Simulation of hadronic interactions, examples from FLUKA

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h-A interactions

The approach to hadronic interaction modelling presented in the following is the one adopted by most state-of-the-art codes. This “microscopic” approach,

- uses **theoretical models** to describe physical processes *whenever possible*
- preserves **correlations** and reproduces **fluctuations**
- Performances are optimized comparing with particle production data at single interaction level
- final predictions are obtained with minimal free parameters, fixed for all energies and target/projectile combinations
- results in complex cases as well as scaling laws and properties come out naturally from the physical models. The basic conservation laws are fulfilled “a priori”

All the examples/results presented in the following have been obtained with **FLUKA** and should be typical of codes adopting similar approaches.
FLUKA: generalities

FLUKA
Interaction and transport Monte Carlo code

- Hadron-hadron and hadron-nucleus interactions 0-10000 TeV
- Nucleus-nucleus interactions 0-10000 TeV/n
- Electromagnetic and $\mu$ interactions 1 keV-10000 TeV
- Neutrino interactions and nucleon decays
- Charged particle transport including all relevant processes
- Transport in magnetic field
- Combinatorial and Voxel geometry + interface to G4 geom
- Neutron multigroup transport and interactions 0-20 MeV
- Analog calculations, or with variance reduction

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Topics

- Hadron nucleon collisions
  - Low energies: elastic, charge and strangeness exchange
  - Intermediate energies: resonance production
  - High energies: quark string models

- Hadron nucleus collisions
  - High energies: Glauber cascade
  - Fast stage: (G)INC
  - Intermediate stage: preequilibrium emission
  - Slow stage: evaporation, fission and fragmentation

- Low energy neutrons

- Multi-group transport
Main steps of h-A interactions

h-A interactions can be schematically described as a sequence of the following steps:

- **Glauber-Gribov cascade and high energy collisions**
- **(Generalized)-IntraNuclear cascade**
- **Preequilibrium emission**
- **Evaporation/Fragmentation/Fission and final deexcitation**

Some of these steps can be missing if the energy of the projectile is low enough.

Basic building block is of course the hadron-nucleon interaction model.
Hadron-nucleon interaction models

Elastic, charge exchange and strangeness exchange reactions:

- Available phase-shift analysis and/or fits of experimental differential data
- At high energies, standard eikonal approximations are used

Particle production interactions: two kind of models

- Those based on “resonance” production and decays, which cover the energy range up to 3–5 GeV
- Those based on quark/parton string models, which provide reliable results up to several tens of TeV
Total and elastic cross section for $p-p$ and $p-n$ scattering, together with experimental data (left), isospin decomposition in the $T=0$ and $T=1$ components (right)
Nonelastic hN interactions at intermediate energies

- $N_1 + N_2 \rightarrow N'_1 + N'_2 + \pi$ threshold around 290 MeV, important above 700 MeV,
- $\pi + N \rightarrow \pi' + \pi'' + N'$ opens at 170 MeV.
- double pion production opens at 350 MeV ($\pi$-N) or 600 MeV (N-N).

Dominance of the $\Delta$ resonance and of the $N^*$ resonances $\rightarrow$ reactions treated in the framework of the isobar model $\rightarrow$ all reactions proceed through an intermediate state containing at least one resonance. Examples:

$$N_1 + N_2 \rightarrow N'_1 + \Delta(1232) \rightarrow N'_1 + N'_2 + \pi$$
$$\pi + N \rightarrow \Delta(1600) \rightarrow \pi' + \Delta(1232) \rightarrow \pi' + \pi'' + N'$$
$$N_1 + N_2 \rightarrow \Delta_1(1232) + \Delta_2(1232) \rightarrow N'_1 + \pi_1 + N'_2 + \pi_2$$

Partial cross sections from one-boson exchange theories and/or folding of Breit-Wigner with matrix elements fixed by N-N scattering or expt. data. Full exploitation of isospin relations. Resonance energies and widths from data, whenever possible.
$K^{-} \bar{K}^{0}$ -nucleon interactions at medium-low energies

Multichannel analysis needed  Many partial waves contribute
Kaon nuclear potential non-negligible  Hyperons can be bound in nuclei

In FLUKA
Multichannel partial wave expansion

s wave at low momenta $^{1};$  \[ 0 < l < 5 \] up to 1.8 GeV/c $^{2}$
Isospin relations to link different charge states
Mass differences taken into account (charge exchange)

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Kaon-nucleon interactions examples

Inelastic hN at high energies: (DPM, QGSM, ...)

- Problem: “soft” interactions $\rightarrow$ no perturbation theory.
- Solution: Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string)
- Interactions treated in the Reggeon-Pomeron framework
- At sufficiently high energies the leading term corresponds to a Pomeron ($IP$) exchange (a closed string exchange)
- Each colliding hadron splits into two colored partons $\rightarrow$ combination into two color neutral chains $\rightarrow$ two back-to-back jets
- Physical particle exchange produce single chains at low energies
- Higher order contributions with multi-Pomeron exchanges important at $E_{lab} \geq 1$ TeV
**strings in MC codes**

**NON-exhaustive compilation:**

DPM is used in FLUKA, but also in

- **DPMJET:** h-A and A-A event generator (in FLUKA for A-A)
- **MARS** (FNAL) through DPMJET-III interface
- **MCNPX** (LOS ALAMOS) through very old (1989) FLUKA pieces
- **GEANT3** (CERN): GEANT-FLUKA and GEANT-CALOR through very old (1992) FLUKA pieces

**QGSM** (quark-gluon string model) in

- **SHIELD** (DUBNA)
- **GEANT4** (CERN) depending on physics list
**DPM and hadronization**

<table>
<thead>
<tr>
<th>from DPM:</th>
<th>Chain hadronization</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Number of chains</td>
<td>• Assumes chain universality</td>
</tr>
<tr>
<td>• Chain composition</td>
<td>• Fragmentation functions from hard processes and $e^+e^-$</td>
</tr>
<tr>
<td>• Chain energies and momenta</td>
<td>• Transverse momentum from uncertainty considerations</td>
</tr>
<tr>
<td>• Diffractive events</td>
<td>• Mass effects at low energies</td>
</tr>
<tr>
<td>Almost No Freedom</td>
<td>The same functions and (few) parameters for all reactions and energies</td>
</tr>
</tbody>
</table>
Invariant cross section spectra, as a function of Feynman $x_F^*$ of negative (left), and positive (right) pions emitted for $\pi^+$ on protons at various momenta. Data from M.E Law et al. LBL80 (1972).
Double differential cross section for $K^- p \rightarrow \Lambda X$ at 10 GeV/c (left), $p_T$ spectra of $\pi^+$ and $\pi^-$ produced by 16 GeV/c $\pi^-$ incident on an hydrogen target. Data from M.E Law et al. LBL80 (1972).
High energies: evolution of Fluka generators

Invariant cross section spectra, as a function of Feynman $x_F^*$ of negative, neutral, and positive pions emitted for $\pi^+$ on protons at 6 GeV/c. left: the old “FLUKA92”, right, the present FLUKA generator.
Glauber

★ Glauber cascade
- Quantum mechanical method to compute all relevant cross sections from hadron–nucleon scattering and nuclear ground state wave function
- Elastic, Quasi-elastic and Absorption $hA$ cross sections derived from Free hadron-Nucleon cross section + Nuclear ground state ONLY

★ Glauber-Gribov
- Field theory formulation of Glauber model
- Multiple collision terms $\Leftrightarrow$ Feynman graphs
- High energies: exchange of one or more Pomerons ($IP$) with one or more target nucleons (a closed string exchange)

No freedom, except in the treatment of mass effects at low energies.
Fermi motion included $\rightarrow$ smearing of $p_T$ distributions

(G)INC follows
(Generalized) IntraNuclear Cascade basic assumptions

1. Primary and secondary particles moving in the nuclear medium
2. Target nucleons motion and nuclear well according to the Fermi gas model
3. Interaction probability from $\sigma_{\text{free}} + \text{Fermi motion} \times \rho(r) + \text{exceptions (ex. } \pi)$
4. Glauber cascade at high energies
5. Classical trajectories (+) nuclear mean potential (resonant for $\pi$’s!!)
6. Curvature from nuclear potential $\rightarrow$ refraction and reflection.
7. Interactions are incoherent and uncorrelated
8. Interactions in projectile–target nucleon CMS $\rightarrow$ Lorentz boosts
9. Multibody absorption for $\pi, \mu^-, K^-$
10. Quantum effects (Pauli blocking, Formation zone, antisymmetrization, Nucleon-nucleon hard-core correlations, Coherence length)
11. Exact conservation of energy, momenta and all additive quantum numbers, including nuclear recoil
(non exhaustive list)
The ancestor: **BERTINI** model ($\approx$ 1970): no curvature, constant binding energy, no formation zone, no preequilibrium...

- Originally implemented in HETC (Oak Ridge), then in HERMES
- with improvements in LAHET and MCNPX
- in the CALOR version of GEANT3
- in GEANT4 (depending on physics list)

The **DUBNA INC** ($\approx$ 1972)

- in SHIELD
- in MARS through the CEM generator
- in MCNPX through the CEM generator

**BEWARE:** huge progresses from Bertini to full GINC !
Nucleon emission: BERTINI and GINC

The FLUKA nuclear module

★ a full GINC is implemented in FLUKA below 4 GeV, in the PEANUT module:

- **PEANUT** (PreEquilibrium Approach to NUclear Thermalization):
  GINC + preequilibrium stage + Equilibrium stage

- Nucleon, pion and kaon induced reactions
- Photonuclear reactions
- Stopping $\mu^-$ absorption
- Proton decay
- Neutrino interactions
Nucleon Fermi Motion

Fermi momentum depends on the local density

\[ k_{F}^{p,n}(r) = \left(3\pi^{2}\rho^{p,n}(r)\right)^{\frac{1}{3}} \]  

(1)

Nuclear density given by symmetrized Woods-Saxon for A>16,

\[ \rho(r) = \rho_{0} \frac{\sinh(R_{0}/a)}{\cosh(r/a) + \cosh(R_{0}/a)} \approx \frac{\bar{\rho}_{0}}{1 + \exp \frac{r-R_{0}}{a}} \]  

(2)

and by a harmonic oscillator shell model for light isotopes.

Simulated as 16 zones of constant density

Effect of nuclear and coulomb potentials also outside \( R_{nuc} \)

Proton and neutron densities are different (shell or droplet model)

Momentum smearing according to uncertainty principle
Test of Fermi motion: positive kaons

$K^+ \quad K^0$

in PEANUT: $\frac{d\sigma}{d\Omega}$ from phase shift analysis

$(K^+, K^0)$ on Pb vs residual excitation, 705 MeV/c, at $24^\circ$ and $43^\circ$.


On free nucleon: recoil energy:

43 MeV at $24^\circ$, 117 MeV at $43^\circ$. 

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Naively: “materialization” time. Qualitative estimate: in the frame where $p_{||} = 0$

$$\tilde{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}$$

particle proper time

$$\tau = \frac{M}{E_T} \tilde{t} = \frac{\hbar M}{p_T^2 + M^2}$$

Going to lab system

$$t_{lab} = \frac{E_{lab}}{E_T} \tilde{t} = \frac{E_{lab}}{M} \tau = \frac{\hbar E_{lab}}{p_T^2 + M^2}$$

As a function of particle rapidity $y$

$$t_{lab} = \tilde{t} \cosh y = \frac{\hbar}{\sqrt{p_T^2 + M^2}} \cosh y$$

Condition for possible reinteraction inside a nucleus:

$$v \cdot t_{lab} \leq R_A \approx r_0 A^{1/3}$$
Formation zone+ coherence length effect

Effect of different formation time ($\tau$) values on the charged hadron spectra and multiplicity in $\nu_\mu$ CC interactions.
Preequilibrium

For $E > \pi$ production threshold $\rightarrow$ only (G)INC models
At lower energies $\rightarrow$ a variety of preequilibrium models

Two leading approaches

- the quantum-mechanical multistep model
- the exciton model

- Very good theoretical background
- statistical assumptions
- complex, difficulties for multiple emission
- simple and fast

Exciton model: chain of steps, each ($n_{th}$) step corresponding to $N_n$
“excitons” $\Rightarrow$ either a particle above or a hole below the Fermi surface

Statistical assumption: any partition of the excitation energy $E$ among $N$, $N = N_h + N_p$, excitons has the same probability to occur

Step: nucleon-nucleon collision with $N_{n+1} = N_n + 2$ (“never come back” approximation)

Chain end = equilibrium $= N_n$ sufficiently high or excitation energy below threshold

$N_1$ depends on the reaction type and on the cascade history
Example of angle integrated $^{90}\text{Zr}(p,xn)$ at 80.5 MeV calculations with the full algorithm (right), and without the INC stage (left). The various lines show the total, INC, preeq. and evaporation contributions, the exp. data have been taken from M.Trabandt et al. **PRC39** (1989) 452.
Nonelastic interactions at intermediate energies: examples

Double differential distributions of charged pions produced by neutrons of $<E_n> = 383$ MeV (left) and 542 MeV (right).

Exp. data have been taken from Buchle et al. NPA515, (1990) 541
Double differential distributions of pions produced by 730 MeV protons. $\pi^-$’s from Be (left) and $\pi^+$ from Pb (right). Exp. data (symbols) have been taken from D.R.F. Cochran et al., PRD6, (1972)
Nonelastic interactions at intermediate energies: examples III

Double differential distributions of $\pi^+$'s produced by 1.6 GeV protons on Pb (left, exp. data from M.C.Lemaire et al., CEA-N-2670) and $\pi^-$ produced by 4 GeV/c protons on Al (right, exp data from H. En’yo et al, PL 159B, 1 (1985)).
Pion interactions in nuclei

The description of pion interactions on nuclei in the sub-GeV energy range must take into account:

- The resonant nature of the $\pi - N$ interaction, mostly dominated by the $\Delta(1232)$.
- The effect of the nuclear medium on the $\pi - N$ interaction
- The possibility of absorption (both s-wave and p-wave) on two or more nucleons
- The resonant nature of the pion-nucleus potential, which is rapidly varying with the pion energy
Pions: nuclear medium effects

Pion-nucleon interactions: non-resonant + p-wave resonant $\Delta$’s.

\[ \Delta \text{ in nuclear medium} \quad \text{decay} \]
\[ \text{reinteraction} \quad \text{elastic scattering or charge exchange} \quad \text{pion absorption} \]
\[ \rightarrow \Delta \text{ width different from the free one} \]

Assuming a Breit-Wigner for the free resonant cross section with width $\Gamma_F$

\[ \sigma_{res}^{\text{Free}} = \frac{8\pi}{p_{cm}^2} \frac{M_\Delta^2 \Gamma_F(p_{cm})^2}{(s - M_\Delta^2)^2 + M_\Delta^2 \Gamma_F(p_{cm})^2} \]

Add “in medium” width \( \frac{1}{2} \Gamma_T = \frac{1}{2} \Gamma_F - \text{Im} \Sigma_\Delta, \quad \Sigma_\Delta = \Sigma_{qe} + \Sigma_2 + \Sigma_3 \)

\( \Sigma_{qe}, \Sigma_2, \Sigma_3 = \text{widths for quasielastic scattering, two and three body absorption} \)
Add two-body s-wave absorption cross section from optical model
Pion absorption cross sections: examples

Computed and exp. pion absorption cross section on Aluminum as a function of energy

Computed and exp. pion absorption cross section on Gold or Bismuth as a function of energy

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Pion interactions: examples

Computed and exp. pion charge exchange angular distribution for $^{58}\text{Ni}(\pi^+, \pi^0)$ at 160 MeV

(Exp. data: W.J. Burger et al., PRC41, (1990) 2215, R.D. McKeown et al., PRC24, (1981) 211)
Pions in $\nu$ interactions

Charged pion spectra after $\nu_\mu$ interaction.

Pions are produced in $\nu$-nucleon interaction, and handled by PEANUT inside the nucleus
Initial State == particles still inside the nucleus
Final State == particles outside the nucleus
No reinteractions == only the effect of potentials
Only 55% escape at 1 GeV on Fe, 75% on Oxygen

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Equilibrium particle emission

- **Evaporation:** Weisskopf-Ewing approach
  - $\approx 600$ (in test!) possible evaporated particles/states ($A \leq 24$)
  - Full level density formula with level density parameter $A, Z$ and excitation dependent
  - Inverse cross-sections with proper sub-barrier
  - Analytic solution for the emission widths (neglecting the $a$ dependence on $U$, taken into account by rejection)
  - Emission energies from the width expression with no approx.

- **Fission:** improved version of the Atchison algorithm
  - Improved mass and charge widths
  - Full competition with evaporation

- **Fermi Break-up for $A \leq 17$ nuclei**
  - $\approx 50,000$ combinations included with up to 6 ejectiles

- **\(\gamma\) de-excitation:** statistical + rotational + tabulated levels
Nucleon emission: thin target examples I

Computed (light symbols) and experimental (symbols with lines) double differential distributions for $^{90}$Zr(p,xn) (left) and $^{90}$Zr(p,xp) at 80.5 MeV. The exp. data have been taken from M. Trabandt et al. PRC39 (1989) 452 and A.A. Cowley et al., PRC43, (1991) 678
Nucleon emission: the various effects
Nucleon emission: thin target examples II

Neutron transport below 20 MeV

All low energy neutron transport codes are based on DATABASES providing cross sections and INCLUSIVE distributions of reaction products. Two approaches: pointwise and multigroup.

The Multigroup technique

- Widely used in low-energy neutron transport programs (not only Monte Carlo, but also Discrete Ordinate codes)
- Energy range of interest divided in a given number of discrete intervals ("energy groups")
- Elastic and inelastic reactions simulated not as exclusive processes, but by group-to-group transfer probabilities (downscattering matrix)
- The scattering transfer probability between different groups represented by a Legendre polynomial expansion truncated at the $(N+1)^{th}$ term:
Low-energy neutrons in FLUKA

- ENEA multigroup cross-sections: 72 groups, $\approx 100$ elements/isotopes
- Gamma-ray generation, different temperatures, Doppler broadening, self-shielding
- Transport: standard multigroup transport with photon and fission neutron generation
- Detailed kinematics and recoil transport for elastic and inelastic scattering on hydrogen and for $^{14}N(n, p)$, $^{10}B(n, \alpha)$ and $^{6}Li(n, x)$
- Correlated capture gamma generation for selected isotopes
- Photons transported with EMF
- Kerma factors to calculate energy deposition
- Residual nuclei production
The FLUKA cross section library

- Prepared originally by experts of ENEA using a specialized code (NJOY) and several ad-hoc programs written to adjust the output to the particular structure of this library.
- Continuously enriched and updated on the basis of the most recent evaluations (ENDF/B, JEF, JENDL etc.)

**Materials**

- The library contains more than 140 different materials, selected for their interest in physics, dosimetry and accelerator engineering.
- The cross sections of some of the materials are available at two or three different temperatures (0, 87, and 293 K) for simulations of calorimeters containing cryogenic liquids or SC devices.
- Doppler broadening at the relevant temperature is taken into account.
Neutron detector calibration

Calibration of the LINUS rem counter (left) and of three Bonner spheres (right) with monoenergetic neutron beams at PTB–Braunschweig and with semi-monoenergetic neutron beams at PSI (full symbols), compared with simulations (dashed histos and open circles)