A Tutorial on Retroreflectors and Arrays for SLR

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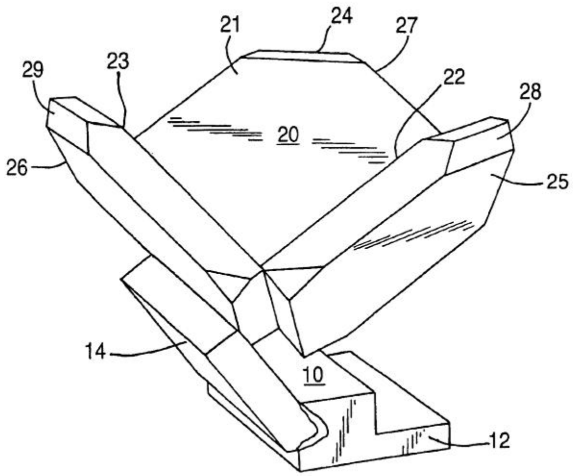
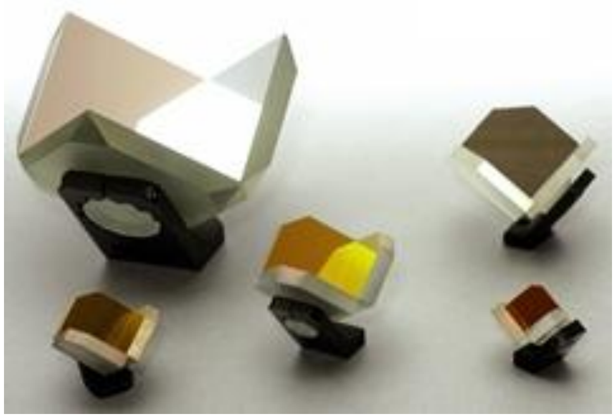
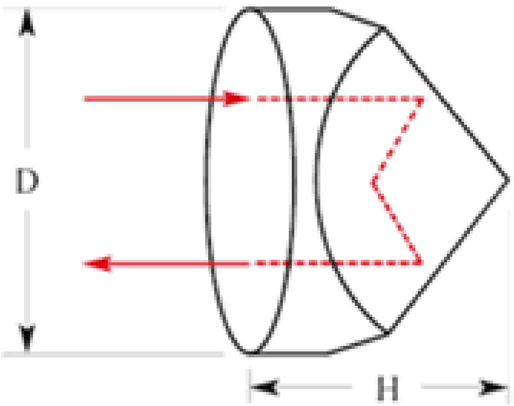
ILRS Workshop, Frascati, Italy
November 5, 2012

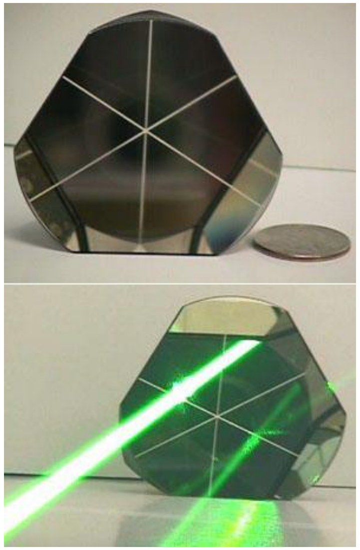
Corner Cube Retroreflectors





Hollow Cube Corner





Solid Cube Corner

Cube corner retroreflectors reflect light back to the point of origin in a narrow beam.

Three Types of Cube Corners

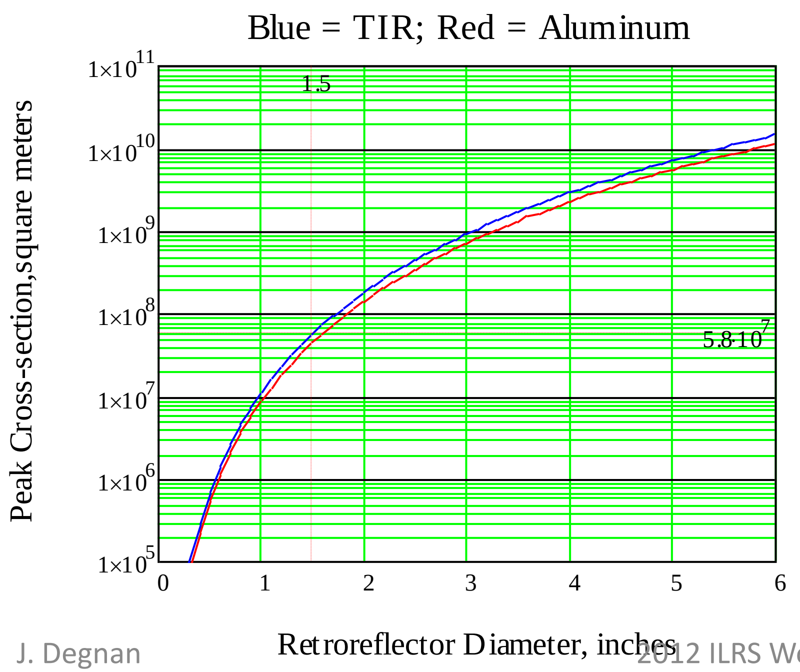
Type	Al Back-Coated Solid	Uncoated Solid (TIR)	Hollow
Frequency of Use	Most Common	Occasional Use	Not currently used in the visible
Satellite Examples	Most satellites	Apollo, LAGEOS, AJISAI, ETS-VIII	ADEOS RIS, REM, TES
Reflectivity, ρ	0.78	0.93	Can approach 1.0
Polarization Sensitive	No	Yes	No – metal coating Yes-dielectric coating
Weight	Heavy	Heavy	Light
Far Field Pattern	Wide	Wide	Narrow
Issues	Metal coatings absorb sunlight and create thermal gradients. Not as well shielded at high altitudes.	Fewer thermal problems but TIR "leaks" at incidence angles > 17^o. Polarization effects reduce cross-section by factor of 4.	Thermal heating and gradient effects on joints

Peak Cross-Section of a Perfect Cube Corner

For normally incident light, a single unspoiled retroreflector (cube corner) has a peak, on-axis, optical cross-section defined by

$$\sigma_{cc} = \rho A_{cc}\left(\frac{4\pi}{\Omega}\right) = \rho\left(\frac{4\pi A_{cc}^2}{\lambda^2}\right) = \frac{\pi^3 \rho D^4}{4\lambda^2}$$

where the reflectivity of the cube corner, ρ, is typically equal to 0.78 or 0.93 for aluminum-coated back faces and uncoated Total Internal Reflection (TIR) surfaces respectively , A_{cc} is the collecting aperture of the corner cube, D is the cube diameter, and $4\pi/\Omega$ is the on-axis reflector gain and Ω is the effective solid angle occupied by the Far Field Diffraction Pattern (FFDP) of the retroreflector.



The peak optical cross-section rises rapidly as the retroreflector diameter to the fourth power. For the popular 1.5 in (38 mm) diameter cube, the peak cross-section is about 5.8 x 10^7 m^2.

Retroreflector Far Field Diffraction Pattern (FFDP)

Since the retroreflector aperture is illuminated by a uniform plane wave from the distant SLR station, the electric field strength in the far field , *E(x',y'),* is equal to the 2D Fourier Transform of the retroreflector entrance aperture as seen by the plane wave source while the Intensity distribution is the electric field multiplied by its complex conjugate, *E*(x',y'),* i.e.

$$E(x',y') = \int dx \int dy \exp(ikx'x)\exp(iky'y)$$

$$I(x',y') = E(x',y')E^*(x',y')$$

where k = 2π/λ and λ is the optical wavelength (532 nm). For a circular aperture we can use cylindrical coordinates

Retroreflector Far Field Diffraction Pattern

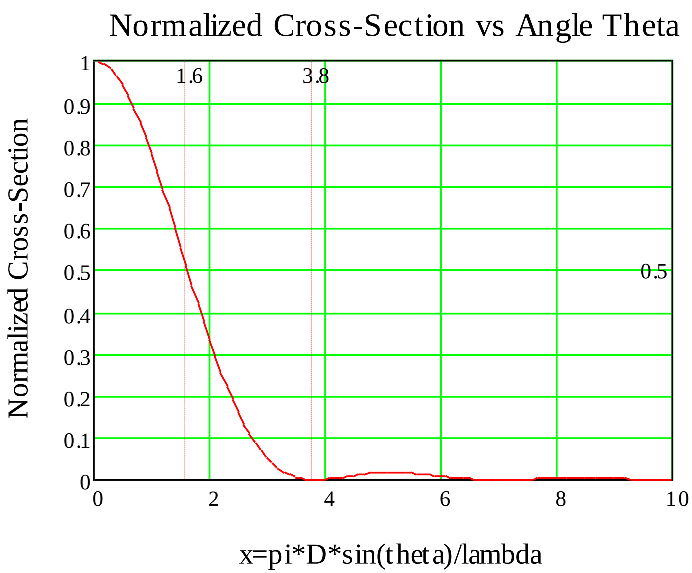
For a circular aperture, the FFDP of the reflected wave is the familiar Airy Function given by

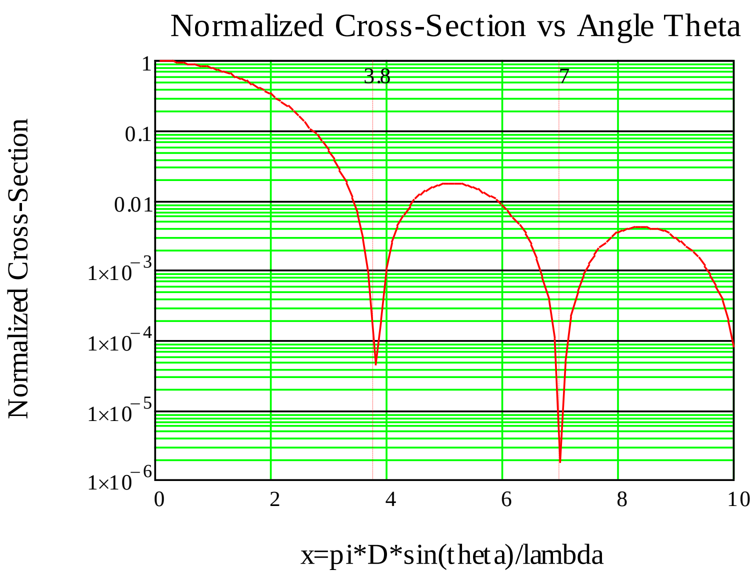
$$\sigma(x) = \sigma_{cc}\left[\frac{2J_1(x)}{x}\right]^2$$

where J_1 (x) is a Bessel function and the argument x is related to the off-axis angle θ by

$$x = \frac{\pi D}{\lambda}\sin\theta$$

λ = 532 nm is the most widely used SLR laser wavelength and D is the cube aperture diameter.





The half-power and first null occur at x = 1.6 and 3.8 respectively. For the popular 1.5 in (38 mm) diameter cube at 532 nm, this corresponds to θ = 7.1 and 16.9 microradians (1.5 and 3.5 arcsec) respectively.

Peak Cross-Section vs Incidence Angle (Hollow Cube vs Coated Fused Silica)

At arbitrary incidence angle, the effective area of the cube is reduced by the factor

$$\eta(\theta_{inc}) = \frac{2}{\pi}\left(\sin^{-1}\mu - \sqrt{2}\tan\theta_{ref}\right)\cos\theta_{inc}$$

where θ_{inc} is the incidence angle and θ_{ref} is the internal refracted angle as determined by Snell's Law, i .e.

$$\theta_{ref} = \sin^{-1}\left(\frac{\sin\theta_{inc}}{n}\right)$$

where n is the cube index of refraction. The quantity μ is given by the formula

$$\mu = \sqrt{1-\tan^2\theta_{ref}}$$

Thus, the peak optical cross-section in the center of the reflected lobe falls off as

$$\sigma_{eff}(\theta_{inc}) = \eta^2(\theta_{inc})\sigma_{cc}$$

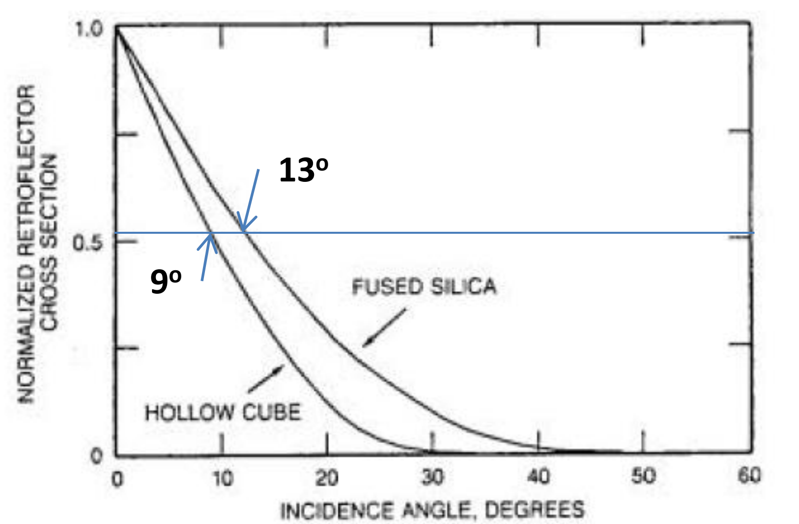


Fig. 23. Normalized cross-section as a function of incidence angle for hollow and fused silica retroreflectors.

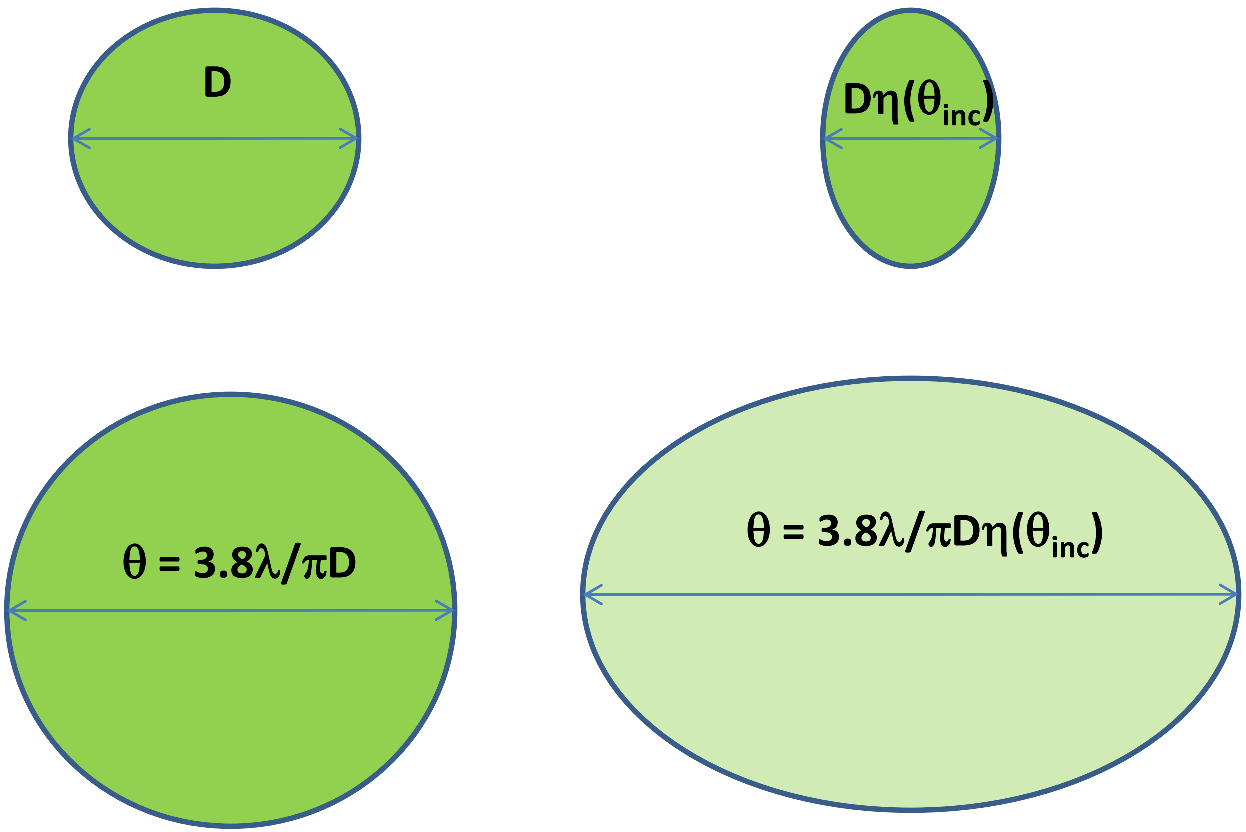
•The 50% and 0% efficiency points for fused silica (n=1.455) are 13° and 45° respectively.
• The 50% and 0% efficiency points for a hollow cube (n=1) are 9° and 31° respectively.
•In short, hollow cubes have a narrower angular response range than solid cubes.

Effect of Incidence Angle on the FFDP

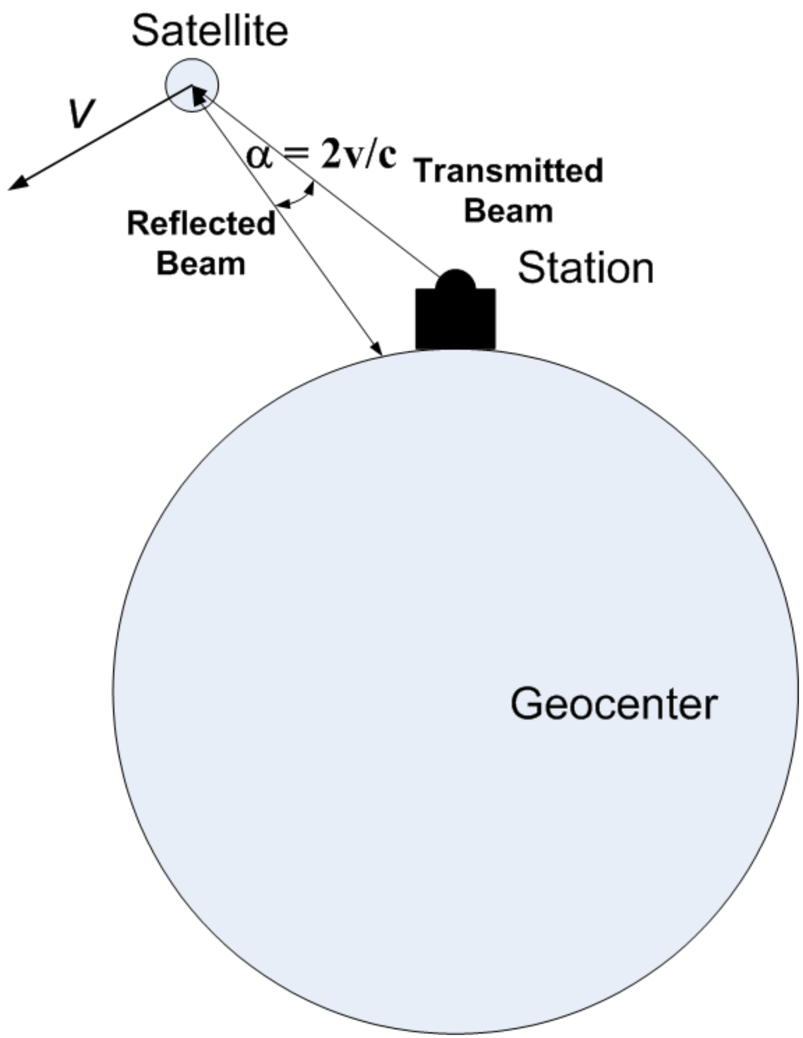
Normal Incidence | **Non- Normal Incidence**

Retro Face Seen by Incident Radiation

Far Field Diffraction Pattern



Introduction to Velocity Aberration



•If there is no relative velocity between the station and satellite, the beam reflected by the retroreflector will fall directly back onto the station .

•A relative velocity, v, between the satellite and station causes the reflected beam to be angularly deflected from the station in the forward direction of the satellite by an amount $\alpha = 2v/c$.

• We have seen that small diameter cubes have small cross-sections but large angle FFDPs , and therefore the signal at the station is not significantly reduced by velocity aberration.

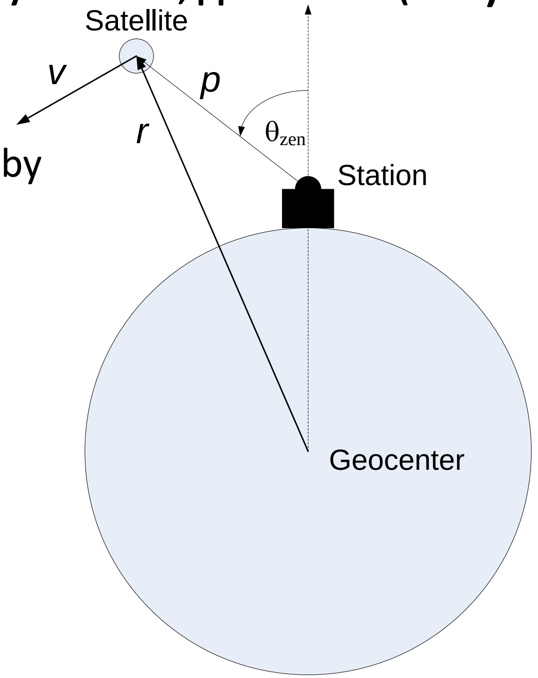
•Similarly, large diameter cubes with high cross-sections have small angle FFDPs, and the signal at the station is therefore substantially reduced by velocity aberration.

•In general, the signal is reduced by half or more if the cube diameter, D_{cc}, satisfies the inequality

$$\alpha = \frac{2v}{c} > \theta_{1/2} = \frac{\lambda x_{1/2}}{\pi D_{cc}} \quad \text{or} \quad D_{cc} > D_{1/2} = \frac{1.6\lambda c}{\pi\alpha}$$

Velocity Aberration: General Earth Orbit

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)



If there is a relative velocity between the satellite and the station, the coordinates of the FFDP are translated in the direction of the velocity vector. The magnitude of the angular displacement in the FFDP is given by

$$\alpha(h_s,\theta_{zen},\omega)=\alpha_{\max}(h_s)\sqrt{\cos^2\omega+\Gamma^2(h_s,\theta_{zen})\sin^2\omega}$$

where the maximum and minimum values are given by

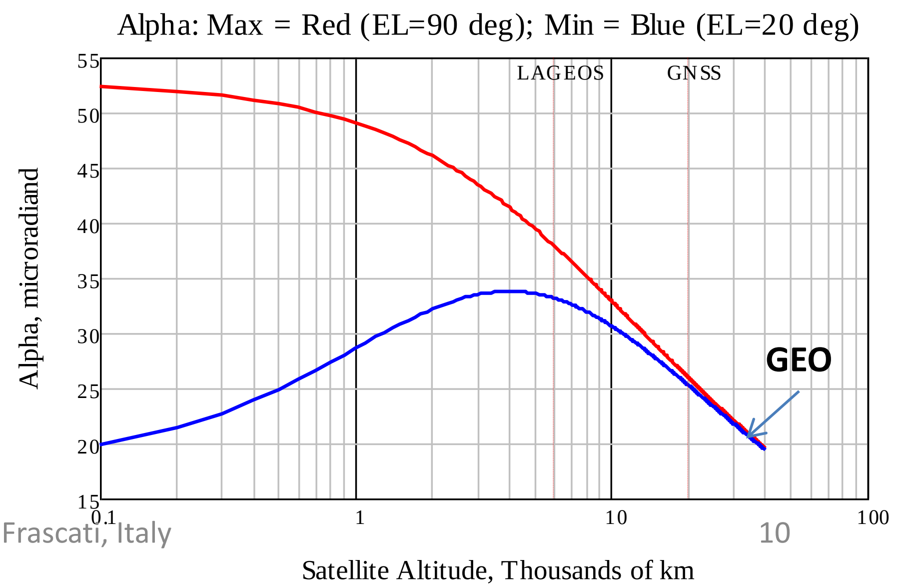
$$\alpha_{\max}(h_s)=\alpha(h_s,0,0)=\frac{2v_s}{c}=\frac{2}{c}\sqrt{\frac{gR_E^2}{R_E+h_s}}$$

$$\alpha_{\min}(h_s)=\alpha(h_s,70^o,90^o)=\alpha_{\max}(h_s)\Gamma(h_s,70^o)$$

$$\Gamma(h_s,\theta_{zen})=\sqrt{1-\left(\frac{R_E\sin\theta_{zen}}{R_E+h_s}\right)^2}<1$$

$$\omega=\cos^{-1}\left[\left(\hat{r}\,x\,\hat{p}\right)\bullet\hat{v}\right]$$

v_s = satellite velocity at altitude h_s
R_E = Earth radius = 6378 km
g = surface gravity acceleration =9.8m/sec^2
h_s=satellite height above sea level
c = velocity of light = 3x10^8 m/sec
θ_{zen} = largest satellite zenith angle for tracking = 70^o
r = unit vector to satellite from the geocenter
p = unit vector from station to satellite
v= unit vector in direction of satellite velocity



Why is α Non-Zero for Geostationary Satellites?

The angular velocities about the Earth's rotation axis are the same for a ground station and a Geostationary satellite but the physical velocities are different resulting in a relative velocity between them. The common angular velocity is given by

$$\omega_E = \frac{2\pi}{day} = 7.27x10^{-5}\,Hz$$

The physical velocity of the geostationary satellite in the Earth-centered frame is

$$v_{GEO} = \omega_E(R_E + h_{GEO}) = 7.27x10^{-5}(6378km + 35786km) = 3.066km/\sec$$

while the physical velocity of the ground station due to Earth rotation is latitude dependent and given by

$$v_{station} = \omega_E R_E \cos(lat) \le 0.464km/\sec$$

However, as shown on the previous slide, the relative velocity also depends on the observing zenith angle at the station (assumed maximum of 70°). The resulting velocity aberration has a very narrow range , i.e.

$$20.29\mu rad \le \alpha = \frac{2v}{c} \le 20.46\mu rad$$

Apollo 15 Lunar Example

Earth-Moon Distance : R_{EM} = h+R_E = 384.4 x10^6 m. From the previous equations

α_{max} =6.74 μrad or 1.40 arcsec

α_{min} = 6.68 μrad or 1.39 arcsec at an elevation angle of 20 degrees

v = relative velocity between target and station due to lunar orbital motion = 1km/sec

However, the latter equations ignore the small contribution of station motion due to Earth rotation (~0.46 km/sec) to the relative velocity which typically reduces α to 4 or 5 μrad for LLR but is negligible for LEO to GEO satellites.

If the Apollo reflector arrays are pointed at the center of the Earth, the maximum beam incidence angle on the array from any Earth station (ignoring lunar libration) is

$$\theta_{inc} = a\tan\left(\frac{R_E}{R_{EM}}\right) = 0.95\deg$$

The unspoiled cube diameter for which the cross-section falls to half its peak value is

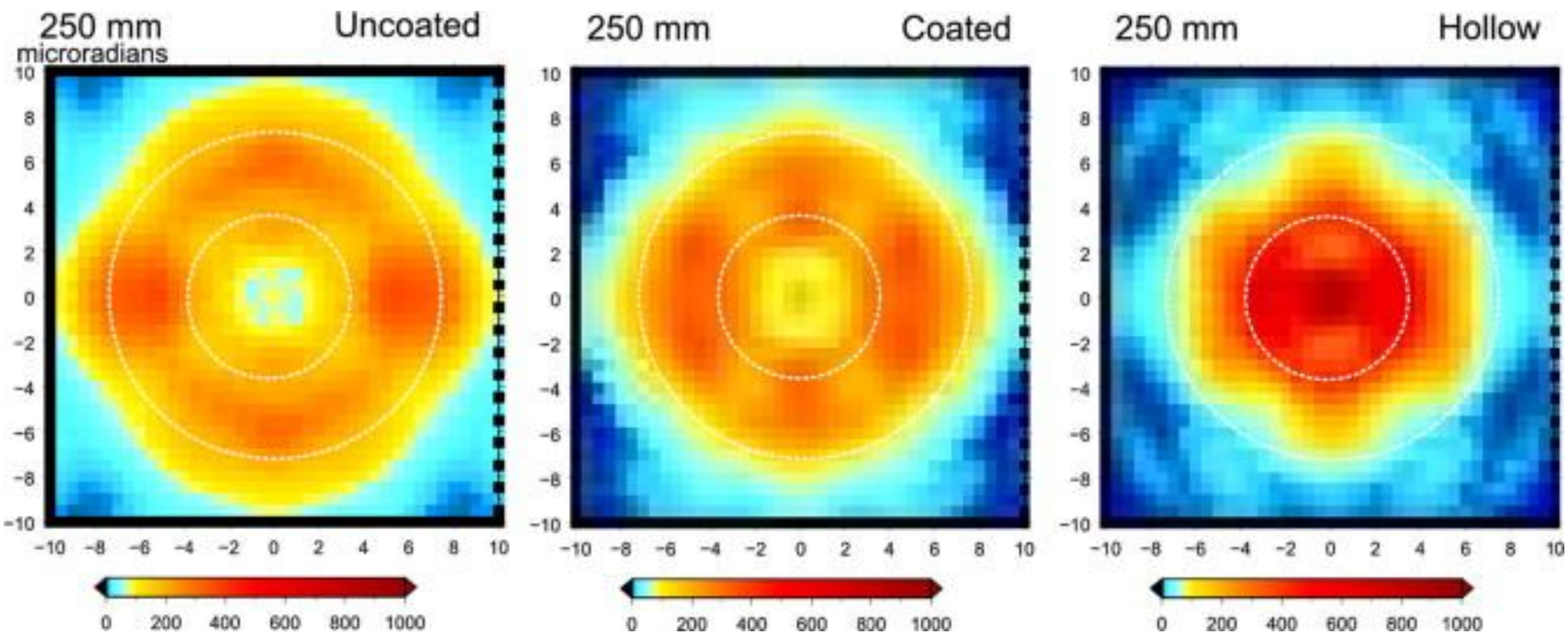
$$D_{1/2} = 40.6mm = 1.6in$$

Typical manufacturing tolerances are 0.5 arcsec for dihedral angles and λ/10 for surface flatness.

Apollo 15 has a flat array of 300 38 mm fused quartz cubes each with an unspoiled peak cross-section of 5.8 x 10^7 m^2. Thus, the theoretical array cross-section, ignoring manufacturing tolerances and local environment effects, is $\Sigma \cong 300(0.5)(5.8 \times 10^7\ m^2) = 8.7 \times 10^9\ m^2$. According to Dave Arnold, polarization losses due to uncoated TIR faces reduce cross-section by factor of 4, leaving]Σ~2.
2x10^9 m^2 . The tabulated ILRS value is 1.4x10^9 m^2.

Lunar Alternative to Apollo Array

Otsubo et al, Advances in Space Research, Vol. 45, pp. 733-740, 2010.
According to the authors, simulations indicate that a single reflector with a diameter of 150 to 250 mm has similar performance to Apollo arrays. No dihedral angle is required for small diameter reflectors (<150 mm for coated and <100 mm for uncoated and hollow reflectors) . Larger diameters required dihedral angles 0f 0.20, 0.25, and 0.35 arcsec for coated, uncoated, and hollow reflectors respectively.



250 mm reflectors with 0.25 arcsec dihedral angles, incidence angle = 6 degrees

GNSS and Geostationary Satellites

GNSS and Geosynchronous Satellites have features in common with LLR:

1. Their orbital altitudes correspond to several Earth radii
2. They generally perform a utilitarian function (Earth observation, communications, navigation, etc.) which keeps the nadir side of the satellite approximately facing the Earth CoM
3. The difference $\Delta\alpha = \alpha_{max} - \alpha_{min}$ is very small (see previous graph).

Their differences from LLR are :

1. The velocity aberration α is 4 to 5 times larger (20 to 25 μrad)
2. For a maximum zenith tracking angle of 70°, beam Incidence angles can vary from 0 to β where

$$\beta = a\sin\left[\frac{R_E}{R_E + h}\sin(110^0)\right]$$

=13.1 deg for GNSS satellites at 20,000 km
= 8.2 deg for GEO satellites at 36,000 km

The smaller range of incidence angles implies limited pulse spreading from a flat array, especially if the array is compact in size and the retros are densely packed together to achieve the necessary cross-section. Nevertheless, the maximum flat panel induced spreading per linear foot of array due to zenith tracking angle is still 474 and 292 psec for GNSS and GEO satellites respectively. This spreading can increase further if satellite attitude deviations from true nadir extend the range of incidence angles.

Retroreflector Arrays for High Altitude Satellites

Ref: D.A. Arnold, "Retroreflector Studies",

http://nercslr.nmt.ac.uk/sig/signature.html

Tables 2 and 3 show the area and mass of the cube corners needed to obtain a cros section of 100 million sq meters at the altitude of the GNSS satellites and a cros section of one billion sq meters at geosynchronous altitude.

***Table 2**: GNSS*

Design	# of cubes	Diam. in	Area sq cm	Mass g
uncoated	50	1.3	428	1000
coated	400	0.5	508	460
hollow	400	0.5	508	201
hollow	36	1.4	356	400
GPS	160	1.06	1008	1760

***Table 3**: Geosynchronous*

Design	# of cubes	Diam. In.	Area sq cm	Mass g
Uncoated	165	1.7	2415	7457
Coated	1153	.7	2863	3638
Hollow	1153	.7	2863	1590
Hollow	122	1.8	2003	2863
Single dihedral	22	2.0	446	708

“Spoiled” Retroreflectors

•“Spoiling” is used to compensate for velocity aberration and improve the signal return from the satellite.

• If we offset one or more (N = 1 to 3) of the cube angles from 90° by an amount δ, the central lobe of the FFDP splits into 2N spots.

• If n is the cube index of refraction, the mean angular distance of the lobe from the center of the original Airy pattern increases linearly with the dihedral angle offset, δ, according to

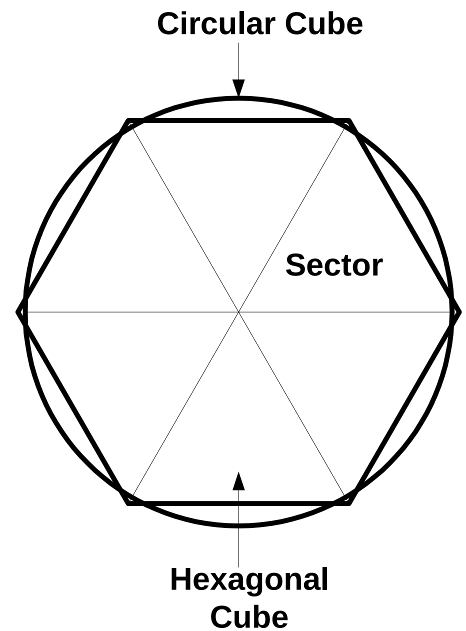
$$\gamma = \frac{4}{3}\sqrt{6}n\delta$$

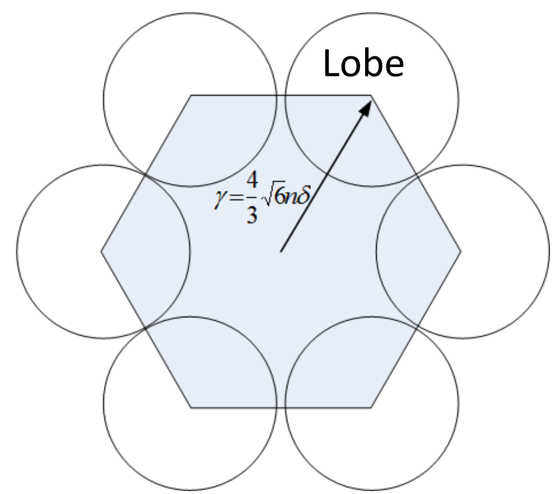
•As before, the angular size of any given lobe decreases as the cube diameter gets larger.

•The FFDP of each lobe is the 2D Fourier transform of an individual 60° sector. The energy distribution is complex but has hexagonal symmetry if all δs are equal.

•Furthermore, the effective area and peak cross-section of each lobe is reduced to

$$A_{eff} = \eta(\theta_{inc})\frac{A_{cc}}{2N} \qquad \sigma_{peak} = \eta^2(\theta_{inc})\frac{\sigma_{cc}}{(2N)^2}$$

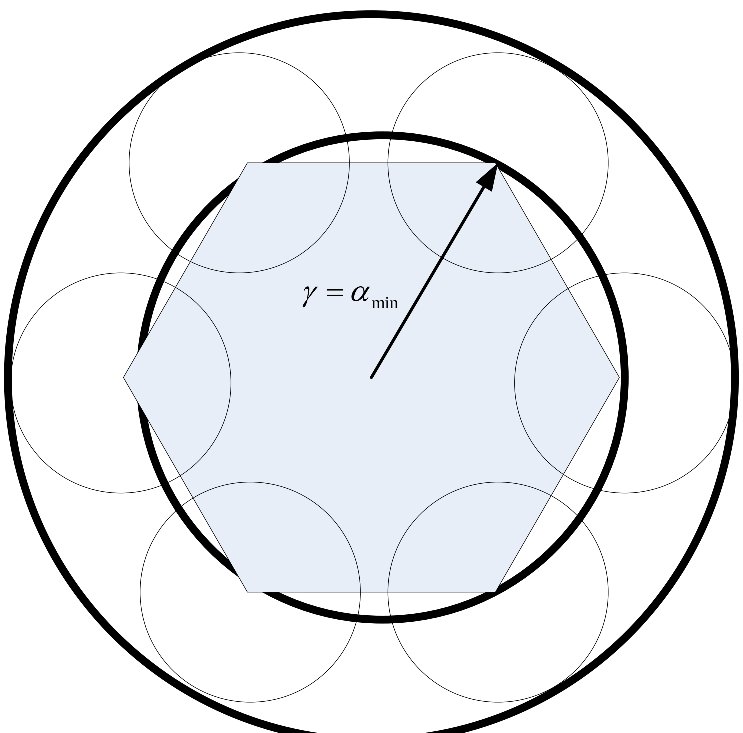




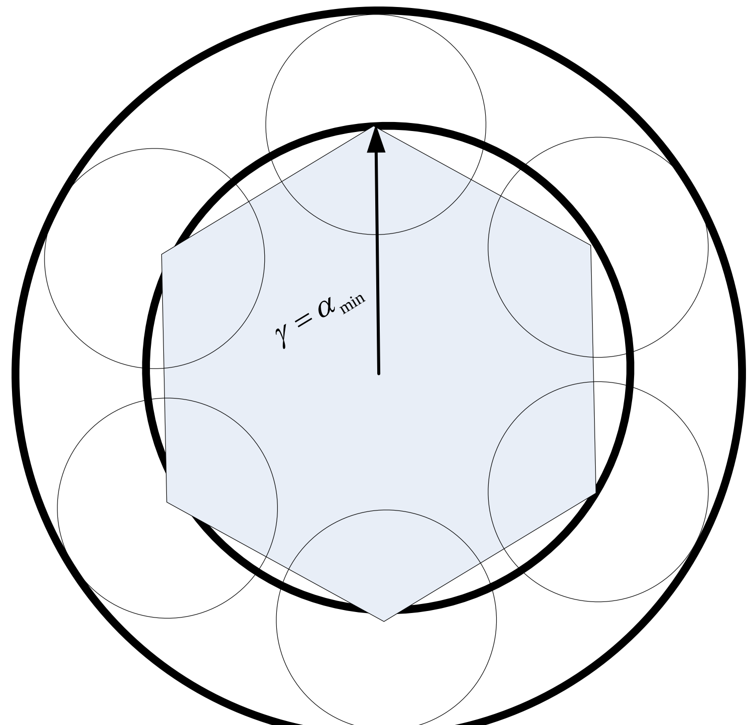
Clocking

Filling in circumferential gaps between lobes can be accomplished by rotating an adjacent cube by an amount not divisible by 60°. The bigger the gap and/or the smaller the lobe diameter , the more rotational positions are needed. This may allow the use of larger diameter cubes which don't overfill the annulus thereby reducing array efficiency.

0° Clocking



30° Clocking



Signal Strength vs Zenith Angle

For a constant target cross-section, the signal strength decreases with zenith angle via two effects:

Space Loss:

$$\frac{1}{R^4} = \left[\frac{\cos(\theta_{zen})}{h}\right]^4$$

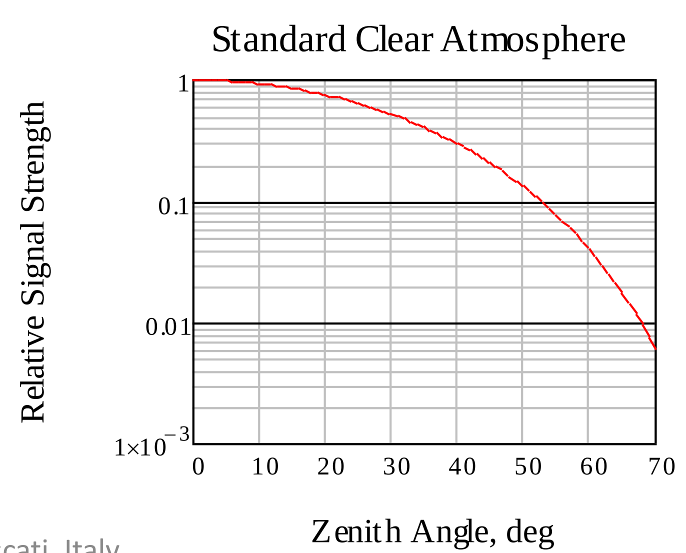
Atmospheric Transmission Loss:

$$T_0^{2\sec(\theta_{zen})}$$

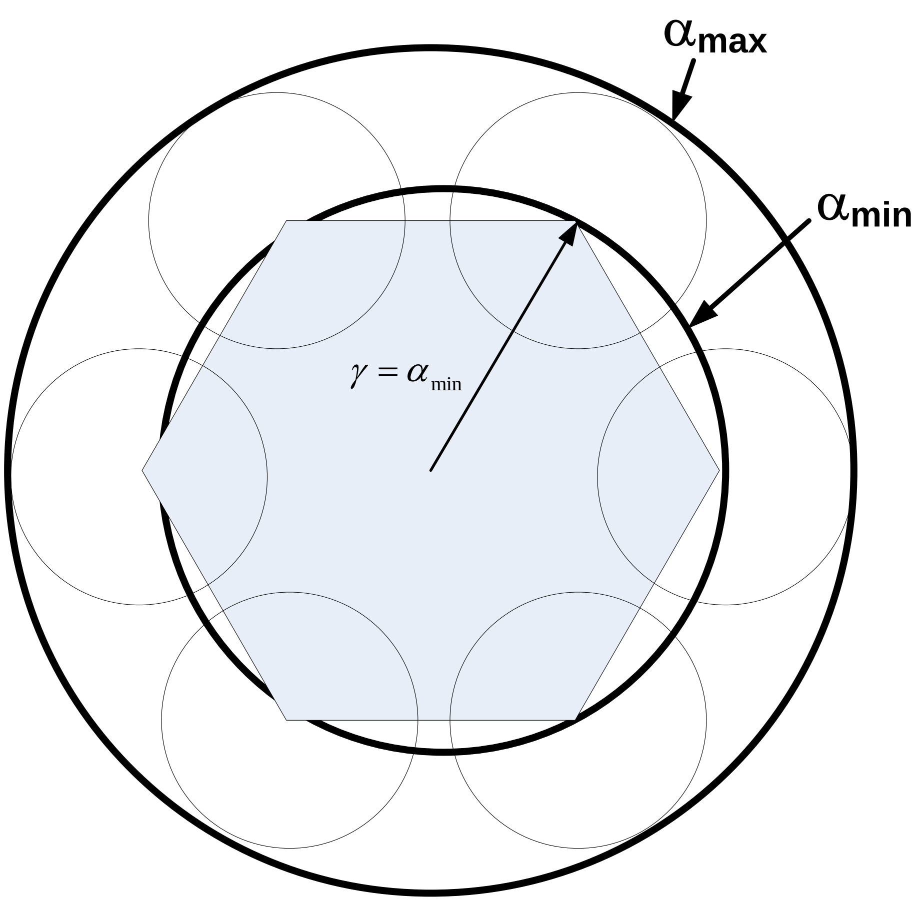
The signal strengths at θ_{zen} = 70° relative to zenith (θ_{zen} = 0°) is therefore

$$\cos^4(\theta_{zen})T_0^{2[\sec(\theta_{zen})-1]}$$

where, for a Standard Clear Atmosphere, the one-way atmospheric transmission is T_0 =0.8 at 532 nm. Thus, from the plot, the signal strength at α_{min} (θ_{zen} = 70°) is about two orders of magnitude smaller than at α_{max} (θ_{zen} = 0°) so **<u>the array design should bias the return toward high zenith angles (low elevation angles).</u>**



LEO to MEO Satellites
LAGEOS Example



- Setting $\gamma = \alpha_{min}$ =33 μrad places the peak of the lobe where the energy is needed most, i.e. at low elevation angles. (θ_{zen} = 70^o)
- The corresponding dihedral offset is given by

$$\delta = \frac{3\alpha_{\min}}{4\sqrt{6}n} = 7.1\mu rad$$

- To fill the annulus quasi-uniformly in the circumferential direction, the transverse angular radius of the lobe must satisfy

$$\theta_{1/2} = \frac{1.6\lambda}{\pi D_c} > \frac{\pi\alpha_{\min}}{6} = 17.3\mu rad$$

or D_c >15.7 mm

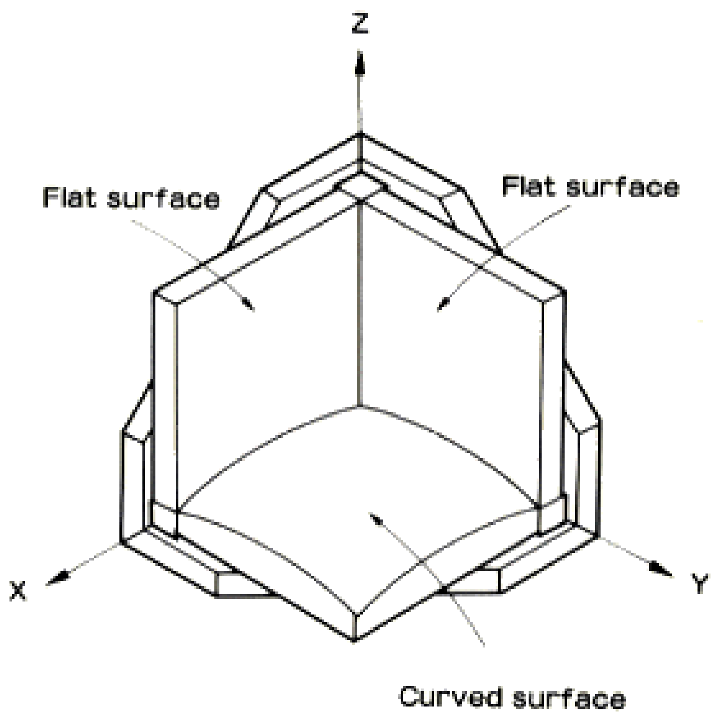
- To fill the annulus in the radial direction, the radial angular radius of the lobe must satisfy

$$\theta_r = \frac{3.8\lambda}{\pi D_r} > \alpha_{\max} - \alpha_{\min} = 5\mu rad$$

or D_r>129 mm

Any reflected energy outside the annulus is wasted!

Retroreflector in Space



Retroreflector in Space (RIS)
Effective Diameter = 50 cm
One dihedral offset
One curved reflecting face
Effective Divergence 60 μrad
Designed for Thermal Infrared
Visible performance was poor.

MM ACCURACY GEODETIC SATELLITES

- **Technical Challenges**
  - The satellite must present a high enough cross-section, consistent with its altitude, to support ranging by the entire ILRS network. Velocity aberration limits the size of the retroreflector.
  - To simultaneously achieve maximum range accuracy, the satellite impulse response must be made as short as possible.
- Planar LLR arrays present a common face to all Earth ranging stations provided it is initially pointed approximately toward the Earth Center-of-Mass (COM). The number or density of small diameter retroreflectors (< 38 mm) can be increased without limit to increase the target cross-section, and the near perpendicularity of the flat panel to the incoming optical wavefronts introduces little or no pulse spreading for maximum range accuracy.
- On the other hand, geodetic satellites (LAGEOS, Starlette, etc.) are in much lower orbits,have large velocity aberrations, rotate freely in space, and are spherical in shape to permit simultaneous and unbiased ranging from multiple SLR stations.
- Spherical geodetic satellites can be made to mimic their LLR cousins by:
  - Building larger supporting spheres or flat panel polyhedrons .
  - Improving the packing density to increase the number of retros per unit surface area and hence the effective array cross-section.
  - For spheres, limit target pulse spreading by restricting returns over a smaller range of incidence angles through the use of hollow or recessed solid retros .
  - For flat panel polyhedrons, ensure that returns from adjacent panels cannot be seen by the ground station.

MM ACCURACY GEODETIC SATELLITES

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

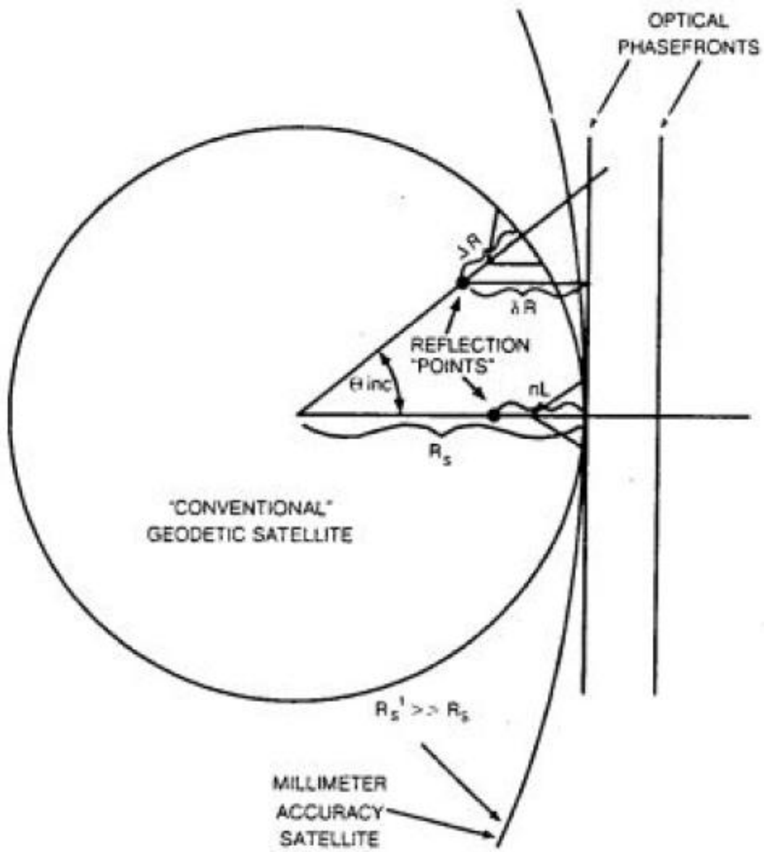


Fig. 25. Definitions of quantities used in the discussion of satellite impulse response. Satellites capable of supporting millimeter accuracy two color measurements are characterized by larger radii, high retroreflector densities, and limited angular field of view. The large satellite radius provides a better "match" to the incoming planar phasefront.

Basic Design Guidelines

- Increase radius of sphere to better approximate a flat surface
- Recess the individual cubes in their holders to restrict the response at large incidence angles and minimize the target signature
- Increase packing density of the cube corners on the satellite surface (ratio of cube aperture within a given surface area). Hexagonal arrays have the greatest packing density but do not permit "clocking". (LAGEOS has a packing density of 0.435.)
- Choose retro diameter and clocking scheme to optimally fill the annular FOV, Ω, between α_{max} and α_{min}

Cross-Section in the Large Satellite Limit (R_s>>nL)

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

The optical cross-section in the large satellite limit is

$$\sigma = \frac{2\pi^2\rho}{3\Omega}\frac{\beta R_s c\Delta t}{\left[1-\frac{nL}{R_s}\left(1+\frac{1}{n^2}\right)\right]} \qquad (6.6.6)$$

Where
ρ is the cube corner reflectance
c is the speed of light
R_s is the satellite radius
n is the corner cube index of refraction
nL is the optical depth of the cube (face to vertex)
β is the packing density (active area/total area)
Ω Is the array FOV , ideally well matched to the annular FOV between α_{max} and α_{min}

Impulse Response in the Large Satellite Limit (R_s>>nL)

J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology, Geodynamics 25, pp. 133- 162 (1993)

In the following graph, τ is a time normalized to the time it takes a light pulse to travel the diameter of the satellite, i.e. $2R_s/c$. Increasing the radius of the satellite to make it appear flatter will increase the impulse response. However, narrowing the incidence angle range by using hollow cubes or recessing the solid cubes reduces the impulse response of the satellite and improve range accuracy.

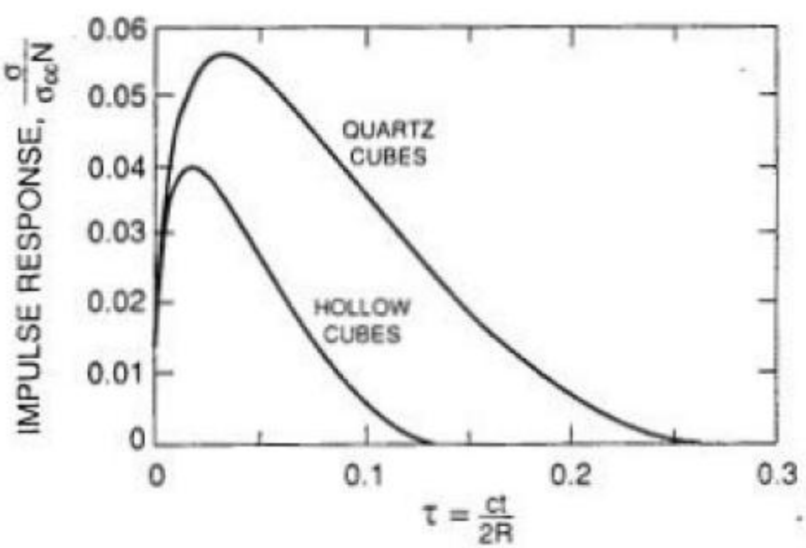
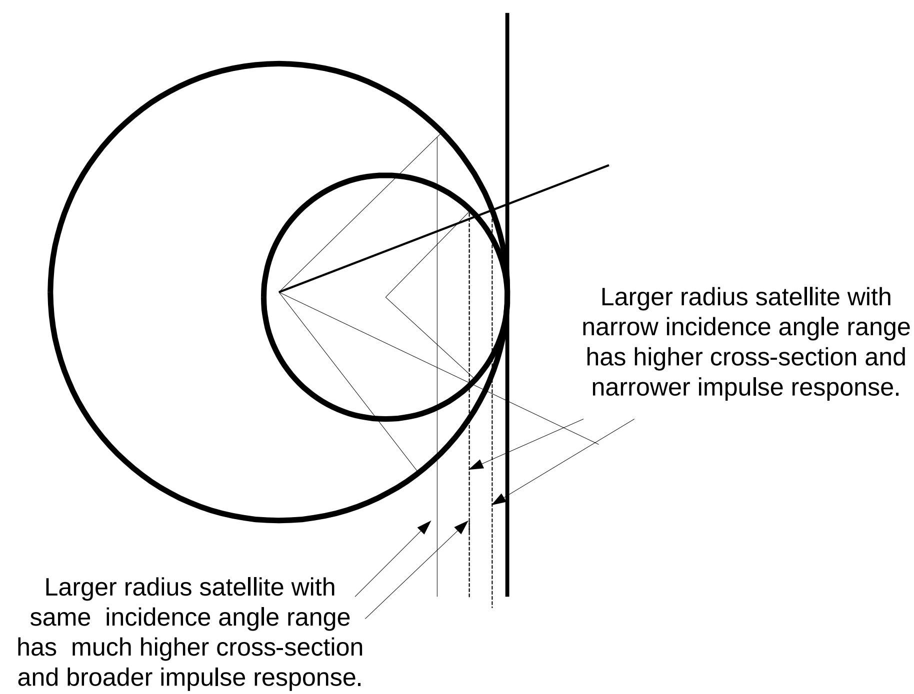
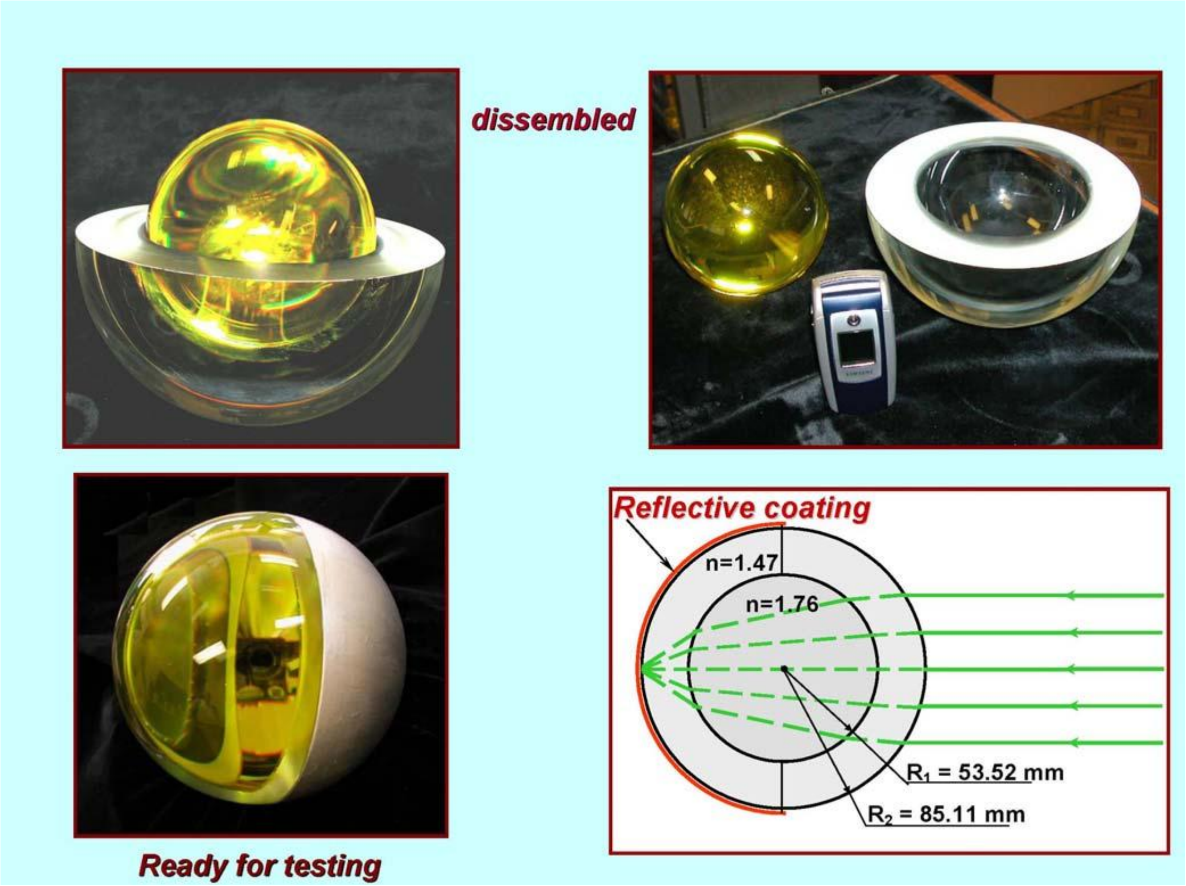


Fig. 27. Impulse response in the large satellite limit ($R_s \gg nL$) for both hollow and solid quartz cube corners.



BLITZ Retroreflector



Inner Sphere: High Index ТФ105 glass
Outer Sphere: ПК6 glass
Reflective Coating: Aluminum protected by a varnish layer
Mass: 7.53 kg
σ ~10^5 m^2 (equivalent to a 0.3 in or 7.6 mm corner cube)

Summary

<u>mm Accuracy LEO to MEO Geodetic Satellites</u>

- Large radius satellites
  - to better match the incoming plane wave
  - Allow more reflectors in the active area to increase cross-section
- Reduce range of accepted incidence angles to minimize satellite impulse response
  - Hollow cubes
  - Recessed hollow or solid cubes
- Also incidence angles < 17^{o} do not leak light in solid TIR reflectors
- Selection of cube diameters and clocking to best match the "α annulus" while favoring high zenith (low elevation) angles is key to efficient array design

<u>GNSS and GEO Satellites</u>

- Typically have nadir face pointed near Earth center due to other functions (Earth observation, communications, navigatios, etc.)
- Flat panels OK but still several hundred psecs of temporal spread at large zenith angles.
- Range accuracy might benefit from using a small segment of a large sphere following LEO/MEO design guidelines

<u>LLR</u>

- Small incidence angles (<1 deg ignoring lunar librations) and velocity aberrations (<1.0 arcsec) suggest the possible use of large diameter cubes provided thermal issues on the lunar surface can be resolved.