Beam microbunching in bunch compressor due to CSR wake

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Introduction

- A relativistic electron beam moving on a circular orbit in free space can radiate coherently at the wavelengths that exceed the length of the bunch.
- Coherent radiation at shorter wavelengths can result from density fluctuations in the beam with characteristic length much shorter than the bunch length.
- If the radiation reaction force drives the growth of the initial fluctuation, one can expect an instability which leads to micro-bunching of the beam and increased coherent radiation at short wavelengths.

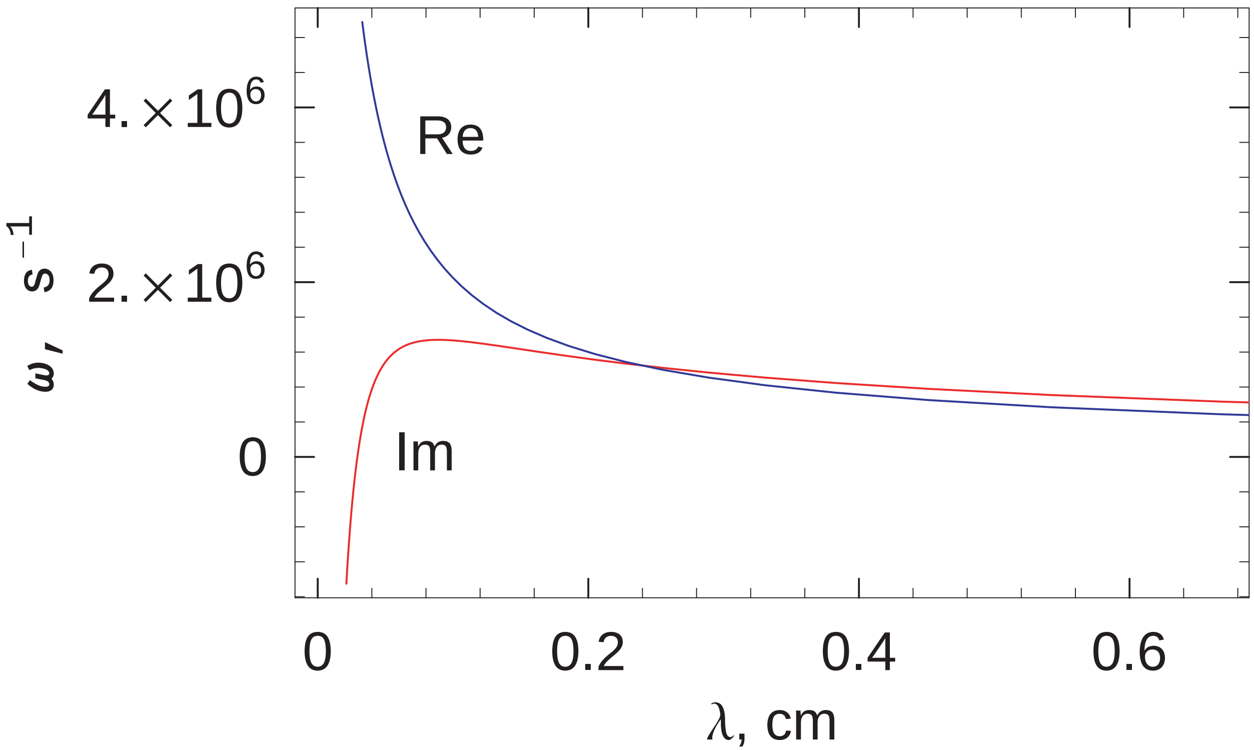
Mechanism of the instability

Let us assume a small initial sinusoidal density perturbation on the beam, $\delta n = \epsilon \sin kz$

- Due to the CSR wake, δn induces energy modulation in the beam δE_1
- Momentum compaction of the ring (or R_{56}) translates δE_1 into δn. Under certain conditions, the final δn is greater than the initial one.
- Energy spread introduces Landau damping and stabilizes short wavelengths.
- Transverse beam emittance mixes the particle over the wavelength and has a stabilizing effect on the instability
- Wall shielding of CSR and finite length of the bunch limits the instability at large wavelength.

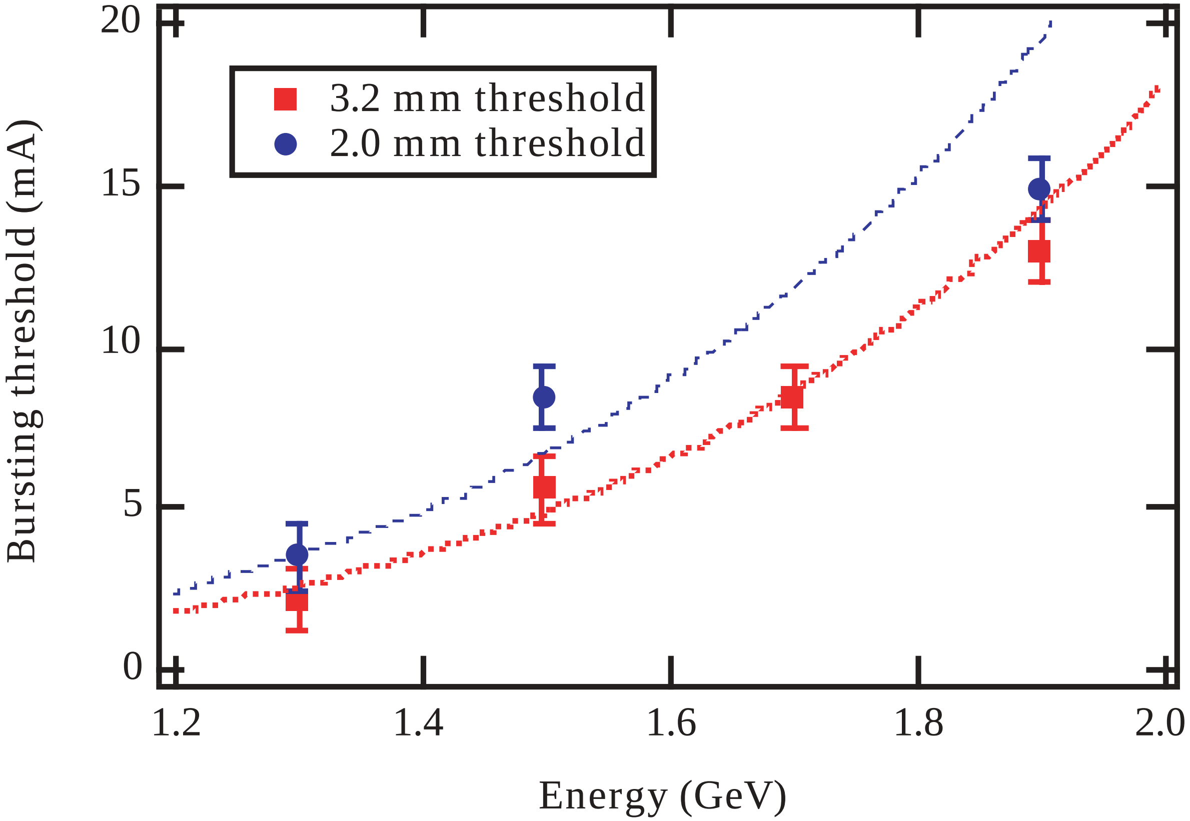
CSR instability in the ALS ring

Theory: S. Heifets, G. Stupakov. PRST-AB, 5, 054402 (2002).



$I_b = 30$ mA, zero transverse emittance.

CSR instability in the ALS ring, cont'd



From J. Byrd et al. EPAC2002 paper.

CSR instability in bunch compressor

The effect of microbunching caused by CSR has been observed in computer simulations by Borland (2001), and later, in simulation of LCLS BC2, by Borland and Emma.

Theory:

1. S. Heifets, G. Stupakov, SLAC Preprint SLAC-PUB-8988, 2001; S. Heifets, G. Stupakov and S. Krinsky PRST-AB 5, 064401 (2002)
2. E. Saldin, E. Schneidmiller and M. Yurkov. In Proceedings of the FEL2001 Conference, 2001; DESY Preprint FEL 2002-02
3. Z. Huang, K.-J. Kim. Formulae for CSR Microbunching in a Bunch Compressor Chicane, 2002

Assumptions in the theory

- CSR wake, no shielding, neglect the transients
- coasting beam approximation ($k\sigma_z \ll 1$)
- linear theory
- include transverse motion, effect of the transverse emittance

Equations of motion, no wake

We take into account transverse motion of the particles in the horizontal plane: $p = \Delta E/E$, $\theta = x'$

$$\begin{aligned} \frac{dx}{ds} &= \theta, \\ \frac{d\theta}{ds} &= -k_\beta(s)^2 x + \frac{p}{R(s)}, \\ \frac{dz}{ds} &= -\frac{x}{R(s)}, \\ \frac{dp}{ds} &= 0. \end{aligned}$$

$R(s)$ – bending radius, k_β – external focusing

Coasting beam model. Initial distribution function

$$\rho_0(x,\theta,z,p) = \frac{n_b}{2\pi\epsilon_0} \exp\left(-\frac{x^2+(\beta_0\theta)^2}{2\epsilon_0\beta_0}\right)\rho_G(p+uz),$$

with

$$\rho_G(p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{p^2}{2\sigma_p^2}\right),$$

where n_b – number of particles per unit length, ϵ_0 – horizontal emittance, β_0 – initial beta function, u – energy chirp, σ_p – rms energy spread.

Beam density inside the compressor

$$n_0(s,z) = \int d\delta\rho_0(\delta,z,s) = n_b C(s)$$

where

$$C(s) = \frac{1}{1-uR_{56}(s)}.$$

Vlasov equation

The Vlasov equation for the distribution function $\rho(x, \theta, z, p, s)$ includes the wake:

$$\begin{aligned}\frac{\partial \rho}{\partial s} - \frac{x}{R}\frac{\partial \rho}{\partial z} + \theta\frac{\partial \rho}{\partial x} + \left(-k_\beta^2 x + \frac{p}{R}\right)\frac{\partial \rho}{\partial \theta} \\ = \frac{r_e}{\gamma}\frac{\partial \rho}{\partial p}\int dz' W(z - z', s) n(z', s),\end{aligned}$$

where $n(z, s) = \int dx d\theta dp\, \rho(x, \theta, z, p, s)$.

CSR wake

Neglect the shielding effect, and assume a steady-state wake

$$W(\zeta) = \frac{2}{(3R^2)^{1/3}} \frac{\partial}{\partial\zeta} \frac{1}{\zeta^{1/3}} \quad \text{for } \zeta > 0,$$

and $W(\zeta) = 0$ for $\zeta \leq 0$. The radiation wakefield is localized in front of the moving charge.

Impedance has real and imaginary parts

$$Z(k) = \int_0^\infty d\zeta W(\zeta) e^{-ik\zeta} = iA \frac{k^{1/3}}{R^{2/3}}.$$

$$A = 3^{-1/3}\Gamma\left(\frac{2}{3}\right)\left(\sqrt{3}i - 1\right) = 1.63i - 0.94$$

Approach: linearize Vlasov equation, $\rho = \rho_0 + \rho_1$, and convert it to an integral equation (alternative derivation – Z. Huang & K.-J. Kim).

For the initial perturbation of the distribution function we use:

$$\rho_1(x, \theta, z, p) = \frac{n_1}{2\pi\epsilon_0} \exp\left(-\frac{x^2 + (\beta_0\theta)^2}{2\epsilon_0\beta_0} \right) \rho_G(p + uz) e^{-ikz}$$

Integral equation

Definition of function $g_k(s)$

$$n_{1,k}(z,s) = C(s) g_k(s) e^{ikC(s)z}.$$

$g_k(s)$ satisfies the integral equation

$$g_k(s) = g_k^{(0)}(s) + \int_0^s K(s,s') g_k(s') ds',$$

$$\begin{aligned} K(s,s') &= \frac{ikr_e n_b}{\gamma} C(s') C(s) Z(kC(s'),s') R_{56}(s' \to s) \\ &\times \quad e^{-(k^2\epsilon_0/2\beta_0)[\beta_0^2 R_{51}^2(s,s') + R_{52}^2(s,s')] - (k^2\sigma_p^2/2) R_{56}^2(s,s')}. \end{aligned}$$

Mathematica code

The integral equation is solved numerically using a code written in Mathematica.

The input for the code includes optical functions for the compressor ($\beta(s)$, $\alpha(s)$, $R_{56}(s)$, $\ldots$) and the beam parameters (N_b, σ_z, γ, ϵ, δ, u).

The integral equation is discretized and solved using either trapezoidal or rectangular rule for integration.

The gain factor G is defined as

$$G = \frac{|n_1^{\text{final}}|/n_b^{\text{final}}}{n_1^{\text{init}}/n_b^{\text{init}}}$$

LCLS BC2, beam parameters

Beam energy $E = 4.54$ GeV

RMS bunch length $\sigma_z = 193$ microns

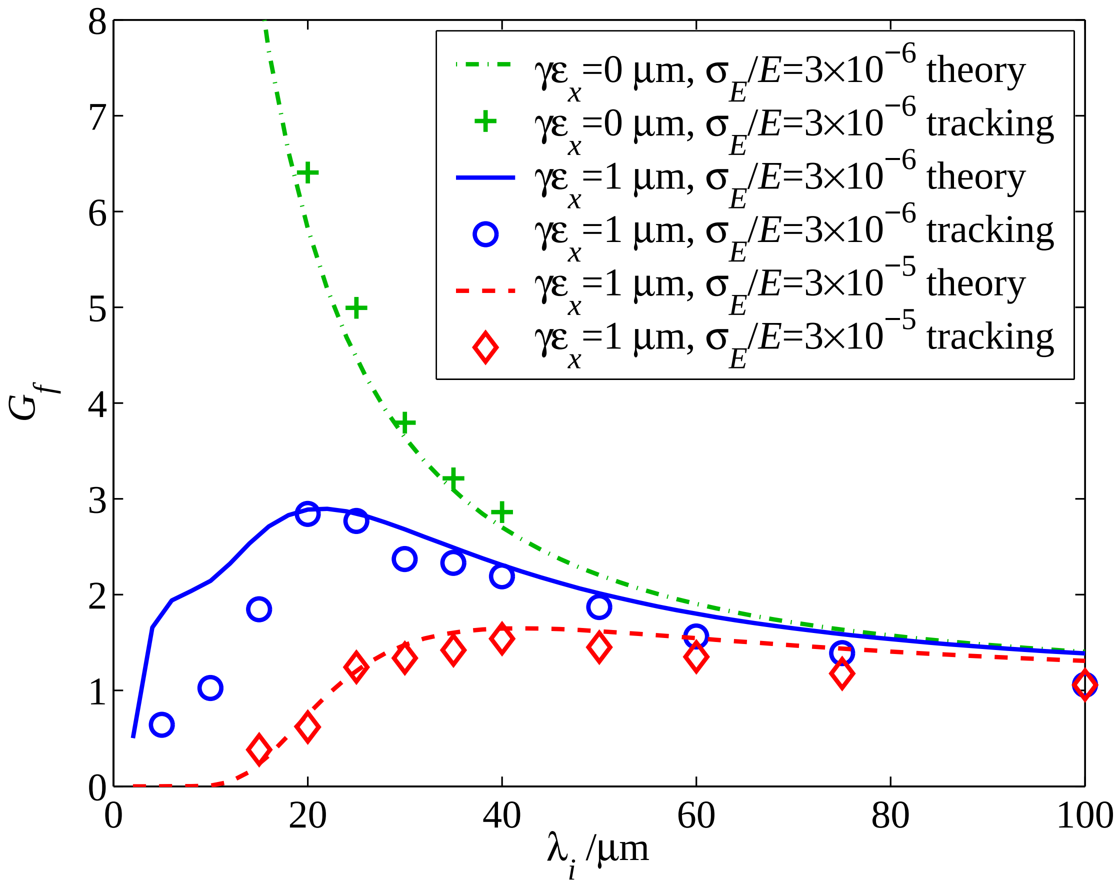
Compression – 8.3

$N_b = 6.2 \cdot 10^9$

Transverse normalized emittance $\epsilon = 1\ \mu$, or 0

Energy spread $\Delta E/E = 3 \cdot 10^{-5}$ of $3 \cdot 10^{-6}$.

LCLS BC2 – theory vs simulations



P. Emma's simulation with `elegant`

Benchmark BC, beam parameters

Beam energy $E = 5$ GeV

RMS bunch length $\sigma_z = 200$ microns

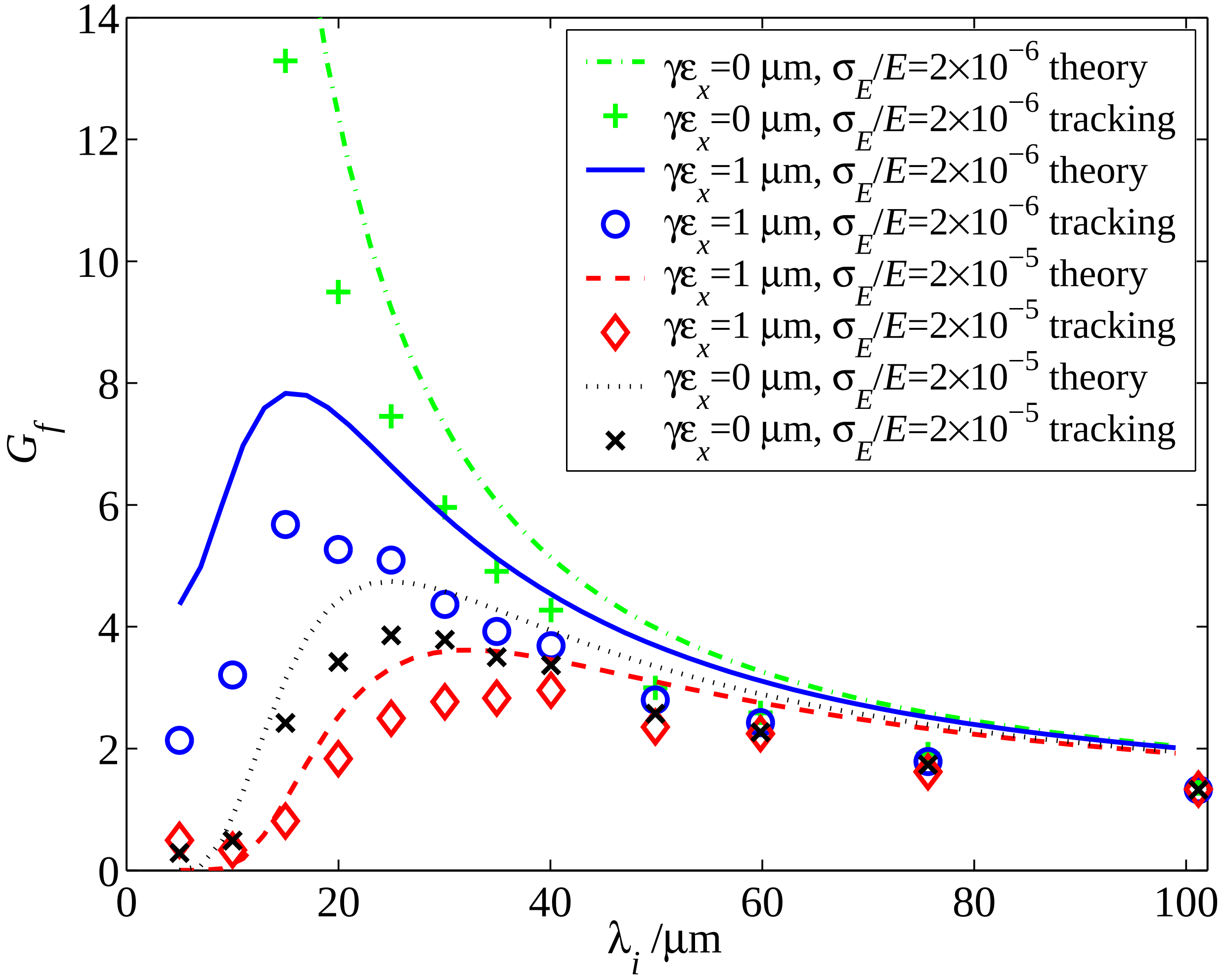
Compression – 10

$N_b = 6.2 \cdot 10^9$

Transverse normalized emittance $\epsilon = 1\ \mu$, or 0

Energy spread $\Delta E/E = 2 \cdot 10^{-5}$ of $2 \cdot 10^{-6}$.

Benchmark BC – theory vs simulations



P. Emma's simulation with `elegant`

Discussion

1. SCR wake – shielding. For finite aperture b of the beam pipe CSR is suppressed due to the shielding effect at

$$k \lesssim R^{-1}\left(\frac{\pi R}{2b}\right)^{3/2}$$

2. SCR wake – short distances. The wake was derived for infinitely thin beam. Due to finite σ_x, at very short distance, the wake actually does not have a singularity

$$\Delta s \sim R\left(\frac{\sigma_x}{R}\right)^{3/2}$$

3. Initial perturbation. The model is somewhat limited to the specific initial distribution function.

Conclusion

- A linear theory of CSR instability in bunch compressors has been developed that takes into account bunch compression, energy spread and the transverse emittance. The gain factor for a given bunch compressor is calculated by solving numerically the integral equation.
- Results show good agreement with `elegant` simulations for LCLS BC2, but not so good agreement for the benchmark BC ...