Wakefield due to roughness in a pipe of rectangular cross section

Gennady Stupakov and Karl Bane

SLAC

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Motivation

- In a recent paper, A. Mostacci et al. calculated wakefield for a rectangular waveguide with corrugated walls. The result—loss factor proportional to δ/a—does not agree with what one expects from the round pipe model, earlier studied by K. Bane and A. Novokhatski (BN) .
- The result of this paper was used to estimate the roughness impedance for LCLS, with the conclusion that the "the result differs by 2 orders of magnitude" from BN calculations.
- It was also used to estimate the impedance of the LHC beam screen.
- We do not discuss here if this is a good model for the real roughness.

Reference 1

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Wakefields due to surface waves in a beam pipe with a periodic rough surface

A. Mostacci* and F. Ruggiero
CERN, Geneva, Switzerland

M. Angelici, M. Migliorati, L. Palumbo, and S. Ugoli†
Dipartimento di Energetica–Universitá La Sapienza, Roma, Italy
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The problem of the wake elds generated by an ultrarelativistic particle traveling in a long beam tube with a periodic rough surface has been revisited by means of a standard theory based on the hybrid modes excited in a periodically corrugated rectangular waveguide. Slow surface waves synchronous with the particle can be excited in the structure, producing wake elds whose frequency and amplitude depend on the depth of the corrugation. We apply our results to the case of the CERN Large Hadron Collider beam screen and the Linac Coherent Light Source undulator.

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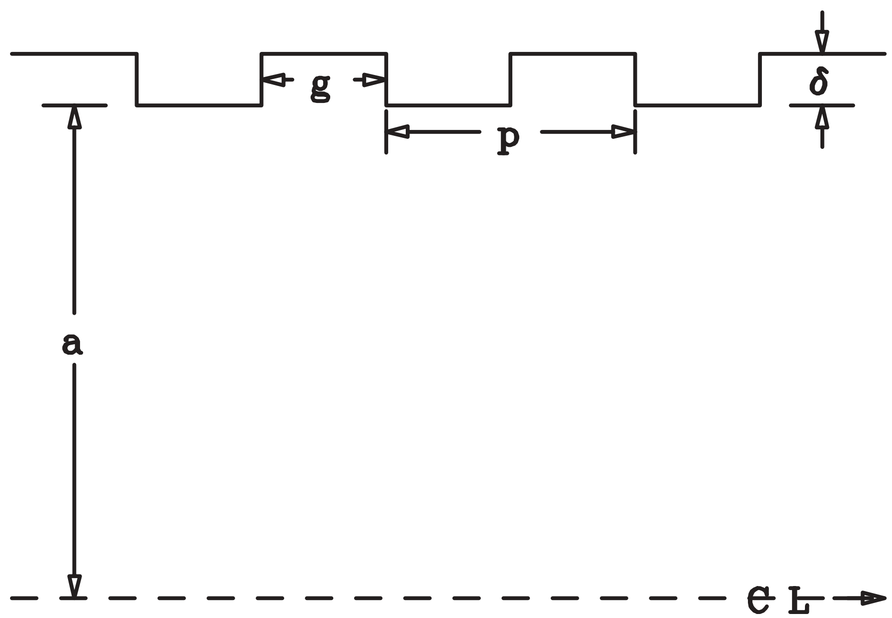
Round pipe with corrugations



K. Bane & A. Novokhatski (1999) modeled roughness as axisymmetric steps on the surface, assuming that $\delta, g, p \ll b$. They found that there exists a synchronous mode with $\omega/k = c$ which has the wavelength

$$\lambda = 2\pi \sqrt{\frac{\delta a g}{2p}}\,.$$

Round pipe with corrugations, cont'd

Wakefield

$$w(s) = 2\kappa \cos\left(\frac{2\pi s}{\lambda}\right)$$

where the loss factor κ (per unit length) is

$$\kappa = \frac{Z_0 c}{2\pi a^2} = \frac{2\pi}{(\pi a^2)}$$

Surprisingly, κ does not depend on the roughness properties. Moreover, it is equal exactly to the loss factor due to the resistive wall impedance.

Group velocity

$$1 - \frac{v_g}{c} = \frac{4\delta g}{ap} \sim \frac{\delta}{a}$$

This result can be also obtained in a model that treats the corrugation as a thin dielectric layer of thickness δ with

$$\epsilon = \frac{p}{p - g}$$

Rectangular pipe (from Ref. 1)

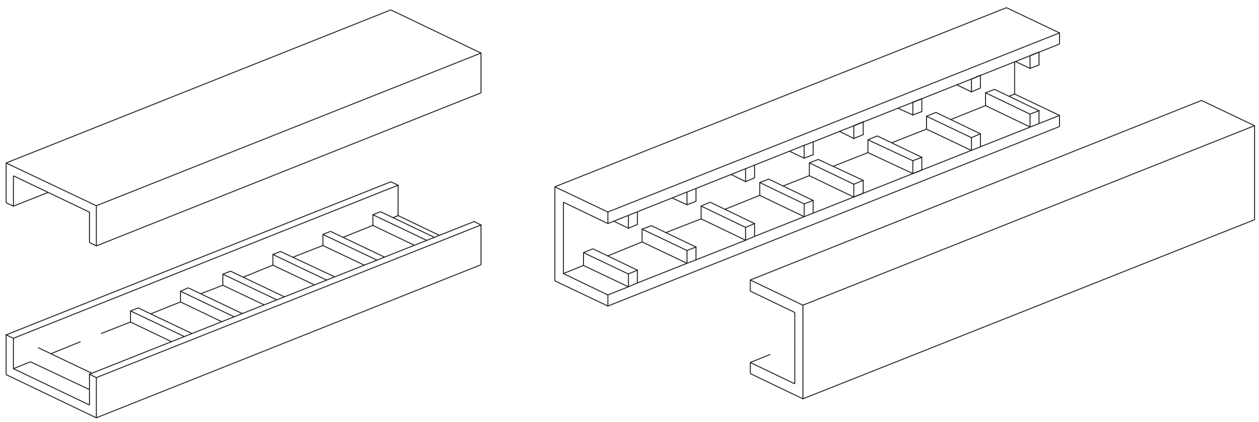


FIG. 1.   Relevant geometry.

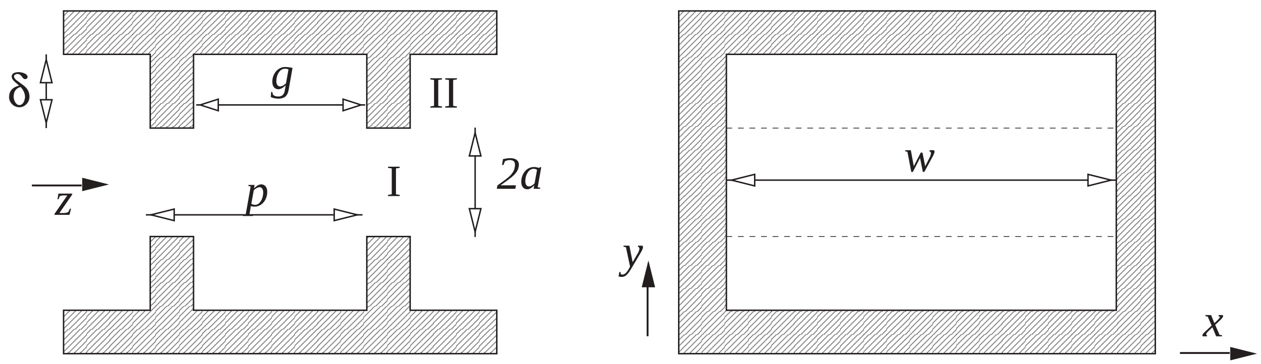


FIG. 2.   Schematic view of the waveguide and notations adopted.

$$\delta \sim g \sim p \ll a \sim w$$

EM field in the rectangular wageguide with corrugations

Two Hertz vectors, $\mathbf{\Pi}_m \propto e^{jkct}$ and $\mathbf{\Pi}_e \propto e^{jkct}$:

$$\begin{aligned}\mathbf{E} &= \nabla \times \nabla \times \mathbf{\Pi}_m - jk\nabla \times \mathbf{\Pi}_e \\ \mathbf{H} &= \nabla \times \nabla \times \mathbf{\Pi}_e + jk\nabla \times \mathbf{\Pi}_m \quad .\end{aligned}$$

In the tube region,

$$\begin{aligned}\Pi^I_{mx} &= \sum_{n=-\infty}^{\infty} \left[A_n \sinh(k^I_{yn} y) + B_n \cosh(k^I_{yn} y)\right] \sin(k_x x)\, e^{-j\beta_n z} \\ \Pi^I_{ex} &= \sum_{n=-\infty}^{\infty} \left[C_n \sinh(k^I_{yn} y) + D_n \cosh(k^I_{yn} y)\right] \cos(k_x x)\, e^{-j\beta_n z} \quad ,\end{aligned}$$

with

$$\beta_n = \beta_0 + \frac{2\pi n}{p}\,, \quad k^I_{yn} = \sqrt{\beta_n^2 - k^2 + k_x^2}\,, \quad k_x = \frac{m\pi}{w}$$

where m is an odd integer.

In the cavity region:

$$\begin{aligned}
\Pi_{mx}^{II} &= \sum_{s=0}^{\infty} E_s \sin[k_{ys}^{II}(a+\delta-y)] \sin(k_x x) \sin[\alpha_s(z+g/2)] \\
\Pi_{ex}^{II} &= \sum_{s=0}^{\infty} F_s \cos[k_{ys}^{II}(a+\delta-y)] \cos(k_x x) \cos[\alpha_s(z+g/2)] \quad ,
\end{aligned}$$

with

$$\alpha_s = \frac{\pi s}{g} \quad , \qquad k_{ys}^{II} = \sqrt{k^2 - \alpha_s^2 - k_x^2} \quad .$$

We need to match the tangential electric and magnetic fields at $y = \pm a$

$$\begin{aligned}
E_{z,x}^{I} &= \begin{cases} E_{z,x}^{II} & : \quad |z| < g/2 \\ 0 & : \quad g/2 < |z| < p/2 \end{cases} \\
H_{z,x}^{I} &= \quad H_{z,x}^{II} \quad : \quad |z| < g/2 \quad .
\end{aligned}$$

Small Corrugations

Assume that the corrugations are small, with $\delta \sim g \sim p \ll a \sim w$. Analysis shows that only one term in the Π vector sums, with $n = 0$ and $s = 0$, suffices to give a consistent solution to the field matching equations.

Setting $\alpha = 0$ implies that $\Pi_{mx}^{II} = 0$.

$$\begin{aligned} \Pi_{mx}^{II} &\approx 0 \\ \Pi_{ex}^{II} &\approx C \cos[k_{y0}^{II}(a + \delta - y)] \cos(k_x x) \quad , \end{aligned}$$

and

$$\begin{aligned} \Pi_{mx}^{I} &\approx 0 \\ \Pi_{ex}^{I} &\approx B \sinh(k_{y0}^{I} y) \cos(k_x x)\, e^{-j\beta_0 z} \quad , \end{aligned}$$

with C, B, constants.

Matching the boundary conditions, we find the dispersion relation for the synchronous mode

$$k^2 = m\pi \left(\frac{1}{\delta w}\right) \left(\frac{p}{g}\right) \coth\left(k_x a\right) \ ,$$

This agrees with Ref. 1.

$$1 - \frac{v_g}{c} = 2m\pi \left(\frac{\delta}{w}\right) \left(\frac{g}{p}\right) \frac{\sinh^2(k_x a)}{\sinh(k_x a)\cosh(k_x a) - k_x a} \quad .$$

where

$$k_x = \frac{m\pi}{w} \, .$$

The wakefield of one mode, at position s behind the (driving) point charge, can be written as

$$w(s) = 2\kappa\cos(ks)\,,$$

with κ the loss factor of the mode. The loss factor is given by

$$\kappa = \frac{|E_z|^2}{4u(1-v_g/c)}\,,$$

with E_z the longitudinal field on axis, and u the (per unit length) stored energy in the mode. The factor $1-v_g/c$ is often missed in the literature.

We find that

$$\kappa = \frac{2\pi}{aw} F\left(\frac{m\pi a}{w}\right)$$

where

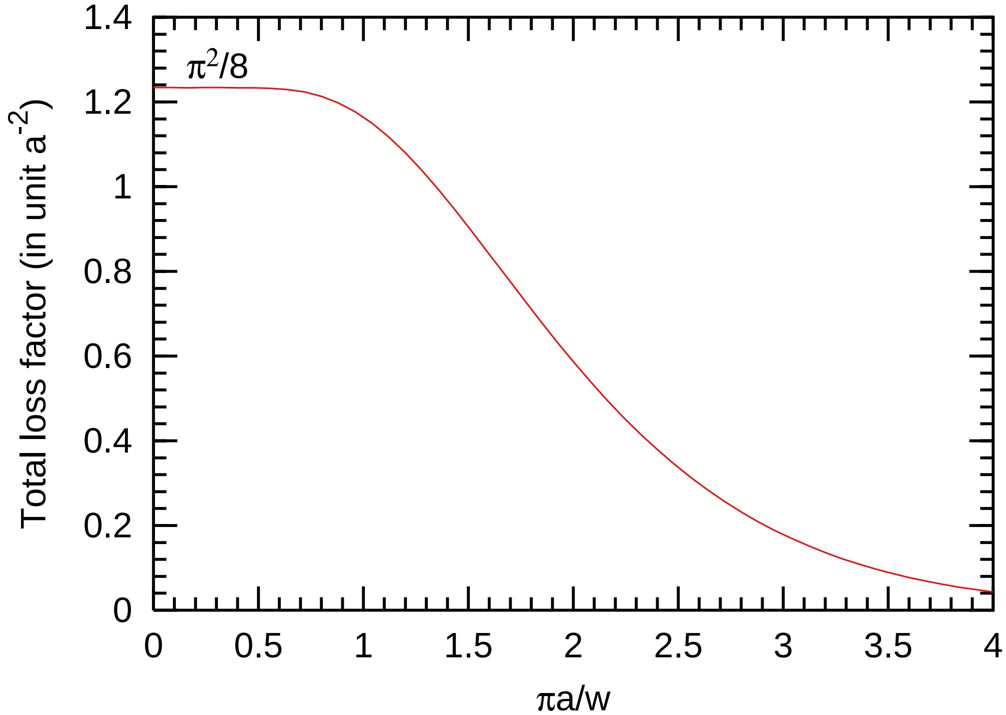
$$F(\zeta) = \frac{\zeta}{\sinh(\zeta)\cosh(\zeta)}$$

The loss factor does not depend on the roughness parameters, as in the case of the round pipe.

This result can be also obtained in a model that treats the corrugation as a thin dielectric layer of thickness δ with

$$\epsilon = \frac{p}{p-g}$$

Loss factor as a function of a/w



In the limit $w \to \infty$ this loss factor is equal to the loss factor of two resistive planes (H. Henke and O. Napoli, EPAC1990).

Conclusion

- We found synchronous modes and calculated the loss factors for the waveguide of rectangular cross section with 2 corrugated walls. Our result for the loss factor is a factor $\sim w/\delta$ larger than published by A. Mostacci et al.

- By order of magnitude, it agrees with the case of the round pipe ($w\,, a \to$ pipe radius). It also agrees with the problem where the corrugation is imitated by a thin layer of the dielectric coating.