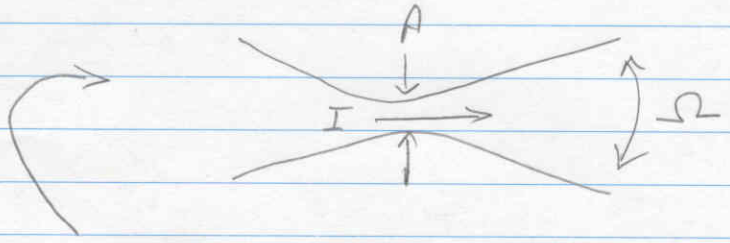
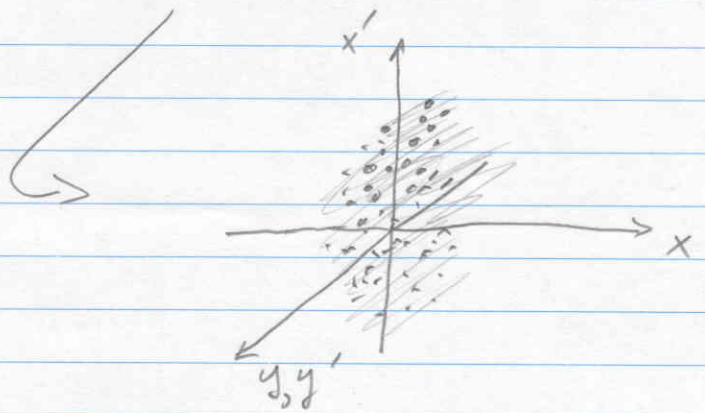
WHAT BRIGHTNESS MEANS



$$B = \frac{d^2 I}{dA\, d\Omega} = \frac{d^4 I}{dx\, dx'\, dy\, dy'}$$



Remarks:

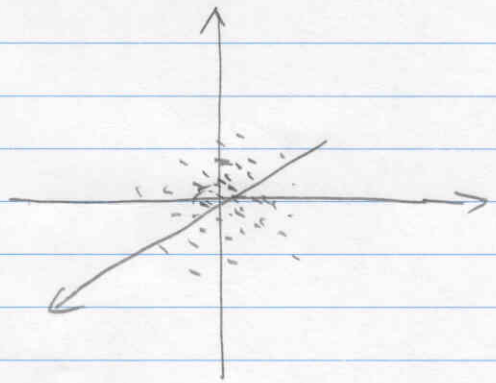
(1) B ≠ property of beam (ϵ = property of beam)
= function of phase space position
– can improve emittance + brightness by filtering + throwing away bad beam

(2) Can speak of
$\hat{B}$ = peak brightness (best point in phase space)
$\overline{B}$ = average brightness (over part of beam used)

(3) Relation to emittance depends on shape of beam

(4) "Beach effect" ≤ 2

GAUSSIAN BEAM IN 4-D TRANSVERSE PHASE SPACE



DENSITY

$$\rho = \hat{B}\, \exp\left[-\frac{x^2+x'^2+y^2+y'^2}{R^2}\right]$$

$\hat{B}$: peak brightness

CURRENT

$$I = \int dx \int dx' \int dy \int dy' \rho = \pi^2 R^4 B$$

EMITTANCE

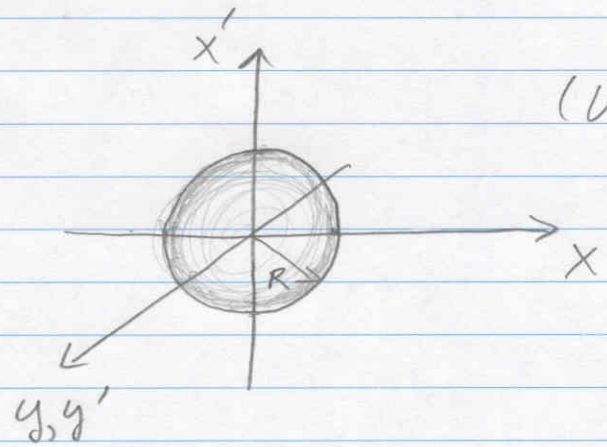
$$I\,\epsilon_{rms} = \int dx \int dx' \int dy \int dy' \, x^2 \rho = \frac{1}{2}\pi^2 R^6 B$$

BRIGHTNESS

$$\hat{B} = \frac{I}{4\pi^2 \epsilon_{x\,rms}\,\epsilon_{y\,rms}} = \frac{1}{8\pi^2}\frac{2I}{\epsilon_{x\,rms}\,\epsilon_{y\,rms}}$$

factor of 80 (~160 for $\bar{B}$)

ELLIPSOIDAL BEAM IN 4-D TRANSVERSE PHASE SPACE



(Use sphere of radius R)

Volume of 4-D sphere

$$V = \frac{\pi^2}{2} R^4$$

RMS emittance

$$\langle x^2 \rangle V = \frac{\pi^2}{12} R^6 = \epsilon_{x\,rms} V$$

Edge emittance

$$\epsilon_x = \text{area} = \pi R^2 = 6\pi \epsilon_{rms}$$

Brightness

$$B = \frac{I}{V} = \frac{2I}{\pi^2 R^4} = \frac{2I}{\epsilon_x \epsilon_y}$$

$$= \frac{2I}{\pi^2 6\epsilon_{x\,rms} 6\epsilon_{y\,rms}} = \frac{1}{36\pi^2} \frac{2I}{\epsilon_{x\,rms} \epsilon_{y\,rms}}$$

↑ factor of 360