THE CHRONO-GEOMETRICAL STRUCTURE OF SPACE-TIME AND DYNAMICAL CLOCK SYNCHRONIZATION IN EINSTEIN'S THEORY

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WAS EINSTEIN RIGHT?

WAS EINSTEIN WRONG?

A BETTER STATEMENT

DO WE UNDERSTAND ALL THE IMPLICATIONS

OF EINSTEIN'S GENERAL RELATIVITY

?

PN APPROXIMATION FOR SOLAR SYSTEM

LINEARIZATION ON BACKGROUND IN HARMONIC COORDINATES

→ SPIN 2 THEORY FOR GRAVITATIONAL WAVES

(DESER → INCONSISTENT IN PRESENCE OF MATTER

→ BACK TO EINSTEIN'S THEORY)

BINARIES → EFFACEMENT OF INTERNAL GRAVITY (BUT TIDAL EFFECTS?)

↳ TOGETHER WITH CASSINI ($1/c^2$ REL. EFFECT) KILLS MOST OF SCALAR-TENSOR THEORIES

↑

NUMERICAL GRAVITY (GAUGE FIXINGS, HYPERBOLICITY)?

GRAVITATIONAL COLLAPSE?

↓

MATHEMATICAL BLACK HOLES ⇒ KILLING VECTORS

NEWTON THEORY → GALILEI SPACETIME

ABSOLUTE TIME
ABSOLUTE EUCLIDEAN INSTANTANEOUS 3-SPACE
ABSOLUTE SIMULTANEITY AND SPATIAL DISTANCE

] ABSOLUTE CHRONO-GEOMETRICAL STRUCTURE

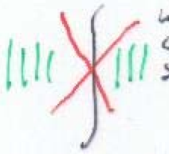
RELATIVITY PRINCIPLE — GLOBAL INERTIAL FRAMES PREFERRED

EQUIVALENCE PRINCIPLE → $m_{INERTIAL} = m_{GRAV.}$

NEWTON EQS $m \ddot{\vec{x}} = \vec{F}$ PREDICTABILITY

NON-INERTIAL FRAMES — $m \ddot{\vec{x}}' = \vec{F} + m \vec{g}$ INERTIAL FORCES

1905 — ANNUS MIRABILIS — EINSTEIN SPECIAL RELATIVITY



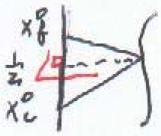
NO NOTION OF INSTANTANEOUS 3-SPACE
" OF SPATIAL DISTANCE
" OF SIMULTANEITY (SYNCHRONIZATION OF DISTANT CLOCKS)
" OF 1-WAY VELOCITY OF LIGHT
" OF CAUCHY SURFACE FOR MAXWELL EQS.

LIGHT POSTULATES → THE 2-WAY (OR ROUND-TRIP ; ONLY 1 CLOCK INVOLVED)

VELOCITY OF LIGHT IS A) CONSTANT B) ISOTROPIC c

RELATIVITY PRINCIPLE → GLOBAL REL. INERTIAL FRAMES PREFERRED

EINSTEIN'S CONVENTION



$x^o_P = x^o_i + \frac{1}{2}(x^o_f - x^o_i)$

⇒ INERTIAL FRAMES EUCLIDEAN 3-SPACES

⇒ 1-WAY = 2-WAY

ABSOLUTE CHRONOGEOMETRICAL STRUCTURE
OF MINKOWSKI SPACE-TIME

IN REAL WORLD ONLY ACCELERATED OBSERVERS

LOCALITY PRINCIPLE

NOT WORKING WITH EM WAVES WITH $\lambda \sim \mathcal{L}$ ACCELERATION RADII

1+3 POINT OF VIEW

A) FERMI COORDINATES

TANGENTPLANE $\perp$ 4-VELOCITY = = ISTANTANEOUS 3-SPACE

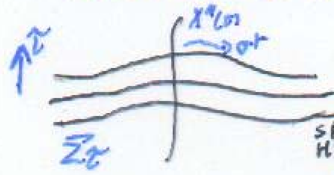
B) RIGIDLY ROTATING FRAMES $\quad X^\mu \mapsto Y^\mu(X)$ $\quad g_{oo}(Y^o, \vec{Y}) = 0$ WHERE

$\eta_{\mu\nu} \mapsto g_{\mu\nu}(X)$ $\quad \omega r = c$ (HORIZON PROBLEM)

$\Rightarrow$ COORDINATE SINGULARITIES $\Rightarrow$ NO CAUCHY SURFACE FOR MAXWELL EQS

PREDICTABILITY $\longrightarrow$ 3+1 POINT OF VIEW

3+1 SPLITTINGS OF MINKOWSKI SPACETIME = GLOBAL NON-RIGID NON-INERTIAL FRAMES



RIGID ROTATIONS FORBIDDEN (ONLY DIFFERENTIAL ONES)

FROM MØLLER CONDITIONS

$\varepsilon g_{oo} > 0 \quad \varepsilon g_{ii} < 0 \quad \begin{vmatrix} g_{ii} & g_{ij} \\ g_{ji} & g_{jj} \end{vmatrix} > 0 \quad \varepsilon \det g_{ij} < 0$

OBSERVER-DEPENDENT LORENTZ SCALAR RADAR COORDINATES (BONDI)

Σ_τ – SIMULTANEITY SURFACE (CLOCK SYNCHR. CONVENTION $\neq$ EINSTEIN'S ONE)

ISTANTANEOUS RIEMANNIAN 3-SPACE WITH GEODESIC SPATIAL DISTANCE

CAUCHY SURFACE FOR MAXWELL EQS.

POINT-DEPENDENT ANISOTROPIC 1-WAY VELOCITY OF LIGHT

$X^\mu \mapsto \sigma^A = (\tau, \sigma^r) \quad \sigma^A \mapsto X^\mu = z^\mu(\tau, \vec{\sigma})$ EMBEDDING. $\eta_{\mu\nu} \mapsto g_{AB}[z(\tau, \vec{\sigma})]$

PARAMETRIZED MINKOWSKI THEORIES FOR ISOLATED SYSTEMS

$$S = \int d\tau \, d^3\sigma \, \mathcal{L}(\text{matter}, g[z])$$

INVARIANT UNDER FRAME-PRESERVING DIFFEOMORPHISMS

$\Rightarrow z^\mu(\tau, \vec{\sigma})$ GAUGE VARIABLE

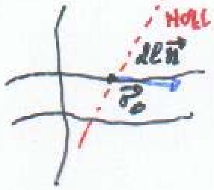
THE CHANGE OF CLOCK SYNCHR. CONVENTION IS A GAUGE TRANSF.

1-WAY VELOCITY OF LIGHT

GIVEN A 3+1 SPLITTING AND ITS RADAR 4-COORDINATES WITH RESPECT TO AN OBSERVER

$$x^\mu \mapsto \sigma^A = (\tau, \sigma^r) \qquad \sigma^A \mapsto x^\mu = z^\mu(\tau,\vec\sigma) \quad \text{EMBEDDING}$$

$${}^4g_{AB}(\tau,\vec\sigma)[z] = z^\mu_A(\tau,\vec\sigma)\,\eta_{\mu\nu}\, z^\nu_B(\tau,\vec\sigma) \qquad z^\mu_A = \frac{\partial z^\mu}{\partial \sigma^A}$$



NULL GEODESIC = RAY OF LIGHT

$$\eta^A(\tau) = (c\tau;\ \eta^r(\tau)) \qquad g_{rs}(\tau,\vec\eta(\tau))\, n^r n^s = -\epsilon$$

SPATIAL LINE ELEMENT IN 3-DIRECTION $n^r(\tau)$

$$d\eta^r(\tau) = n^r(\tau)\, d\ell(\tau) \qquad d\ell(\tau) = \sqrt{-\epsilon\, {}^4g_{rs}(\tau,\vec\eta(\tau))\, d\eta^r(\tau)\, d\eta^s(\tau)}$$

NULL GEODESIC $\quad {}^4g_{AB}(\tau,\vec\eta(\tau))\, \frac{d\eta^A(\tau)}{d\tau}\, \frac{d\eta^B(\tau)}{d\tau} = 0$

INSTANTANEOUS 1-WAY LIGHT VELOCITY at τ and in $\vec\eta(\tau)$

$\begin{cases} \text{MODULUS} & V(\tau) = \frac{d\ell(\tau)}{d\tau} \\ \text{DIRECTION} & n^r(\tau) \end{cases}$

$$\Rightarrow \quad c^2\, {}^4g_{\tau\tau}(\tau,\vec\eta(\tau)) + 2c\, g_{\tau r}(\tau,\vec\eta(\tau))\, n^r(\tau)\, V(\tau) - V^2(\tau) = 0$$

in $(\tau_o, \vec\sigma_o = \vec\eta(\tau_o))$

$$V(\tau_o) = V(\tau;\vec\sigma_o;\vec n(\tau_o)) = \frac{c\, {}^4g_{\tau\tau}(\tau_o,\vec\sigma_o)}{-{}^4g_{\tau s}(\tau_o,\vec\sigma_o)\, n^s(\tau_o) + \sqrt{[g_{\tau s}(\tau_o,\vec\sigma_o)\, n^s(\tau_o)]^2 + {}^4g_{\tau\tau}(\tau_o,\vec\sigma_o)}}$$

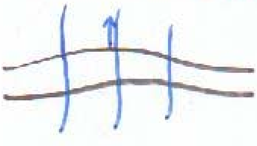
$$\frac{1}{\sqrt{{}^4g(\sigma)}}\, \partial_A \left(\sqrt{{}^4g(\sigma)}\, {}^4g^{AB}(\tau,\vec\sigma)\, \frac{\partial}{\partial\sigma^B}\, A_C(\tau,\vec\sigma) \right) = 0$$

MAXWELL EQS. IN RADAR 4-COORDINATES

IN THE GEOMETRICAL OPTICS APPROXIMATION, I.E. WITH AN EIKONAL APPROXIMATION

$V(\tau,\vec\sigma,\vec n(\tau))\ \vec n(\tau)$ IS THE LOCAL PROPAGATION VELOCITY OF PLANE WAVES AROUND $(\tau,\vec\sigma)$

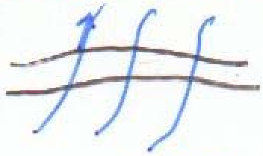
EACH 3+1 SPLITTING HAS TWO ASSOCIATED CONGRUENCES OF TIMELIKE OBSERVERS



EULERIAN OBSERVERS

NORMAL VECTOR FIELD

NON-ROTATING, SURFACE-FORMING

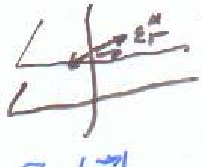


VELOCITY FIELD OF A PERFECT FLUID

$\frac{\partial z^\mu(\tau,\sigma^r)}{\partial \tau}$ EVOLUTION VECTOR FIELD

ROTATING, NON-SURFACE FORMING

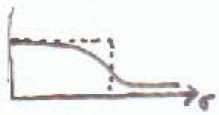
SIMPLEST ADMISSIBLE NON-RIGIDLY ROTATING COORDINATES ON HYPERPLANES Σ_τ



$$z^\mu(\tau,\sigma^r) = x^\mu(\tau) + \epsilon^\mu_r\, R^r{}_s(\tau,\sigma)\, \sigma^s$$

$\sigma = |\vec{\sigma}|$

$$R(\tau,\sigma) = R[\alpha_i(\tau,\sigma)] \quad \text{WITH} \quad \alpha_i(\tau,\sigma) = F(\sigma)\, \hat{\alpha}_i(\tau)$$



$$0 < F(\sigma) < \frac{1}{A\sigma}\ , \quad \frac{dF(\sigma)}{d\sigma} \neq 0 \qquad \text{MØLLER CONDITIONS}$$

DIFFERENTIAL ROTATIONS NEEDED IN ROTATING STARS

ALBA-LUSANNA gr-qc/0501090

OPERATIONAL DEFINITION OF RADAR COORDINATES POSSIBLE WITH GPS METHODS (FERMI COORD. ONLY THEORETICAL)

ACES - ESA EXPERIMENT - LASER COOLED CLOCKS (10^{16} PRECISION)
ON SPACE STATION AND ON EARTH - SYNCHRONIZATION AT ORDER $\frac{1}{c^3}$
WITH MICROWAVE SIGNALS (5 ps) - SENSIBLE TO EARTH ROTATION
AND SHAPIRO TIME DELAY OF THE GEOID

LATER - $1/c^4$

PROBLEM — SATELLITE NASA COORDINATES

? ↙↗ ?

IAU SOLAR AND GEOCENTRIC COORD. + PN METRIC ⟷ THEORETICAL COORD. (HARMONIC, ---)

DYNAMICS IN NON-INERTIAL FRAMES ?

GENERAL RELATIVITY AND THE GEOMETRIC VIEW OF THE GRAVITATIONAL FIELD

DOUBLE ROLE OF THE METRIC TENSOR ${}^4g_{\mu\nu}(x)$

1) POTENTIAL OF THE GRAVITATIONAL INTERACTION

2) DYNAMICAL CHRONO-GEOMETRICAL STRUCTURE OF SPACETIME

$$ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

IT TEACHES RELATIVISTIC CAUSALITY TO ALL THE OTHER FIELDS!

IN EACH POINT A DIFFERENT LIGHT-CONE AND DIFFERENT TRAJECTORIES

FOR LIGHT RAYS (FOR PHOTONS, GLUONS ... AT THE QUANTUM LEVEL)

PREDICTABILITY – GLOBALLY HYPERBOLIC SPACETIMES → CAUCHY SURFACES

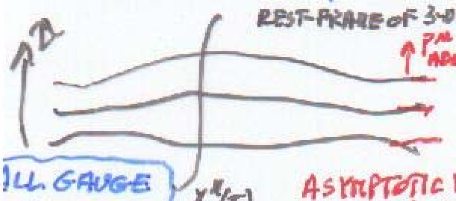
EQUIVALENCE PRINCIPLE → NO GLOBAL INERTIAL FRAME!

BUT IN GLOBALLY HYPERBOLIC SPACETIME

⇒ GLOBAL NON-INERTIAL FRAMES

↳ NEEDED FOR CAUCHY PROBLEM ⇒ PREDICTABILITY DETERMINISTIC EVOLUTION

3+1 SPLITTINGS, TIMELIKE OBSERVER AS ORIGIN, RADAR 4-COORDINATES



REST-FRAME OF 3-UNIVERSE

CLASS OF CHRISTODOULOU-KLAINERMAN SPACETIMES WITH ASYMPTOTIC ADM POINCARÉ GROUP

ASYMPTOTIC INERTIAL OBSERVER = FIXED STARS

DEPARAMETRIZATION

ALL GAUGE EQUIVALENT

ASYMPTOTIC MINKOWSKI METRIC (ASYMPTOTIC BACKGROUND)

$H \approx E_{ADM} = \int d^3\sigma\, \mathcal{E}_{ADM}(\tau, \vec{\sigma})$

↳ GAUGE-DEPENDENT (ENERGY PROBLEM IN GR)

LINEARIZATION AROUND A BACKGROUND SPACETIME (MINKOWSKI, DESITTER)

${}^4g_{\mu\nu}(x) = {}^4\eta_{\mu\nu} + {}^4h_{\mu\nu}(x)$ PROPERTY 2) LOST
ABSOLUTE STRUCTURE OF BACKGROUND

LIGHT RAYS (PHOTONS, GLUONS ...) PROPAGATE IN AN INERTIAL FRAME OF THE BACKGROUND

GRAVITY → SPIN 2 (MASSLESS GRAVITON) IN THE INERTIAL FRAME

INCONSISTENT WHEN MATTER PRESENT (DESER – GR-QC/0411023)

EINSTEIN'S EQS $G^{\mu\nu}(x) = 8\pi G\, T^{\mu\nu}(x)$ – ONLY 2 INDEPENDENT

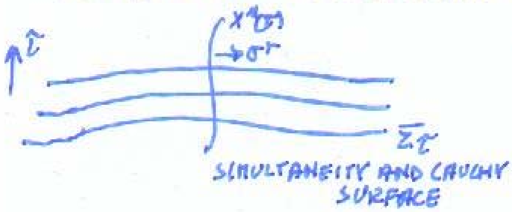
WHAT IS ARBITRARY (GAUGE-DEPENDENT) AND WHAT HAS A DETERMINISTIC EVOLUTION INSIDE ${}^4g_{\mu\nu}$?

CANONICAL ADM GRAVITY

ADM METRIC GRAVITY $\longrightarrow$ ADM TETRAD GRAVITY (THEORY OF TIMELIKE OBSERVERS NEEDED FOR FERMIONS)

METRIC TENSOR 10 FIELDS $\quad {}^4g_{\mu\nu}(x) \underset{\text{PUT INSIDE ADM ACTION}}{=} {}^4E^{(\alpha)}_{\mu}(x)\, {}^4\eta_{(\alpha)(\beta)}\, {}^4E^{(\beta)}_{\nu}(x)$ COTETRAD 16 FIELDS

3+1 SPLITTING OF SPACETIME + ACCELERATED OBSERVER



ADAPTED RADAR 4-COORDINATES

$$\begin{cases} x^\mu \mapsto \sigma^A = (\tau, \sigma^r) \\ \sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r) \end{cases}$$ EMBEDDING OF Σ_τ

$$ {}^4g_{AB}(\tau, \sigma^r) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A}\, {}^4g_{\mu\nu}(x)\, \underbrace{\frac{\partial z^\nu(\tau,\sigma)}{\partial \sigma^B}}_{\text{NOT COTETRADS}} = $$

$$ = {}^4E^{(\alpha)}_{A}(\sigma)\, {}^4\eta_{(\alpha)(\beta)}\, {}^4E^{(\beta)}_{B}(\sigma) = \varepsilon \begin{pmatrix} N^2 - {}^3g_{rs}N^rN^s & -{}^3g_{su}N^u \\ -{}^3g_{ru}N^u & -{}^3g_{rs} \end{pmatrix} $$

$\eta = \varepsilon\,(+---)$, $\varepsilon = \pm 1$, ${}^3g = (+++)$ $\qquad {}^3g_{rs} = \sum_a {}^3e_{(a)r}\, {}^3e_{(a)s}$

CLASS OF SPACETIMES (FOR SOLAR SYSTEM, GALAXY ; MAYBE COSMOLOGY)

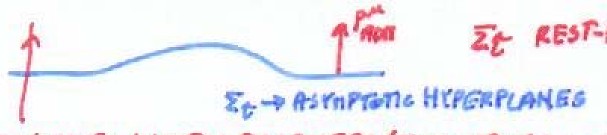
- GLOBALLY HYPERBOLIC, NON-COMPACT, TOPOLOGICALLY TRIVIAL ($\Sigma_\tau \sim R^3$; GLOBAL COORD. & UNIQUE 3-GEODESIC)

- ASYMPTOTICALLY FLAT AT SPATIAL INFINITY

  α) ASYMPTOTIC MINKOWSKI METRIC (ASYMPTOTIC BACKGROUND)

  β) BOUNDARY CONDITIONS : NO SUPERTRANSLATIONS (DIRECTION-INDEPENDENT) ($\exists$ OF ANGULAR MOMENTUM IN GR) REGGE-TEITELBOIM BEIG-O'MURCHADHA

  $\Rightarrow$ ASYMPTOTIC SYMMETRIES $\quad$ SPI GROUP $\rightarrow$ ADM POINCARÉ GROUP

  γ) 

  Σ_τ REST-FRAME OF 3-UNIVERSE

  WIGNER $\xrightarrow[G\to 0]{}$ REST-FRAME INSTANT FORM

  ASYMPTOTIC INERTIAL OBSERVERS (FIXED STARS ; COBE FRAME IN COSMOLOGY ?) $\rightarrow$ KLIONER

  δ) Σ_τ LICHNEROWICZ 3-MANIFOLDS (INSTANTANEOUS FOCK SPACE)

  ε) ALL THE FIELDS IN SUITABLE WEIGHTED SOBOLEV SPACES (NO KILLING VECTORS, NO GRIBOV)

  CHRISTODOULOU-KLAINERMANN SPACETIMES

HAMILTONIAN TETRAD GRAVITY

THEORY OF OBSERVERS WITH GYROSCOPES

WITH CANONICAL TRANSFORMATIONS WE CAN SEPARATE THE GAUGE VARIABLES FROM THE PHYSICAL DEGREES OF FREEDOM OF THE GRAVITATIONAL FIELD (TIDAL EFFECTS)

YORK MAP

MATTER	N	$N_{(a)}$	$\varphi_{(a)}$	$\alpha_{(a)}$	θ^r	ϕ	$R_{\bar{a}}$
	≈ 0	≈ 0	≈ 0	≈ 0	π_{θ}^{r}	π_{ϕ}	$\pi_{\bar{a}}$

π_θ^r → FROM SUPERMOMENTUM CONSTRAINTS

$\bar{a} = 1,2$

$R_{\bar{a}}, \pi_{\bar{a}}$ → PHYSICAL DEGREES OF FREEDOM

$$H = E_{ADM} + \int d^3\sigma \, [N(\tau,\vec{\sigma})\, \mathcal{H}(\tau,\vec{\sigma}) + N_{(a)}(\tau,\vec{\sigma})\, \mathcal{H}_{(a)}(\tau,\vec{\sigma})]$$

$\phi = (\det {}^3g)^{1/12}$ CONFORMAL FACTOR OF 3-METRIC

UNKNOWN IN THE SUPER-HAM. CONSTRAINT (LICHNEROWICZ EQ.) $\mathcal{H}(\tau,\vec{\sigma}) \approx 0$

O GAUGE VARIABLES (GENERALIZED RELATIVISTIC INERTIAL EFFECTS)

A) $\alpha_{(a)}(\tau,\vec{\sigma}), \varphi_{(a)}(\tau,\vec{\sigma})$ — DESCRIBE THE TETRAD OF AN OBSERVER: UNIT 4-VELOCITY +3 SPATIAL GYROSCOPES WITH A CONVENTION FOR THEIR TRANSPORT ALONG THE WORLDLINE

B) $\xi^r(\tau,\vec{\sigma})$ — DESCRIBE THE ARBITRARINESS OF THE 3-COORDINATES ON Σ_τ
- THEIR CHOICE WILL DETERMINE THE PATTERN OF RELATIVISTIC STANDARD INERTIAL FORCES (CENTRIFUGAL, CORIOLIS, ...)

C) $N_{(a)}(\tau,\vec{\sigma})$ — SHIFT FUNCTIONS — THEY TELL WHICH POINTS ON DIFFERENT Σ_τ's HAVE THE SAME NUMERICAL 3-COORD.
- INERTIAL ~~FORE~~ GRAVITO-MAGNETIC POTENTIALS DETERMINING THE DRAGGING OF INERTIAL FRAMES (LENSE-THIRRING EFFECT) LAGEOS, GRAVITY PROBE B

D) $\boxed{\pi_\phi(\tau,\vec{\sigma}) \propto {}^3K(\tau,\vec{\sigma})}$ IN YORK BASIS — ARBITRARINESS OF SHAPE OF Σ_τ
- ~~RE~~ RELATIVISTIC INERTIAL POTENTIAL (ATTRACTIVE AND REPULSIVE) DESCRIBING THE CHOICE OF THE INSTANTANEOUS 3-SPACE (I.E. THE CLOCK SYNCHRONIZATION CONVENTION)
- → IT DOES NOT EXIST IN NEWTON GRAVITY (ABSOLUTE EUCLIDEAN 3-SPACE)
- DARK MATTER AS AN INERTIAL EFFECT? (COOPERSTOCK-TIEU)

E) $N(\tau,\vec{\sigma})$ LAPSE FUNCTION → INERTIAL POTENTIAL DESCRIBING THE ARBITRARINESS IN THE CHOICE OF THE UNIT OF PROPER TIME IN EACH POINT OF Σ_τ

AT THIS STAGE HAMILTON EQS FOR $R_{\bar{a}}, \pi_{\bar{a}}$ DEPEND ON ALL THE GAUGE VARIABLES NO DETERMINISM

IN A COMPLETELY FIXED GAUGE (GLOBAL NON-INERTIAL FRAME
= EXTENDED PHYSICAL LAB WITH ITS METROLOGY)

$\varphi_{(a)}, \alpha_{(a)}, N, N_{(a)}, \xi^r, \pi_\phi$ ARE GIVEN FUNCTIONS OF $\tau, \vec{\sigma}, R_{\bar{a}}(\tau,\vec{\sigma}), \Pi_{\bar{a}}(\tau,\vec{\sigma})$

IF ONE IS ABLE TO SOLVE THE LICHNEROWICZ EQ $\quad \underline{\Sigma_\tau \text{ DEPENDS ON } R_{\bar{a}}, \Pi_{\bar{a}}}$

$\Rightarrow$ ALSO ϕ IS A ~~...~~ FUNCTION OF $\tau, \vec{\sigma}, R_{\bar{a}}(\tau,\vec{\sigma}), \Pi_{\bar{a}}(\tau,\vec{\sigma})$

$\Downarrow$

R-P COORD. DEPENDENT TERM = RELATIVISTIC VERSION OF STANDARD INERTIAL POTENTIALS

$$H = E_{ADM} = \int d^3\sigma\, \mathcal{E}_{ADM}(\tau,\vec{\sigma})$$

$\hookrightarrow$ WELL DEFINED FUNCTION OF $\tau, \vec{\sigma}, R_{\bar{a}}(\tau,\vec{\sigma}), \Pi_{\bar{a}}(\tau,\vec{\sigma})$

$$\Rightarrow \begin{cases} \partial_\tau R_{\bar{a}}(\tau,\vec{\sigma}) = \{R_{\bar{a}}(\tau,\vec{\sigma}), H\} \\ \partial_\tau \Pi_{\bar{a}}(\tau,\vec{\sigma}) = \{\Pi_{\bar{a}}(\tau,\vec{\sigma}), H\} \end{cases}$$

HYPERBOLIC EQS

$\Rightarrow$ DETERMINISTIC EVOLUTION OF TIDAL VARIABLES IN THE GLOBAL NON-INERTIAL FRAME

HOLE ARGUMENT $-\quad \chi^A = \sigma^A - F^A[R_{\bar{a}}(\tau,\vec{\sigma}), \Pi_{\bar{a}}(\tau,\vec{\sigma})] \approx 0$

SPACETIME $\equiv$ GRAVITATIONAL FIELD

GAUGE-DEPENDENT (I.E. NON-INERTIAL-FRAME-DEPENDENT)

SPATIAL DISTRIBUTION OF THE GRAVITATIONAL ENERGY DENSITY

$\mathcal{E}_{ADM}(\tau,\vec{\sigma})$ — HAS IT ANY RELEVANCE FOR THE DARK ENERGY PROBLEM?

A SOLUTION WITH A CAUCHY SURFACE Σ_{τ_0} GIVES A DYNAMICAL DETERMINATION OF

THE NON-INERTIAL FRAME (Σ_{τ_0} IS ON OF ITS SIMULTANEITY SURFACES)

NAMELY THE ISTANTANEOUS 3-SPACES AND THE CLOCK SYNCHR. CONVENTION

${}^4g_{\mu\nu}$ AND THE WHOLE CHRONO-GEOMETRICAL STRUCTURE

INVERSE PROBLEM

FIX COMPLETELY THE GAUGE BUT WITH ${}^3K(\tau,\vec{\sigma})$ AN ARBITRARY FUNCTION (ISTANT. 3-SPACE AND CLOCK SYNCHR. CONVENTION ARBITRARY)

$\Rightarrow$ FAMILY OF GAUGES WITH LAPSE AND SHIFT FUNCTIONS DEPENDING ON ${}^3K(\tau,\vec{\sigma})$

1) LET THE MATTER BE A BALL OF DUST (A GALAXY)

STUDY THE HAMILTON EQS FOR $R_{\bar{a}}, \Pi_{\bar{a}}$, DUST

LOOK FOR A ${}^3K(\tau,\vec{\sigma})$, NAMELY FOR A GLOBAL NON-INERTIAL FRAMES, IN WHICH WE GET THE OBSERVED ROTATION CURVE OF THE GALAXY LIKE IT HAPPENS WITH MOND OR IN THE SIMPLE COOPERSTOCK-TIEU MODEL.

DARK MATTER AS AN INERTIAL EFFECT ?

2) LET THE MATTER BE THE ELECTROMAGNETIC FIELD

HOW THE DISTRIBUTION OF THE MAGNETIC FIELD VARIES WITH ${}^3K(\tau,\vec{\sigma})$?

(COSMIC MAGNETIC FIELD PROBLEM)

OPEN PROBLEMS IN ASTROPHYSICS AND COSMOLOGY

1) PARALLAX DISTANCE OF NEARBY STARS
= EINSTEIN'S CONVENTION OF CLOCK SYNCHRONIZATION
IN INERTIAL BARYCENTRIC CELESTIAL REFERENCE SYSTEM
Σ_t INSTANTANEOUS 3-SPACE OF SR INERTIAL FRAMES ($x^o = const.$)

2) LUMINOSITY DISTANCE AND PROPER DISTANCE IN COSMOLOGY
AND IN GRAVITATIONAL LENSING
↳ WEINBERG – INTEGRAL OVER RADIAL DISTANCE OF A SUCCESSION OF COMOVING OBSERVERS

WHICH Σ_t? WHICH SYNCHRONIZATION CONVENTION?

3) IN EINSTEIN'S THEORY $c_{GRAVITY} = c$ (2-WAY)
↳ GRAVITATIONAL WAVES; RETARDED OR INSTANTANEOUS AAAD INFLUENCE OF EARTH
KOPEIKIN – $c_g \neq c$ – BIMETRIC THEORY
LIGHT FROM QUASAR GRAZING JUPITER $c_g \simeq c$ at 20% level

PROBLEM OF PROPAGATION OF LIGHT ~~AND~~ IN GENERAL RELATIVITY +
+ LINEARIZATION OF GRAVITY + PN APPROXIMATION
IN HARMONIC GAUGES

c, c_g → 1-WAY VELOCITY OF THE CHOSEN 3+1 SPLITTING
OF SPACETIME

4) BINARY PULSAR TIMING PROBLEM
SPECTRA OF LIGHT FROM PULSARS → THEIR ANALYSIS DEPENDS ON THE
LOCATION OF THE OBSERVATORY ON THE GEOID (GPS AND BCRS used)

IAU 2000 RESOLUTIONS

ASTRO-PH/0303376

$\varepsilon = \pm 1$

1) BARYCENTRIC CELESTIAL REFERENCE SYSTEM (BCRS)

↳ SOLAR SYSTEM   SPATIAL AXES → FIXED STARS (ICRS, HYPPARCOS)

~ INERTIAL (IGNORE EXTERNAL GALACTIC AND EXTRAGALACTIC MATTER)

$$\begin{cases} t = TCB & \text{BARYCENTRIC COORD-TIME} \\ x^i & \\ g_{\mu\nu} \xrightarrow[r\to\infty]{} \varepsilon(+---) & \text{ASYMPTOTICALLY FLAT} \\ w, w^i & \text{GRAV. POTENTIALS} \end{cases}$$

$$\begin{cases} g_{00} = \varepsilon\left[1 - \frac{2w}{c^2} + \frac{2w^2}{c^4}\right] + O(c^{-5}) \\ g_{0i} = -\varepsilon\left[-\frac{4}{c^3} w^i\right] + O(c^{-5}) \\ g_{ij} = -\varepsilon\left[\left(1 + \frac{2w}{c^2}\right)\delta_{ij}\right] + O(c^{-4}) \end{cases}$$

HARMONIC GAUGE

$$\begin{cases} \Box w = -4\pi G\, \sigma + O(c^{-4}) \\ \Delta w^i = -4\pi G\, \sigma^i + O(c^{-2}) \end{cases}$$

$$\begin{cases} w(t,\vec{x}) = G \int d^3x' \frac{\sigma(t,\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{G}{2c^2}\frac{\partial^2}{\partial t^2}\int d^3x' |\vec{x}-\vec{x}'| \sigma(t,\vec{x}') \\ w^i(t,\vec{x}) = G \int d^3x' \frac{\sigma^i(t,\vec{x}')}{|\vec{x}-\vec{x}'|} \end{cases}$$

$\sigma = \frac{1}{c^2}(T^{00} + T^{ss})$, $\sigma^i = \frac{1}{c} T^{0i}$ SOLAR SYSTEM $T^{\mu\nu}$ RETARDED SOLUTION

2) GEOCENTRIC CELESTIAL REFERENCE SYSTEM (GCRS)

QUASI-INERTIAL · { SPATIAL AXES NON-ROTATING W.R.T. BCRS
ACCELERATED GEOCENTER OF THE GEOID

$$\begin{cases} T = TCG & \text{GEOCENTRIC COORD-TIME} \\ X^a & \\ G_{\alpha\beta} & \\ W, W^a & \end{cases}$$

$$\begin{cases} G_{00} = \varepsilon\left[1 - \frac{2W}{c^2} + \frac{2W^2}{c^4}\right] + O(c^{-5}) \\ G_{0a} = -\varepsilon\left[-\frac{4}{c^3} W^a\right] + O(c^{-5}) \\ G_{ab} = -\varepsilon\left[\left(1 + \frac{2W}{c^2}\right)\delta_{ab}\right] + O(c^{-4}) \end{cases}$$

$$\begin{cases} W(T,\vec{X}) = W_{EARTH}(T,\vec{X}) + W_{TIDAL}(T,\vec{X}) + W_{INERTIAL}(T,\vec{X}) \\ W^a(T,\vec{X}) = W^a_{EARTH}(T,\vec{X}) + W^a_{TIDAL}(T,\vec{X}) + W^a_{INERTIAL}(T,\vec{X}) \end{cases}$$

→ GEODESIC, THOMAS, LENSE-THIRRING PRECESSIONS

QUADRATIC IN $\vec{X}$ LINEAR IN $\vec{X}$

↳ ACCELERATION GEOID

A) COORD. TRANSF. $(t, x^i) \leftrightarrow (T, X^a)$ TIME-DEP. LORENTZ TRANS. IN PRESENCE OF PN GRAVITY

B) POT. TRANSF $(w, w^i) \leftrightarrow (W, W^a)$

C) $TCB - TCG = \frac{1}{c^2}\left[\int_{t_0}^{t}\left(\frac{\vec{v}_E^2}{2} + U_{ext}(\vec{x}_E)\right)dt + \vec{v}_E \cdot \vec{r}_E\right] + O(c^{-4})$ $r_E^i = x^i - x_E^i$ GEOCENTER

$TCG - TT = L_G \cdot (JD - 2443144.5) \cdot 86400$ $L_G \simeq 6.9\ldots 10^{-10}$

TERRESTRIAL TIME ~ TAI   STANDARD OF TIME = COORD. TIME NOT PROPER TIME
(EQS. OF MOTION NOT KNOWN TO THE EXPERIMENTALIST)

$^4g_{\mu\nu}(x)$ SOLUTION OF EINSTEIN'S EQUATIONS
IN A GIVEN 4-COORDINATE SYSTEM x^μ
IN AN EINSTEIN'S SPACETIME

$$\begin{cases} g_{oo} = \epsilon\,[N^2 - {}^3g^{rs} N_r N_s] \\ g_{or} = -\epsilon N_r \end{cases} \Rightarrow N, N_r$$

$$\Rightarrow {}^3K_{rs}(x) = \frac{1}{2N}\left(N_{r|s} + N_{s|r} - \partial_o {}^3g_{rs} \right)(x)$$

$$\Downarrow$$

THE 3+1 SPLITTING OF EINSTEIN SPACETIMES WHOSE
SIMULTANEITY SURFACES Σ_τ HAVE THIS EXTRINSIC CURVATURE
(INVERSE PROBLEM $\overset{1^n}{\frown}$ $L_n\, {}^3g_{rs} = {}^3K_{rs}$)

IDENTIFY 1) THE CAUCHY SURFACE OF THE SOLUTION

2) THE INSTANTANEOUS 3-SPACE Σ_τ, NAMELY A DYNAMICAL CONVENTION FOR THE SYNCHRONIZATION OF DISTANT CLOCKS

PROBLEM - HOW TO IDENTIFY THIS DYNAMICAL SYNCHRONIZATION WITH SOME DEDICATED EXPERIMENT ?

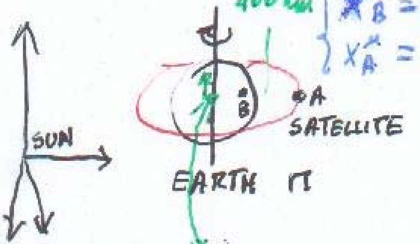
$\frac{1}{c^3}, \frac{1}{c^4}$ RELATIVISTIC EFFECTS IN THE PN GRAVITATIONAL FIELD OF THE SOLAR SYSTEM

PROBLEM OF THE STANDARD OF SPACETIME - GPS AND NASA COORDINATES FOR SATELLITES (PN-HARMONIC BY DEFINITION ?) -
- ONLY DEVIATIONS FROM THE STANDARD MEASURABLE

1-WAY ~~[illegible]~~ TIME TRANSFER FROM EARTH TO A SATELLITE

ACES (ESA) LASER COOLED CLOCKS 10^{-15}

400 km $\begin{cases} x_B^\mu = (x_B^o = ct;\ \vec{x}_B(t)) \\ x_A^\mu = (x_A^o = ct;\ \vec{x}_A(t)) \end{cases}$



INERTIAL SOLAR SYSTEM BARYCENTRIC CELESTIAL REFERENCE SYSTEM

NON-INERTIAL BUT NON-ROTATING GEOCENTIC CELESTIAL REFERENCE FRAME

BLANCHET, SALOMON, TEYSSANDIER, WOLF

RELATIVISTIC THEORY FOR TIME AND FREQUENCY TRANSFER TO ORDER $1/c^3$

ASTRON. ASTROPHYS. **370**, 320 (2000)

APPROSSIMATO CON UN SISTEMA INERZIALE (FREE FALL IN POST-NEWTONIAN GRAVITY)

LA ROTAZIONE DELLA TERRA È APPROSSIMATA CON UN TERMINE $\omega_E^2\, |\vec{x}_B(t)|$ NELL'ACCELERAZIONE DI B

SIMULTANEITÀ $x^o = ct = \text{const.}$

$$\Delta^2_{AB} = (x_A - x_B)^2 = \epsilon\left[c^2 (t_A - t_B)^2 - \vec{\Delta}^2_{AB}\right] = 0$$

RADAR SIGNAL DA B AD A

$\vec{\Delta}_{AB} = R_{AB}\, \hat{N}_{AB}$ RETARDED DISTANCE

$T_{AB} = t_B - t_A = \frac{1}{c} R_{AB}$ → VA RIESPRESSO IN TERMINI DELLA DISTANZA ISTANTANEA

$\boxed{\text{ACES} \rightarrow 5\ \text{ps}}$

$\vec{D}_{AB} = \vec{x}_A(t_A) - \vec{x}_B(t_A)$ → MEGLIO NOTA DI $\vec{x}_B(t_B)$

$$T_{AB} = \frac{R_{AB}}{c} + \frac{2GM}{c^3} \ln \frac{|\vec{x}_A(t_A)| + |\vec{x}_B(t_B)| + R_{AB}}{|\vec{x}_A(t_A)| + |\vec{x}_B(t_B)| - R_{AB}}$$

POST-NEWTONIAN SHAPIRO TIME DELAY OF LIGHT DUE TO THE GEOID

$$= \underbrace{\frac{1}{c} |\vec{D}_{AB}| + \frac{1}{c^2} \vec{D}_{AB}\cdot\vec{v}_B(t_A)}_{\text{1st SAGNAC TERM (200 ns)}} + \frac{1}{c^3}\Big\{ |\vec{D}_{AB}| \Big(\vec{v}_B^{\,2}(t_A) + \frac{(\vec{D}_{AB}\cdot\vec{v}_B(t_A))^2}{|\vec{D}_{AB}|^2} + \vec{D}_{AB}\cdot\vec{a}_B(t_A)\Big) +$$

$$+ 2GM \ln \frac{|\vec{x}_A(t_A)| + |\vec{x}_B(t_A)| + |\vec{D}_{AB}|}{|\vec{x}_A(t_A)| + |\vec{x}_B(t_A)| - |\vec{D}_{AB}|} \Big\}$$

2nd SAGNAC TERM (5 ps)

EARTH ROTATION

11 ps SHAPIRO TIME DELAY (GEOID)

INERTIAL SOLAR SYSTEM FRAME – SIMULTANEITY (HYPERPLANES) MØLLER ADMISSIBLE

$$z^\mu(\tau, \vec{\sigma}) = \underbrace{x_o^\mu + \ell^\mu \tau}_{\text{SUN}} + R\, \epsilon^\mu_{\ r}\, {}^{r}{}_{s}(\tau, \sigma)\, \sigma^s$$

EARTH ROTATION $\alpha_i(\tau, \vec{\sigma}) = F(\sigma)\, \tilde{\alpha}_i(\tau)$

REFORMULATED WITH ADMISSIBLE COORDINATES CENTERED ON THE NON-INERTIAL C.O.M OF THE EARTH $x^\mu(\tau)$

⇒ $F(\sigma)$-DEPENDENT T_{AB} WITH EXACT EARTH ROTATION EFFECT TO ORDER $1/c^3$

LISA, VLBI ....

The ACES Mission

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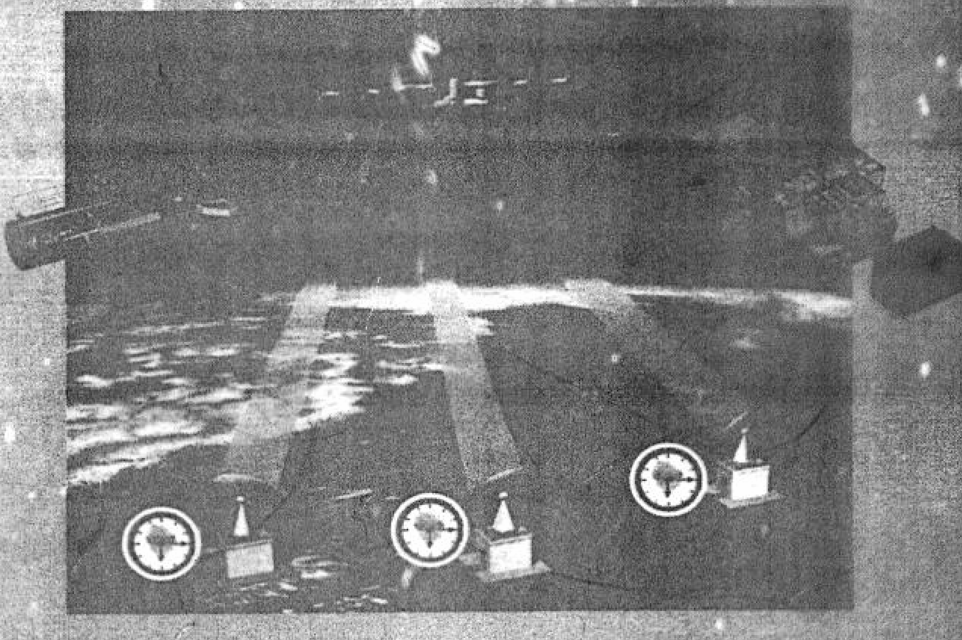
ACES Workshop
Atomic Time in Space

European Space Agency
ESTEC, Noordwijk, The Netherlands
10 – 11 October 2005



The Aces mission concept.
The stable and accurate time base generated by ACES clocks on-board the International Space Station is delivered on Earth through a high-performance two way time and frequency transfer link. The clock signal is used to perform space-to-ground as well as ground-to-ground comparisons of atomic frequency standards. These comparisons will allow accurate tests of Einstein's theory of General Relativity and other applications in universal time scales, global positioning and navigation, geodesy, etc.

For a correct analysis of the results of the ACES experiment, the frequency shift that corresponds to the above equation must be subtracted from the proper frequency to obtain the coordinate frequency.
In the two-way frequency transfer technique, a tracking signal is sent from the ground station B at instant $t_{B'}$, received on the ISS at instant t_A, and re-emitted by a transponder towards B where it will be received at instant t_B (see Fig. 6). The downward clock signal is emitted simultaneously with the re-emitted tracking signal at t_A and received at the ground station at t_B. This scheme provides one-way and two-way Doppler information by simultaneous transmission of three microwave signals and allows to remove the first-order Doppler shift [4]. In a geocentric inertial coordinate frame (GRS) where $\vec{r}_{A,B}$ and $\vec{v}_{A,B}$ respectively define the position and the velocity of the two clocks, the relative frequency difference between the on-board ACES clock (A) and the reference clock on the geoid (B) obtained with the two-way frequency transfer method [5] is

$$\frac{\delta f}{f} = \frac{1}{c^2}\left(U_{BA} - \frac{v_{BA}^{\;2}}{2} - \vec{R}_{BA}\cdot\vec{a}_B\right)\left(1 - \frac{1}{c}\frac{\vec{R}_{BA}\cdot\vec{v}_{BA}}{R_{BA}}\right) +$$
$$+\frac{R_{BA}}{c^3}(-\vec{v}_A\cdot\vec{a}_B + \vec{R}_{BA}\cdot\vec{b}_B + 2\vec{v}_B\cdot\vec{a}_B - \vec{v}_B\cdot\vec{\nabla}U_B) + O\left(\frac{1}{c^4}\right).$$

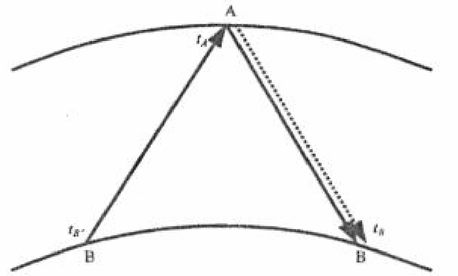


Fig. 6: Two-way frequency transfer in the non-rotating frame.

In this expression, the first line contains the main contributions to the relative frequency difference. The first term, depending on the difference of the Earth potential $U_{BA} \equiv U_B - U_A = U_E[\vec{r}_B(t_B)] - U_E[\vec{r}_A(t_A)]$ at the ground clock (B) and the ACES (A) locations, is the Einstein's gravitational red-shift. The second term, depending on the modulus of the relative velocity $\vec{v}_{BA} = \vec{v}_B(t_B) - \vec{v}_A(t_A)$, is the second order Doppler shift. The third term, proportional to the scalar product between the relative position of the clocks $\vec{R}_{BA} = \vec{r}_B(t_B) - \vec{r}_A(t_A)$ and the acceleration of the ground clock $\vec{a}_B = \vec{a}_B(t_B)$, is the Sagnac effect. In order to reach the required $1/c^3$ precision, the three terms must be corrected by a factor which looks like a first order Doppler effect. Finally, the four last terms introduce a correction of the order $1/c^3$, which is negligible for the ACES mission.
As a consequence, errors in the determination of the gravity field or in the position and velocity of the clock will induce errors in the applied correction to the ACES frequency. It is thus extremely important to estimate this error and to take steps to reduce it to a level compatible with performance objectives of the mission.
Here we simply mention the requirements imposed on the precise orbit determination of the ACES payload. This information will be useful to define the ultimate performances of the clock signal delivered on Earth by ACES.
To verify the performances of the space clocks it is necessary to calculate and compensate relativistic frequency shifts. The frequency noise due to errors in the orbit determination is quantified using the Allan standard deviation of the POD induced error $\sigma_y^{POD}(\tau)$.
Its contribution is required to be lower than 20 % of the frequency noise of the best ACES clock. In terms of Allan variance,

$$\sigma_y^{ACES}(\tau)^2 + \sigma_y^{POD}(\tau)^2 < 1.2\cdot\sigma_y^{ACES}(\tau)^2 .$$

Over one pass, SHM is more stable than PHARAO and the short term predictive orbit determination errors must lead to a corresponding frequency noise lower than

$$\sigma_y^{POD}(10\,\mathrm{s} \le \tau \le 300\,\mathrm{s}) \le 2.3\cdot 10^{-14}\cdot\tau^{-1/2}.$$

On this time scales, real time or almost real time POD data are necessary.
The requirements on the medium and long term stability of the orbit determination must be satisfied over an integration time from 1000 s to 10 days, but do not need real time. Considering the mission constraints, it is difficult to have very long observation periods with enough data to compute a variance of integration times longer than 10 days (the orbit re-boost every 90 days will perturb clock operations and orbit restitution, possibly together with power supply cuts so that clocks continuous operations are not guaranteed for durations longer than 90 days). For medium and long term, the orbit determination error must lead to corresponding frequency noise lower than

$$\sigma_y^{POD}(10000\,\mathrm{s} \le \tau \le 10\,\mathrm{days}) \le 4.5\cdot 10^{-14}\cdot\tau^{-1/2}.$$

Due to relativistic corrections, requirements formulated on the stability of the clock shift translate into conditions on the ACES orbital parameters. The gravitational potential $U_{BA} \equiv U_B - U_A = U_E[\vec{r}_B(t_B)] - U_E[\vec{r}_A(t_A)]$ and the relative velocity $\vec{v}_{BA} = \vec{v}_B(t_B) - \vec{v}_A(t_A)$ are not independent quantities, but turn out to be strongly related by the Keplerian laws governing the relative motion of the ground station and the ISS. Nevertheless, with the purpose of estimating the required stability on the determination of ACES orbital parameters, we suppose to have equal and correlated errors on position and velocity of the ACES payload. Under this assumption, we obtain:

$\sigma_{position}(\tau = 300\,\mathrm{s}) \le 10\,\mathrm{m}$ $\quad\sigma_{velocity}(\tau = 300\,\mathrm{s}) \le 12\,\mathrm{mm/s}$
$\sigma_{position}(\tau = 3000\,\mathrm{s}) \le 5.2\,\mathrm{m}$ $\quad\sigma_{velocity}(\tau = 3000\,\mathrm{s}) \le 6.5\,\mathrm{mm/s}$
$\sigma_{position}(\tau = 1\,\mathrm{day}) \le 0.7\,\mathrm{m}$ $\quad\sigma_{velocity}(\tau = 1\,\mathrm{day}) \le 0.9\,\mathrm{mm/s}$
$\sigma_{position}(\tau = 10\,\mathrm{day}) \le 0.2\,\mathrm{m}$ $\quad\sigma_{velocity}(\tau = 10\,\mathrm{day}) \le 0.3\,\mathrm{mm/s}.$

At this point, we are ready to discuss the performances of the ACES clock signal on ground. Fig. 7 shows the expected fractional frequency instability of the ACES clocks and subsystems (FCDP and MWL), together with the requirements on the orbit determination, both in Allan and time deviation. The green curve with solid squares represents the expected fractional frequency instability of the ACES clock signal out of the MWL ground terminal (ground restitution).

POST NEWTONIAN 4-METRIC $\qquad \varepsilon = \pm 1 \quad \varepsilon(+---)$

${}^3g_{rs} \quad (+++)$

$$\begin{cases} {}^4g_{\tau\tau} = \varepsilon\left(1 - \frac{2}{c^2}W + \frac{2}{c^4}W^2\right) = \varepsilon\left(N^2 - {}^3g_{rs}N^rN^s\right) \\ {}^4g_{\tau r} = \varepsilon \frac{4}{c^3}W_r = -\varepsilon N_r = -\varepsilon\, {}^3g_{rs}N^s \\ {}^4g_{rs} = -\varepsilon\, {}^3g_{rs} = -\varepsilon\left(1 + \frac{2}{c^2}W + \frac{A}{c^4}\right) \end{cases}$$

$$\begin{cases} W = W(\tau, \vec\sigma) \\ W_r = W_r(\tau, \vec\sigma) \end{cases}$$

$${}^4g^{\tau\tau} = \varepsilon\left(1 + \frac{2}{c^2}W + \frac{2}{c^4}W^2\right)$$

$${}^4g^{\tau r} = -\varepsilon N^r = \varepsilon \frac{4}{c^3}W_r$$

$${}^4g^{rs} = -\varepsilon\left({}^3g^{rs} - \frac{N^rN^s}{N^2}\right) = -\varepsilon\left(1 - \frac{2W}{c^2} - \frac{A - 4W^2}{c^4}\right)$$

$$\begin{cases} N = 1 - \frac{W}{c^2} + \frac{3}{2}\frac{W^2}{c^4} \\ N^r = -\frac{4}{c^3}W_r \end{cases}$$

$${}^3K_{rs} = -\frac{\partial_\tau W}{c^2}\delta_{rs} - \frac{2}{c^3}\left(\partial_r W_s + \partial_s W_r\right) - \frac{1}{2c^4}\left(\partial_\tau A + 2W\partial_\tau W\right)\delta_{rs}$$

$${}^4\Gamma^v_{rs} = {}^3\Gamma^v_{rs} = \frac{2}{c^2}\left[\delta^v_r\partial_s W + \delta^v_s\partial_r W - \partial_v W\delta_{rs}\right] +$$
$$+ \frac{1}{c^4}\left[\delta^v_r\left(\partial_s A - 4W\partial_s W\right) + \delta^v_s\left(\partial_r A - 4W\partial_r W\right) - \delta_{rs}\left(\partial_v A - 4W\partial_v W\right)\right]$$

$\Sigma_\tau \qquad \hat{e}^r_a = \delta^r_a$ FIXED STARS

SEN-WITTEN TRIADS

$$\begin{cases} {}^3\nabla_r \hat{e}^r_1 = {}^3\nabla_r \hat{e}^r_2 = 0 \\ {}^3\nabla_r \hat{e}^r_3 = -\alpha\, {}^3K = \alpha\left[\frac{\partial_\tau W}{c^2} + \frac{2}{c^3}\partial_r W_r + \frac{3}{2c^4}\left(\partial_\tau A - 2W\partial_\tau W\right)\right] \\ \hat{e}^r_1 \hat{e}^s_2 \; {}^3\nabla_r \hat{e}_{3s} + \text{circul} = 0 \end{cases}$$

ACES

$$Z^\mu(\tau, \vec\sigma) = X^\mu(\tau) + \frac{\partial Z^\mu(\tau, \vec\sigma)}{\partial \sigma^A} F^A(\tau, \vec\sigma)$$

$$F^\tau(\tau, \vec\sigma) = \tau \qquad F^r(\tau, \vec\sigma) = \hat{e}^r_a(\tau, \vec\sigma)\,\sigma^a + \tau N_r$$

$$V_{1\text{-WAY}} \cong c\left(1 - \frac{W}{c^2}\right)$$

$$\Rightarrow \; Z^\mu(\tau, \vec\sigma) = \underbrace{X^\mu(\tau) + \epsilon^\mu_\tau \tau + \epsilon^\mu_a \sigma^a}_{\text{INERTIAL SPACELIKE HYPER-PLANE}} + \frac{1}{c^2}(\cdots\cdots) + \frac{1}{c^3}(\cdots\cdots)$$

INTERPRETATION

NO SPIN-2 IN INERTIAL FRAME OF THE BACKGROUND FOR THE LINEARIZED THEORY

$\boxed{\text{FRW COSMOLOGY}}$

$V_{1\text{-WAY}} = c$

$g_{\tau\tau} = \varepsilon, \; g_{\tau r} = 0$

$g_{rs} = -\varepsilon\delta_{rs}A(\tau)$

$A(\tau) = \frac{k^2(\tau)}{\left(1 + \eta\frac{r^2}{4}\right)^2}$

$\eta = -1$ OPEN

${}^3K_{rs} = \frac{\varepsilon}{2}\frac{dA(\tau)}{d\tau}\delta_{rs}$

UNDER INVESTIGATION WITH D. BINI

$$\hat{e}^{\,r}_a = \delta^r_a + \frac{1}{c^2}\, f^r_a + \frac{1}{c^3}\, g^r_a$$

1) $$ {}^3\nabla_r\, \hat{e}^{\,r}_a = \Big(\partial_r + \frac{1}{c^2}\Gamma_r\Big)\Big(\delta^r_a + \frac{1}{c^2} f^r_a + \frac{1}{c^3} g^r_a\Big) = \delta_{a3}\Big(\frac{K_2}{c^2} + \frac{K_3}{c^3}\Big)$$

$$\begin{cases} f^r_a(\vec\sigma) = \int d^3\sigma_1\, c^r(\vec\sigma - \vec\sigma_1)\big(\delta_{a3} K_2 - \Gamma_a\big)(\vec\sigma_1) + P^{rs}(\vec\sigma)\, h_{as}(\vec\sigma) \\ g^r_a(\vec\sigma) = \int d^3\sigma_1\, c^r(\vec\sigma - \vec\sigma_1)\, \delta_{a3} K_3 \qquad\qquad + P^{rs}(\vec\sigma)\, t_{as}(\vec\sigma) \end{cases}$$

$$c^r(\vec\sigma) = \frac{\sigma^r}{4\pi |\vec\sigma|^3} \qquad \partial_r c^r(\vec\sigma) = \delta^3(\vec\sigma) \qquad P^{rs} = \delta^{rs} - \frac{\partial^r \partial^s}{\Delta}$$

2) $$\hat{e}^{\,r}_1\, \hat{e}^{\,s}_2\, {}^3\nabla_r\, \hat{e}_{3s} + (\text{CYCLIC}) = 0$$

$$\begin{cases} \big(\partial_1 P^{2s} + \partial_3 P^{1s} + \partial_2 P^{3s}\big)\, h_{as} = \\ \qquad = -\int d^3\sigma_1 \Big[\partial_1 c^2(\vec\sigma - \vec\sigma_1)(K_2 - \Gamma_3)(\vec\sigma_1) + \partial_3 c^1(\vec\sigma - \vec\sigma_1)\Gamma_2(\vec\sigma_1) - \partial_2 c^3(\vec\sigma - \vec\sigma_1)\Gamma_1(\vec\sigma_1)\Big] \\ \big(\partial_1 P^{2s} + \partial_3 P^{1s} + \partial_2 P^{3s}\big)\, t_{as} = -\int d^3\sigma_1\, \partial_1 c^2(\vec\sigma - \vec\sigma_1)\, K_3(\vec\sigma_1) \end{cases}$$

3) $$z^\mu(\tau, \vec\sigma) = X^\mu(0) + \frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\tau} + \frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\sigma^r}\Big[\hat{e}^{\,r}_a\, \sigma^a + \tau\, n_r\Big](0, \vec\sigma)$$

Σ_τ $$\stackrel{\text{def}}{=} \underbrace{X^\mu(0) + \epsilon^\mu_\tau\, \tau + \epsilon^\mu_r\, \sigma^r}_{\text{INERTIAL PLANE}} + \frac{1}{c^2}\, C^\mu_1(\tau, \vec\sigma) + \frac{1}{c^3}\, C^\mu_2(\tau, \vec\sigma)$$

PN INSTANTANEOUS 3-SPACE (DYNAMICAL PN CONVENTION FOR CLOCK SYNCHRONIZATION)

$$\begin{cases} \Big(1 - \tau\frac{\partial}{\partial\tau} - \sigma^r \frac{\partial}{\partial\sigma^r}\Big)\, C^\mu_1(\tau, \vec\sigma) = \epsilon^\mu_r\, f^r_a(\tau, \vec\sigma)\, \sigma^a \\ \Big(1 - \tau\frac{\partial}{\partial\tau}\Big)\, C^\mu_2(\tau, \vec\sigma) = \epsilon^\mu_s \Big[g^s_a(\tau, \vec\sigma)\, \sigma^a - 4\tau\, W_s(\tau, \vec\sigma)\Big] \end{cases}$$

SIMILAR CALCULATIONS FOR THE INSTANTANEOUS 3-SPACE OF FRW COSMOLOGY

$$z^\mu(\tau, \vec\sigma) = \text{PLANE OF CONSTANT COSMIC TIME} \; +$$
$$+ \;\ldots\ldots$$

Σ_τ WITH $\;{}^3K_{rs}(\tau, \vec\sigma) \propto \frac{dA(\tau)}{d\tau}\, \delta_{rs} \qquad A(\tau) = \frac{a^2(\tau)}{(1 + \eta\, r^2)^2}$

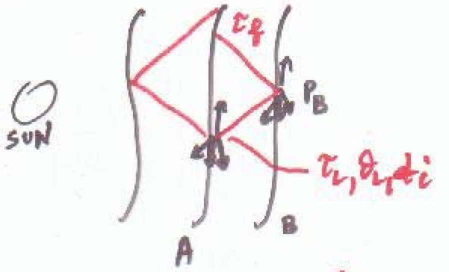
EINSTEIN SPACETIMES – EACH OF THEM HAS A DYNAMICALLY DETERMINED FAMILY OF ADMISSIBLE NOTIONS OF SIMULTANEITY (CONNECTED BY ON-SHELL GAUGE TRANSF.)

– THEY ARE MUCH LESS IN NUMBER THAN THOSE POSSIBLE IN SPECIAL RELATIVITY

MINKOWSKI SPACETIME AS A SPECIAL SOLUTION OF EINSTEIN'S EQS. (NO MATTER) –
→ ITS DYNAMICALLY DETERMINED ADMISSIBLE NOTIONS OF SIMULTANEITY HAVE
3-CONFORMALLY FLAT Σ_τ [$r_{\bar a} = \pi_{\bar a} = 0 \Rightarrow$ COTTON-YORK TENSOR $= 0$]
↳ NO SUCH RESTRICTION IN SPECIAL RELATIVITY AS AN AUTONOMOUS THEORY

PROTOCOL FOR AN EMPIRICAL DEFINITION OF SPACETIME

(LIKE, IN METROLOGY, "TIME" IS A REFERENCE ATOMIC CLOCK)



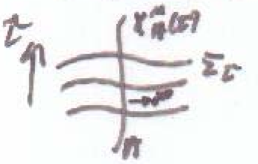
SET OF SPACECRAFTS

WORLD LINES
4-VELOCITIES } KNOWN FROM GPS SPACE NAVIGATION

EACH SPACECRAFT IS A TIMELIKE OBSERVER WITH A TETRAD (GYROSCOPES → SPATIAL TRIADS)

A SENDS RADAR SIGNALS WHICH ARE REFLECTED BACK TO A
A KNOWS $\tau_i, \theta_i, \phi_i, \tau_f$ FOR EACH SIGNAL

GIVEN 4 SUITABLE FUNCTIONS $\mathcal{E}(\tau_i, \theta_i, \phi_i, \tau_f)$, $\vec{\xi}(\tau_i, \theta_i, \phi_i, \tau_f)$
A ESTABLISH AN ADMISSIBLE RADAR 4-COORDINATE SYSTEM



$$\begin{cases} \tau(P_B) = \tau_i + \mathcal{E}(\tau_i, \theta_i, \phi_i, \tau_f)\,(\tau_f - \tau_i) \\ \vec{\sigma}(P_B) = \vec{\xi}(\tau_i, \theta_i, \phi_i, \tau_f) \end{cases} \qquad 0 < \mathcal{E} < 1$$

FROM MEASUREMENTS WITH EITHER TEST PARTICLES OR POLARIZED LIGHT
A DETERMINES ${}^4g_{AB}(\tau, \vec{\sigma})$ IN THESE RADAR 4-COORD.

$$\begin{cases} \text{IF EINSTEIN'S EQS SATISFIED} \Rightarrow \Sigma_\tau \text{ IS A DYNAMICAL SIMULTANEITY 3-SURFACE} \\ \text{OTHERWISE CHANGE } \mathcal{E}, \vec{\xi} \text{ AND TRY AGAIN – SUCCESSIVE APPROXIMATIONS} \\ \qquad \text{TO THE UNKNOWN IDENTIFICATION OF POINT-EVENTS} \end{cases}$$

OPEN PROBLEM – DYNAMICAL THEORY OF MEASUREMENT
REPLACE THE TEST MATTER (EHLERS-PIRANI-SCHILD AXIOMATIC THEORY)
WITH DYNAMICAL MATTER