not known accurately, the ratio of the peak NPE counts during the scan can be used in the expression for divergence estimation

Consider scan end points where NPE ~ 0

Solve the equation of $NPE_1 = NPE_2$ at the end points of the scans for θ_t:

$$K*(\sigma_1/R_1^4)* \exp[-2(\theta_1/\theta_t)^2] = K*(\sigma_2/R_2^4)* \exp[-2(\theta_2/\theta_t)^2]$$

$$(\sigma_1/\sigma_2)*(R_2^4/R_1^4) = \exp[2(\theta_1/\theta_t)^2 - 2(\theta_2/\theta_t)^2]$$

Take natural logarithm of both sides

$$\ln[(\sigma_1/\sigma_2)*(R_2^4/R_1^4)] = 2(\theta_1^2/\theta_t^2) - 2(\theta_2^2/\theta_t^2)$$

$$\boxed{\theta_t^2 = 2(\theta_1^2 - \theta_2^2) \,/\, \ln[(\sigma_1/\sigma_2)*(R_2^4/R_1^4)]}$$

The result is the divergence estimate in terms of known quantities: the satellite cross-section, satellite range, and the measured scan angles

LRCS and altitude data used

Satellite	***Altitude***	***LRCS (Mm^2) - current***	***LRCS (Mm^2) - revised***	
Starlette	815	0.65	1.8	Cross section of ILRS satellites
Lageos-1	5850	7	15	David A. Arnold
Lageos-2	5625	assume = Lageos-1	assume = Lageos-1	
Etalon-1	19105	60	55	Altitudes from ILRS web site
Etalon-2	19135	assume = Etalon-1	assume = Etalon-2	
Topex	1350	2	33	
BeaconC	927	3.6	13	
Ajisai	1485	12	23	
Gfo-1	800	2	0.5	
Stella	815	0.65	1.8	
Jason-1	1336	0.3	0.8	
GPS	20030	40	19	
Champ	474	1.8	1	
Westpac	835	0.03	0.04	
ERS-1	780	0.3	0.85	
Glonass396	19140	360	240	
Glonass132	19140	80*	80	
Envisat	800	0.3	0.85	
LRE	250-36000	1.25	2	
SUNSAT	400	0.2	0.4	
GIOVE-B**	23916	56	26.6	**estimated from 1.4 x GPS
Glonass109	19140	80*	80*	* Glonass100,109,132 assumed
Glonass100	19140	80*	80*	LRCS of 80

Calculation example: Stafford

Stafford data divergence estimation example: Ajisai and Lageos

Measured AZ half angle in radians

$$\theta1\_meas := 30 \cdot 10^{-6} \qquad \theta2\_meas := 75 \cdot 10^{-6}$$

Elevation angle in radians

$$\alpha1 := 40 \cdot \frac{\pi}{180} \qquad \alpha2 := 40 \cdot \frac{\pi}{180}$$

$$\theta1 := \theta1\_meas \cdot \cos(\alpha1) \qquad \theta2 := \theta2\_meas \cdot \cos(\alpha2)$$

angle corrected for elevation

$$R1 := 7055 \qquad R2 := 2080$$

One way slant range in km

$$\sigma1 := 15 \cdot 10^{6} \qquad \sigma2 := 23 \cdot 10^{6}$$

LRCS in square meters

$$\theta t\_sqr := \left| \frac{2 \cdot \left(\theta1^2 - \theta2^2\right)}{\ln\left(\frac{\sigma1 \cdot R2^4}{\sigma2 \cdot R1^4}\right)} \right|$$

Full angle divergence = 64.6 µrad

$$Div\_half := \sqrt{\theta t\_sqr} \qquad Div\_half = 3.231 \times 10^{-5}$$

$$Div\_full := 2 \cdot (Div\_half) \qquad Div\_full = 6.461 \times 10^{-5}$$

Full angle divergence in radians, 1/e2

Calculation example: Shanghai

Shanghai data divergence calculation: Ajisai (2) and Lageos (1)

Measured AZ half angle in radians

$\theta1\_meas := 62.5 \cdot 10^{-6}$ $\quad\quad$ $\theta2\_meas := 200 \cdot 10^{-6}$

Elevation angle in radians

$\alpha1 := 66 \cdot \frac{\pi}{180}$ $\quad\quad$ $\alpha2 := 60 \cdot \frac{\pi}{180}$

$\theta1 := \theta1\_meas \cdot \cos(\alpha1)$ $\quad\quad$ $\theta2 := \theta2\_meas \cdot \cos(\alpha2)$ $\quad$ angle corrected for elevation

One way slant range in km; they didn't record slant range so use altitude from ILRS, then divide by sin of their recorded EL angle.

$A1 := 5850$ $\quad\quad$ $A2 := 1485$ $\quad\quad$ Sat altitudes from ILRS website

$$R1 := \frac{A1}{\sin(\alpha1)} \quad\quad R2 := \frac{A2}{\sin(\alpha2)}$$

$\sigma1 := 15 \cdot 10^{6}$ $\quad\quad$ $\sigma2 := 23 \cdot 10^{6}$ $\quad\quad$ LRCS in square meters

$$\theta t\_sqr := \frac{2 \cdot \left(\theta1^{2} - \theta2^{2}\right)}{\ln\left(\frac{\sigma1 \cdot R2^{4}}{\sigma2 \cdot R1^{4}}\right)}$$

Full angle divergence = 114.6 μrad

$Div\_half := \sqrt{\theta t\_sqr}$ $\quad\quad$ $Div\_half = 5.73 \times 10^{-5}$

$Div\_full := 2 \cdot (Div\_half)$ $\quad\quad$ $Div\_full = 1.146 \times 10^{-4}$ $\quad\quad$ Full angle divergence in radians, 1/e2

Calculation example: Graz

Graz data divergence calculation: Envisat (2) and Lageos-2 (1)

Measured AZ half angle in radians

$\theta1\_meas := 26.2 \cdot 10^{-6}$ $\theta2\_meas := 69.8 \cdot 10^{-6}$

Elevation angle in radians

$\alpha1 := 61 \cdot \frac{\pi}{180}$ $\alpha2 := 55 \cdot \frac{\pi}{180}$

$\theta1 := \theta1\_meas \cdot \cos(\alpha1)$ $\theta2 := \theta2\_meas \cdot \cos(\alpha2)$ angle corrected for elevation

One way slant range in km; they didn't record slant range so use altitude from ILRS, then divide by sin of their recorded EL angle.

$A1 := 5625$ $A2 := 800$ Sat altitudes from ILRS website

$R1 := \frac{A1}{\sin(\alpha1)}$ $R2 := \frac{A2}{\sin(\alpha2)}$

$\sigma1 := 15 \cdot 10^{6}$ $\sigma2 := .85 \cdot 10^{6}$ LRCS in square meters

$$\theta t\_sqr := \left| \frac{2 \cdot \left(\theta1^2 - \theta2^2\right)}{\ln\left(\frac{\sigma1 \cdot R2^4}{\sigma2 \cdot R1^4}\right)} \right|$$

$Div\_half := \sqrt{\theta t\_sqr}$ $Div\_half = 2.485 \times 10^{-5}$

Full angle divergence = 49.7 μrad

$Div\_full := 2 \cdot (Div\_half)$ $Div\_full = 4.97 \times 10^{-5}$ Full angle divergence in radians, 1/e2

The Divergence setting for these measurements was 0.002 degrees = 35 microradians

Calculation example: Herstmonceux

Herstmonceaux data divergence calculation: Lageos-2 (2) and Glonass100 (1)

Measured AZ half angle in radians

$\theta1\_meas := 43.6 \cdot 10^{-6}$ $\theta2\_meas := 58.2 \cdot 10^{-6}$

Elevation angle in radians

$\alpha 1 := 50 \cdot \frac{\pi}{180}$ $\alpha 2 := 50 \cdot \frac{\pi}{180}$

$\theta1 := \theta1\_meas \cdot \cos(\alpha 1)$ $\theta2 := \theta2\_meas \cdot \cos(\alpha 2)$ angle corrected for elevation

Slant range not recorded; Elevation angle not recorded; assume all EL angles ~ 50 degrees.

$A1 := 19140$ $A2 := 5625$ Sat altitudes from ILRS website

$R1 := \frac{A1}{\sin(\alpha 1)}$ $R2 := \frac{A2}{\sin(\alpha 2)}$

$\sigma 1 := 80 \cdot 10^{6}$ $\sigma 2 := 15 \cdot 10^{6}$ LRCS in square meters

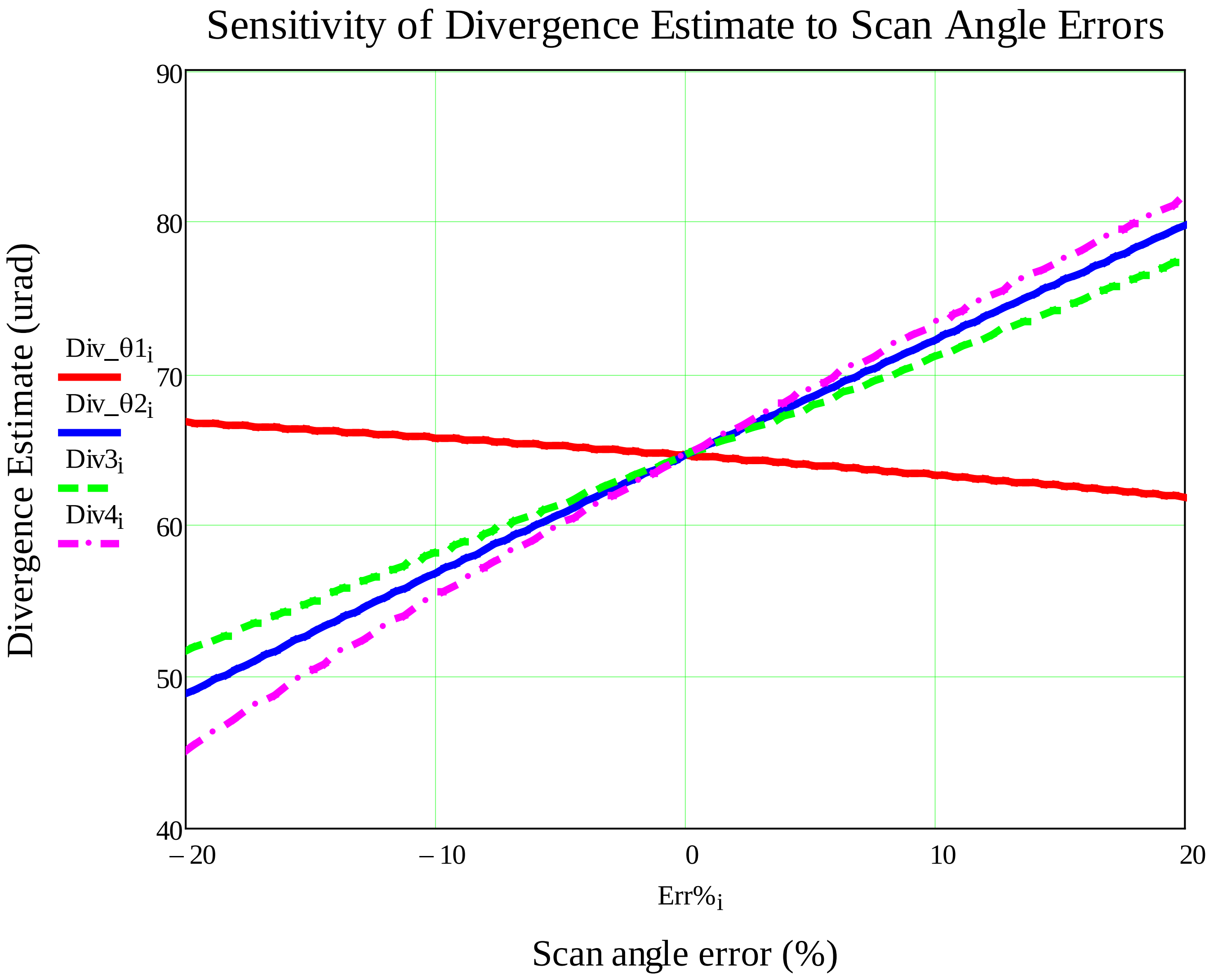
$$\theta t\_sqr := \left| \frac{2 \cdot \left(\theta 1^2 - \theta 2^2\right)}{\ln\left(\frac{\sigma 1 \cdot R2^4}{\sigma 2 \cdot R1^4}\right)} \right|$$

Full angle divergence = 39 μrad

$Div\_half := \sqrt{\theta t\_sqr}$ $Div\_half = 1.952 \times 10^{-5}$

$Div\_full := 2 \cdot (Div\_half)$ $Div\_full = 3.903 \times 10^{-5}$ Full angle divergence in radians, 1/e2

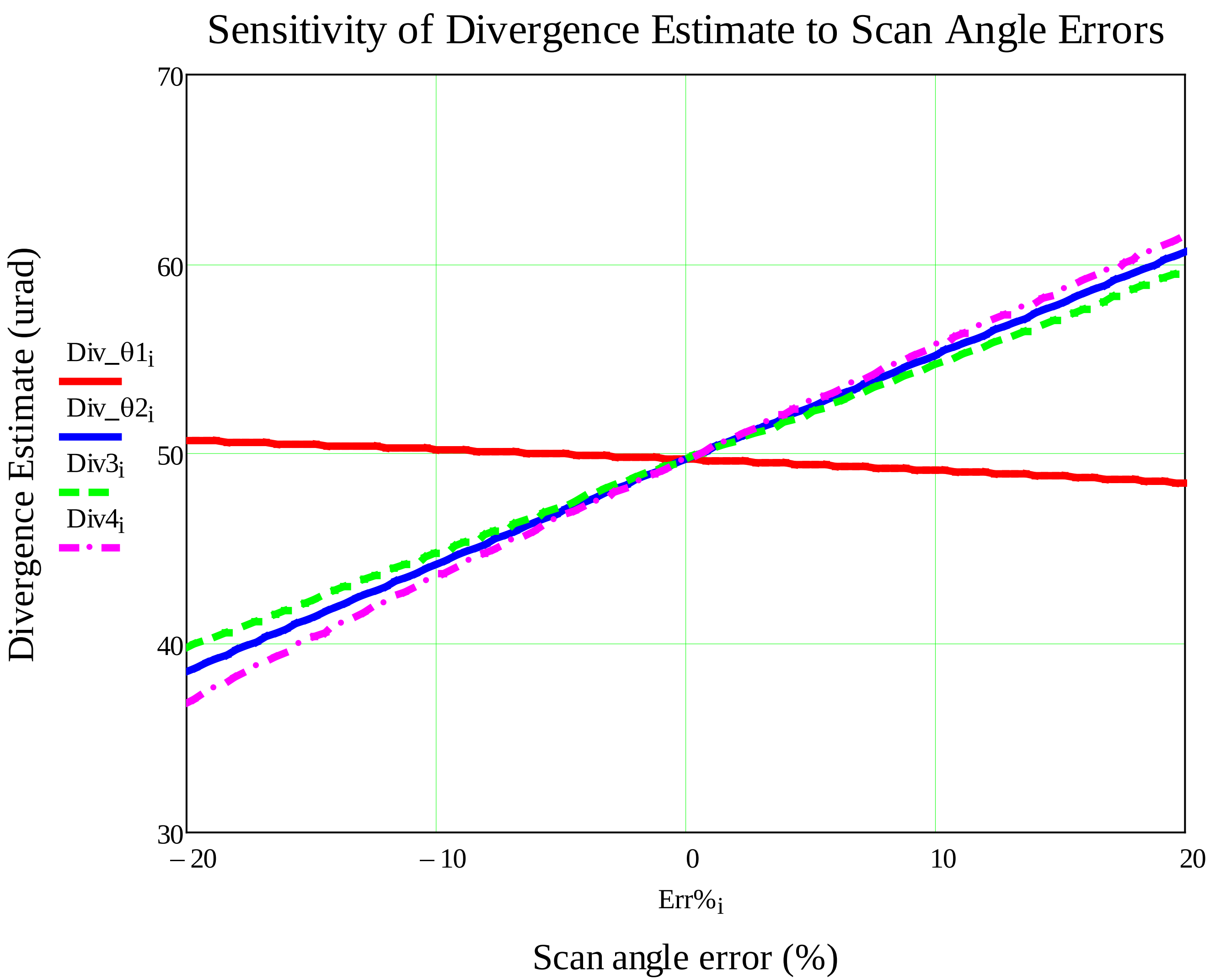
Effect of Scan Angle Errors



1) <u>Red solid</u>: error in longest range sat
2) <u>Blue solid</u>: error in shortest range sat
3) <u>Green dashed</u>: errors in both varying in same direction
4) <u>Purple dashed</u>: errors in both varying in opposite direction

Done with Stafford data on Ajisai and Lageos

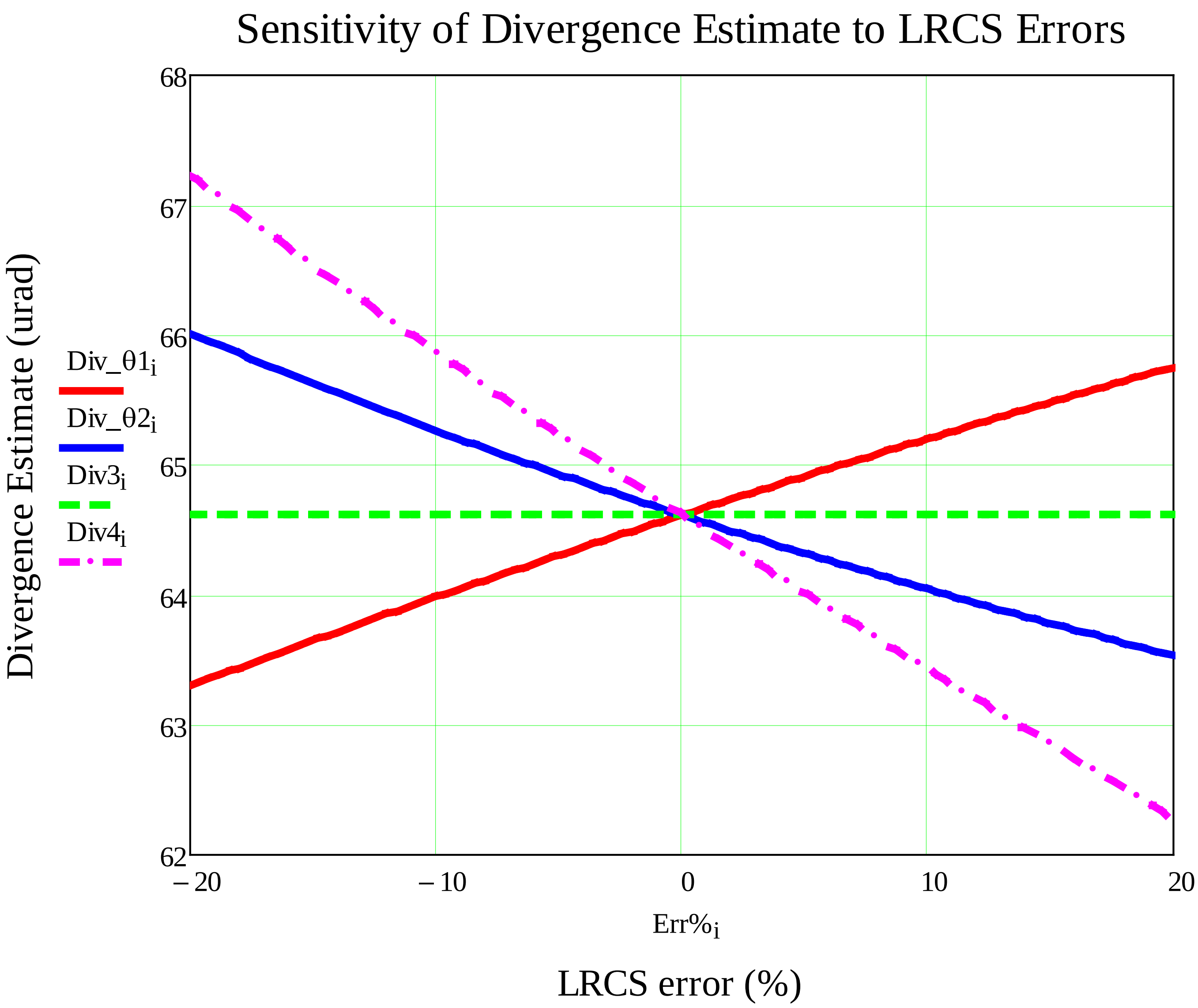
Effect of Scan Angle Errors



1) Red solid: error in longest range sat
2) Blue solid: error in shortest range sat
3) Green dashed: errors in both varying in same direction
4) Purple dashed: errors in both varying in opposite direction

Done with Graz data on Envisat and Lageos-2

Effect of LRCS Errors



1) Red solid: error in longest range sat LRCS
2) Blue solid: error in shortest range sat LRCS
3) Green dashed: errors in both varying in same direction
4) Purple dashed: errors in both varying in opposite direction

Done with Stafford data on Ajisai and Lageos

Divergence estimates from data reported by Chinese Stations

STATION	***SATELLITES***	***Full Angle DIV (μ rad)***	***Full Angle DIV (μ rad)***
Shanghai	Lageos & Ajisai	113.5	114.6
Shanghai	Etalon & Ajisai	90.6	87.2
Shanghai	Etalon & Lageos	9.9 ???	8.8 ???
Changchun	Etalon & Lageos	90.7	73.1
Changchun	Lageos & Starlette	126.1	123.6
Changchun	Etalon & Starlette	119.8	112
Yunnan	Lageos & Starlette	41.7	40.7
Yunnan	Lageos & Stella	37.9	37
Yunnan	Lageos & Ajisai	24.5	24.7
		Current LRCS	Revised LRCS

Divergence estimates from data reported by Graz, Stafford, & Herstmonceux

STATION	***SATELLITES***	***Full Angle DIV (μ rad)***	***Full Angle DIV (μ rad)***	***DIV Setting (μ rad)***
Graz	Envisat & Lageos-2	51.3	49.7	35
Graz	Giove-B & Lageos-2	22.5	18.9	17.5
Graz	Giove-B & Lageos-3	14.4	12.5	17.5
		Current LRCS	Revised LRCS	

STATION	***SATELLITES***	***Full Angle DIV (μ rad)***	***Full Angle DIV (μ rad)***
Stafford	Ajisai & Lageos	64	64.6
Stafford	Ajisai & Etalon	57.8	55.1
		Current LRCS	Revised LRCS

STATION	***SATELLITES***	***Full Angle DIV (μ rad)***	***Full Angle DIV (μ rad)***
Herstmonceaux	Lageos-2 & Glonass100	44.7	39
Herstmonceaux	Lageos-2 & Etalon-2	39	34.1
		Current LRCS	Revised LRCS

Concluding Remarks

- A simple calculation for estimation of divergence has been derived from the standard link budget equation for number of photoelectrons
- Assumptions made in derivation require care in taking the data for the estimation to be valid
- Results will differ depending on atmospheric transmission and other conditions at the SLR station
- Method should be used as an estimate to obtain values for average divergence, maximum and minimum, and to determine health of station optical train

BACKUP SLIDES

Gaussian beam propagation

$$I(r,z) = [2*P/\pi*\omega(z)^2]*\exp(-2r^2/\omega(z)^2)$$

- $I(r,z)$ = intensity at axial distance z from beam waist and at radial distance r from beam center axis
- P = total power in beam
- r = radial distance from the beam center axis
- z = axial distance from the beam waist
- ω_o = beam radius at the waist
- $\omega(z)$ = radius at which the intensity drops to $1/e^2$ of the intensity on axis

Link Budget Differences at 40 deg

	Starlette /Stella	Ajisai	Lageos	Etalon	Qzss	
1-way Range (km)	1159	2080	7055	20804	35817	
Range wrt Lageos	6.28	3.43	1.0	0.343	0.200	
Flux wrt Lageos	39.4	11.8	1.0	0.118	0.039	
Log (flux ratio to lag)	1.6	1.05	1.0	-0.93	-1.40	Decade Shift
Avg LRCS (millions sq meters)	1.80	23	15	55	253	
Avg NPE wrt Lageos	185	214	1	0.051	0.028	
Log (avg NPE)	2.3	2.3	0	-1.3	-1.6	Decade Shift

* Cross Sections from Arnold – "Cross section of ILRS Satellites"