COMMENTS ON OPTICAL STOCHASTIC COOLING

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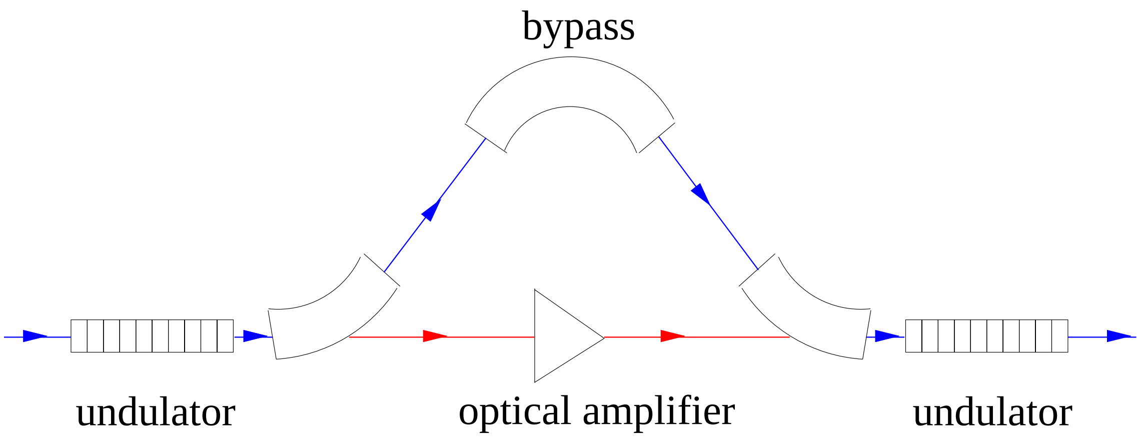
1 Introduction

- Transit-time optical stochastic cooling (OSC) was first introduced by Zolotorev and Zholents (ZZ).
- Optical frequency of $\sim 3 \times 10^5$ GHz ($\lambda \sim 1$ μm) is used.
  This provides a bandwidth $>$ 10000 times than microwave stochastic cooling.
- OSC can be used in low energy electron rings to provide high brightness beams.
  It can also be used in proton collider rings to increase luminosity by counteracting intra-beam scattering.



- Particles with different energies have slightly different path lengths and interact with the optical beam at slightly different phases,

$$\Delta\phi_i = k(\ell_i - \ell_0) = k(x_i I_1 + x_i' I_2 + \delta_i I_D)$$

$$I_1 = \int_{s_1}^{s_2} \frac{M_{11}(s,s_1)\,ds}{\rho(s)}, \quad I_2 = \int_{s_1}^{s_2} \frac{M_{12}(s,s_1)\,ds}{\rho(s)}, \quad I_D = \int_{s_1}^{s_2} \frac{D(s)\,ds}{\rho(s)}.$$

The correction is

$$\Delta\delta_i = -\operatorname{sgn}(I_D) G \sin(\Delta\phi_i)$$

Gain factor : $G = \dfrac{gq\mathcal{E}_0 N_u \lambda_u K}{2\gamma E_b}$

- Through the dispersion D_2 at 2nd undulator, there are horizontal corrections $\Delta x_i = -D_2 \Delta\delta_i$, $\Delta x'_i = -D'_2 \Delta\delta_i$.
  Thus there is also horizontal cooling.

- ZZ computed the horizontal cooling decrement

$$\alpha_x = \frac{1}{2}\left(\frac{\overline{\Delta(x^2)}}{\overline{x^2}} + \frac{\overline{\Delta(x'^2)}}{\overline{x'^2}}\right)$$
$$= \frac{1}{2}\left[4GD_0\eta'_0 k \exp\left\{-\frac{\overline{\Delta\phi_i}}{2}\right\} - \frac{G^2 N_s}{2}\left({\eta'_0}^2 + \frac{D_0^2}{\beta^2}\right)\frac{\beta}{\epsilon_x}\right]$$

  where D_0 and $-\eta'_0$ are D and D' at the 2nd undulator.

  Thus, there is no horizontal cooling if $\eta'_0 = 0$, which can hardly be correct.

  In order to understand OSC, we rederived all the equations.

2 OSC Decrements

- Longitudinal damping decrement:

$$\alpha_\delta \equiv -\frac{\langle \delta_{ic}^2 - \delta_i^2 \rangle}{\sigma_\delta^2} = 2GkI_D e^{-u} - \frac{G^2 N_s}{2\sigma_\delta^2} \ ,$$

where

$$u = \frac{1}{2}k^2[(\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2)\epsilon_x + I_D^2 \sigma_\delta^2]$$

is the total thermal energy of the system.

- Horizontal damping decrement:

$$\alpha_x \equiv -\frac{\langle P_{x2c}^2 + x_{2c}^2 - (P_{x2}^2 + x_2^2) \rangle}{\sigma_{x2}^2} = 2GkI_\perp e^{-u} - \frac{G^2 N_s \mathcal{H}_2}{2\epsilon_x} \ ,$$

where

$$\mathcal{H}_2 = \frac{D_2^2 + P_{D2}^2}{\beta_2}$$

is the $\mathcal{H}$-function at 2nd undulator, and

$$I_\perp = -\frac{\beta_1}{\beta_2}\left\{P_{D2}\left[\left((\beta_2 M_{21}+\alpha_2 M_{11}) - \frac{\alpha_1}{\beta_1}(\beta_2 M_{22}+\alpha_2 M_{12})\right)\left(I_1 - \frac{\alpha_1}{\beta_1} I_2\right) + \frac{\beta_2 M_{22}+\alpha_2 M_{12}}{\beta_1^2} I_2\right] + D_2\left[\left(M_{11} - \frac{\alpha_1}{\beta_1} M_{12}\right)\left(I_1 - \frac{\alpha_1}{\beta_1} I_2\right) + \frac{M_{12}}{\beta_1^2} I_2\right]\right\}$$

Thus the necessary condition of horizontal damping is $I_\perp > 0$.

3 OSC Dynamic

$$\frac{d\epsilon_x}{dt} = -\frac{2GkI_\perp \epsilon_x}{T_0}e^{-u} + \frac{G^2 N_s \mathcal{H}_2}{2T_0},$$
$$\frac{d\sigma_\delta^2}{dt} = -\frac{2GkI_D\sigma_\delta^2}{T_0}e^{-u} + \frac{G^2 N_s}{2T_0},$$

Equal gain condition: $I_D = I_\perp$. Then obtain

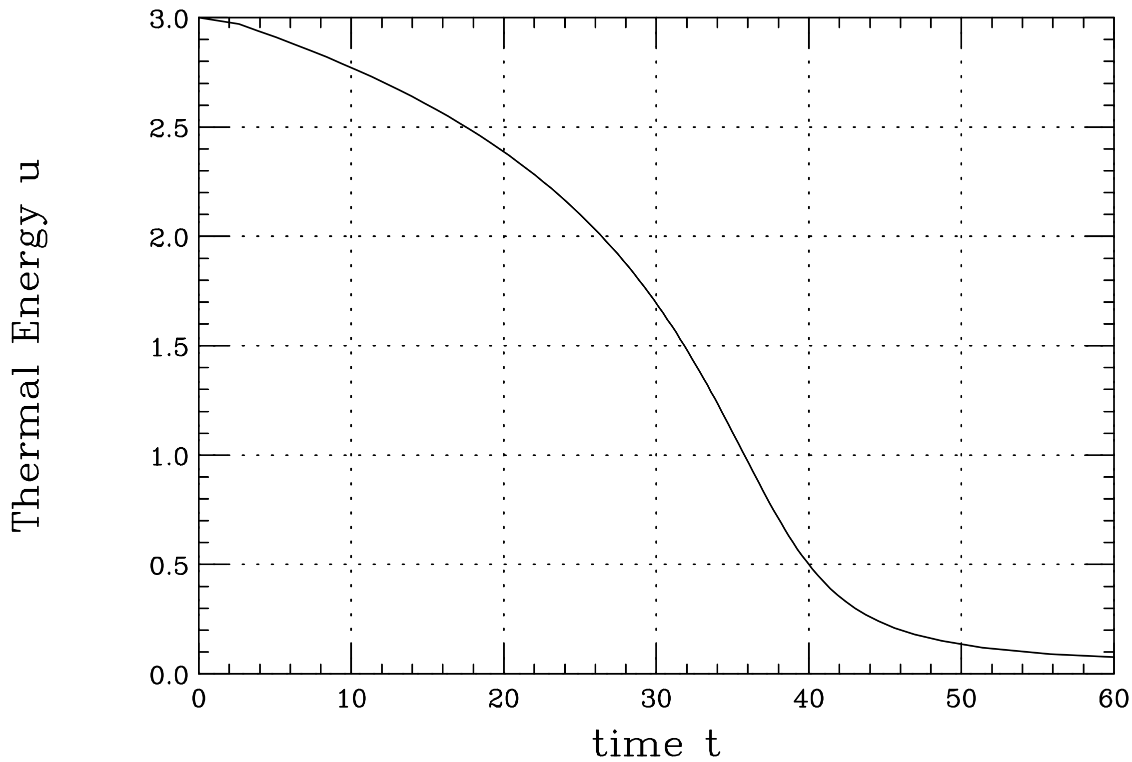
$$\frac{du}{dt} = -\frac{2G_0 kI_D}{T_0}ue^{-u} + \frac{G_0^2 N_s v}{2T_0},$$

where

$$v = \frac{1}{2}k^2[(\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2)\mathcal{H}_2 + I_D^2].$$

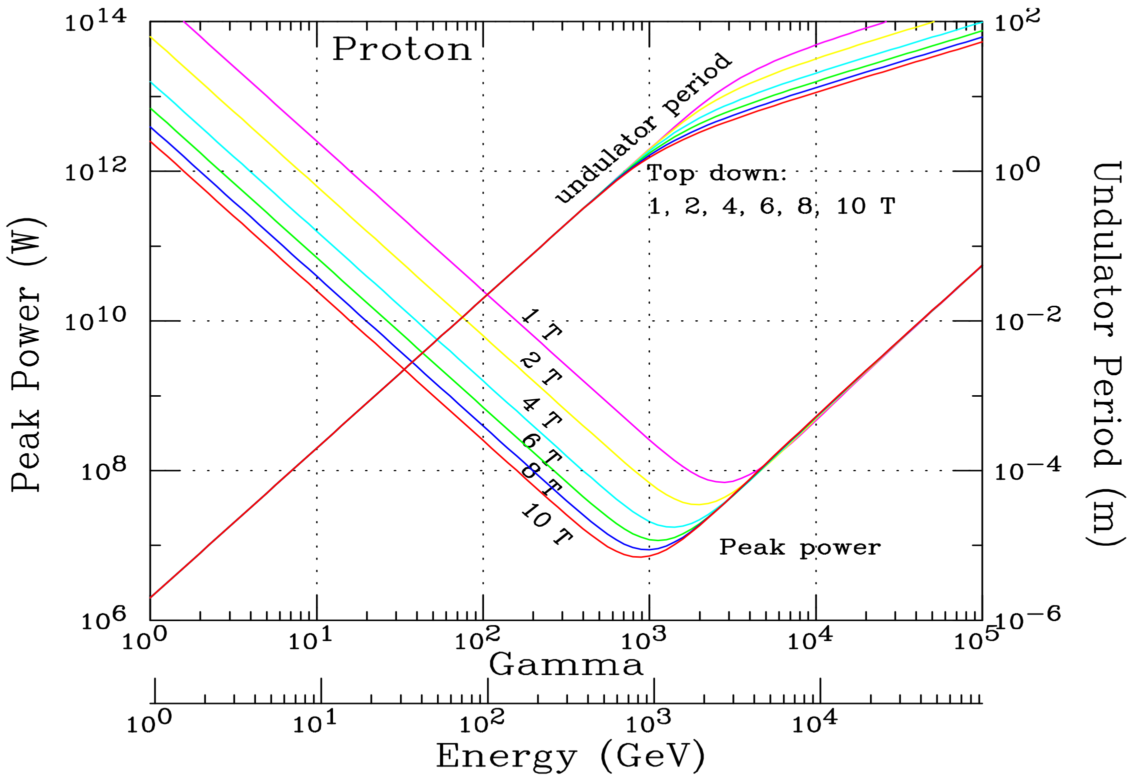
- Optimum gain gives

$$G_{\mathrm{opt}} = \frac{2kI_D}{vN_s}ue^{-u} \quad \longrightarrow \quad \frac{du}{dt} = -\frac{2k^2 I_D^2}{vN_sT_0}u^2 e^{-2u}.$$



- Because of the $u^2 e^{-2u}$ factor, cooling slows down.

- Actually $G_{\rm opt}$ is usually very large; so is output power of the laser amplifier.



Tevatron:

with $B = 10$ T

$\hat{P} \approx 6.5$ MW

$\bar{P} \approx 67$ kW

$\tau = 74$ s

$N_s = 1.5\ 10^6$.

- OSC does not favor small γ because less photons will be emitted.
  OSC also does not favor very large γ because the beam becomes too stiff to be bent inside the undulators. Optimum proton energy is $\gamma \approx 1$ to 2 TeV.

4 Non-Optimum Cooling

- The degradation of the Tevatron luminosity is of order hours. Therefore we can allow a much longer cooling time and employ a laser amplifier of much less gain (or lower output power).
- When $G \ll G_{\rm opt}$, the heating term can be neglected, leaving

$$\frac{du}{dt} = -\frac{2G_0 k I_D}{T_0} u e^{-u} \; .$$

- Max. cooling rate is at $u = 1$. But that will not give the shortest cooling time.
- Since the interaction of beam with photons is

$$\Delta\delta_i = -\,{\rm sgn}(I_D) G \sin(\Delta\phi_i)$$

In order that the energy-offset particles receive the right correction, we must have

$$\Delta\phi|_{\pm 3\sigma_\delta} < \frac{\pi}{2} \; .$$

Since

$$\Delta\phi_i = k(x_i I_1 + x' I_2 + \delta_i I_D)$$

and I_1 and I_2 can be made very small in the bypass design, the phase restriction gives

$$k I_D \sigma_\delta = \frac{\pi}{6} \; .$$

Therefore

$$u = \frac{1}{2} k^2 [(\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2)\epsilon_x + I_D^2 \sigma_\delta^2] \approx \frac{\pi^2}{72} \; .$$

4.1 Average Power

- Average power is

$$\bar{P} = g^2 \frac{W_0}{T_0} N_{\rm tot}$$

where photon energy emitted by one particle is

$$W_0 = \frac{1}{2}\epsilon_0 \mathcal{E}_0^2 A c \Delta t_R \ .$$

This photon emission has area of cross section at the waist

$$\boxed{A \approx \frac{N_u \lambda_u \lambda}{8}}$$

assuming $Z_R \approx \frac{1}{4} N_u \lambda_u$, and duration

$$\Delta t_r \approx \frac{N_u \lambda}{c} \ .$$

- Get

$$\bar{P} = \frac{(E_b/q)^2 N_{\rm tot} \lambda G^2}{16 Z_0 c T_0 F \xi [JJ]^2}$$

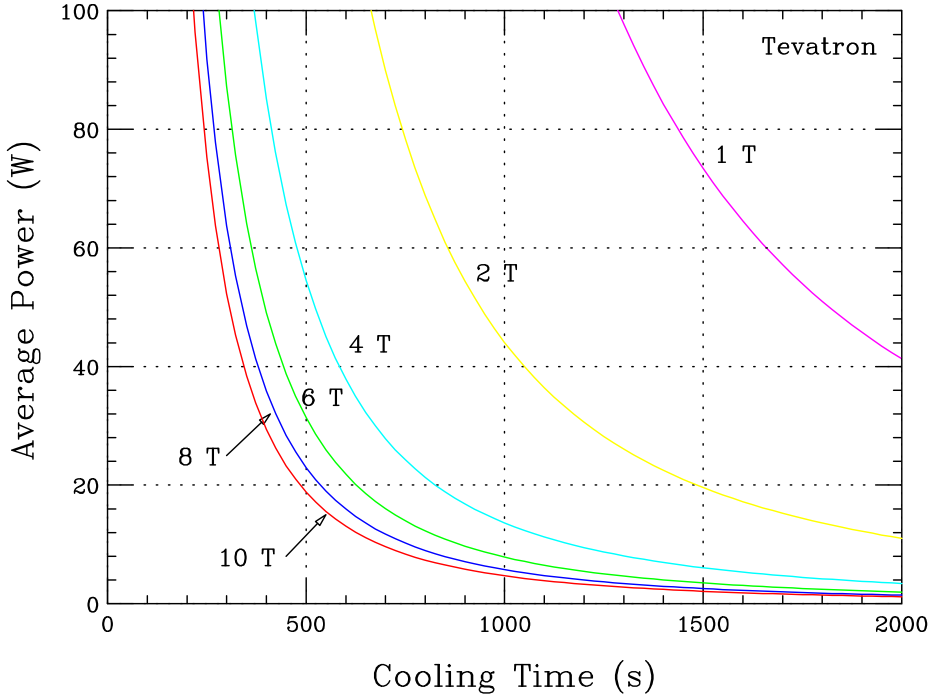
Cooling time is

$$\frac{\tau}{T_0} = \frac{3a\sigma_\delta}{\pi G} \qquad \text{with} \qquad a = \frac{e^u}{\sqrt{1+2u-u^2}} \ .$$

Eliminating G,

$$\left(\frac{\tau}{T_0}\right)^2 \bar{P} = \frac{(\sigma_\delta E_b/q)^2 N_{\rm tot} \lambda}{Z_0 c T_0 F \xi [JJ]^2} \left(\frac{3a}{4\pi}\right)^2$$

Application to Tevatron



- Normal temperature undulator: $B \leq 1$ T.

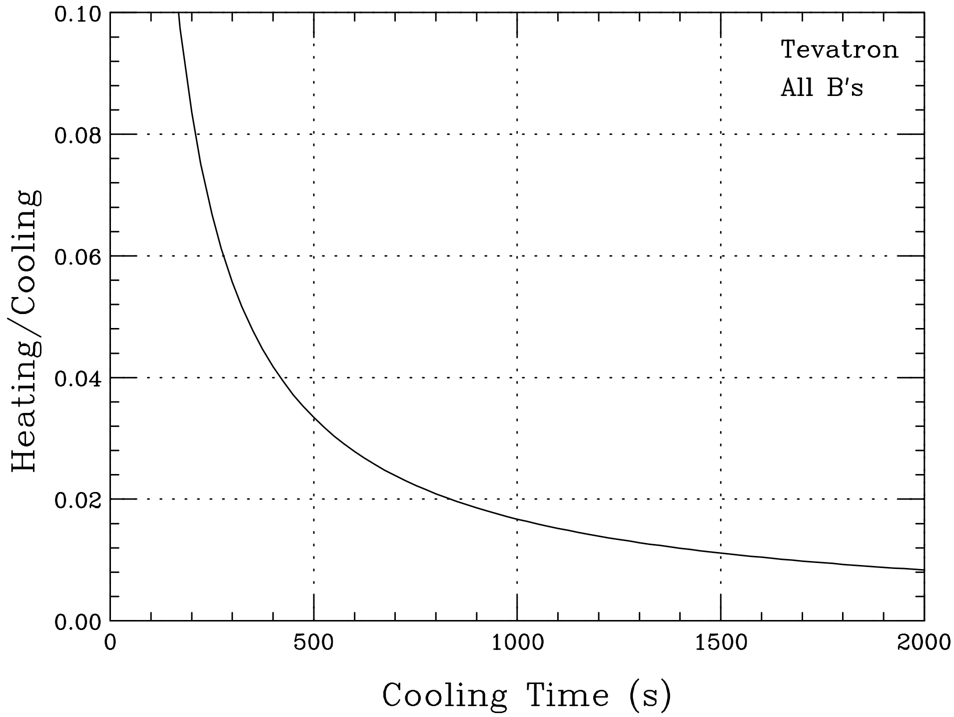
  Superconducting undulator: $B \leq 6$ T.

  Super-fluid undulator $B > 6$ T.

  For example, at $B = 6$ T, $\lambda_u = 1.93$ m. $\tau = 1200$ s $\longrightarrow \bar{P} = 5.5$ W.

  Notice that further increases B does not help much.

- The ratio of heating to cooling in the cooling equation is indeed small



RHIC:

Gold at 100 GeV/u

$$\text{Power} \propto \frac{A^4}{Z^4} = 38.7$$

with $A = 197$, $Z = 79$

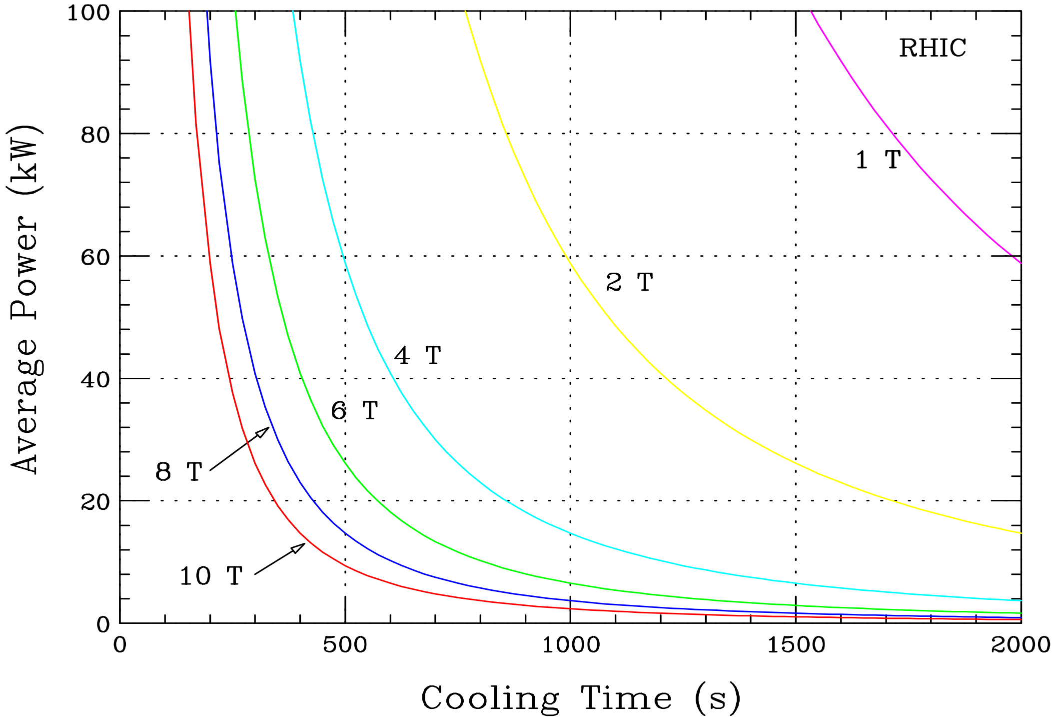
$\text{Power} \propto \gamma^{-2}$

$\gamma = 106$

10 times < Tevatron

$N = 1 \times 10^9$

90 bunches



- Because γ is 10 times smaller, undulator period decreases 100 times to $\lambda_u \approx 2.3$ cm. Thus, only normal temperature undulator, or $B = 1$ T.

- $\tau = 1200$ s $\longrightarrow$ $\bar{P} = 163$ kW.

  OSC is not suitable for heavy ions.

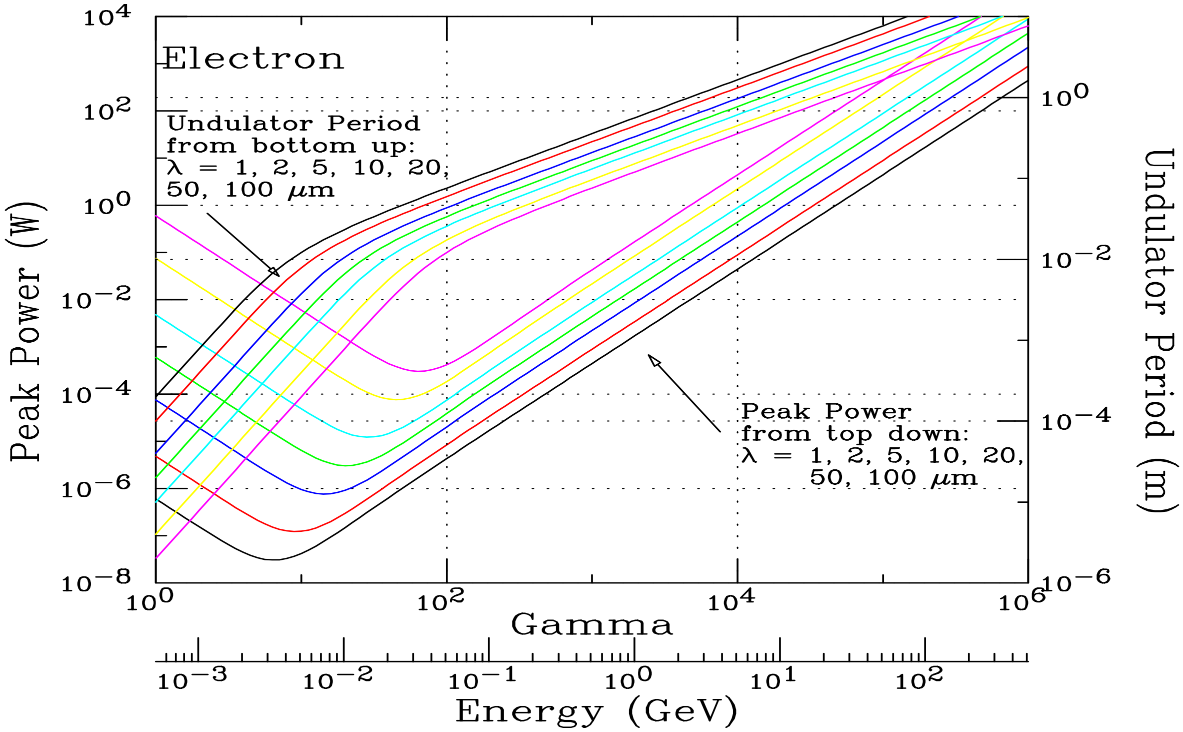
5 Electron Rings

$\sigma_\delta = 1 \times 10^{-4}$

$\sigma_\ell = 1.0$ cm

$N = 2.7 \times 10^{11}$

$B = 1$ T



- With $\lambda = 1$ μm and $B = 1$ T,

$$\gamma_{\min} = \sqrt{\frac{4\sqrt{2}\pi}{3\sqrt{3}}}\sqrt{\frac{mc}{qB\lambda}} = 76.3 \quad \text{or} \quad E_b = 39 \text{ MeV} \quad \text{and} \quad \lambda_u \approx 0.9 \text{ cm}$$

- Superconducting undulator cannot be used, needs $B \approx 1$ T.

  Most electron rings are on high γ side.

  Power is low because it scales as

$$\text{Power} \propto \left(\frac{\gamma m}{q}\right)^2$$

  Minimum peak power is ~ 2 mW.

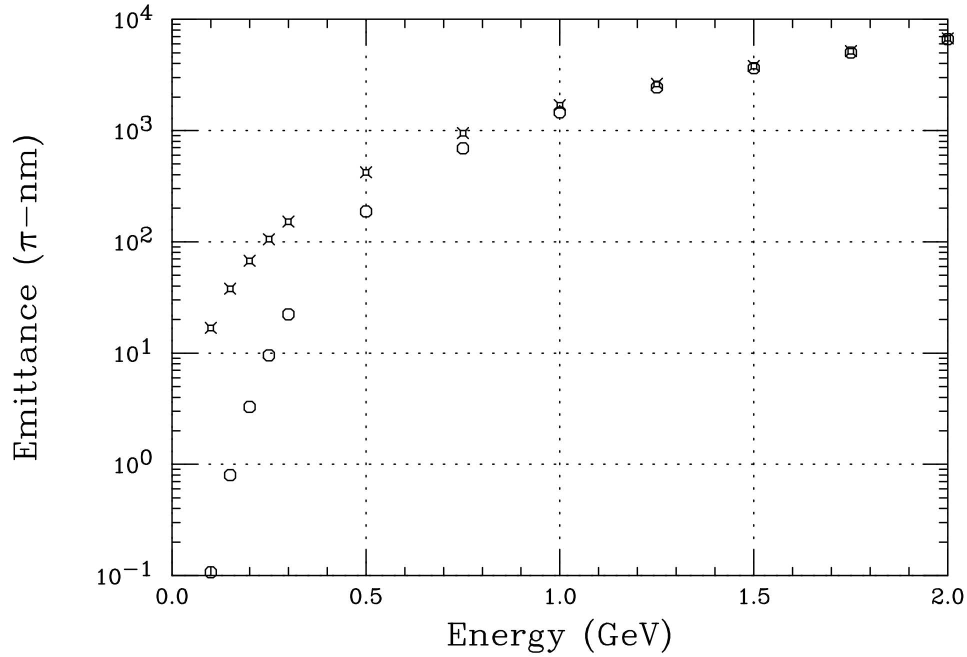
- OSC seems to favor high energy electron rings.

  However, OSC damping time $\propto N_s \sim \dfrac{N N_u \lambda}{\text{bunch length}}$ is energy independent.

  On the other hand, radiation damping time decreases rapidly with energy.

  Thus, OSC is really good for low energy electron rings only.

- Convert the IUCF 85 m Cooler Ring into an electron storage ring. For a transverse OSC cooling time of 0.1 s, the change in emittance is



- For electron less than 0.3 GeV, the OSC reduces the emittance by more than 1 order of magnitude. But is not efficient at all for higher energies.

6 Conclusion

- A necessary condition has been derived for transverse OSC: $I_\perp > 0$.
- Power $\propto \gamma^{-2}$ for small γ and $\propto \gamma^2$ for large γ.
  Thus, OSC may not be possible for VLHC.
- Power can be further reduced if the photons can be focused to a size $Z_R = \frac{1}{4} N_u \lambda_u$.
  But this must be larger than the particle beam size.
- Output power for proton rings can be reduced by using superconducting undulators operating at below optimum gain.
- OSC is not suitable for heavy ions because of $\left(\frac{A}{Z}\right)^4$ factor, especially RHIC where γ is not large enough.
- For electrons, λ_u is small and only $B = 1$ T is possible.
  Output power is mostly $\propto (\gamma m/q)^2$.
  Thus, OSC favors very low energy electron storage rings which have small radiation damping times.