Wilson Lines in **Warped** Space

Dynamical Symmetry Breaking and Restoration

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`hep-ph/0410061`

Outline

1. Motivation: Warped SUSY GUT and SUSY Breaking
2. Setup: 5-Dim $SU(N)$ with Warped Compactific'n on S^1/Z_2
3. Analysis: KK Expansions and Effective Potential

Orbifold Compactification on S^1/Z_2

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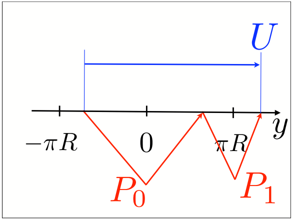

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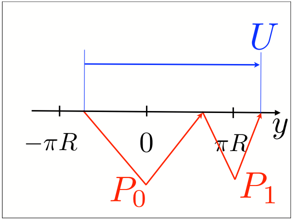

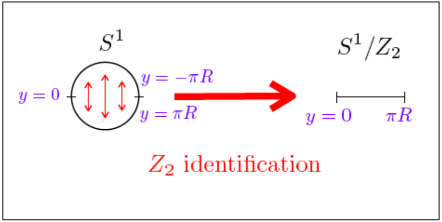

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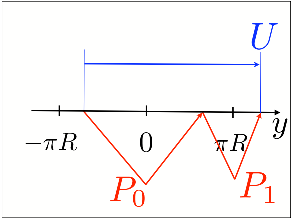

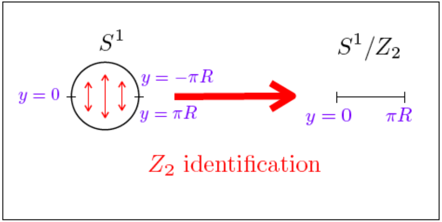

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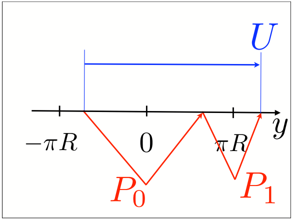

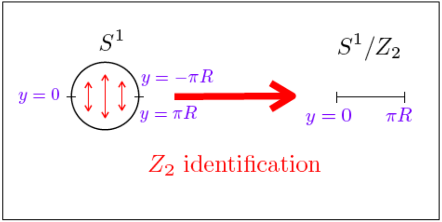

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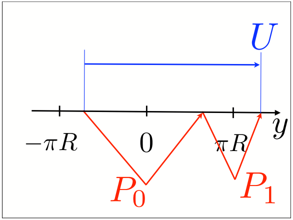

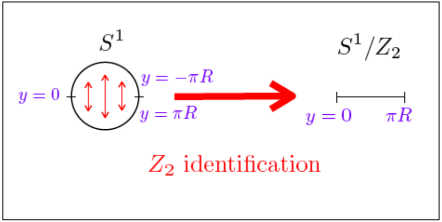

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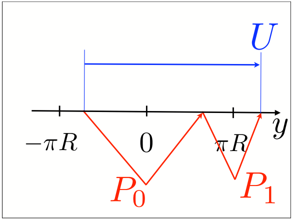

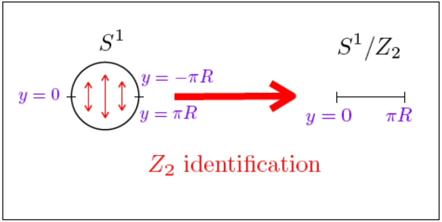

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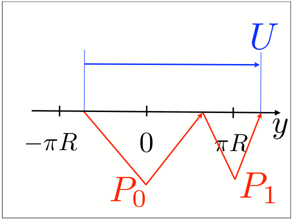

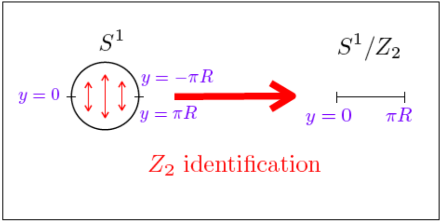

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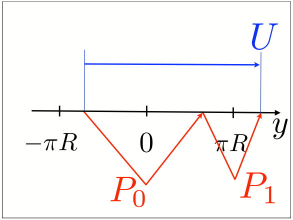

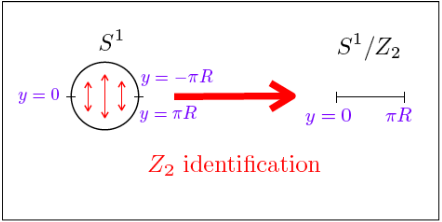

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- P_i: $\partial_y \to -\partial_y$

5-dim **SU(5)** GUT on S^1/Z_2

Kawamura's realization of **doublet-triplet (2-3) splitting**

Take [Kawamura '00, '01; Altarelli, Feruglio '01; Hall, Nomura '01, '02; Hebecker, March-Russell '01]

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Then, (P_0, P_1) parities are

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- **2-3 splitting** realized!

Gauge Twist and Scherk-Schwarz SUSY Breaking

Similarly, for $SU(2)_R$ auxiliary gauge field, **continuous twists**

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- SUSY unbroken or broken at GUT scale!

Motivation

We can use large hierarchy
in Randall-Sundrum
warped compactification,
which is also on S^1/Z_2,
if we learn how to treat
continuous Wilson lines
associated with continuous twists!

Randall-Sundrum Geometry

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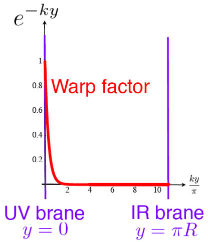
AdS Metric for $-\pi R < y \leq \pi R$:

$$e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

(k: [AdS radius]$^{-1} \lesssim M_{\text{Planck}}$)



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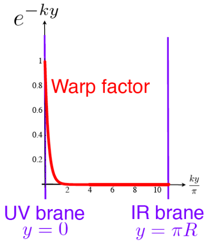

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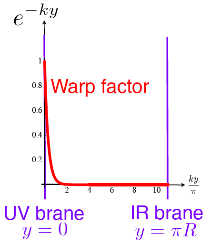

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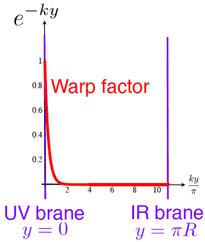

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- If, say, $kR \simeq 11$, large hierarchy generated

$$a \equiv e^{-k\pi R} \simeq 10^{-15}.$$

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Slogan

Make use of zero modes of A_y as adjoint Higgs fields.

General Twists, Zero Modes

- Most general $SU(N)$ gauge twists can be block-diagonalized into $SU(2)$ subspaces:

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- Hereafter concentrate on a $SU(2)$ block with above twists.

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Large gauge transformation

$$\Omega = e^{i\varphi y T} : \quad g\langle A_y \rangle \to g\langle A_y \rangle - \frac{\varphi}{\pi R}, \quad \theta \to \theta - \varphi, \quad m_n \to m_n$$

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- Symmetry breaking pattern
  determined by dynamics rather than by hand!

Kaluza-Klein Expansions, Simple when Even or Odd

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- As in flat case, there is zero mode of A_y
  contributing to Wilson lines
- This time, it is **not flat** but
  exponentially localized on IR brane $\propto z(y)^2$, where

$$z(y) = e^{k|y|}.$$

(Recall: k is [AdS curvature radius]$^{-1} \lesssim M_{\text{Planck}}$)

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$\longrightarrow$ Then we can **freely** take $\theta = 0$ gauge (or whatever)

KK Exp'ns for Most General Twist and B.G. (Cont'd)

KK expansions in $\theta = 0$ gauge:

$$A_\mu^\pm \sim \left(\cos\frac{vz^2}{2} \pm i\epsilon(y)\sin\frac{vz^2}{2}\right) z\left[\alpha_n J_1(\mu_n z) + \beta_n Y_1(\mu_n z)\right],$$

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where $z = e^{k|y|}$, $v \sim \langle A_y \rangle$ and $\epsilon(y) = \begin{cases} +1 & \text{for } y > 0, \\ -1 & \text{for } y < 0. \end{cases}$

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- Wave functions continuous, even if accompanied by $\epsilon(y)$

Calculation of Effective Potential for $\langle A_y \rangle$

Effective potential for $v = \langle A_y \rangle / k$: (Recall: k = [AdS curvature radius]$^{-1} \lesssim M_{\text{Planck}}$)

$$V_{\text{eff}}(v) = a^4 I_{IR} + I_{UV}$$

$$+ \frac{(ka)^4}{(4\pi)^2} \int_0^\infty dx\, x^3 \log \left[1 - \frac{I_0(x)K_1(x) - K_0(x)I_1(x) - \frac{1}{x} \cos a^{-2} v}{2 I_0(x) I_1(x) \left(\gamma + \log \frac{ax}{2} \right)} \right],$$

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   $\rightarrow$ Can be renormalized by **brane tensions**

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Normalized effective potential $\sim a^{-4} V_{\text{eff}}$

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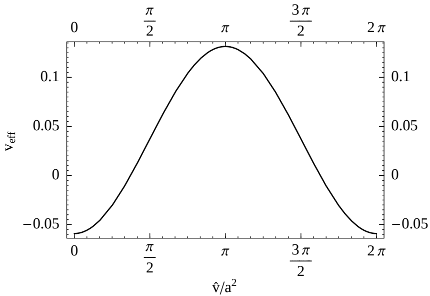

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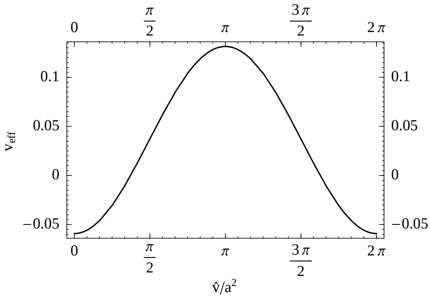

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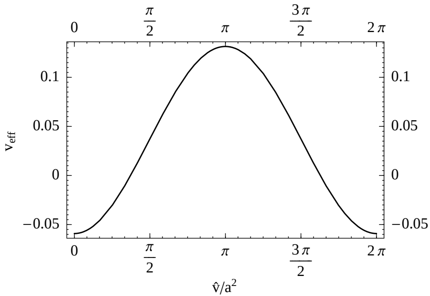

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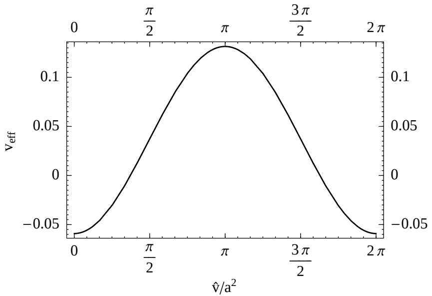

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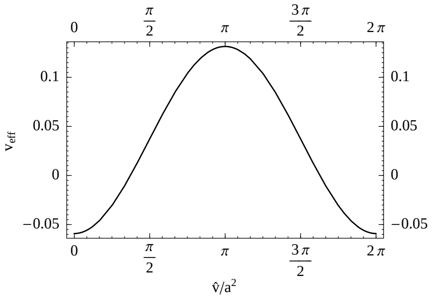

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  - **Dynamical Schwerk-Schwarz SUSY breaking**... Necessary to find a supergravity configuration proposed by Abe and Sakamura

    [Recently appeared toy models: Eto, Maru, Sakai '03; Correia, Schmidt, Tavartkiladze '04]