COMMENTS ON OPTICAL STOCHASTIC COOLING

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1 Introduction

- Transit-time optical stochastic cooling (OSC) was first introduced by Zolotorev and Zholents (ZZ).

- Optical frequency of $\sim 3 \times 10^5$ GHz ($\lambda \sim 1$ $\mu$m) is used. This provides a bandwidth $> 10000$ times than microwave stochastic cooling.

- OSC can be used in low energy electron rings to provide high brightness beams. It can also be used in proton collider rings to increase luminosity by counteracting intra-beam scattering.
- Particles with different energies have slightly different path lengths and interact with the optical beam at slightly different phases,

\[ \Delta \phi_i = k(\ell_i - \ell_0) = k(x_i I_1 + x'_i I_2 + \delta_i I_D) \]

\[ I_1 = \int_{s_1}^{s_2} \frac{M_{11}(s, s_1) \, ds}{\rho(s)}, \quad I_2 = \int_{s_1}^{s_2} \frac{M_{12}(s, s_1) \, ds}{\rho(s)}, \quad I_D = \int_{s_1}^{s_2} \frac{D(s) \, ds}{\rho(s)}. \]

The correction is

\[ \Delta \delta_i = -\text{sgn}(I_D)G \sin(\Delta \phi_i) \]

Gain factor: \[ G = \frac{ggq E_0 N_u \lambda u K}{2 \gamma E_b} \]
• Through the dispersion $D_2$ at 2nd undulator, there are horizontal corrections
\[ \Delta x_i = -D_2 \Delta \delta_i \] \[ \Delta x'_i = -D'_2 \Delta \delta_i. \]
Thus there is also horizontal cooling.

• ZZ computed the horizontal cooling decrement
\[
\alpha_x = \frac{1}{2} \left( \frac{\Delta (x^2)}{x^2} + \frac{\Delta (x'^2)}{x'^2} \right) \\
= \frac{1}{2} \left[ 4GD_0 \eta_0' k \exp \left\{ -\frac{\Delta \phi_i}{2} \right\} - \frac{G^2 N_s}{2} \left( \eta_0'^2 + \frac{D_0^2}{\beta^2} \right) \frac{\beta}{\epsilon_x} \right]
\]
where $D_0$ and $-\eta_0'$ are $D$ and $D'$ at the 2nd undulator.
Thus, there is no horizontal cooling if $\eta_0' = 0$, which can hardly be correct.
In order to understand OSC, we rederived all the equations.
2 OSC Decrement

- Longitudinal damping decrement:

\[ \alpha_\delta \equiv -\frac{\langle \delta_{ic}^2 - \delta_{c}^2 \rangle}{\sigma_{\delta}^2} = 2GkI_D e^{-u} - \frac{G^2 N_s}{2\sigma_{\delta}^2}, \]

where

\[ u = \frac{1}{2} k^2 [ (\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2) \epsilon_x + I_D^2 \sigma_{\delta}^2 ] \]

is the total thermal energy of the system.

- Horizontal damping decrement:

\[ \alpha_x \equiv -\frac{\langle P_{x2c}^2 + x_{2c}^2 - (P_{x2}^2 + x_2^2) \rangle}{\sigma_{x2}^2} = 2GkI_{\perp} e^{-u} - \frac{G^2 N_s \mathcal{H}_2}{2\epsilon_x}, \]
where

\[ \mathcal{H}_2 = \frac{D_2^2 + P_{D2}^2}{\beta_2} \]

is the \( \mathcal{H} \)-function at 2nd undulator, and

\[
I_\perp = -\frac{\beta_1}{\beta_2} \left\{ P_{D2} \left[ \left( \beta_2 M_{21} + \alpha_2 M_{11} \right) - \frac{\alpha_1}{\beta_1} \left( \beta_2 M_{22} + \alpha_2 M_{12} \right) \right] \left( I_1 - \frac{\alpha_1}{\beta_1} I_2 \right) + \frac{\beta_2 M_{22} + \alpha_2 M_{12}}{\beta_1^2} I_2 \right\} + D_2 \left[ \left( M_{11} - \frac{\alpha_1}{\beta_1} M_{12} \right) \left( I_1 - \frac{\alpha_1}{\beta_1} I_2 \right) + \frac{M_{12}}{\beta_1^2} I_2 \right] \}
\]

Thus the necessary condition of horizontal damping is \( I_\perp > 0 \).
3 OSC Dynamic

\[
\begin{align*}
\frac{d\varepsilon_x}{dt} &= -\frac{2GkI_\perp \varepsilon_x}{T_0} e^{-u} + \frac{G^2 N_s \mathcal{H}_2}{2T_0}, \\
\frac{d\sigma_\delta^2}{dt} &= -\frac{2GkI_D \sigma_\delta^2}{T_0} e^{-u} + \frac{G^2 N_s}{2T_0},
\end{align*}
\]

**Equal gain condition:** \( I_D = I_\perp \). Then obtain

\[
\frac{du}{dt} = -\frac{2G_0 k I_D}{T_0} u e^{-u} + \frac{G_0^2 N_s v}{2T_0},
\]

where

\[
v = \frac{1}{2} k^2 [ (\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2) \mathcal{H}_2 + I_D^2 ].
\]

**Optimum gain** gives

\[
G_{opt} = \frac{2k I_D}{v N_s} u e^{-u} \quad \rightarrow \quad \frac{du}{dt} = -\frac{2k^2 I_D^2}{v N_s T_0} u^2 e^{-2u}.
\]
Because of the $u^2 e^{-2u}$ factor, cooling slows down.
• Actually $G_{\text{opt}}$ is usually very large; so is output power of the laser amplifier.

**Tevatron:**
with $B = 10$ T
$\hat{P} \approx 6.5$ MW
$\bar{P} \approx 67$ kW
$\tau = 74$ s
$N_s = 1.5 \times 10^6$.

• OSC does not favor small $\gamma$ because less photons will be emitted.
OSC also does not favor very large $\gamma$ because the beam becomes too stiff to be bent inside the undulators. Optimum proton energy is $\gamma \approx 1$ to 2 TeV.
4 Non-Optimum Cooling

- The degradation of the Tevatron luminosity is of order hours. Therefore we can allow a much longer cooling time and employ a laser amplifier of much less gain (or lower output power).

- When \( G \ll G_{\text{opt}} \), the heating term can be neglected, leaving

\[
\frac{du}{dt} = -\frac{2G_0 k I_D}{T_0} u e^{-u}.
\]

- Max. cooling rate is at \( u = 1 \). But that will not give the shortest cooling time.

- Since the interaction of beam with photons is

\[
\Delta \delta_i = -\text{sgn}(I_D) G \sin(\Delta \phi_i)
\]

In order that the energy-offset particles receive the right correction, we must have

\[
\Delta \phi |_{\pm 3 \sigma} < \frac{\pi}{2}.
\]
Since

$$\Delta \phi_i = k(x_i I_1 + x' I_2 + \delta_i I_D)$$

and $I_1$ and $I_2$ can be made very small in the bypass design, the phase restriction gives

$$k I_D \sigma_\delta = \frac{\pi}{6}.$$ 

Therefore

$$u = \frac{1}{2} k^2 [ (\beta_1 I_1^2 - 2 \alpha_1 I_1 I_2 + \gamma I_2^2) \epsilon_x + I_D^2 \sigma_\delta^2 ] \approx \frac{\pi^2}{72}.$$
4.1 Average Power

- Average power is
  \[ \bar{P} = g^2 \frac{W_0}{T_0} N_{tot} \]

where photon energy emitted by one particle is
  \[ W_0 = \frac{1}{2} \varepsilon_0 \varepsilon^2 A c \Delta t_R \, . \]

This photon emission has area of cross section at the waist

\[ A \approx \frac{N_u \lambda u \lambda}{8} \]

assuming \( Z_R \approx \frac{1}{4} N_u \lambda_u \), and duration
  \[ \Delta t_r \approx \frac{N_u \lambda}{c} \, . \]
\[ \bar{P} = \frac{(E_b/q)^2 N_{\text{tot}} \lambda G^2}{16 Z_0 c T_0 F \xi [J J]^2} \]

Cooling time is
\[ \frac{\tau}{T_0} = \frac{3a \sigma_\delta}{\pi G} \quad \text{with} \quad a = \frac{e^u}{\sqrt{1 + 2u - u^2}}. \]

Eliminating \( G \),
\[ \left( \frac{\tau}{T_0} \right)^2 \bar{P} = \frac{(\sigma_\delta E_b/q)^2 N_{\text{tot}} \lambda}{Z_0 c T_0 F \xi [J J]^2} \left( \frac{3a}{4\pi} \right)^2 \]
Application to Tevatron

- Normal temperature undulator: $B \leq 1$ T.
- Superconducting undulator: $B \leq 6$ T.
- Super-fluid undulator $B > 6$ T.

For example, at $B = 6$ T, $\lambda_u = 1.93$ m. $\tau = 1200$ s $\rightarrow \bar{P} = 5.5$ W.
Notice that further increases $B$ does not help much.
The ratio of heating to cooling in the cooling equation is indeed small.
RHIC:
Gold at 100 GeV/u
Power $\propto \frac{A^4}{Z^4} = 38.7$
with $A=197$, $Z=79$
Power $\propto \gamma^{-2}$
$\gamma = 106$
10 times < Tevatron
$N = 1 \times 10^9$
90 bunches

- Because $\gamma$ is 10 times smaller, undulator period decreases 100 times to $\lambda_u \approx 2.3$ cm. Thus, only normal temperature undulator, or $B = 1$ T.

- $\tau = 1200 \text{ s} \rightarrow \bar{P} = 163 \text{ kW}$.

OSC is not suitable for heavy ions.
5 Electron Rings

$\sigma_\delta = 1 \times 10^{-4}$

$\sigma_l = 1.0 \text{ cm}$

$N = 2.7 \times 10^{11}$

$B = 1 \text{ T}$

- With $\lambda = 1 \mu\text{m}$ and $B = 1 \text{ T}$,

$$\gamma_{\text{min}} = \sqrt{\frac{4\sqrt{2\pi}}{3\sqrt{3}}} \sqrt{\frac{mc}{qB\lambda}} = 76.3 \quad \text{or} \quad E_b = 39 \text{ MeV} \quad \text{and} \quad \lambda_u \approx 0.9 \text{ cm}$$
• Superconducting undulator cannot be used, needs $B \approx 1$ T.
  
  Most electron rings are on high $\gamma$ side.

  Power is low because it scales as

  \[ \text{Power} \propto \left( \frac{\gamma m}{q} \right)^2 \]

  Minimum peak power is $\sim 2$ mW.

• OSC seems to favor high energy electron rings.

  However, OSC damping time $\propto N_s \sim \frac{N N_u \lambda}{\text{bunch length}}$ is energy independent.

  On the other hand, radiation damping time decreases rapidly with energy.

  Thus, OSC is really good for low energy electron rings only.
- Convert the IUCF 85 m Cooler Ring into an electron storage ring. For a transverse OSC cooling time of 0.1 s, the change in emittance is

- For electron less than 0.3 GeV, the OSC reduces the emittance by more than 1 order of magnitude. But is not efficient at all for higher energies.
6 Conclusion

- A necessary condition has been derived for transverse OSC: $I_\perp > 0$.

- Power $\propto \gamma^{-2}$ for small $\gamma$ and $\propto \gamma^2$ for large $\gamma$.
  Thus, OSC may not be possible for VLHC.

- Power can be further reduced if the photons can be focused to a size $Z_R = \frac{1}{4} N_u \lambda_u$.
  But this must be larger than the particle beam size.

- Output power for proton rings can be reduced by using superconducting undulators operating at below optimum gain.

- OSC is not suitable for heavy ions because of $\left(\frac{A}{Z}\right)^4$ factor, especially RHIC where $\gamma$ is not large enough.

- For electrons, $\lambda_u$ is small and only $B = 1$ T is possible.
  Output power is mostly $\propto (\gamma m/q)^2$.
  Thus, OSC favors very low energy electron storage rings which have small radiation damping times.