$K^+ \to \pi^+ \nu \bar{\nu}$ from standard to new physics

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Content of the talk :

- $K^+ \to \pi^+ \nu \bar{\nu}$ in the Standard Model
- Generic SUSY with unbroken R-parity
- R-parity violation
- Constraints on the R_p violating couplings
- Summary - Conclusions

$K \to \pi\nu\bar{\nu}$ in the Standard Model

(A.Buras *et al.* [hep-ph/0405132]; G.D'Ambrosio, G.Isidori [hep-ph/0112135])

$\longrightarrow$ There is **no tree-level contribution** to the process.

$\longrightarrow$ Only significative contribution in the loops by the **charm** and the **top** quarks.

$\longrightarrow$ Effective Hamiltonian governing $K \to \pi\nu\bar{\nu}$:

$$H_{eff} = \frac{G_f}{\sqrt{2}} \frac{2\alpha_e}{\pi \sin^2\theta_w} \sum_l \left(\lambda_c X_c^l + \lambda_t X_t \right) \bar{s_L} \gamma^\mu d_L \; \bar{\nu_L^l} \gamma_\mu \nu_L^l + h.c.$$

- $\lambda_i = V_{is}^* V_{id}$ is a product of CKM elements.

- X_c^l is the charm contribution for flavour l.

- X_t is the top contribution (no lepton flavour dependence because lepton masses can be neglected with respect to the top mass)

$\longrightarrow$ **EW** calculations at the **one-loop level**.

$\longrightarrow$ **QCD** corrections at the **NLO level**

$\longrightarrow$ The branching ratio for $K \to \pi\nu\bar{\nu}$ can be expressed as :

$$BR^{SM} = \bar{\kappa}_+ \left[\left(\frac{Im(\lambda_t)X_t}{\lambda} \right)^2 + \left(\lambda^4 P_c\,(X)\, \frac{Re(\lambda_c)}{\lambda} + \frac{Re(\lambda_t)}{\lambda} X_t \right)^2 \right]$$

- $\lambda = |V_{us}|$

- $P_c(X)$ is the charm loop-function $P_c(X) = \frac{1}{3\lambda^4}(2\,X_c^e + X_c^\tau)$.

- The **hadronic matrix element** is **related via isospin to** the experimentally well known decay $K^+ \to \pi^0 e^+ \nu_e$ (Marciano, Parsa).

- $\bar{\kappa}_+ = r_+ \frac{3\alpha^2(m_Z)\, BR(K^+ \to \pi^0 e^+ \nu_e)}{2\pi^2 \sin^4(\theta_w)}$ with $r_+ = 0.901$ an isospin violation correction factor.

$\Rightarrow$ The branching rate becomes **theoretically clean**.

Numerical results:

$\longrightarrow$ We used the **Wolfenstein parametrization** for the CKM matrix, and the **fits** of (λ, $|V_{cb}|$, $\bar{\rho}$, $\bar{\eta}$) with the **latest top mass value**.

$\longrightarrow$ Moreover, the PDG value in July 2004 for $BR(K^+ \to \pi^0 e^+ \nu_e)$ is :

$$BR(K^+ \to \pi^0 e^+ \nu_e) = (4.87 \pm 0.06) \times 10^{-2}$$

But **did not include** the E865 result: (A.Sher *et al.* [hep-ex/0305042])

$$BR(K^+ \to \pi^0 e^+ \nu_e) = (5.13 \pm 0.15) \times 10^{-2}$$

We combined the two results, the **average** is:

$$BR(K^+ \to \pi^0 e^+ \nu_e) = (5.08 \pm 0.13) \times 10^{-2}$$

$\Rightarrow$ The **central value increases** by 4.4$\%$ and there are **larger errors**.

The **prediction in the SM** for the branching ratio of $K \to \pi\nu\bar{\nu}$ is then:

$$BR^{SM} = (8.18 \pm 1.22) \times 10^{-11}$$

Our BR is **slightly larger** than the recent prediction by Buras *et al.*

$$BR^{SM} = (7.8 \pm 1.2) \times 10^{-11}$$

The E787 and E949 collaborations give the **experimental result**:

$$BR^{EXP} = (1.47\,^{+1.3}_{-0.8}) \times 10^{-10}$$

$\Rightarrow$ This is **compatible with the Standard Model** but there is **enough place for new physics.**

$\Rightarrow$ We can **constraint new physics parameters** with $K \to \pi\nu\bar{\nu}$.

Which new physics ? (A.Buras *et al.* [hep-ph/0405132])

- General SUSY Models with R_p conserved?
- SUSY with LFV ?
- Universal Extra Dimensions ?
- SUSY $\not{R}_p$? $\rightarrow$ **this work**

$K \to \pi\nu\bar{\nu}$ in the general MSSM

(Buras, Romanino,Silvestrini [hep-ph/9712398]; Colangelo, Isidori [hep-ph/9808487])

$\longrightarrow$ **No tree-level contribution** just as in the SM.

$\longrightarrow$ Significant contribution in the loops by **Charged Higgses, Charginos, Neutralinos**.

$\longrightarrow$ **SUSY contribution** can be **of the same order as the SM ones**.

Features and assumptions of the analysis ?

- One-loop level calculations.
- Only dimension six operators ($\bar{s}_L \gamma^\mu d_L \ \bar{\nu}^l_L \gamma_\mu \nu^l_L$)
- Minimal field content (MSSM-like)
- Unbroken R_p.
- New physics contribution to $K^+ \to \pi^0 e^+ \nu_e$ neglected.

$\longrightarrow$ Same effective Hamiltonian as in the SM but X_t^{SM} **replaced by** $X_t^{new} = r_K e^{-i\theta_K} X_t^{SM}$

$$H_{eff} = \frac{G_f}{\sqrt{2}} \frac{2\alpha_e}{\pi \sin^2 \theta_w} \sum_l \left(\lambda_c X_c^l + \lambda_t X_t^{new}\right) \bar{s_L} \gamma^\mu d_L \; \bar{\nu_L^l} \gamma_\mu \nu_L^l + h.c.$$

$\Rightarrow$ r_K **and** θ_K **parameterize new physics contributions**. They are functions of **masses and couplings** of the new particles.

$\longrightarrow$ We have $X_t^{new} = X_t^{(SM)} + X_{H^\pm} + X_{\tilde{C}} + X_{\tilde{N}}$

<u>Remarks:</u>

- The **SM is included** as a special case: $r_K = 1$ and $\theta_K = 0$
- New physics effects proportional to λ_c are included in X_t^{new}.

The **flavour structure** of SUSY theories is **complicated** and **unknown**,

$\Rightarrow$ the calculation of the Feynman graphs is done in the **Mass Insertion Approximation** = diagonalization of the mass matrices perturbatively around the diagonal (Hall, Kostelecky, Rabi)

- Chargino/squark and neutralino/squark contributions are:

$$
\begin{aligned}
X_{\tilde{C}} &= C^0 + C_{LL}\frac{(\delta^U_{12})_{LL}}{\lambda_t} + C_{LR}\frac{(\delta^U_{23})_{LR}}{\lambda_t m_t} V_{td} + C_{RL}\frac{(\delta^U_{31})_{RL}}{\lambda_t m_t} V^*_{ts} \\
X_{\tilde{N}} &= \tilde{N}\frac{(\delta^D_{12})_{LL}}{\lambda_t}
\end{aligned}
$$

$\hookrightarrow$ the δs are typically $\frac{(m^2_Q)_{ij}}{m^2_{\tilde{q}_L}}$, with $(m^2_Q)_{ij}$ off-diagonal elements of the squark mass matrices.

$\hookrightarrow$ C_i and $\tilde{N}$ are functions at the one loop level.

$\longrightarrow$ Using various experimental results it is possible to derive **upper limits** on the δ's -Buras, Romanino, Silvestrini; Colangelo, Isidori; Gabbiani, Gabrielli, Masiero, Silvestrini; Misiak, Pokorski, Rosiek.

$\longrightarrow$ By varying the δ's and all the SUSY parameters ($\tan\beta$, $m_{\tilde{q}_L}$...), we have a **typical range** (=the most probable values) for r_K and θ_K:

$$0.5 < r_K < 1.3, \ -25^o < \theta_K < 25^o$$

$\longrightarrow$ That makes at most a change of $\sim \pm$50% for the branching ratio of $K^+ \to \pi^+ \nu \bar{\nu}$.

$$BR^{SUSY}(K^+ \to \pi^+ \nu\bar{\nu}) = (8.18 \ ^{+4.26}_{-5.23}) \times 10^{-11}$$

(the central value corresponds to the SM value, $r_K = 1$ and $\theta_K = 0$.

Variations correspond to the maximal and minimal SUSY contribution)

Contribution of $\not R_p$ violating SUSY

(R.Barbier *et al.*, R-parity violating supersymmetry [hep-ph/0406039])

R-parity:

$$R_p = (-1)^{3(B-L)+2S}$$

$\longrightarrow$ **New terms** allowing **R-parity violation** in the superpotential:

$$W_{\not R_p} = \underbrace{\lambda_{ijk} L_i L_j E_k}_{\not L} + \underbrace{\lambda'_{ijk} L_i Q_j D^c_k}_{\not L,\ \not B} + \underbrace{\lambda''_{ijk} U^c_i D^c_j D^c_k}_{\not B}$$

$\longrightarrow$ 45 unknown complex couplings.

$\longrightarrow$ We focus on λ'_{ijk} couplings. They induce **tree level** contributions *via* **squark exchanges** to $K^+ \to \pi^+ \nu\bar{\nu}$.

$$\mathcal{L}_{L_i Q_j D^c_k} = -\lambda'_{ijk} \left(\tilde{d}^*_{kR} \overline{\nu^c}_i d_{jL} + \tilde{d}_{jL} \nu_i \bar{d}_{kR} \right) + \mathrm{h.}c$$

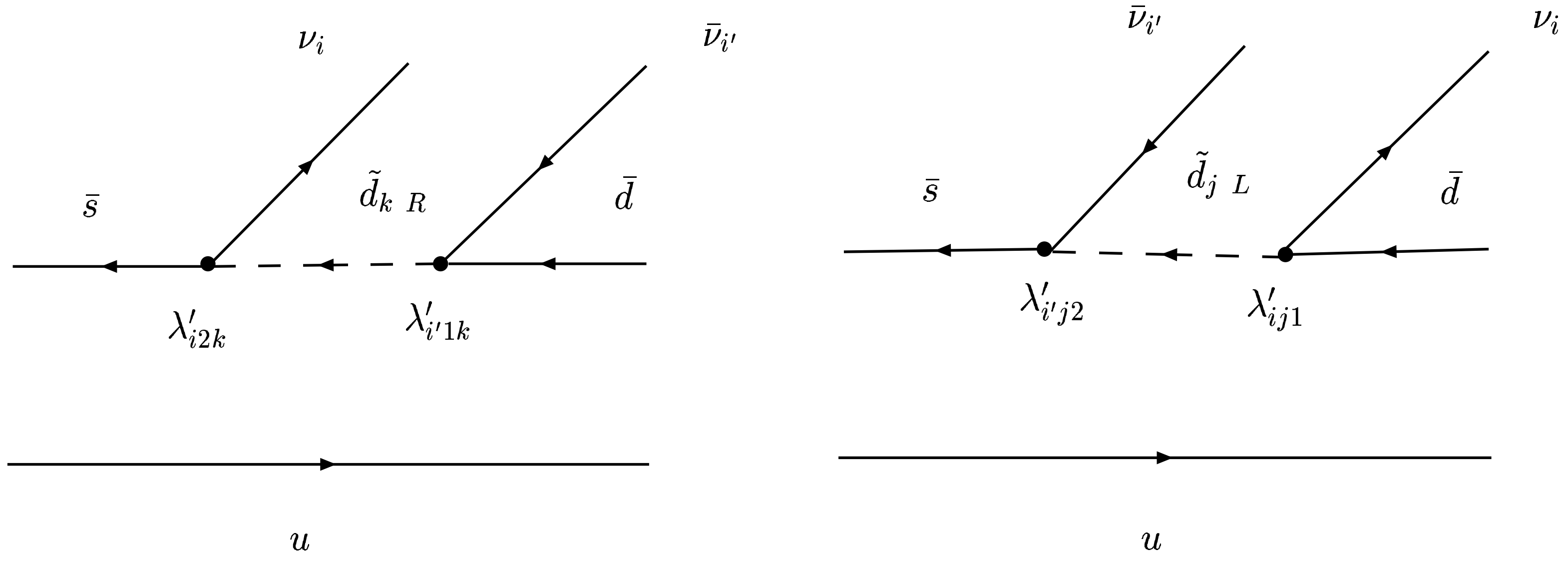

Figure 1: *R-parity violating tree level diagrams contributing to the process $K^+ \longrightarrow \pi^+ \nu\bar{\nu}$.*

$\Rightarrow$ R-parity violating couplings induce a contribution to $K^+ \longrightarrow \pi^+ \nu\bar{\nu}$ with a **neutrino and an antineutrino of different flavour** in the final state ! (*in contrast to the SM and the MSSM*)

$\rightarrow$ Including these 2 diagrams, the branching ratio can be written as:

$$BR = \frac{\bar{\kappa}_+}{3\lambda^2}\left(\sum_l \mid C_l^{\mathrm{SUSY}} + \frac{\epsilon_{ll}}{4k\,(200\ \mathrm{GeV})^2}\mid^2 + \sum_{b\neq l}\frac{\mid \epsilon_{bl}\mid^2}{16k^2\,(200\ \mathrm{GeV})^4}\right)$$

(the sum is over ν's and $\bar{\nu}$'s flavours)

Where:

$$C_l^{\mathrm{SUSY}} = \lambda_c X_c^l + \lambda_t\, X_{new}$$

$$k = \frac{G_f\alpha(m_Z)}{\sqrt{2}\pi\sin^2(\theta_w)} = 8.88\times 10^{-8}\ \mathrm{GeV}^{-2}$$

The RPV couplings appear in ϵ_{ij}:

$$\epsilon_{ij} = \sum_n\left(\frac{\lambda'^{*}_{i2n}\lambda'_{j1n}}{m^2_{\tilde{d}n_R}} - \frac{\lambda'^{*}_{in1}\lambda'_{jn2}}{m^2_{\tilde{d}n_L}}\right)(200\ \mathrm{GeV})^2$$

$\longrightarrow$ If we develop BR:

$$BR(K^+ \to \pi^+ \nu\bar{\nu}) = \underbrace{BR_{SUSY}}_{part2} + \underbrace{BR_{\not R_p} + BR_{int}}_{\to now}$$

$\longrightarrow$ we have a "Pure" R_p violating contribution:

$$BR_{\not R_p} = \frac{\bar{\kappa}_+}{48\lambda^2\ k^2(200\ \text{GeV})^4} \sum_{i,j} \mid \epsilon_{ij} \mid^2$$

$\longrightarrow$ There are **interferences between** SUSY and the $\not R_p$ part:

$$BR^{Int} = -2\frac{\bar{\kappa}_+}{12\lambda^2\ k(200\ \text{GeV})^2} \sum_{l} Re(C_l^{SUSY}\ \epsilon_{ll})$$

Constraints on the λ'

- First case: neglecting interferences (approximation)

As we want an **upper-bound**, we assume the SUSY contribution to be **minimal** (= the resulting BR is minimal).

$\rightarrow$ We compare the sum $BR_{\not R_p}$ + $BR_{SUSY}|_{min}$ with the experimental value $(1.47\,^{+1.3}_{-0.8})\,10^{-10}$.

$\Rightarrow$ this gives for squarks masses at 200 GeV:

$$\sum_{i,j} \mid \epsilon_{ij} \mid^2 < 4.45 \times 10^{-10}$$

Recalling that:

$$\epsilon_{ij} = \sum_n \left(\frac{\lambda'^*_{i2n} \lambda'_{j1n}}{m^2_{\tilde{d}n_R}} - \frac{\lambda'^*_{in1} \lambda'_{jn2}}{m^2_{\tilde{d}n_L}} \right) (200 \text{ GeV})^2$$

How to translate this into an **upper-bound on** $\lambda^{'}$?

$\longrightarrow$ we naively set **all the couplings to zero except one product** (single coupling dominance hypothesis)

We have then:

$$\left| \frac{\lambda^{'*}_{i2n}\lambda^{'}_{j1n}}{m^2_{\tilde{d}n_R}} \right| < 2.11 \times 10^{-5}$$

$$\left| \frac{\lambda^{'*}_{in1}\lambda^{'}_{jn2}}{m^2_{\tilde{d}n_L}} \right| < 2.11 \times 10^{-5}$$

- Second case: full analysis.

$\longrightarrow$ Interferences make the extraction of upper-bounds harder. They occur if the final neutrino and antineutrino are of the same flavour, $i = j$.

$\longrightarrow$ The general branching ratio formula compared with the experimental value gives:

$$\sum_{i=e,\mu,\tau} \left(Re(\epsilon_{ii}) + \frac{\alpha_i}{2} \right)^2 + \sum_{i=e,\mu,\tau} \left(Im(\epsilon_{ii}) + \frac{\beta}{2} \right)^2 = R^2$$

-α and β contains the CKM inputs and the loop functions X_t and X_c^l.

-The radius R is proportional to $(BR^{exp} - BR^{SUSY}|_{min})$ and contains the shifts α and β.

$\Rightarrow$ For only one non-zero ϵ_{ii}, this equation describes a **circle in the complex plane**.

$\Rightarrow$ The resulting constraint for ϵ_{11} is the following circle:

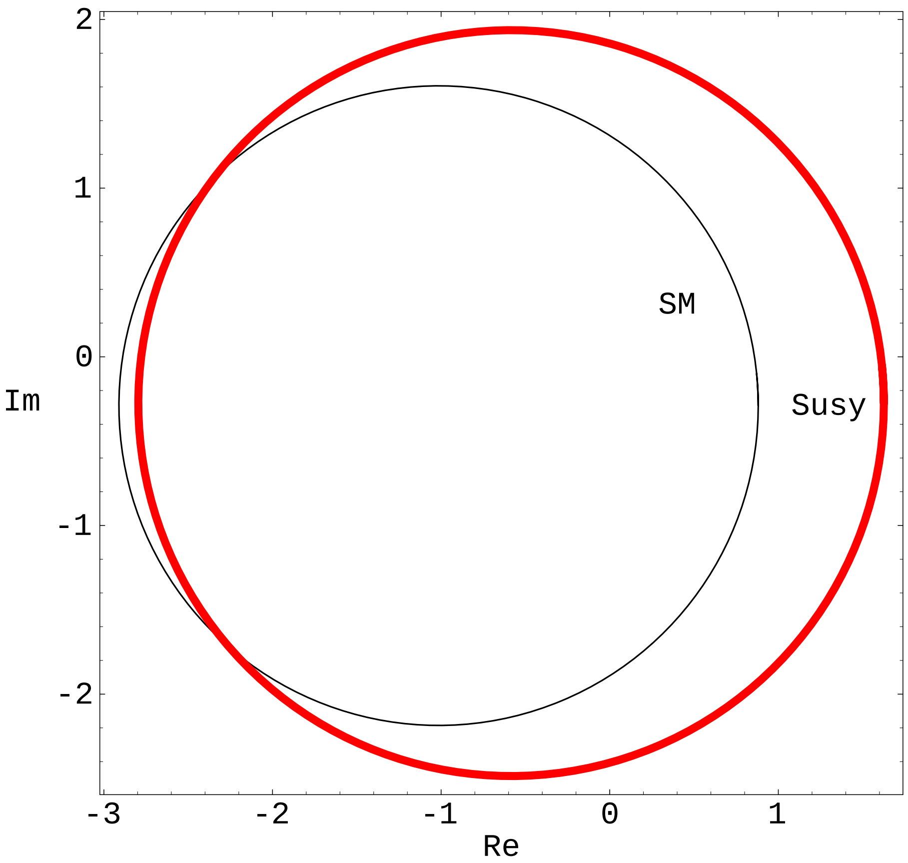

Figure 2: Allowed region for $Re(\epsilon_{11})$ and $Im(\epsilon_{11})$ in units of 10^{-5}. We take 200 GeV as reference value for the mass of the squarks.

$\Rightarrow$ If we choose the point ($Re(\epsilon_{11}) = -2\ 10^{-5}$, $Im(\epsilon_{11}) = -2\ 10^{-5}$) to have an **numerical idea of the interferences**, we have :

$$| \epsilon_{11} | = 2.8 \times 10^{-5}$$

Then,

$$| \frac{\lambda'^{*}_{i2n}\lambda'_{i1n}}{m^2_{\tilde{d}n_R}} | < 2.8 \times 10^{-5}$$

$$| \frac{\lambda'^{*}_{in1}\lambda'_{in2}}{m^2_{\tilde{d}n_L}} | < 2.8 \times 10^{-5}$$

$\Rightarrow$ **upper-bounds including interferences are 30$\%$ bigger than without**.

Summary

We have investigated the decay $K^+ \to \pi^+ \nu \bar{\nu}$ to obtain stringent **limits on the R-parity violating couplings**.

- First we updated the standard model value of the branching ratio:

$$B = (8.18 \pm 1.22) \times 10^{-11}$$

- We have then analyzed the general SUSY contribution and corrected few misprints present in the literature.

- Finally, we obtained more realistic upper-bound on the products of the RPV couplings $\lambda^{'}$.

Conclusions

In establishing limits on RPV couplings involved in $K^+ \to \pi^+ \nu \bar{\nu}$:

- One loop SM contribution should be taken into account.
- One loop SUSY can contribute up to 50$\%$ of the SM and should be taken into account too.
- Interferences between SUSY and "pure" RPV part do have a significant influence.