Radiative Corrections in the MSSM

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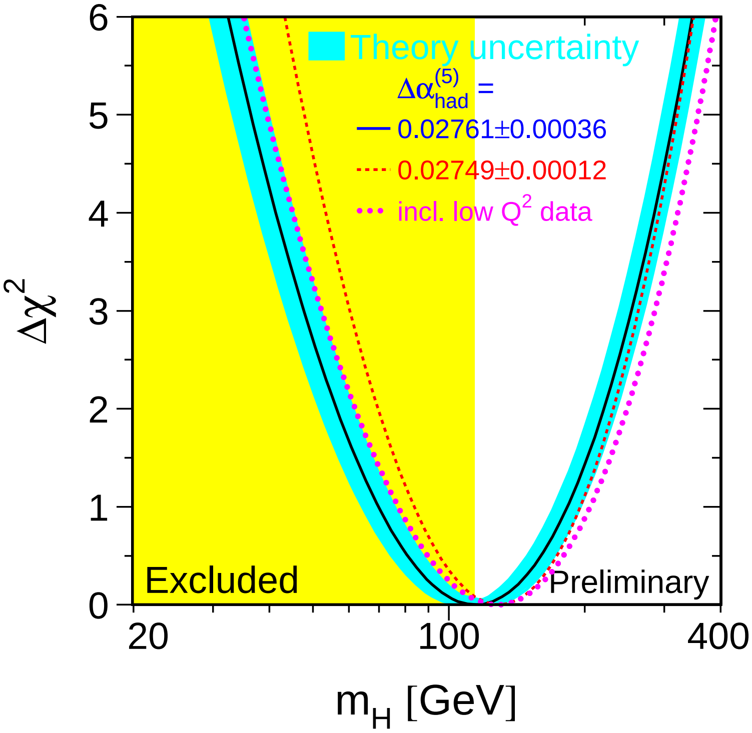
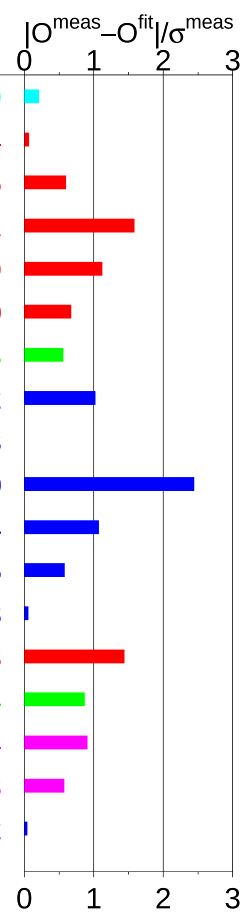
- The LEP legacy
- Higgs mass
- $B \to X_s \gamma$
- Precision physics: $(g-2)_\mu$, $\delta\rho$
- Conclusions

The LEP legacy



Summer 2004

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02769
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4966
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01650
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1483
R_b	0.21630 ± 0.00066	0.21562
R_c	0.1723 ± 0.0031	0.1723
$A^{0,b}_{fb}$	0.0998 ± 0.0017	0.1040
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0744
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1483
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.394
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.2



- Mechanisms of electroweak symmetry breaking with a "light" Higgs are clearly favored.
- The success of the SM fit places strong constraints on new-physics.
- Non-decoupling physics can exist (effects that do not vanish as $\Lambda \to \infty$ where Λ is the new physics scale). However it always need some conspiracy to pass the SM fit constraints.
- New physics of the decoupling type (effects that scale as M_Z^2/Λ^2) can avoid "naturally" ($\Lambda \to \infty$) the SM fit constraints.

Best (most fashionable) candidate for new physics is the Minimal Supersymmetric extension of the Standard Model (MSSM).

SUSY Virtue: SUSY *is* theory, *not* a scenario.

Radiative corrections to m_h

Higgs sector of the MSSM

- Two $SU(2) \times U(1)$ doublets:

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$$H_i^0 = \frac{v_i + S_i + i\,P_i}{\sqrt{2}} \qquad \tan\beta = \frac{v_2}{v_1}$$

  Soft SUSY-breaking mass terms for H_1^0 and H_2^0 responsible for electroweak symmetry breaking (EWSB):

$$V_{\rm tree} = (m_{H_1}^2 + \mu^2)\,|H_1^0|^2 + (m_{H_2}^2 + \mu^2)\,|H_2^0|^2 + m_3^2\,\left(H_1^0 H_2^0 + {\rm h.c.}\right) + \frac{1}{8}\,(g^2 + g'^{\,2})\,\left(|H_1^0|^2 - |H_2^0|^2\right)^2$$

- Five physical states: $h\,,\ H\,,\ A^0\,,\ H^+\,,\ H^-$

  CP-even Higgs h, H $(m_h < M_H)$ linear combination through an angle α of the H_1, H_2 doublets; ($H_1 \Rightarrow$ down–fermions, $H_2 \Rightarrow$ up–fermions).

Tree–level mass matrix for the CP–even sector:

$$\left(\mathcal{M}_S^2\right)^{\rm tree} = \left(\begin{array}{cc} M_Z^2\, c_\beta^2 + M_A^2\, s_\beta^2 & -\left(M_Z^2 + M_A^2\right) s_\beta\, c_\beta \\ & \\ -\left(M_Z^2 + M_A^2\right) s_\beta\, c_\beta & M_Z^2\, s_\beta^2 + M_A^2\, c_\beta^2 \end{array} \right)$$

$\rightarrow$ m_h and M_H predicted in terms of M_Z, M_A and $\tan\beta$

At the tree-level

$$\cos^2(\beta-\alpha) = \frac{m_h^2\,(M_Z - m_h^2)}{M_A^2\,(M_H^2 - m_h^2)}$$

As $M_A \gg M_Z$, $\cos^2(\beta-\alpha) \rightarrow 0$, decoupling limit:

- lightest Higgs has SM couplings, $m_h^2 \approx M_Z^2 \cos^2 2\beta$;
- $M_A \simeq M_H \simeq M_H^\pm$.

Decoupling limit corrections $\mathcal{O}(M_Z^2/M_A^2)$.

Radiative corrections to the MSSM Higgs sector

Tree-level result $m_h < M_Z$ (ruled out by LEP) corrected by important quantum contributions:

$$\left(\mathcal{M}_S^2\right)^{\rm tree} \to \mathcal{M}_S^2\,(p^2) = \left(\mathcal{M}_S^2\right)^{\rm tree} + \left(\Delta\mathcal{M}_S^2\right)^{\rm eff} + \left(\Delta\mathcal{M}_S^2\right)^{p^2}$$

(m_h^2, M_H^2) solutions of $\quad \det\left[p^2 - \mathcal{M}_S^2(p^2)\right] = 0$

SUSY breaking $\rightarrow$ incomplete cancellation between loops of particles and susy partners.

main effects: top and stop loops.

Corrections to m_h:

- scale as m_t^4;
- depend on the stop mixing parameter $X_t = A_t - \mu/\tan\beta$;
- have a logarithmic sensitivity to $M_{\tilde{t}_i}$

$$\Delta m_h \approx \frac{3\,\alpha_t\,m_t^2\sin^2\beta}{\pi}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12\,M_S^2}\right)\right] + \ldots$$

$\left(\alpha_t = \frac{h_t^2}{(4\,\pi)} = \frac{g^2\,m_t^2}{(8\,\pi\,\sin^2\beta)},\; M_S^2 = \frac{1}{2}\,(M_{\tilde{t}_1} + M_{\tilde{t}_2})\right)$

Large $\tan\beta$ scenario: (s)bottom loops also relevant.

Summary of more than ten years of calculations

Corrections to $\mathcal{M}_S^2(p^2)$:

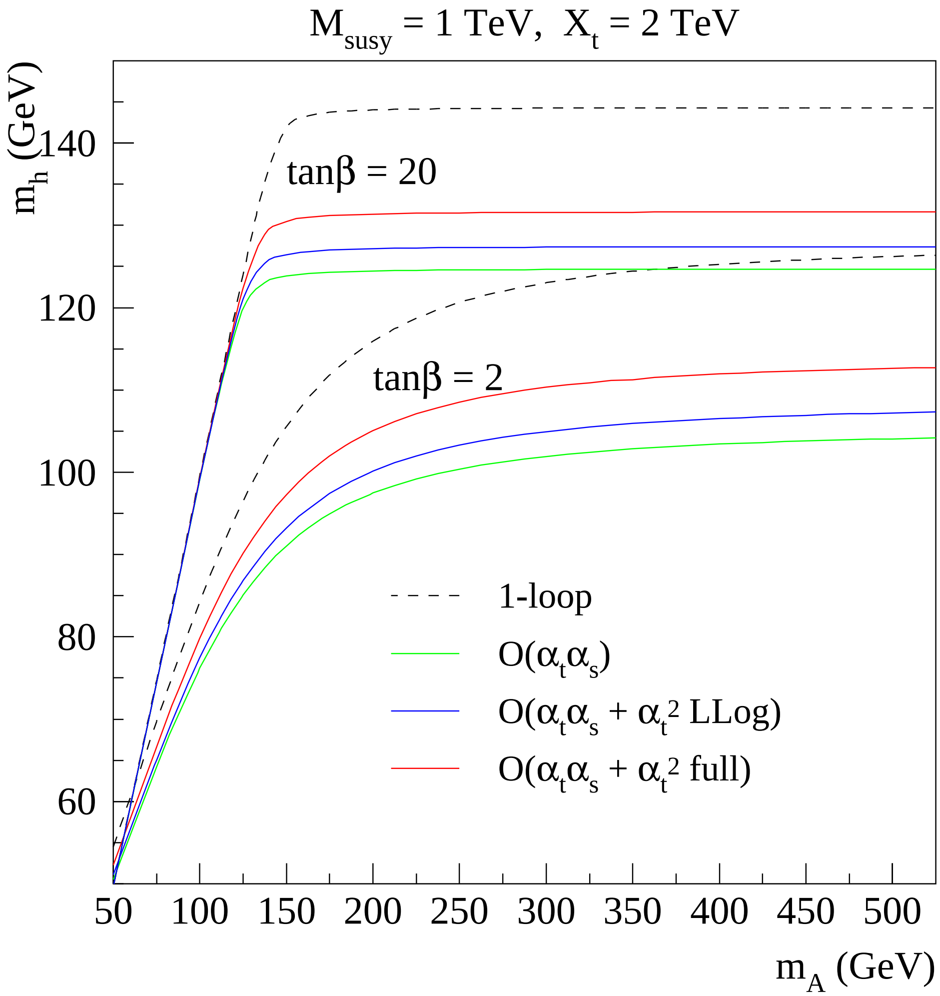
- one-loop: completely known;
  Ellis, Ridolfi, Zwirner (91); Okada, Yamaguchi, Yanagida (91); Haber, Hemplfling (91)......
- two-loop: dominant contributions to $\left(\Delta\mathcal{M}_S^2\right)^{\text{eff}}$ (effective potential approximation) known.
  - strong corrections to the one-loop (s)top term $(\mathcal{O}(\alpha_t\alpha_s))$;
    Heinemeyer, Hollik, Weiglein (98); Espinosa, Zhang (00); Slavich, Zwirner, GD (01)
  - Yukawa corrections to the one-loop (s)top term $(\mathcal{O}(\alpha_t^2))$;
    Espinosa, Zhang (00); Brignole, Slavich, Zwirner, GD (02)
  - strong corrections to the one-loop (s)bottom term $(\mathcal{O}(\alpha_b\alpha_s))$, including resummation of $\tan\beta$-enhanced terms $(\mathcal{O}(\alpha_b(\alpha_s\tan\beta)^n))$.
    Brignole, Slavich, Zwirner, GD (02); Carena, Garcia, Nierste, Wagner (00); Heinemeyer, Hollik, Rzehak, Weiglein (04)
  - mixed two-loop Yukawa corrections $(\mathcal{O}(\alpha_t\alpha_s))$;
    Dedes, Slavich, GD (03)

Corrections to the minimization condition for V^{eff}:

same accuracy

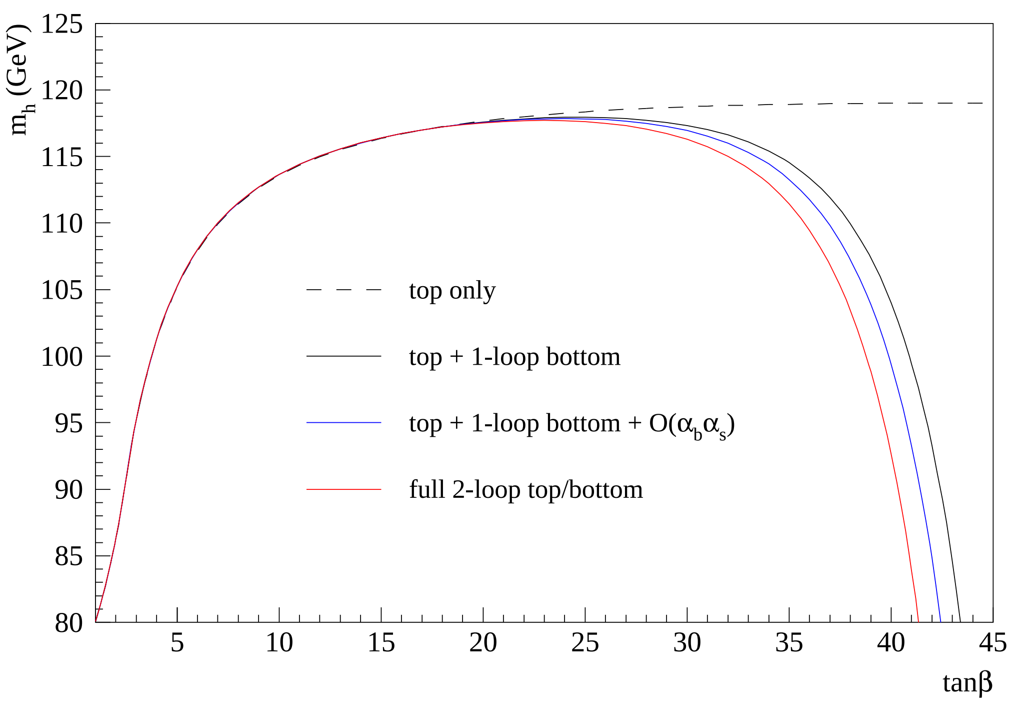
Dedes, Slavich (02); Dedes, Slavich, GD (03)

Top



M_susy = 1 TeV, X_t = 2 TeV
m_h (GeV)
140
120
100
80
60
tanβ = 20
tanβ = 2
1-loop
O(α_t α_s)
O(α_t α_s + α_t^2 LLog)
O(α_t α_s + α_t^2 full)
50
100
150
200
250
300
350
400
450
500
m_A (GeV)

Bottom



m_h (GeV)
125
120
115
110
105
100
95
90
85
80
top only
top + 1-loop bottom
top + 1-loop bottom + O(α_b α_s)
full 2-loop top/bottom
5
10
15
20
25
30
35
40
45
tanβ

Recent progress

- Complete two-loop effective potential, including electroweak effects, in the Landau gauge and in the $\overline{\mathrm{DR}}$ scheme.
- Complete effective potential approximation of the CP-even mass matrix.
- Strong and Yukawa momentum dependent effects in the Higgs scalar boson self-energies.
  Martin (02-04)

However

- Results expressed in terms of Lagrangian parameters $m^2_{H_{1,2}} + \mu^2$ not of M_Z and M_A
  $\longrightarrow$ tree-level squared Higgs masses strongly dependent on the renormalization scale, Q, and can become negative for small Q.
- Calculations are applicable only for $Q \gtrsim 550$ GeV.
- Results cannot be easily implemented (and in fact they are not yet) in the public computer codes that already contain all dominant contributions:
  - FeynHiggs 2.2 (Heinemeyer, Hollik, Weiglein, Frank, Hahn)
  - SuSpect 2.3 (Djoudi, Kneur, Moultaka)
  - SoftSusy 1.8.7 (Allanach)
  - SPheno 2.2.1 (Porod)

$$B \to X_s \gamma$$

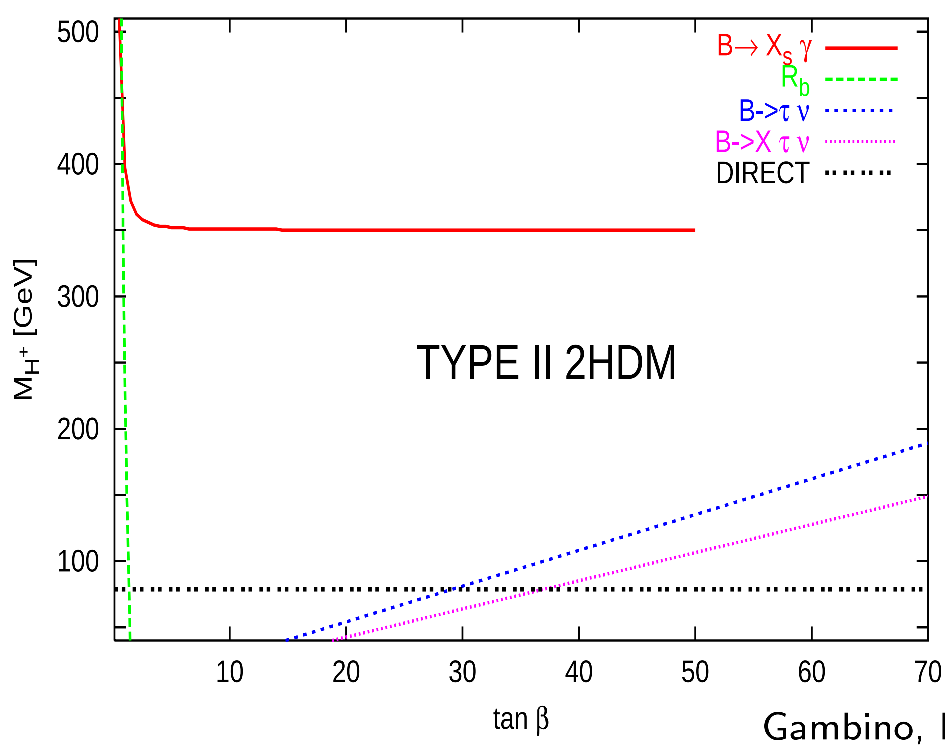
$B \to X_s\gamma$: a new physics killer

- Best measured rare decay:
  $\mathcal{B}(B \to X_s\gamma) = (3.52 \pm 0.30) \times 10^{-4}$
- Solid SM prediction.
  NLO QCD calculation completed and checked (both Wilson coeff.'s and ADM), EW and power corrections known:
  $\mathcal{B}(B \to X_s\gamma)_{SM} = (3.70 \pm 0.30) \times 10^{-4}$
- Not too much room for new physics.

THDM: NLO QCD calculation completed (type I and II)

Ciuchini, Gambino, Giudice, GD (98); Borzumati, Greub (98); Ciafaloni, Romanino, Strumia (98)

Strong bound on H^+



Gambino, Misiak (01)

SUSY

- Susy exact: no magnetic operators allowed
  $\rightarrow \mathcal{B}(B \rightarrow X_s\gamma) = 0$
  Ferrara, Remiddi (74)

- Susy *is* broken: however cancellations among the various contributions may be partially effective;
  $\rightarrow$ the bound on H^+ is weaken.
  Barbieri, Giudice (93)

- Susy novelties:
  - new source of FV from the mismatch between quarks and squarks mass eigenstates.
  - chargino contribution enhanced by $\tan\beta$.

LO QCD results: completely known both in:

- MFV: CKM matrix is the only source of FV at the weak scale
  Bertolini, Borzumati, Masiero, Ridolfi (91); Barbieri, Giudice (93)...

- GFV: flavor violating gluino interactions.
  Gabbiani, Gabrielli, Masiero, Silvestrini (96); Borzumati, Greub, Hurth, Wyler (00)

NLO QCD results: partial knowledge

Status of the SUSY NLO QCD calculation

MFV scenario

- possible large contributions identified.
  - terms enhanced by $\tan\beta$ factors

    main source: modified relation between m_b and y_b

    $g\, m_b = \sqrt{2} M_W y_b \cos\beta (1 + \epsilon_b \tan\beta)$

    Hall, Rattazzi, Sarid (94)

    * $\mathcal{O}(\tan\beta^2)$: only from chargino contribution
    * $\mathcal{O}(\tan\beta)$: several sources, full diagrammatic calculation required (not yet available);

      but H^+ contribution known

    Gambino, Giudice, GD (00); Carena, Garcia, Nierste, Wagner (01); Borzumati,Greub Yamada (03);
  - terms enhanced by $\log(\mu_{\mathrm{SUSY}}/\mu_W)$
    * different renormalization of y_t and $\tilde{y}_t$ ($\chi \tilde{t} \tilde{b}$ coup.).

      SUSY: $\tilde{y}_t(\mu_{\mathrm{SUSY}}) = y_t(\mu_{\mathrm{SUSY}})$

      ~~SUSY~~: $\tilde{y}_t$ frozen, y_t run

      $\rightarrow m_t(\mu_{\mathrm{SUSY}})$ in chargino contribution
    * evolution of the Wilson coefficients ($\mu_{\mathrm{SUSY}} \rightarrow \mu_W$)

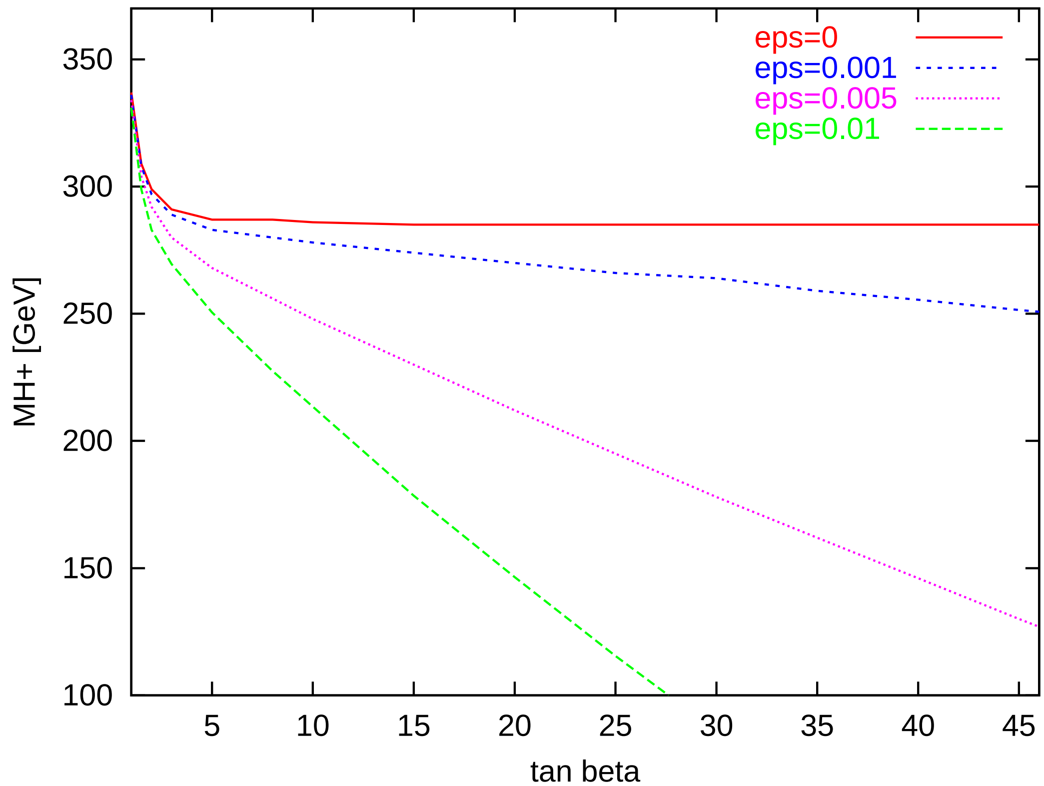
    Gambino, Giudice, GD (00);

- specific mass scenario:

  charginos and one stop (mainly R.H.) light

  Wilson coefficients computed

  Ciuchini, Gambino, Giudice, GD (98)



Lower bound on H^+ in a THDM scenario as a function of $\tan\beta$ for different values of ϵ

GFV scenario

Generalization of the MFV analyses with the inclusion of NLO dominant gluino flavor changing effects, that generally reduce the SUSY contribution

Okumura, Roszkowski (03)

Precision physics

MSSM contributions to $(g-2)_\mu$

$(g-2)_\mu$ SM prediction depends upon the value of the hadronic vacuum polarization and the light-by-light (l.b.l.) contribution:

$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (6-25) \times 10^{-10}$ [0.7-2.8] σ, l.b.l.=136(25)
([1.2- 3.2] σ, l.b.l.=8(4)).

SUSY contribution:

one-loop: $a_\mu^{\rm SUSY,1l} \simeq 13 \times 10^{-10} \left(\frac{100 \rm GeV}{M_{\rm SUSY}}\right)^2 \tan\beta \, {\rm sign}(\mu)$;
Moroi (96); Carena, Giudice, Wagner (97)...

$(g-2)_\mu$ measurements place strong constraints on SUSY parameter space.

$${\rm sign}(\mu) = +, \qquad M_{\rm SUSY} \simeq 72 \sqrt{\tan\beta} \, {\rm GeV}$$

Known two-loop contributions:

- Leading Logs ($\log(m_\mu/M_{\rm SUSY})$);
  Degrassi, Giudice (98), Czarnecki, Marciano (01)
- closed fermion/sfermion loop inserted into a one-loop THDM diagram: $a_\mu^{\tilde{f},2l} \simeq 2.5 \times 10^{-10}$
  (old estimate: $a_\mu^{\tilde{f},2l} \simeq 20 \times 10^{-10}$)
- Higgs scalar sector contribution (THDM) (L.L. result);
- chargino and neutralino loop inserted into a one-loop THDM diagram
  $a_\mu^{\chi,2l} \simeq 11 \times 10^{-10} \left(\frac{100 \rm GeV}{M_{\rm SUSY}}\right)^2 \left(\frac{\tan\beta}{50}\right) \, {\rm sign}(\mu)$;
  Heinemeyer, Stöckinger, Weiglein (04)

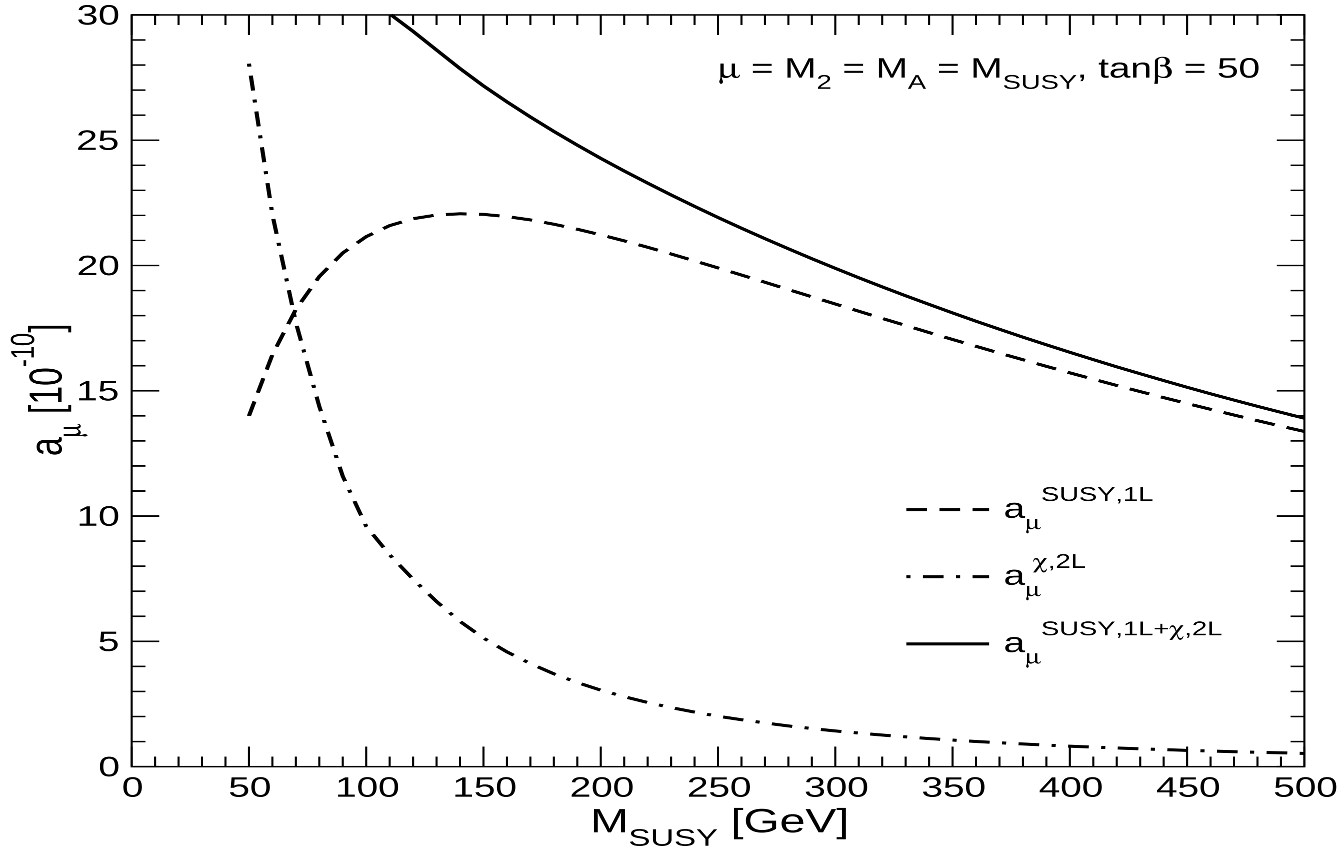


Figure 1: Comparison of the supersymmetric one-loop result, $a_\mu^{\mathrm{SUSY},1l}$, (dashed) with the two-loop chargino/neutralino contributions, $a_\mu^{\chi,2l}$ and the sum (full line). The sfermion mass parameters are set to 1 TeV.
(from Heinemeyer, Stöckinger, Weiglein)

MSSM contributions to the ρ parameter

The $\delta\rho$ correction is very important in the predictions of M_W and $\sin^2\theta_{eff}^{lept}$.

$$\sin^2\theta_{eff}^{lept} \sim \frac{1}{2}\left\{1 - \left[1 - \frac{4(\pi\,\alpha/\sqrt{2}\,G_\mu)}{M_Z^2\,\hat{\rho}\,(1-\Delta\hat{r}_W)}\right]^{1/2}\right\}$$

$$M_W^2 = \frac{\hat{\rho}}{2}M_Z^2\left\{1 + \left[1 - \frac{4(\pi\,\alpha/\sqrt{2}\,G_\mu)}{M_Z^2\,\hat{\rho}\,(1-\Delta\hat{r}_W)}\right]^{1/2}\right\}$$

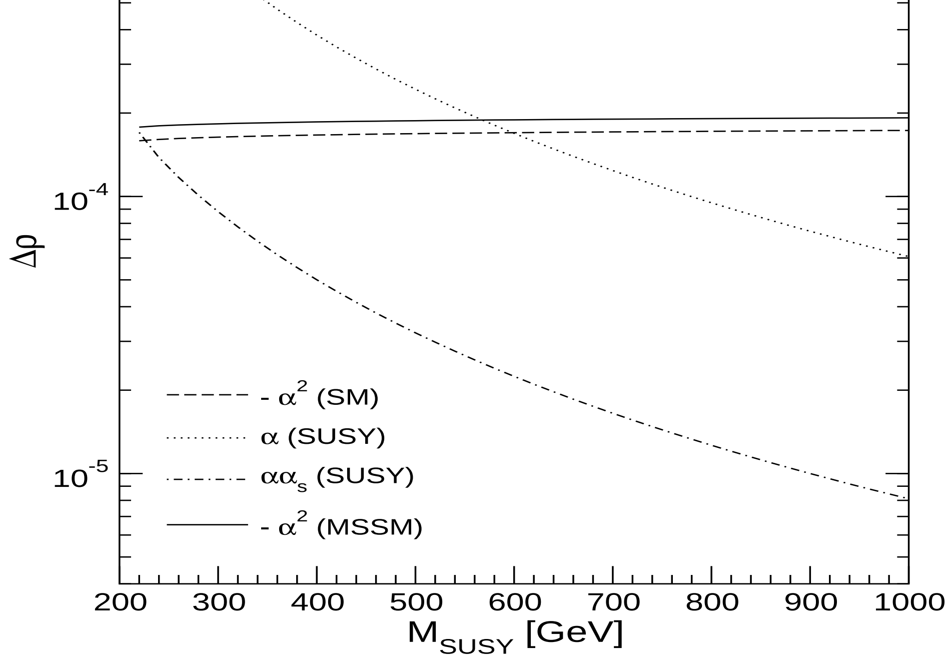
$\hat{\rho} = \rho_0 + \delta\rho \leftrightarrow (\epsilon_1,\, T)\ \ (\rho_0^{\mathrm{SM}} = 1)$
$\Delta\hat{r}_W \leftrightarrow (\epsilon_3,\, S)$

SUSY contribution:

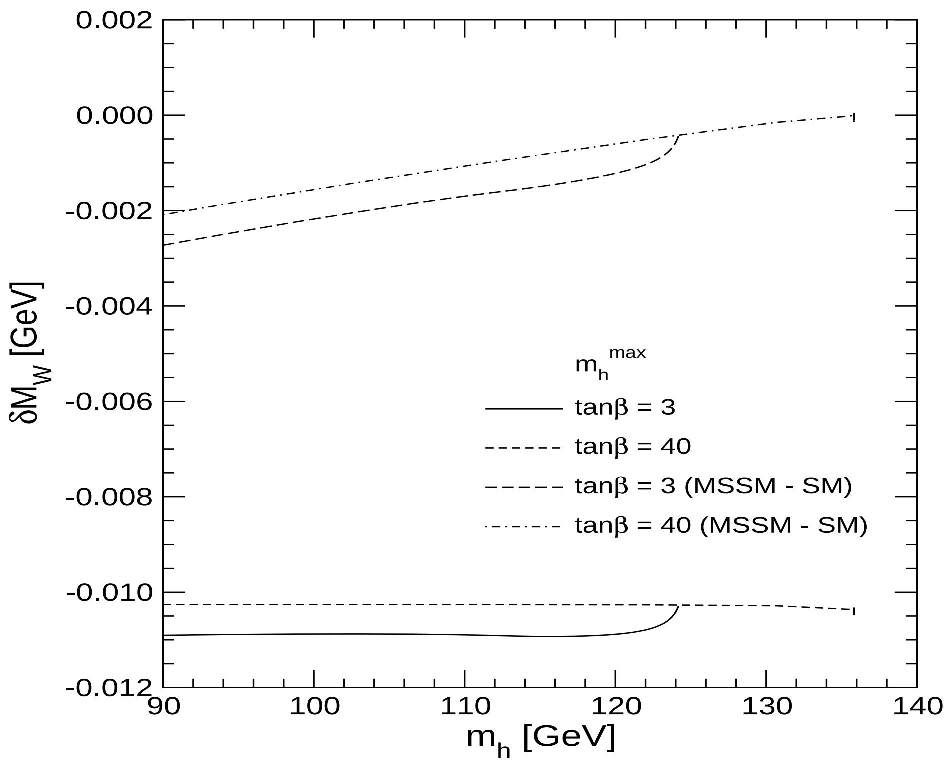
one-loop: large effect (few per mille) related to the splitting between $\tilde{t}$'s and $\tilde{b}$'s from the $\tilde{t}$ mass matrix $\longrightarrow m_t^2$ in the LL entry or via the LR entry.
Barbieri, Maiani (83); Lim, Inami, Sakai (84); Grifols, Sola (85)....

two-loop:

- $\mathcal{O}(\alpha_s)$ corrections to $\tilde{t}$, $\tilde{b}$ loops
  Djoudi, Gambino, Heinemeyer, Hollik, Jünger, Weiglein (97)
- $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ in the limit of heavy squarks, i.e. THDM with MSSM restrictions.
  Heinemeyer, Weiglein (02)



$\mathcal{O}(\alpha_t^2)$ contributions negative, shown with reversed sign.



$M_{susy} = 1000$ GeV, $\mu = 200$ GeV $A_b = A_t$, M_A varying from 50 GeV to 1000 GeV.

$\delta \sin^2 \theta_{\rm eff} = +6 \times 10^{-5}$; (MSSM - SM: $+3 \times 10^{-5} \to 0$)

Conclusions

- Clear indication for a light Higgs boson (weakly-coupled scalar dynamics) and for new physics of the decoupling type $\rightarrow$ MSSM.
- Very accurate evaluation of the MSSM neutral Higgs sector:
  uncertainty on m_h: $\delta m_h \approx 3 - 5$ GeV from unknown h.o., same size from experimental errors in the SM input parameters.
- Complete SUSY NLO calculation of $B \rightarrow X_s \gamma$ is important
- Complete SUSY two-loop calculation of $(g-2)_\mu$ is desirable
- One-loop studies of SUSY processes are under way: many results already available