Rare Kaon Decays: a Theoretical Overview

Gino Isidori [*INFN-Frascati*]

A non-trivial challenge...

Is there something I can tell you
about rare K decays that you
have not already heard many times ?

Rare Kaon Decays: a ~~Theoretical~~ Personal Overview

Gino Isidori [*INFN-Frascati*]

The field of rare K decays is very alive: several new works on this subject in the last few months [12 preprints in the last 24 months; in particular, several new theoretical papers addressing both refinements in SM predictions & new beyond-SM expectations]...

.. but instead of discussing in detail these latest developments, I will concentrate on what I think is the *heart of the problem*, namely

Why we are still interested in rare K decays

After the large amount of data on *flavour physics* from *B* factories, and the great success of the SM in this sector,

given the tremendous experimental difficulties in measuring branching ratios below 10^{-10} (possibly with invisible final states...),

is it still worth to plan new experiments about rare K decays ?

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given the tremendous experimental difficulties in measuring branching ratios below 10^{-10} (possibly with invisible final states...),

is it still worth to plan new experiments about rare K decays ?

I will try to provide an articulated answer to this question, discussing:

- Why we (a certain number of crazy theoreticians...) are convinced that a few rare *K* decays are still very interesting
- Which are the decays modes which is still interesting to measure
- At which level of precision it would be useful to measure theses rare decays modes
- How this conclusions is affected by the developments at the high-energy frontier (LHC)

Why we are convinced that a few rare K decays are still very interesting:

- Because the SM cannot be the end of the story, and we have convincing indications that there are new degrees of freedom not too far from the electroweak scale
- Because we have not understood yet the underlying mechanism of flavour mixing

The information coming form rare K decays is a key element to understand the *flavour structure* of *physics beyond the SM*

The flavour sector of the SM:

$$\mathscr{L}_{\mathrm{SM}} = \mathscr{L}_{\mathrm{gauge}}(A_i, \psi_i) + \mathscr{L}_{\mathrm{Higgs}}(\phi_i, A_i, \psi_i ; Y, v)$$

3 identical replica of the basic fermion family
[$\psi_i = Q_L , U_R, D_R, L_L, E_R$]

huge flavour-degeneracy [$U(3)^5$ group]

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huge flavour-degeneracy [$U(3)^5$ group] broken by the Yukawa interaction:

$$Q_i \, Y_d^{ij} \, d_j \, \phi \rightarrow Q_i \, M_d^{ij} \, d_j$$

$$Q_i \, Y_u^{ij} \, u_j \, \phi_c \rightarrow Q_i \, M_u^{ij} \, d_j$$

$$M_d = \mathrm{diag}(m_d, m_s, m_b)$$

$$M_u^{+} = \mathrm{diag}(m_u, m_c, m_t) \times V_{CKM}$$

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huge flavour-degeneracy [$U(3)^5$ group] broken by the Yukawa interaction:

Nowadays we have a good knowledge
of all the 10 observables entries
[6 masses + 4 CKM angles]
of the quark mass matrices:

$$Q_i\, Y_d^{ij}\, d_j\, \phi \rightarrow Q_i \boxed{M_d^{ij}} d_j$$

$$Q_i\, Y_u^{ij}\, u_j\, \phi_c \rightarrow Q_i \boxed{M_u^{ij}} d_j$$

- strong hierarchical structure
- no clear symmetric pattern

$$M_d = \mathrm{diag}(m_d, m_s, m_b)$$

$$M_u^{+} = \mathrm{diag}(m_u, m_c, m_t) \times V_{\mathrm{CKM}}$$

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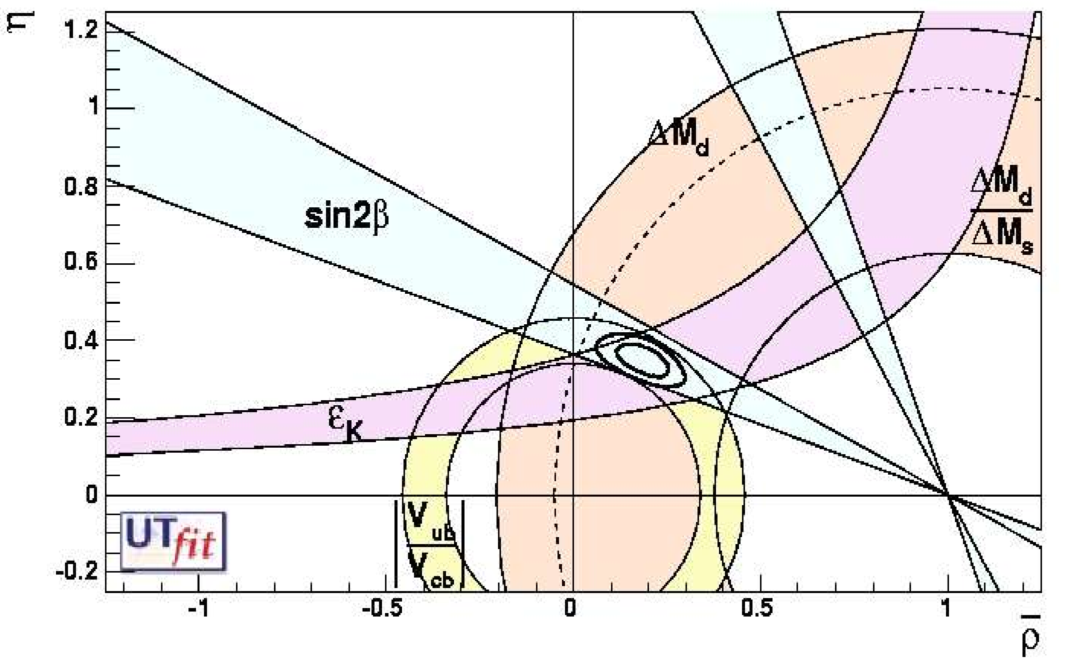
huge flavour-degeneracy [$U(3)^5$ group] broken by the Yukawa interaction

Nowadays we have a good knowledge of all the 10 observables entries [6 masses + 4 CKM angles] of the quark mass matrices:

$$V_{CKM} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$



The flavour sector beyond the SM:

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_i, \psi_i) + \mathscr{L}_{Higgs}(\phi_i, A_i, \psi_i; Y, v) + \boxed{\sum \frac{c_n}{\Lambda^{d-4}} O_n^{(d\geq5)}}$$

3 identical replica of the basic fermion family
[$\psi_i = Q_L, U_R, D_R, L_L, E_R$]

huge flavour-degeneracy [$U(3)^5$ group] broken by the Yukawa int.

Nowadays we have a good knowledge
of all the 10 observables entries
[6 masses + 4 CKM angles]
of the quark mass matrices

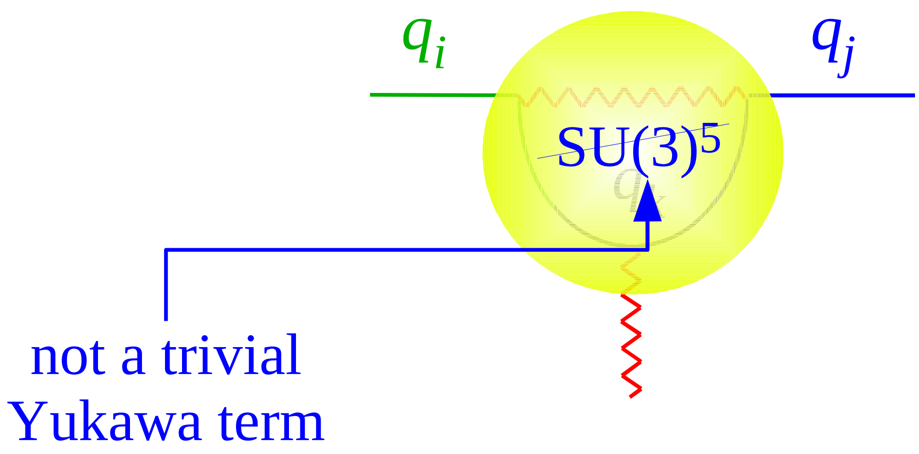
What we still need to investigate is the
flavour structure of the new degrees of freedom
which hopefully will show
up above the electroweak scale

several new sources of
$U(3)^5$ breaking are possible

Rare decays mediated by Flavor Changing Neutral Currents are the main experimental tool to probe the flavour structure of physics beyond the SM

$$q_i \to q_j + l^+ l^-,\ \nu\nu$$

- No SM tree-level contribution



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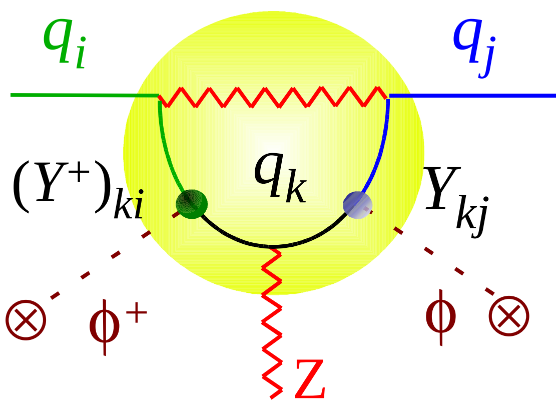
- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy
- Predicted with high precision within the SM if dominated by short-distance dynamics [*key point*]



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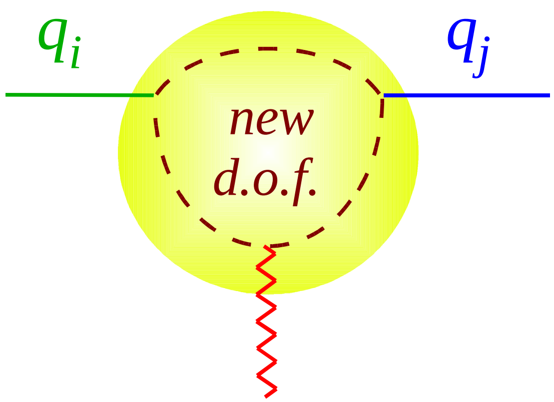


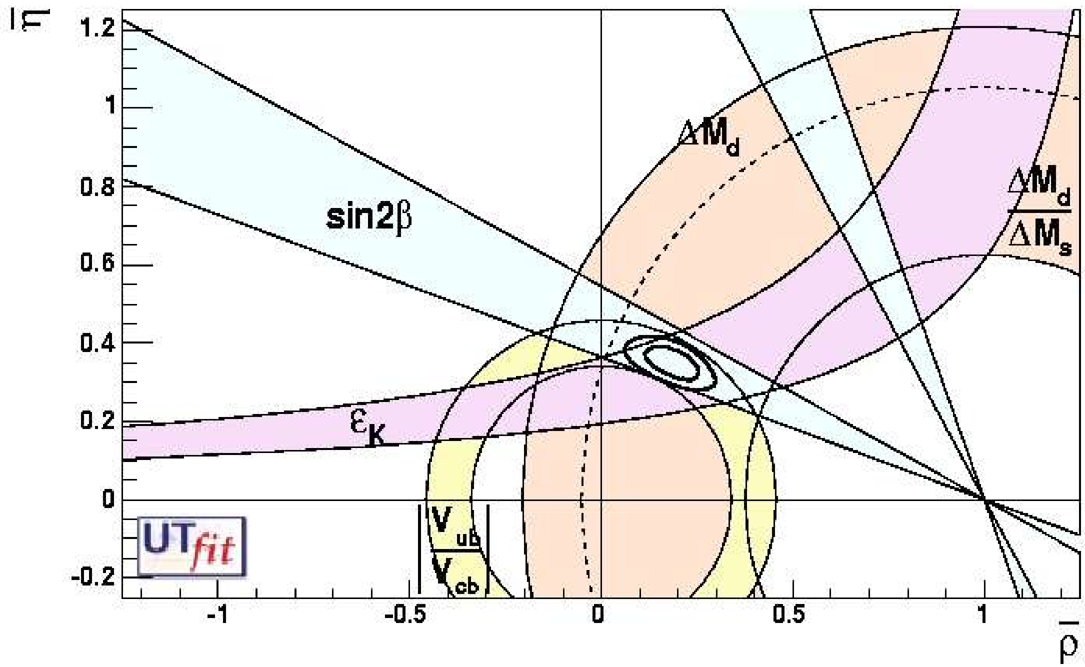
enhanced sensitivity to

[*the flavour structure of*]

physics beyond the SM

The flavour problem:

Precise data on loop-induced flavour-changing processes of $\Delta F=2$ type [K-$\bar{K}$ & B-$\bar{B}$ *mixing*] already provide stringent bounds on possible new degrees of freedom beyond the SM



$$\mathscr{L}_{eff} = \mathscr{L}_{\text{gauge}}(A_i, \psi_i) + \mathscr{L}_{\text{Higgs}}(\phi_i, A_i, \psi_i\,; Y, v) + \boxed{\sum \frac{c_n}{\Lambda^{d-4}} O_n^{(d\geq 5)}}$$

E.g.: K^0-$\bar{K}^0$ mixing $\Rightarrow$ $\Lambda > 10^3$ TeV for $O^{(6)} \sim (\bar{s}d)^2$

...while a natural stabilization of the Higgs sector $\Rightarrow$ $\Lambda \sim 1$ TeV

Two possible solutions:

- *pessimistic* [very unnatural]: $\Lambda > 100$ TeV [the nightmare of LHC...]

  $\Rightarrow$ rare decays not necessarily sensitive to NP, but potentially more interesting than LHC: on pure dimensional grounds a 10% meas. of $B(K_L \to \pi^0 \nu\nu)$ would probe NP scales around 1000 TeV !

- *natural*: $\Lambda \sim 1$ TeV + flavor-mixing protected by additional symmetries

  $\Rightarrow$ still a lot to learn from rare decays:

  - present fits of the CKM unitarity triangle involve only $\Delta F=2$ loops + tree-level amplitudes $\Rightarrow$ present knowledge about rare $\Delta F=1$ FCNC transitions is still very limited

  - CKM fits provide mainly a consistency check of the SM hypothesis but do not provide a bound on the NP parameter space $\Rightarrow$ only with the help of rare decays we can study the underlying flavour symmetry in a model-independent way

Towards a model independent approach to the flavour problem:

Anatomy of a typical $O_i^{(6)}$ relevant to FCNC rare decays:

$$Q_\gamma^{bs} \;=\; W_\gamma^{bs}\; \underbrace{D_R^{\,b}\; \sigma_{\mu\nu} F^{\mu\nu}\, H\, Q_L^{\,s}} \;\sim\; m_b\, b_R\, \sigma_{\mu\nu} F^{\mu\nu}\, s_L$$

flavour coupling

e.g.: $W_\gamma^{bs} \sim y_b\, y_t^2\, V_{tb}^{\,*}\, V_{ts}$
for the SM short-distance contr.

The most restrictive choice is
the so-called MFV hypothesis

= same CKM / Yukawa
suppression as in the SM

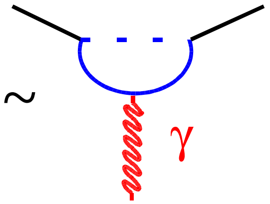
it cannot be worse than this without
serious fine-tuning problems

[Chivukula & Georgi, '86; Buras *et al*. '00;
D'Ambrosio, Giudice, G.I., Strumia '02]

flavour-blind electroweak structure

Limited number of independent
terms once we impose
$SU(3)_c \times SU(2)_L \times U(1)_Y$
gauge invariance

closely related to specific
loop topologies, e.g.:

$D_R \sigma_{\mu\nu} F^{\,\mu\nu} H\, Q_L \quad \sim$ 

Towards a model independent approach to the flavour problem:

th. error $\lesssim$ 10%
(grey) = exp. error $\lesssim$ 10%
(white) = exp. error ~ 30%

FLAVOUR COUPLING:

ELECTROWEAK STRUCTURE	$b \to s$ $(\sim\lambda^2)$	$b \to d$ $(\sim\lambda^3)$	$s \to d$ $(\sim\lambda^5)$
ΔF=2 box	ΔM_{Bs}, $A_{CP}(B_s \to \psi\phi)$	ΔM_{Bd}, $A_{CP}(B_d \to \psi K)$	ΔM_K, ε_K
ΔF=1 4-quark box	$B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to \pi\pi$, $B_d \to \rho\pi$, ...	ε'/ε, $K \to 3\pi$, ...
gluon penguin	$B_d \to X_s \gamma$, $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d \gamma$, $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, ...
γ penguin	$B_d \to X_s l^+ l^-$, $B_d \to X_s \gamma$, $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d l^+ l^-$, $B_d \to X_d \gamma$, $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, ...
Z^0 penguin	$B_d \to X_s l^+ l^-$, $B_s \to \mu\mu$, $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d l^+ l^-$, $B_d \to \mu\mu$, $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, $K \to \pi\nu\nu$, $K \to \mu\mu$, ...
H^0 penguin	$B_s \to \mu\mu$	$B_d \to \mu\mu$	

<u>Mandatory to explore this corner of the table!</u>

Which are the decays modes which is still very interesting to measure:

The four golden modes:

$K^+ \to \pi^+ \nu\nu$ $\quad$ $K_L \to \pi^0 e^+e^-$

$K_L \to \pi^0 \nu\nu$ $\quad$ $K_L \to \pi^0 \mu^+\mu^-$

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General properties of $K \to \pi + (ll, \nu\nu)$ decay amplitudes:

I. Clean electroweak short-distance amplitude

[similar -within the SM- for all the channels]

Here is where NP can show up

II. Long-distance contributions due to light quarks

- potentially large effects of e.m. origin in $K \to \pi\, ll$
  [but under good th. control in the $K_L \to \pi^0$ case]
- small effects in $K \to \pi\, \nu\nu$ modes
  [totally negligible in the $K_L \to \pi^0$ case]

These are the contributions which can *obscure* possible NP effects

I. The clean electroweak short-distance amplitude

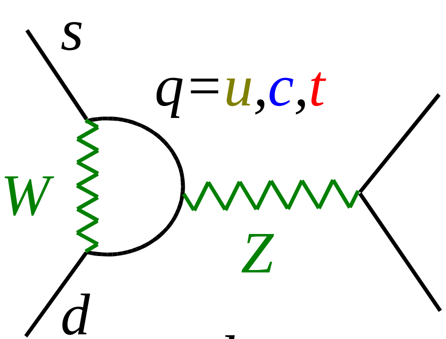
Electroweak penguins and box diagrams determine the *initial conditions* of the effective FCNC Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \Sigma_i \, C_i(M_W) \, Q_i$$

$$Q_\nu = (s\,d)_{V-A} (\nu\,\nu)_{V-A}$$
$$Q_{9V} = (s\,d)_{V-A} (l\,l)_{V}$$
$$Q_{10A} = (s\,d)_{V-A} (l\,l)_{A}$$

The $O(G_F^2)$ Z-penguin and box diagrams are subject to a *power-like* GIM mechanism

$\Rightarrow$ scale-independent amplitude dominated by the top-quark exchange:



+ box

$$\Rightarrow C_i(M_W) \sim m_q^2 \underbrace{V_{qs}^* V_{qd}}_{\lambda_q} \sim \begin{cases} \Lambda_{QCD}^2 \, \lambda & (u) \\ m_c^2 \, \lambda + i \, m_c^2 \, \lambda^5 & (c) \\ m_t^2 \, \lambda^5 + i \, m_t^2 \, \lambda^5 & (t) \end{cases}$$

- QCD corr. small and known beyond LO
- large CPV-phase

[$\lambda = \sin \theta_c$]

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$$Q_\nu = (s\,d)_{V-A}(\nu\,\nu)_{V-A}$$
$$Q_{9\,V} = (s\,d)_{V-A}(l\,l)_{V}$$
$$Q_{10\,A} = (s\,d)_{V-A}(l\,l)_{A}$$

- Hadronic matrix element: $\langle\, \pi \,|\, (sd)_{V\text{-}A} \,|\, K \,\rangle$ known (from K_{l3}) with excellent accuracy
- Lepton pair in a CP eigenstate: the contrib. of $\mathcal{H}_{\rm eff}$ to $K_L \to \pi^0 + ll\ (\nu\nu)$ is CPV

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Electroweak penguins and box diagrams determine the *initial conditions* of the effective FCNC Hamiltonian:

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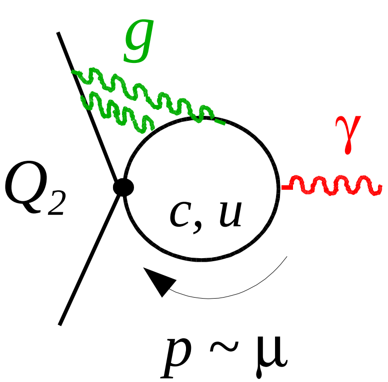
QCD corrections below the e.w. scale

[RGE]

mixing with 4-quarks operators:



large effect in CPC γ-penguin amplitudes

$$\mathcal{H}_{\text{eff}} = \Sigma_i \, C_i(\mu \sim 1 \text{ GeV}) \, Q_i$$

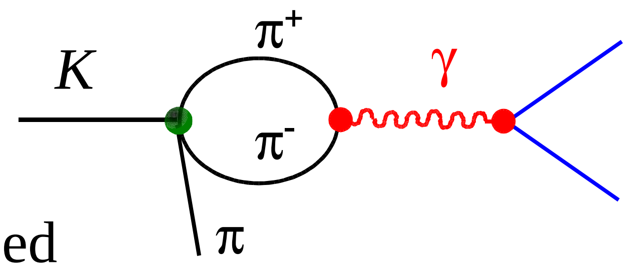
NB.: the mixing with 4-quark operators *dilute* the interesting short-distance info

- Negligible corrections for $\text{Im}(C_\nu)$ — $K_L \to \pi^0 \nu\nu$
- Small & calculable [*charm loops*] for $\text{Re}(C_\nu)$ — $K^+ \to \pi^+ \nu\nu$ $\quad K_S \to \pi^0 \nu\nu$
- Small & calculable [*charm loops*] for $\text{Im}(C_{9V})$ — $K_L \to \pi^0 l^+ l^-$
- Huge and not stable [*true long distance*] for $\text{Re}(C_{9V})$ — $K^+ \to \pi^+ l^+ l^-$ $\quad K_S \to \pi^0 l^+ l^-$

II.a The e.m. long-distance amplitude in $K \to (\pi)\, ll$ modes

Qualitative picture:

A) $K^{\pm}(K_S) \to \pi^{\pm}(\pi^0) l^+ l^-$

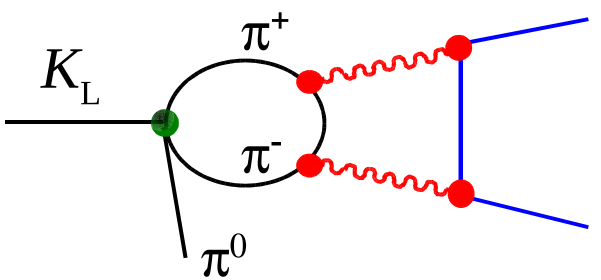


$A_{\text{short}}/A_{\text{long}} \sim 10^{-2}$

One-photon exchange not suppressed

$\Rightarrow$ hopeless to disentangle short–distance effects

B) $K_L \to \pi^0 l^+ l^-$



$A_{\text{short}}/A_{\text{long}} \gtrsim 1$

One-photon exchange suppressed by CP invariance

$\Rightarrow$ possible to perform precision tests of short-distance dynamics ?

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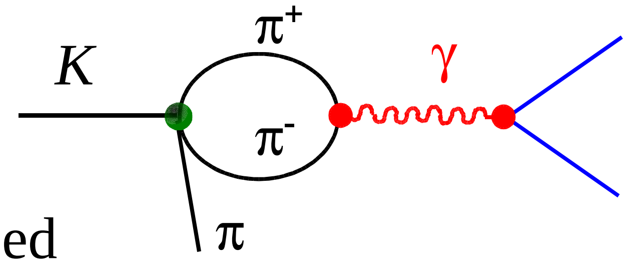

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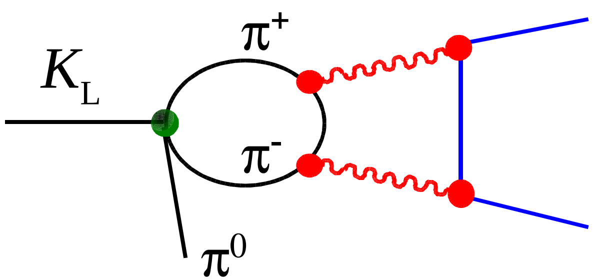


$A_{\text{short}}/A_{\text{long}} \gtrsim 1$

One-photon exchange suppressed
by CP invariance

$\Rightarrow$ possible to perform precision
tests of short-distance dynamics ? YES !

A rare example of a finite,
counterterm-free amplitude
at the two-loop level in CHPT

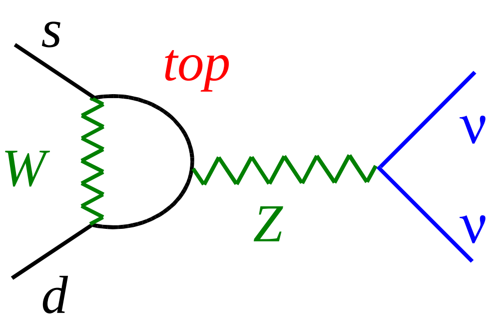
Buchalla, D'Ambrosio, G.I. '03
Friot, Grenat, de Rafael '04

G.I., Smith, Unterdorfer '04

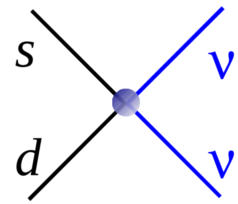
*Quantitative analysis possible by
means of low-energy EFT approaches (CHPT):*

II.b Light-quark loops (& power-corrections) in $K \to \pi\nu\nu$

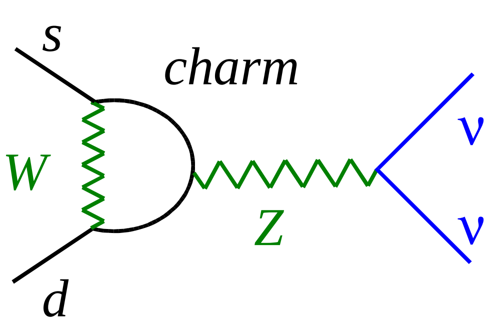
Qualitative picture:



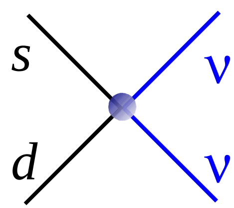
$\lambda_t G_F^{\,2} m_t^{\,2} Q_\nu^{(6)}$



The *power-like* GIM mechanism holds only for the leading dim-6 operators ⇒ non-negligible *scale-dependent* effect due to dim-8 ops in the charm case:

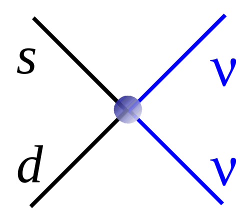


$\lambda_c G_F^{\,2} m_c^{\,2} Q_\nu^{(6)}$



+

$\lambda_c G_F^{\,2} Q_\nu^{(8)} \longrightarrow \sim (s\partial d)(\nu\partial\nu)$



In the $K^+ \to \pi^+\nu\nu$ case:

30% of the leading top amplitude

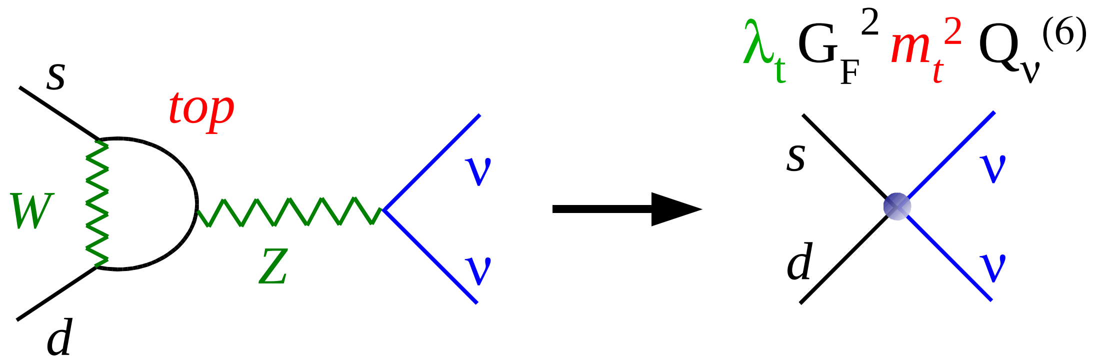
$O(q^2/m_c^{\,2}) \sim$ 10-20 % of the dim-6 charm contribution

Falk *et al.* '00

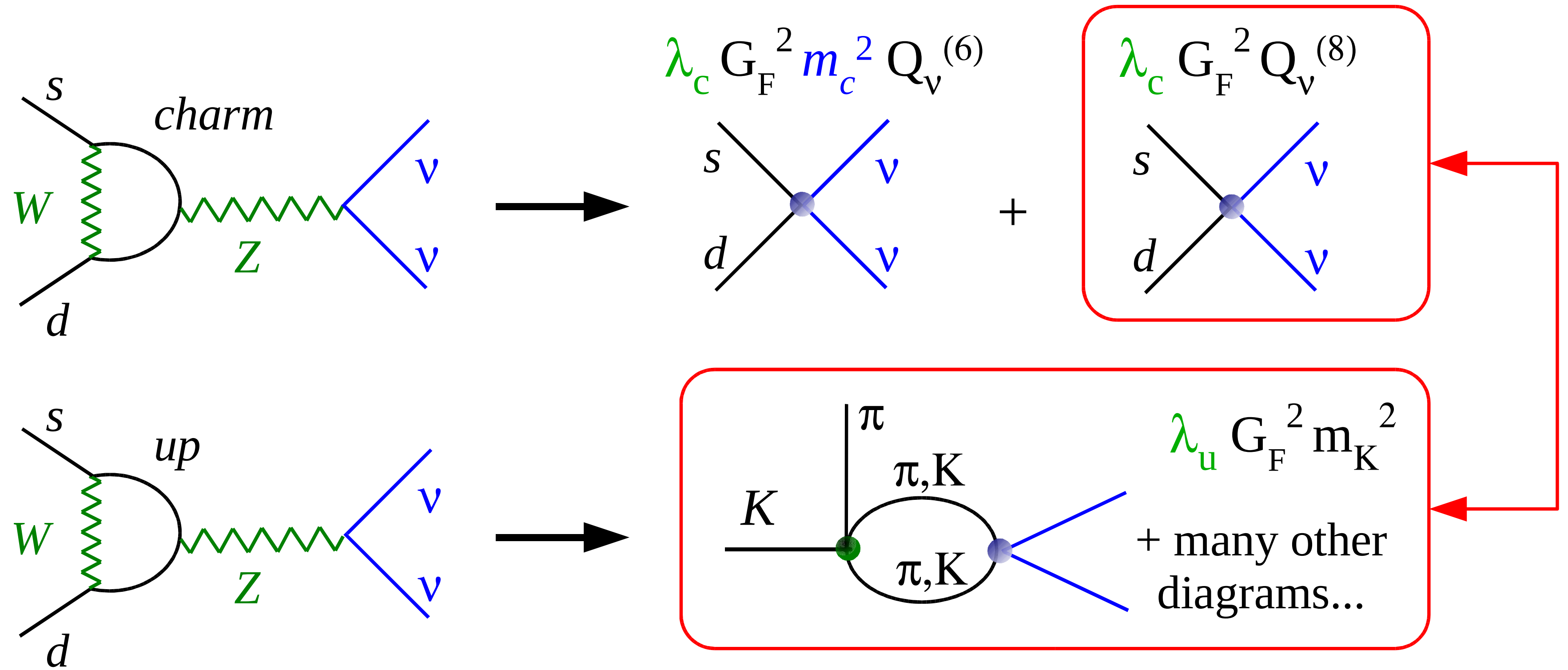
Aiming at few % precision we need to control the $O(q^2/m_c^{\,2})$ corrections

II.b Light-quark loops (& power-corrections) in $K \to \pi\nu\nu$

Qualitative picture:



The *scale-dependence* induced by the dim-8 ops. should match the one appearing in the genuine *long-distance component* of the amplitude



Quantitative analysis possible within CHPT

G.I, Mescia, Smith '05

II.b Light-quark loops (& power-corrections) in $K \to \pi\nu\nu$

An important point about Z-mediated FCNCs in CHPT:

- At $O(G_F^2)$ the Z current cannot be treated as an external field in the weak chiral Lagrangian: the $SU(2)_L \times U(1)_Y$ breaking allows a new chiral structure already at $O(p^2)$ [because of this missing term, all previous attempts to evaluate long-distance effects in $K \to \pi\nu\nu$ were not correct]
- The leading new chiral operator can be fixed by an appropriate matching condition with the short-distance partonic Hamiltonian

Numerical results:

- The dominant effect is due to the long-distance component of the Z penguin ($\Delta I=1/2$ enhancement), which scale as $O(\pi^2 F_\pi^2/m_c^2) \approx 10\%$ with respect to the dim-6 charm contribution

$$P_c \rightarrow P_c + \delta P_c$$

$$\left[\begin{array}{l} P_c = 0.39 \pm 0.06 \quad [\text{ @ NLO }] \\ \delta P_c = 0.04 \pm 0.02 \end{array}\right.$$

G.I, Mescia, Smith '05

▶ At which level of precision it would be useful to measure the 4 golden modes:

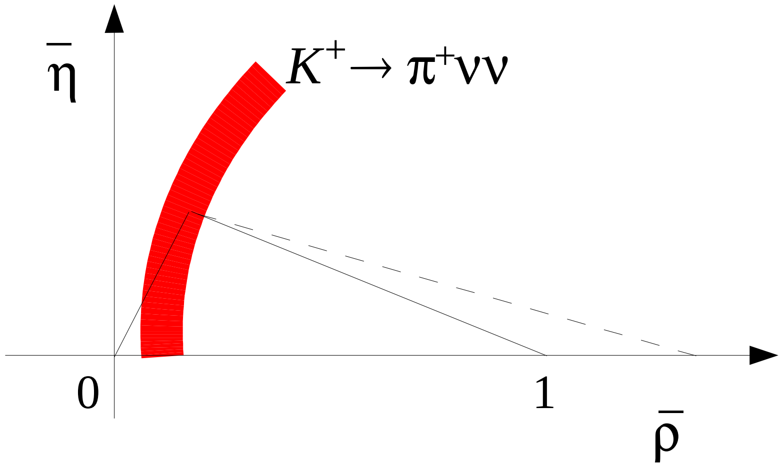
$\boxed{K^+ \to \pi^+ \nu\nu}$ $\quad \mathrm{BR}(K^+)^{[\mathrm{SM}]} = C_+ \, |V_{cb}|^4 \, [(\bar\rho - \rho_c)^2 + (\sigma\bar\eta)^2] = (8.0 \pm 1.0) \times 10^{-11}$

sizable fraction of the present error still due to parametric CKM uncertainties

Dominant error due to the perturbative charm contribution (dim.-6) presently known at NLO :

$\rho_c = 1.27 \pm 0.04 \quad \Rightarrow \quad \delta\, \mathrm{BR}_{th} \approx 8\%$

Buchalla & Buras, '97-'99
Misiak & Urban, '99



At which level of precision it would be useful to measure the 4 golden modes:

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sizable fraction of the present error still due to parametric CKM uncertainties

Dominant error due to the perturbative charm contribution (dim.-6) presently known at NLO :

$\rho_c = 1.27 \pm 0.04 \quad \Rightarrow \quad \delta\,\mathrm{BR}_{th} \approx 8\%$

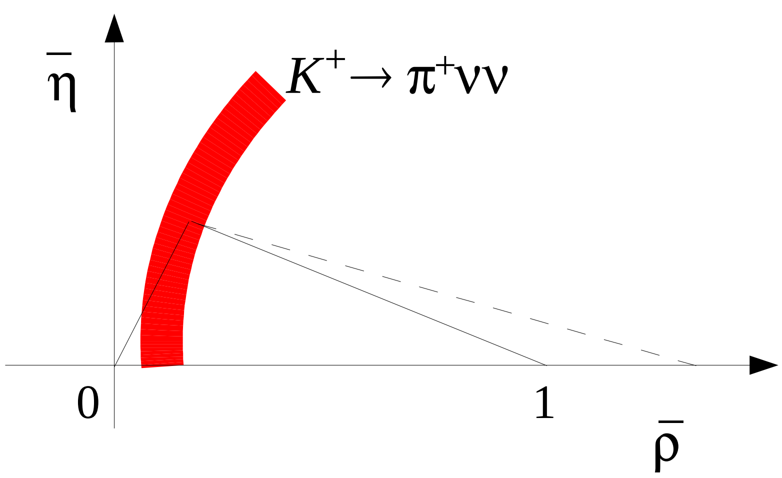
Buchalla & Buras, '97-'99
Misiak & Urban, '99

on-going theoretical activity to substantially reduce this error with a NNLO calculation

Buras et al. [Munich - FNAL]

on-going theoretical activity to substantially reduce this error with Lattice calculations

G.I., Martinelli, Turchetti, '05



→ ultimate th. precision of ~ 2% on BR possible within a few years

At which level of precision it would be useful to measure the 4 golden modes:

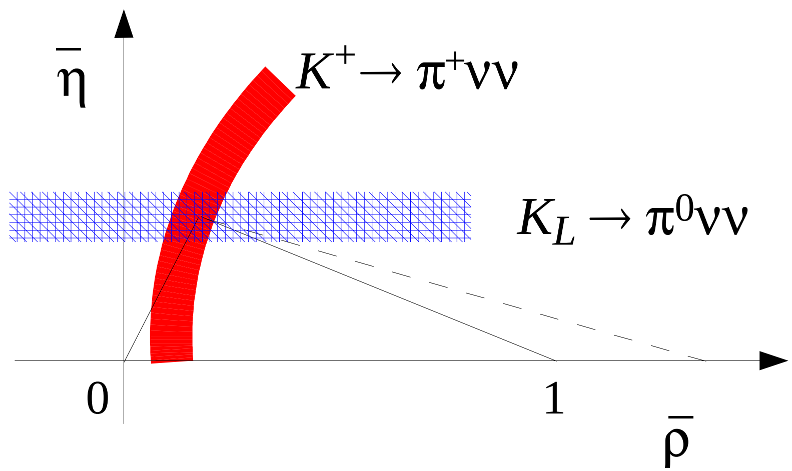
$\boxed{K^+ \to \pi^+ \nu\nu}$ $\quad \mathrm{BR}(K^+)^{[SM]} = C_+ \, |V_{cb}|^4 \, [(\bar\rho - \rho_c)^2 + (\sigma\bar\eta)^2] = (8.0 \pm 1.0) \times 10^{-11}$

$\boxed{K_L \to \pi^0 \nu\nu}$ $\quad \mathrm{BR}(K_L)^{[SM]} = C_0 \left[\frac{\mathrm{Im}(V_{ts}{}^* V_{td})}{10^{-4}} \right]^2 = (3.0 \pm 0.6) \times 10^{-11}$

Littenberg, '89
Buchalla & Buras '97
Buchalla & G.I. '98

irreducible th. error already @ 2% !

control the amount of CPV within the SM



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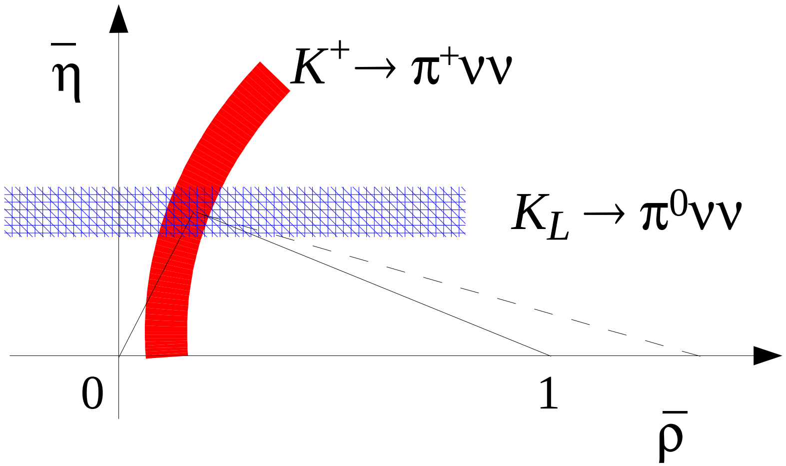
At this level of th. precision, the parametric uncertainty on m_t & V_{td} is likely to be the dominant source of uncertainty

Certainly worth to push the experimental sensitivity on both modes at least down to the 5% level



$\boxed{K_L \to \pi^0\, l^+ l^-}$

The 3 components of the $K_L \to \pi^0\, l^+ l^-$ amplitude:

A. direct CPV amplitude

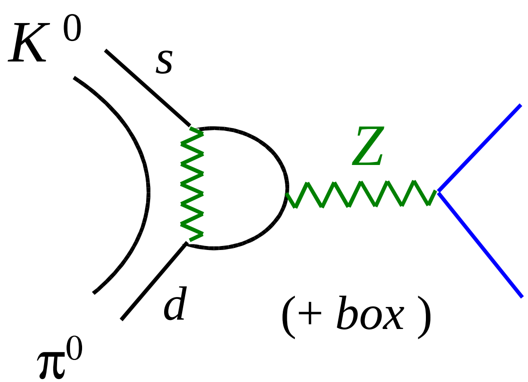
- short-distance dominated
- very similar to $K_L \to \pi^0\, \nu\nu$

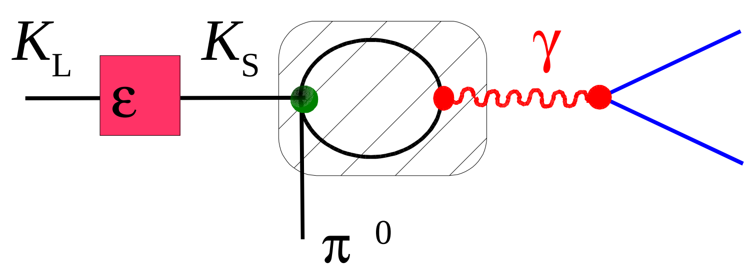
interference

B. indirect CPV

- determined by $\boxed{K_S \to \pi^0\, l^+ l^-}$
  + theory to fix the sign

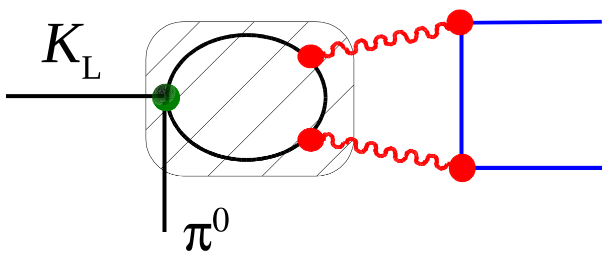
need exp. input





C. CPC amplitude

- no interference & different Dalitz plot
- predicted by theory with good accuracy
  in terms of rate & spectrum of $\boxed{K_L \to \pi^0\, \gamma\gamma}$



need exp. input

$K_L \to \pi^0 l^+ l^-$

Thanks to some recent results by NA48-NA48/1:

$$B(K_S \to \pi^0 e^+ e^-)_{m_{ee} > 165\ \mathrm{MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9}$$

$$B(K_S \to \pi^0 \mu^+ \mu^-) = (2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9}$$

$$B(K_L \to \pi^0 \gamma\gamma)_{m_{\gamma\gamma} < 110\ \mathrm{MeV}} < 0.9 \times 10^{-8}$$

\+

Some related th. works:

Buchalla, D'Ambrosio, G.I. '03
Friot, Grenat, de Rafael '04

G.I., Smith, Unterdorfer '04

We finally have a clear picture of the various terms:

$$B(K_L \to \pi^0 l^+ l^-)^{[SM]} = [\ C_{mix} + C_{int}\, y_t + C_{dir}\, y_t^2 + C_{CPC}\ ] \times 10^{-12} \qquad y_t = \frac{\mathrm{Im}(V_{ts}^* V_{td})}{10^{-4}}$$

$$(e^+e^-) \approx 23 + (10 + 4) + 0 \Rightarrow (3.7 \pm 1.0) \times 10^{-11}$$

$$(\mu^+\mu^-) \approx 5.4 + (2.5 + 1.8) + 5.2 \Rightarrow (1.5 \pm 0.3) \times 10^{-11}$$

$B(K_L \to \pi^0 e^+ e^-)^{[SM]} = (3.7 \pm \boxed{1.0}) \times 10^{-11}$ [≈ 40% due to short dist.]

$B(K_L \to \pi^0 \mu^+ \mu^-)^{[SM]} = (1.5 \pm \boxed{0.3}) \times 10^{-11}$ [≈ 30% due to short dist.]

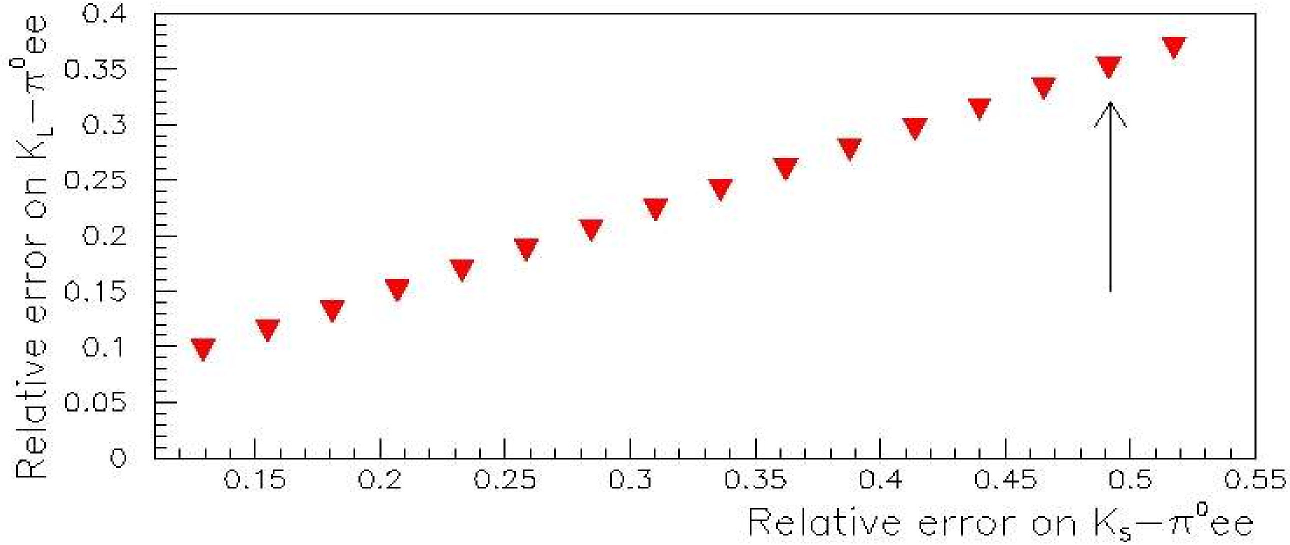
<u>Irreducible theoretical error below 10%</u>

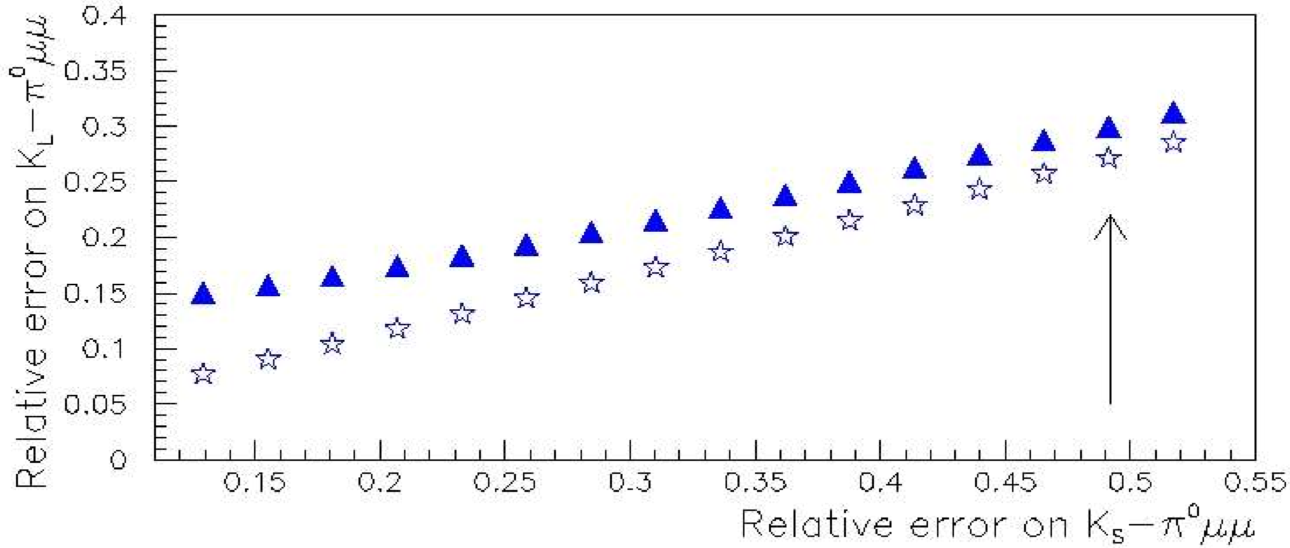
present large errors due to the large exp. uncertainty on $B(K_S \to \pi^0 l^+ l^-)$:

$B_S(e^+e^-) \approx (6.0 \pm 2.9) \times 10^{-9}$

$B_S(\mu^+\mu^-) \approx (2.9 \pm 1.4) \times 10^{-9}$

NA48/1 '03-'04





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$B(K_L \to \pi^0 \mu^+\mu^-)^{[SM]} = (1.5 \pm 0.3) \times 10^{-11}$ [≈ 30% due to short dist.]

Errors on SM predictions dominated by the large (exp.) uncertainty on $B(K_S \to \pi^0\, l^+l^-)$, but irreducible theoretical error below 10%

⇕

$B(K_L \to \pi^0 e^+e^-)^{\exp} < 2.8\times10^{-10}$ [90% CL] KTeV '03

$B(K_L \to \pi^0 \mu^+\mu^-)^{\exp} < 3.8\times10^{-10}$ [90% CL] KTeV '00

not too far...

⇓

Very interesting candidates for future dedicated experiments

- More observables to be studied [Dalitz plot]
- Different sensitivity to NP with respect to $K_L \to \pi^0 \nu\nu$

the 3 decay modes $K_L \to \pi^0 + e^+e^-, \mu^+\mu^-, \nu\nu$
are sensitive to different short-distance structures
⇒ 3 independent info on CPV beyond the SM

$$Q_\nu = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$
$$Q_{9V} = (\bar{s}d)_{V-A}(\bar{l}l)_V$$
$$Q_{10A} = (\bar{s}d)_{V-A}(\bar{l}l)_A$$

Rare K decays beyond the SM and the connection with the high-energy frontier (LHC)

Within the natural solution of the flavour (+hierarchy) problem:

$\Lambda \sim 1$ TeV & flavor-mixing is protected by additional symmetries

As long as we are interested only in low-energy rare processes, the most important feature of the NP model is the nature of this symmetry

▶ Rare K decays beyond the SM and the connection with the high-energy frontier (LHC)

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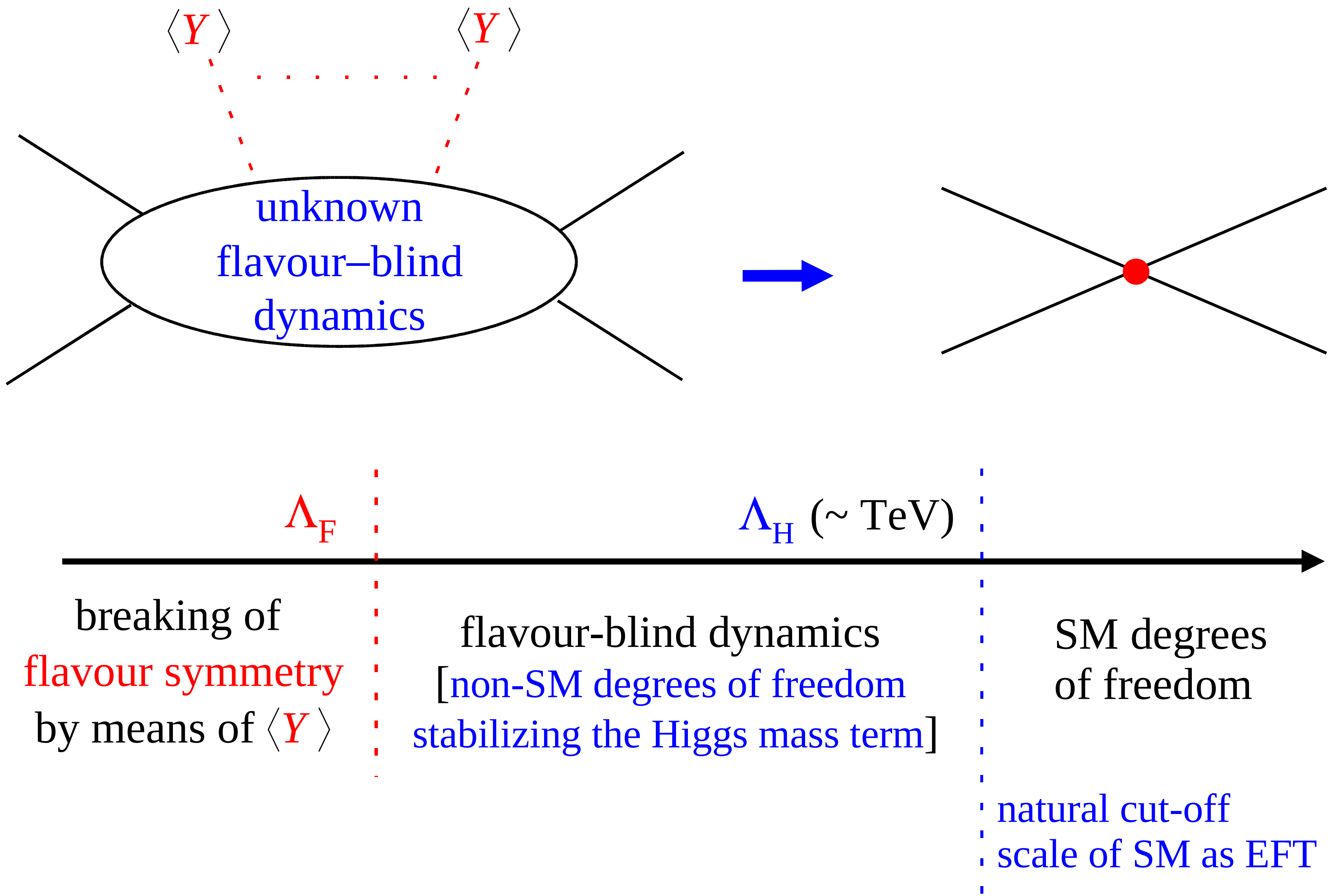
most restrictive possibility

Minimal Flavour Violation (MFV) hypothesis:

The breaking of the flavour symmetry occurs at very high scales and is mediated at low energies only by terms prop. to SM Yukawa couplings

- natural implementation in many consistent scenarios [SUSY, technicolour, extra dimensions,...]
- possible to build a predictive low-energy EFT model-independent approach

Chivukula & Georgi, '86
D'Ambrosio, Giudice, G.I., Strumia '02



The MFV hypothesis can be considered as the <u>most pessimistic scenario</u>:

$\Rightarrow$ deviations from the SM in rare K decays bounded by flavour-conserving e.w. precision observables and/or rare B decays

$\Rightarrow$ deviations from the SM in rare K decays bounded by flavour-conserving e.w. precision observables and/or rare B decays :

- Even within this pessimistic NP scenario, up to O(50%) deviations from SM are still possible in $B(K \to \pi \nu\nu)$ and $B(K_L \to \pi^0 e^+ e^-)$

Bona *et al.* '05

- O(10%) measurements of both $B(K \to \pi \nu\nu)$ would probe a NP parameter space not accessible by any other experiment in the field of flavour physics

$\Downarrow$

Key information to prove the validity of the MFV hypothesis

th. error $\lesssim 10\%$
= exp. error $\lesssim 10\%$
= exp. error $\sim 30\%$

FLAVOUR COUPLING:

ELECTROWEAK STRUCTURE

	$b \to s$ $(\sim\lambda^2)$	$b \to d$ $(\sim\lambda^3)$	$s \to d$ $(\sim\lambda^5)$
$\Delta F=2$ box	ΔM_{Bs} $A_{CP}(B_s \to \psi\phi)$	ΔM_{Bd} $A_{CP}(B_d \to \psi K)$	ΔM_K, ε_K
$\Delta F=1$ 4-quark box	$B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to \pi\pi$, $B_d \to \rho\pi$, ...	ε'/ε, $K \to 3\pi$, ...
gluon penguin	$B_d \to X_s\, \gamma$, $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d\, \gamma$, $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, ...
γ penguin	$B_d \to X_s\, l^+ l^-$, $B_d \to X_s\, \gamma$ $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d\, l^+ l^-$, $B_d \to X_d\, \gamma$ $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, ...
Z^0 penguin	$B_d \to X_s\, l^+ l^-$, $B_s \to \mu\mu$ $B_d \to \phi K$, $B_d \to K\pi$, ...	$B_d \to X_d\, l^+ l^-$, $B_d \to \mu\mu$ $B_d \to \pi\pi$, ...	ε'/ε, $K_L \to \pi^0 l^+ l^-$, $K \to \pi\nu\nu$, $K \to \mu\mu$, ...
H^0 penguin	$B_s \to \mu\mu$	$B_d \to \mu\mu$	

Beyond Minimal Flavour Violation

[new sources of flavour symmetry breaking at the TeV scale]

- A priori the most natural possibility
  naturally appearing in several specific scenarios [e.g. SUSY: huge literature]
- challenged -at present- by the good agreement with SM in $\Delta F=2$ sector, but still room for sizable effects

Beyond Minimal Flavour Violation

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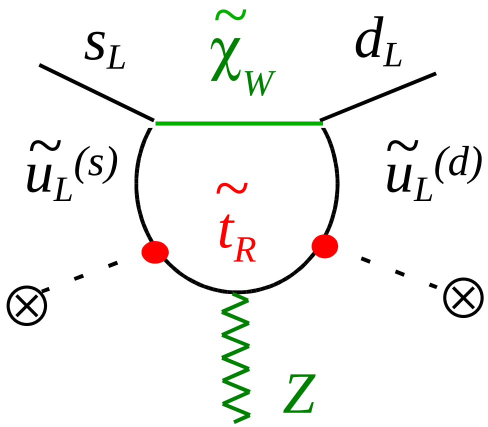
General features:



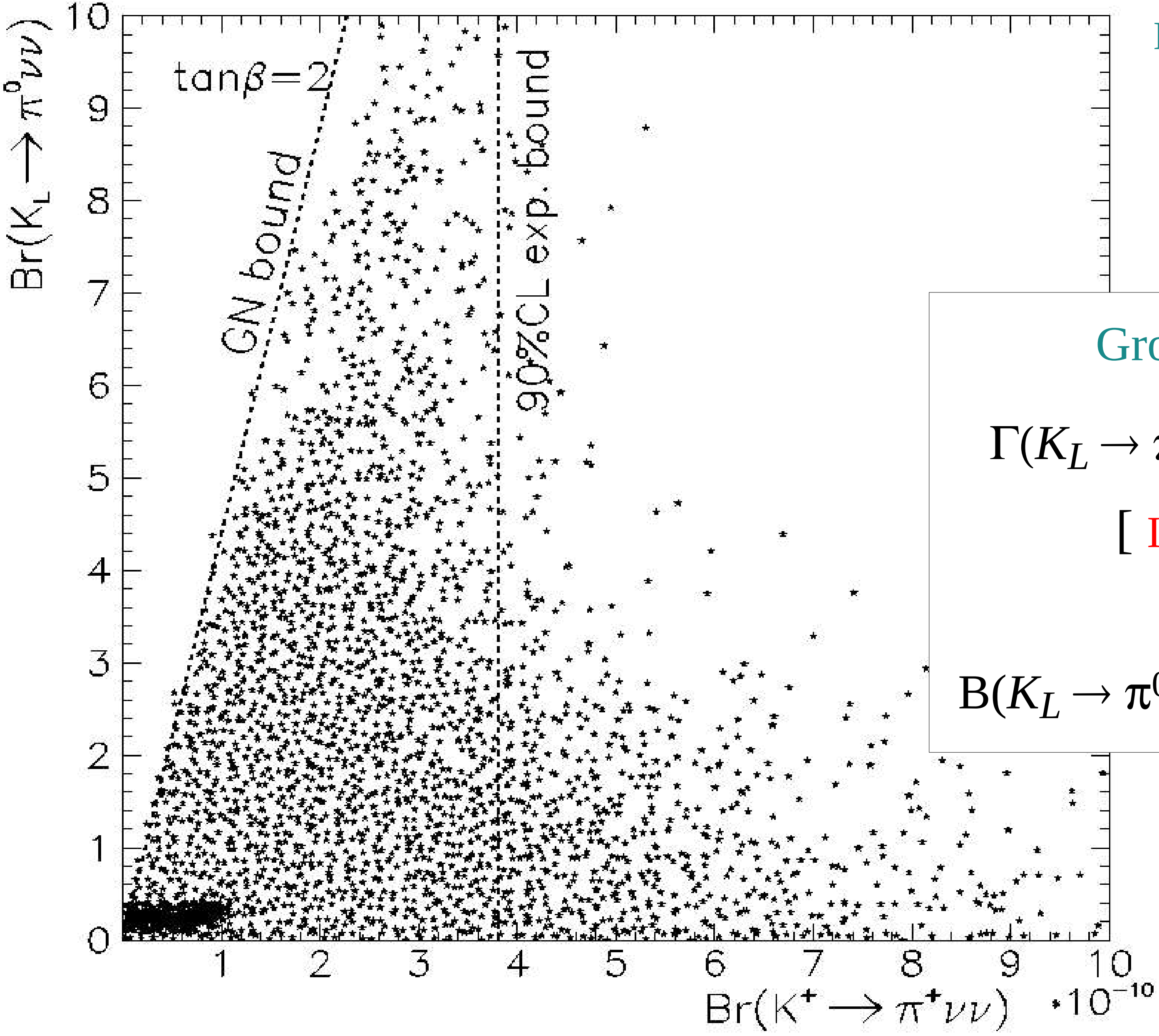
- Some decoupling between $\Delta F=2$ & $\Delta F=1$
  [i.e.: $\delta_{NP}(\Delta F=1) \sim 100\%$ vs. $\delta_{NP}(\Delta F=2) \sim 10\%$]
  possible thanks to the interplay between $SU(2)_L \cdot U(1)$ & flavour symm. breaking

Colangelo & G.I. '98,
Nir & Worah '97;
Buras, Romanino & Silvestrini, '97

- Rare kaon decays are particularly sensitive to new sources of flavour symm. breaking because of the severe CKM suppression [$V_{ts}^{*}V_{td} \sim \lambda^5$]

E.g.: B($K\to\pi\nu\nu$) within generic MSSM

[including all the present constraints from ε_K, ΔM_K, $b\to s\gamma$, ...]



Buras *et al.* '04

Grossman-Nir bound:

$$\Gamma(K_L\to\pi^0\nu\nu) < \Gamma(K^+\to\pi^+\nu\nu)$$

$$[\ \mathrm{Im}(A) < |A|\ ]$$

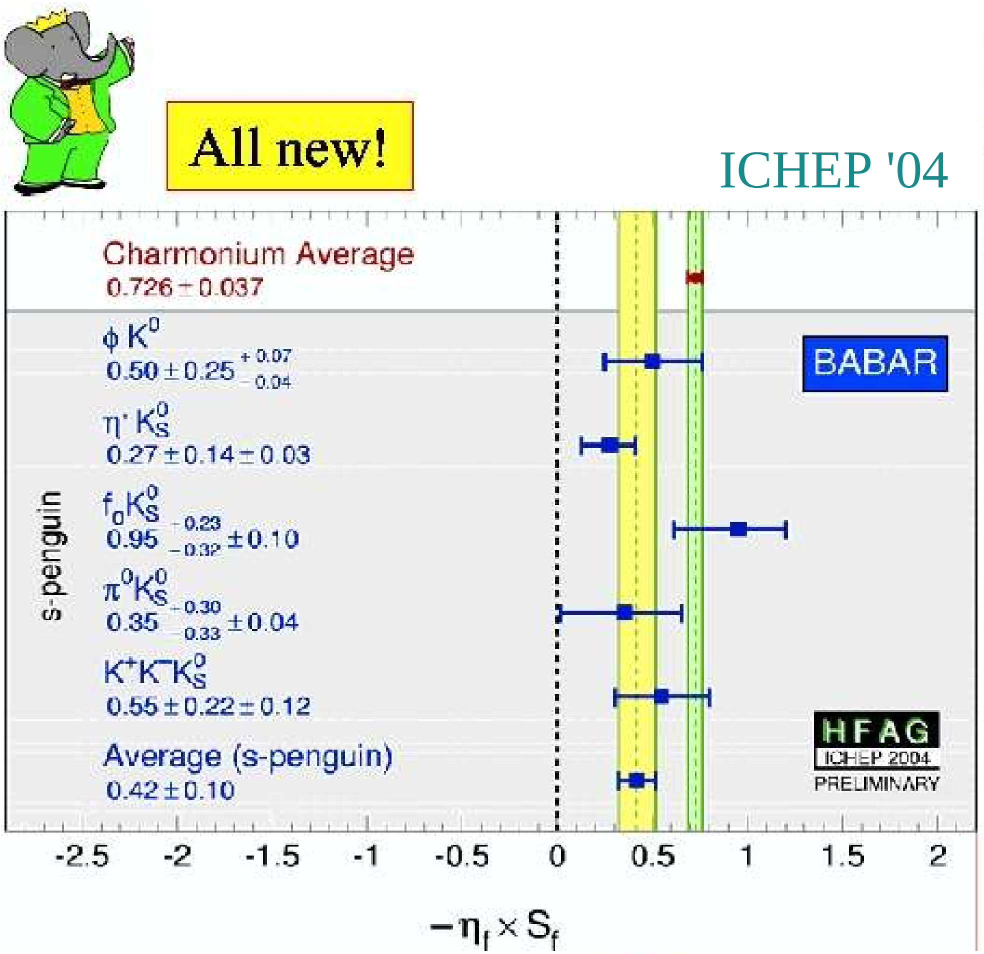
$$\Downarrow$$

$$B(K_L\to\pi^0\nu\nu) < 4.4\ B(K^+\to\pi^+\nu\nu)$$

More about non-MFV models:

- Rare K decays particularly sensitive to new sources of flavour-symm. Breaking
  [$\Leftrightarrow$ λ^5 suppression]

If a 10% deviation from SM is clearly established in time-dependent CPV asymmetries in B decays



2.7σ from s-penguin to sin2β ($c\bar{c}$)

- high chances to find O(1) non-SM effects in rare K decays

- clean electroweak processes [such as $K \to \pi + \nu\nu, ee$] are crucial to identify the nature of the effect [time-dependent CP asymmetries usually not clean beyond SM] th. error $\lesssim 10\%$

= exp. error $\lesssim 10\%$

= exp. error $\sim 30\%$

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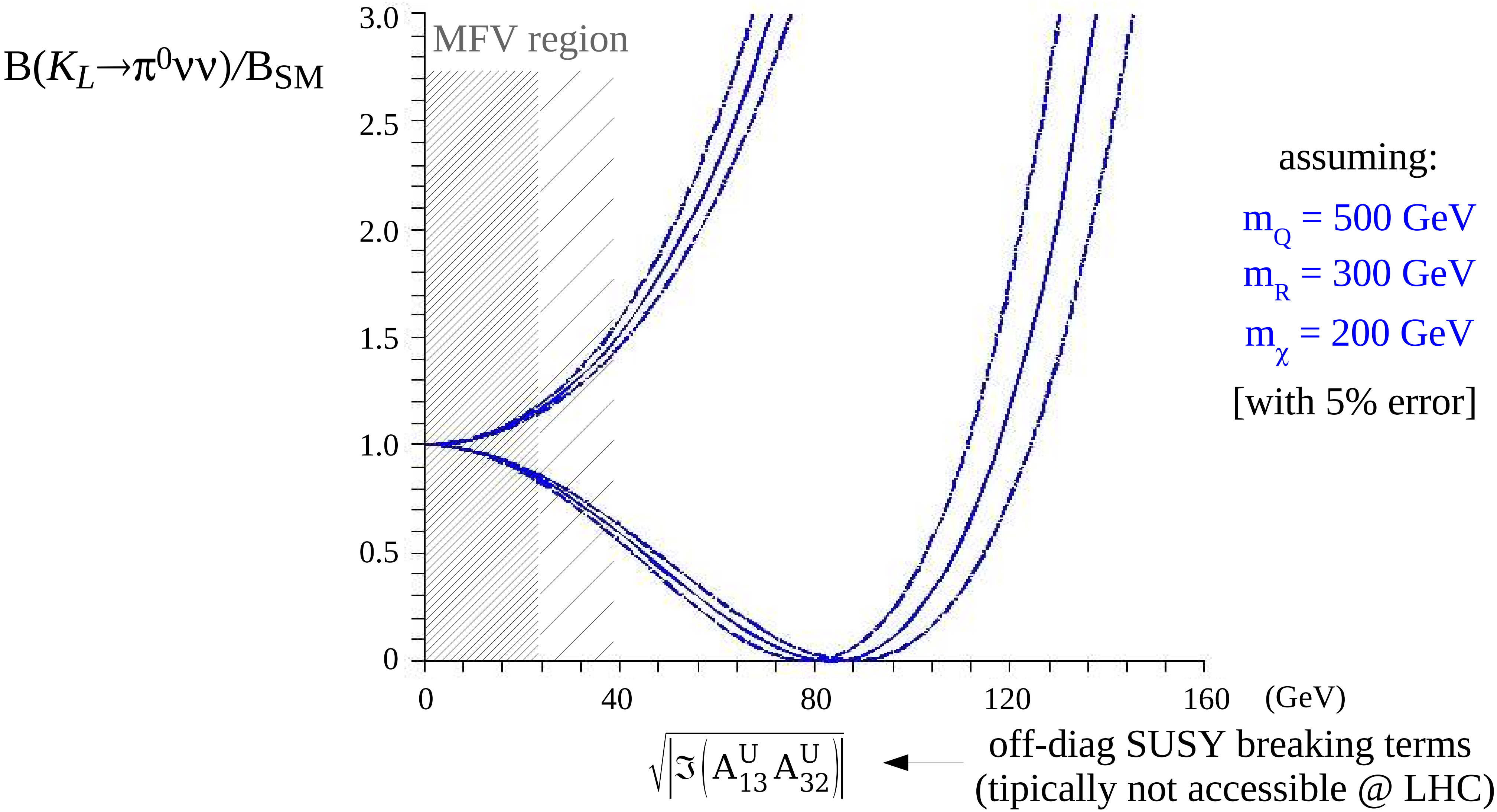
H^0 penguin	$B_s \to \mu\mu$	$B_d \to \mu\mu$	

A possible realistic scenario in 2012:

- LHC has seen NP ! It looks like low-energy SUSY
- Squark and chargino masses are measured with good accuracy, but we are still far from a complete determination of all the soft-breaking terms



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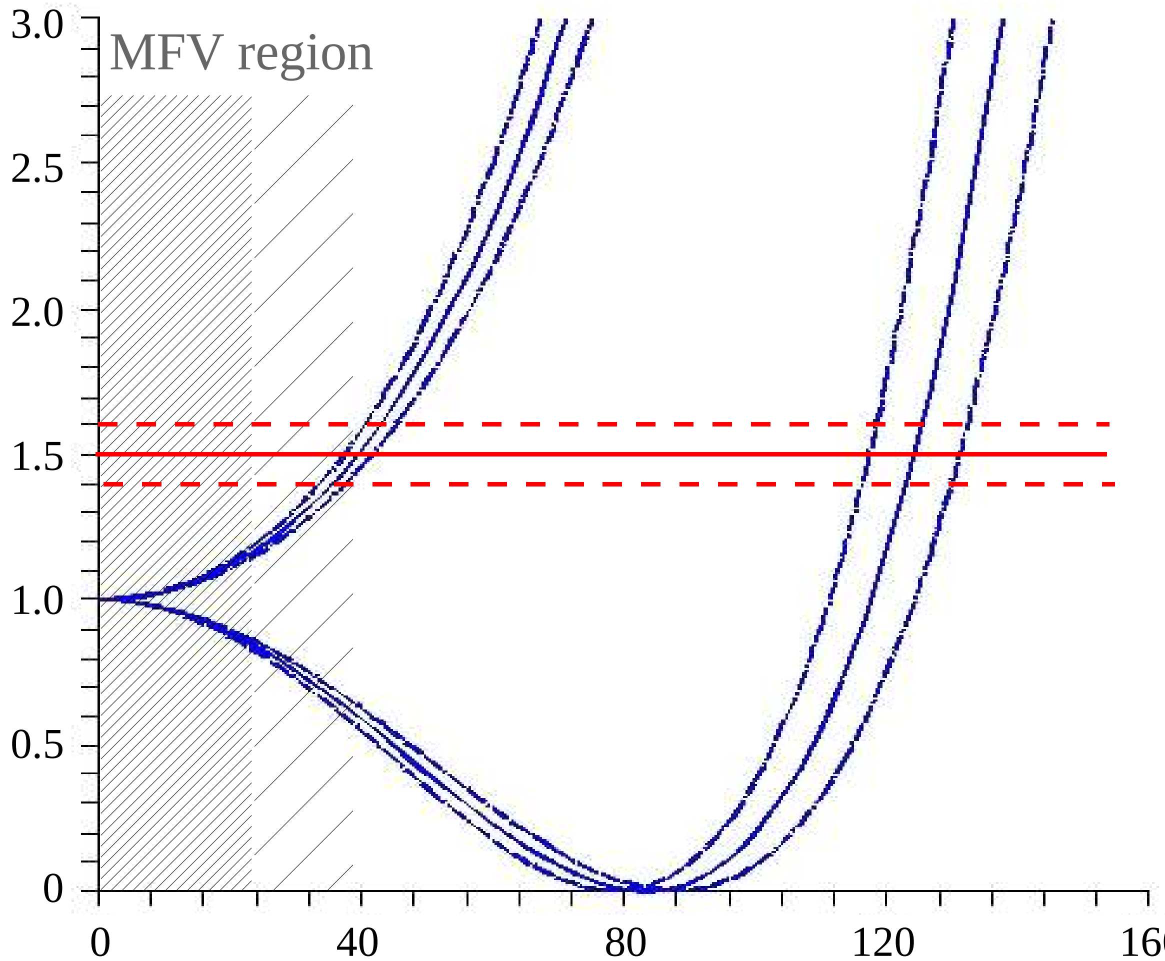
$B(K_L \to \pi^0 \nu\nu)/B_{SM}$



10% measurements of the two $B(K \to \pi\nu\nu)$ would provide unique & precise new inputs to determine the structure of the (new) model

assuming:

m_Q = 500 GeV

m_R = 300 GeV

m_χ = 200 GeV

[with 5% error]

$\sqrt{\left|\Im\left(A^U_{13} A^U_{32}\right)\right|}$ ← off-diag SUSY breaking terms (tipically not accessible @ LHC)

Conclusions

- Why rare K decays are still very interesting:

  The information coming form rare K decays is a key element to understand the *flavour structure* of *physics beyond the SM*

- Which are the decays modes which is still interesting to measure in this perspective:

$$K^+ \to \pi^+ \nu\nu \qquad K_L \to \pi^0 e^+e^-$$
$$K_L \to \pi^0 \nu\nu \qquad K_L \to \pi^0 \mu^+\mu^-$$

- At which level of precision it would be useful to measure them:

  The dream is a 5% accuracy (especially on the $\nu\nu$ modes)...

- How these conclusions are affected by the developments at LHC:

  These measurements are interesting even if LHC does not see anything new

  but if there are new particle below 1 TeV, carrying flavour quantum numbers, the game become much more exciting...