

Review of applications to laser accelerators in vacuum

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IFEL experiments

IFELA: Wernick & Marshall 1992 (*PRA* 46, 3566)

- First proof-of-principle IFEL experiment
- 5 MW at $\lambda = 1.6$ mm, gradient 0.7 MV/m, gain 0.2 MeV

BNL-IFEL: Van Steenbergen, Gallardo et al. 1996 (*PRL* 77, 2690)

- Microbunching observed 1998 (*PRL* 80 4418)
- 1-2 GW at $\lambda = 10.6$ μ m, gradient 2.5 MV/m, gain 1 MeV

MIFELA: Yoder, Marshall, Hirshfield 2001 (*PRL* 86, 1765)

- All electrons accelerated, phase dependency of the acceleration
- 6 MW at $\lambda = 10$ cm, gradient 0.43 MV/m, gain 0.35 MeV

STELLA: Kimura et al. 2001 (~~to be published~~, AAC proceedings) 2000

- First staging of two IFEL modules. (*Rhys. Rev. Spec. Top.* 4, 101301, 2000)
- 0.1-0.5 GW at $\lambda = 10.6$ μ m, gain up to 2 MeV

STELLA 2

- 5 GW $\lambda = 10.6$ μ m gain ~ 4 MeV. AAC 2002
 $\Delta W/W = 12\%$

Laser fields for Gaussian fundamental TEM₀₀ mode.

$$\vec{E} = E_x \vec{e}_x + E_z \vec{e}_z \quad \vec{B} = B_y \vec{e}_y + B_z \vec{e}_z$$

$$E_x = E_0 \frac{e^{-\frac{x^2+y^2}{w(z)^2 w_0^2}}}{w(z)} \cdot \sin \left(kz - \omega t + \varphi_0 + \frac{k(x^2+y^2)}{2R(z)} - \arctg \frac{z}{z_R} \right)$$

$$E_z = E_0 \frac{e^{-\frac{x^2+y^2}{w(z)^2 w_0^2}}}{k \cdot w(z)^2 \cdot w_0^2} \cdot 2x \cdot \cos \left(kz - \omega t + \varphi_0 + \frac{k(x^2+y^2)}{2R(z)} - 2 \cdot \arctg \frac{z}{z_R} \right)$$

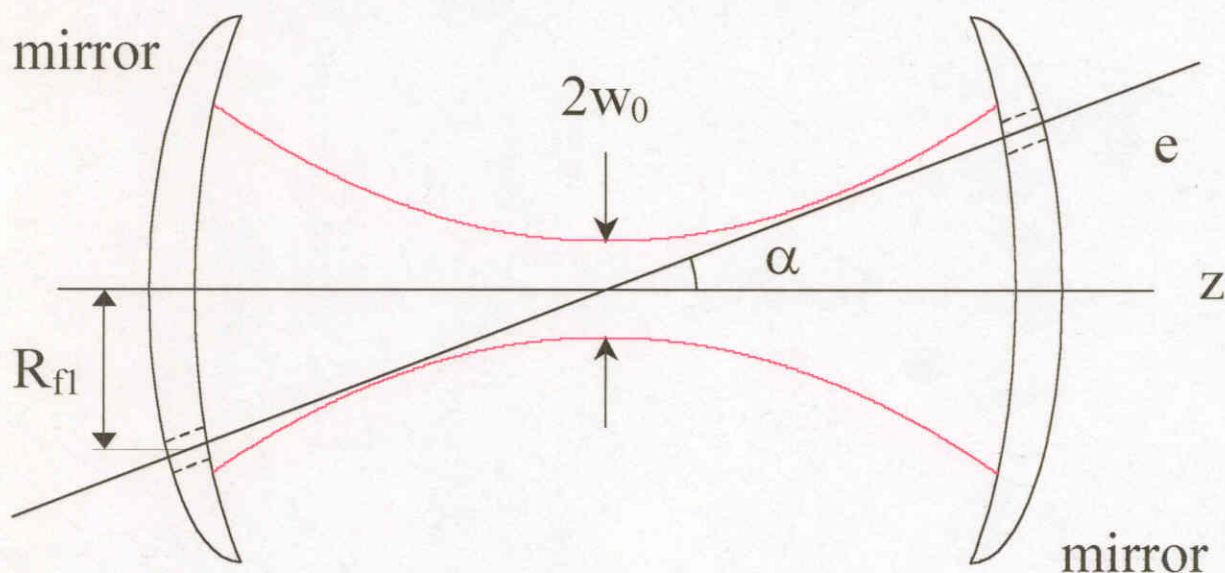
$$B_y = B_0 \frac{e^{-\frac{x^2+y^2}{w(z)^2 w_0^2}}}{w(z)} \cdot \sin \left(kz - \omega t + \varphi_0 + \frac{k(x^2+y^2)}{2R(z)} - \arctg \frac{z}{z_R} \right)$$

$$B_z = B_0 \frac{e^{-\frac{x^2+y^2}{w(z)^2 w_0^2}}}{k \cdot w(z)^2 \cdot w_0^2} \cdot 2y \cdot \cos \left(kz - \omega t + \varphi_0 + \frac{k(x^2+y^2)}{2R(z)} - 2 \cdot \arctg \frac{z}{z_R} \right)$$

where

$$k = \frac{2\pi}{\lambda}; \quad z_R = \frac{k w_0^2}{2}; \quad w(z)^2 = 1 + \frac{z^2}{z_R^2}; \quad R(z) = z + \frac{z_R}{z}$$

w_0 – radius of the field E waist; z_R – Rayleigh length



Phase synchronization improvement

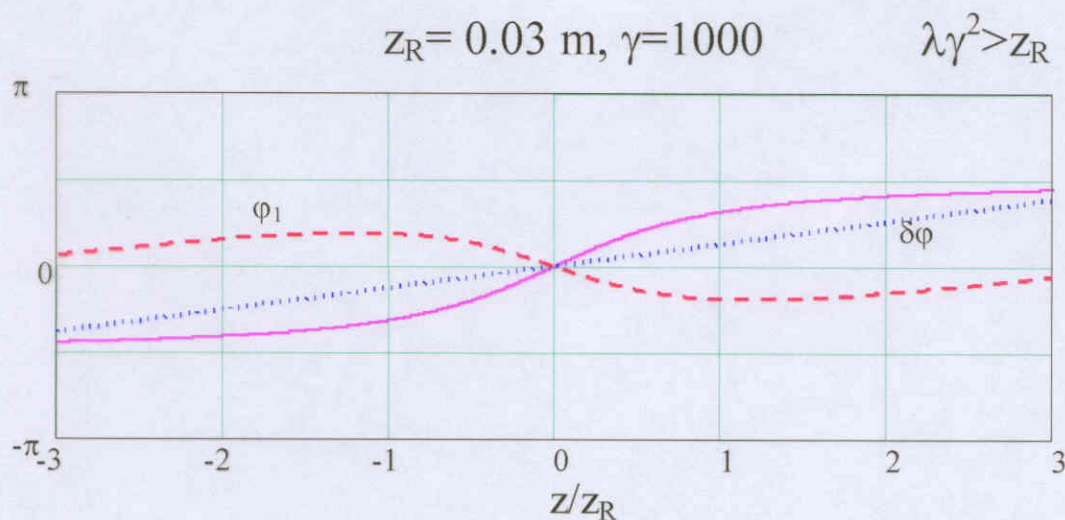
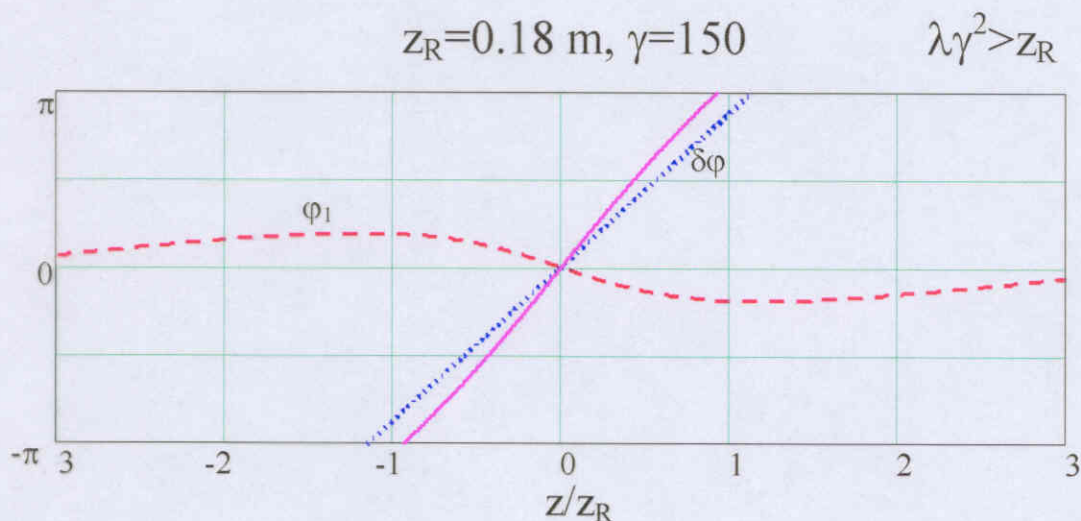
For particle crossing the resonator at angle α

$$\delta\varphi = \frac{2\pi}{\lambda} z \frac{1 + \alpha^2 \gamma^2}{2\gamma^2} \quad \text{- phase slippage with respect to a plane wave}$$

$$\varphi_1 = \frac{k(x^2 + y^2)}{2R_z} - \text{arctg} \frac{z}{z_R} \quad \text{- additional phase of Gaussian mode}$$

$\varphi_1 - \delta\varphi$ - relative phase seeng by particle,
can be more smooth due to $k(x^2 + y^2)/2R_z$ term.

$$\alpha_{\text{opt}} \cong \frac{1}{2} \sqrt{\frac{\lambda_s}{2z_R}}$$



Accelerating field profiles E_α in Gaussian beam

$$\text{Noncollinearity angle } \alpha_{opt} = \frac{1}{2} \sqrt{\frac{\lambda_s}{2z_R}}$$

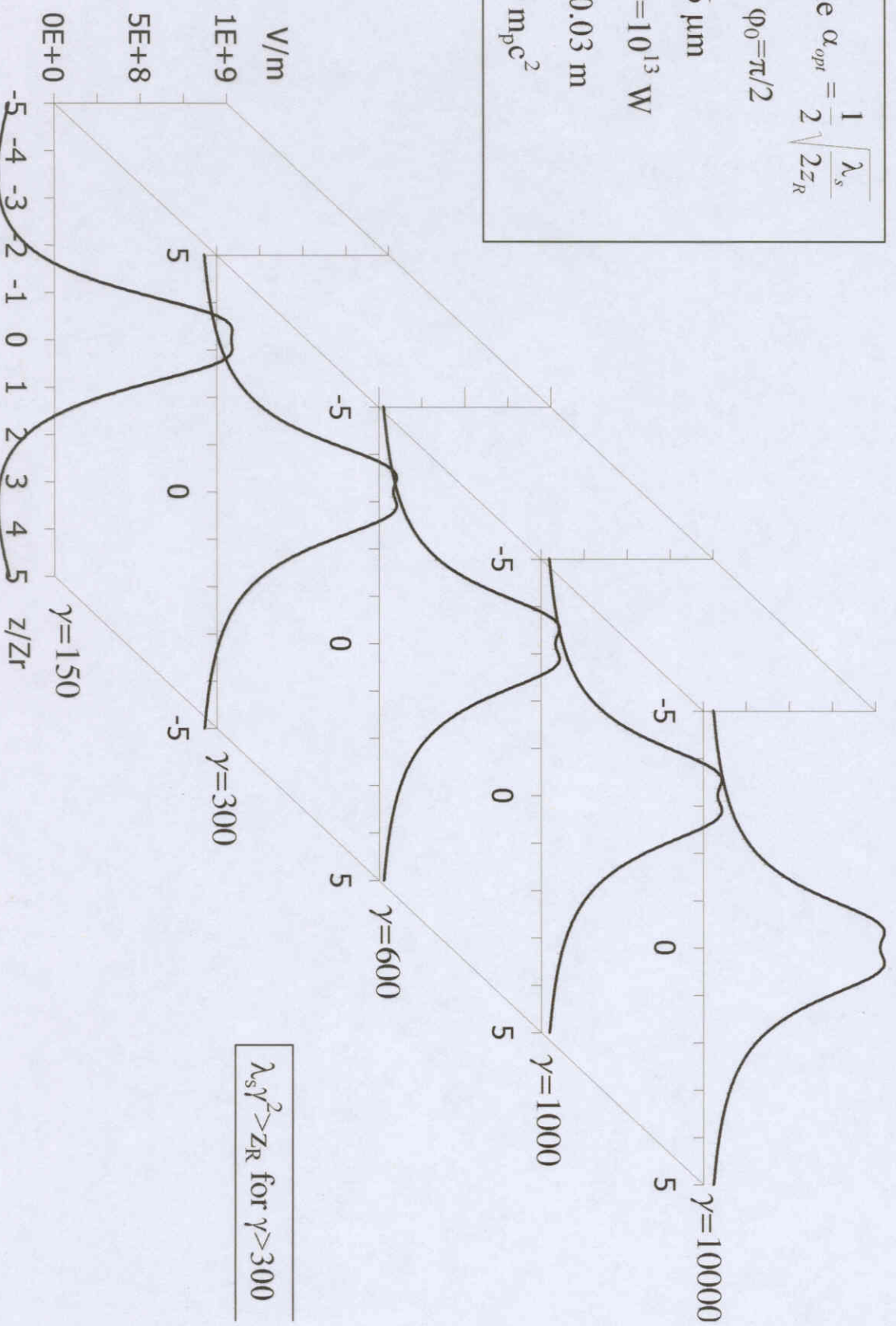
Initial particle phase $\varphi_0 = \pi/2$

Wavelength $\lambda_s = 10.6 \text{ } \mu\text{m}$

Laser beam power $P = 10^{13} \text{ W}$

Rayleigh length $z_R = 0.03 \text{ m}$

Particle energy $E_p = \gamma m_p c^2$



$$\lambda_s \gamma^2 > z_R \text{ for } \gamma > 300$$

The shapes are similar to that for $z_R = 0.18$ with 6 times higher peak fields.

Accelerating field profiles E_α in Gaussian beam

$$\text{Noncollinearity angle } \alpha_{opt} = \frac{1}{2} \sqrt{\frac{\lambda_s}{2z_R}}$$

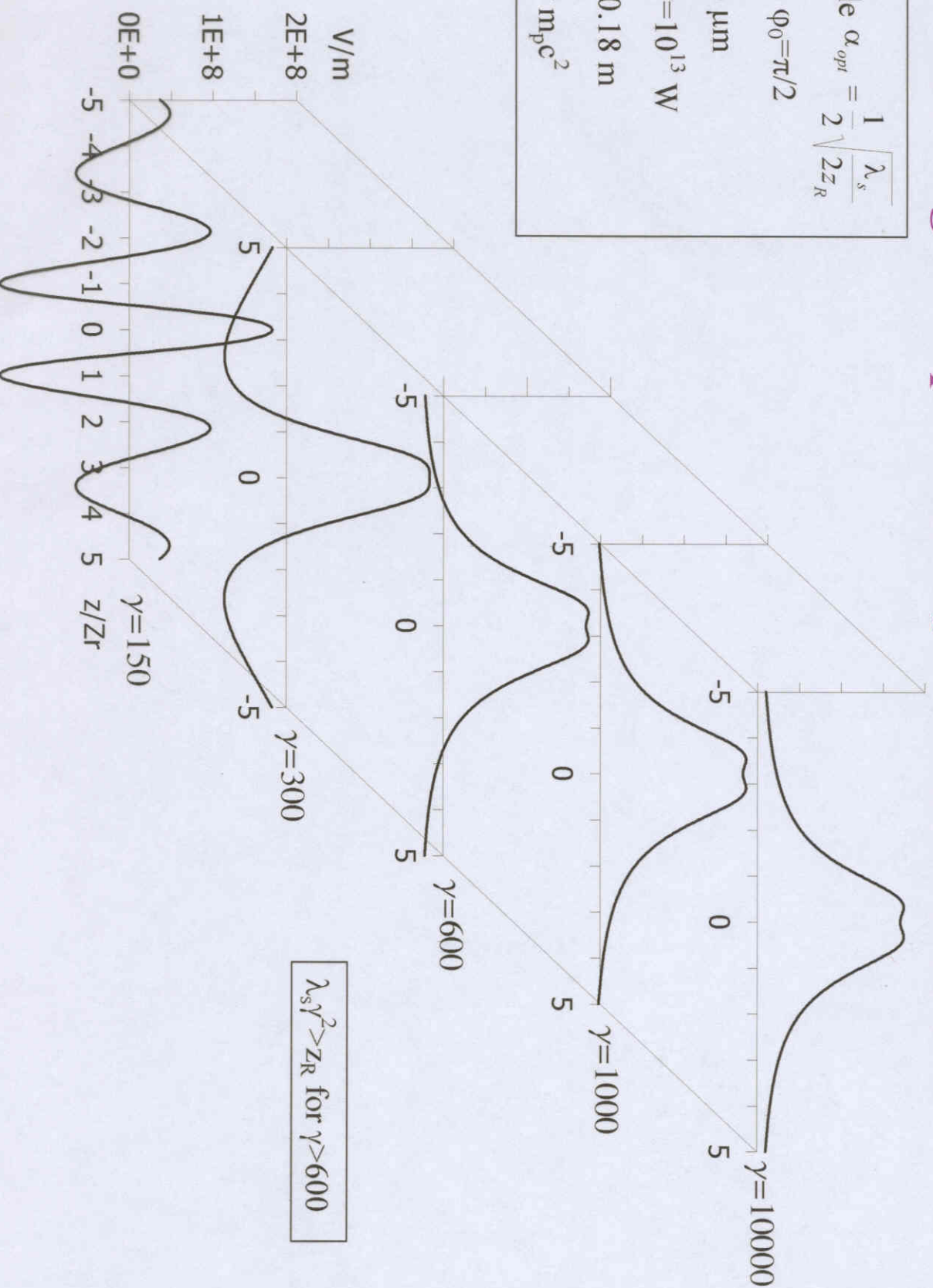
Initial particle phase $\varphi_0 = \pi/2$

Wavelength $\lambda_s = 10.6 \mu\text{m}$

Laser beam power $P = 10^{13} \text{ W}$

Rayleigh length $z_R = 0.18 \text{ m}$

Particle energy $E_p = \gamma m_p c^2$

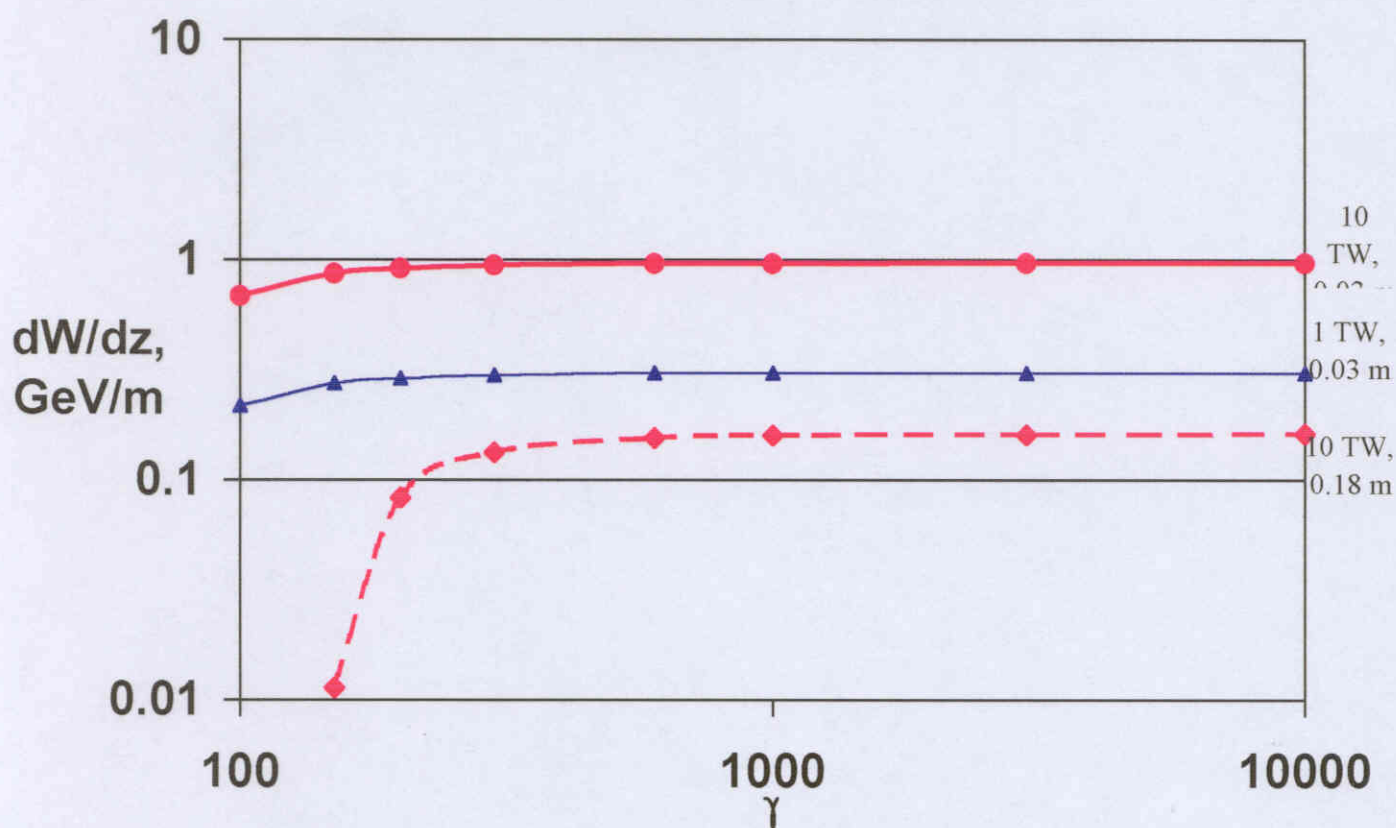


For $\gamma > 600$ acceleration is positive in wide range $-2z_R \div 2z_R$.

Profile shapes are not dependent on γ (at $\gamma > 600$) and on laser power P .

Acceleration rate in Gaussian beam as a function of particle energy.

Laser beam power $P=10^{13}$ W - red line
 $P=10^{12}$ W - blue line
 Rayleigh length $z_R=0.03$ m - solid line
 $z_R=0.18$ m - dashed line
 Noncollinearity $\alpha=\alpha_{\text{opt}}$

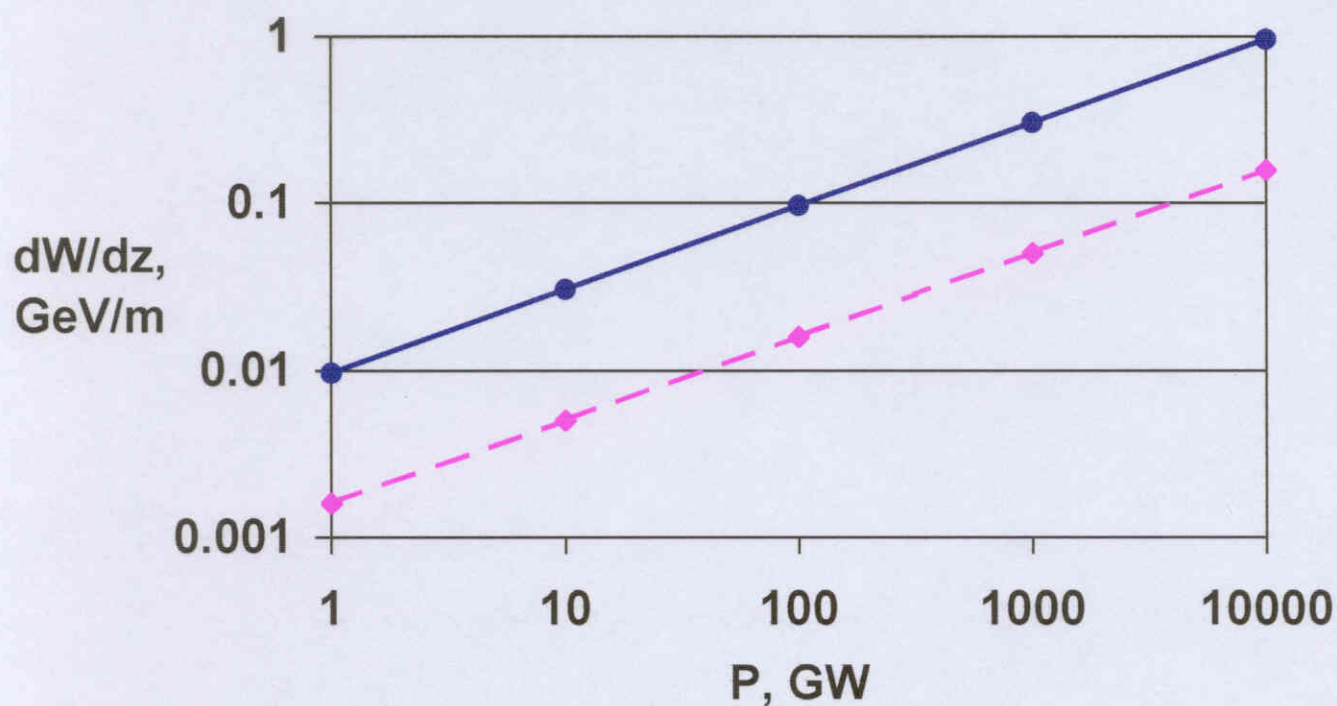


For $\gamma > 300$ the following relation is valid:

$$\frac{dW}{dz} \left(\frac{\text{eV}}{\text{m}} \right) = 9.15 \cdot \frac{\sqrt{P(\text{W})}}{z_R(\text{m})}$$

Acceleration rate in Gaussian beam as a function of the laser beam power P

Particle energy $\gamma=10^3$
 Rayleigh length $z_R=0.03$ m - solid line
 $z_R=0.18$ m - dashed line
 Noncollinearity $\alpha=\alpha_{\text{opt}}$



$$\frac{dW}{dz} \left(\frac{\text{eV}}{\text{m}} \right) = 9.15 \cdot \frac{\sqrt{P(\text{W})}}{z_R(\text{m})}$$

Relation is valid in wide range of power P.

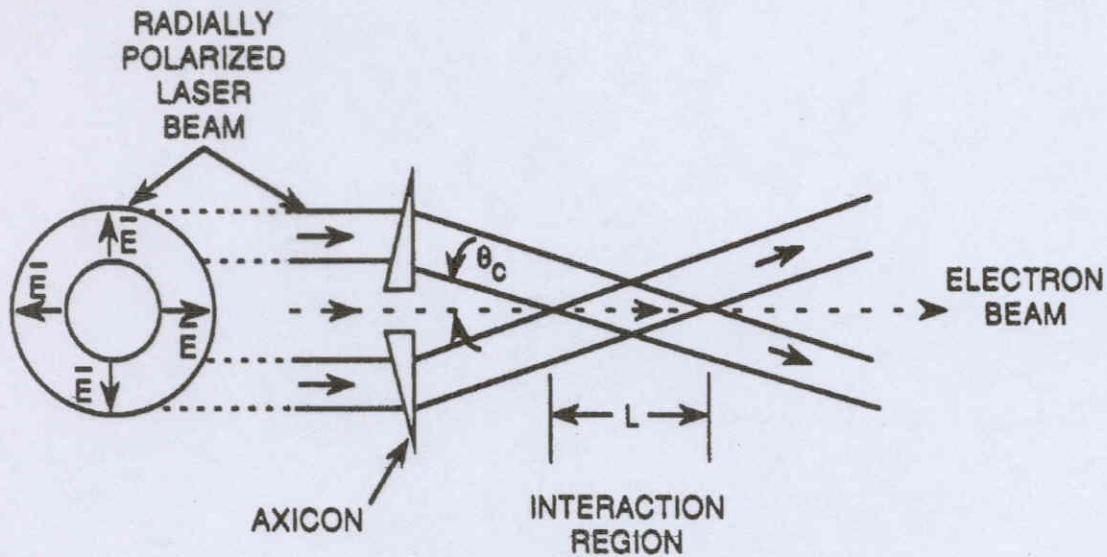
Comment on particle beam quality requirements

- For $\varepsilon_n = 1$ mm mrad the acceleration gain per one section is $\Delta W/W = 0.01$
- Focusing and crossing angle α
 - For $\alpha = 1/\gamma$ laser field induce no net transverse deviation of particle in vicinity of the focus

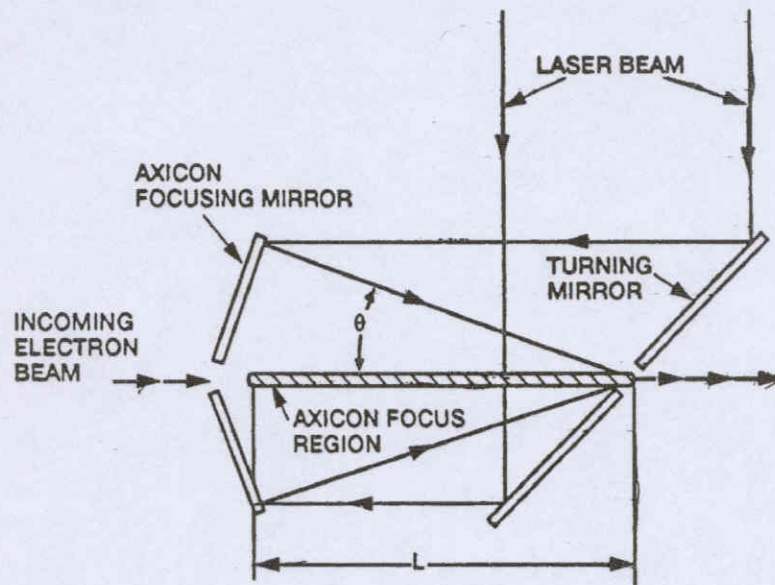
$$e \left(E_0 \cos \alpha - \frac{v}{c} H_0 \right) \cong 0$$
 - Acceleration and beam focusing are correlated
 - $\alpha > 1/\gamma$ perfect acceleration along with focusing
deacceleration along with defocusing.
 - $\alpha < 1/\gamma$ correlation is opposite: acceleration is
accompanying by defocusing
 - $\alpha = \alpha_{\text{opt}}, \gamma = 10^3$ acceleration and focusing take place simultaneously.
- Acceleration rate as a function of initial particle phase φ
 - $\Delta\varphi = 0 \quad dW/dz = (dW/dz)_{\text{max}}$
 - $\Delta\varphi = \pm\pi/2 \quad dW/dz = 0$
 - $\Delta\varphi = 0 \div \pi/2$ slope, feed back possible for beam stability
- Multiple section acceleration, possibilities for tuning
 - Intersection phase shifts
 - Intersection crossing angle correction and dispersion
 - Small transverse displacements of laser beam lines in sections stability and beam acceptances

Vacuum laser acceleration with axicon

- Axicon laser beam



- Schematic of optical system and termination of interaction



- Special properties

- Radially-polarized axisymmetric electric field
- Line axicon focus with pure longitudinal field E_z on Z axis
- Limited interaction length L_{int} (for nonzero net acceleration)

Axicon focus field

- Field on the of the focus (z axis)

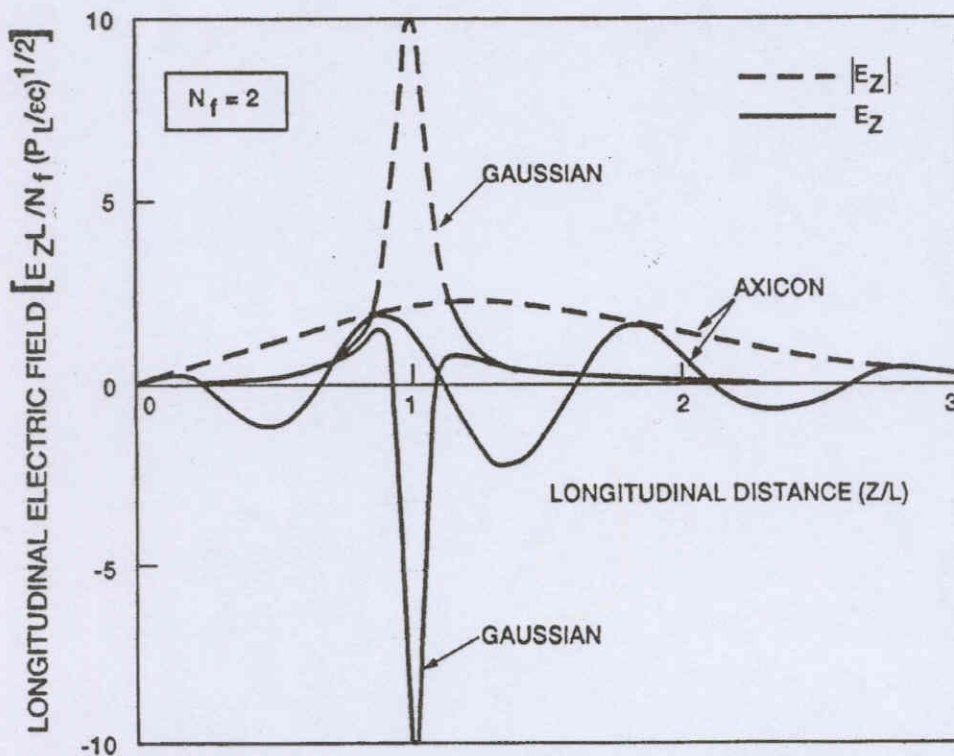
$$E_z(0, z, t) = E_0 \theta \frac{2\sqrt{e\theta ka}}{\left[\frac{z}{L} + i\theta ka\right]^2} Q(\xi) \exp\left[i(\omega t - kz) + i\frac{\pi}{2}\right]$$

where

$$Q(\xi) = 1 + \xi^2 + \sqrt{\pi}\xi\left(\frac{3}{2} + \xi^2\right)e^{\xi^2}[1 + \operatorname{erf}(\xi)];$$

$$\xi = ika\left[\frac{z/L}{2(z/L + i\theta ka)}\right]^{\frac{1}{2}}$$

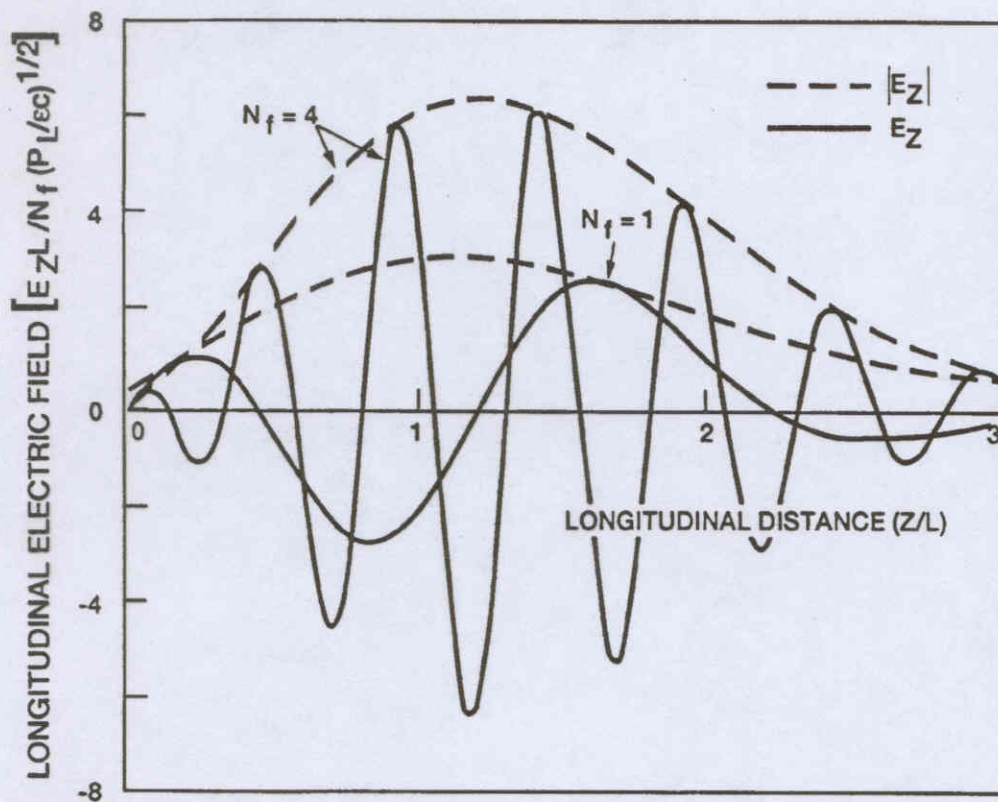
- Structure of the longitudinal electric field along z-axis at r=0



- The factor $\exp[i(\omega t - kz)]$ is dropped
- $L_{\text{int}} = L$ contains some peaks,

$$z_R = L / 2\pi N_f$$

- Longitudinal electric field structure for an axicon focus with two different Fresnel numbers



The factor $\exp[i(\omega t - kz)]$ is dropped. Oscillation comes from field complex amplitude structure mainly, the fact that rays cross the axis at angle θ

$$\Delta\varphi = \frac{2\pi\Delta z}{2\lambda_s \gamma^2 / (1 + \theta^2)} \cong \frac{2\pi}{\lambda_s} \cdot \frac{\Delta z \theta^2}{2} ;$$

- $z_{\text{eff}} = L/N_f$ caused by finite $\theta \gg 1/\gamma$
 $z_R = L/2\pi N_f$ takes into account phase of the focused wave (not plane wave)

- Acceleration rate achievable with axicon.

- $E_{z \max} = (E_0/ka) \cdot N_f^{3/2}$;

- $\left\langle \frac{dW}{dz} \right\rangle = q(E_{z \max}) \cdot \frac{\Delta z_{eff}}{2L} = 5.94 \left[\frac{P_L \lambda \theta}{a^3} \right]^{1/2} \frac{eV}{m}$

- Tendency

Larger θ gives larger $E_{z \max}$ but smaller $\Delta z_{eff} = L/N_f$.

(It is actually less if the wave phase is taken into account more correctly).

- Comparision

- Axicon (C. Steinhauer, W. Kimura, J. of Appl. Phys. 72, 3237, 1992)

P_L	Crossing angle	λ	$4a$	L_{int}	dW/dz
W	mrad	μm	cm	cm	MeV/m
10^{13}	100	10.6	1	5	155
10^{13}	100	10.6	0.4	2	612

- Gaussian mode

P_L	Crossing angle	λ	Waist w_0	$L_{int} = 2z_R$	dW/dz
W	mrad	μm	cm	cm	MeV/m
10^{13}	6.6	10.6	0.03	6	1200

- Gaussian mode provides twice higher acceleration rate with the same laser power on comparable path length.
Smooth phase channel (small crossing angles) provides single isolated peak of the field

IFEL in Gaussian beams

- Accelerator equations for planar wiggler, strong laser fields, plane wave approximation

$$\frac{d\gamma}{dz} = \frac{eE_0}{2mc^2} \cdot \frac{k}{\gamma} [J_0(G) - J_1(G)] \sin \psi + \frac{eE_0}{2mc^2} \frac{K_L}{\gamma} \sin 2k_w z - \frac{2}{3} r_e \gamma^2 k_w^2 \left\{ \frac{K^2}{2} + \frac{K_L^2}{2} + K_L K [J_0(G) + J_1(G)] \cos \psi - \frac{K^2 + K_L^2}{2} \cos 2k_w z \right\}$$

$$\frac{d\psi}{dz} = k_w - k_s \frac{1}{2\gamma^2} \left\{ 1 + \frac{K^2}{2} + \frac{K_L^2}{2} + K_L K [J_0(G) - J_1(G)] \cos \psi \right\}$$

Here

$$G(z) = \frac{k_s}{k_w} \cdot \frac{K^2 + K_L^2}{8\gamma^2}; \quad K = \frac{eB_{w0}}{mc^2 k_w}; \quad K_L = \frac{eE_0}{mc^2 k_s}; \quad k_s = \frac{\omega}{c}; \quad k_w = \frac{2\pi}{\lambda_w};$$

$$\psi(z) = \int_0^z k_w dz - k_s \int_0^z \frac{dz}{2\gamma^2} \left\{ 1 + \frac{K^2}{2} + \frac{K_L^2}{2} + K_L K [J_0(G) - J_1(G)] \cos \psi \right\} - \psi_0$$

J_0, J_1 – Bessel functions

- Adiabatic approximation

Small changing of parameters $k_w(z)$, $K(z)$, $K_L(z)$, $E_0(z)$, $G(z)$, $B_{w0}(z)$ on one period $2\pi/k_w$.

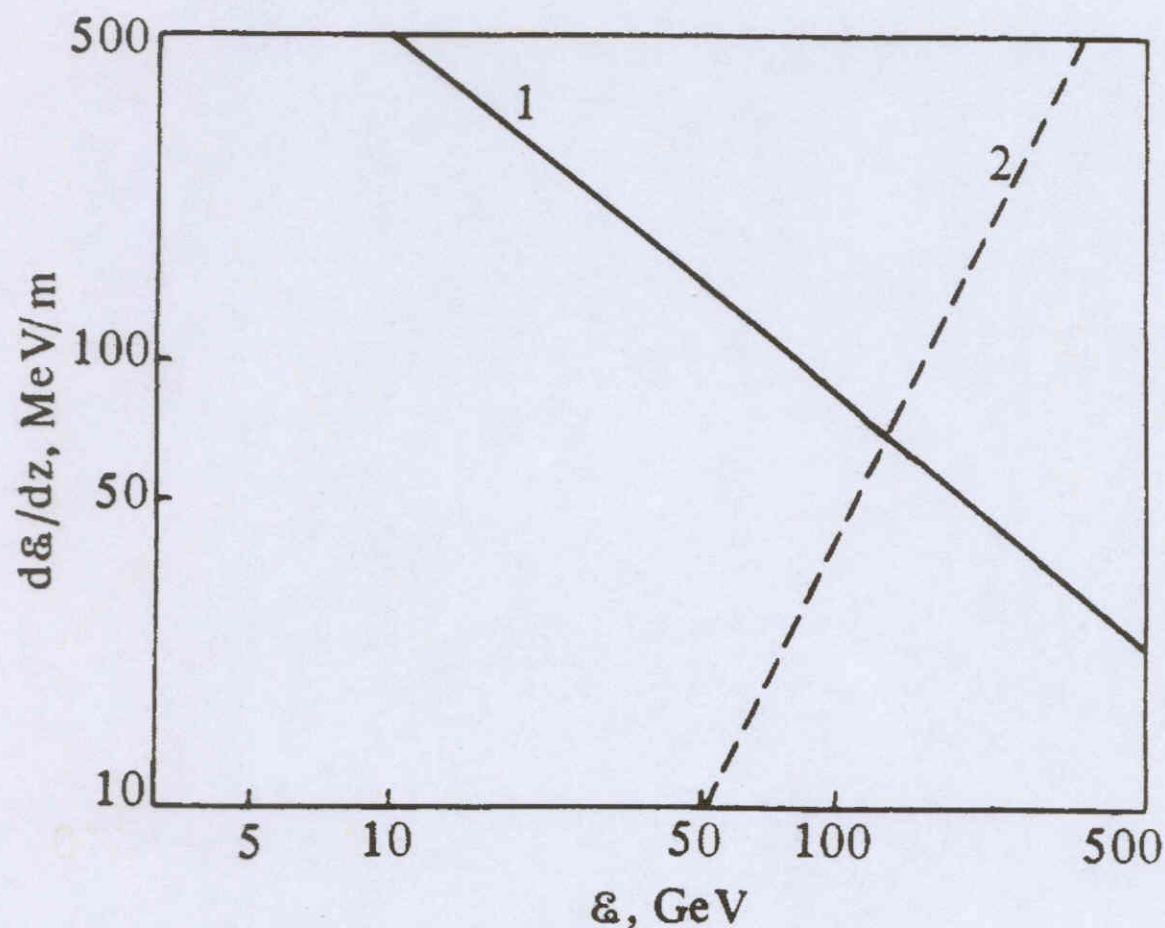
Laser field is not necessary small, $K_L \neq 0$.

Radiation losses limit the net acceleration to zero at high energies $\gamma > 10^6$.

- Plane wave and adiabatic approaches are not adequate for real performances.
- Beam emittance required

$$\begin{aligned} \beta &> \lambda_s \gamma^2 & \varepsilon_n &< 0.1 \cdot \lambda_s \gamma \\ z_R &> \lambda_s \gamma^2 & \sigma &= 0.1 \cdot w_0^2 \end{aligned}$$

Maximum acceleration rate in a collinear IFEL for a field of 10^{11} V/m and $\lambda_s=1 \mu\text{m}$ (solid line) and synchrotron-radiation energy losses per unit length in a bending-magnet field 5 T (dashed line)



IFEL in Gaussian beam, small z_R

- Analytical equations for Gaussian mode with diffraction in 1D approach ($\sigma_{\perp}=0$)

$$\frac{d\gamma}{dz} = \frac{eE_0}{2mc^2} \cdot \frac{k}{\gamma} [J_0(G) - J_1(G)] \frac{\sin \psi}{\left(1 - \frac{z^2}{z_R^2}\right)^{\frac{1}{2}}} + \frac{eE_0}{mc^2} \frac{K_L}{\gamma} \sin 2k_w z \frac{1}{\left(1 - \frac{z^2}{z_R^2}\right)^{\frac{1}{2}}};$$

$$\frac{d\psi}{dz} = k_w - k_s \frac{1}{2\gamma^2} \left\{ 1 + \frac{K^2}{2} + \frac{K_L^2}{2} + K_L K [J_0(G) - J_1(G)] \cos \psi + \gamma^2 \theta^2 \right\} - \frac{1}{z_R \left(1 + \frac{z^2}{z_R^2}\right)^{\frac{1}{2}}};$$

Approximation $r_{\perp}=0$ is not valid for focus region

• Conclusion

More correct consideration of Gaussian mode field and optimization $k(z)$ parameters for real performance numerical simulations are needed with using equations of motions and 3D field maps of both laser and undulator.

Inverse Free Electron Laser Project

UCLA – Kurchatov Institute

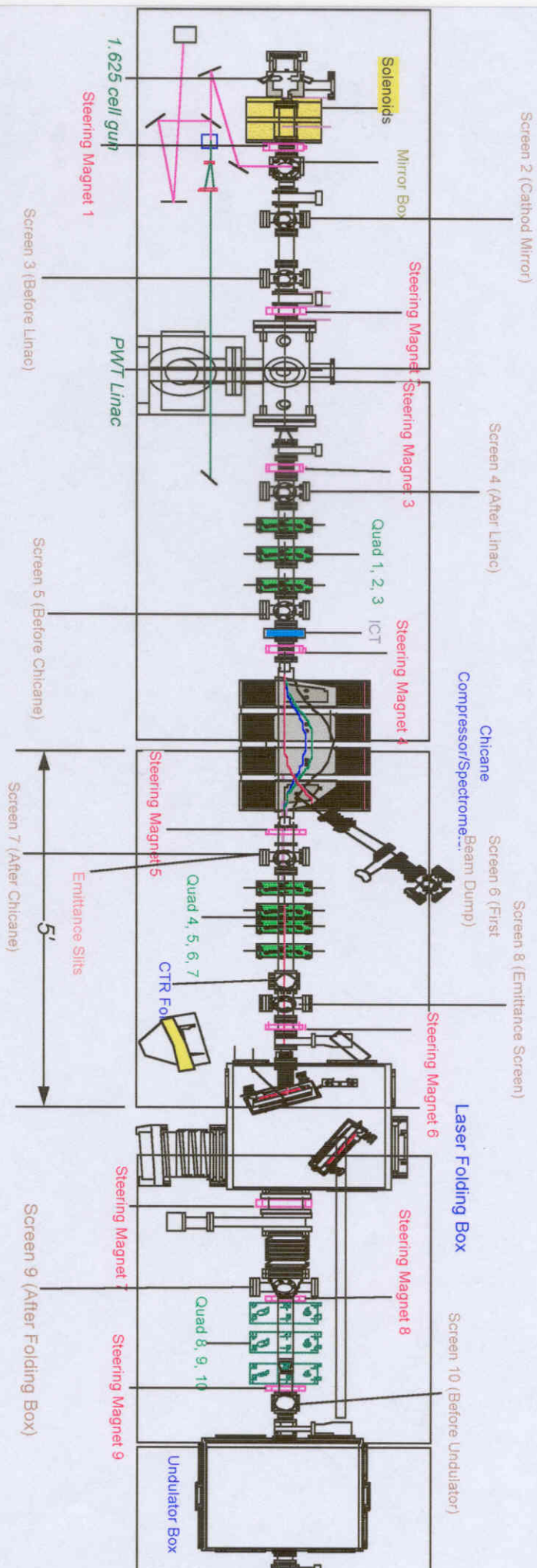
Non-adiabatic, diffraction dominated regime

- Powerfull (Twatt) CO₂ laser (most powerfull used for IFELs)
- Gaussian mode, short Rayleigh range, 3.6 cm, small focal waist 0.35 mm
- Short accelerator length, 50 cm
- Focal region is also using for acceleration
- Initial electron energy 14 MeV
- Normalized e.b. emittance 10 mm·mrad
- Charge 0.3 nC; pulse length 3 ps

Extraordinary characteristics

- High electron acceleration gain, 41 MeV
- High percentage of accelerated electrons, >20%

Neptune beamline



Non adiabatically tapered undulator for IFEL experiment at UCLA

- Basic parameters of IFEL for the undulator construction.
 - Laser wavelength $10.6 \mu\text{m}$
 - Laser power P $0.4 - 0.8 \text{ TW}$
 - Rayleigh length z_R 3.6 cm
 - Laser beam waist w_0 0.35 mm
 - Laser beam size at the undulator entrance ww_0 2.5 mm
 - Initial electron energy W_0 14 MeV
 - E. b. emittance ϵ_n 10 mm mrad
 - E. b. rms radius at focus 0.15 mm
 - E. b. rms radius at the undulator entrance 0.50 mm
 - Total acceleration length 50 cm
 - Total acceleration gain 40 MeV
- Basic undulator requirement
 - Transparency to laser and electron beams
 - Strong fields with strong tapering of both $\lambda_w(z)$ and $B_w(z)$
 - Optimized acceleration in Gaussian mode including focus area.

Numerical simulation procedure and magnet construction optimization

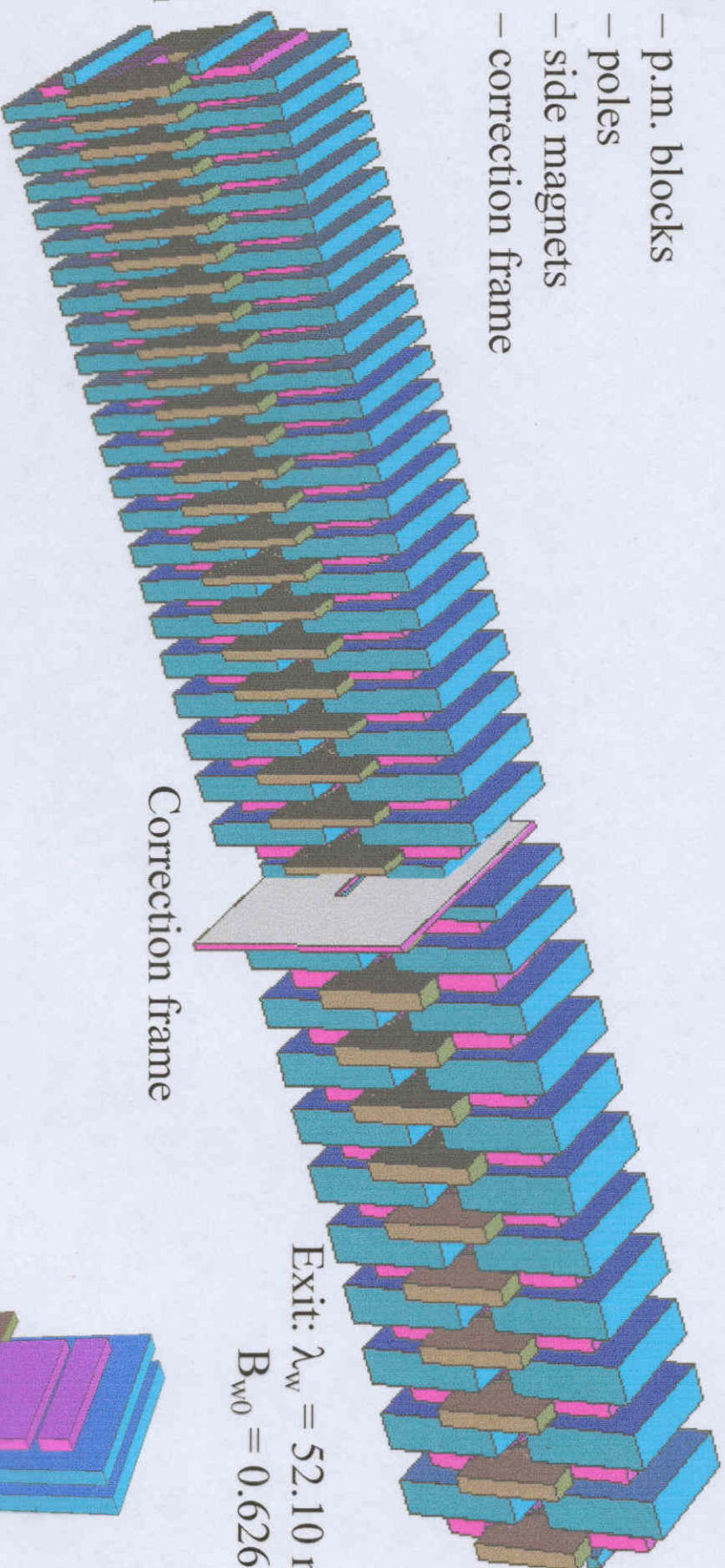
- Codes
 - Radia code for 3D magnetic field maps
 - Lorenz equation code with MathCAD for reference electron trajectory
 - 3D code TREDI, IFEL version 2
(Lienard Wiechert potential approach)
- Successive approximations
 - Reference electron trajectory, analysis of synchronizations
 - Corrections of the field
 - Correction limit 0.1%
- Optimization goal
 - Synchronism with the laser field
 - Maximum acceleration rate
 - Maximum acceptance (capture and acceleration of an electron bunch)
 - Small sensitivity to laser jitter

Schematic construction of planar hybrid undulator for IFEL project

- Blue – p.m. blocks
- Red – poles
- Brown – side magnets
- Gray – correction frame

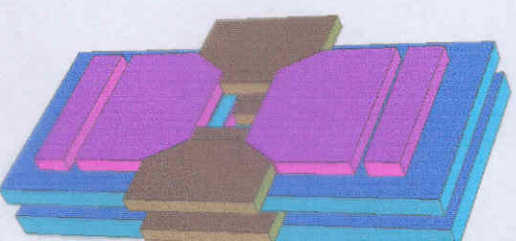
Gap 12 mm

Entrance: $\lambda_w = 15.16$ mm
 $B_{w0} = 0.115$ T



Correction frame

Exit: $\lambda_w = 52.10$ m
 $B_{w0} = 0.626$ T



Magnet cell

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Перв. примен.

Справ. №

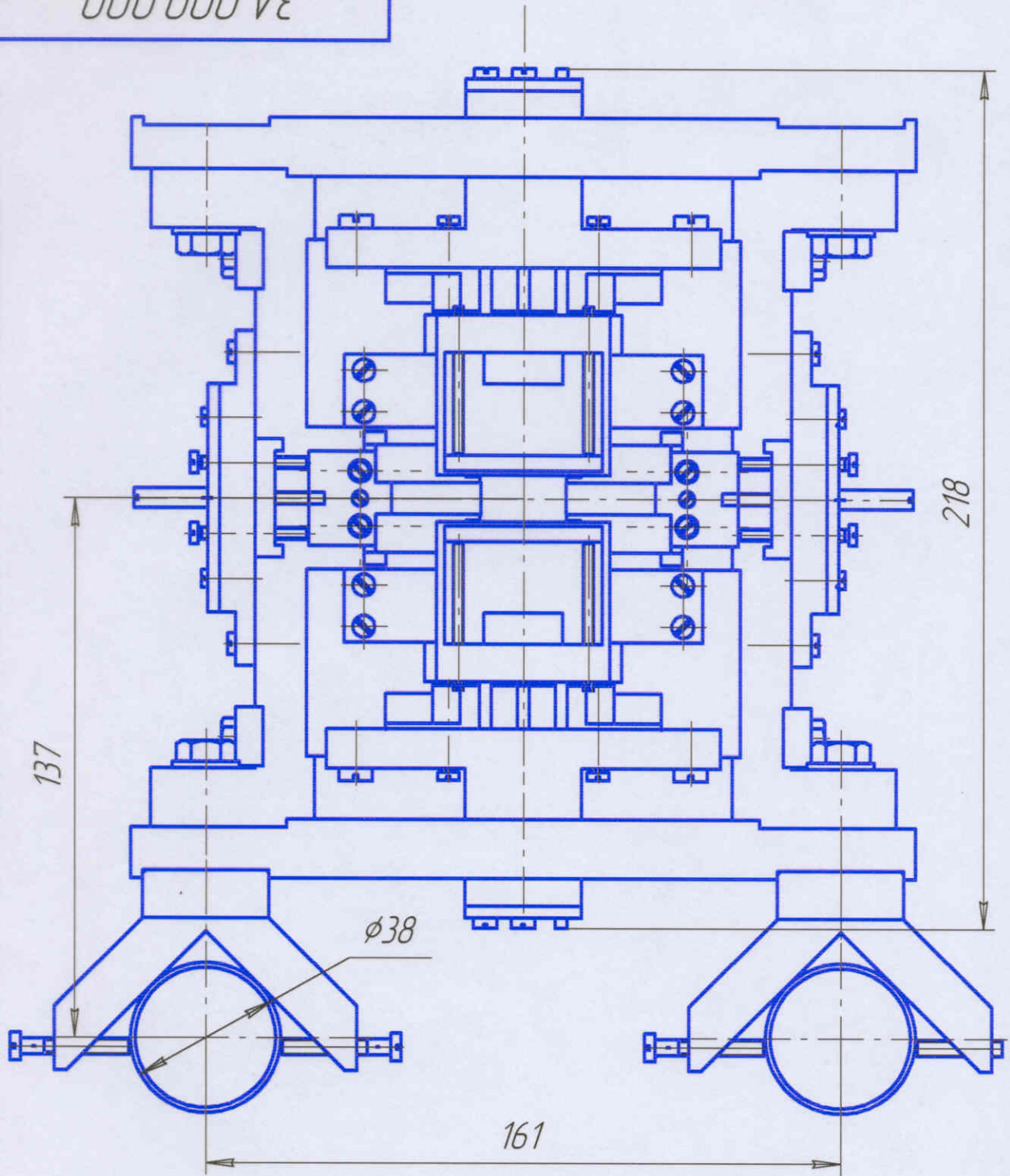
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Инв. № подл.



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Undulator
for Neptune Lab.

Лист Масса Масштаб

Лист Листов

RRC Kurchatov Institute

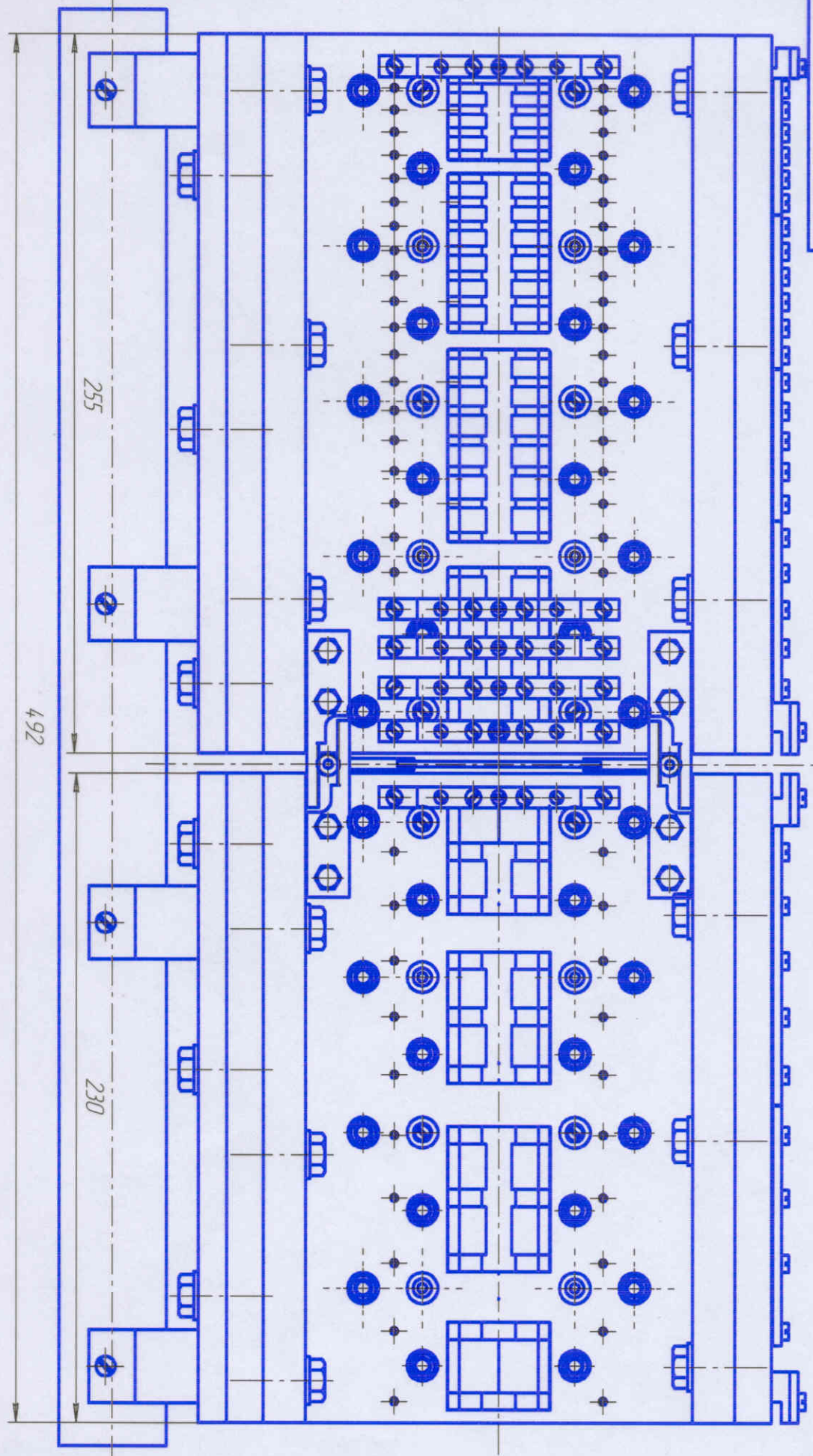
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Пров.				
Т.контр.				
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Формат А4

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Undulator
for Neptune Lab.

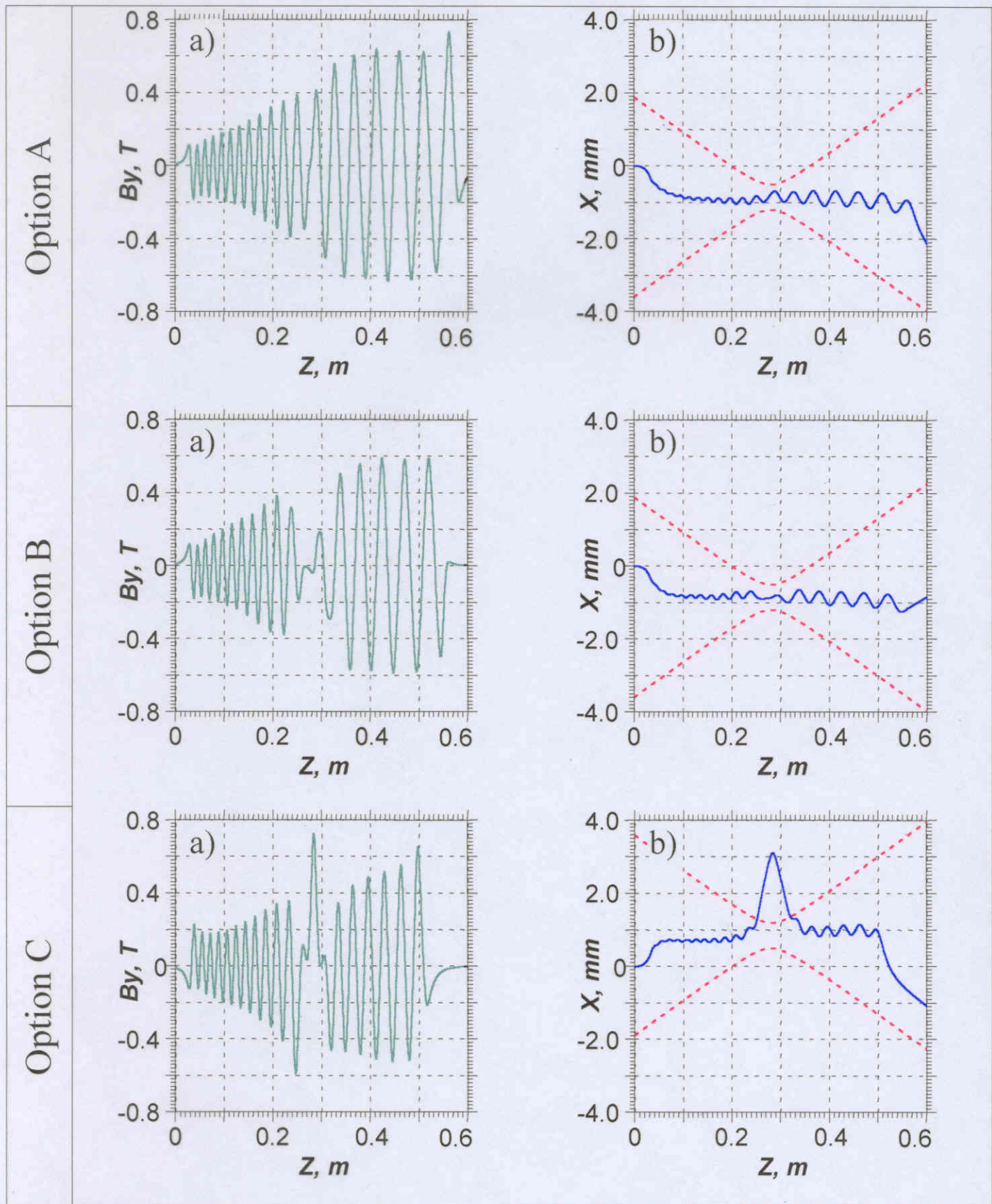
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RRC Kurchatov Institute

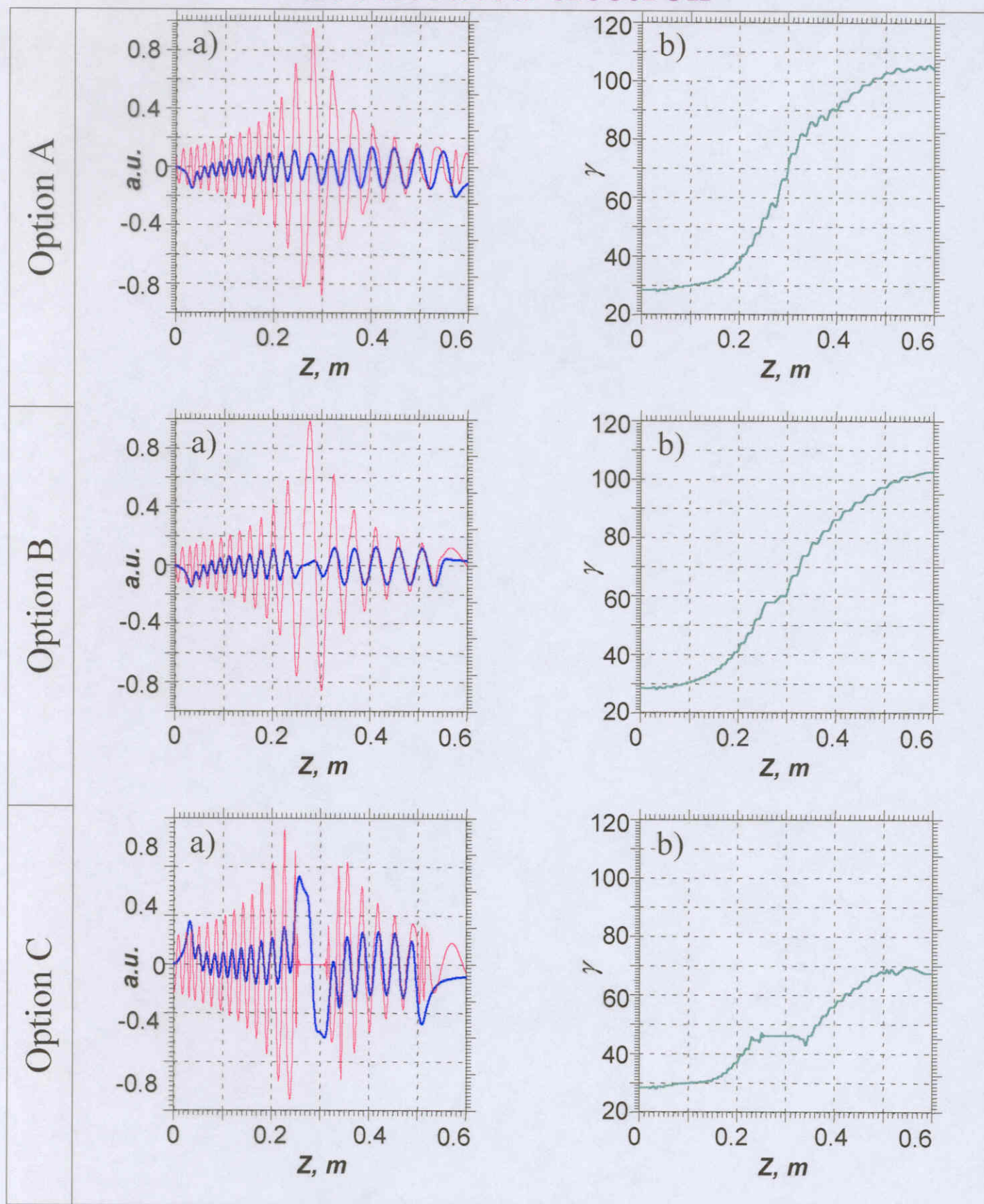
Копирован

Формат А3

Magnetic fields (a) and trapped electron trajectories (b) in the undulator

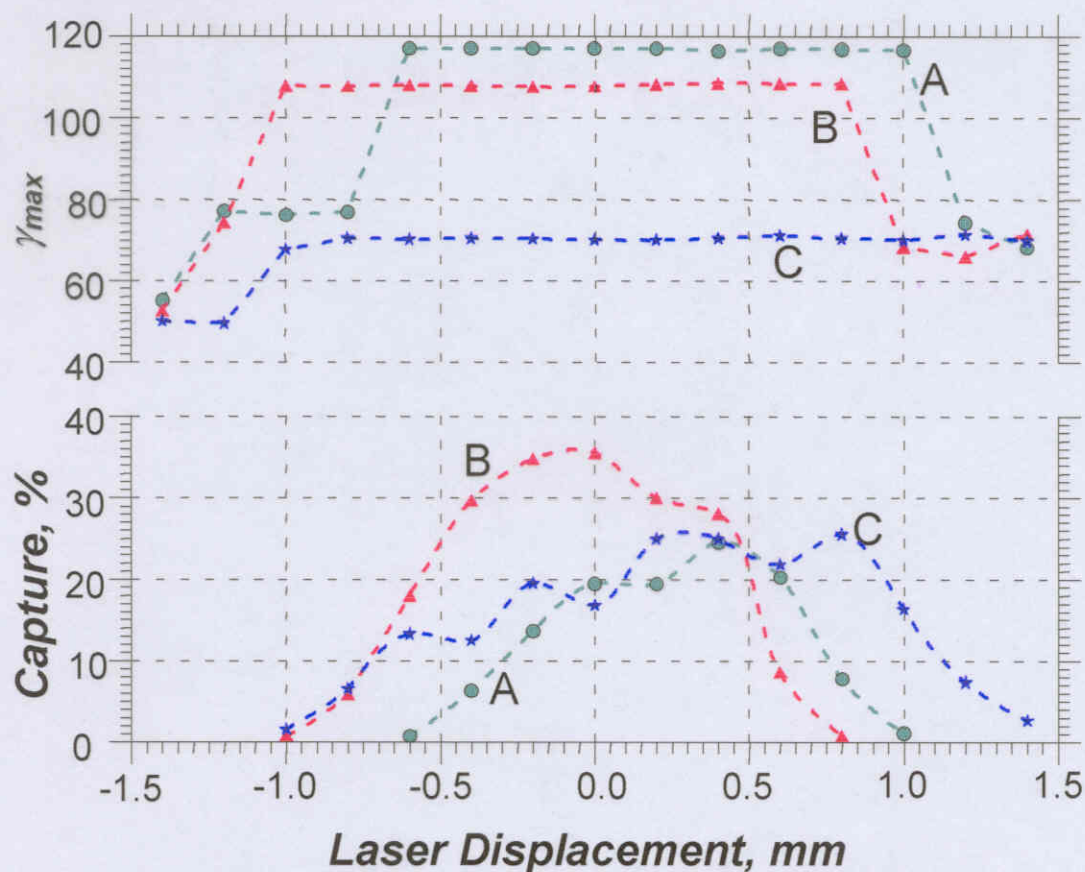


Synchronization curves and energies of accelerated electron



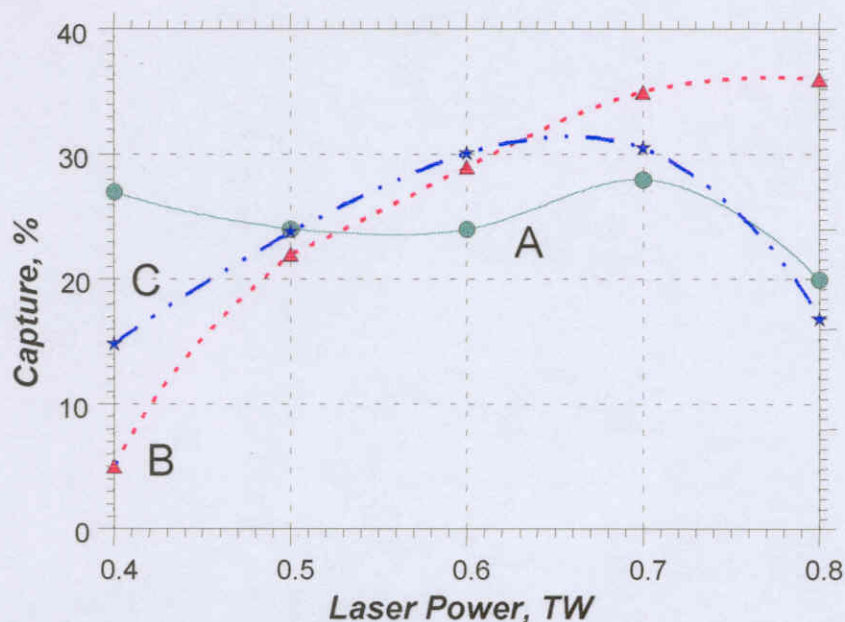
On the left: thin red line – laser field seen by electron
solid blue line – transverse velocity of the electron
On the right: accelerated electron energies along z .

• Sensitivity to laser beam jitter



The higher $\gamma_{\max}$ the more sensitivity to jitter.
 Displacement ± 0.25 mm can be acceptable.
 Option A was chosen for construction.

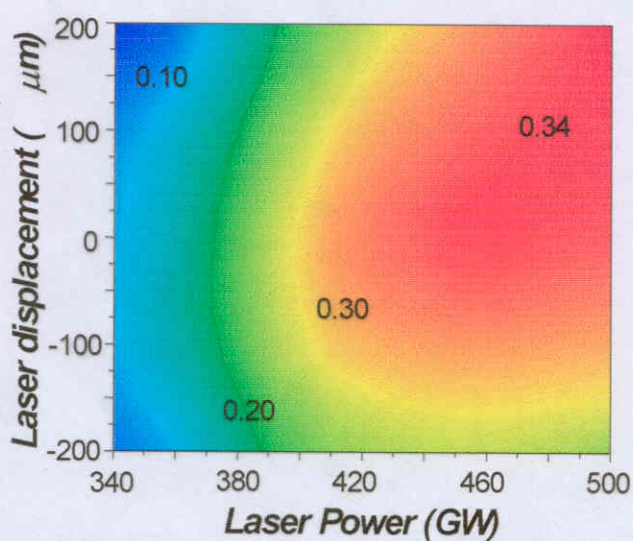
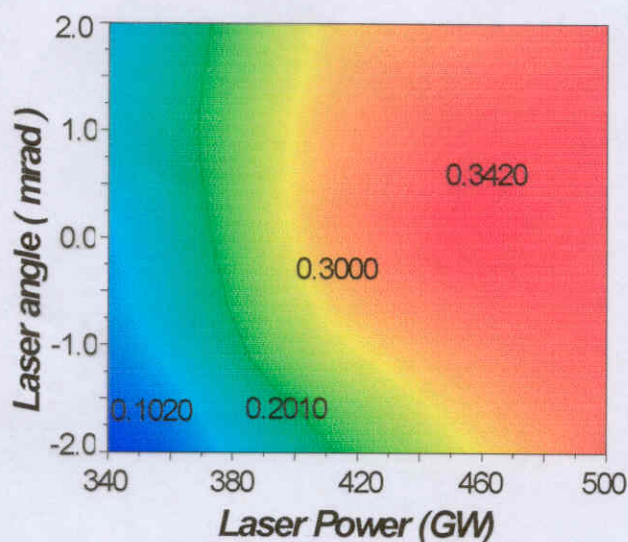
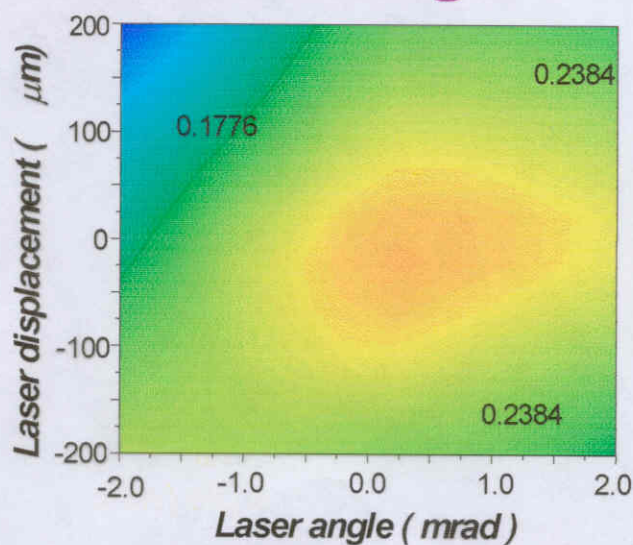
• Dependence on laser power



Capture $\sim 25\%$ in wide range of P.

The higher γ the less capture fraction at $P > 0.5$ TW

- Fraction of trapped particles when laser beam properties are changed.



Fraction $> 20\%$ is within acceptable limits.

Laser power $0.35 \div 0.5$ TW.

Initial energy γ $28.5 \div 29.0$

Laser angle $-0.001 \div 0.001$.

Laser displacement $-0.2 \div 0.2$ m

Output beam from IFEL accelerator

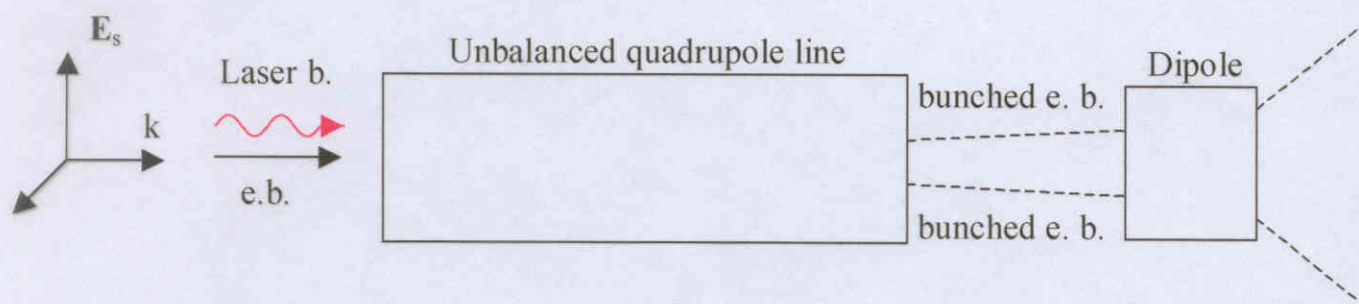
Energy	55 MeV
Energy spread	2.5%
Pulse length	3 fs
Peak current	3 kA
Transverse emittance	10 mm·mrad
Trapped fraction	> 20%

Conclusion

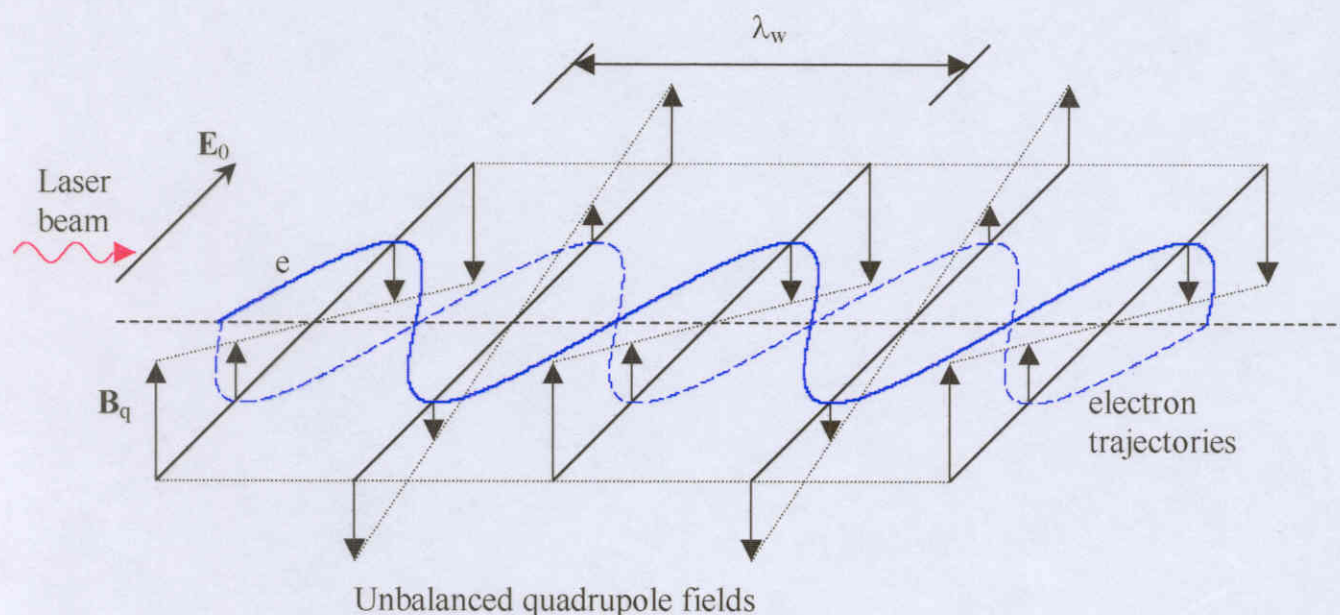
- Vacuum laser acceleration is potential for high rate acceleration in wide energy range of accelerated particles
- Presently achievable beam qualities are sufficient for the experiment to be done on next generation on laser acceleration including first of all
 - Acceleration of electron beams up to total energy gain >100 MeV with captured fraction of beam bunch some tens %
 - Development of staging systems with efficient prebunching providing increased beam trapping for acceleration as well as higher total energy gain due to multistage acceleration

Transverse beam bunching by laser field

- Schematic of transverse prebuncher

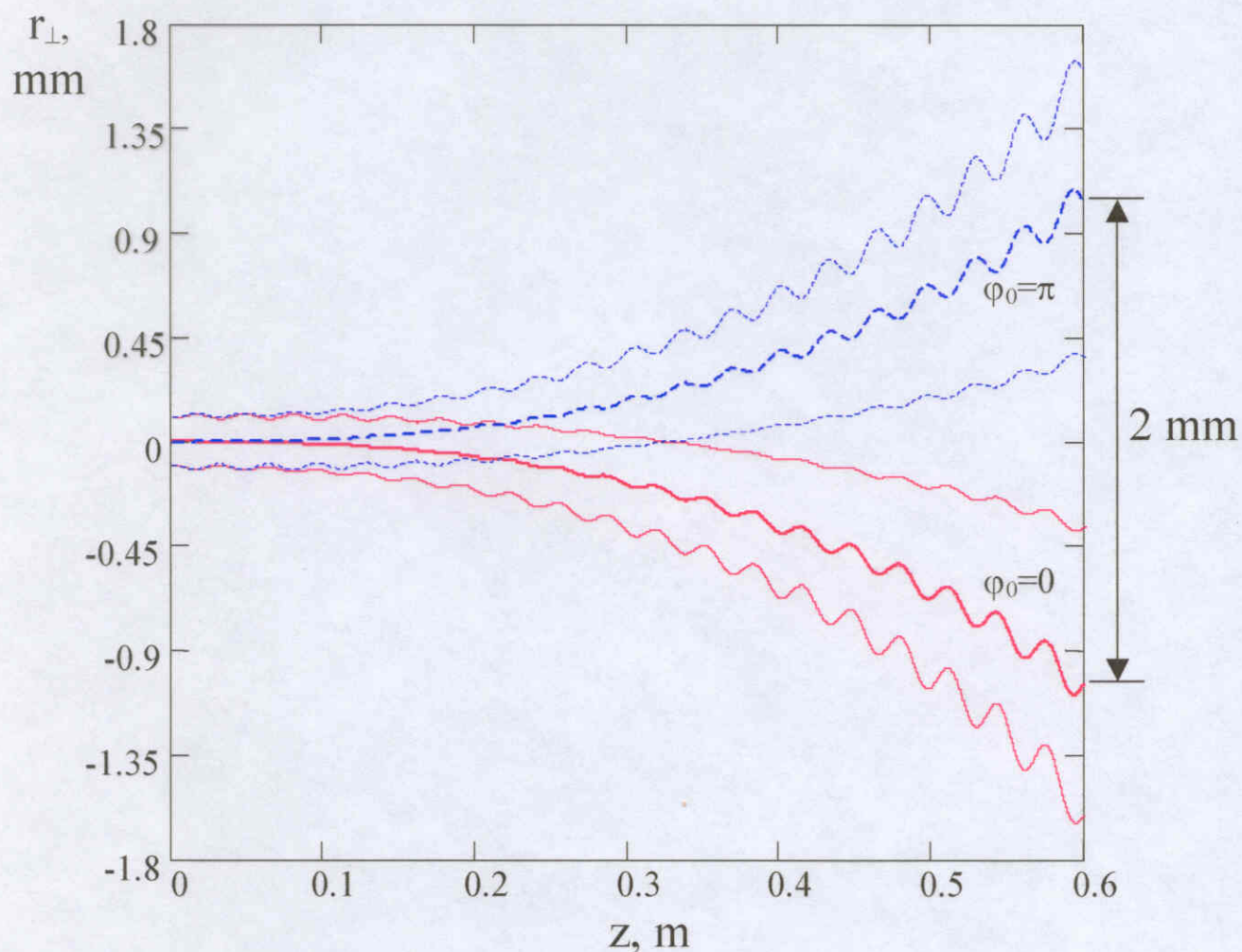


- Geometry of the fields of laser and unbalanced quadrupole causing transverse bunching.



- Simulation of transverse deviations for beam bunching

$$\begin{array}{ll}
 P=0.7 \text{ TW}, & E_0=7 \cdot 10^9 \text{ V/m}, \\
 w_0=1.8 \text{ mm}, & \lambda_w=32 \text{ mm}, \\
 \gamma_0=39, & \Delta B_q/B_q=14\%, \\
 r_{\perp 0}=0; \pm 0.1 \text{ mm}, & \varphi_0=0; \pi
 \end{array}$$

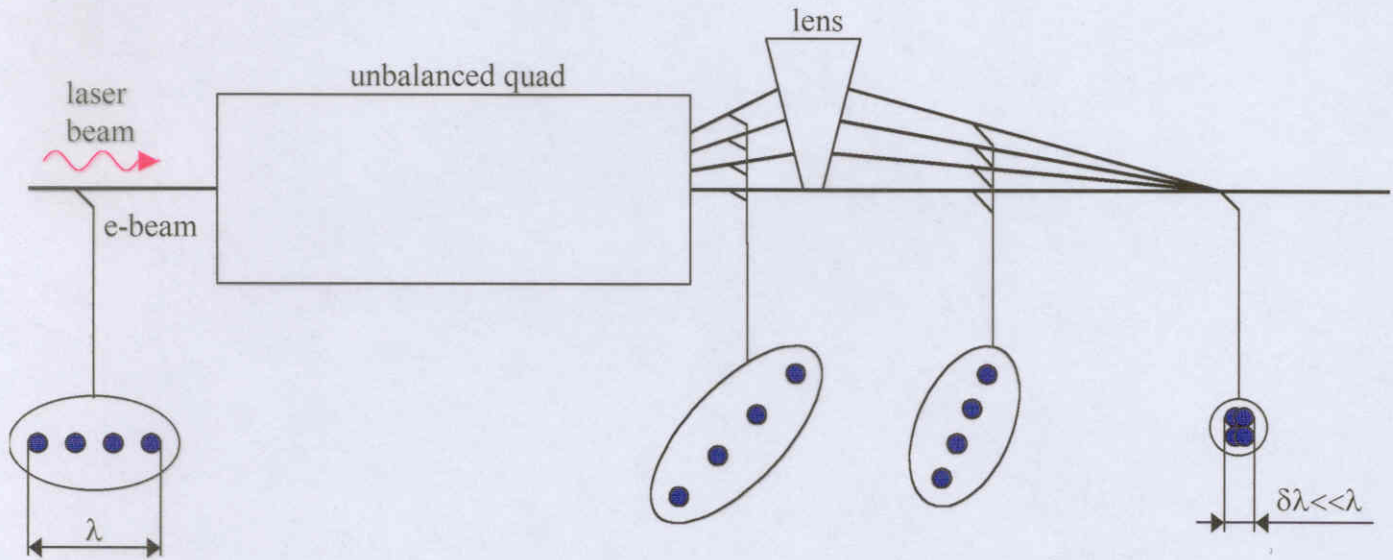


Prebunched beams separation is 2 mm

Acceptable emittance $\varepsilon_n < 10 \text{ mm mrad}$

energy spread $\Delta\gamma/\gamma < 1.5\%$

Debuncher principle scheme based on transverse bunching



Beam density structure

Initial



Transversally bunched at an angle of deviation



Transversally bunched and focused from all deviation angles

